STAT 2011

Q1 n trails $\quad$ k_1 outcome 1 (P_1) $\quad$ k_2 outcome 2 (P_2) $\quad$ $n-k_1-k_2$ out 3 ($1-P_1-P_2$)

$$\mathbb{P}(k_1 \text{ out } 1, k_2 \text{ out } 2) = P_1^{k_1} \cdot P_2^{k_2} \cdot (1-P_1-P_2)^{n-k_1-k_2} \frac{n!}{k_1!\,k_2!\,(n-k_1-k_2)!}$$

$$\boxed{\frac{n!}{n_1! \cdots n_i!}} = \frac{n!}{k_1!\,k_2!\,(n-k_1-k_2)!} \cdot P_1^{k_1} P_2^{k_2} \cdot (1-P_1-P_2)^{n-k_1-k_2}$$

Multinomial Distribution.

2. Repair calls for central air conditioners fall into three general categories: coolant leakage, compressor failure, and electrical malfunction. Experience has shown that the probabilities associated with the three are 0.5, 0.3, and 0.2, respectively. Suppose that a dispatcher has logged in ten service requests for tomorrow morning. Use the answer to Question 3.2.18 to calculate the probability that three of those ten will involve coolant leakage and five will be compressor failures.

Q2 $n=10 \quad k_1=3 \quad k_2=5 \quad P_1=0.5 \quad P_2=0.3 \quad P_3=1-P_1-P_2 = 0.2$

$$\mathbb{P} = \frac{10!}{3!\,5!\,2!} \cdot 0.5^3\, 0.3^5\, 0.2^2$$

$$= 0.031$$

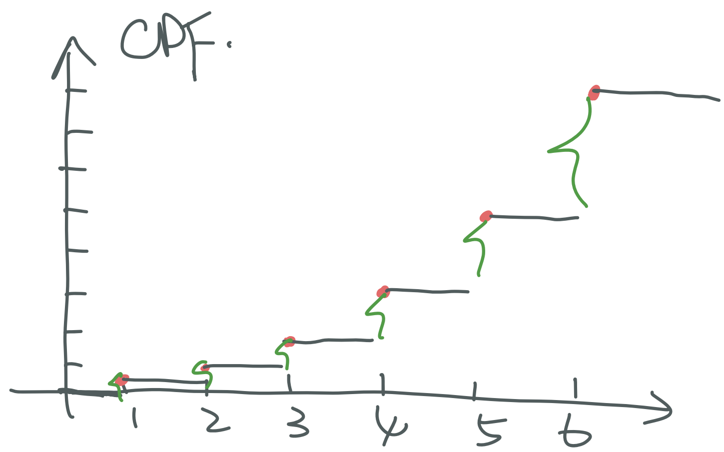
3. Find the probability mass function (pmf) for the discrete random variable X whose cumulative distribution function (cdf) at the points $x = 0, 1, \ldots, 6$ is given by $F_X(x) = x^2/36$.

Q3 $F_X(x) = \mathbb{P}(X \le x)$

$P_X(x) = \mathbb{P}(X = x)$

$F_X(x) = x^2/36$

r.v. $X \in \{0, 1, \ldots, 6\}$



$F_X(0) = 0 \quad F_X(1) = 1/36 \quad F_X(6) = 1$

$F_X(2) = 4/36 \quad F_X(3) = 9/36$

$F_X(4) = 16/36 \quad F_X(5) = 25/36$

$p_X(x) = \mathbb{P}(X = x) = \mathbb{P}(X \le x) - \mathbb{P}(X \le \boxed{x^-})$

$p_X(0) = \mathbb{P}(X \le 0) = 0$

$p_X(1) = \mathbb{P}(X \le 1) - \mathbb{P}(X \le 0) = 1/36$

$p_X(2) = \mathbb{P}(X \le 2) - \mathbb{P}(X \le 1) = 4/36 - 1/36 = 3/36$

$p_X(3) = 5/36 \quad p_X(4) = 7/36 \quad p_X(5) = 9/36 \quad p_X(6) = 11/36$

$$p_X(x) = \begin{cases} \cdots & x = \cdots \\ \cdots & x = \cdots \end{cases}$$

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7. Independent trials consisting of the flipping of a coin having probability $\boxed{p}$ of coming up heads are continually performed until either a head occur or a total of n flips is made. Let X denote the number of times the coin is flipped, find $p_X(k)$, and check that it is a pmf.

Q7 X : # of times the coin is flipped.

$\{\underbrace{\boxed{T \cdots T}}_{k-1}, H\}$ $\quad k$

$\{\underbrace{T \cdots \cdots T}_{n}\}$

$p_X(k) = \mathbb{P}(X = k)$

$= (1-p)^{k-1} \cdot p \qquad k < n$

$p_X(k) = (1-p)^{n-1} \cdot p + (1-p)^n \qquad k = n$

$\sum_{k=1}^{n} p_X(k) = 1 \, ?$

$\sum_{k=1}^{n-1} (1-p)^{k-1} \cdot p + (1-p)^{n-1} p + (1-p)^n$

$$= \sum_{k=1}^{n} (1-p)^{k-1} \cdot p + \underline{(1-p)^n}.$$

$$p + p(1-p) + \cdots + p(1-p)^{n-1}$$

$$= \cancel{p} \frac{(1 - \cancel{(1-p)^n})}{\cancel{1-(1-p)}} + \underline{\cancel{(1-p)^n}} = 1$$

8. Find E(X) and Var(X), where X is the outcome when we roll a fair die.

Q8 $X \{1, 2, 3, 4, 5, 6\}$ $\quad \mathbb{P}_X(x) = 1/6$ for all x.

$$\mathbb{E}(X) = \sum_x x \, P_X(x)$$

$$= \frac{1}{6}(1 + 2 + \cdots + 6)$$

$$= 7/2$$

$$\mathrm{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$$

$$= \frac{91}{6} - \left(\frac{7}{2}\right)^2$$

$$= 35/12$$

$$\underline{\sum x^2 \cdot P_X(x)}$$

$$= \frac{1}{6}(1 + 4 + 9 + 16 + 25 + 36)$$