STAT 2011
Tutorial 11

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Maximum Likelihood Estimation

- Likelihood Function:

Definition 4.2.1 (Likelihood function). *Let $X_1, X_2, \ldots, X_n$ be a random (i.i.d.) sample of size n from the discrete pmf $p_X(x;\theta)$, where θ is an unknown parameter. The likelihood function $L(\theta)$ is proportional to the joint pmf evaluated at $k_1, k_2, \ldots, k_n$*

$$L(\theta) \propto \prod_{i=1}^{n} p_X(k_i;\theta).$$

Let $Y_1, Y_2, \ldots, Y_n$ be a random (i.i.d.) sample of size n from a continuous pdf $f_Y(y;\theta)$ where θ is an unknown parameter. The likelihood function $L(\theta)$ is proportional to the joint pdf evaluated at $y_1, y_2, \ldots, y_n$

$$L(\theta) \propto \prod_{i=1}^{n} f_Y(y_i;\theta).$$

Definition 4.2.2. *Let $L(\theta)$ be the likelihood function of a random sample $X_1, X_2, \ldots, X_n$ drawn from either a discrete pmf or a continuous pdf. Let $\theta*$ be a value of the parameter such that*

$$L(\theta^*) \geq L(\theta)$$

for all possible values of θ. Then θ^* is called the ***maximum likelihood estimator*** *of θ and it is usually indicated as $\widehat{\theta}_{ML}$.*

Method of moments

- Definition:

Definition 4.2.3 (Method of Moments). *Let $X_1, X_2, \ldots, X_n$ be a random (i.i.d.) sample from a discrete pmf $p_X(x; \theta_1, \theta_2, \ldots, \theta_s)$.*
The method of moments estimates, $\widehat{\theta}_{1,MM}, \widehat{\theta}_{2,MM}, \ldots, \widehat{\theta}_{s,MM}$ for the model unknown parameters are the solutions of the s equations

$$\sum_{\text{all } k} k \cdot p_X(x; \theta_1, \theta_2, \ldots, \theta_s) = \mathrm{E}(X) = \frac{1}{n} \sum_{\text{all } k} k$$
$$\sum_{\text{all } k} k^2 \cdot p_X(x; \theta_1, \theta_2, \ldots, \theta_s) = \mathrm{E}(X^2) = \frac{1}{n} \sum_{\text{all } k} k^2$$
$$\vdots$$
$$\sum_{\text{all } k} k^s \cdot p_X(x; \theta_1, \theta_2, \ldots, \theta_s) = \mathrm{E}(X^s) = \frac{1}{n} \sum_{\text{all } k} k^s$$

Method of moments

- Continuous:

Let $Y_1, Y_2, \ldots, Y_n$ be a random (i.i.d.) sample from a continuous pdf $f_Y(y; \theta_1, \theta_2, \ldots, \theta_s)$. The method of moments estimates, $\widehat{\theta}_{1,MM}, \widehat{\theta}_{2,MM}, \ldots, \widehat{\theta}_{s,MM}$ for the model unknown parameters are the solutions of the s equations

$$\int_{-\infty}^{\infty} y \cdot f_Y(y; \theta_1, \theta_2, \ldots, \theta_s) dy = \mathrm{E}(Y) = \frac{1}{n} \sum_{i=1}^{n} y_i$$

$$\int_{-\infty}^{\infty} y^2 \cdot f_Y(y; \theta_1, \theta_2, \ldots, \theta_s) dy = \mathrm{E}(Y^2) = \frac{1}{n} \sum_{i=1}^{n} y_i^2$$

$$\vdots$$

$$\int_{-\infty}^{\infty} y^s \cdot f_Y(y; \theta_1, \theta_2, \ldots, \theta_s) dy = \mathrm{E}(Y^s) = \frac{1}{n} \sum_{i=1}^{n} y_i^s$$