STAT 2011
Tutorial 2

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Counting Permutation

- Multiplication Rule:

Theorem 2.1.1 (Multiplication rule). *If operation A_i, $i = 1, \ldots, k$, can be performed in n_i different ways, respectively, then the ordered sequence*

$$(\text{operation } A_1,\ \text{operation } A_2,\ \ldots,\ \text{operation } A_k)$$

can be performed in

$$n_1 n_2 \ldots n_k = \prod_{i=1}^{k} n_i \text{ ways.} \qquad \square$$

- Permutation:

Theorem 2.1.2. *The number of permutations of length k that can be formed from a set of n distinct elements, repetitions not allowed, is denoted by the symbol ${}_nP_k$, where*

$$ {}_nP_k = n(n-1)(n-2)\ldots(n-k+1) = \frac{n!}{(n-k)!} \qquad \square$$

- Stirlings formula:

Corollary 2.1.3. *The number of ways to permute an entire set of n distinct objects is* ${}_nP_n = n!$. $\square$

Note, $n!$ can be approximated by **Stirlings formula** for large n:

$$n! \approx \sqrt{2\pi} n^{\frac{2n+1}{2}} e^{-n}.$$

- Arranging n objects with r classes:

Theorem 2.1.4. *The number of ways to arrange n objects, n_1 being of one kind, n_2 of a second kind, etc to n_r of an rth kind, is*

$$\frac{n!}{n_1!\, n_2!\, \ldots\, n_r!}, \quad \textit{where } n = \sum_{i=1}^{r} n_i \qquad \square$$

Counting Combination

- Combination is when order doesn't matter

Theorem 2.2.1. *The number of ways to form combinations of size k from a set of n distinct objects, repetitions not allowed, is denoted by the symbols $\binom{n}{k}$ or ${}_nC_k$, where*

$$\binom{n}{k} = {}_nC_k = \frac{n!}{k!(n-k)!} \qquad \square$$

- Properties:

Theorem 2.2.2. *For $n \geq k \geq 1$,*

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1} \qquad \square$$

Theorem 2.2.3. *Let $n \geq k \geq 0$,*

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n \qquad \square$$