# STAT 2011
# Tutorial 5

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# Bernoulli and Binomial

- Definition:

**Definition 3.2.8** (Bernoulli trials). *A series of $n$ independent trials, each resulting in one of two possible outcomes "success" or "failure" with constant success probability $p$ is called a Bernoulli trial of length $n$.* $\square$

**Theorem 3.2.5.** *Let $X$ denote the number of successes in a Bernoulli trial of length $n$. Then*

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \ k = 0, 1, \ldots, n$$

*which is known as the binomial distribution.* $\square$

- Mean value and variance:

**Theorem 3.2.6.** *Suppose $X$ is a binomial RV with parameters $n$ and $p$, $X \sim Bin(n,p)$. Then $\mathrm{E}(X) = np$ and the variance is $\mathrm{Var}(X) = np(1-p)$.* $\square$

# Hypergeometric

- Definition:

**Theorem 3.2.7.** *Suppose an urn contains a total of $N$ chips of which $r$ are red and $w$ are white. If $n$ chips are drawn out at random, without replacement, and if RV $X$ denotes the number of red chips selected, then*

$$P(X = k) = \frac{\binom{r}{k}\binom{w}{n-k}}{\binom{N}{n}}, \tag{6}$$

*where $k$ varies over all integers for which $\binom{r}{k}$ and $\binom{w}{n-k}$ are defined. The probabilities on the RHS of Equation (6) are known as the hypergeometric distribution.* □

- Mean value and variance:

**Theorem 3.2.8.** *If $X$ is a hypergeometric RV with parameters $r$, $w$ and $n$, that is with pdf*

$$p_X(k) = P(X = k) = \frac{\binom{r}{k}\binom{w}{n-k}}{\binom{r+w}{n}} \quad \textit{and} \quad \mathrm{E}(X) = \frac{rn}{r+w}$$ □

# Poisson

- Definition:

**Theorem 3.2.9.** *Suppose $X$ is a binomial random variable where*

$$P(X=k)=p_X(k)=\binom{n}{k}p^k(1-p)^{n-k}, \qquad k=0,1,2,\ldots.$$

*If $n\to\infty$ and $p\to 0$ in such a way that $\lambda=np$ remains constant then*

$$\lim_{n\to\infty}P(X=k)=\lim_{n\to\infty}\binom{n}{k}p^k(1-p)^{n-k}=\frac{e^{-np}(np)^k}{k!}.$$

- Mean value and variance:

**Theorem 3.2.10.** *The random variable $X$ is said to have a Poisson distribution if*

$$p_X(k)=P(X=k)=\frac{e^{-\lambda}\lambda^k}{k!} \qquad k=0,1,2,\ldots$$

*where $\lambda$ is a positive constant. Also, for any Poisson random variable $E(X)=\lambda$ and $Var(X)=\lambda$.*