Assignment 5: Recurrences and Hash Tables

1) $T(n) = 3T(n/4) + 4n \leftarrow 1^{st}$

$T(n/4) = 3T(n/16) + 4(n/4) \leftarrow 2^{nd}$

$T(n) = 3[3T(n/16) + 4(n/4)] + 4n$

$= 9T(n/16) + 3(4n/4) + 4n$

$= 9T(n/16) + 3n + 4n$

$= 9T(n/16) + 7n$

$T(n) = 9[3T(n/64) + 4(n/16)] + 7n \leftarrow 3^{rd}$

$= 27T(n/64) + 9 \cdot 4(n/16) + 7n$

$= 27T(n/64) + 9(n/4) + 7n$

$= 27T(n/64) + \frac{9}{4}n + 7n$

$= 27T(n/64) + 37/4\, n$

k-iterations:

$$T(n) = 3^k T(n/4^k) + 4n \sum_{i=0}^{k-1} (3^i/4^i)$$

$$\sum_{i=0}^{k-1} (3^i/4^i) = \frac{1-(3/4)^k}{1-3/4} = \frac{1-(3/4)^k}{1/4} = 4(1-(3/4)^k)$$

$T(n) = 3^k T(n/4^k) + 4n \cdot 4(1-(3/4)^k)$

$T(n) = 3^k T(n/4^k) + 16n(1-(3/4)^k)$

When $n/4^k = 1$, solving for k:

$$k = \log_4 n$$

$T(1) = c$ (constant)

$T(n) = 3^{\log_4 n} c + 16n(1-(3/4)^{\log_4 n})$

Using $3^{\log_4 n} = n^{\log_4 3}$

$T(n) = O(n^{\log_4 3})$

Master Theorem:

$T(n) = 3T(n/4) + 4n$

$a = 3,\ b = 4,\ f(n) = 4n = \Theta(n^1)$

$\log_b a = \log_4 3 \approx 0.792$

$f(n) = \Theta(n^1)$

$1 > 0.792$

Case 3: $T(n) = \Theta(n)$