STAT 2011
Tutorial 3

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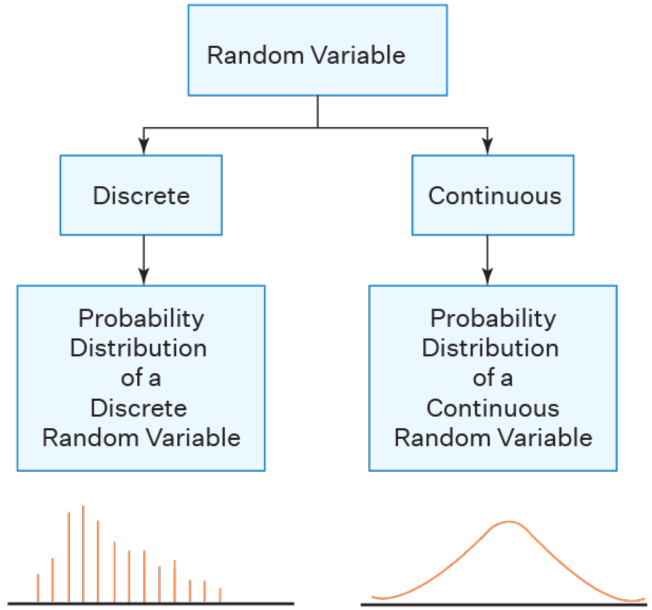
Random Variable

- Definition:

Definition 3.1.1 (Random variable). *A random variable X is a function, mapping each sample outcome s in some probability space S to a real number, i.e.*

$$for\ s \in S : X(s) = t_s = t \in \mathbb{R} \quad or \quad X : S \mapsto \mathbb{R}$$



- Discrete random variables :

Definition 3.2.1. *A function whose domain is a sample space S and whose values form a finite or countably infinite set of real numbers is called a discrete random variable.* □

PDF and CDF

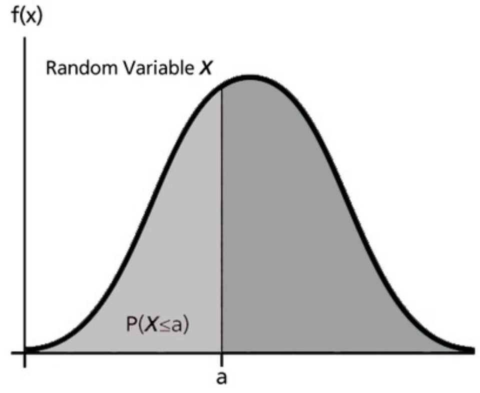
- Probability mass function:

Definition 3.2.2 (Probability mass function). *Let S be a finite or countably infinite sample space, X be a discrete random variable, and p be a real-valued function defined for each element of S satisfying. Define $p(s) = P(X = s)$, then*

a. $0 \leq p(s)$ *for each* $s \in S$

b. $\sum_{s \in S} p(s) = 1$

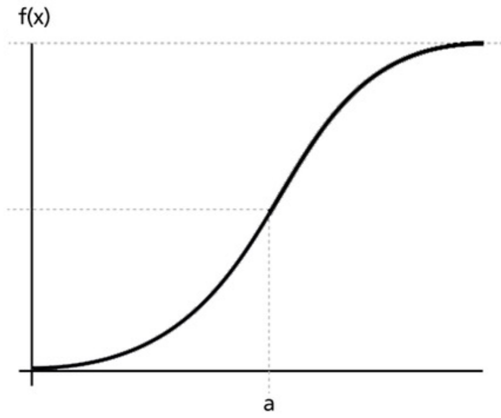
The function $p(\cdot)$ is said to be a probability mass function. We define it also as $p_X(k) = P(\{s \in S : X(s) = k\}) = P(X = k)$. □



- Cumulative distribution function

Definition 3.2.3 (Cumulative distribution function). *The cumulative distribution function (cdf) of X is defined as*

$$F_X(t) = P(X \leq t)$$ □



The cdf has many properties (proofs left as an exercise for now) such as

- $F_X(t)$ is monotone increasing with values between 0 and 1;
- $F_X(t)$ is continuous from right.

Expectation and Variance

- Expectation

Definition 3.2.4 (Expected value of a discrete RV X). *Let discrete RV X have probability mass function (pmf) $p_X(k)$. The expected value of X, denoted by* $\mathrm{E}(X)$ *(or sometimes μ or μ_X), is given by*

$$\mathrm{E}(X) = \sum_{all\ k} k \cdot p_X(k). \qquad \square$$

- Variance:

Definition 3.2.6. *The variance of a RV X is the expected value of its squared deviation from* $\mathrm{E}(X) = \mu$ *and is only defined when* $\mathrm{E}(X^2)$ *is finite,*

$$\mathrm{Var}(X) = \sigma^2 = \mathrm{E}[(X-\mu)^2] \qquad \square$$