STAT 2011

Q1 Six chips 1. 2. 3. 4. 5. 6

A : " Second largest chip is a 3"

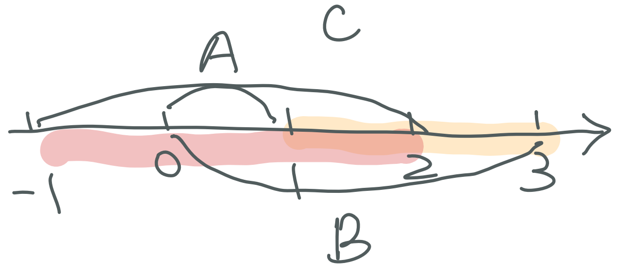
$A = \{ (i, j, k) : 1 \le i < j < k \le 6 , j = 3 \}$

$\underline{A = \{ (1,3,4) , (1,3,5) , (1,3,6) , (2,3,4) (2,3,5) (2,3,6) \}}$

Q3.

$B = \{ (9,9) , (9, \underbrace{\overset{\text{not } 7 \text{ or } 9}{\cdots\cdots}}_{n} , 9) \}$

Q4



$A^C \cap B \cap C$

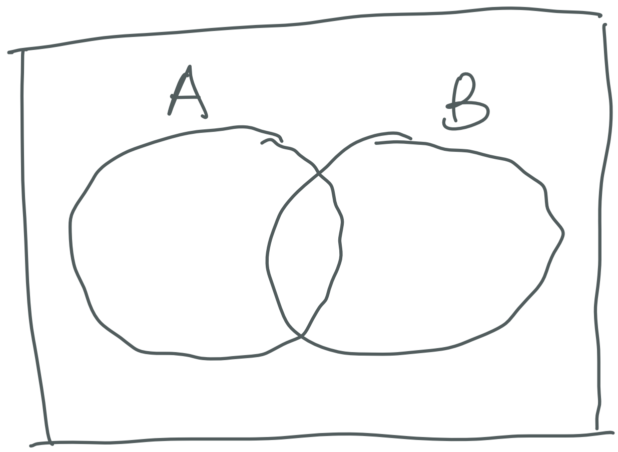
$A^C = \mathbb{R} \setminus [0, 1]$

$= (-\infty, 0) \cup (1, \infty)$

$\underline{A^C \cap B} = (1, 3]$

$A^C \cap B \cap C = (1, 2]$

Q6 $\overset{M}{(A \cap B)^C} = \overset{N}{A^C \cup B^C}$



$\underline{s \in M}$

$s \in (A \cap B)^C$

$s \notin (A \cap B)$

$s \notin A$ or $s \notin B$

$s \in A^C$ or $s \in B^C$

$s \in A^C \cup B^C$ $\Rightarrow$ $\underline{s \in N}$ $M \subseteq N$

$s \in N$

$s \in A^c \cup B^c$

$s \in A^c$ or $s \in B^c$

$s \notin A$ or $s \notin B$

$s \notin (A \cap B)$

$s \in (A \cap B)^c$ $\quad s \in M$

$$\Downarrow$$

$$M = N$$

$$\Uparrow$$

$$N \subseteq M$$

Q7 $\mathbb{P}(A_1) = \mathbb{P}(A_2) = \mathbb{P}(A_3) = 1/6$

$\mathbb{P}(A_1 \cup A_2 \cup A_3) = \mathbb{P}(\text{at least a 6 on three dice})$

$= 1 - \mathbb{P}(\text{no } 6)$

$= 1 - \left(\frac{5}{6}\right)^3$

$= 0.4212$

Q13

K : knows

R : get right

$\mathbb{P}(R \mid K) = 1$

$\mathbb{P}(R \mid \bar{K}) = 0.2$

$\mathbb{P}(K \mid R) = 0.92$

$\mathbb{P}(K)$

$$\mathbb{P}(K \mid R) = \frac{\mathbb{P}(K \cap R)}{\mathbb{P}(R)} = \frac{\mathbb{P}(R \mid K) \cdot \mathbb{P}(K)}{\mathbb{P}(R \mid K) \cdot \mathbb{P}(K) + \mathbb{P}(R \mid \bar{K}) \cdot \mathbb{P}(\bar{K})}$$

$$\frac{1 \cdot \mathbb{P}(K)}{1 \cdot \mathbb{P}(K) + 0.2(1 - \mathbb{P}(K))} = 0.92$$

$\mathbb{P}(K) = 0.697$

$\mathbb{P}(K) \cdot 20$

$= 14$

Q14 $\mathbb{P}(A) = 1/4$ $\quad \mathbb{P}(B) = 1/8$

(i) $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$

$= 1/4 + 1/8$

(ii) $\mathbb{P}(AB) = \mathbb{P}(A) \cdot \mathbb{P}(B)$

$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$

$= \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A) \cdot \mathbb{P}(B)$

$= 1/4 + 1/8 - 1/4 \cdot 1/8 = 11/32$

Q15

$\mathbb{P}(A_1) = \frac{2}{6} = 1/3$

$\mathbb{P}(A_2) = 3/6 = 1/2$

$\mathbb{P}(A_3) = 6/36 = 1/6$

$A_3 : \{(1,3)\ (3,1)\ (2,2),\ (5,6),(6,5)\ (6,6)\}$

$(1,3)\ (1,4)\ (1,5)\ (2,3)\ (2,4)\ (2,5)$

$1/6$

$\mathbb{P}(A_1 \cap A_2) = \mathbb{P}(A_1) \cdot \mathbb{P}(A_2)$

$\mathbb{P}(A_1 \cap A_2 \cap A_3) = 1/36$

$\mathbb{P}(A_1 \cap A_2 \cap A_3) = \mathbb{P}(A_1) \cdot \mathbb{P}(A_2) \cdot \mathbb{P}(A_3)$ #

$i \neq j$ $\quad \mathbb{P}(A_i \cap A_j) \neq \mathbb{P}(A_i) \cdot \mathbb{P}(A_j)$

$\mathbb{P}(A_2 \cap A_3) = 2/36 = 1/18$ $\neq$

$\mathbb{P}(A_2) \cdot \mathbb{P}(A_3) = \frac{1}{2} \cdot \frac{1}{6} = 1/12$ #