

Stanford University ACM Team Notebook (2014-15)

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Dinic.cc 1/35

```
// Adjacency list implementation of Dinic's blocking flow algorithm.  
// This is very fast in practice, and only loses to push-relabel flow.  
//  
// Running time:  
//       $O(|V|^2 |E|)$   
//  
// INPUT:
```

```

//      - graph, constructed using AddEdge()
//      - source and sink
//
// OUTPUT:
//      - maximum flow value
//      - To obtain actual flow values, look at edges with capacity > 0
//      (zero capacity edges are residual edges).

```

```

#include <iostream>
#include <vector>

```

```

using namespace std;
typedef long long LL;

```

```

struct Edge {
    int from, to, cap, flow, index;
    Edge(int from, int to, int cap, int flow, int index) :
        from(from), to(to), cap(cap), flow(flow), index(index) {}
    LL rcap() { return cap - flow; }
};

```

```

struct Dinic {
    int N;
    vector<vector<Edge> > G;
    vector<vector<Edge *> > Lf;
    vector<int> layer;
    vector<int> Q;

    Dinic(int N) : N(N), G(N), Q(N) {}

    void AddEdge(int from, int to, int cap) {
        if (from == to) return;
        G[from].push_back(Edge(from, to, cap, 0, G[to].size()));
        G[to].push_back(Edge(to, from, 0, 0, G[from].size() - 1));
    }
}

```

```

LL BlockingFlow(int s, int t) {
    layer.clear(); layer.resize(N, -1);
    layer[s] = 0;
    Lf.clear(); Lf.resize(N);

```

```

    int head = 0, tail = 0;
    Q[tail++] = s;
    while (head < tail) {
        int x = Q[head++];

```

```

for (int i = 0; i < G[x].size(); i++) {
    Edge &e = G[x][i]; if (e.rcap() <= 0) continue;
    if (layer[e.to] == -1) {
        layer[e.to] = layer[e.from] + 1;
        Q[tail++] = e.to;
    }
    if (layer[e.to] > layer[e.from]) {
        Lf[e.from].push_back(&e);
    }
}
}
if (layer[t] == -1) return 0;

LL totflow = 0;
vector<Edge *> P;
while (!Lf[s].empty()) {
    int curr = P.empty() ? s : P.back()->to;
    if (curr == t) { // Augment
        LL amt = P.front()->rcap();
        for (int i = 0; i < P.size(); ++i) {
            amt = min(amt, P[i]->rcap());
        }
        totflow += amt;
        for (int i = P.size() - 1; i >= 0; --i) {
            P[i]->flow += amt;
            G[P[i]->to][P[i]->index].flow -= amt;
            if (P[i]->rcap() <= 0) {
                Lf[P[i]->from].pop_back();
                P.resize(i);
            }
        }
    }
    else if (Lf[curr].empty()) { // Retreat
        P.pop_back();
        for (int i = 0; i < N; ++i)
            for (int j = 0; j < Lf[i].size(); ++j)
                if (Lf[i][j]->to == curr)
                    Lf[i].erase(Lf[i].begin() + j);
    }
    else { // Advance
        P.push_back(Lf[curr].back());
    }
}
return totflow;
}

LL GetMaxFlow(int s, int t) {

```

```

LL totflow = 0;
while (LL flow = BlockingFlow(s, t))
    totflow += flow;
return totflow;
}
};

```

MinCostMaxFlow.cc 2/35

```

// Implementation of min cost max flow algorithm using adjacency
// matrix (Edmonds and Karp 1972). This implementation keeps track of
// forward and reverse edges separately (so you can set cap[i][j] !=
// cap[j][i]). For a regular max flow, set all edge costs to 0.
//
// Running time,  $O(|V|^2)$  cost per augmentation
//      max flow:  $O(|V|^3)$  augmentations
//      min cost max flow:  $O(|V|^4 * MAX\_EDGE\_COST)$  augmentations
//
// INPUT:
//      - graph, constructed using AddEdge()
//      - source
//      - sink
//
// OUTPUT:
//      - (maximum flow value, minimum cost value)
//      - To obtain the actual flow, look at positive values only.

```

```

#include <cmath>
#include <vector>
#include <iostream>

```

```

using namespace std;

```

```

typedef vector<int> VI;
typedef vector<VI> VVI;
typedef long long L;
typedef vector<L> VL;
typedef vector<VL> VVL;
typedef pair<int, int> PII;
typedef vector<PII> VPII;

```

```

const L INF = numeric_limits<L>::max() / 4;

struct MinCostMaxFlow {
    int N;
    VVL cap, flow, cost;
    VI found;
    VL dist, pi, width;
    VPII dad;

    MinCostMaxFlow(int N) :
        N(N), cap(N, VL(N)), flow(N, VL(N)), cost(N, VL(N)),
        found(N), dist(N), pi(N), width(N), dad(N) {}

    void AddEdge(int from, int to, L cap, L cost) {
        this->cap[from][to] = cap;
        this->cost[from][to] = cost;
    }

    void Relax(int s, int k, L cap, L cost, int dir) {
        L val = dist[s] + pi[s] - pi[k] + cost;
        if (cap && val < dist[k]) {
            dist[k] = val;
            dad[k] = make_pair(s, dir);
            width[k] = min(cap, width[s]);
        }
    }

    L Dijkstra(int s, int t) {
        fill(found.begin(), found.end(), false);
        fill(dist.begin(), dist.end(), INF);
        fill(width.begin(), width.end(), 0);
        dist[s] = 0;
        width[s] = INF;

        while (s != -1) {
            int best = -1;
            found[s] = true;
            for (int k = 0; k < N; k++) {
                if (found[k]) continue;
                Relax(s, k, cap[s][k] - flow[s][k], cost[s][k], 1);
                Relax(s, k, flow[k][s], -cost[k][s], -1);
                if (best == -1 || dist[k] < dist[best]) best = k;
            }
            s = best;
        }
    }
}

```

```

    for (int k = 0; k < N; k++)
        pi[k] = min(pi[k] + dist[k], INF);
    return width[t];
}

pair<L, L> GetMaxFlow(int s, int t) {
    L totflow = 0, totcost = 0;
    while (L amt = Dijkstra(s, t)) {
        totflow += amt;
        for (int x = t; x != s; x = dad[x].first) {
            if (dad[x].second == 1) {
                flow[dad[x].first][x] += amt;
                totcost += amt * cost[dad[x].first][x];
            } else {
                flow[x][dad[x].first] -= amt;
                totcost -= amt * cost[x][dad[x].first];
            }
        }
    }
    return make_pair(totflow, totcost);
};

```

PushRelabel.cc 3/35

```

// Adjacency list implementation of FIFO push relabel maximum flow
// with the gap relabeling heuristic. This implementation is
// significantly faster than straight Ford-Fulkerson. It solves
// random problems with 10000 vertices and 1000000 edges in a few
// seconds, though it is possible to construct test cases that
// achieve the worst-case.
//
// Running time:
//       $O(|V|^3)$ 
//
// INPUT:
//      - graph, constructed using AddEdge()
//      - source
//      - sink
//

```

```

// OUTPUT:
//      - maximum flow value
//      - To obtain the actual flow values, look at all edges with
//        capacity > 0 (zero capacity edges are residual edges).

#include <cmath>
#include <vector>
#include <iostream>
#include <queue>

using namespace std;

typedef long long LL;

struct Edge {
    int from, to, cap, flow, index;
    Edge(int from, int to, int cap, int flow, int index) :
        from(from), to(to), cap(cap), flow(flow), index(index) {}
};

struct PushRelabel {
    int N;
    vector<vector<Edge> > G;
    vector<LL> excess;
    vector<int> dist, active, count;
    queue<int> Q;

    PushRelabel(int N) : N(N), G(N), excess(N), dist(N), active(N), count(2*N) {}

    void AddEdge(int from, int to, int cap) {
        G[from].push_back(Edge(from, to, cap, 0, G[to].size()));
        if (from == to) G[from].back().index++;
        G[to].push_back(Edge(to, from, 0, 0, G[from].size() - 1));
    }

    void Enqueue(int v) {
        if (!active[v] && excess[v] > 0) { active[v] = true; Q.push(v); }
    }

    void Push(Edge &e) {
        int amt = int(min(excess[e.from], LL(e.cap - e.flow)));
        if (dist[e.from] <= dist[e.to] || amt == 0) return;
        e.flow += amt;
        G[e.to][e.index].flow -= amt;
        excess[e.to] += amt;
    }
};

```



```

    excess[e.from] -= amt;
    Enqueue(e.to);
}

void Gap(int k) {
    for (int v = 0; v < N; v++) {
        if (dist[v] < k) continue;
        count[dist[v]]--;
        dist[v] = max(dist[v], N+1);
        count[dist[v]]++;
        Enqueue(v);
    }
}

void Relabel(int v) {
    count[dist[v]]--;
    dist[v] = 2*N;
    for (int i = 0; i < G[v].size(); i++)
        if (G[v][i].cap - G[v][i].flow > 0)
            dist[v] = min(dist[v], dist[G[v][i].to] + 1);
    count[dist[v]]++;
    Enqueue(v);
}

void Discharge(int v) {
    for (int i = 0; excess[v] > 0 && i < G[v].size(); i++) Push(G[v][i]);
    if (excess[v] > 0) {
        if (count[dist[v]] == 1)
            Gap(dist[v]);
        else
            Relabel(v);
    }
}

LL GetMaxFlow(int s, int t) {
    count[0] = N-1;
    count[N] = 1;
    dist[s] = N;
    active[s] = active[t] = true;
    for (int i = 0; i < G[s].size(); i++) {
        excess[s] += G[s][i].cap;
        Push(G[s][i]);
    }

    while (!Q.empty()) {

```

```

        int v = Q.front();
        Q.pop();
        active[v] = false;
        Discharge(v);
    }

    LL totflow = 0;
    for (int i = 0; i < G[s].size(); i++) totflow += G[s][i].flow;
    return totflow;
}
};

```

MinCostMatching.cc 4/35

```

/////////////////////////////////////////////////////////////////
// Min cost bipartite matching via shortest augmenting paths
//
// This is an O(n^3) implementation of a shortest augmenting path
// algorithm for finding min cost perfect matchings in dense
// graphs. In practice, it solves 1000x1000 problems in around 1
// second.
//
// cost[i][j] = cost for pairing left node i with right node j
// Lmate[i] = index of right node that left node i pairs with
// Rmate[j] = index of left node that right node j pairs with
//
// The values in cost[i][j] may be positive or negative. To perform
// maximization, simply negate the cost[][] matrix.
/////////////////////////////////////////////////////////////////

#include <algorithm>
#include <cstdio>
#include <cmath>
#include <vector>

using namespace std;

typedef vector<double> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

```

```

double MinCostMatching(const VVD &cost, VI &Lmate, VI &Rmate) {
    int n = int(cost.size());

    // construct dual feasible solution
    VD u(n);
    VD v(n);
    for (int i = 0; i < n; i++) {
        u[i] = cost[i][0];
        for (int j = 1; j < n; j++) u[i] = min(u[i], cost[i][j]);
    }
    for (int j = 0; j < n; j++) {
        v[j] = cost[0][j] - u[0];
        for (int i = 1; i < n; i++) v[j] = min(v[j], cost[i][j] - u[i]);
    }

    // construct primal solution satisfying complementary slackness
    Lmate = VI(n, -1);
    Rmate = VI(n, -1);
    int mated = 0;
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            if (Rmate[j] != -1) continue;
            if (fabs(cost[i][j] - u[i] - v[j]) < 1e-10) {
                Lmate[i] = j;
                Rmate[j] = i;
                mated++;
                break;
            }
        }
    }

    VD dist(n);
    VI dad(n);
    VI seen(n);

    // repeat until primal solution is feasible
    while (mated < n) {

        // find an unmatched left node
        int s = 0;
        while (Lmate[s] != -1) s++;

        // initialize Dijkstra
        fill(dad.begin(), dad.end(), -1);
        fill(seen.begin(), seen.end(), 0);
    }
}

```

```

for (int k = 0; k < n; k++)
    dist[k] = cost[s][k] - u[s] - v[k];

int j = 0;
while (true) {

    // find closest
    j = -1;
    for (int k = 0; k < n; k++) {
        if (seen[k]) continue;
        if (j == -1 || dist[k] < dist[j]) j = k;
    }
    seen[j] = 1;

    // termination condition
    if (Rmate[j] == -1) break;

    // relax neighbors
    const int i = Rmate[j];
    for (int k = 0; k < n; k++) {
        if (seen[k]) continue;
        const double new_dist = dist[j] + cost[i][k] - u[i] - v[k];
        if (dist[k] > new_dist) {
            dist[k] = new_dist;
            dad[k] = j;
        }
    }
}

// update dual variables
for (int k = 0; k < n; k++) {
    if (k == j || !seen[k]) continue;
    const int i = Rmate[k];
    v[k] += dist[k] - dist[j];
    u[i] -= dist[k] - dist[j];
}
u[s] += dist[j];

// augment along path
while (dad[j] >= 0) {
    const int d = dad[j];
    Rmate[j] = Rmate[d];
    Lmate[Rmate[j]] = j;
    j = d;
}

```

```

    Rmate[j] = s;
    Lmate[s] = j;

    mated++;
}

double value = 0;
for (int i = 0; i < n; i++)
    value += cost[i][Lmate[i]];

return value;
}

```

MaxBipartiteMatching.cc 5/35

```

// This code performs maximum bipartite matching.
//
// Running time:  $O(|E| |V|)$  -- often much faster in practice
//
// INPUT: w[i][j] = edge between row node i and column node j
// OUTPUT: mr[i] = assignment for row node i, -1 if unassigned
//         mc[j] = assignment for column node j, -1 if unassigned
//         function returns number of matches made

```

```

#include <vector>

```

```

using namespace std;

```

```

typedef vector<int> VI;
typedef vector<VI> VVI;

```

```

bool FindMatch(int i, const VVI &w, VI &mr, VI &mc, VI &seen) {
    for (int j = 0; j < w[i].size(); j++) {
        if (w[i][j] && !seen[j]) {
            seen[j] = true;
            if (mc[j] < 0 || FindMatch(mc[j], w, mr, mc, seen)) {
                mr[i] = j;
                mc[j] = i;
                return true;
            }
        }
    }
}

```

```

    return false;
}

int BipartiteMatching(const VVI &w, VI &mr, VI &mc) {
    mr = VI(w.size(), -1);
    mc = VI(w[0].size(), -1);

    int ct = 0;
    for (int i = 0; i < w.size(); i++) {
        VI seen(w[0].size());
        if (FindMatch(i, w, mr, mc, seen)) ct++;
    }
    return ct;
}

```

MinCut.cc 6/35

```

// Adjacency matrix implementation of Stoer-Wagner min cut algorithm.
//
// Running time:
//      O(|V|^3)
//
// INPUT:
//      - graph, constructed using AddEdge()
//
// OUTPUT:
//      - (min cut value, nodes in half of min cut)

```

```

#include <cmath>
#include <vector>
#include <iostream>

```

```

using namespace std;

```

```

typedef vector<int> VI;
typedef vector<VI> VVI;

```

```

const int INF = 1000000000;

```

```

pair<int, VI> GetMinCut(VVI &weights) {
    int N = weights.size();
    VI used(N), cut, best_cut;

```

```

int best_weight = -1;

for (int phase = N-1; phase >= 0; phase--) {
    VI w = weights[0];
    VI added = used;
    int prev, last = 0;
    for (int i = 0; i < phase; i++) {
        prev = last;
        last = -1;
        for (int j = 1; j < N; j++)
            if (!added[j] && (last == -1 || w[j] > w[last])) last = j;
        if (i == phase-1) {
            for (int j = 0; j < N; j++) weights[prev][j] += weights[last][j];
            for (int j = 0; j < N; j++) weights[j][prev] = weights[prev][j];
            used[last] = true;
            cut.push_back(last);
            if (best_weight == -1 || w[last] < best_weight) {
                best_cut = cut;
                best_weight = w[last];
            }
        } else {
            for (int j = 0; j < N; j++)
                w[j] += weights[last][j];
            added[last] = true;
        }
    }
}
return make_pair(best_weight, best_cut);
}

```

GraphCutInference.cc 7/35

```

// Special-purpose {0,1} combinatorial optimization solver for
// problems of the following by a reduction to graph cuts:
//
//      minimize      sum_i psi_i(x[i])
//      x[1]...x[n] in {0,1}      + sum_{i < j} phi_{ij}(x[i], x[j])
//
// where
//      psi_i : {0, 1} --> R
//      phi_{ij} : {0, 1} x {0, 1} --> R

```

```

//
// such that
//  $\phi_{ij}(0,0) + \phi_{ij}(1,1) \leq \phi_{ij}(0,1) + \phi_{ij}(1,0)$  (*)
//
// This can also be used to solve maximization problems where the
// direction of the inequality in (*) is reversed.
//
// INPUT: phi -- a matrix such that  $\phi[i][j][u][v] = \phi_{ij}(u, v)$ 
//         psi -- a matrix such that  $\psi[i][u] = \psi_i(u)$ 
//         x -- a vector where the optimal solution will be stored
//
// OUTPUT: value of the optimal solution
//
// To use this code, create a GraphCutInference object, and call the
// DoInference() method. To perform maximization instead of minimization,
// ensure that #define MAXIMIZATION is enabled.

```

```

#include <vector>
#include <iostream>

```

```
using namespace std;
```

```

typedef vector<int> VI;
typedef vector<VI> VVI;
typedef vector<VVI> VVVI;
typedef vector<VVVI> VVVVI;

```

```
const int INF = 1000000000;
```

```

// comment out following line for minimization
#define MAXIMIZATION

```

```

struct GraphCutInference {
    int N;
    VVI cap, flow;
    VI reached;

    int Augment(int s, int t, int a) {
        reached[s] = 1;
        if (s == t) return a;
        for (int k = 0; k < N; k++) {
            if (reached[k]) continue;
            if (int aa = min(a, cap[s][k] - flow[s][k])) {
                if (int b = Augment(k, t, aa)) {
                    flow[s][k] += b;

```



```

        flow[k][s] -= b;
        return b;
    }
}
}
return 0;
}

int GetMaxFlow(int s, int t) {
    N = cap.size();
    flow = VVI(N, VI(N));
    reached = VI(N);

    int totflow = 0;
    while (int amt = Augment(s, t, INF)) {
        totflow += amt;
        fill(reached.begin(), reached.end(), 0);
    }
    return totflow;
}

int DoInference(const VVVVI &phi, const VVI &psi, VI &x) {
    int M = phi.size();
    cap = VVI(M+2, VI(M+2));
    VI b(M);
    int c = 0;

    for (int i = 0; i < M; i++) {
        b[i] += psi[i][1] - psi[i][0];
        c += psi[i][0];
        for (int j = 0; j < i; j++)
            b[i] += phi[i][j][1][1] - phi[i][j][0][1];
        for (int j = i+1; j < M; j++) {
            cap[i][j] = phi[i][j][0][1] + phi[i][j][1][0] - phi[i][j][0][0] - phi[i][j][1][1];
            b[i] += phi[i][j][1][0] - phi[i][j][0][0];
            c += phi[i][j][0][0];
        }
    }
}

#ifdef MAXIMIZATION
    for (int i = 0; i < M; i++) {
        for (int j = i+1; j < M; j++)
            cap[i][j] *= -1;
        b[i] *= -1;
    }
}

```

```

    c *= -1;
#endif

    for (int i = 0; i < M; i++) {
        if (b[i] >= 0) {
            cap[M][i] = b[i];
        } else {
            cap[i][M+1] = -b[i];
            c += b[i];
        }
    }

    int score = GetMaxFlow(M, M+1);
    fill(reached.begin(), reached.end(), 0);
    Augment(M, M+1, INF);
    x = VI(M);
    for (int i = 0; i < M; i++) x[i] = reached[i] ? 0 : 1;
    score += c;
#ifdef MAXIMIZATION
    score *= -1;
#endif

    return score;
}

};

int main() {

    // solver for "Cat vs. Dog" from NWERC 2008

    int numcases;
    cin >> numcases;
    for (int caseno = 0; caseno < numcases; caseno++) {
        int c, d, v;
        cin >> c >> d >> v;

        VVVVI phi(c+d, VVVI(c+d, VVI(2, VI(2))));
        VVI psi(c+d, VI(2));
        for (int i = 0; i < v; i++) {
            char p, q;
            int u, v;
            cin >> p >> u >> q >> v;
            u--; v--;
            if (p == 'c') {

```

```

        phi[u][c+v][0][0]++;
        phi[c+v][u][0][0]++;
    } else {
        phi[v][c+u][1][1]++;
        phi[c+u][v][1][1]++;
    }
}

GraphCutInference graph;
VI x;
cout << graph.DoInference(phi, psi, x) << endl;
}

return 0;
}

```

ConvexHull.cc 8/35

```

// Compute the 2D convex hull of a set of points using the monotone chain
// algorithm. Eliminate redundant points from the hull if REMOVE_REDUNDANT is
// #defined.
//
// Running time:  $O(n \log n)$ 
//
// INPUT:  a vector of input points, unordered.
// OUTPUT: a vector of points in the convex hull, counterclockwise, starting
//         with bottommost/leftmost point

```

```

#include <cstdio>
#include <cassert>
#include <vector>
#include <algorithm>
#include <cmath>

```

```
using namespace std;
```

```
#define REMOVE_REDUNDANT
```

```

typedef double T;
const T EPS = 1e-7;
struct PT {
    T x, y;

```

```

PT() {}
PT(T x, T y) : x(x), y(y) {}
bool operator<(const PT &rhs) const { return make_pair(y,x) < make_pair(rhs.y,rhs.x); }
bool operator==(const PT &rhs) const { return make_pair(y,x) == make_pair(rhs.y,rhs.x); }
};

T cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
T area2(PT a, PT b, PT c) { return cross(a,b) + cross(b,c) + cross(c,a); }

#ifdef REMOVE_REDUNDANT
bool between(const PT &a, const PT &b, const PT &c) {
    return (fabs(area2(a,b,c)) < EPS && (a.x-b.x)*(c.x-b.x) <= 0 && (a.y-b.y)*(c.y-b.y) <= 0);
}
#endif

void ConvexHull(vector<PT> &pts) {
    sort(pts.begin(), pts.end());
    pts.erase(unique(pts.begin(), pts.end()), pts.end());
    vector<PT> up, dn;
    for (int i = 0; i < pts.size(); i++) {
        while (up.size() > 1 && area2(up[up.size()-2], up.back(), pts[i]) >= 0) up.pop_back();
        while (dn.size() > 1 && area2(dn[dn.size()-2], dn.back(), pts[i]) <= 0) dn.pop_back();
        up.push_back(pts[i]);
        dn.push_back(pts[i]);
    }
    pts = dn;
    for (int i = (int) up.size() - 2; i >= 1; i--) pts.push_back(up[i]);

#ifdef REMOVE_REDUNDANT
    if (pts.size() <= 2) return;
    dn.clear();
    dn.push_back(pts[0]);
    dn.push_back(pts[1]);
    for (int i = 2; i < pts.size(); i++) {
        if (between(dn[dn.size()-2], dn[dn.size()-1], pts[i])) dn.pop_back();
        dn.push_back(pts[i]);
    }
    if (dn.size() >= 3 && between(dn.back(), dn[0], dn[1])) {
        dn[0] = dn.back();
        dn.pop_back();
    }
    pts = dn;
#endif
}

```

Geometry.cc 9/35

```
// C++ routines for computational geometry.
```

```
#include <iostream>
#include <vector>
#include <cmath>
#include <cassert>
```

```
using namespace std;
```

```
double INF = 1e100;
double EPS = 1e-12;
```

```
struct PT {
    double x, y;
    PT() {}
    PT(double x, double y) : x(x), y(y) {}
    PT(const PT &p) : x(p.x), y(p.y) {}
    PT operator + (const PT &p) const { return PT(x+p.x, y+p.y); }
    PT operator - (const PT &p) const { return PT(x-p.x, y-p.y); }
    PT operator * (double c) const { return PT(x*c, y*c); }
    PT operator / (double c) const { return PT(x/c, y/c); }
};
```

```
double dot(PT p, PT q) { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q) { return dot(p-q,p-q); }
double cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
    os << "(" << p.x << ", " << p.y << ")";
}
```

```
// rotate a point CCW or CW around the origin
```

```
PT RotateCCW90(PT p) { return PT(-p.y,p.x); }
PT RotateCW90(PT p) { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
    return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}
```

```
// project point c onto line through a and b
// assuming a != b
```

```
PT ProjectPointLine(PT a, PT b, PT c) {
```

```

    return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}

// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
    double r = dot(b-a, b-a);
    if (fabs(r) < EPS) return a;
    r = dot(c-a, b-a)/r;
    if (r < 0) return a;
    if (r > 1) return b;
    return a + (b-a)*r;
}

// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
    return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
}

// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane(double x, double y, double z,
                           double a, double b, double c, double d)
{
    return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}

// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
    return fabs(cross(b-a, c-d)) < EPS;
}

bool LinesCollinear(PT a, PT b, PT c, PT d) {
    return LinesParallel(a, b, c, d)
        && fabs(cross(a-b, a-c)) < EPS
        && fabs(cross(c-d, c-a)) < EPS;
}

// determine if line segment from a to b intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
    if (LinesCollinear(a, b, c, d)) {
        if (dist2(a, c) < EPS || dist2(a, d) < EPS ||
            dist2(b, c) < EPS || dist2(b, d) < EPS) return true;
        if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 && dot(c-b, d-b) > 0)
            return false;
        return true;
    }

```

```

}
if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
return true;
}

// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
// intersection exists; for segment intersection, check if
// segments intersect first
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
    b=b-a; d=c-d; c=c-a;
    assert(dot(b, b) > EPS && dot(d, d) > EPS);
    return a + b*cross(c, d)/cross(b, d);
}

// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
    b=(a+b)/2;
    c=(a+c)/2;
    return ComputeLineIntersection(b, b+RotateCW90(a-b), c, c+RotateCW90(a-c));
}

// determine if point is in a possibly non-convex polygon (by William
// Randolph Franklin); returns 1 for strictly interior points, 0 for
// strictly exterior points, and 0 or 1 for the remaining points.
// Note that it is possible to convert this into an *exact* test using
// integer arithmetic by taking care of the division appropriately
// (making sure to deal with signs properly) and then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
    bool c = 0;
    for (int i = 0; i < p.size(); i++){
        int j = (i+1)%p.size();
        if ((p[i].y <= q.y && q.y < p[j].y ||
            p[j].y <= q.y && q.y < p[i].y) &&
            q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y - p[i].y))
            c = !c;
    }
    return c;
}

// determine if point is on the boundary of a polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {
    for (int i = 0; i < p.size(); i++)

```

```

    if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) < EPS)
        return true;
    return false;
}

```

*// compute intersection of line through points a and b with
// circle centered at c with radius r > 0*

```

vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
    vector<PT> ret;
    b = b-a;
    a = a-c;
    double A = dot(b, b);
    double B = dot(a, b);
    double C = dot(a, a) - r*r;
    double D = B*B - A*C;
    if (D < -EPS) return ret;
    ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
    if (D > EPS)
        ret.push_back(c+a+b*(-B-sqrt(D))/A);
    return ret;
}

```

*// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R*

```

vector<PT> CircleCircleIntersection(PT a, PT b, double r, double R) {
    vector<PT> ret;
    double d = sqrt(dist2(a, b));
    if (d > r+R || d+min(r, R) < max(r, R)) return ret;
    double x = (d*d-R*R+r*r)/(2*d);
    double y = sqrt(r*r-x*x);
    PT v = (b-a)/d;
    ret.push_back(a+v*x + RotateCCW90(v)*y);
    if (y > 0)
        ret.push_back(a+v*x - RotateCCW90(v)*y);
    return ret;
}

```

*// This code computes the area or centroid of a (possibly nonconvex)
// polygon, assuming that the coordinates are listed in a clockwise or
// counterclockwise fashion. Note that the centroid is often known as
// the "center of gravity" or "center of mass".*

```

double ComputeSignedArea(const vector<PT> &p) {
    double area = 0;
    for(int i = 0; i < p.size(); i++) {
        int j = (i+1) % p.size();

```



```

        area += p[i].x*p[j].y - p[j].x*p[i].y;
    }
    return area / 2.0;
}

double ComputeArea(const vector<PT> &p) {
    return fabs(ComputeSignedArea(p));
}

PT ComputeCentroid(const vector<PT> &p) {
    PT c(0,0);
    double scale = 6.0 * ComputeSignedArea(p);
    for (int i = 0; i < p.size(); i++){
        int j = (i+1) % p.size();
        c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
    }
    return c / scale;
}

// tests whether or not a given polygon (in CW or CCW order) is simple
bool IsSimple(const vector<PT> &p) {
    for (int i = 0; i < p.size(); i++) {
        for (int k = i+1; k < p.size(); k++) {
            int j = (i+1) % p.size();
            int l = (k+1) % p.size();
            if (i == l || j == k) continue;
            if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
                return false;
        }
    }
    return true;
}

int main() {

    // expected: (-5,2)
    cerr << RotateCCW90(PT(2,5)) << endl;

    // expected: (5,-2)
    cerr << RotateCW90(PT(2,5)) << endl;

    // expected: (-5,2)
    cerr << RotateCCW(PT(2,5),M_PI/2) << endl;

    // expected: (5,2)

```

```

cerr << ProjectPointLine(PT(-5,-2), PT(10,4), PT(3,7)) << endl;

// expected: (5,2) (7.5,3) (2.5,1)
cerr << ProjectPointSegment(PT(-5,-2), PT(10,4), PT(3,7)) << " "
      << ProjectPointSegment(PT(7.5,3), PT(10,4), PT(3,7)) << " "
      << ProjectPointSegment(PT(-5,-2), PT(2.5,1), PT(3,7)) << endl;

// expected: 6.78903
cerr << DistancePointPlane(4,-4,3,2,-2,5,-8) << endl;

// expected: 1 0 1
cerr << LinesParallel(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
      << LinesParallel(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
      << LinesParallel(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;

// expected: 0 0 1
cerr << LinesCollinear(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
      << LinesCollinear(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
      << LinesCollinear(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;

// expected: 1 1 1 0
cerr << SegmentsIntersect(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << " "
      << SegmentsIntersect(PT(0,0), PT(2,4), PT(4,3), PT(0,5)) << " "
      << SegmentsIntersect(PT(0,0), PT(2,4), PT(2,-1), PT(-2,1)) << " "
      << SegmentsIntersect(PT(0,0), PT(2,4), PT(5,5), PT(1,7)) << endl;

// expected: (1,2)
cerr << ComputeLineIntersection(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << endl;

// expected: (1,1)
cerr << ComputeCircleCenter(PT(-3,4), PT(6,1), PT(4,5)) << endl;

vector<PT> v;
v.push_back(PT(0,0));
v.push_back(PT(5,0));
v.push_back(PT(5,5));
v.push_back(PT(0,5));

// expected: 1 1 1 0 0
cerr << PointInPolygon(v, PT(2,2)) << " "
      << PointInPolygon(v, PT(2,0)) << " "
      << PointInPolygon(v, PT(0,2)) << " "
      << PointInPolygon(v, PT(5,2)) << " "
      << PointInPolygon(v, PT(2,5)) << endl;

```

```

// expected: 0 1 1 1 1
cerr << PointOnPolygon(v, PT(2,2)) << " "
      << PointOnPolygon(v, PT(2,0)) << " "
      << PointOnPolygon(v, PT(0,2)) << " "
      << PointOnPolygon(v, PT(5,2)) << " "
      << PointOnPolygon(v, PT(2,5)) << endl;

// expected: (1,6)
//           (5,4) (4,5)
//           blank line
//           (4,5) (5,4)
//           blank line
//           (4,5) (5,4)
vector<PT> u = CircleLineIntersection(PT(0,6), PT(2,6), PT(1,1), 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
u = CircleLineIntersection(PT(0,9), PT(9,0), PT(1,1), 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
u = CircleCircleIntersection(PT(1,1), PT(10,10), 5, 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
u = CircleCircleIntersection(PT(1,1), PT(8,8), 5, 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 10, sqrt(2.0)/2.0);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 5, sqrt(2.0)/2.0);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;

// area should be 5.0
// centroid should be (1.1666666, 1.166666)
PT pa[] = { PT(0,0), PT(5,0), PT(1,1), PT(0,5) };
vector<PT> p(pa, pa+4);
PT c = ComputeCentroid(p);
cerr << "Area: " << ComputeArea(p) << endl;
cerr << "Centroid: " << c << endl;

return 0;
}

```

JavaGeometry.java 10/35

```

// In this example, we read an input file containing three lines, each
// containing an even number of doubles, separated by commas. The first two
// lines represent the coordinates of two polygons, given in counterclockwise

```

```
// (or clockwise) order, which we will call "A" and "B". The last line
// contains a list of points, p[1], p[2], ...
//
// Our goal is to determine:
// (1) whether B - A is a single closed shape (as opposed to multiple shapes)
// (2) the area of B - A
// (3) whether each p[i] is in the interior of B - A
//
// INPUT:
// 0 0 10 0 0 10
// 0 0 10 10 10 0
// 8 6
// 5 1
//
// OUTPUT:
// The area is singular.
// The area is 25.0
// Point belongs to the area.
// Point does not belong to the area.
```

```
import java.util.*;
import java.awt.geom.*;
import java.io.*;

public class JavaGeometry {

    // make an array of doubles from a string
    static double[] readPoints(String s) {
        String[] arr = s.trim().split("\\s+");
        double[] ret = new double[arr.length];
        for (int i = 0; i < arr.length; i++) ret[i] = Double.parseDouble(arr[i]);
        return ret;
    }

    // make an Area object from the coordinates of a polygon
    static Area makeArea(double[] pts) {
        Path2D.Double p = new Path2D.Double();
        p.moveTo(pts[0], pts[1]);
        for (int i = 2; i < pts.length; i += 2) p.lineTo(pts[i], pts[i+1]);
        p.closePath();
        return new Area(p);
    }

    // compute area of polygon
    static double computePolygonArea(ArrayList<Point2D.Double> points) {
```

```

Point2D.Double[] pts = points.toArray(new Point2D.Double[points.size()]);
double area = 0;
for (int i = 0; i < pts.length; i++){
    int j = (i+1) % pts.length;
    area += pts[i].x * pts[j].y - pts[j].x * pts[i].y;
}
return Math.abs(area)/2;
}

```

// compute the area of an Area object containing several disjoint polygons

```

static double computeArea(Area area) {
    double totArea = 0;
    PathIterator iter = area.getPathIterator(null);
    ArrayList<Point2D.Double> points = new ArrayList<Point2D.Double>();

    while (!iter.isDone()) {
        double[] buffer = new double[6];
        switch (iter.currentSegment(buffer)) {
            case PathIterator.SEG_MOVETO:
            case PathIterator.SEG_LINETO:
                points.add(new Point2D.Double(buffer[0], buffer[1]));
                break;
            case PathIterator.SEG_CLOSE:
                totArea += computePolygonArea(points);
                points.clear();
                break;
        }
        iter.next();
    }
    return totArea;
}

```

*// notice that the main() throws an Exception -- necessary to
 // avoid wrapping the Scanner object for file reading in a
 // try { ... } catch block.*

```

public static void main(String args[]) throws Exception {

    Scanner scanner = new Scanner(new File("input.txt"));
    // also,
    // Scanner scanner = new Scanner (System.in);

    double[] pointsA = readPoints(scanner.nextLine());
    double[] pointsB = readPoints(scanner.nextLine());
    Area areaA = makeArea(pointsA);
    Area areaB = makeArea(pointsB);
}

```

```

areaB.subtract(areaA);
// also,
//   areaB.exclusiveOr (areaA);
//   areaB.add (areaA);
//   areaB.intersect (areaA);

// (1) determine whether B - A is a single closed shape (as
//      opposed to multiple shapes)
boolean isSingle = areaB.isSingular();
// also,
//   areaB.isEmpty();

if (isSingle)
    System.out.println("The area is singular.");
else
    System.out.println("The area is not singular.");

// (2) compute the area of B - A
System.out.println("The area is " + computeArea(areaB) + ".");

// (3) determine whether each p[i] is in the interior of B - A
while (scanner.hasNextDouble()) {
    double x = scanner.nextDouble();
    assert(scanner.hasNextDouble());
    double y = scanner.nextDouble();

    if (areaB.contains(x,y)) {
        System.out.println ("Point belongs to the area.");
    } else {
        System.out.println ("Point does not belong to the area.");
    }
}

// Finally, some useful things we didn't use in this example:
//
//   Ellipse2D.Double ellipse = new Ellipse2D.Double (double x, double y,
//                                                    double w, double h);
//
//   creates an ellipse inscribed in box with bottom-left corner (x,y)
//   and upper-right corner (x+y,w+h)
//
//   Rectangle2D.Double rect = new Rectangle2D.Double (double x, double y,
//                                                      double w, double h);
//
//   creates a box with bottom-left corner (x,y) and upper-right

```

```

    //      corner (x+y,w+h)
    //
    // Each of these can be embedded in an Area object (e.g., new Area (rect)).
}
}

```

Geom3D.java 11/35

```

public class Geom3D {
    // distance from point (x, y, z) to plane  $aX + bY + cZ + d = 0$ 
    public static double ptPlaneDist(double x, double y, double z,
        double a, double b, double c, double d) {
        return Math.abs(a*x + b*y + c*z + d) / Math.sqrt(a*a + b*b + c*c);
    }

    // distance between parallel planes  $aX + bY + cZ + d1 = 0$  and
    //  $aX + bY + cZ + d2 = 0$ 
    public static double planePlaneDist(double a, double b, double c,
        double d1, double d2) {
        return Math.abs(d1 - d2) / Math.sqrt(a*a + b*b + c*c);
    }

    // distance from point (px, py, pz) to line (x1, y1, z1)-(x2, y2, z2)
    // (or ray, or segment; in the case of the ray, the endpoint is the
    // first point)
    public static final int LINE = 0;
    public static final int SEGMENT = 1;
    public static final int RAY = 2;
    public static double ptLineDistSq(double x1, double y1, double z1,
        double x2, double y2, double z2, double px, double py, double pz,
        int type) {
        double pd2 = (x1-x2)*(x1-x2) + (y1-y2)*(y1-y2) + (z1-z2)*(z1-z2);

        double x, y, z;
        if (pd2 == 0) {
            x = x1;
            y = y1;
            z = z1;
        } else {
            double u = ((px-x1)*(x2-x1) + (py-y1)*(y2-y1) + (pz-z1)*(z2-z1)) / pd2;
            x = x1 + u * (x2 - x1);

```

```

y = y1 + u * (y2 - y1);
z = z1 + u * (z2 - z1);
if (type != LINE && u < 0) {
    x = x1;
    y = y1;
    z = z1;
}
if (type == SEGMENT && u > 1.0) {
    x = x2;
    y = y2;
    z = z2;
}
}

return (x-px)*(x-px) + (y-py)*(y-py) + (z-pz)*(z-pz);
}

public static double ptLineDist(double x1, double y1, double z1,
    double x2, double y2, double z2, double px, double py, double pz,
    int type) {
    return Math.sqrt(ptLineDistSq(x1, y1, z1, x2, y2, z2, px, py, pz, type));
}
}

```

Delaunay.cc 12/35

```

// Slow but simple Delaunay triangulation. Does not handle
// degenerate cases (from O'Rourke, Computational Geometry in C)
//
// Running time: O(n^4)
//
// INPUT:      x[] = x-coordinates
//             y[] = y-coordinates
//
// OUTPUT:     triples = a vector containing m triples of indices
//                  corresponding to triangle vertices

```

```

#include<vector>
using namespace std;

```

```

typedef double T;

```



```

struct triple {
    int i, j, k;
    triple() {}
    triple(int i, int j, int k) : i(i), j(j), k(k) {}
};

vector<triple> delaunayTriangulation(vector<T>& x, vector<T>& y) {
    int n = x.size();
    vector<T> z(n);
    vector<triple> ret;

    for (int i = 0; i < n; i++)
        z[i] = x[i] * x[i] + y[i] * y[i];

    for (int i = 0; i < n-2; i++) {
        for (int j = i+1; j < n; j++) {
            for (int k = i+1; k < n; k++) {
                if (j == k) continue;
                double xn = (y[j]-y[i])*(z[k]-z[i]) - (y[k]-y[i])*(z[j]-z[i]);
                double yn = (x[k]-x[i])*(z[j]-z[i]) - (x[j]-x[i])*(z[k]-z[i]);
                double zn = (x[j]-x[i])*(y[k]-y[i]) - (x[k]-x[i])*(y[j]-y[i]);
                bool flag = zn < 0;
                for (int m = 0; flag && m < n; m++)
                    flag = flag && ((x[m]-x[i])*xn +
                                     (y[m]-y[i])*yn +
                                     (z[m]-z[i])*zn <= 0);
                if (flag) ret.push_back(triple(i, j, k));
            }
        }
    }
    return ret;
}

int main()
{
    T xs[]={0, 0, 1, 0.9};
    T ys[]={0, 1, 0, 0.9};
    vector<T> x(&xs[0], &xs[4]), y(&ys[0], &ys[4]);
    vector<triple> tri = delaunayTriangulation(x, y);

    //expected: 0 1 3
    //           0 3 2

    int i;
    for(i = 0; i < tri.size(); i++)

```

```
        printf("%d %d %d\n", tri[i].i, tri[i].j, tri[i].k);
    return 0;
}
```

Euclid.cc 13/35

*// This is a collection of useful code for solving problems that
// involve modular linear equations. Note that all of the
// algorithms described here work on nonnegative integers.*

```
#include <iostream>
#include <vector>
#include <algorithm>
```

```
using namespace std;
```

```
typedef vector<int> VI;
typedef pair<int,int> PII;
```

// return a % b (positive value)

```
int mod(int a, int b) {
    return ((a%b)+b)%b;
}
```

// computes gcd(a,b)

```
int gcd(int a, int b) {
    int tmp;
    while(b){a%=b; tmp=a; a=b; b=tmp;}
    return a;
}
```

// computes lcm(a,b)

```
int lcm(int a, int b) {
    return a/gcd(a,b)*b;
}
```

// returns d = gcd(a,b); finds x,y such that d = ax + by

```
int extended_euclid(int a, int b, int &x, int &y) {
    int xx = y = 0;
    int yy = x = 1;
    while (b) {
        int q = a/b;
```

```

    int t = b; b = a%b; a = t;
    t = xx; xx = x-q*xx; x = t;
    t = yy; yy = y-q*yy; y = t;
}
return a;
}

// finds all solutions to ax = b (mod n)
VI modular_linear_equation_solver(int a, int b, int n) {
    int x, y;
    VI solutions;
    int d = extended_euclid(a, n, x, y);
    if (!(b%d)) {
        x = mod (x*(b/d), n);
        for (int i = 0; i < d; i++)
            solutions.push_back(mod(x + i*(n/d), n));
    }
    return solutions;
}

// computes b such that ab = 1 (mod n), returns -1 on failure
int mod_inverse(int a, int n) {
    int x, y;
    int d = extended_euclid(a, n, x, y);
    if (d > 1) return -1;
    return mod(x,n);
}

// Chinese remainder theorem (special case): find z such that
// z % x = a, z % y = b. Here, z is unique modulo M = lcm(x,y).
// Return (z,M). On failure, M = -1.
PII chinese_remainder_theorem(int x, int a, int y, int b) {
    int s, t;
    int d = extended_euclid(x, y, s, t);
    if (a%d != b%d) return make_pair(0, -1);
    return make_pair(mod(s*b*x+t*a*y,x*y)/d, x*y/d);
}

// Chinese remainder theorem: find z such that
// z % x[i] = a[i] for all i. Note that the solution is
// unique modulo M = lcm_i (x[i]). Return (z,M). On
// failure, M = -1. Note that we do not require the a[i]'s
// to be relatively prime.
PII chinese_remainder_theorem(const VI &x, const VI &a) {
    PII ret = make_pair(a[0], x[0]);

```

```

for (int i = 1; i < x.size(); i++) {
    ret = chinese_remainder_theorem(ret.second, ret.first, x[i], a[i]);
    if (ret.second == -1) break;
}
return ret;
}

// computes x and y such that ax + by = c; on failure, x = y = -1
void linear_diophantine(int a, int b, int c, int &x, int &y) {
    int d = gcd(a,b);
    if (c%d) {
        x = y = -1;
    } else {
        x = c/d * mod_inverse(a/d, b/d);
        y = (c-a*x)/b;
    }
}

int main() {

    // expected: 2
    cout << gcd(14, 30) << endl;

    // expected: 2 -2 1
    int x, y;
    int d = extended_euclid(14, 30, x, y);
    cout << d << " " << x << " " << y << endl;

    // expected: 95 45
    VI sols = modular_linear_equation_solver(14, 30, 100);
    for (int i = 0; i < (int) sols.size(); i++) cout << sols[i] << " ";
    cout << endl;

    // expected: 8
    cout << mod_inverse(8, 9) << endl;

    // expected: 23 56
    //           11 12
    int xs[] = {3, 5, 7, 4, 6};
    int as[] = {2, 3, 2, 3, 5};
    PII ret = chinese_remainder_theorem(VI (xs, xs+3), VI(as, as+3));
    cout << ret.first << " " << ret.second << endl;
    ret = chinese_remainder_theorem (VI(xs+3, xs+5), VI(as+3, as+5));
    cout << ret.first << " " << ret.second << endl;
}

```

```

// expected: 5 -15
linear_diophantine(7, 2, 5, x, y);
cout << x << " " << y << endl;
}

```

GaussJordan.cc 14/35

```

// Gauss-Jordan elimination with full pivoting.
//
// Uses:
//   (1) solving systems of linear equations (AX=B)
//   (2) inverting matrices (AX=I)
//   (3) computing determinants of square matrices
//
// Running time: O(n^3)
//
// INPUT:   a[][] = an nxn matrix
//          b[][] = an nxm matrix
//
// OUTPUT:  X      = an nxm matrix (stored in b[][])
//          A^{-1} = an nxn matrix (stored in a[][])
//          returns determinant of a[][]

#include <iostream>
#include <vector>
#include <cmath>

using namespace std;

const double EPS = 1e-10;

typedef vector<int> VI;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

T GaussJordan(VVT &a, VVT &b) {
    const int n = a.size();
    const int m = b[0].size();
    VI irow(n), icol(n), ipiv(n);
    T det = 1;

```

```

for (int i = 0; i < n; i++) {
    int pj = -1, pk = -1;
    for (int j = 0; j < n; j++) if (!ipiv[j])
        for (int k = 0; k < n; k++) if (!ipiv[k])
            if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk = k; }
    if (fabs(a[pj][pk]) < EPS) { cerr << "Matrix is singular." << endl; exit(0); }
    ipiv[pk]++;
    swap(a[pj], a[pk]);
    swap(b[pj], b[pk]);
    if (pj != pk) det *= -1;
    irow[i] = pj;
    icol[i] = pk;

    T c = 1.0 / a[pk][pk];
    det *= a[pk][pk];
    a[pk][pk] = 1.0;
    for (int p = 0; p < n; p++) a[pk][p] *= c;
    for (int p = 0; p < m; p++) b[pk][p] *= c;
    for (int p = 0; p < n; p++) if (p != pk) {
        c = a[p][pk];
        a[p][pk] = 0;
        for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
        for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
    }
}

for (int p = n-1; p >= 0; p--) if (irow[p] != icol[p]) {
    for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
}

return det;
}

int main() {
    const int n = 4;
    const int m = 2;
    double A[n][n] = { {1,2,3,4}, {1,0,1,0}, {5,3,2,4}, {6,1,4,6} };
    double B[n][m] = { {1,2}, {4,3}, {5,6}, {8,7} };
    VVT a(n), b(n);
    for (int i = 0; i < n; i++) {
        a[i] = VT(A[i], A[i] + n);
        b[i] = VT(B[i], B[i] + m);
    }
}

```

```

double det = GaussJordan(a, b);

// expected: 60
cout << "Determinant: " << det << endl;

// expected: -0.233333 0.166667 0.133333 0.0666667
//           0.166667 0.166667 0.333333 -0.333333
//           0.233333 0.833333 -0.133333 -0.0666667
//           0.05 -0.75 -0.1 0.2
cout << "Inverse: " << endl;
for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++)
        cout << a[i][j] << ' ';
    cout << endl;
}

// expected: 1.63333 1.3
//           -0.166667 0.5
//           2.36667 1.7
//           -1.85 -1.35
cout << "Solution: " << endl;
for (int i = 0; i < n; i++) {
    for (int j = 0; j < m; j++)
        cout << b[i][j] << ' ';
    cout << endl;
}
}

```

ReducedRowEchelonForm.cc 15/35

```

// Reduced row echelon form via Gauss-Jordan elimination
// with partial pivoting. This can be used for computing
// the rank of a matrix.
//
// Running time:  $O(n^3)$ 
//
// INPUT:    a[][] = an nxm matrix
//
// OUTPUT:   rref[][] = an nxm matrix (stored in a[][])
//           returns rank of a[][]

```

```

#include <iostream>

```

```

#include <vector>
#include <cmath>

using namespace std;

const double EPSILON = 1e-10;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

int rref(VVT &a) {
    int n = a.size();
    int m = a[0].size();
    int r = 0;
    for (int c = 0; c < m && r < n; c++) {
        int j = r;
        for (int i = r + 1; i < n; i++)
            if (fabs(a[i][c]) > fabs(a[j][c])) j = i;
        if (fabs(a[j][c]) < EPSILON) continue;
        swap(a[j], a[r]);

        T s = 1.0 / a[r][c];
        for (int j = 0; j < m; j++) a[r][j] *= s;
        for (int i = 0; i < n; i++) if (i != r) {
            T t = a[i][c];
            for (int j = 0; j < m; j++) a[i][j] -= t * a[r][j];
        }
        r++;
    }
    return r;
}

int main() {
    const int n = 5, m = 4;
    double A[n][m] = {
        {16, 2, 3, 13},
        { 5, 11, 10, 8},
        { 9, 7, 6, 12},
        { 4, 14, 15, 1},
        {13, 21, 21, 13}};
    VVT a(n);
    for (int i = 0; i < n; i++)
        a[i] = VT(A[i], A[i] + m);
}

```



```

int rank = rref(a);

// expected: 3
cout << "Rank: " << rank << endl;

// expected: 1 0 0 1
//           0 1 0 3
//           0 0 1 -3
//           0 0 0 3.10862e-15
//           0 0 0 2.22045e-15
cout << "rref: " << endl;
for (int i = 0; i < 5; i++) {
    for (int j = 0; j < 4; j++)
        cout << a[i][j] << ' ';
    cout << endl;
}
}

```

FFT_new.cpp 16/35

```

#include <cassert>
#include <cstdio>
#include <cmath>

struct cpx
{
    cpx(){}
    cpx(double aa):a(aa),b(0){}
    cpx(double aa, double bb):a(aa),b(bb){}
    double a;
    double b;
    double modsq(void) const
    {
        return a * a + b * b;
    }
    cpx bar(void) const
    {
        return cpx(a, -b);
    }
};

cpx operator +(cpx a, cpx b)

```

```

{
    return cpx(a.a + b.a, a.b + b.b);
}

cpx operator *(cpx a, cpx b)
{
    return cpx(a.a * b.a - a.b * b.b, a.a * b.b + a.b * b.a);
}

cpx operator /(cpx a, cpx b)
{
    cpx r = a * b.bar();
    return cpx(r.a / b.modsq(), r.b / b.modsq());
}

cpx EXP(double theta)
{
    return cpx(cos(theta), sin(theta));
}

const double two_pi = 4 * acos(0);

// in:      input array
// out:      output array
// step:     {SET TO 1} (used internally)
// size:     length of the input/output {MUST BE A POWER OF 2}
// dir:      either plus or minus one (direction of the FFT)
// RESULT:   out[k] = \sum_{j=0}^{size - 1} in[j] * exp(dir * 2pi * i * j * k / size)
void FFT(cpx *in, cpx *out, int step, int size, int dir)
{
    if(size < 1) return;
    if(size == 1)
    {
        out[0] = in[0];
        return;
    }
    FFT(in, out, step * 2, size / 2, dir);
    FFT(in + step, out + size / 2, step * 2, size / 2, dir);
    for(int i = 0 ; i < size / 2 ; i++)
    {
        cpx even = out[i];
        cpx odd = out[i + size / 2];
        out[i] = even + EXP(dir * two_pi * i / size) * odd;
        out[i + size / 2] = even + EXP(dir * two_pi * (i + size / 2) / size) * odd;
    }
}

```

```
}
```

```
// Usage:  
// f[0...N-1] and g[0...N-1] are numbers  
// Want to compute the convolution h, defined by  
//  $h[n] = \text{sum of } f[k]g[n-k] \text{ (} k = 0, \dots, N-1 \text{)}.$   
// Here, the index is cyclic;  $f[-1] = f[N-1]$ ,  $f[-2] = f[N-2]$ , etc.  
// Let  $F[0...N-1]$  be  $\text{FFT}(f)$ , and similarly, define  $G$  and  $H$ .  
// The convolution theorem says  $H[n] = F[n]G[n]$  (element-wise product).  
// To compute  $h[]$  in  $O(N \log N)$  time, do the following:  
// 1. Compute  $F$  and  $G$  (pass  $\text{dir} = 1$  as the argument).  
// 2. Get  $H$  by element-wise multiplying  $F$  and  $G$ .  
// 3. Get  $h$  by taking the inverse FFT (use  $\text{dir} = -1$  as the argument)  
// and *dividing by  $N$ !. DO NOT FORGET THIS SCALING FACTOR.
```

```
int main(void)  
{  
    printf("If rows come in identical pairs, then everything works.\n");  
  
    cpx a[8] = {0, 1, cpx(1,3), cpx(0,5), 1, 0, 2, 0};  
    cpx b[8] = {1, cpx(0,-2), cpx(0,1), 3, -1, -3, 1, -2};  
    cpx A[8];  
    cpx B[8];  
    FFT(a, A, 1, 8, 1);  
    FFT(b, B, 1, 8, 1);  
  
    for(int i = 0 ; i < 8 ; i++)  
    {  
        printf("%7.2lf%7.2lf", A[i].a, A[i].b);  
    }  
    printf("\n");  
    for(int i = 0 ; i < 8 ; i++)  
    {  
        cpx Ai(0,0);  
        for(int j = 0 ; j < 8 ; j++)  
        {  
            Ai = Ai + a[j] * EXP(j * i * two_pi / 8);  
        }  
        printf("%7.2lf%7.2lf", Ai.a, Ai.b);  
    }  
    printf("\n");  
  
    cpx AB[8];  
    for(int i = 0 ; i < 8 ; i++)  
        AB[i] = A[i] * B[i];  
}
```

```

cpx aconvb[8];
FFT(AB, aconvb, 1, 8, -1);
for(int i = 0 ; i < 8 ; i++)
    aconvb[i] = aconvb[i] / 8;
for(int i = 0 ; i < 8 ; i++)
{
    printf("%7.21f%7.21f", aconvb[i].a, aconvb[i].b);
}
printf("\n");
for(int i = 0 ; i < 8 ; i++)
{
    cpx aconvbi(0,0);
    for(int j = 0 ; j < 8 ; j++)
    {
        aconvbi = aconvbi + a[j] * b[(8 + i - j) % 8];
    }
    printf("%7.21f%7.21f", aconvbi.a, aconvbi.b);
}
printf("\n");

return 0;
}

```

Simplex.cc 17/35

```

// Two-phase simplex algorithm for solving linear programs of the form
//
//      maximize      c^T x
//      subject to    Ax <= b
//                   x >= 0
//
// INPUT: A -- an m x n matrix
//        b -- an m-dimensional vector
//        c -- an n-dimensional vector
//        x -- a vector where the optimal solution will be stored
//
// OUTPUT: value of the optimal solution (infinity if unbounded
//        above, nan if infeasible)
//
// To use this code, create an LPSolver object with A, b, and c as
// arguments. Then, call Solve(x).

```

```

#include <iostream>
#include <iomanip>
#include <vector>
#include <cmath>
#include <limits>

using namespace std;

typedef long double DOUBLE;
typedef vector<DOUBLE> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

const DOUBLE EPS = 1e-9;

struct LPSolver {
    int m, n;
    VI B, N;
    VVD D;

    LPSolver(const VVD &A, const VD &b, const VD &c) :
        m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2)) {
        for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] = A[i][j];
        for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[i][n + 1] = b[i]; }
        for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
        N[n] = -1; D[m + 1][n] = 1;
    }

    void Pivot(int r, int s) {
        for (int i = 0; i < m + 2; i++) if (i != r)
            for (int j = 0; j < n + 2; j++) if (j != s)
                D[i][j] -= D[r][j] * D[i][s] / D[r][s];
        for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] /= D[r][s];
        for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] /= -D[r][s];
        D[r][s] = 1.0 / D[r][s];
        swap(B[r], N[s]);
    }

    bool Simplex(int phase) {
        int x = phase == 1 ? m + 1 : m;
        while (true) {
            int s = -1;
            for (int j = 0; j <= n; j++) {
                if (phase == 2 && N[j] == -1) continue;

```

```

        if (s == -1 || D[x][j] < D[x][s] || D[x][j] == D[x][s] && N[j] < N[s]) s = j;
    }
    if (D[x][s] > -EPS) return true;
    int r = -1;
    for (int i = 0; i < m; i++) {
        if (D[i][s] < EPS) continue;
        if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] ||
            (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r][s]) && B[i] < B[r]) r = i;
    }
    if (r == -1) return false;
    Pivot(r, s);
}
}

DOUBLE Solve(VD &x) {
    int r = 0;
    for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
    if (D[r][n + 1] < -EPS) {
        Pivot(r, n);
        if (!Simplex(1) || D[m + 1][n + 1] < -EPS) return -numeric_limits<DOUBLE>::infinity();
        for (int i = 0; i < m; i++) if (B[i] == -1) {
            int s = -1;
            for (int j = 0; j <= n; j++)
                if (s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s] && N[j] < N[s]) s = j;
            Pivot(i, s);
        }
    }
    if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
    x = VD(n);
    for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n + 1];
    return D[m][n + 1];
}

};

int main() {

    const int m = 4;
    const int n = 3;
    DOUBLE _A[m][n] = {
        { 6, -1, 0 },
        { -1, -5, 0 },
        { 1, 5, 1 },
        { -1, -5, -1 }
    };
    DOUBLE _b[m] = { 10, -4, 5, -5 };

```

```

DOUBLE _c[n] = { 1, -1, 0 };

VVD A(m);
VD b(_b, _b + m);
VD c(_c, _c + n);
for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] + n);

LPSolver solver(A, b, c);
VD x;
DOUBLE value = solver.Solve(x);

cerr << "VALUE: " << value << endl; // VALUE: 1.29032
cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1
for (size_t i = 0; i < x.size(); i++) cerr << " " << x[i];
cerr << endl;
return 0;
}

```

FastDijkstra.cc 18/35

```

// Implementation of Dijkstra's algorithm using adjacency lists
// and priority queue for efficiency.
//
// Running time:  $O(|E| \log |V|)$ 

```

```

#include <queue>
#include <stdio.h>

```

```

using namespace std;
const int INF = 2000000000;
typedef pair<int,int> PII;

```

```

int main(){

```

```

    int N, s, t;
    scanf ("%d%d%d", &N, &s, &t);
    vector<vector<PII> > edges(N);
    for (int i = 0; i < N; i++){
        int M;
        scanf ("%d", &M);
        for (int j = 0; j < M; j++){
            int vertex, dist;

```

```

        scanf ("%d%d", &vertex, &dist);
        edges[i].push_back (make_pair (dist, vertex)); // note order of arguments here
    }
}

// use priority queue in which top element has the "smallest" priority
priority_queue<PII, vector<PII>, greater<PII> > Q;
vector<int> dist(N, INF), dad(N, -1);
Q.push (make_pair (0, s));
dist[s] = 0;
while (!Q.empty()){
    PII p = Q.top();
    if (p.second == t) break;
    Q.pop();

    int here = p.second;
    for (vector<PII>::iterator it=edges[here].begin(); it!=edges[here].end(); it++){
        if (dist[here] + it->first < dist[it->second]){
            dist[it->second] = dist[here] + it->first;
            dad[it->second] = here;
            Q.push (make_pair (dist[it->second], it->second));
        }
    }
}

printf ("%d\n", dist[t]);
if (dist[t] < INF)
    for(int i=t;i!=-1;i=dad[i])
        printf ("%d%c", i, (i==s?'\n':' '));

return 0;
}

```

SCC.cc 19/35

```

#include<memory.h>
struct edge{int e, nxt;};
int V, E;
edge e[MAXE], er[MAXE];
int sp[MAXV], spr[MAXV];
int group_cnt, group_num[MAXV];
bool v[MAXV];

```



```

int stk[MAXV];
void fill_forward(int x)
{
    int i;
    v[x]=true;
    for(i=sp[x];i;i=e[i].nxt) if(!v[e[i].e]) fill_forward(e[i].e);
    stk[++stk[0]]=x;
}
void fill_backward(int x)
{
    int i;
    v[x]=false;
    group_num[x]=group_cnt;
    for(i=spr[x];i;i=er[i].nxt) if(v[er[i].e]) fill_backward(er[i].e);
}
void add_edge(int v1, int v2) //add edge v1->v2
{
    e[++E].e=v2; e[E].nxt=sp[v1]; sp[v1]=E;
    er[E].e=v1; er[E].nxt=spr[v2]; spr[v2]=E;
}
void SCC()
{
    int i;
    stk[0]=0;
    memset(v, false, sizeof(v));
    for(i=1;i<=V;i++) if(!v[i]) fill_forward(i);
    group_cnt=0;
    for(i=stk[0];i>=1;i--) if(v[stk[i]]){group_cnt++; fill_backward(stk[i]);}
}

```

EulerianPath.cc 20/35

```

struct Edge;
typedef list<Edge>::iterator iter;

struct Edge
{
    int next_vertex;
    iter reverse_edge;

    Edge(int next_vertex)

```

```

        :next_vertex(next_vertex)
        { }
};

const int max_vertices = ;
int num_vertices;
list<Edge> adj[max_vertices];           // adjacency list

vector<int> path;

void find_path(int v)
{
    while(adj[v].size() > 0)
    {
        int vn = adj[v].front().next_vertex;
        adj[vn].erase(adj[v].front().reverse_edge);
        adj[v].pop_front();
        find_path(vn);
    }
    path.push_back(v);
}

void add_edge(int a, int b)
{
    adj[a].push_front(Edge(b));
    iter ita = adj[a].begin();
    adj[b].push_front(Edge(a));
    iter itb = adj[b].begin();
    ita->reverse_edge = itb;
    itb->reverse_edge = ita;
}

```

SuffixArray.cc 21/35

```

// Suffix array construction in  $O(L \log^2 L)$  time. Routine for
// computing the length of the longest common prefix of any two
// suffixes in  $O(\log L)$  time.
//
// INPUT:   string s
//
// OUTPUT:  array suffix[] such that suffix[i] = index (from 0 to L-1)
//          of substring s[i...L-1] in the list of sorted suffixes.

```

```

//          That is, if we take the inverse of the permutation suffix[],
//          we get the actual suffix array.

#include <vector>
#include <iostream>
#include <string>

using namespace std;

struct SuffixArray {
    const int L;
    string s;
    vector<vector<int>> > P;
    vector<pair<pair<int,int>,int> > M;

    SuffixArray(const string &s) : L(s.length()), s(s), P(1, vector<int>(L, 0)), M(L) {
        for (int i = 0; i < L; i++) P[0][i] = int(s[i]);
        for (int skip = 1, level = 1; skip < L; skip *= 2, level++) {
            P.push_back(vector<int>(L, 0));
            for (int i = 0; i < L; i++)
                M[i] = make_pair(make_pair(P[level-1][i], i + skip < L ? P[level-1][i + skip] : -1000), i);
            sort(M.begin(), M.end());
            for (int i = 0; i < L; i++)
                P[level][M[i].second] = (i > 0 && M[i].first == M[i-1].first) ? P[level][M[i-1].second] : i;
        }
    }

    vector<int> GetSuffixArray() { return P.back(); }

    // returns the length of the longest common prefix of s[i...L-1] and s[j...L-1]
    int LongestCommonPrefix(int i, int j) {
        int len = 0;
        if (i == j) return L - i;
        for (int k = P.size() - 1; k >= 0 && i < L && j < L; k--) {
            if (P[k][i] == P[k][j]) {
                i += 1 << k;
                j += 1 << k;
                len += 1 << k;
            }
        }
        return len;
    }
};

int main() {

```

```

// bobocel is the 0'th suffix
// obocel is the 5'th suffix
//  bocel is the 1'st suffix
//   ocel is the 6'th suffix
//    cel is the 2'nd suffix
//     el is the 3'rd suffix
//      l is the 4'th suffix
SuffixArray suffix("bobocel");
vector<int> v = suffix.GetSuffixArray();

// Expected output: 0 5 1 6 2 3 4
//                  2
for (int i = 0; i < v.size(); i++) cout << v[i] << " ";
cout << endl;
cout << suffix.LongestCommonPrefix(0, 2) << endl;
}

```

BIT.cc 22/35

```

#include <iostream>
using namespace std;

#define LOGSZ 17

int tree[(1<<LOGSZ)+1];
int N = (1<<LOGSZ);

// add v to value at x
void set(int x, int v) {
    while(x <= N) {
        tree[x] += v;
        x += (x & -x);
    }
}

// get cumulative sum up to and including x
int get(int x) {
    int res = 0;
    while(x) {
        res += tree[x];
        x -= (x & -x);
    }
}

```

```

}
return res;
}

// get largest value with cumulative sum less than or equal to x;
// for smallest, pass x-1 and add 1 to result
int getind(int x) {
    int idx = 0, mask = N;
    while(mask && idx < N) {
        int t = idx + mask;
        if(x >= tree[t]) {
            idx = t;
            x -= tree[t];
        }
        mask >>= 1;
    }
    return idx;
}

```

UnionFind.cc 23/35

```

//union-find set: the vector/array contains the parent of each node
int find(vector<int>& C, int x){return (C[x]==x) ? x : C[x]=find(C, C[x]);} //C++
int find(int x){return (C[x]==x)?x:C[x]=find(C[x]);} //C

```

KDTree.cc 24/35

```

// -----
// A straightforward, but probably sub-optimal KD-tree implmentation
// that's probably good enough for most things (current it's a
// 2D-tree)
//
// - constructs from n points in O(n lg^2 n) time
// - handles nearest-neighbor query in O(lg n) if points are well
//   distributed
// - worst case for nearest-neighbor may be linear in pathological
//   case
//
// Sonny Chan, Stanford University, April 2009

```

```
// -----  
  
#include <iostream>  
#include <vector>  
#include <limits>  
#include <cstdlib>  
  
using namespace std;  
  
// number type for coordinates, and its maximum value  
typedef long long ntype;  
const ntype sentry = numeric_limits<ntype>::max();  
  
// point structure for 2D-tree, can be extended to 3D  
struct point {  
    ntype x, y;  
    point(ntype xx = 0, ntype yy = 0) : x(xx), y(yy) {}  
};  
  
bool operator==(const point &a, const point &b)  
{  
    return a.x == b.x && a.y == b.y;  
}  
  
// sorts points on x-coordinate  
bool on_x(const point &a, const point &b)  
{  
    return a.x < b.x;  
}  
  
// sorts points on y-coordinate  
bool on_y(const point &a, const point &b)  
{  
    return a.y < b.y;  
}  
  
// squared distance between points  
ntype pdist2(const point &a, const point &b)  
{  
    ntype dx = a.x-b.x, dy = a.y-b.y;  
    return dx*dx + dy*dy;  
}  
  
// bounding box for a set of points  
struct bbox
```

```

{
    ntype x0, x1, y0, y1;

    bbox() : x0(sentry), x1(-sentry), y0(sentry), y1(-sentry) {}

    // computes bounding box from a bunch of points
    void compute(const vector<point> &v) {
        for (int i = 0; i < v.size(); ++i) {
            x0 = min(x0, v[i].x);    x1 = max(x1, v[i].x);
            y0 = min(y0, v[i].y);    y1 = max(y1, v[i].y);
        }
    }

    // squared distance between a point and this bbox, 0 if inside
    ntype distance(const point &p) {
        if (p.x < x0) {
            if (p.y < y0)    return pdist2(point(x0, y0), p);
            else if (p.y > y1) return pdist2(point(x0, y1), p);
            else             return pdist2(point(x0, p.y), p);
        }
        else if (p.x > x1) {
            if (p.y < y0)    return pdist2(point(x1, y0), p);
            else if (p.y > y1) return pdist2(point(x1, y1), p);
            else             return pdist2(point(x1, p.y), p);
        }
        else {
            if (p.y < y0)    return pdist2(point(p.x, y0), p);
            else if (p.y > y1) return pdist2(point(p.x, y1), p);
            else             return 0;
        }
    }
};

// stores a single node of the kd-tree, either internal or leaf
struct kdnode
{
    bool leaf;           // true if this is a leaf node (has one point)
    point pt;           // the single point of this is a leaf
    bbox bound;         // bounding box for set of points in children

    kdnode *first, *second; // two children of this kd-node

    kdnode() : leaf(false), first(0), second(0) {}
    ~kdnode() { if (first) delete first; if (second) delete second; }
}

```

```

// intersect a point with this node (returns squared distance)
ntype intersect(const point &p) {
    return bound.distance(p);
}

// recursively builds a kd-tree from a given cloud of points
void construct(vector<point> &vp)
{
    // compute bounding box for points at this node
    bound.compute(vp);

    // if we're down to one point, then we're a leaf node
    if (vp.size() == 1) {
        leaf = true;
        pt = vp[0];
    }
    else {
        // split on x if the bbox is wider than high (not best heuristic...)
        if (bound.x1-bound.x0 >= bound.y1-bound.y0)
            sort(vp.begin(), vp.end(), on_x);
        // otherwise split on y-coordinate
        else
            sort(vp.begin(), vp.end(), on_y);

        // divide by taking half the array for each child
        // (not best performance if many duplicates in the middle)
        int half = vp.size()/2;
        vector<point> vl(vp.begin(), vp.begin()+half);
        vector<point> vr(vp.begin()+half, vp.end());
        first = new kdnode(); first->construct(vl);
        second = new kdnode(); second->construct(vr);
    }
};

// simple kd-tree class to hold the tree and handle queries
struct kdtree
{
    kdnode *root;

    // constructs a kd-tree from a points (copied here, as it sorts them)
    kdtree(const vector<point> &vp) {
        vector<point> v(vp.begin(), vp.end());
        root = new kdnode();
        root->construct(v);
    }
};

```



```

}
~kdtree() { delete root; }

// recursive search method returns squared distance to nearest point
ntype search(kdnode *node, const point &p)
{
    if (node->leaf) {
        // commented special case tells a point not to find itself
        // if (p == node->pt) return sentry;
        // else
        return pdist2(p, node->pt);
    }

    ntype bfirst = node->first->intersect(p);
    ntype bsecond = node->second->intersect(p);

    // choose the side with the closest bounding box to search first
    // (note that the other side is also searched if needed)
    if (bfirst < bsecond) {
        ntype best = search(node->first, p);
        if (bsecond < best)
            best = min(best, search(node->second, p));
        return best;
    }
    else {
        ntype best = search(node->second, p);
        if (bfirst < best)
            best = min(best, search(node->first, p));
        return best;
    }
}

// squared distance to the nearest
ntype nearest(const point &p) {
    return search(root, p);
}

};

// -----
// some basic test code here

int main()
{
    // generate some random points for a kd-tree
    vector<point> vp;

```

```

for (int i = 0; i < 100000; ++i) {
    vp.push_back(point(rand()%100000, rand()%100000));
}
kdtree tree(vp);

// query some points
for (int i = 0; i < 10; ++i) {
    point q(rand()%100000, rand()%100000);
    cout << "Closest squared distance to (" << q.x << ", " << q.y << ")"
         << " is " << tree.nearest(q) << endl;
}

return 0;
}

// -----

```

splay.cpp 25/35

```

#include <cstdio>
#include <algorithm>
using namespace std;

const int N_MAX = 130010;
const int oo = 0x3f3f3f3f;
struct Node
{
    Node *ch[2], *pre;
    int val, size;
    bool isTurned;
} nodePool[N_MAX], *null, *root;

Node *allocNode(int val)
{
    static int freePos = 0;
    Node *x = &nodePool[freePos++];
    x->val = val, x->isTurned = false;
    x->ch[0] = x->ch[1] = x->pre = null;
    x->size = 1;
    return x;
}

```

```
inline void update(Node *x)
{
    x->size = x->ch[0]->size + x->ch[1]->size + 1;
}
```

```
inline void makeTurned(Node *x)
{
    if(x == null)
        return;
    swap(x->ch[0], x->ch[1]);
    x->isTurned ^= 1;
}
```

```
inline void pushDown(Node *x)
{
    if(x->isTurned)
    {
        makeTurned(x->ch[0]);
        makeTurned(x->ch[1]);
        x->isTurned ^= 1;
    }
}
```

```
inline void rotate(Node *x, int c)
{
    Node *y = x->pre;
    x->pre = y->pre;
    if(y->pre != null)
        y->pre->ch[y == y->pre->ch[1]] = x;
    y->ch[!c] = x->ch[c];
    if(x->ch[c] != null)
        x->ch[c]->pre = y;
    x->ch[c] = y, y->pre = x;
    update(y);
    if(y == root)
        root = x;
}
```

```
void splay(Node *x, Node *p)
{
    while(x->pre != p)
    {
        if(x->pre->pre == p)
            rotate(x, x == x->pre->ch[0]);
        else
```

```

{
    Node *y = x->pre, *z = y->pre;
    if(y == z->ch[0])
    {
        if(x == y->ch[0])
            rotate(y, 1), rotate(x, 1);
        else
            rotate(x, 0), rotate(x, 1);
    }
    else
    {
        if(x == y->ch[1])
            rotate(y, 0), rotate(x, 0);
        else
            rotate(x, 1), rotate(x, 0);
    }
}
}
update(x);
}

```

```

void select(int k, Node *fa)
{
    Node *now = root;
    while(1)
    {
        pushDown(now);
        int tmp = now->ch[0]->size + 1;
        if(tmp == k)
            break;
        else if(tmp < k)
            now = now->ch[1], k -= tmp;
        else
            now = now->ch[0];
    }
    splay(now, fa);
}

```

```

Node *makeTree(Node *p, int l, int r)
{
    if(l > r)
        return null;
    int mid = (l + r) / 2;
    Node *x = allocNode(mid);
    x->pre = p;

```

```

x->ch[0] = makeTree(x, l, mid - 1);
x->ch[1] = makeTree(x, mid + 1, r);
update(x);
return x;
}

int main()
{
    int n, m;
    null = allocNode(0);
    null->size = 0;
    root = allocNode(0);
    root->ch[1] = allocNode(0);
    root->ch[1]->pre = root;
    update(root);

    scanf("%d%d", &n, &m);
    root->ch[1]->ch[0] = makeTree(root->ch[1], 1, n);
    splay(root->ch[1]->ch[0], null);

    while(m --)
    {
        int a, b;
        scanf("%d%d", &a, &b);
        a ++, b ++;
        select(a - 1, null);
        select(b + 1, root);
        makeTurned(root->ch[1]->ch[0]);
    }

    for(int i = 1; i <= n; i ++)
    {
        select(i + 1, null);
        printf("%d ", root->val);
    }
}

```

SegmentTreeLazy.java 26/35

```

public class SegmentTreeRangeUpdate {
    public long[] leaf;
    public long[] update;

```

```

public int origSize;
public SegmentTreeRangeUpdate(int[] list)    {
    origSize = list.length;
    leaf = new long[4*list.length];
    update = new long[4*list.length];
    build(1,0,list.length-1,list);
}
public void build(int curr, int begin, int end, int[] list)    {
    if(begin == end)
        leaf[curr] = list[begin];
    else
    {
        int mid = (begin+end)/2;
        build(2 * curr, begin, mid, list);
        build(2 * curr + 1, mid+1, end, list);
        leaf[curr] = leaf[2*curr] + leaf[2*curr+1];
    }
}
public void update(int begin, int end, int val) {
    update(1,0,origSize-1,begin,end,val);
}
public void update(int curr, int tBegin, int tEnd, int begin, int end, int val)    {
    if(tBegin >= begin && tEnd <= end)
        update[curr] += val;
    else
    {
        leaf[curr] += (Math.min(end,tEnd)-Math.max(begin,tBegin)+1) * val;
        int mid = (tBegin+tEnd)/2;
        if(mid >= begin && tBegin <= end)
            update(2*curr, tBegin, mid, begin, end, val);
        if(tEnd >= begin && mid+1 <= end)
            update(2*curr+1, mid+1, tEnd, begin, end, val);
    }
}
public long query(int begin, int end)    {
    return query(1,0,origSize-1,begin,end);
}
public long query(int curr, int tBegin, int tEnd, int begin, int end)    {
    if(tBegin >= begin && tEnd <= end)    {
        if(update[curr] != 0)    {
            leaf[curr] += (tEnd-tBegin+1) * update[curr];
            if(2*curr < update.length){
                update[2*curr] += update[curr];
                update[2*curr+1] += update[curr];
            }
            update[curr] = 0;
        }
    }
}

```

```

        return leaf[curr];
    }
    else {
        leaf[curr] += (tEnd-tBegin+1) * update[curr];
        if(2*curr < update.length){
            update[2*curr] += update[curr];
            update[2*curr+1] += update[curr];
        }
        update[curr] = 0;
        int mid = (tBegin+tEnd)/2;
        long ret = 0;
        if(mid >= begin && tBegin <= end)
            ret += query(2*curr, tBegin, mid, begin, end);
        if(tEnd >= begin && mid+1 <= end)
            ret += query(2*curr+1, mid+1, tEnd, begin, end);
        return ret;
    }
}
}

```

LCA.cc 27/35

```

const int max_nodes, log_max_nodes;
int num_nodes, log_num_nodes, root;

vector<int> children[max_nodes];           // children[i] contains the children of node i
int A[max_nodes][log_max_nodes+1];       // A[i][j] is the 2^j-th ancestor of node i, or -1 if that ancestor does not exist
int L[max_nodes];                         // L[i] is the distance between node i and the root

// floor of the binary logarithm of n
int lb(unsigned int n)
{
    if(n==0)
        return -1;
    int p = 0;
    if (n >= 1<<16) { n >>= 16; p += 16; }
    if (n >= 1<< 8) { n >>= 8; p += 8; }
    if (n >= 1<< 4) { n >>= 4; p += 4; }
    if (n >= 1<< 2) { n >>= 2; p += 2; }
    if (n >= 1<< 1) { n >>= 1; p += 1; }
    return p;
}

```

```

void DFS(int i, int l)
{
    L[i] = l;
    for(int j = 0; j < children[i].size(); j++)
        DFS(children[i][j], l+1);
}

int LCA(int p, int q)
{
    // ensure node p is at least as deep as node q
    if(L[p] < L[q])
        swap(p, q);

    // "binary search" for the ancestor of node p situated on the same level as q
    for(int i = log_num_nodes; i >= 0; i--)
        if(L[p] - (1<<i) >= L[q])
            p = A[p][i];

    if(p == q)
        return p;

    // "binary search" for the LCA
    for(int i = log_num_nodes; i >= 0; i--)
        if(A[p][i] != -1 && A[p][i] != A[q][i])
        {
            p = A[p][i];
            q = A[q][i];
        }

    return A[p][0];
}

int main(int argc, char* argv[])
{
    // read num_nodes, the total number of nodes
    log_num_nodes=lb(num_nodes);

    for(int i = 0; i < num_nodes; i++)
    {
        int p;
        // read p, the parent of node i or -1 if node i is the root

        A[i][0] = p;
        if(p != -1)

```



```

        children[p].push_back(i);
    else
        root = i;
}

// precompute A using dynamic programming
for(int j = 1; j <= log_num_nodes; j++)
    for(int i = 0; i < num_nodes; i++)
        if(A[i][j-1] != -1)
            A[i][j] = A[A[i][j-1]][j-1];
        else
            A[i][j] = -1;

// precompute L
DFS(root, 0);

return 0;
}

```

LongestIncreasingSubsequence.cc 28/35

```

// Given a list of numbers of length n, this routine extracts a
// longest increasing subsequence.
//
// Running time: O(n log n)
//
// INPUT: a vector of integers
// OUTPUT: a vector containing the longest increasing subsequence

```

```

#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

typedef vector<int> VI;
typedef pair<int,int> PII;
typedef vector<PII> VPII;

#define STRICTLY_INCREASNG

```

```

VI LongestIncreasingSubsequence(VI v) {
    VPII best;
    VI dad(v.size(), -1);

    for (int i = 0; i < v.size(); i++) {
#ifdef STRICTLY_INCREASNG
        PII item = make_pair(v[i], 0);
        VPII::iterator it = lower_bound(best.begin(), best.end(), item);
        item.second = i;
#else
        PII item = make_pair(v[i], i);
        VPII::iterator it = upper_bound(best.begin(), best.end(), item);
#endif
        if (it == best.end()) {
            dad[i] = (best.size() == 0 ? -1 : best.back().second);
            best.push_back(item);
        } else {
            dad[i] = dad[it->second];
            *it = item;
        }
    }

    VI ret;
    for (int i = best.back().second; i >= 0; i = dad[i])
        ret.push_back(v[i]);
    reverse(ret.begin(), ret.end());
    return ret;
}

```

Dates.cc 29/35

```

// Routines for performing computations on dates. In these routines,
// months are expressed as integers from 1 to 12, days are expressed
// as integers from 1 to 31, and years are expressed as 4-digit
// integers.

```

```

#include <iostream>
#include <string>

```

```
using namespace std;
```

```
string dayOfWeek[] = {"Mon", "Tue", "Wed", "Thu", "Fri", "Sat", "Sun"};
```

```

// converts Gregorian date to integer (Julian day number)
int dateToInt (int m, int d, int y){
    return
        1461 * (y + 4800 + (m - 14) / 12) / 4 +
        367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
        3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
        d - 32075;
}

// converts integer (Julian day number) to Gregorian date: month/day/year
void intToDate (int jd, int &m, int &d, int &y){
    int x, n, i, j;

    x = jd + 68569;
    n = 4 * x / 146097;
    x -= (146097 * n + 3) / 4;
    i = (4000 * (x + 1)) / 1461001;
    x -= 1461 * i / 4 - 31;
    j = 80 * x / 2447;
    d = x - 2447 * j / 80;
    x = j / 11;
    m = j + 2 - 12 * x;
    y = 100 * (n - 49) + i + x;
}

// converts integer (Julian day number) to day of week
string intToDay (int jd){
    return dayOfWeek[jd % 7];
}

int main (int argc, char **argv){
    int jd = dateToInt (3, 24, 2004);
    int m, d, y;
    intToDate (jd, m, d, y);
    string day = intToDay (jd);

    // expected output:
    //      2453089
    //      3/24/2004
    //      Wed
    cout << jd << endl
        << m << "/" << d << "/" << y << endl
        << day << endl;
}

```

LogLan.java 30/35

```
// Code which demonstrates the use of Java's regular expression libraries.  
// This is a solution for  
//  
// Loglan: a logical language  
// http://acm.uva.es/p/v1/134.html  
//  
// In this problem, we are given a regular language, whose rules can be  
// inferred directly from the code. For each sentence in the input, we must  
// determine whether the sentence matches the regular expression or not. The  
// code consists of (1) building the regular expression (which is fairly  
// complex) and (2) using the regex to match sentences.
```

```
import java.util.*;  
import java.util.regex.*;  
  
public class LogLan {  
  
    public static String BuildRegex (){  
        String space = " +";  
  
        String A = "([aeiou])";  
        String C = "([a-z&[^aeiou]])";  
        String MOD = "(g" + A + ")";  
        String BA = "(b" + A + ")";  
        String DA = "(d" + A + ")";  
        String LA = "(l" + A + ")";  
        String NAM = "([a-z]*" + C + ")";  
        String PREDA = "(" + C + C + A + C + A + "|" + C + A + C + C + A + ")";  
  
        String predstring = "(" + PREDA + "(" + space + PREDA + ")*";  
        String predname = "(" + LA + space + predstring + "|" + NAM + ")";  
        String preds = "(" + predstring + "(" + space + A + space + predstring + ")*";  
        String predclaim = "(" + predname + space + BA + space + preds + "|" + DA + space +  
            preds + ")";  
        String verbpred = "(" + MOD + space + predstring + ")";  
        String statement = "(" + predname + space + verbpred + space + predname + "|" +  
            predname + space + verbpred + ")";  
        String sentence = "(" + statement + "|" + predclaim + ")";  
  
        return "^" + sentence + "$";  
    }  
}
```

```

}

public static void main (String args[]){

String regex = BuildRegex();
Pattern pattern = Pattern.compile (regex);

Scanner s = new Scanner(System.in);
while (true) {

    // In this problem, each sentence consists of multiple lines, where the last
    // line is terminated by a period. The code below reads lines until
    // encountering a line whose final character is a '.'. Note the use of
    //
    // s.length() to get length of string
    // s.charAt() to extract characters from a Java string
    // s.trim() to remove whitespace from the beginning and end of Java string
    //
    // Other useful String manipulation methods include
    //
    // s.compareTo(t) < 0 if s < t, lexicographically
    // s.indexOf("apple") returns index of first occurrence of "apple" in s
    // s.lastIndexOf("apple") returns index of last occurrence of "apple" in s
    // s.replace(c,d) replaces occurrences of character c with d
    // s.startsWith("apple") returns (s.indexOf("apple") == 0)
    // s.toLowerCase() / s.toUpperCase() returns a new lower/uppercased string
    //
    // Integer.parseInt(s) converts s to an integer (32-bit)
    // Long.parseLong(s) converts s to a long (64-bit)
    // Double.parseDouble(s) converts s to a double

String sentence = "";
while (true){
    sentence = (sentence + " " + s.nextLine()).trim();
    if (sentence.equals("#")) return;
    if (sentence.charAt(sentence.length()-1) == '.') break;
}

// now, we remove the period, and match the regular expression

String removed_period = sentence.substring(0, sentence.length()-1).trim();
if (pattern.matcher (removed_period).find()){
    System.out.println ("Good");
} else {
    System.out.println ("Bad!");
}
}

```

```
}  
}  
}  
}
```

Primes.cc 31/35

```
// O(sqrt(x)) Exhaustive Primality Test  
#include <cmath>  
#define EPS 1e-7  
typedef long long LL;  
bool IsPrimeSlow (LL x)  
{  
    if(x<=1) return false;  
    if(x<=3) return true;  
    if (!(x%2) || !(x%3)) return false;  
    LL s=(LL)(sqrt((double)(x))+EPS);  
    for(LL i=5;i<=s;i+=6)  
    {  
        if (!(x%i) || !(x%(i+2))) return false;  
    }  
    return true;  
}  
  
// Primes less than 1000:  
//      2      3      5      7      11      13      17      19      23      29      31      37  
//      41      43      47      53      59      61      67      71      73      79      83      89  
//      97     101     103     107     109     113     127     131     137     139     149     151  
//     157     163     167     173     179     181     191     193     197     199     211     223  
//     227     229     233     239     241     251     257     263     269     271     277     281  
//     283     293     307     311     313     317     331     337     347     349     353     359  
//     367     373     379     383     389     397     401     409     419     421     431     433  
//     439     443     449     457     461     463     467     479     487     491     499     503  
//     509     521     523     541     547     557     563     569     571     577     587     593  
//     599     601     607     613     617     619     631     641     643     647     653     659  
//     661     673     677     683     691     701     709     719     727     733     739     743  
//     751     757     761     769     773     787     797     809     811     821     823     827  
//     829     839     853     857     859     863     877     881     883     887     907     911  
//     919     929     937     941     947     953     967     971     977     983     991     997  
  
// Other primes:  
//      The largest prime smaller than 10 is 7.  
//      The largest prime smaller than 100 is 97.
```

```
// The largest prime smaller than 1000 is 997.
// The largest prime smaller than 10000 is 9973.
// The largest prime smaller than 100000 is 99991.
// The largest prime smaller than 1000000 is 999983.
// The largest prime smaller than 10000000 is 9999991.
// The largest prime smaller than 100000000 is 99999989.
// The largest prime smaller than 1000000000 is 999999937.
// The largest prime smaller than 10000000000 is 9999999967.
// The largest prime smaller than 100000000000 is 9999999977.
// The largest prime smaller than 1000000000000 is 99999999989.
// The largest prime smaller than 10000000000000 is 999999999971.
// The largest prime smaller than 100000000000000 is 9999999999973.
// The largest prime smaller than 1000000000000000 is 99999999999989.
// The largest prime smaller than 10000000000000000 is 999999999999937.
// The largest prime smaller than 100000000000000000 is 999999999999997.
// The largest prime smaller than 1000000000000000000 is 9999999999999989.
```

IO.cpp 32/35

```
#include <iostream>
#include <iomanip>

using namespace std;

int main()
{
    // Output a specific number of digits past the decimal point,
    // in this case 5
    cout.setf(ios::fixed); cout << setprecision(5);
    cout << 100.0/7.0 << endl;
    cout.unsetf(ios::fixed);

    // Output the decimal point and trailing zeros
    cout.setf(ios::showpoint);
    cout << 100.0 << endl;
    cout.unsetf(ios::showpoint);

    // Output a '+' before positive values
    cout.setf(ios::showpos);
    cout << 100 << " " << -100 << endl;
    cout.unsetf(ios::showpos);
```

```
// Output numerical values in hexadecimal
cout << hex << 100 << " " << 1000 << " " << 10000 << dec << endl;
}
```

KMP.cpp 33/35

```
/*
Searches for the string w in the string s (of length k). Returns the
0-based index of the first match (k if no match is found). Algorithm
runs in O(k) time.
*/

#include <iostream>
#include <string>
#include <vector>

using namespace std;

typedef vector<int> VI;

void buildTable(string& w, VI& t)
{
    t = VI(w.length());
    int i = 2, j = 0;
    t[0] = -1; t[1] = 0;

    while(i < w.length())
    {
        if(w[i-1] == w[j]) { t[i] = j+1; i++; j++; }
        else if(j > 0) j = t[j];
        else { t[i] = 0; i++; }
    }
}

int KMP(string& s, string& w)
{
    int m = 0, i = 0;
    VI t;

    buildTable(w, t);
    while(m+i < s.length())
    {
```



```

    if(w[i] == s[m+i])
    {
        i++;
        if(i == w.length()) return m;
    }
    else
    {
        m += i-t[i];
        if(i > 0) i = t[i];
    }
}
return s.length();
}

int main()
{
    string a = (string) "The example above illustrates the general technique for assembling "+
        "the table with a minimum of fuss. The principle is that of the overall search: "+
        "most of the work was already done in getting to the current position, so very "+
        "little needs to be done in leaving it. The only minor complication is that the "+
        "logic which is correct late in the string erroneously gives non-proper "+
        "substrings at the beginning. This necessitates some initialization code.";

    string b = "table";

    int p = KMP(a, b);
    cout << p << ": " << a.substr(p, b.length()) << " " << b << endl;
}

```

LatLong.cpp 34/35

```

/*
Converts from rectangular coordinates to latitude/longitude and vice
versa. Uses degrees (not radians).
*/

```

```

#include <iostream>
#include <cmath>

```

```
using namespace std;
```

```
struct ll
```

```

{
    double r, lat, lon;
};

struct rect
{
    double x, y, z;
};

ll convert(rect& P)
{
    ll Q;
    Q.r = sqrt(P.x*P.x+P.y*P.y+P.z*P.z);
    Q.lat = 180/M_PI*asin(P.z/Q.r);
    Q.lon = 180/M_PI*acos(P.x/sqrt(P.x*P.x+P.y*P.y));

    return Q;
}

rect convert(ll& Q)
{
    rect P;
    P.x = Q.r*cos(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
    P.y = Q.r*sin(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
    P.z = Q.r*sin(Q.lat*M_PI/180);

    return P;
}

int main()
{
    rect A;
    ll B;

    A.x = -1.0; A.y = 2.0; A.z = -3.0;

    B = convert(A);
    cout << B.r << " " << B.lat << " " << B.lon << endl;

    A = convert(B);
    cout << A.x << " " << A.y << " " << A.z << endl;
}

```

EmacsSettings.txt 35/35

```
;; Jack's .emacs file
```

```
(global-set-key "\C-z"      'scroll-down)
(global-set-key "\C-x\C-p"  '(lambda() (interactive) (other-window -1)) )
(global-set-key "\C-x\C-o"  'other-window)
(global-set-key "\C-x\C-n"  'other-window)
(global-set-key "\M-."      'end-of-buffer)
(global-set-key "\M-,"      'beginning-of-buffer)
(global-set-key "\M-g"      'goto-line)
(global-set-key "\C-c\C-w"  'compare-windows)
```

```
(tool-bar-mode 0)
(scroll-bar-mode -1)
```

```
(global-font-lock-mode 1)
(show-paren-mode 1)
```

```
(setq-default c-default-style "linux")
```

```
(custom-set-variables
 '(compare-ignore-whitespace t)
)
```

Generated by [GNU Enscript 1.6.5.90](#).