STAT 2011

Q1.

If two fair dice are tossed, what is the smallest number of throws, n, for which the probability of getting at least one double 6 exceeds 0.5? *(Note: This was one of the first problems that de Méré communicated to Pascal in 1654.)*

A_i : get double 6 on i^{th} trail. $\mathbb{P}(A_i) = 1/36$

$\mathbb{P}(A) = \mathbb{P}($get at least one double 6$)$

$= 1 - \mathbb{P}($no double 6 for all n times$)$

$= 1 - (35/36)^n \geq 0.5$

$0.5 \geq \left(\frac{35}{36}\right)^n$

$n \geq 24.6$

$\Rightarrow$ at least 25 times

Q3

3. Suppose that ten people, including you and a friend, line up for a group picture. How many ways can the photographer rearrange the line if they want to keep exactly three people between you and your friend?

① $\boxed{\text{O \_\_ \_\_ \_\_ O}}$ __ __ __ __ __

10 in total.

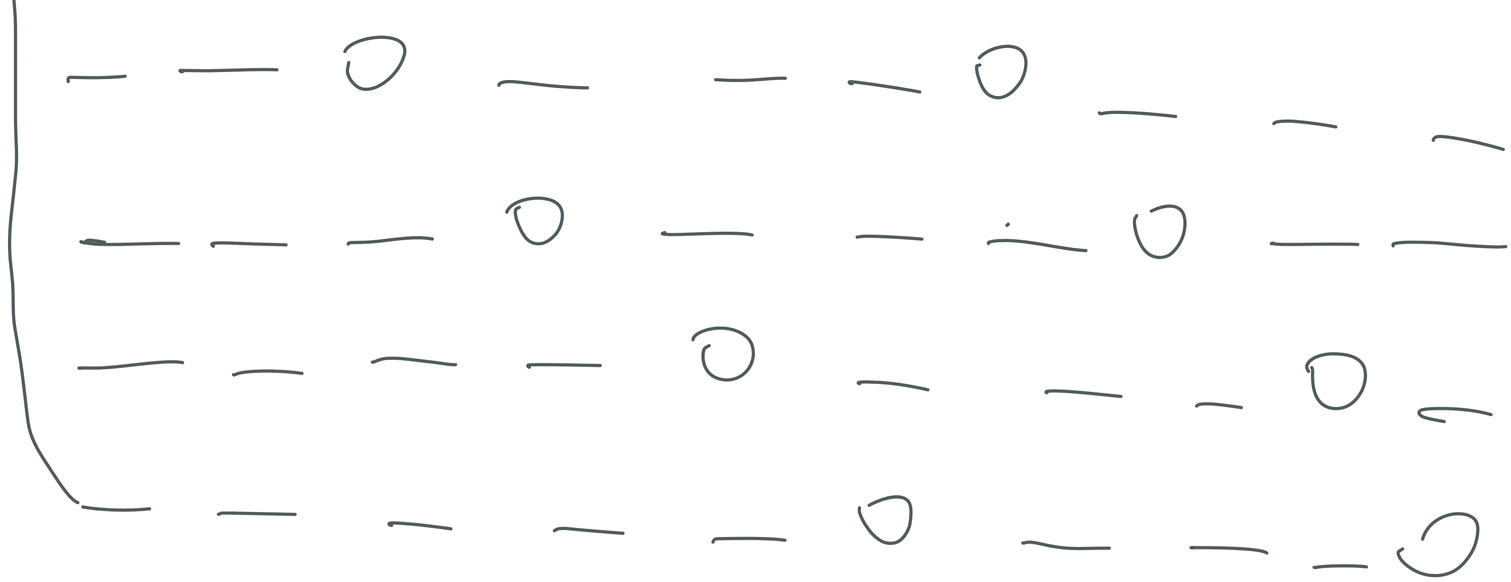
$\binom{8}{3} \cdot 3! \times 2 \times 6!$

$= 483840$

② $\boxed{\text{O \_\_ \_\_ \_\_ O}}$ __ __ __ __ __

__ O __ __ __ O __ __ __ __

6



$6 \times 8! \times 2!$

$= 483840$

Q5 $$\binom{n}{0}^2 + \binom{n}{1}^2 + \ldots + \binom{n}{n}^2 = \binom{2n}{n}$$

$$\binom{n}{k}^2 = \binom{n}{k}\binom{n}{k}$$

$$= \binom{n}{k}\binom{n}{n-k}$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$= \frac{n!}{(n-k)!\cdot k!} = \binom{n}{n-k}$$

$$\binom{n}{0}\binom{n}{n} + \binom{n}{1}\binom{n}{n-1} + \cdots + \binom{n}{n}\binom{n}{0}$$

$$= \sum_{i=0}^{n} \binom{n}{i}\binom{n}{n-i}$$

$$= \binom{2n}{n}$$

Q7

7. Consider a set of ten urns, nine of which contain three white chips and three red chips each. The tenth contains five white chips and one red chip. An urn is picked at random. Then a sample of size 3 is drawn without replacement from that urn. If all three chips drawn are white, what is the probability that the urn being sampled is the one with five white chips?

A : all three chips are white.

B : urn sampled is the tenth.

$\mathbb{P}(B) = 1/10$

$$\mathbb{P}(A \mid B) = \frac{5}{6} \times \frac{4}{5} \times \frac{3}{4} = \frac{1}{2}$$

$$\mathbb{P}(A \mid \overline{B}) = \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} = 1/20$$

$$\mathbb{P}(B \mid A) = \frac{\mathbb{P}(BA)}{\mathbb{P}(A)}$$

$$= \frac{\mathbb{P}(A \mid B) \cdot \mathbb{P}(B)}{\mathbb{P}(A \mid B) \cdot \mathbb{P}(B) + \mathbb{P}(A \mid \overline{B}) \cdot \mathbb{P}(\overline{B})}$$

$$= \frac{\frac{1}{2} \times \frac{1}{10}}{\frac{1}{2} \times \frac{1}{10} + \frac{1}{20} \times \frac{9}{10}}$$

$$= 10/19$$

Q8

8. Suppose that a randomly selected group of k people are brought together. What is the probability that exactly one pair has the same birthday? (Suppose that the 29th of February is not included).

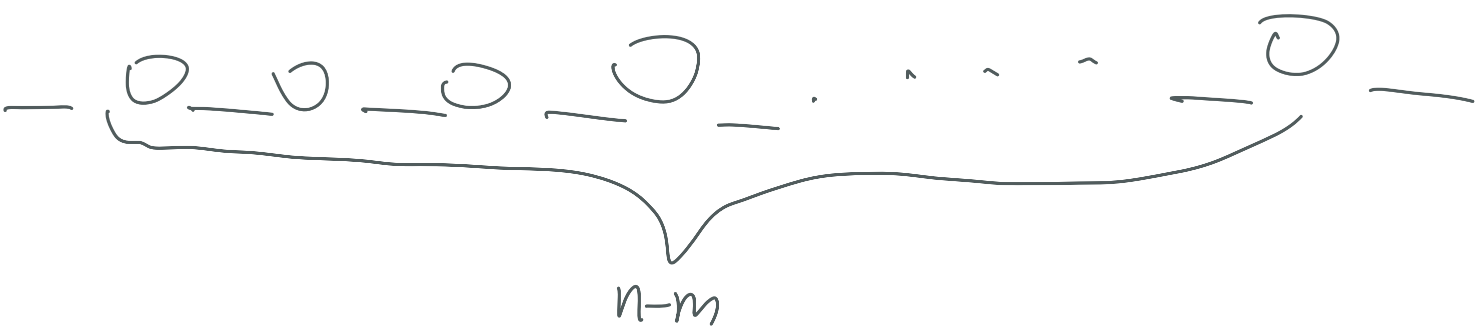
$$\frac{\binom{k}{2} \times 365 \times {}_{364}P_{(k-2)}}{(365)^k} = \frac{\binom{k}{2} \times 365 \times 364 \times 363 \times \cdots \times (364-k+3)}{(365)^k}$$

Q10

10. Consider a set of n antennas of which m are defective and $n-m$ are functional and assume that all of the defective and all of the functionals are considered indistinguishable. How many linear orderings are there in which no two defectives are consecutive?

n — m defective

n — $n-m$ functional



$$\binom{n-m+1}{m}$$