STAT 2011 Tutorial 6

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Geometric distribution

- Definition:

Define X to be the **trial at which the first success occurs**. Then the pmf of X is

$$\begin{aligned} p_X(k) = P(X = k) &= P(\text{first success occurs on the k-th trial}) \\ &= P(\text{first k-1 trials end in failure and k-th trial ends in success}) \\ &= P(\text{first k-1 trials end in failure}) \cdot P(\text{k-th trial ends in success}) \\ &= (1-p)^{k-1}p \end{aligned}$$

Theorem 3.2.11. *Let X be a geometric random variable with pmf*

$$p_X(k) = (1-p)^{k-1}p \qquad k = 1, 2, \ldots.$$

Then

(i) $E(X) = \frac{1}{p}$

(ii) $Var(X) = \frac{1-p}{p^2}$.

Negative binomial distribution

- Definition:

$$
\begin{aligned}
p_X(k) = P(X = k) &= P(\underline{r\text{-th success occurs at the } k\text{-th trial}}) \\
&= P(r-1 \text{ successes in first} k-1 \text{ trials and success occurs on the } k\text{-th trial}) \\
&= P(r-1 \text{ successes occur in first } k-1 \text{ trials}) \cdot P(\text{success occurs in } k\text{-th trial}) \\
&= \binom{k-1}{r-1} p^{r-1}(1-p)^{k-1-(r-1)} \cdot p \\
&= \binom{k-1}{r-1} p^r (1-p)^{k-r}.
\end{aligned}
$$

Theorem 3.2.12. *Let X have a negative binomial distribution with $p_X(k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}$, with $k = r, r+1, \ldots$*

(i) $E(X) = \frac{r}{p}$

(ii) $Var(X) = \frac{r(1-p)}{p^2}$

Continuous Random Variable

- Definition:

Definition 3.3.1 (Continuous probability function). *A probability function P on a set of real numbers S is called continuous if there exists a (probability density) function $f(t)$ such that for any closed interval $[a, b] \subset S$*

$$P([a,b]) = \int_a^b f(t)dt.$$

- Pdf and pmf:

Definition 3.3.2 (Probability density function). *Let Y be a function from a sample space S to $\mathbb{R}$. The function Y is called a continuous RV if there exist a function $f_Y(y)$ such that for any $a, b \in \mathbb{R}$ with $a < b$*

$$P(a \le Y \le b) = \int_a^b f_Y(y)dy.$$

The function $f_Y(y)$ is called the probability density function (pdf) for Y. □

Theorem 3.3.1. *Let $F_Y(y)$ be the cdf of a continuous RV. Then*

$$\frac{d}{dy}F_Y(y) = f_Y(y).$$ □

- Properties:

Theorem 3.3.4. *Let Y be a continuous RV with cdf $F_Y(y)$. Then,*

a. $P(Y > y) = 1 - P(Y \leq y) = 1 - F_Y(y)$

b. $P(r < Y \leq s) = F_Y(s) - F_Y(r)$

c. $\lim_{y\to\infty} F_Y(y) = 1$

d. $\lim_{y\to(-\infty)} F_Y(y) = 0$ $\square$