Team notebook

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7 FastIO	8		
8 FenwickTree	10	1 BellmanFord	
9 FenwickTree2D	11	// This function runs the Bellman-Ford algorithm for single source	
10 FloydWar	11	// shortest paths with negative edge weights. The function returns // false if a negative weight cycle is detected. Otherwise, the	
11 Geom3D	12	<pre>// function returns true and dist[i] is the length of the shortest // path from start to i. //</pre>	
12 Geometry	12	<pre>// // Running time: O(V ^3) //</pre>	
13 Kadane1DMaxSum	16	<pre>// // INPUT: start, w[i][j] = cost of edge from i to j // OUTPUT: dist[i] = min weight path from start to i</pre>	
14 Kruskal	16	// prev[i] = previous node on the best path from the // start node	
15 LCA	17	#include <iostream></iostream>	
16 LongestIncreasingSubsequence	18	<pre>#include <queue> #include <cmath></cmath></queue></pre>	
10 10116000111101 cubiii160 ubboquoiice	10 1	"Inotago 'omgon'	

```
#include <vector>
using namespace std;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;
typedef vector<int> VI;
typedef vector<VI> VVI;
bool BellmanFord (const VVT &w, VT &dist, VI &prev, int start){
 int n = w.size();
 prev = VI(n, -1);
 dist = VT(n, 100000000);
 dist[start] = 0;
 for (int k = 0; k < n; k++){
   for (int i = 0: i < n: i++){
     for (int j = 0; j < n; j++){
       if (dist[j] > dist[i] + w[i][j]){
        if (k == n-1) return false;
        dist[j] = dist[i] + w[i][j];
        prev[j] = i;
     }
   }
 return true;
```

2 ConvexHull

```
#include <bits/stdc++.h>
using namespace std;

// Compute the 2D convex hull of a set of points using the monotone chain
// algorithm. Eliminate redundant points from the hull if
    REMOVE_REDUNDANT is
// #defined.
//
```

```
// Running time: O(n log n)
11
// INPUT: a vector of input points, unordered.
// OUTPUT: a vector of points in the convex hull, counterclockwise,
    starting
11
             with bottommost/leftmost point
#define REMOVE REDUNDANT
typedef double T;
const T EPS = 1e-7:
struct PT {
 T x, y;
 PT() {}
 PT(T x, T y) : x(x), y(y) {}
 bool operator<(const PT &rhs) const { return make_pair(y,x) <</pre>
      make_pair(rhs.y,rhs.x); }
 bool operator==(const PT &rhs) const { return make_pair(y,x) ==
      make_pair(rhs.y,rhs.x); }
};
T cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
T area2(PT a, PT b, PT c) { return cross(a,b) + cross(b,c) + cross(c,a); }
#ifdef REMOVE_REDUNDANT
bool between(const PT &a, const PT &b, const PT &c) {
 return (fabs(area2(a,b,c)) < EPS && (a.x-b.x)*(c.x-b.x) <= 0 &&
      (a.y-b.y)*(c.y-b.y) <= 0);
}
#endif
void ConvexHull(vector<PT> &pts) {
  sort(pts.begin(), pts.end());
  pts.erase(unique(pts.begin(), pts.end()), pts.end());
  vector<PT> up, dn;
 for (int i = 0; i < pts.size(); i++) {</pre>
   while (up.size() > 1 && area2(up[up.size()-2], up.back(), pts[i]) >=
        0) up.pop_back();
   while (dn.size() > 1 && area2(dn[dn.size()-2], dn.back(), pts[i]) <=</pre>
        0) dn.pop_back();
   up.push_back(pts[i]);
   dn.push_back(pts[i]);
  pts = dn;
  for (int i = (int) up.size() - 2; i >= 1; i--) pts.push_back(up[i]);
```

```
#ifdef REMOVE_REDUNDANT
 if (pts.size() <= 2) return;</pre>
 dn.clear();
 dn.push_back(pts[0]);
 dn.push_back(pts[1]);
 for (int i = 2; i < pts.size(); i++) {</pre>
   if (between(dn[dn.size()-2], dn[dn.size()-1], pts[i])) dn.pop_back();
   dn.push_back(pts[i]);
 if (dn.size() >= 3 && between(dn.back(), dn[0], dn[1])) {
   dn[0] = dn.back();
   dn.pop_back();
 pts = dn;
#endif
int main()
 vector<PT> pts(10);
 pts[0] = (PT(0,0));
 pts[1] = (PT(1,5));
 pts[2] = (PT(-4,7));
 pts[3] = (PT(8,9));
 pts[4] = (PT(20,20));
 pts[5] = (PT(1,1));
 pts[6] = (PT(-16,4));
 pts[7] = (PT(0,-20));
 pts[8] = (PT(30,45));
 pts[9] = (PT(20,0));
 ConvexHull(pts);
 for(PT pt:pts)
   cout << pt.x << " " << pt.y << endl;
 should print:
 0 -20
 20 0
 30 45
```

```
-16 4
*/

return 0;
}
```

3 Dijkstra

```
// Implementation of Dijkstra's algorithm using adjacency lists
// and priority queue for efficiency.
// Running time: O(|E| log |V|)
#include <queue>
#include <stdio.h>
using namespace std;
const int INF = 2000000000;
typedef pair<int,int> PII;
int main(){
 int N, s, t;
 scanf ("%d%d%d", &N, &s, &t);
 vector<vector<PII> > edges(N);
 for (int i = 0; i < N; i++){</pre>
   int M;
   scanf ("%d", &M);
   for (int j = 0; j < M; j++){
     int vertex, dist;
     scanf ("%d%d", &vertex, &dist);
     edges[i].push_back (make_pair (dist, vertex)); // note order of
         arguments here
   }
 // use priority queue in which top element has the "smallest" priority
 priority_queue<PII, vector<PII>, greater<PII> > Q;
 vector<int> dist(N, INF), dad(N, -1);
 Q.push (make_pair (0, s));
 dist[s] = 0;
```

```
while (!Q.empty()){
   PII p = Q.top();
   if (p.second == t) break;
   Q.pop();
   int here = p.second;
   for (vector<PII>::iterator it=edges[here].begin();
        it!=edges[here].end(); it++){
     if (dist[here] + it->first < dist[it->second]){
       dist[it->second] = dist[here] + it->first;
       dad[it->second] = here:
       Q.push (make_pair (dist[it->second], it->second));
     }
   }
 }
 printf ("%d\n", dist[t]);
 if (dist[t] < INF)</pre>
   for(int i=t;i!=-1;i=dad[i])
     printf ("%d%c", i, (i==s?'\n':' '));
 return 0;
}
```

4 Dinic

```
// Adjacency list implementation of Dinic's blocking flow algorithm.
// This is very fast in practice, and only loses to push-relabel flow.
//
// Running time:
      O(|V|^2 |E|)
//
// INPUT:
      - graph, constructed using AddEdge()
      - source and sink
//
// OUTPUT:
//
      - maximum flow value
      - To obtain actual flow values, look at edges with capacity > 0
        (zero capacity edges are residual edges).
#include <iostream>
```

```
#include <vector>
using namespace std;
typedef long long LL;
struct Edge {
 int from, to, cap, flow, index;
 Edge(int from, int to, int cap, int flow, int index) :
   from(from), to(to), cap(cap), flow(flow), index(index) {}
 LL rcap() { return cap - flow; }
struct Dinic {
 int N;
 vector<vector<Edge> > G;
 vector<vector<Edge *> > Lf;
 vector<int> layer;
 vector<int> Q;
 Dinic(int N) : N(N), G(N), Q(N) {}
 void AddEdge(int from, int to, int cap) {
   if (from == to) return;
   G[from].push_back(Edge(from, to, cap, 0, G[to].size()));
   G[to].push_back(Edge(to, from, 0, 0, G[from].size() - 1));
 LL BlockingFlow(int s, int t) {
   layer.clear(); layer.resize(N, -1);
   layer[s] = 0;
   Lf.clear(); Lf.resize(N);
   int head = 0, tail = 0;
   Q[tail++] = s;
   while (head < tail) {</pre>
     int x = Q[head++];
     for (int i = 0; i < G[x].size(); i++) {</pre>
       Edge &e = G[x][i]; if (e.rcap() <= 0) continue;</pre>
       if (layer[e.to] == -1) {
         layer[e.to] = layer[e.from] + 1;
         Q[tail++] = e.to;
       }
       if (layer[e.to] > layer[e.from]) {
         Lf[e.from].push_back(&e);
```

```
}
   }
   if (layer[t] == -1) return 0;
   LL totflow = 0;
   vector<Edge *> P;
   while (!Lf[s].empty()) {
     int curr = P.empty() ? s : P.back()->to;
     if (curr == t) { // Augment
       LL amt = P.front()->rcap();
       for (int i = 0; i < P.size(); ++i) {</pre>
         amt = min(amt, P[i]->rcap());
       }
       totflow += amt;
       for (int i = P.size() - 1; i >= 0; --i) {
         P[i]->flow += amt;
         G[P[i]->to][P[i]->index].flow -= amt;
         if (P[i]->rcap() <= 0) {</pre>
           Lf[P[i]->from].pop_back();
           P.resize(i);
         }
       }
     } else if (Lf[curr].empty()) { // Retreat
       P.pop_back();
       for (int i = 0; i < N; ++i)</pre>
         for (int j = 0; j < Lf[i].size(); ++j)</pre>
           if (Lf[i][j]->to == curr)
            Lf[i].erase(Lf[i].begin() + j);
     } else { // Advance
       P.push_back(Lf[curr].back());
     }
   }
   return totflow;
 }
 LL GetMaxFlow(int s, int t) {
   LL totflow = 0;
   while (LL flow = BlockingFlow(s, t))
     totflow += flow;
   return totflow;
 }
};
// BEGIN CUT
```

```
// The following code solves SPOJ problem #4110: Fast Maximum Flow
    (FASTFLOW)
int main() {
 int n, m;
  scanf("%d%d", &n, &m);
 Dinic flow(n);
  for (int i = 0; i < m; i++) {</pre>
   int a, b, c;
   scanf("%d%d%d", &a, &b, &c);
   if (a == b) continue;
   flow.AddEdge(a-1, b-1, c);
   flow.AddEdge(b-1, a-1, c);
 printf("%Ld\n", flow.GetMaxFlow(0, n-1));
 return 0;
}
// END CUT
```

5 Euclid

```
// This is a collection of useful code for solving problems that
// involve modular linear equations. Note that all of the
// algorithms described here work on nonnegative integers.

#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

typedef vector<int> VI;
typedef pair<int, int> PII;

// return a % b (positive value)
int mod(int a, int b) {
    return ((a%b) + b) % b;
}

// computes gcd(a,b)
```

```
int gcd(int a, int b) {
       while (b) { int t = a%b; a = b; b = t; }
       return a;
}
// computes lcm(a,b)
int lcm(int a, int b) {
       return a / gcd(a, b)*b;
}
// (a^b) mod m via successive squaring
int powermod(int a, int b, int m)
{
       int ret = 1;
       while (b)
              if (b & 1) ret = mod(ret*a, m);
              a = mod(a*a, m);
              b >>= 1;
       return ret;
}
// returns g = gcd(a, b); finds x, y such that d = ax + by
int extended_euclid(int a, int b, int &x, int &y) {
       int xx = y = 0;
       int yy = x = 1;
       while (b) {
              int q = a / b;
              int t = b; b = a%b; a = t;
              t = xx; xx = x - q*xx; x = t;
              t = yy; yy = y - q*yy; y = t;
       }
       return a;
}
// finds all solutions to ax = b (mod n)
VI modular_linear_equation_solver(int a, int b, int n) {
       int x, y;
       VI ret;
       int g = extended_euclid(a, n, x, y);
       if (!(b%g)) {
              x = mod(x*(b / g), n);
              for (int i = 0; i < g; i++)</pre>
                      ret.push_back(mod(x + i*(n / g), n));
```

```
}
       return ret;
}
// computes b such that ab = 1 (mod n), returns -1 on failure
int mod_inverse(int a, int n) {
       int x, y;
       int g = extended_euclid(a, n, x, y);
       if (g > 1) return -1;
       return mod(x, n);
}
// Chinese remainder theorem (special case): find z such that
// z % m1 = r1, z % m2 = r2. Here, z is unique modulo M = lcm(m1, m2).
// Return (z, M). On failure, M = -1.
PII chinese_remainder_theorem(int m1, int r1, int m2, int r2) {
       int g = extended_euclid(m1, m2, s, t);
       if (r1%g != r2%g) return make_pair(0, -1);
       return make_pair(mod(s*r2*m1 + t*r1*m2, m1*m2) / g, m1*m2 / g);
}
// Chinese remainder theorem: find z such that
// z % m[i] = r[i] for all i. Note that the solution is
// unique modulo M = lcm_i (m[i]). Return (z, M). On
// failure, M = -1. Note that we do not require the a[i]'s
// to be relatively prime.
PII chinese_remainder_theorem(const VI &m, const VI &r) {
       PII ret = make_pair(r[0], m[0]);
       for (int i = 1; i < m.size(); i++) {</pre>
              ret = chinese_remainder_theorem(ret.second, ret.first,
                   m[i], r[i]):
              if (ret.second == -1) break;
       }
       return ret;
}
// computes x and y such that ax + by = c
// returns whether the solution exists
bool linear_diophantine(int a, int b, int c, int &x, int &y) {
       if (!a && !b)
       {
              if (c) return false;
              x = 0; y = 0;
              return true;
```

```
}
       if (!a)
               if (c % b) return false;
               x = 0; y = c / b;
               return true;
       }
       if (!b)
       {
               if (c % a) return false;
               x = c / a; y = 0;
               return true;
       }
       int g = gcd(a, b);
       if (c % g) return false;
       x = c / g * mod_inverse(a / g, b / g);
       y = (c - a*x) / b;
       return true;
}
int main() {
       // expected: 2
       cout << gcd(14, 30) << endl;</pre>
       // expected: 2 -2 1
       int x, y;
       int g = extended_euclid(14, 30, x, y);
       cout << g << " " << x << " " << y << endl;
       // expected: 95 451
       VI sols = modular_linear_equation_solver(14, 30, 100);
       for (int i = 0; i < sols.size(); i++) cout << sols[i] << " ";</pre>
       cout << endl;</pre>
       // expected: 8
       cout << mod_inverse(8, 9) << endl;</pre>
       // expected: 23 105
                   11 12
       PII ret = chinese_remainder_theorem(VI({ 3, 5, 7 }), VI({ 2, 3, 2})
       cout << ret.first << " " << ret.second << endl;</pre>
       ret = chinese_remainder_theorem(VI({ 4, 6 }), VI({ 3, 5 }));
       cout << ret.first << " " << ret.second << endl;</pre>
```

```
// expected: 5 -15
if (!linear_diophantine(7, 2, 5, x, y)) cout << "ERROR" << endl;
cout << x << " " << y << endl;
return 0;
}</pre>
```

6 FastExpo

```
Uses powers of two to exponentiate numbers and matrices. Calculates
n^k in O(\log(k)) time when n is a number. If A is an n x n matrix,
calculates A^k in O(n^3*log(k)) time.
*/
#include <iostream>
#include <vector>
using namespace std;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;
T power(T x, int k) {
 T ret = 1:
  while(k) {
   if(k & 1) ret *= x;
   k >>= 1; x *= x;
 return ret;
VVT multiply(VVT& A, VVT& B) {
  int n = A.size(), m = A[0].size(), k = B[0].size();
  VVT C(n, VT(k, 0));
  for(int i = 0; i < n; i++)</pre>
   for(int j = 0; j < k; j++)
     for(int 1 = 0; 1 < m; 1++)</pre>
       C[i][i] += A[i][1] * B[1][i];
```

```
return C;
}
VVT power(VVT& A, int k) {
 int n = A.size();
 VVT ret(n, VT(n)), B = A;
 for(int i = 0; i < n; i++) ret[i][i]=1;</pre>
 while(k) {
   if(k & 1) ret = multiply(ret, B);
   k >>= 1; B = multiply(B, B);
 return ret;
}
int main()
  /* Expected Output:
    2.37^48 = 9.72569e + 17
    376 264 285 220 265
    550 376 529 285 484
    484 265 376 264 285
    285 220 265 156 264
    529 285 484 265 376 */
 double n = 2.37;
 int k = 48;
  cout << n << "^" << k << " = " << power(n, k) << endl;
 double At[5][5] = {
   { 0, 0, 1, 0, 0 },
   { 1, 0, 0, 1, 0 },
   \{0, 0, 0, 0, 1\},\
   { 1, 0, 0, 0, 0 },
   { 0, 1, 0, 0, 0 } };
  vector <vector <double> > A(5, vector <double>(5));
 for(int i = 0; i < 5; i++)
   for(int j = 0; j < 5; j++)
     A[i][j] = At[i][j];
 vector <vector <double> > Ap = power(A, k);
  cout << endl;</pre>
```

```
for(int i = 0; i < 5; i++) {
  for(int j = 0; j < 5; j++)
    cout << Ap[i][j] << " ";
  cout << endl;
}</pre>
```

7 FastIO

```
import java.io.*;
import java.util.InputMismatchException;
/**
* Created by Shreyans on $DATE at $TIME using IntelliJ IDEA (Fast IO
     Template)
class $NAME
   public static void main(String[] args) throws Exception
       InputReader in = new InputReader(System.in);
       OutputWriter out = new OutputWriter(System.out);
       out.printLine("Hello World");
          out.close();
   }
   //FAST IO
   private static class InputReader
       private InputStream stream;
       private byte[] buf = new byte[1024];
       private int curChar;
       private int numChars;
       private SpaceCharFilter filter;
       public InputReader(InputStream stream)
           this.stream = stream;
       }
```

```
public int read()
   if (numChars == -1)
       throw new InputMismatchException();
   if (curChar >= numChars)
   {
       curChar = 0;
       try
           numChars = stream.read(buf);
       } catch (IOException e)
           throw new InputMismatchException();
       if (numChars <= 0)</pre>
           return -1;
   }
   return buf[curChar++];
public int readInt()
   int c = read();
   while (isSpaceChar(c))
       c = read();
   int sgn = 1;
   if (c == '-')
       sgn = -1;
       c = read();
   }
   int res = 0;
   do
       if (c < '0' || c > '9')
           throw new InputMismatchException();
       res *= 10;
       res += c - '0';
       c = read();
   } while (!isSpaceChar(c));
   return res * sgn;
}
public String readString()
```

```
{
   int c = read();
   while (isSpaceChar(c))
       c = read();
   StringBuilder res = new StringBuilder();
       res.appendCodePoint(c);
       c = read();
   } while (!isSpaceChar(c));
   return res.toString();
public double readDouble() {
   int c = read();
   while (isSpaceChar(c))
       c = read();
   int sgn = 1;
   if (c == '-') {
       sgn = -1;
       c = read();
   }
   double res = 0;
   while (!isSpaceChar(c) && c != '.') {
       if (c == 'e' || c == 'E')
           return res * Math.pow(10, readInt());
       if (c < '0' || c > '9')
           throw new InputMismatchException();
       res *= 10;
       res += c - '0';
       c = read();
   if (c == '.') {
       c = read();
       double m = 1;
       while (!isSpaceChar(c)) {
           if (c == 'e' || c == 'E')
              return res * Math.pow(10, readInt());
           if (c < '0' || c > '9')
              throw new InputMismatchException();
           m /= 10;
           res += (c - '0') * m;
           c = read();
   return res * sgn;
```

```
}
   public long readLong() {
       int c = read();
       while (isSpaceChar(c))
          c = read();
       int sgn = 1;
       if (c == '-') {
          sgn = -1;
          c = read();
       long res = 0;
       do {
          if (c < '0' || c > '9')
              throw new InputMismatchException();
          res *= 10;
          res += c - '0';
          c = read();
       } while (!isSpaceChar(c));
       return res * sgn;
   public boolean isSpaceChar(int c)
       if (filter != null)
          return filter.isSpaceChar(c);
       return c == ' ' || c == '\n' || c == '\r' || c == '\t' || c ==
           -1;
   }
   public String next()
       return readString();
   public interface SpaceCharFilter
       public boolean isSpaceChar(int ch);
private static class OutputWriter
   private final PrintWriter writer;
   public OutputWriter(OutputStream outputStream)
```

}

```
writer = new PrintWriter(new BufferedWriter(new
               OutputStreamWriter(outputStream)));
       }
       public OutputWriter(Writer writer)
           this.writer = new PrintWriter(writer);
       }
       public void print(Object... objects)
          for (int i = 0; i < objects.length; i++)</pre>
              if (i != 0)
                  writer.print(' ');
              writer.print(objects[i]);
          }
       }
       public void printLine(Object... objects)
           print(objects);
           writer.println();
       public void close()
       {
           writer.close();
       }
       public void flush()
           writer.flush();
       }
   }
/* USAGE
//initialize
       InputReader in
                             = new InputReader(System.in);
       OutputWriter out
                                     new OutputWriter(System.out);
//read int
  int i = in.readInt();
//read string
```

```
String s = in.readString();
//read int array of size N
  int[] x = IOUtils.readIntArray(in,N);
//printline
  out.printLine("X");
//flush output
  out.flush();
//remember to close the
//outputstream, at the end
  out.close();
   /*class IOUtils {
       public static int[] readIntArray(InputReader in, int size) {
           int[] array = new int[size];
          for (int i = 0; i < size; i++)
              array[i] = in.readInt();
          return array; */
```

8 FenwickTree

```
class FenwickTree {
private: vi ft;
    // recall that vi is: typedef vector<int> vi;
public:
    FenwickTree(long n) { ft.assign(n + 1, 0); }
    // init n + 1 zeroes
    long LSOne(long n){return (n & (-n));}
    long long rsq(long b)
    {
        // returns RSQ(1, b)
        long long sum = 0; for (; b; b -= LSOne(b)) sum += ft[b];
        return sum;
    }
    long long rsq(long a, long b)
    {
        return rsq(b) - (a == 1 ? 0 : rsq(a - 1));
}
```

9 FenwickTree2D

```
long tree[max_xy] [max_xy];
void update(long x , long y , long val){
    long y1;
    while (x <= max_xy){</pre>
       v1 = v;
       while (y1 <= max_xy){</pre>
           tree[x][y1] += val;
           y1 += (y1 \& -y1);
       }
       x += (x \& -x);
}
long long read(long x,long y){ // return sum from 1,1 to x,y.
  long long sum= 0;
    while(x){
         long y1 = y;
        while(y1){
            sum += tree[x][y1];
              y1 -= y1 \& -y1;
       }
       x -= x \& -x;
       return sum;
}
int main()
```

```
long long suma = read(x2+1,y2+1) + read(x1,y1) - read(x2+1,y1) -
    read(x1,y2+1); //suma de rango entre x1,y1 y x2,y2
    update(x+1,y+1, num ); //aumenta en num
}
```

10 FloydWar

```
//ASSP!!!
// This function runs the Floyd-Warshall algorithm for all-pairs
// shortest paths. Also handles negative edge weights. Returns true
// if a negative weight cycle is found.
// Running time: O(|V|^3)
//
// INPUT: w[i][j] = weight of edge from i to j
    OUTPUT: w[i][j] = shortest path from i to j
            prev[i][j] = node before j on the best path starting at i
bool FloydWarshall (VVT &w, VVI &prev){
 int n = w.size();
 prev = VVI (n, VI(n, -1));
 for (int k = 0; k < n; k++){
   for (int i = 0; i < n; i++){</pre>
     for (int j = 0; j < n; j++){
       if (w[i][j] > w[i][k] + w[k][j]){
         w[i][j] = w[i][k] + w[k][j];
         prev[i][j] = k;
       }
     }
   }
  // check for negative weight cycles
 for(int i=0;i<n;i++)</pre>
   if (w[i][i] < 0) return false;</pre>
 return true;
}
```

11 Geom3D

```
public class Geom3D {
 // distance from point (x, y, z) to plane aX + bY + cZ + d = 0
 public static double ptPlaneDist(double x, double y, double z,
     double a, double b, double c, double d) {
   return Math.abs(a*x + b*y + c*z + d) / Math.sqrt(a*a + b*b + c*c);
 // distance between parallel planes aX + bY + cZ + d1 = 0 and
 // aX + bY + cZ + d2 = 0
 public static double planePlaneDist(double a, double b, double c,
     double d1, double d2) {
   return Math.abs(d1 - d2) / Math.sqrt(a*a + b*b + c*c);
 // distance from point (px, py, pz) to line (x1, y1, z1)-(x2, y2, z2)
 // (or ray, or segment; in the case of the ray, the endpoint is the
 // first point)
 public static final int LINE = 0;
 public static final int SEGMENT = 1;
 public static final int RAY = 2;
 public static double ptLineDistSq(double x1, double y1, double z1,
     double x2, double y2, double z2, double px, double py, double pz,
     int type) {
   double pd2 = (x1-x2)*(x1-x2) + (y1-y2)*(y1-y2) + (z1-z2)*(z1-z2);
   double x, y, z;
   if (pd2 == 0) {
     x = x1;
     y = y1;
     z = z1:
   } else {
     double u = ((px-x1)*(x2-x1) + (py-y1)*(y2-y1) + (pz-z1)*(z2-z1)) /
         pd2;
     x = x1 + u * (x2 - x1);
     y = y1 + u * (y2 - y1);
     z = z1 + u * (z2 - z1);
     if (type != LINE && u < 0) {</pre>
      x = x1;
       y = y1;
      z = z1:
     if (type == SEGMENT && u > 1.0) {
```

12 Geometry

```
// C++ routines for computational geometry.
#include <iostream>
#include <vector>
#include <cmath>
#include <cassert>
using namespace std;
double INF = 1e100:
double EPS = 1e-12;
struct PT {
 double x, y;
 PT() {}
 PT(double x, double y) : x(x), y(y) {}
 PT(const PT &p) : x(p.x), y(p.y) {}
 PT operator + (const PT &p) const { return PT(x+p.x, y+p.y); }
 PT operator - (const PT &p) const { return PT(x-p.x, y-p.y); }
 PT operator * (double c) const { return PT(x*c, y*c ); }
 PT operator / (double c) const { return PT(x/c, y/c ); }
}:
```

```
double dot(PT p, PT q) { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q) { return dot(p-q,p-q); }
double cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
  os << "(" << p.x << "," << p.y << ")";
// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p) { return PT(-p.y,p.x); }
PT RotateCW90(PT p) { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
 return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
 return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}
// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
 double r = dot(b-a,b-a);
 if (fabs(r) < EPS) return a:</pre>
 r = dot(c-a, b-a)/r;
 if (r < 0) return a;
 if (r > 1) return b:
 return a + (b-a)*r;
// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
 return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane(double x, double y, double z,
                       double a, double b, double c, double d)
 return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
 return fabs(cross(b-a, c-d)) < EPS;</pre>
```

```
}
bool LinesCollinear(PT a, PT b, PT c, PT d) {
 return LinesParallel(a, b, c, d)
     && fabs(cross(a-b, a-c)) < EPS
     && fabs(cross(c-d, c-a)) < EPS;
}
// determine if line segment from a to b intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
 if (LinesCollinear(a, b, c, d)) {
   if (dist2(a, c) < EPS || dist2(a, d) < EPS ||</pre>
     dist2(b, c) < EPS || dist2(b, d) < EPS) return true;
   if (dot(c-a, c-b) > 0 \&\& dot(d-a, d-b) > 0 \&\& dot(c-b, d-b) > 0)
     return false:
   return true;
 if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
 if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
 return true:
}
// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
// intersection exists; for segment intersection, check if
// segments intersect first
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
 b=b-a; d=c-d; c=c-a;
 assert(dot(b, b) > EPS && dot(d, d) > EPS);
 return a + b*cross(c, d)/cross(b, d);
}
// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
 b=(a+b)/2;
 c=(a+c)/2:
 return ComputeLineIntersection(b, b+RotateCW90(a-b), c,
      c+RotateCW90(a-c));
}
// determine if point is in a possibly non-convex polygon (by William
// Randolph Franklin); returns 1 for strictly interior points, 0 for
// strictly exterior points, and 0 or 1 for the remaining points.
// Note that it is possible to convert this into an *exact* test using
```

```
// integer arithmetic by taking care of the division appropriately
// (making sure to deal with signs properly) and then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
 bool c = 0;
 for (int i = 0; i < p.size(); i++){</pre>
   int j = (i+1)%p.size();
   if ((p[i].y <= q.y && q.y < p[j].y ||</pre>
     p[j].y \le q.y && q.y \le p[i].y) &&
     q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y -
         p[i].y))
     c = !c;
 }
 return c;
// determine if point is on the boundary of a polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {
 for (int i = 0; i < p.size(); i++)</pre>
   if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) < EPS)</pre>
     return true:
   return false;
}
// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a. PT b. PT c. double r) {
 vector<PT> ret;
  b = b-a:
  a = a-c:
 double A = dot(b, b);
  double B = dot(a, b);
 double C = dot(a, a) - r*r;
 double D = B*B - A*C;
 if (D < -EPS) return ret;</pre>
 ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
  if (D > EPS)
   ret.push_back(c+a+b*(-B-sqrt(D))/A);
 return ret:
}
// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R
vector<PT> CircleCircleIntersection(PT a, PT b, double r, double R) {
  vector<PT> ret;
```

```
double d = sqrt(dist2(a, b));
 if (d > r+R \mid | d+min(r, R) < max(r, R)) return ret;
 double x = (d*d-R*R+r*r)/(2*d);
 double y = sqrt(r*r-x*x);
 PT v = (b-a)/d;
 ret.push_back(a+v*x + RotateCCW90(v)*y);
 if (y > 0)
   ret.push_back(a+v*x - RotateCCW90(v)*y);
 return ret;
}
// This code computes the area or centroid of a (possibly nonconvex)
// polygon, assuming that the coordinates are listed in a clockwise or
// counterclockwise fashion. Note that the centroid is often known as
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
 double area = 0;
 for(int i = 0; i < p.size(); i++) {</pre>
   int j = (i+1) % p.size();
   area += p[i].x*p[j].y - p[j].x*p[i].y;
 return area / 2.0;
}
double ComputeArea(const vector<PT> &p) {
 return fabs(ComputeSignedArea(p));
}
PT ComputeCentroid(const vector<PT> &p) {
 PT c(0,0);
 double scale = 6.0 * ComputeSignedArea(p);
 for (int i = 0; i < p.size(); i++){</pre>
   int j = (i+1) % p.size();
   c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
 return c / scale;
}
// tests whether or not a given polygon (in CW or CCW order) is simple
bool IsSimple(const vector<PT> &p) {
 for (int i = 0; i < p.size(); i++) {</pre>
   for (int k = i+1; k < p.size(); k++) {</pre>
     int j = (i+1) % p.size();
     int 1 = (k+1) % p.size();
     if (i == 1 || j == k) continue;
```

```
if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
       return false;
   }
 }
 return true;
int main() {
 // expected: (-5,2)
 cerr << RotateCCW90(PT(2,5)) << endl;</pre>
 // expected: (5,-2)
  cerr << RotateCW90(PT(2,5)) << endl;</pre>
 // expected: (-5,2)
  cerr << RotateCCW(PT(2,5),M_PI/2) << endl;</pre>
 // expected: (5,2)
  cerr << ProjectPointLine(PT(-5,-2), PT(10,4), PT(3,7)) << endl;</pre>
 // expected: (5,2) (7.5,3) (2.5,1)
  cerr << ProjectPointSegment(PT(-5,-2), PT(10,4), PT(3,7)) << " "</pre>
      << ProjectPointSegment(PT(7.5,3), PT(10,4), PT(3,7)) << " "</pre>
      << ProjectPointSegment(PT(-5,-2), PT(2.5,1), PT(3,7)) << endl;</pre>
 // expected: 6.78903
  cerr << DistancePointPlane(4,-4,3,2,-2,5,-8) << endl;</pre>
 // expected: 1 0 1
 cerr << LinesParallel(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "</pre>
      << LinesParallel(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "</pre>
      << LinesParallel(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;</pre>
 // expected: 0 0 1
  cerr << LinesCollinear(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "</pre>
      << LinesCollinear(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "</pre>
      << LinesCollinear(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;</pre>
 // expected: 1 1 1 0
  cerr << SegmentsIntersect(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << " "</pre>
      << SegmentsIntersect(PT(0,0), PT(2,4), PT(4,3), PT(0,5)) << " "
      << SegmentsIntersect(PT(0,0), PT(2,4), PT(2,-1), PT(-2,1)) << " "
      << SegmentsIntersect(PT(0,0), PT(2,4), PT(5,5), PT(1,7)) << endl;</pre>
```

```
// expected: (1,2)
cerr << ComputeLineIntersection(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) <</pre>
    endl;
// expected: (1,1)
cerr << ComputeCircleCenter(PT(-3,4), PT(6,1), PT(4,5)) << endl;</pre>
vector<PT> v;
v.push_back(PT(0,0));
v.push_back(PT(5,0));
v.push_back(PT(5,5));
v.push_back(PT(0,5));
// expected: 1 1 1 0 0
cerr << PointInPolygon(v, PT(2,2)) << " "</pre>
     << PointInPolygon(v, PT(2,0)) << " "</pre>
     << PointInPolygon(v, PT(0,2)) << " "</pre>
     << PointInPolygon(v, PT(5,2)) << " "</pre>
     << PointInPolygon(v, PT(2,5)) << endl;</pre>
// expected: 0 1 1 1 1
cerr << PointOnPolygon(v, PT(2,2)) << " "</pre>
     << PointOnPolygon(v, PT(2,0)) << " "
     << PointOnPolygon(v, PT(0,2)) << " "
     << PointOnPolygon(v, PT(5,2)) << " "
     << PointOnPolygon(v, PT(2,5)) << endl;</pre>
// expected: (1,6)
//
            (5,4)(4,5)
            blank line
11
            (4,5) (5,4)
            blank line
            (4.5)(5.4)
vector<PT> u = CircleLineIntersection(PT(0,6), PT(2,6), PT(1,1), 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;</pre>
u = CircleLineIntersection(PT(0,9), PT(9,0), PT(1,1), 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;</pre>
u = CircleCircleIntersection(PT(1,1), PT(10,10), 5, 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;</pre>
u = CircleCircleIntersection(PT(1,1), PT(8,8), 5, 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;</pre>
u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 10, sqrt(2.0)/2.0);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;</pre>
u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 5, sqrt(2.0)/2.0);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;</pre>
```

```
// area should be 5.0
// centroid should be (1.1666666, 1.166666)
PT pa[] = { PT(0,0), PT(5,0), PT(1,1), PT(0,5) };
vector<PT> p(pa, pa+4);
PT c = ComputeCentroid(p);
cerr << "Area: " << ComputeArea(p) << endl;
cerr << "Centroid: " << c << endl;
return 0;
}</pre>
```

13 Kadane1DMaxSum

```
#include <bits/stdc++.h>
using namespace std;
int max1DSum(int* arr,int size)
       int max_ending_here,max_so_far;
       max_ending_here = max_so_far = arr[0];
       for (int i = 1; i < size; ++i)</pre>
               max_ending_here = max(arr[i],max_ending_here + arr[i]);
               max_so_far = max(max_so_far, max_ending_here);
       return max_so_far;
}
int main()
Ł
       int arr[9] = \{5,3, -16,7, -8,9,10, -14,33\};
       //print: max 1D sum = 38
       cout << "max 1D sum = " << max1DSum(arr,9) << endl;</pre>
       return 0;
```

14 Kruskal

```
/*
Uses Kruskal's Algorithm to calculate the weight of the minimum spanning
forest (union of minimum spanning trees of each connected component) of
a possibly disjoint graph, given in the form of a matrix of edge weights
(-1 if no edge exists). Returns the weight of the minimum spanning
forest (also calculates the actual edges - stored in T). Note: uses a
disjoint-set data structure with amortized (effectively) constant time per
union/find. Runs in O(E*log(E)) time.
#include <iostream>
#include <vector>
#include <algorithm>
#include <queue>
using namespace std;
typedef int T;
struct edge
 int u, v;
 Td;
};
struct edgeCmp
 int operator()(const edge& a, const edge& b) { return a.d > b.d; }
};
int find(vector <int>& C, int x) { return (C[x] == x) ? x : C[x] =
    find(C, C[x]); }
T Kruskal(vector <vector <T> >& w)
{
 int n = w.size();
 T weight = 0;
 vector <int> C(n), R(n);
 for(int i=0; i<n; i++) { C[i] = i; R[i] = 0; }</pre>
 vector <edge> T;
 priority_queue <edge, vector <edge>, edgeCmp> E;
```

```
for(int i=0; i<n; i++)</pre>
   for(int j=i+1; j<n; j++)</pre>
     if(w[i][j] >= 0)
     {
       edge e;
       e.u = i; e.v = j; e.d = w[i][j];
       E.push(e);
     }
  while(T.size() < n-1 && !E.empty())</pre>
   edge cur = E.top(); E.pop();
   int uc = find(C, cur.u), vc = find(C, cur.v);
   if(uc != vc)
     T.push_back(cur); weight += cur.d;
     if(R[uc] > R[vc]) C[vc] = uc;
     else if(R[vc] > R[uc]) C[uc] = vc;
     else { C[vc] = uc; R[uc]++; }
   }
 }
 return weight;
int main()
{
 int wa[6][6] = {
   \{ 0, -1, 2, -1, 7, -1 \},
   \{-1, 0, -1, 2, -1, -1\},\
   \{ 2, -1, 0, -1, 8, 6 \},\
   \{-1, 2, -1, 0, -1, -1\},\
   \{7, -1, 8, -1, 0, 4\},\
   \{-1, -1, 6, -1, 4, 0\};
  vector <vector <int> > w(6, vector <int>(6));
 for(int i=0; i<6; i++)</pre>
   for(int j=0; j<6; j++)
     w[i][j] = wa[i][j];
  cout << Kruskal(w) << endl;</pre>
  cin >> wa[0][0];
```

}

$15 \quad LCA$

```
const int max_nodes, log_max_nodes;
int num_nodes, log_num_nodes, root;
vector<int> children[max_nodes];
                                    // children[i] contains the children
    of node i
int A[max_nodes][log_max_nodes+1]; // A[i][j] is the 2^j-th ancestor of
    node i, or -1 if that ancestor does not exist
int L[max_nodes];
                                    // L[i] is the distance between node
    i and the root
// floor of the binary logarithm of n
int lb(unsigned int n)
   if(n==0)
       return -1;
   int p = 0;
   if (n >= 1<<16) { n >>= 16; p += 16; }
   if (n >= 1<< 8) { n >>= 8; p += 8; }
   if (n >= 1<< 4) { n >>= 4; p += 4; }
   if (n >= 1<< 2) { n >>= 2; p += 2; }
   if (n >= 1<< 1) {
                           p += 1; }
   return p;
}
void DFS(int i, int 1)
   L[i] = 1;
   for(int j = 0; j < children[i].size(); j++)</pre>
       DFS(children[i][j], l+1);
}
int LCA(int p, int q)
{
   // ensure node p is at least as deep as node q
   if(L[p] < L[q])
       swap(p, q);
```

```
// "binary search" for the ancestor of node p situated on the same
        level as q
   for(int i = log_num_nodes; i >= 0; i--)
       if(L[p] - (1 << i) >= L[q])
           p = A[p][i];
   if(p == q)
       return p;
   // "binary search" for the LCA
   for(int i = log_num_nodes; i >= 0; i--)
       if(A[p][i] != -1 && A[p][i] != A[q][i])
           p = A[p][i];
           q = A[q][i];
   return A[p][0];
int main(int argc,char* argv[])
   // read num_nodes, the total number of nodes
   log_num_nodes=lb(num_nodes);
   for(int i = 0; i < num_nodes; i++)</pre>
       int p;
       // read p, the parent of node i or -1 if node i is the root
       A[i][0] = p;
       if(p != -1)
           children[p].push_back(i);
       else
           root = i;
   }
   // precompute A using dynamic programming
   for(int j = 1; j <= log_num_nodes; j++)</pre>
       for(int i = 0; i < num_nodes; i++)</pre>
           if(A[i][j-1] != -1)
              A[i][j] = A[A[i][j-1]][j-1];
           else
              A[i][j] = -1;
```

```
// precompute L
DFS(root, 0);

return 0;
```

16 LongestIncreasingSubsequence

```
// Given a list of numbers of length n, this routine extracts a
// longest increasing subsequence.
// Running time: O(n log n)
//
    INPUT: a vector of integers
    OUTPUT: a vector containing the longest increasing subsequence
#include <iostream>
#include <vector>
#include <algorithm>
using namespace std;
typedef vector<int> VI;
typedef pair<int,int> PII;
typedef vector<PII> VPII;
#define STRICTLY_INCREASNG
VI LongestIncreasingSubsequence(VI v) {
 VPII best;
 VI dad(v.size(), -1);
 for (int i = 0; i < v.size(); i++) {</pre>
#ifdef STRICTLY_INCREASNG
   PII item = make_pair(v[i], 0);
   VPII::iterator it = lower_bound(best.begin(), best.end(), item);
   item.second = i;
#else
   PII item = make_pair(v[i], i);
   VPII::iterator it = upper_bound(best.begin(), best.end(), item);
#endif
```

```
if (it == best.end()) {
   dad[i] = (best.size() == 0 ? -1 : best.back().second);
   best.push_back(item);
} else {
   dad[i] = dad[it->second];
   *it = item;
}

VI ret;
for (int i = best.back().second; i >= 0; i = dad[i])
   ret.push_back(v[i]);
   reverse(ret.begin(), ret.end());
   return ret;
}
```

17 MaxFlow

```
vector<vector<long> > res; //adj
vector<vi> edgeflows; //edge flow
long mf, f, s, t; // global variables
vi p; // p stores the BFS spanning tree from s
void augment(long v, long minEdge)
{ // traverse BFS spanning tree from s->t
      if (v == s) { f = minEdge; return; } // record minEdge in a global
          var f
      else if (p[v] != -1)
            augment(p[v], min(minEdge,edgeflows[p[v]][v]));
            edgeflows[p[v]][v] -= f; edgeflows[v][p[v]] += f;
      }
}
appropriate values
{
      while (1)
```

```
f = 0:
              vi dist(MAX_V, INF); dist[s] = 0; queue<long> q; q.push(s);
              p.assign(MAX_V, -1); // record the BFS spanning tree, from
                   s to t!
              while (!q.empty())
              {
                      long u = q.front(); q.pop();
                      if (u == t) break; // immediately stop BFS if we
                          already reach sink t
                      for (long v:res[u]) // note: this part is slow
                             if (edgeflows[u][v] > 0 && dist[v] == INF)
                                    dist[v] = dist[u] + 1, q.push(v),
                                        p[v] = u; // 3 lines in 1!
              }
              augment(t, INF); // find the min edge weight f in this
                   path, if any
              if (f == 0) break; // we cannot send any more flow (f =
                   0), terminate
              mf += f; // we can still send a flow, increase the max
                   flow!
       }
       printf("%ld\n", mf);
}
```

18 Primes

```
// O(sqrt(x)) Exhaustive Primality Test
#include <cmath>
#define EPS 1e-7
typedef long long LL;
bool IsPrimeSlow (LL x)
{
   if(x<=1) return false;
   if(x<=3) return true;
   if (!(x%2) || !(x%3)) return false;
   LL s=(LL)(sqrt((double)(x))+EPS);
   for(LL i=5;i<=s;i+=6)
   {
    if (!(x%i) || !(x%(i+2))) return false;</pre>
```

```
}
 return true;
}
// Primes less than 1000:
                               13
                                                              37
           43
                47
                     53
                          59
                               61
                                    67
                                         71
                                              73
                                                   79
                                                         83
                                                              89
      97
          101
               103
                    107
                         109
                              113
                                   127
                                        131
                                             137
                                                  139
                                                        149
                                                             151
//
     157
          163
               167
                    173
                         179
                              181
                                   191
                                        193
                                              197
                                                   199
                                                             223
                                                        211
//
     227
          229
               233
                    239
                         241
                              251
                                   257
                                         263
                                              269
                                                   271
                                                        277
                                                             281
     283
          293
               307
                    311
                         313
                              317
                                   331
                                        337
                                             347
                                                  349
                                                        353
                                                             359
//
          373
                    383
     367
               379
                         389
                              397
                                   401
                                        409
                                              419
                                                   421
                                                        431
                                                             433
     439
          443
               449
                    457
                         461
                              463
                                   467
                                        479
                                              487
                                                   491
                                                        499
                                                             503
//
    509
          521
               523
                    541
                                        569
                                             571
                         547
                              557
                                   563
                                                  577
                                                        587
                                                             593
//
          601
               607
                    613
                                   631
                                        641
                                             643
     599
                         617
                              619
                                                   647
                                                        653
                                                             659
//
          673
                    683
                                   709
                                             727
     661
               677
                         691
                              701
                                        719
                                                  733
                                                        739
                                                             743
     751
          757
                    769
                         773
                                   797
                                        809
                                             811
               761
                              787
                                                   821
                                                        823
                                                             827
//
     829
          839
               853
                    857
                         859
                              863
                                   877
                                        881
                                             883
                                                   887
                                                        907
                                                             911
     919
          929
                    941
                         947
                              953
                                         971
                                                   983
                                                        991
                                                             997
// Other primes:
     The largest prime smaller than 10 is 7.
     The largest prime smaller than 100 is 97.
//
     The largest prime smaller than 1000 is 997.
     The largest prime smaller than 10000 is 9973.
     The largest prime smaller than 100000 is 99991.
11
     The largest prime smaller than 1000000 is 999983.
//
     The largest prime smaller than 10000000 is 9999991.
//
     The largest prime smaller than 100000000 is 99999989.
11
     The largest prime smaller than 1000000000 is 999999937.
11
     The largest prime smaller than 1000000000 is 9999999967.
//
     The largest prime smaller than 10000000000 is 9999999977.
//
     The largest prime smaller than 100000000000 is 99999999999.
//
     The largest prime smaller than 1000000000000 is 99999999971.
11
     The largest prime smaller than 1000000000000 is 999999999973
11
     The largest prime smaller than 10000000000000 is 9999999999999999.
     The largest prime smaller than 1000000000000000 is
    99999999999937.
```

19 RollingHash

```
#include <bits/stdc++.h>
using namespace std;
struct RollingHash
{
       string s;
       const long L;
       long long hashValue;
       long ini;
       long end;
       long long base;
       RollingHash(const string &s,long
           hlength):L(hlength),s(s),ini(0),end(L-1),hashValue(0)
       {
              base = 1;
              for (long i = L-1; i >= 0; --i)
                      //hashValue += (s[i]) << ((L-1) - i);
                      hashValue += s[i] * base;
                      if(i!=0)base*=127;
              }
       }
       long long getHashValue(){return hashValue;}
       //returns the next hash value. If the last value is reached, next
            calls will return the last value.
       long long nextHashValue()
       {
              if(end == s.length()) return hashValue;
              hashValue -= (s[ini]) * base;//<< (L-1);
              hashValue*=127:
              hashValue+= (s[end+1]);
              ini++;
              end++;
              return hashValue;
       }
       void reset()
              hashValue = 0;
```

```
for (long i = 0; i < L; ++i)</pre>
              {
                      hashValue += (s[i]) << ((L-1) - i);
              end = end - ini;
              ini = 0;
       }
       bool finished()
              return end==s.length();
       }
};
int main()
{
       string s1 = "aggg";
       string s2 = "aggtmreaggaggtmreagg";
       RollingHash RH(s1,s1.length());
       long long h1 = RH.getHashValue();
       RollingHash RH2(s2,s1.length());
       long long count = RH2.getHashValue()==h1?1:0;
       while(!RH2.finished())
       {
              if(RH2.nextHashValue()==h1)count++;
       }
       //ocurrences of "agg" in "aggtmreaggaggtmreagg" are 4
       cout << "ocurrences of \""<< s1 << "\" in \""<< s2<< "\" are "<<
            count << endl:
```

20 SCC

```
#define INF INT_MAX;

vector<vii> AdjList;
vi dfs_num, dfs_low, S, visited;
```

```
int dfsNumberCounter,numSCC,UNVISITED=-1,VISITED=1;
void tarjanSCC(int u)
       dfs_low[u] = dfs_num[u] = dfsNumberCounter++; // dfs_low[u] <=</pre>
           dfs num[u]
       S.push_back(u);
       // stores u in a vector based on order of visitation
       visited[u] = 1:
       for (int j = 0; j < (int)AdjList[u].size(); j++)</pre>
              ii v = AdjList[u][j];
              if (dfs_num[v.first] == UNVISITED)
              tarjanSCC(v.first);
              if (visited[v.first])
              // condition for update
              dfs_low[u] = min(dfs_low[u], dfs_low[v.first]);
       if (dfs low[u] == dfs num[u])
              bool agg = false;
              while (1)
              {
                      int v = S.back(); S.pop_back(); visited[v] = 0;
                      //aca se ouede imprimir la visita si es necesario
                      if (u == v) break;
              }
       }
```

21 SegmentTree

```
#include <bits/stdc++.h>
using namespace std;

typedef vector<long long> vi;

#define MAX (1+(1<<6)) // Why? :D
#define inf 0x7fffffff //9223372036854775807 max llong
//st para max queries</pre>
```

```
class SegmentTree
private:
       int N;
       vi arr;
       long long tree[MAX];
       long long lazy[MAX];
       /**
        * Build and init tree
       void build_tree(long long node, long long a, long long b) {
              if(a > b) return; // Out of range
              if(a == b) { // Leaf node
                     tree[node] = arr[a]; // Init value
                     return;
              }
              build_tree(node*2, a, (a+b)/2); // Init left child
              build_tree(node*2+1, 1+(a+b)/2, b); // Init right child
              tree[node] = max(tree[node*2], tree[node*2+1]); // Init
                   root value
       }
       /**
        * Increment elements within range [i, j] with value value
       void update_tree(long long node, long long a, long long b, long
           long i, long long j, long long value) {
              if(lazy[node] != 0) { // This node needs to be updated
                     tree[node] += lazy[node]; // Update it
                     if(a != b) {
                             lazy[node*2] += lazy[node]; // Mark child as
                             lazy[node*2+1] += lazy[node]; // Mark child
                                 as lazy
                     lazy[node] = 0; // Reset it
              }
```

```
if(a > b || a > j || b < i) // Current segment is not</pre>
           within range [i, j]
              return;
       if(a >= i && b <= j) { // Segment is fully within range</pre>
              tree[node] += value:
              if(a != b) { // Not leaf node
                      lazy[node*2] += value;
                      lazy[node*2+1] += value;
              }
              return;
       }
       update_tree(node*2, a, (a+b)/2, i, j, value); // Updating
           left child
       update_tree(1+node*2, 1+(a+b)/2, b, i, j, value); //
           Updating right child
       tree[node] = max(tree[node*2], tree[node*2+1]); //
           Updating root with max value
}
/**
 * Query tree to get max element value within range [i, j]
long long query_tree(long long node, long long a, long long b,
    long long i, long long j) {
       if(a > b || a > j || b < i) return -inf; // Out of range</pre>
       if(lazy[node] != 0) { // This node needs to be updated
              tree[node] += lazy[node]; // Update it
              if(a != b) {
                      lazy[node*2] += lazy[node]; // Mark child as
                      lazy[node*2+1] += lazy[node]; // Mark child
                          as lazy
              }
              lazy[node] = 0; // Reset it
       }
```

```
if(a >= i && b <= j) // Current segment is totally within</pre>
                   range [i, j]
                      return tree[node];
              long long q1 = query_tree(node*2, a, (a+b)/2, i, j); //
                   Query left child
              long long q2 = query_tree(1+node*2, 1+(a+b)/2, b, i, j);
                   // Query right child
              long long res = max(q1, q2); // Return final result
              return res;
       }
public:
       SegmentTree(long long N):N(N)
              arr.assign(N,0);
              build_tree(1, 0, N-1);
              memset(lazy, 0, sizeof lazy);
       }
       void update(long long i,long long j, long long v)
       {
              update_tree(1, 0, N-1, i, j, v);
       }
       long long rmq(long long i,long long j)
              query_tree(1, 0, N-1, i, j);
       }
};
//st para sum queries
class SegmentTreeSum
private:
       int N;
       vi arr;
       long long tree[MAX];
       long long lazy[MAX];
       /**
        * Build and init tree
```

```
void build_tree(long long node, long long a, long long b) {
       if(a > b) return; // Out of range
       if(a == b) { // Leaf node
              tree[node] = arr[a]; // Init value
              return;
       }
       build_tree(node*2, a, (a+b)/2); // Init left child
       build_tree(node*2+1, 1+(a+b)/2, b); // Init right child
       tree[node] = tree[node*2] + tree[node*2+1]; // Init root
           value
}
/**
 * Increment elements within range [i, j] with value value
 */
void update_tree(long long node, long long a, long long b, long
    long i, long long j, long long value) {
       if(lazy[node] != 0) { // This node needs to be updated
              tree[node] += (b-a+1)*lazy[node]; // Update it
              if(a != b) {
                      lazy[node*2] += lazy[node]; // Mark child as
                      lazy[node*2+1] += lazy[node]; // Mark child
                          as lazy
              }
              lazy[node] = 0; // Reset it
       }
       if(a > b \mid \mid a > j \mid \mid b < i) // Current segment is not
           within range [i, j]
              return;
       if(a >= i && b <= j) { // Segment is fully within range</pre>
              tree[node] += (b-a+1)*value;
              if(a != b) { // Not leaf node
                      lazv[node*2] += value;
                      lazy[node*2+1] += value;
              }
```

```
return:
       }
       update_tree(node*2, a, (a+b)/2, i, j, value); // Updating
           left child
       update_tree(1+node*2, 1+(a+b)/2, b, i, j, value); //
           Updating right child
       tree[node] = (tree[node*2] + tree[node*2+1]); // Updating
           root with max value
}
/**
 * Query tree to get max element value within range [i, j]
long long query_tree(long long node, long long a, long long b,
    long long i, long long j) {
       if(a > b || a > j || b < i) return 0; // Out of range</pre>
       if(lazy[node] != 0) { // This node needs to be updated
              tree[node] += (b-a+1)*lazy[node]; // Update it
              if(a != b) {
                     lazy[node*2] += lazy[node]; // Mark child as
                     lazy[node*2+1] += lazy[node]; // Mark child
                          as lazy
              }
              lazy[node] = 0; // Reset it
       }
       if(a >= i && b <= j) // Current segment is totally within
           range [i, j]
              return tree[node];
       long long q1 = query_tree(node*2, a, (a+b)/2, i, j); //
           Query left child
       long long q2 = query_tree(1+node*2, 1+(a+b)/2, b, i, j);
           // Query right child
       long long res = (q1 + q2); // Return final result
```

```
return res;
       }
public:
       SegmentTreeSum(long long N):N(N)
              arr.assign(N,0);
              build_tree(1, 0, N-1);
              memset(lazy, 0, sizeof lazy);
       }
       void update(long long i,long long j, long long v)
              update_tree(1, 0, N-1, i, j, v);
       }
       long long rsq(long long i,long long j)
       {
              return query_tree(1, 0, N-1, i, j);
       }
};
int main()
{
       int N = 20;
       SegmentTree st(N);
       st.update(0, 6, 5); // Increment range [0, 6] by 5. here 0, N-1
           represent the current range.
       st.update(7, 10, 12); // Increment range [7, 10] by 12. here 0,
           N-1 represent the current range.
       st.update(11, N-1, 100); // Increment range [10, N-1] by 100. here
           0, N-1 represent the current range.
       cout << st.rmq(0, N-1) << endl; // Get max element in range [0,
           N-1
       return 0;
}
```

22 SuffixArray

```
// Suffix array construction in O(L \log^2 L) time. Routine for // computing the length of the longest common prefix of any two
```

```
// suffixes in O(log L) time.
//
// INPUT: string s
// OUTPUT: array suffix[] such that suffix[i] = index (from 0 to L-1)
           of substring s[i...L-1] in the list of sorted suffixes.
11
           That is, if we take the inverse of the permutation suffix[],
//
           we get the actual suffix array.
#include <bits/stdc++.h>
using namespace std;
struct SuffixArray {
 const int L;
 string s;
 vector<vector<int> > P;
 vector<pair<int,int>,int> > M;
 SuffixArray(const string &s) : L(s.length()), s(s), P(1, vector<int>(L,
      0)), M(L) {
   for (int i = 0; i < L; i++) P[0][i] = int(s[i]);</pre>
   for (int skip = 1, level = 1; skip < L; skip *= 2, level++) {</pre>
     P.push_back(vector<int>(L, 0));
     for (int i = 0; i < L; i++)</pre>
       M[i] = make_pair(make_pair(P[level-1][i], i + skip < L ?</pre>
           P[level-1][i + skip] : -1000), i);
     sort(M.begin(), M.end());
     for (int i = 0; i < L; i++)</pre>
       P[level][M[i].second] = (i > 0 && M[i].first == M[i-1].first) ?
           P[level][M[i-1].second] : i;
   }
 }
 vector<int> GetSuffixArray() { return P.back(); }
  // returns the length of the longest common prefix of s[i...L-1] and
      s[j...L-1]
 int LongestCommonPrefix(int i, int j) {
   int len = 0;
   if (i == j) return L - i;
   for (int k = P.size() - 1; k >= 0 && i < L && j < L; k--) {
     if (P[k][i] == P[k][j]) {
       i += 1 << k;
       i += 1 << k;
```

```
len += 1 << k;
     }
   }
   return len;
 }
};
int main() {
   // bobocel is the 0'th suffix
   // obocel is the 5'th suffix
   // bocel is the 1'st suffix
   // ocel is the 6'th suffix
       cel is the 2'nd suffix
          el is the 3'rd suffix
           l is the 4'th suffix
       string s = "amandamandamandamanda";
   SuffixArray suffix(s);
   vector<int> v = suffix.GetSuffixArray();
   // Expected output: 0 5 1 6 2 3 4
   for (int i = 0; i < v.size(); i++) cout << v[i] << " ";</pre>
       cout << endl:</pre>
   vector<int> v2(v.size(),0);
   for (int i = 0; i < v.size(); i++)</pre>
   {
         v2[v[i]] = i;
   }
   for(int n:v2)
         cout << s.substr(n) << endl;</pre>
   }
```

23 UnionFind

```
#include <iostream>
#include <vector>
using namespace std;
//map implementation, requires c++11
```

```
class UnionFind
{
       private: unordered_map<int,int> p, rank; // remember: vi is
       public:
              UnionFind(int N)
                     //rank.assign(N, 0);
                     //p.assign(N, 0);
                     //for (int i = 0; i < N; i++) p[i] = i;
              int findSet(int i) { if(p.count(i)==0)p[i]=i; return (p[i]
                   == i) ? i : (p[i] = findSet(p[i])); }
              bool isSameSet(int i, int j) { return findSet(i) ==
                   findSet(j); }
              void unionSet(int i, int j)
              {
                     if (!isSameSet(i, j))
                     { // if from different set
                             int x = findSet(i), y = findSet(j);
                             if (rank[x] > rank[y]) p[y] = x; // rank
                                 keeps the tree short
                             else
                             {
                                    p[x] = y;
                                    if (rank[x] == rank[y]) rank[y] =
                                        rank[y]+1;
                     }
              }
};
```