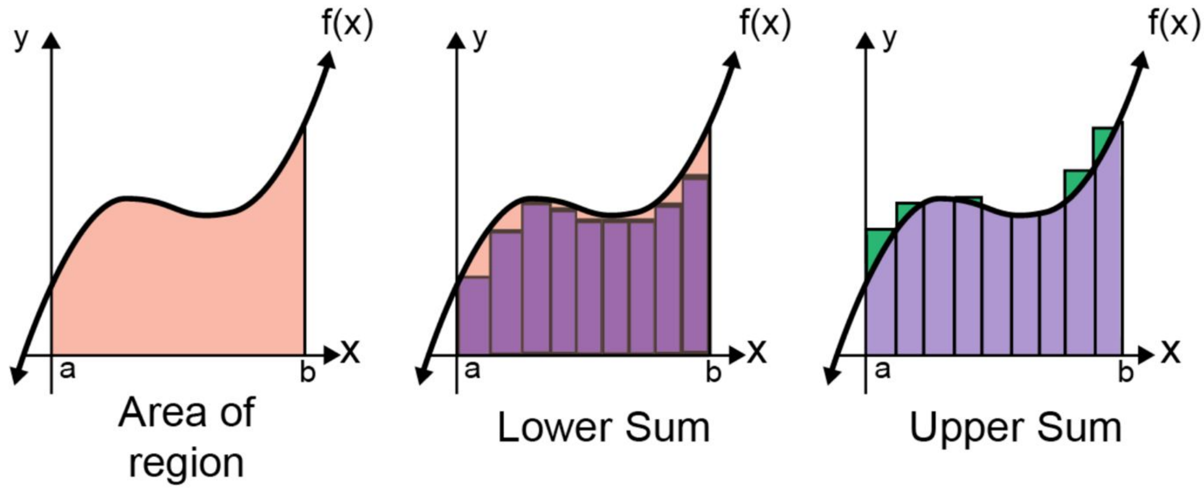
Assignment 3: Integration

In this assignment we will explore numerical methods of integration. As usual, you will be judged by the correctness of your response as well as the quality of your code.

Problem 1

Write a function that performs an approximation of an integral via a Riemann sum, ie: given an array of x- values, and an array of y- values, calculate the integral using the rectangle method:



You may use either upper or lower sums. Note that unlike the picture above, the x values and y values need not be given on equally spaced intervals. **For reference, 99% of "integration" you will do in computational physics will be this procedure.** Typically, data will be given to you with a set of x and y values, and if you find the integral, you compute the Riemann sum and get the answer. Usually, the data is evaluated "finely enough" that you don't need the higher order schemes we will discuss below.

1. Test your function against the function y = x**2 from the interval 0 to 1.
    a. Evaluate this integral analytically to get an exact answer.

Generate x and f(x) from 0 to 1 in steps of 0.1, 0.01, to 1e-6 and plot how the relative percent error of the integral:

$$\delta = \left|\frac{True - Approximation}{True}\right| \cdot 100\%$$

    b. scales as the step size h gets smaller. ie, you should have a plot of the step size on the x axis vs the relative error on the y. How much does your approximation improve if you make the step size 1/10th as large? Think about what a reasonable answer to this question means. For full credit, you should use numpy to operate on the x's and f(x)'s as arrays rather than using for loops (why?)
2. Repeat part 1 using the trapezoidal rule instead.
3. Repeat part 1 using Simpson's rule

Problem 3.

Compute, numerically, the value of the integral:

$$\iint_{x,y\ \in A} e^{-x^2+2\sqrt{xy}}\,dxdy, A = x, y\geq 0, x^2 + y^2 \leq 1$$

Problem 4.

In this problem we will analyze a typical solar power dataset to inform our decisions about investing in solar panels and solar batteries. As a bit of background, a typical unit of power is the Watt, which is the power in an electrical circuit running at 1 Volt with a current of 1 Amp. (eg, Power = Voltage*Current). For most household needs, the power draw is more conveniently expressed in kiloWatts, which is approximately the power needed to run a vacuum cleaner.

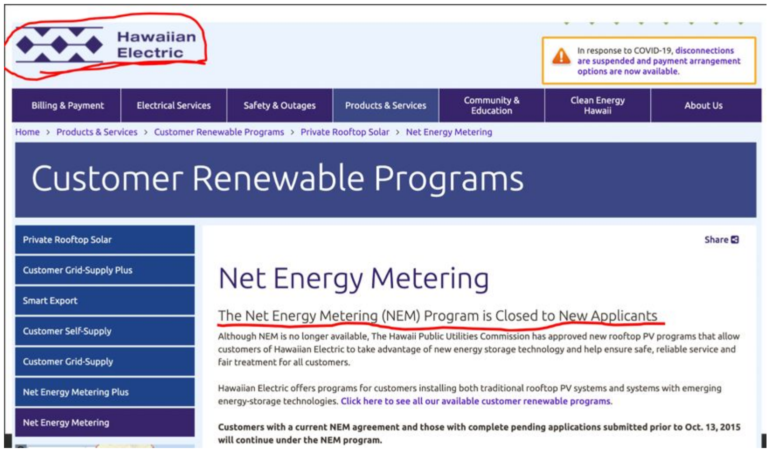
For energy, therefore, the unit is kilowatt-hour (kWh), which is the energy you need to run a vacuum cleaner for 1 hour (eg, E = P*t). Power utilities charge you based on the number of kilowatt-hours you use, HELCO for example will charge you around $0.35 per kWh in Hilo. This is among the highest in the nation, and for that reason a lot of people invest in solar panels.

Open the dataset “solar_household_data.txt”, which contains 3 columns.[1] The first column is time in hours; the second, the *load* (or power drawn, in kilowatts) of a house; the last is the power (in kilowatts) produced by a typical solar panel installation.

1. Make a nice plot of the load and typical solar panel and convince yourself you understand what is going on. Why do the solar power and load curves look the way they do?
2. Calculate the following:
    a. The total energy used by the household over a day. (ignore the solar panels). What would the electric bill be per month?
    b. The total energy produced by the solar panels over the course of a day
3. Consider a situation where the “old-style” net-metering agreement is in place and the excess energy used by the household can be sold back to the utility at retail price. How much profit (profit = gross-cost) would the solar panels generate each month?

As you can see from question 3, net metering turned out to be a disaster for the power companies as people aggressively bought up solar panels. The power companies were still responsible for maintaining the infrastructure, but instead of getting paid, had to pay out.

[1] I used a plot digitizer (https://apps.automeris.io/wpd/) to pull data from this paper: http://tpea.cpss.org.cn/uploads/allimg/180802/EnhancingPVInverterReliabilityWithBattery.pdf



(from current HELCO website)

Without a net-metering program, the power produced by solar panels must be used instantaneously. If it isn't used, it goes back to the grid "for free" (sorry!).

4. Estimate the total energy draw from the grid of the household without a net metering program in place. Here, you only pay when the instantaneous household load exceeds the solar power production, and you pay the difference. Eg, @ 1-2pm, load = 11 kW, solar = 10 kW, delta = 1 kW□draw = 1 kWh, cost = $0.35.
5. (Bonus) The cost of a 5 kWh Tesla Powerwall costs about $10,000. How long would it take to for it to pay for itself?