STAT 2011
Tutorial 8

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Gamma distribution

- Definition:

Theorem 3.3.10. *Suppose that Poisson events are occurring at the constant rate of λ per unit of time. Let the random variable Y denote the waiting time for the r-th event. Then Y has pdf*

$$f_Y(y) = \frac{\lambda^r}{(r-1)!} y^{r-1} e^{-\lambda y} \qquad y > 0.$$

- Mean and variance:

Theorem 3.3.11. *Suppose that Y has a gamma pdf with shape parameter r and rate parameter λ. Then*

(i) $E(Y) = \frac{r}{\lambda}$

(ii) $Var(Y) = \frac{r}{\lambda^2}$.

Higher Moment

- Definition:

Definition 3.3.7. *Let RV W have pdf $f_W(w)$ (or pmf $p_W(w)$). For any positive integer r, the r-th moment of W about the origin, μ_r, is*

$$\mu_r = \mathrm{E}(W^r)$$

and the r-th moment about the mean, μ'_r, is

$$\mu'_r = \mathrm{E}[(W - \mu)^r]$$

provided $\mathrm{E}(|W|^r) < \infty$. $\square$

Joint Density

- Discrete:

Definition 3.4.1. *Suppose S is a discrete sample space on which two RVs, X and Y, are defined. The joint pmf of X and Y is denoted $p_{X,Y}(x,y)$, where*

$$p_{X,Y}(x,y) = P\left(\{s : X(s) = x \ \underline{and} \ Y(s) = y\}\right) = \underbrace{P(X = x, Y = Y)} \qquad \square$$

Theorem 3.4.1. *Let $p_{X,Y}(x,y)$ be the joint pmf of the discrete RV X and Y. Then,*

$$p_X(x) = \sum_{all\ y} p_{X,Y}(x,y) \quad and \quad p_Y(y) = \sum_{all\ x} p_{X,Y}(x,y) \qquad \square$$

- Continuous:

Definition 3.4.3. *Two RVs on the same set of real numbers are jointly continuous if there exists a function $f_{X,Y}(x,y)$ such that for any region R in the xy-plane*

$$P\left[(X,Y) \in R\right] = \iint_R f_{X,Y}(x,y)\,dx\,dy$$

and the function $f_{X,Y}(x,y)$ is called the joint pdf of X and Y. $\square$

Theorem 3.4.2. *Let continuous RV X and Y have joint pdf $f_{X,Y}(x,y)$. Then the marginal pdf's $f_X(x)$ and $f_Y(y)$ are given by*

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy \quad and \quad f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx \qquad \square$$

Basic Transformation

- Properties:

Theorem 3.5.1. *Let RV X be discrete and consider $Y = aX + b$, $a \neq 0$. Then,*

$$p_Y(y) = p_X\left(\frac{y-b}{a}\right). \qquad \square$$

Theorem 3.5.2. *Let RV X be continuous and consider $Y = aX + b$, $a \neq 0$. Then,*

$$f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right) \qquad \square$$

Theorem 3.5.3. *Let RVs X and Y be independent and $W = X + Y$. Then,*

1. *If X and Y are discrete*

$$p_W(w) = \sum_{all\ x} p_X(x) p_Y(w-x)$$

2. *If X and Y are continuous*

$$f_W(w) = \int_{-\infty}^{\infty} f_X(x) f_Y(w-x)\, dx \qquad \square$$