STAT 2011
Tutorial 1

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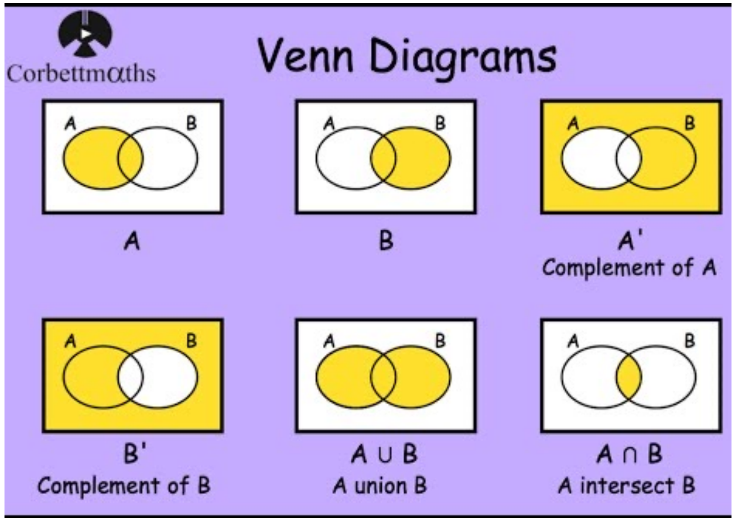
Assessment

• Online Lab report: 5%	Week 3
• Tutorial quiz 1: 10%	Week 5
• Tutorial quiz 2: 10%	Week 11
• Assignment: 15%	Week 12
• Final Exam: 60%	Exam week

Sample space and set theory

- Experiment: a procedure can be repeated and has a well-defined set of possible outcomes.
- Sample space: set of total outcomes.
- Event: a collection of "favourable" outcomes.
- Unions, intersections and complements



- De Morgan's laws

The complement of their intersection is the union of their complements: $(A \cap B)^C = A^C \cup B^C$
The complement of their union is the intersection of their complements: $(A \cup B)^C = A^C \cap B^C$

Probability function

- Probability function: The symbol P(A) will denote the probability of A.
  P(A) is a function, mapping $A \subset S$ to a value in [0,1].

- Basic properties of the probability function

(i) $P(A^C) = 1 - P(A)$

(ii) $P(\emptyset) = 0$

(iii) *If* $A \subset B$ *then* $P(A) \leq P(B)$

(iv) *For any event* A, $P(A) \leq 1$

(v) *Let* $A_1, A_2, \ldots, A_k$ *be (pairwise) mutually exclusive events, then*

$$P(\cup_{i=1}^{k} A_i) = \sum_{i=1}^{k} P(A_i)$$

(vi) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Conditional Probability

- Any probability that is revised to take into account the (known) occurrence of other events is said to be a conditional probability.

Definition 1.4.1 (Conditional probability). *Let $A, B \subset S$ such that $P(B) > 0$. The conditional probability of A given B is defined as*

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \Rightarrow \quad P(A \cap B) = P(A|B)P(B) \qquad \square$$

- Higher order intersections

$$\begin{aligned} P(A_1 \cap A_2 \cap \ldots \cap A_k) = & P(A_k|A_1 \cap A_2 \cap \ldots \cap A_{k-1}) \\ & \times P(A_{k-1}|A_1 \cap A_2 \cap \ldots \cap A_{k-2)} \\ & \vdots \\ & \times P(A_2|A_1)P(A_1) \end{aligned}$$

- "Unconditional" and "inverse" probabilities

Theorem 1.4.1. *Let $\{A_i\}_{i=1}^n$ with $A_i \subset S$ such that $S = \cup_{i=1}^n A_i$ and $A_i \cap A_j = \emptyset$ for $i \neq j$ and $P(A_i) > 0$ for $i = 1, \ldots, n$. For any event B,*

$$P(B) = \sum_{i=1}^{n} P(B|A_i)P(A_i) \qquad \square$$

Independent

Definition 1.5.1 (Independence)**.** *Two events A and B are said to be independent if*

$$P(A \cap B) = P(A)P(B) \qquad \square$$

This definition is equivalent to characterizing independence through

$$P(A \cap B) = P(A|B)P(B) \overset{\text{Def}}{=} P(A)P(B)$$

Thus if A and B are independent then,

$$P(A|B) = P(A) \quad \text{or (by symmetry)} \quad P(B|A) = P(B)$$

provided $P(B)$ and $P(A)$ are nonzero, respectively.

Definition 1.5.2. *Events $A_1, A_2, \ldots, A_n$ are said to be independent if for every set of indices $i_1, i_2, \ldots, i_k$ between 1 and n, inclusive,*

$$P(A_{i_1} \cap A_{i_2} \cap \ldots \cap A_{i_k}) = P(A_{i_1})P(A_{i_2}) \ldots P(A_{i_k}) \qquad \square$$