Spring 2024 ECE 5961/6961 Fundamentals of Wireless Communications Homework 1

Assign date: 1/16/2024 Due date: 1/30/2024 at 11:59pm through Canvas submission.

Problem 1.

Show that the complex baseband channel impulse response to carrier modulated signal is given by

$$C_{d,\theta}(\tau) = \sum_n \beta_n(d,\theta) e^{j\phi_n(d,\theta)} \delta(\tau - \tau_n), \tag{1}$$

where $\phi_n = -2\pi f_c \tau_n$. (Hint: You may use the fact that the passband impulse response of a multipath channel is $h_{d,\theta}(\tau) = \sum_n \beta_n(d,\theta)\delta(\tau - \tau_n)$, and a passband carrier modulated signal $s(t)$ relates to its baseband representation $s_b(t)$ by $s(t) = \mathrm{Re}(s_b(t)e^{j2\pi f_c t})$.

Problem 2.

Consider the communication from a transmitter to a mobile receiver (see textbook Fig. 2.4, page 16), where there is a single perfectly reflecting large fixed wall. The received signal at the receiver is the sum of the signal coming from the transmit antenna plus a reflected signal coming from the wall. As shown in the figure, the distance between the transmitter and the wall is denoted by d and the distance between the transmitter and the mobile is denoted by r. Assume the channel gains $\{\beta_n\}$ associated with all paths are the same and are denoted by β, carrier frequency is denoted by f_c, and the speed of light is denoted by c.

(a) Find the channel impulse response of this wireless communication system for fixed d and r.

(b) Find the delay spread of this system.

(c) Assume the mobile moves at a velocity v towards the wall, Find the Doppler shift associated with each path.

Problem 3.

Consider the same two-path example as in problem 2 with d = 2 km and the receiver at 1.5 km from the transmitter moving at velocity 60 km/hr away from the transmitter. The carrier frequency is 900 MHz.

(a) Plot in MATLAB the magnitudes of the taps of the discrete-time baseband channel at a fixed time $t = 1$. Give a few plots for several bandwidths W = 10 KHz, 100 KHz, 1 MHz, and 3 MHz so as to exhibit both flat and frequency-selective fading. As described in the book, you may assume that the path loss is given by $(r + vt)^{-1}$ for the direct path and by $(2d - r - vt)^{-1}$ for the reflected path.

(b) Plot the time variation of the magnitude of a typical tap of the discrete-time baseband channel for a bandwidth where the channel is (approximately) flat, e.g., the 0-th tap with W = 10 KHz and for a bandwidth where the channel is frequency selective, e.g., the 5-th tap for W = 1 MHz. How does the time-variations depend on the bandwidth? Explain.

Problem 4.

A mobile receiver is moving at speed v and is receiving signals arriving along two reflected paths which are at angles θ_1 and θ_2 from the direction of motion. The transmitted signal is a sinusoid at frequency f.

(a) Is the above information enough for estimating i) the coherence time T_c; ii) the coherence bandwidth W_c? If so, express in terms of the given parameters. If not, specify what additional information would be needed.

(b) Consider an environment in which there are reflectors and scatterers in all directions from the receiver and an environment in which they are clustered within a small angular range. Using part (a), explain how the channel would differ in these two environments.

Problem 5.

Small Scale Fading. Consider the delay profile with the 13 paths, and the following parameters. The sequence of squared path gains $(\alpha_0^2, \alpha_1^2, \cdots, \alpha_{12}^2)$ is given by

$$(0.5, 0.025, 0.075, 0.1, 0.075, 0.025, 0.025, 0.075, 0.025, 0.025, 0.02, 0.02, 0.01)$$

and the corresponding sequence of path delays $(\tau_0, \tau_1, \cdots, \tau_{12})$ is given by:

$$(0, 1.2, 1.4, 1.6, 1.8, 2.0, 3.4, 3.6, 3.8, 4.0, 5.2, 5.4, 5.6)\mu s$$

(a) Carefully sketch the delay profile $|h(t;\xi)|$ with the heights of the δ functions drawn approximately to scale to match the path gains.

(b) Is there an LOS path? If so, what is the Ricean factor?

(c) Argue that a signal with bandwidth $W = 20$ KHz sees the channel as a flat fading channel, and write down an expression for the pdf $p_\alpha(\alpha)$ of the effective channel gain α.

(d) Argue that a signal with bandwidth $W = 1$ MHz sees the channel as a frequency selective channel, and draw the tapped delay line model for $\frac{1}{W}$-spaced taps.

(e) Suppose the carrier frequency $f_c = 1$ GHz, the mobile velocity is 60 km/hr, and the symbol rate is 10^5 symbols/s. Find the coherence time. Is the fading slow?

Problem 6.

Simulating Ricean flat fading. Using the direct approach we discussed in class to simulate a Ricean flat fading process with Rice factor $\kappa = 1$. Assume that the LOS component arrives at angle $\theta_0 = \pi/4$, and that the diffuse components are uniformly distributed in angle and power (isotropic). Let the total number of diffuse components be 12, and assume $f_m = 60$ Hz.

Plot the channel gain $|V(t)|$ in dB as a function of t, for t ranging from 0 to 250 ms.

Problem 7.

Ricean Random Variables. Assume $X \sim N(a, \sigma^2)$ and $Y \sim N(b, \sigma^2)$ are independent Gaussian random variables, where a and b are deterministic constants. Define R and Θ by:

$$R = \sqrt{X^2 + Y^2}, \text{ and } \Theta = \tan^{-1}(\frac{Y}{X})$$

(Assume $\Theta \in [-\pi, \pi]$.) Find the joint pdf of R and Θ and from this the marginal pdf of R. Express the latter in term of the modified Bessel function of the first kind:

$$I_0(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp(x \cos\phi) d\phi$$

The pdf of R in this case is called Ricean.