STAT 2011

Q1 Six chips 1, 2, 3, 4, 5, 6

A : " second largest chip is 3 "

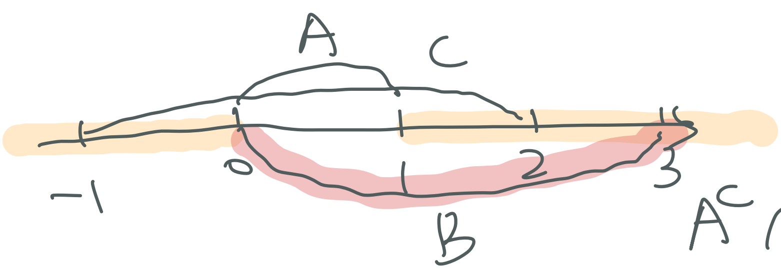
$A = \{ (i, j, k) : 1 \le i < j < k \le 6, \ j = 3 \}$

$A = \{ (1,3,4), (1,3,5), (1,3,6), (2,3,4), (2,3,5), (2,3,6) \}$

Q3 :

$B = \{ (9, 9), (9, \underbrace{\text{not 7 or 9} \cdots\cdots}_{n}, 9 \}$

Q4



(a) $A^c \cap B \cap C$

$A^c = R \setminus [0, 1]$

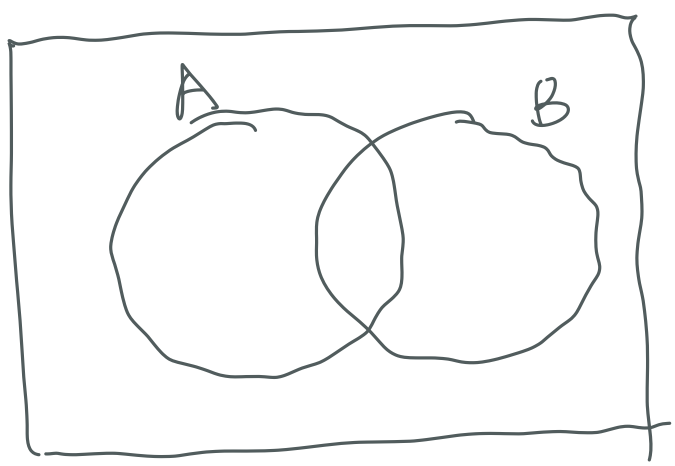
$= (-\infty, 0) \cup (1, \infty)$

$A^c \cap B = (1, 3]$

$A^c \cap B \cap C = (1, 2]$

Q6

(a) $\overset{M}{(A \cap B)^c} = \overset{N}{A^c \cup B^c}$



if $s \in M = (A \cap B)^c$

$s \notin A \cap B$

$s \notin A$ or $s \notin B$

$s \in A^c$ or $s \in B^c$

$s \in A^c \cup B^c$

$M \subseteq N$

$s \in N = A^c \cup B^c$

$s \in A^c$ or B^c

$s \notin A$ or $s \notin B$

$s \notin (A \cap B)$

$s \in (A \cap B)^c \in M$ $\Rightarrow M = N$

$N \subseteq M$

Q7

A_i : 6 on i^{th} die

$\mathbb{P}(A_i) = 1/6$

$\mathbb{P}(A_1 \cup A_2 \cup A_3) = \mathbb{P}(A_1) + \mathbb{P}(A_2) + \mathbb{P}(A_3) = \boxed{\frac{1}{2}}$?

$\mathbb{P}(A_1 \cup A_2 \cup A_3) = \mathbb{P}(\text{at least a 6 on three dice})$

$= 1 - \mathbb{P}(\text{no 6 on three dice})$

$= 1 - (\frac{5}{6})^3$

$\boxed{= 0.4212}$

Q13

K : knows question

R : get it right

$\mathbb{P}(K \mid R) = 0.92$

$\mathbb{P}(R \mid K) = 1$

$\mathbb{P}(R \mid \bar{K}) = 0.2$

$\mathbb{P}(K) = \frac{\#\text{ knows}}{\text{total questions}}$

$$\mathbb{P}(K \mid R) = \frac{\mathbb{P}(K \cap R)}{\mathbb{P}(R)} = \frac{\mathbb{P}(R \mid K) \cdot \mathbb{P}(K)}{\mathbb{P}(R \mid K) \cdot \mathbb{P}(K) + \mathbb{P}(R \mid \bar{K}) \cdot \mathbb{P}(\bar{K})}$$

$= 0.92$

$$\frac{1 \cdot \mathbb{P}(K)}{\mathbb{P}(K) + 0.2 \times (1 - \mathbb{P}(K))} = 0.92$$

$\mathbb{P}(K) = 0.697$ $\times 20 = 14$

Q14

$\mathbb{P}(A) = 1/4 \qquad \mathbb{P}(B) = 1/8$

(i)

$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$

$= 1/4 + 1/8 = 3/8$

(ii) $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B) \qquad \mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$

$= 1/4 + 1/8 - 1/4 \cdot 1/8$

$= 11/32$

Q15

$A_3 = \{(1,3)\ (3,1)\ (2,2)\ (5,6)\ (6,5),\ (6,6)\}$

$\mathbb{P}(A_1) = 2/6 = 1/3$

$\mathbb{P}(A_2) = 1/2$

$\mathbb{P}(A_3) = 6/36 = 1/6$

$\mathbb{P}(A_1 \cap A_2 \cap A_3) = 1/36$

$\mathbb{P}(A_1 \cap A_2 \cap A_3) = \underline{\mathbb{P}(A_1) \cdot \mathbb{P}(A_2) \cdot \mathbb{P}(A_3)}$

for all $i \neq j$ $\underline{\mathbb{P}(A_i \cap A_j) \neq \mathbb{P}(A_i) \cdot \mathbb{P}(A_j)}$

$\mathbb{P}(A_2 \cap A_3) = 2/36 = 1/18 \neq$

$\mathbb{P}(A_2) \cdot \mathbb{P}(A_3) = \frac{1}{2} \cdot \frac{1}{6} = 1/12$

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