1. (Proof of Theorem 3.6.2,, continuous case) Let X and Y be any two random variables with joint  $pdf f_{X,Y}(x,y)$  and let a and b be any two constants. Show that  $E(aX+bY)=aE(X)+\overline{bE(Y)}$ .

$$E(ax+by)$$

$$= S(ax+by) \cdot f_{xy}(x,y) dxdy$$

$$= a.SS x. f_{xy}(x,y) dxdy + b.SS y. f_{xy}(x,y) dxdy$$

$$= aS x S f_{xy}(x,y) dy dx + b.Sy S f_{xy}(x,y) dx dy$$

$$= aS x. f_{x(x)} dx + b.Sy . f_{y}(x,y) dx dy$$

$$= aS x. f_{x(x)} dx + b.Sy . f_{y}(y) dy$$

$$= a.E(x) + b.E(y) #$$

4. Show that

$$Cov(aX + b, cY + d) = ac Cov(X, Y)$$

for any constants a, b, c, d.

= a.c. cov(x, Y)

$$cou(ax+b) \cdot cx+d) \qquad a \cdot E(x)+b$$

$$= E(ax+b) \cdot (cx+d) - E(ax+b) \cdot E(cx+d)$$

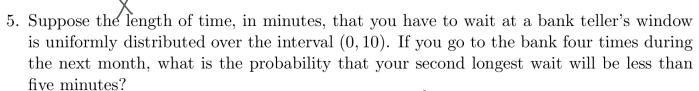
$$= E(a \cdot c \cdot x \cdot x) + a \cdot d \cdot x + b \cdot c \cdot x + b \cdot d - a \cdot c \cdot E(x) + d$$

$$= a \cdot c \cdot E(xx) + a \cdot d \cdot E(x) + b \cdot c \cdot E(x) + b \cdot d - a \cdot c \cdot E(x) \cdot E(x) - a \cdot d$$

$$= a \cdot c \cdot E(xx) - a \cdot c \cdot E(x) \cdot E(x)$$

$$= a \cdot c \cdot E(xx) - a \cdot c \cdot E(x) \cdot E(x)$$

$$= a \cdot c \cdot E(xx) - E(x) \cdot E(x)$$



9. Let  $X_1, X_2, \ldots, X_n$  be independent and identically distributed random variables having expected value  $\mu$  and variance  $\sigma^2$ , and let

$$S^{2} = \sum_{i=1}^{n} \frac{(X_{i} - \overline{X})^{2}}{n-1}$$

be the sample variance, where  $\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$ . Find the  $E(S^2)$ .

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$$S^{2} = \sum_{i=1}^{n} \frac{(Y_{i} - \overline{X})^{2}}{n_{i-1}} \Rightarrow (n_{i} - \overline{X})^{2}$$

$$= \sum_{i=1}^{n} (Y_{i} - \overline{X})^{2} + (Y_$$

$$= \frac{n \cdot \hat{X} - n \cdot M}{n \cdot (\hat{X} - M)^2}$$

$$= \frac{1}{4^{-1}} (\hat{X}_1 - M)^2 + n \cdot (M - \hat{X}_1)^2 + 2 \cdot (M - \hat{X}_1) \cdot n \cdot (\hat{X} - M)$$

$$= \frac{1}{4^{-1}} (\hat{X}_1 - M)^2 - n \cdot (M - \hat{X}_1)^2$$

$$= \frac{1}{n - 1} \cdot \mathbb{E} \left( \frac{n}{4^{-1}} (\hat{X}_1 - \hat{X}_1)^2 \right)$$

$$= \frac{1}{n - 1} \left( \frac{n}{4^{-1}} \mathbb{E} (\hat{X}_1 - M)^2 - n \cdot (M - \hat{X}_1)^2 \right)$$

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$$= \frac{1}{n - 1} \left( \frac{n}{4^{-1}} \cdot \mathbb{E} (\hat{X}_1$$

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