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The persistence of unemployment in the USA and Europe in terms of fractionally ARIMA models

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This article examines the persistence of unemployment in the USA and four European countries by means of fractionally integrated ARMA (ARFIMA) models. In doing so, a type of flexibility in modelling low-frequency dynamics not achieved by non-fractionally ARIMA models can be provided. The results indicate that the unemployment series are much more persistent in some countries such as the UK and France, than in others including Germany or the USA.

I. INTRODUCTION

This article is concerned with the dynamics underlying both the short-run and long-run behaviour of unemployment in the USA and four European countries. Traditionally, two schools of thought exist concerning the behaviour of unemployment. One approach focuses on the determinants of the long-run equilibrium level. The other concentrates on the movements of unemployment between equilibria. Using econometric skills, the method of distinguishing between the approaches is by examination of the order of integration of the series. Thus, if unemployment is $I(1)$, i.e. if it follows a unit root process, the shocks affecting the series will have permanent effects, shifting the unemployment equilibrium from one level to another. If this is the case, a policy action will be required to bring unemployment back to its original level. On the other hand, if unemployment is $I(0)$, the effect of the shocks will be merely transitory and as a result, less need will exist for policy action, since unemployment will in any case return to its equilibrium level sometime in the future.

The $I(0)$ and $I(1)$ specifications are, however, too restrictive in comparison to the huge range of possibilities covered by the fractionally integrated $(I(d))$ models, in which the time series observed is modelled as

$$(1 - L)^d x_t = u_t, \quad t = 1, 2 \tag{1}$$

where u_t is an $I(0)$ process, defined in this context as a covariance stationary process, with spectral density function which is positive and finite at zero frequency, and where d can be any real number. One can write $(1 - L)^d$ in terms of its Binomial expansion

$$(1 - L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j$$

such that

$$(1 - L)^d x_t = x_t - dx_{t-1} + \frac{d(d-1)}{2} x_{t-2}$$
$$\frac{d(d-1)(d-2)}{6} x_{t-3}$$

Thus, d plays a crucial role in explaining the dependence between observations. The higher d is, the higher the dependence between observations will be, even though they are widely separated in time. The process u_t in Equation 1 could itself be a stationary and invertible ARMA sequence, when its autocorrelations decay exponentially. However, they could decay much slower than exponentially. When $d = 0$ in (1), $x_t = u_t$ and thus, x_t is 'weakly dependent'. If $d < 0 < 0.5$, x_t is still stationary, but

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its lag-j autocovariance decreases very slowly, like the power law j^{2d-1} as $j \to \infty$ and so the autocovariances are non-summable. It can then be said that x_t has long memory because of the strong association between observations widely separated in time. Processes like Equation 1 with positive non-integer d are called fractionally integrated and when u_t is ARMA(p, q), x_t has been called a fractionally ARIMA (ARFIMA(p, d, q)) process. Thus, the model becomes

$$\Phi_p(L)(1-L)^d x_t = \Theta_q(L)\varepsilon_t, \quad t = 1, 2 \qquad (2)$$

where $\Phi_p(L)$ and $\Theta q(L)$ are polynomials of orders p and q respectively, with all zeros of $\Phi_p(L)$ outside the unit circle, and all zeros of $\Theta_q(L)$ outside or on the unit circle, and ε_t white noise. These kind of models were introduced by Granger and Joyeux (1980), Granger (1980, 1981) and Hosking (1981) (although earlier work by Adenstedt (1974) and Taqqu (1975) shows an awareness of the representation) and were justified theoretically in terms of aggregation by Robinson (1978) and Granger (1980).

Modelling unemployment within the ARFIMA specification allows study of both its short-run and its long-run dynamics in the same framework, and the estimated value of d can tell us something about the degree of persistence of the series. Thus, if $d \in (0, 0.5)$, the series will be stationary and mean-reverting; if $d \in [0.5, 1)$ it will be non-stationary but still mean-reverting; and finally, if $d \geq 1$, the series will be nonstationary and nonmean-reverting. Sowell (1992) analysed in the time domain the exact maximum likelihood estimates of the parameters of a fractionally ARIMA model Equation 2, using a recursive procedure that allows quick evaluation of the likelihood function.

II. AN EMPIRICAL APPLICATION

The unemployment rates in Germany, France, Italy, the UK and the USA are examined in this section by means of fractionally ARIMA models. The data are quarterly and the starting date is 1962:1 for Germany, 1968:1 for France, 1975:1 for the UK and the USA, and 1978:1 for Italy. All the series end in 1998:3. The first 30 sample autocorrelations for the original time series and its first differences are plotted in Table 1. Looking at the values for the original time series, it is observed that they all start at around 0.96 and then decay slowly. Significant autocorrelations are found at all lags in the cases of Germany and France; at practically all of them in the cases of the UK and the USA; and up to the lag 19 in Italy. This kind of persistent behaviour in the autocorrelations may be consistent even with the simple random walk hypothesis. Looking at the first differences, however, significant autocorrelations are still observed, especially at first lags, but also at some others, with some apparent slow decay and/or oscillation, which could be indicative of fractional integration of greater than or less than a unit root.

Denoting any of the series x_t model Equation 2 is estimated with p and q smaller than or equal to 3, using the Sowell's (1992) procedure of estimating by maximum likelihood in the time domain. This procedure consistently estimates d along with the other parameters of the model when $d < 0.5$. Thus, in order to assure stationarity the models are estimated in second differences, adding later 2 to the estimated values of d.

Table 2 summarizes the results for the unemployment in Germany. It is observed that the estimated values of d vary widely depending on how the short-run component of the series is modelled. These values range from -0.50 (when using an ARMA(2, 2) specification for u_t) to 1.51 (when u_t is white noise). It is also seen that the null hypothesis of $d = 0$ cannot be rejected in six out of the 16 models presented, and the unit root null hypothesis is not rejected in seven. Looking at the last two columns of the table, it is seen that according to the Akaike Information Criterium (AIC), the ARFIMA$(3, -0.13, 0)$ specification is the most adequate model. However, according to the Schwarz Information Criterium, (SIC), the ARFIMA$(1, 0.67, 0)$ model appears more appropriate. In order to choose between these two specifications, a likelihood ratio test was performed and the latter model was preferred. It can be concluded the analysis of this series by saying that the unemployment in Germany may well be described in terms of an ARFIMA$(1, 0.67, 0)$ process and thus, being non-stationary but with a mean-reverting behaviour.

Studying the results for France, displayed in Table 3, it is seen that the estimated values of d are higher than those observed for Germany in Table 1. They range now between 0.48 and 2.31, and while the null $d = 0$ is not rejected in six models, the unit root null hypothesis cannot be rejected in 11. The AIC and the SIC both suggest that the best model specification for this series is an ARFIMA(0, 1.32, 1), rejecting both the $I(0)$ and $I(1)$ specifications. Thus, a higher degree of integration is observed in unemployment in France compared with Germany, suggesting a stronger degree of persistence in its behaviour.

Table 4 displays the results for Italy. It is observed in this table that all the estimated values of d are around 1, and in fact, the unit root null hypothesis cannot be rejected in any of the models. According to the AIC and the SIC, the white noise specification seems to be the most appropriate way of describing the short run dynamics of the series. Thus, one concludes by saying that the unemployment in Italy can be well described as an ARFIMA$(0, 0.95, 0)$ model.

The results for the UK are given in Table 5. It is seen that all the estimated values of d are above 1, ranging from 1.03 when u_t is an ARMA(3,2) to 1.97 when u_t is AR(1). The null hypothesis of $d = 0$ is not rejected in two cases and the unit root cannot be rejected in nine. According to the AIC, the ARFIMA$(2, 1.29, 0)$ model is the best

Table 1. *Sample autocorrelations of the original time series and first differences*

	Germany		France		Italy		UK		USA	
Lag	x_t	$(1-L)x_t$	x_t	$(1-L)x_t$	x_t	$(1-L)x_t$	x_t	$(1-L)x_t$	x_t	$(1-L)x_t$
1	0.971	0.548	0.984	0.606	0.952	−0.146	0.957	0.809	0.951	0.583
2	0.937	0.399	0.967	0.306	0.908	0.121	0.901	0.696	0.862	0.342
3	0.898	0.227	0.947	0.175	0.862	−0.104	0.834	0.548	0.760	0.183
4	0.858	0.045	0.925	0.059	0.819	0.084	0.760	0.388	0.649	0.010
5	0.818	0.028	0.902	−0.010	0.775	0.117	0.680	0.248	0.543	−0.066
6	0.777	−0.012	0.876	−0.008	0.724	−0.044	0.598	0.128	0.444	−0.030
7	0.737	−0.082	0.850	−0.041	0.677	0.077	0.515	0.002	0.348	−0.054
8	0.699	−0.086	0.824	−0.040	0.623	−0.087	0.435	−0.085	0.255	−0.112
9	0.662	−0.090	0.799	0.062	0.573	0.140	0.358	−0.148	0.169	−0.050
10	0.627	−0.129	0.773	0.076	0.514	−0.168	0.286	−0.183	0.091	−0.012
11	0.594	−0.123	0.747	−0.000	0.462	−0.007	0.221	−0.230	0.017	−0.078
12	0.560	−0.180	0.721	−0.017	0.410	−0.093	0.163	−0.262	−0.051	−0.153
13	0.526	−0.161	0.696	−0.066	0.355	−0.117	0.111	−0.227	−0.107	−0.133
14	0.494	−0.152	0.671	−0.027	0.310	−0.145	0.064	−0.210	−0.153	−0.163
15	0.464	−0.101	0.646	−0.023	0.272	−0.052	0.021	−0.162	−0.192	−0.216
16	0.434	−0.080	0.620	−0.057	0.239	−0.082	−0.019	−0.061	−0.216	−0.221
17	0.407	−0.090	0.594	−0.138	0.192	−0.030	−0.063	−0.039	−0.225	−0.167
18	0.382	−0.083	0.566	−0.147	0.152	−0.056	−0.106	0.010	−0.219	−0.191
19	0.361	−0.091	0.538	−0.102	0.115	−0.110	−0.148	0.064	−0.200	−0.140
20	0.354	−0.100	0.510	−0.072	0.086	−0.022	−0.192	0.091	−0.172	−0.030
21	0.353	−0.077	0.481	−0.071	0.062	−0.077	−0.234	0.088	−0.145	−0.052
22	0.355	−0.053	0.453	−0.115	0.046	−0.033	−0.271	0.051	−0.119	−0.112
23	0.359	−0.075	0.426	−0.161	0.040	0.039	−0.300	0.000	−0.090	−0.012
24	0.363	0.007	0.401	−0.205	0.039	0.014	−0.319	−0.027	−0.055	0.010
25	0.366	−0.093	0.376	−0.107	0.038	0.049	−0.330	−0.066	−0.018	−0.088
26	0.369	−0.080	0.351	−0.073	0.040	−0.009	−0.331	−0.105	0.022	−0.064
27	0.371	−0.077	0.328	−0.072	0.039	−0.007	−0.325	−0.135	0.065	−0.104
28	0.372	−0.037	0.307	−0.003	0.040	−0.017	−0.314	−0.157	0.101	−0.127
29	0.372	0.014	0.287	0.025	0.043	0.002	−0.297	−0.151	0.136	−0.127
30	0.369	0.056	0.269	−0.012	0.046	0.096	−0.278	−0.146	0.170	0.023

Note: The large sample standard error under the null hypothesis of no autocorrelation is $T^{-1/2}$ or roughly 0.10 for series of length considered here.

Table 2. *Maximum likelihood estimation of ARFIMA*(p,d,q) *models for the unemployment in Germany*

				AR parameters			MA parameters			Criterions	
ARMA	d	$t_{d=0}$[a]	$t_{d=1}$[a]	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(0, 0)	1.51	18.85	6.37	–	–	–	–	–	–	−20.13	−21.60
(1, 0)[c]	0.67	5.15	−2.53	0.81	–	–	–	–	–	−18.08	−21.03
(0, 1)	1.44	11.07	3.38	–	–	–	0.08	–	–	−20.95	−23.90
(1, 1)	0.77	2.96	−0.88[a]	0.79	–	–	0.07	–	–	−18.98	−23.40
(2, 0)	0.86	3.07	−0.50[a]	0.61	0.11	–	–	–	–	−18.83	−23.25
(0, 2)	1.28	9.84	2.15	–	–	–	0.19	0.23	–	−19.96	−24.39
(2, 1)	0.83	3.32	−0.68[a]	0.43	0.25	–	0.20	–	–	−19.70	−25.60
(1, 2)	0.60	2.00	−1.33[a]	0.79	–	–	0.04	0.20	–	−18.55	−24.45
(2, 2)	−0.50	1.38[a]	−4.16	1.82	−0.84	–	0.04	0.17	–	−18.09	−25.46
(3, 0)	−0.13	−0.37[a]	−3.22	1.53	−0.40	−0.16	–	–	–	−17.70	−23.59
(0, 3)	1.01	7.76	0.07[a]	–	–	–	0.48	0.42	0.25	−18.33	−24.23
(3, 1)	−0.19	−0.50[a]	−3.13	1.48	−0.30	−0.21	0.10	–	–	−18.67	−26.04
(3, 2)	0.76	2.71	−0.85[a]	0.45	−0.32	0.42	0.23	0.68	–	−19.66	−28.51
(1, 3)	0.20	0.80[a]	−3.20	0.88	–	–	0.37	0.35	0.21	−18.23	−25.60
(2, 3)	0.31	1.03[a]	−2.30	−0.51	0.30	–	0.63	0.39	0.26	−19.02	−27.87
(3, 3)	0.47	1.23[a]	−1.39[a]	0.34	0.10	0.25	0.64	0.58	0.23	−19.92	−30.24

Notes: [a] Non-rejection values of the null hypotheses: $d = 1$ and $d = 0$ at the 95% significance level.
[b] Best model specification according to the AIC.
[c] Best model specification according to the SIC.

Table 3. *Maximum likelihood estimation of ARFIMA*(p,d,q) *models for the unemployment in France*

				AR parameters			MA parameters			Criterions	
ARMA	d	$t_{d=0}$[a]	$t_{d=1}$[a]	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(0, 0)	1.61	17.88	6.77	–	–	–	–	–	–	43.51	42.11
(1, 0)	1.19	4.76	0.76[a]	0.56	–	–	–	–	–	46.47	43.66
(0, 1)[b]	1.32	12.00	2.90	–	–	–	0.40	–	–	46.84	44.04
(1, 1)	0.48	1.77[a]	−1.92[a]	0.88	–	–	0.33	–	–	46.46	42.26
(2, 0)	1.06	2.40	0.13[a]	0.65	−0.10	–	–	–	–	46.07	41.86
(0, 2)	2.31	21.00	11.90	–	–	–	1.49	2.49	–	45.84	41.63
(2, 1)	0.58	0.77[a]	−0.56[a]	0.73	0.09	–	0.38	–	–	45.47	39.87
(1, 2)	–	–	–	–	–	–	–	–	–	–	–
(2, 2)	0.77	0.60[a]	−0.17[a]	0.72	0.03	–	0.21	0.06	–	44.48	37.47
(3, 0)	0.89	1.48[a]	−0.18[a]	0.81	−0.18	0.08	–	–	–	45.41	39.80
(0, 3)	–	–	–	–	–	–	–	–	–	–	–
(3, 1)	0.85	1.11[a]	−0.19[a]	0.56	0.04	0.04	0.29	–	–	44.50	37.49
(3, 2)	0.98	2.72	−0.05[a]	−0.60	−0.10	0.55	1.37	1.00	–	46.39	37.98
(1, 3)	1.76	1.36[a]	0.58[a]	0.77	−0.82	−0.25	0.07	–	–	44.48	37.47
(2, 3)	1.44	6.00	1.83[a]	−0.08	0.87	–	0.37	0.95	−0.37	43.87	35.45
(3, 3)	1.98	5.50	2.72	−0.60	−0.10	0.55	0.37	0.37	−0.99	45.39	35.58

Notes: [a] Nonrejection values of the null hypotheses: $d = 1$ and $d = 0$ at the 95% significance level.
[b] Best model specification according to the AIC and SIC.
– The model failed to achieve convergence after 240 iterations

Table 4. *Maximum likelihood estimation of ARFIMA*(p,d,q) *models for the unemployment in Italy*

				AR parameters			MA parameters			Criterions	
ARMA	d	$t_{d=0}$[a]	$t_{d=1}$[a]	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(0, 0)	0.95	10.55	−0.55[a]	–	–	–	–	–	–	−28.73	−29.93
(1, 0)	1.13	7.53	0.86[a]	−0.24	–	–	–	–	–	−28.87	−31.26
(0, 1)	1.15	4.11	0.26[a]	–	–	–	−0.24	–	–	−29.11	−31.51
(1, 1)	1.07	8.23	0.53	−0.58	–	–	0.40	–	–	−29.56	−33.15
(2, 0)	0.98	2.80	−0.05[a]	−0.09	0.12	–	–	–	–	−29.65	−33.24
(0, 2)	0.96	3.00	−0.12[a]	–	–	–	−0.05	0.11	–	−29.88	−33.47
(2, 1)	1.02	4.08	0.08[a]	−0.43	0.06	–	0.31	–	–	−30.52	−35.31
(1, 2)	0.97	4.21	−0.13[a]	−0.50	–	–	0.43	0.10	–	−30.46	−35.25
(2, 2)	–	–	–	–	–	–	–	–	–	–	–
(3, 0)	1.22	4.69	0.84[a]	−0.30	−0.03	−0.13	–	–	–	−30.28	−35.07
(0, 3)	1.07	3.34	0.21[a]	–	–	–	−0.14	0.09	0.11	−30.48	−35.27
(3, 1)	1.25	3.57	0.71[a]	−0.24	−0.03	−0.14	−0.10	–	–	−31.27	−37.25
(3, 2)	1.15	7.18	0.93[a]	−0.50	−0.87	−0.30	0.26	0.84	–	−31.44	−38.62
(1, 3)	1.12	3.39	0.36[a]	−0.36	0.16	–	0.02	−0.10	0.10	−31.24	−37.22
(2, 3)	1.18	3.93	0.60[a]	−0.36	−0.64	–	0.11	0.60	0.28	−31.77	−38.95
(3, 3)	1.12	4.86	0.52[a]	−0.57	−0.90	−0.36	0.36	0.88	0.08	−32.42	−40.81

Notes: [a] Nonrejection values of the null hypothesis: $d = 1$ at the 95% significance level.
[b] Best model specification according to the AIC and SIC.
– The model failed to achieve convergence after 240 iterations.

possible specification but the SIC suggests that the ARFIMA$(0, 1.83, 0)$ is a better model. Performing an LR test in order to choose between these two specifications, the latter model seems to be more appropriate. Thus a strong degree of persistence is observed in the behaviour of UK unemployment.

Finally, Table 6 displays the results for the USA. The values of d range now from -0.45 when u_t is ARMA$(3, 2)$ to 1.62 when u_t is white noise. It is observed in this table that the white noise specification for u_t, along with the MA(1) and MA(2) models, are the only cases in which d is higher than 1. In all the remaining specifications d is smaller than 1, and it is even negative in several cases. It is also observed that the null hypothesis of $d = 0$ cannot be rejected in ten models while the unit root is not rejected in seven. The AIC suggests that the ARFIMA$(3, 0.37, 3)$ is the best model specification but the SIC indicates that the ARFIMA$(1, 0.91, 0)$ might be more appropriate. A visual inspection of the residuals, along with the fact that the standard errors are smaller in the former case, suggest that US unemployment may well be described in terms of an ARFIMA$(3, 0.37, 3)$ model. In this case, the null

Table 5. *Maximum likelihood estimation of ARFIMA*(p,d,q) *models for the unemployment in the UK*

				AR parameters			MA parameters			Criterions	
ARMA	d	$t_{d=0}$ [a]	$t_{d=1}$ [a]	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(0, 0)[c]	1.83	20.33	9.22	–	–	–	–	–	–	14.01	12.74
(1, 0)	1.97	11.58	5.70	−0.17	–	–	–	–	–	13.35	10.82
(0, 1)	1.91	10.61	5.05	–	–	–	−0.09	–	–	13.16	10.63
(1, 1)	1.93	12.06	5.81	−0.40	–	–	0.25	–	–	12.51	8.71
(2, 0)[b]	1.29	5.60	1.26[a]	0.45	0.26	–	–	–	–	14.65	10.85
(0, 2)	1.77	9.83	4.27	–	–	–	−0.01	0.19	–	13.43	9.63
(2, 1)	1.31	4.36	1.03[a]	0.47	0.24	–	−0.03	–	–	13.65	8.59
(1, 2)	1.22	3.48	0.64[a]	0.77	–	–	−0.26	0.18	–	14.16	9.09
(2, 2)	1.15	1.69[a]	0.22[a]	1.71	−0.78	–	−1.14	0.36	–	14.13	7.80
(3, 0)	1.35	3.46	0.89[a]	0.39	0.25	0.03	–	–	–	13.66	8.60
(0, 3)	1.78	19.77	8.66	–	–	–	1928	4916	3564	12.43	7.37
(3, 1)	1.37	4.72	1.27[a]	−0.03	0.40	0.16	0.40	–	–	12.78	6.45
(3, 2)	1.03	6.43	0.18[a]	0.42	−0.51	0.73	0.32	1.00	–	14.25	6.65
(1, 3)	1.10	2.68	0.24[a]	0.77	–	–	−1.16	2.76	0.98	13.47	7.14
(2, 3)	1.82	1.00[a]	0.82	1.74	−0.81	–	−1.84	1.07	0.20	13.49	5.90
(3, 3)	–	–	–	–	–	–	–	–	–	–	–

Notes: [a] Nonrejection values of the null hypotheses: $d = 1$ and $d = 0$ at the 95% significance level.
[b] Best model specification according to the AIC.
[c] Best model specification according to the SIC. – The model failed to achieve convergence after 240 iterations.

Table 6. *Maximum likelihood estimation of ARFIMA*(p,d,q) *models for the unemployment in the USA*

				AR parameters			MA parameters			Criterions	
ARMA	d	$t_{d=0}$ [a]	$t_{d=1}$ [a]	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(0, 0)	1.62	14.72	5.63	–	–	–	–	–	–	−8.47	−9.74
(1, 0)[c]	0.91	3.95	−0.39[a]	0.69	–	–	–	–	–	−6.27	−8.80
(0, 1)	1.34	8.93	2.26	–	–	–	0.34	–	–	−7.63	−10.17
(1, 1)	0.64	1.93[a]	−1.09[a]	0.77	–	–	0.20	–	–	−6.85	−10.65
(2, 0)	−0.16	−0.57[a]	−4.14	1.69	−0.73	–	–	–	–	−5.85	−9.65
(0, 2)	1.25	6.94	1.38[a]	–	–	–	0.41	0.09	–	−8.38	−12.18
(2, 1)	0.74	1.42[a]	−0.50[a]	0.42	0.25	–	0.47	–	–	−7.89	−12.95
(1, 2)	0.60	1.62[a]	−1.08[a]	0.78	–	–	0.23	0.03	–	−7.82	−12.89
(2, 2)	0.68	1.94[a]	−0.91[a]	−0.06	0.63	–	1.01	0.13	–	−8.79	−15.12
(3, 0)	–	–	–	–	–	–	–	–	–	–	–
(0, 3)	0.94	5.87	−0.37[a]	–	–	–	0.66	0.52	0.44	−6.66	−11.72
(3, 1)	−0.33	−0.78[a]	−3.16	1.24	0.12	−0.41	0.63	–	–	−7.55	−13.89
(3, 2)	−0.45	−0.83[a]	−2.68	0.99	0.60	−0.60	0.66	1.33	–	−8.53	−16.13
(1, 3)	0.16	0.47[a]	−2.47	0.83	–	–	0.58	0.44	0.38	−6.82	−13.15
(2, 3)	−0.01	−0.02[a]	−2.19	1.07	−0.17	–	0.52	0.43	0.38	−7.69	−15.29
(3, 3)[b]	0.37	1.19[a]	−2.03	0.98	−0.76	0.52	0.27	0.91	0.46	−5.26	−14.12

Notes: [a] Nonrejection values of the null hypotheses: $d = 0$ and $d = 1$ at the 95% significance level.
[b] Best model specification according to the AIC; [c] Best model specification according to the SIC.
– The model failed to achieve convergence after 240 iterations.

hypothesis of $d = 0$ is not rejected though allowing d to be fractional, a stationary long memory process might also appear appropriate.

III. CONCLUSIONS

This article has put forward several different ARFIMA models for modelling the unemployment rate series in the USA and four European countries. Figure 1 summarizes the estimated values of d for each of the different ARMA representations in each country. It is observed in this figure that the highest orders of integration are given in practically all cases in the UK. Only France, in a couple of cases, presents values which are higher than those given in the UK. Italy displays fairly stable behaviour, with all the orders of integration fluctuating around 1. Finally, Germany and the USA give the lowest values with practi-

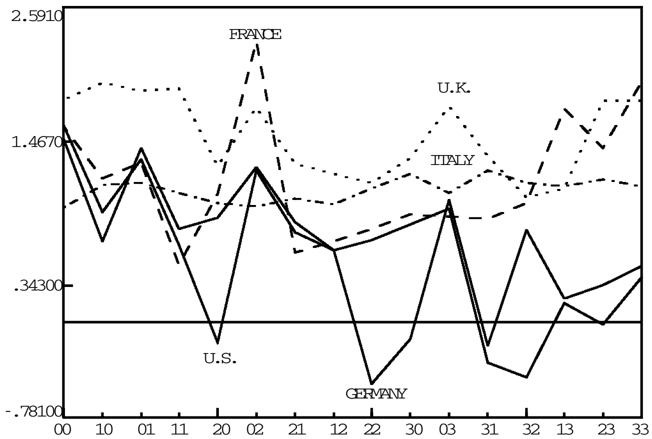

Fig. 1. *Estimates of* d *in each country for each of the different ARMA representations*

cally all of them below 1 and, in some cases, even obtaining negative numbers. These results suggest that the unemployment series are much more persistent in some countries such as the UK and France than in others like Germany or the USA.

In order to choose the best model specification for each country it was decided to follow some likelihood criteria. In particular, the Akaike and Schwarz (AIC and SIC) information criteria were utilized. In those cases where the results between these two criteria were conflicted, a LR test was performed to decide the best possible specification. Table 7 resumes the selected models for each country. It is seen that unemployment in the UK is modelled as an ARFIMA$(0, 1.83, 0)$, and in France as an ARFIMA$(0, 1.32, 1)$. Thus, the order of integration in these two series is much higher than 1, rejecting the null hypothesis of a unit root in both cases. This type of behaviour may be consistent with movements of the unemployment equilibrium from one level to another. Italy is modelled as an ARFIMA$(0, 0.95, 0)$, and thus, it is clearly nonstationary, and though it may present mean-reverting behaviour, the unit root null hypothesis cannot be rejected in this case. Germany is modelled as an ARFIMA$(1, 0.67, 0)$, rejecting both the $I(0)$ and $I(1)$ specifications. The series is therefore nonstationary but mean-reverting. Finally US unemployment is specified as an ARFIMA$(3, 0.37, 3)$ model, which is both stationary and mean-reverting.

The results relating to Germany should be treated with a good deal of caution, following the re-unification in 1990. In order to study this problem in greater depth, the ARFIMA models were estimated separately for the two subsamples 1962:1–1990:4 and 1991:1–1998:3, obtaining as best model specifications the ARFIMA $(1, 0.83, 0)$ in the former and the ARFIMA$(0, 1.39, 0)$ in the latter sample. These results seem to indicate that the persistence of unemployment is higher after the re-unification, though the analysis in this case was carried out with very few observations.

Table 8 displays the impulse response functions for each one of the selected models in Table 7. As was to be expected, explosive behaviour was observed in the most nonstationary series, which correspond to the UK and France. For the remaining countries, the effect of the shocks will tend to disappear in the long run, though this will take very long periods. Thus, for example, we see that for Germany, practically the whole effect of the shock still remains on the series after 50 periods, and for Italy and the USA, 80% and 40% respectively of the effect still remains even 100 periods after the initial shock.

The USA is the only country in which the unemployment series appears stationary. Italy and Germany appear as nonstationary but with a mean-reverting behaviour, and finally, France and the UK are clearly nonstationary and nonmean-reverting, with shocks affecting the series shifting the unemployment equilibrium from one level to another.

This article can be extended in several directions. In particular, given that it is primarily interested in the long-run behaviour of the series, instead of using the ARMA representation for describing the short-run dynamics, the Bloomfield (1973) exponential spectral model could have been employed, which is, in a certain way, a non-parametric approach to modelling the $I(0)$ disturbances of the fractional model Equation 1. Testing the order of integration of the series, either with the ARMA or with the Bloomfield (1973) model for u_t is another approach for describing the long-run properties of unemployment and it can be easily performed using the test statistics proposed

Table 7. *Best model specification for each country according to the AIC and SIC in Tables 2–6*

Country	ARFIMA model (p, d, q)	t–tests[a]		AR parameters			MA parameters		
		$t_{d=0}$	$t_{d=1}$	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3
Germany	(1, 0.67, 0)	5.15	−2.53	0.81	–	–	–	–	–
France	(0, 1.32, 1)	12.00	2.90	–	–	–	0.40	–	–
Italy	(0, 0.95, 0)	10.55	−0.55[a]	–	–	–	–	–	–
UK	(0, 1.83, 0)	20.33	9.22	–	–	–	–	–	–
USA	(3, 0.37, 3)	1.19[a]	−2.03	0.98	−0.76	0.52	0.27	0.91	0.46

Notes: [a] Non-rejection values of the null hypotheses $d = 0$ and $d = 1$ at the 95% significance level.

Table 8. *Impulse response functions of each of the selected ARFIMA* (p, d, q) *models for each country*

	Germany (1, 0.67, 0)	France (0, 1.32, 1)	Italy (0, 0.95, 0)	UK (0, 1.83, 0)	USA (3, 0.37, 3)
0	1.000	1.000	1.000	1.000	1.000
1	1.480	1.720	0.950	1.830	1.620
2	1.758	2.059	0.926	2.589	2.090
3	1.922	2.307	0.910	3.305	2.403
4	2.013	2.507	0.899	3.991	2.232
5	2.057	2.679	0.890	4.654	1.940
6	2.070	2.829	0.883	5.298	1.871
7	2.060	2.965	0.876	5.926	1.888
8	2.037	3.088	0.871	6.541	1.767
9	2.005	3.202	0.866	7.144	1.573
10	1.967	3.307	0.862	7.737	1.460
11	1.926	3.406	0.858	8.321	1.416
12	1.884	3.499	0.854	8.897	1.341
13	1.841	3.588	0.851	9.465	1.230
14	1.799	3.671	0.848	10.026	1.142
15	1.759	3.751	0.845	10.581	1.092
16	1.719	3.828	0.842	11.130	1.043
17	1.682	3.901	0.840	11.673	0.979
18	1.646	3.972	0.837	12.211	0.920
19	1.612	4.039	0.835	12.745	0.879
20	1.581	4.105	0.833	13.274	0.844
21	1.550	4.169	0.831	13.798	0.805
22	1.522	4.230	0.829	14.319	0.766
23	1.495	4.290	0.827	14.836	0.736
24	1.469	4.348	0.826	15.349	0.710
25	1.445	4.404	0.824	15.858	0.684
26	1.423	4.459	0.823	16.365	0.658
27	1.401	4.512	0.821	16.868	0.636
28	1.381	4.564	0.820	17.368	0.617
29	1.362	4.615	0.818	17.865	0.598
30	1.343	4.665	0.817	18.359	0.580
40	1.201	5.110	0.805	23.167	0.458
50	1.106	5.485	0.796	27.777	0.387
60	1.035	5.812	0.789	32.235	0.340
70	0.980	6.104	0.783	36.570	0.305
80	0.935	6.369	0.778	40.801	0.279
90	0.898	6.613	0.773	44.945	0.257
100	0.865	6.839	0.769	49.011	0.240

by Robinson (1994). Finally, semi-parametric methods for estimating and testing the degree of integration of the series might be an additional alternative approach of describing these series.

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