1. Suppose a series of n independent trials can end in one of three possible outcomes. Let k_1 and k_2 denote the number of trials that result in outcomes 1 and 2, respectively. Let p_1 and p_2 denote the probabilities associated with outcomes 1 and 2. Deduce a formula for the probability of getting k_1 and k_2 occurrences of outcomes 1 and 2, respectively.

n : # of total trails

k_1 out 1 P_1

k_2 out 2 P_2

$n-k_1-k_2$ out 3 $1-P_1-P_2$

$$\frac{n!}{n_1!n_2!\cdots n_r!}$$

$$\mathbb{P} = P_1^{k_1} P_2^{k_2} \cdot (1-P_1-P_2)^{n-k_1-k_2} \cdot \frac{n!}{k_1!\,k_2!\,(n-k_1-k_2)!}$$

multinomial Distribution.

2. Repair calls for central air conditioners fall into three general categories: coolant leakage, compressor failure, and electrical malfunction. Experience has shown that the probabilities associated with the three are 0.5, 0.3, and 0.2, respectively. Suppose that a dispatcher has logged in ten service requests for tomorrow morning. Use the answer to Question 3.2.18 to calculate the probability that three of those ten will involve coolant leakage and five will be compressor failures.

$n = 10$

$P_1 = 0.5$ $k_1 = 3$

$P_2 = 0.3$ $k_2 = 5$

$P_3 = 0.2$ $k_3 = 10 - k_1 - k_2 = 2$

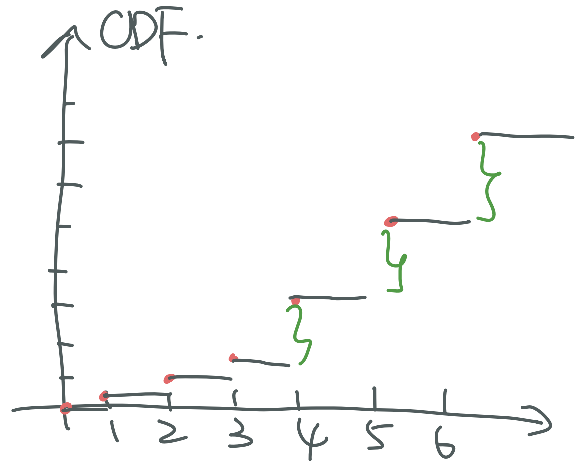
$$\mathbb{P} = \frac{10!}{3!\,5!\,2!} \cdot 0.5^3\, 0.3^5 \cdot 0.2^2$$

$= 0.0306$

3. Find the probability mass function (pmf) for the discrete random variable X whose cumulative distribution function (cdf) at the points $x = 0, 1, \ldots, 6$ is given by $F_X(x) = x^2/36$.

$F_X(x) = \mathbb{P}(X \le x)$

$P_X(x) = \mathbb{P}(X = x)$



$F_X(0) = 0 \quad F_X(1) = 1/36 \quad F_X(2) = 4/36$

$F_X(3) = 9/36 \quad F_X(4) = 16/36 \quad F_X(5) = 25/36$

$F_X(6) = 1$

$P_X(x) = \mathbb{P}(X = x) = \mathbb{P}(X \le x) - \mathbb{P}(X \le x^-)$

$P_X(0) = \mathbb{P}(X \le 0) = 0$

$P_X(1) = \mathbb{P}(X \le 1) - \mathbb{P}(X \le 0) = 1/36$

$P_X(2) = \mathbb{P}(X \le 2) - \mathbb{P}(X \le 1) = 4/36 - 1/36 = 3/36$

$\sum_i \mathbb{P}(X = x_i) = 1$

7. Independent trials consisting of the flipping of a coin having probability p of coming up heads are continually performed until either a head occur or a total of n flips is made. Let X denote the number of times the coin is flipped, find $p_X(k)$, and check that it is a pmf.

X : # of times the coin flipped.

$P_X(k) = \mathbb{P}(X = k)$ $\qquad \{T \cdots T\,\boxed{H}\}$

$= (1-p)^{k-1} \cdot p \qquad k < n \qquad \{\underbrace{T \cdots T}_{n.}\}$

$P_X(k) = (1-p)^{n-1} \cdot p + (1-p)^n \qquad k = n$

$$\boxed{P_X(k) = \begin{cases} & k < n \\ & k = n \end{cases}}$$

Check : $\sum_{k=1}^{\infty} P_X(k) = 1$?

$$\sum_{k=1}^{n} P_X(k)$$

$$= \underline{\sum_{k=1}^{n-1} (1-p)^{k-1} \cdot p} + \underline{(1-p)^{n-1} \cdot p} + (1-p)^n$$

$$= \underline{\sum_{k=1}^{n} (1-p)^{k-1} \cdot p} + (1-p)^n$$

$$p + p(1-p) + \cdots + p(1-p)^{n-1}$$

$$= \cancel{p} \frac{1 \cdot (1 - \cancel{(1-p)^n})}{\cancel{1-(1-p)}} + \cancel{(1-p)^n}$$

$$= 1$$

8. Find E(X) and Var(X), where X is the outcome when we roll a fair die.

$X = \{1, 2, 3, 4, 5, 6\}$ $\quad P_X(x) = 1/6$ for all x

$$\mathbb{E}(X) = \sum_x x \cdot P_X(x)$$

$$= \frac{1}{6}(1 + \cdots + 6)$$

$$= 7/2$$

$$\underline{\mathrm{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2} \geq 0 \qquad \mathbb{E}(X^2) \geq (\mathbb{E}(X))^2$$

$$= \frac{1}{6} \times (1^2 + 2^2 + \cdots + 6^2) - \left(\frac{7}{2}\right)^2$$

$$= 35/12$$

10. Suppose there are m days in a year, and that each person is independently born on day r with probability p_r, with $r = 1, \ldots, m$ and $\sum_{k=1}^{m} p_k = 1$. Let $A_{i,j}$ be the event that person i and person j are born on the same day.

(a) Find $P(A_{1,3})$.

(b) Find $P(A_{1,3}|A_{1,2})$.

(c) Show that $P(A_{1,3}|A_{1,2}) \geq P(A_{1,3})$.

(a) $\mathbb{P}(A_{13}) = P_1 \times P_1 + P_2 \times P_2 + \cdots + P_m P_m$

$$= \boxed{\sum_{r=1}^{m} P_r^2}$$

(b) $\mathbb{P}(A_{13}|A_{12}) = \dfrac{\mathbb{P}(A_{13} A_{12})}{\mathbb{P}(A_{12})} = \boxed{\dfrac{\sum_{r=1}^{m} P_r^3}{\sum_{r=1}^{m} P_r^2}}$

(c) $\mathbb{P}(A_{13} | A_{12}) \geq \mathbb{P}(A_{13})$

$$\frac{\sum_{r=1}^{m} P_r^3}{\boxed{\sum_{r=1}^{m} P_r^2}} \geq \boxed{\sum_{r=1}^{m} P_r^2}$$

$$\boxed{\boxed{\sum_{r=1}^{m} P_r^3} \geq \left(\boxed{\sum_{r=1}^{m} P_r^2}\right)^2}$$

$\mathbb{E}(X^2) \geq (\mathbb{E}(X))^2$

$Y: P_1 \cdots P_r.$

$P_1 \cdots P_r$