STAT 2011

Q1 two dice.

A_i : get double 6 on i^{th} trail $\quad P(A_i) = 1/36$

throw n times

$P(A) = P(\text{get at least a double 6 in } n \text{ trails})$

$= 1 - P(\text{no double 6})$

$= 1 - \left(\frac{35}{36}\right)^n \geq 0.5$

$0.5 \geq \left(\frac{35}{36}\right)^n$

$n \geq 24.6$

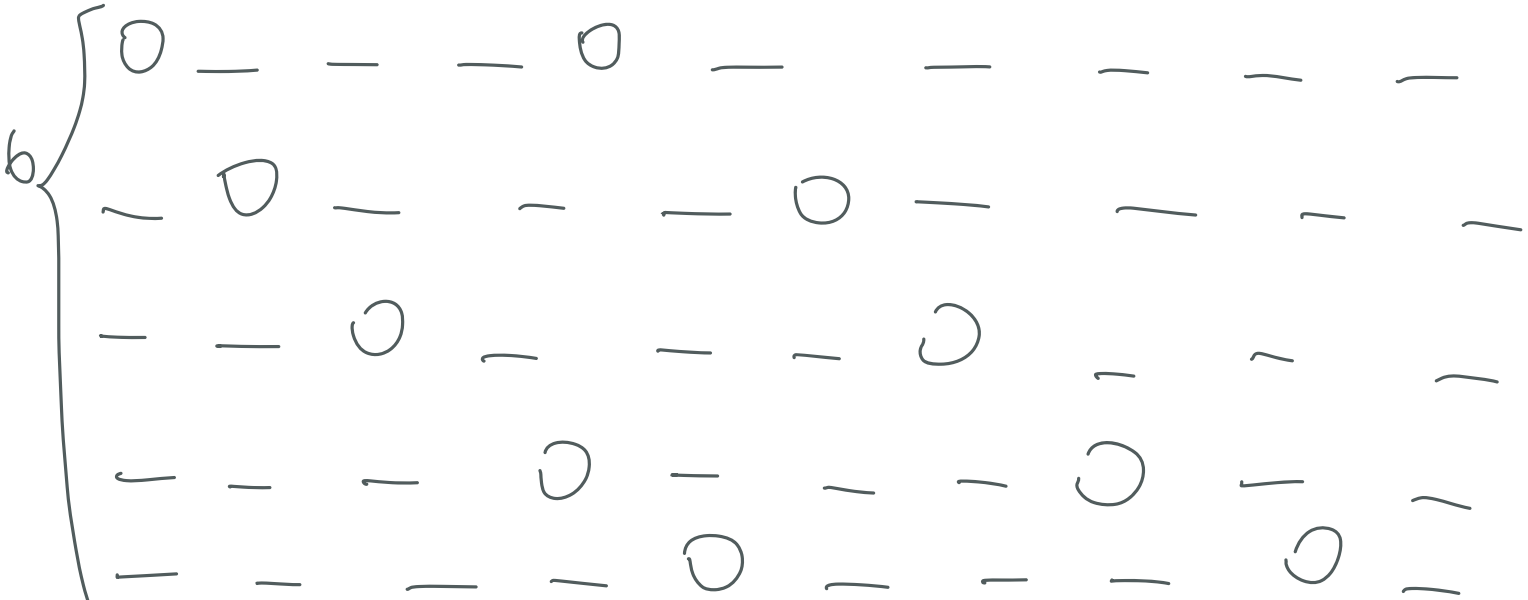
$\Rightarrow$ at least 25 times

Q3

$\binom{8}{3} \times 3! \times 2 \times 6!$

$= 483840$

b)



$6 \times 2! \times 8!$

$= 483840$

Q5.

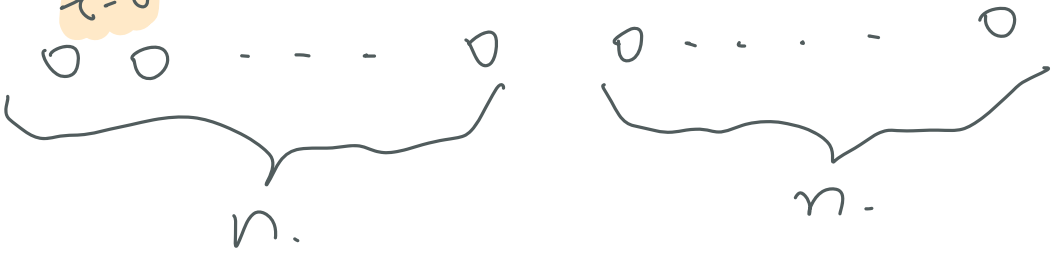
$$\binom{n}{0}^2 + \binom{n}{1}^2 + \dots + \binom{n}{n}^2 = \binom{2n}{n}.$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$\binom{n}{0}\binom{n}{0} \qquad = \frac{n!}{(n-k)!\,k!} = \binom{n}{n-k}$$

$$\Rightarrow \binom{n}{0}\binom{n}{n} + \binom{n}{1}\binom{n}{n-1} + \dots + \binom{n}{n}\binom{n}{0}$$

$$= \sum_{i=0}^{n} \binom{n}{i}\binom{n}{n-i}$$



$$= \binom{2n}{n}$$

Q7 Consider a set of ten urns, nine of which contain three white chips and three red chips each. The tenth contains five white chips and one red chip. An urn is picked at random. Then a sample of size 3 is drawn without replacement from that urn. If all three chips drawn are white, what is the probability that the urn being sampled is the one with five white chips?

A: all three chips are white

B: urn is the tenth.

$P(B) = 1/10$

$$P(A \mid B) = \frac{5}{6} \times \frac{4}{5} \times \frac{3}{4} = \frac{1}{2}$$

$$P(A \mid \bar{B}) = \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} = 1/20$$

$$\mathbb{P}(B \mid A) = \frac{\mathbb{P}(BA)}{\mathbb{P}(A)}$$

$$= \frac{\mathbb{P}(A \mid B) \cdot \mathbb{P}(B)}{\mathbb{P}(A \mid B) \cdot \mathbb{P}(B) + \mathbb{P}(A \mid \overline{B}) \cdot \mathbb{P}(\overline{B})}$$

$$= \frac{\frac{1}{2} \times \frac{1}{10}}{\frac{1}{2} \times \frac{1}{10} + \frac{1}{10} \times \frac{9}{10}}$$

$$= 10/19$$

Q10. n — m defective; $n-m$ functional.

$n-m+1$ positions

$$\boxed{\binom{n-m+1}{m}}$$