Tarea 7

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Link al notebook con las celdas de python para la resolucion de los ejercicios

https://github.com/JesuaVAlc/Deberes-Metodos-Numericos/blob/main/Tarea7_JesuaVillacis.ipynb

Conjunto de ejercios

1) Dados los puntos (0,1), (1,5), (2,3), determine el spline cúbico.

$$\begin{aligned}
S_0(x) &= a_0 + b_0(x - x_0) + c_0(x - x_0)^2 + d_0(x - x_0)^3 \\
&= a_0 + b_0x + c_0x^2 + d_0x^3 \\
S_1(x) &= a_1 + b_1(x - x_1) + c_0(x - x_1)^2 + d_1(x - x_1)^3 \\
&= a_1 + b_1(x - 1) + c_0(x - 1)^2 + d_0(x - 1)^3
\end{aligned}$$

[Ecuacion 1]

$$S_0(x_0) = y_0 \Rightarrow a_0 + b_0x + c_0x^2 + d_0x^3$$
$$a_0 = y_0 \Rightarrow a_0 = 1$$

[Ecuacion 2]

$$S_0(x_1) = y_1 \Rightarrow 1 + b_0x + c_0x^2 + d_0x^3$$
$$b_0x + c_0x^2 + d_0x^3 = 4$$

[Ecuacion 3]

$$S_1(x_1) = y_1 \Rightarrow a_1 + b_1(x-1) + c_0(x-1)^2 + d_1(x-1)^3$$
$$a_0 = y_1 \Rightarrow a_1 = 5$$

[Ecuacion 4]

$$S_1(x_2) = y_2 \Rightarrow 5 + b_1(x-1) + c_0(x-1)^2 + d_1(x-1)^3$$
$$b_1 + c_0 + d_1 = -2$$

[Ecuacion 5]

$S_i' = b_i + 2c_i(x - x_i) + 3d_i(x - x_i)^2$

$$S_0'(x_1) = S_1'(x_1)$$
$$b_0 + 2c_0x + 3d_0x^2 = b_1 + 2c_1(x-1)^2 + 3d_1(x-1)^2$$
$$b_0 + 2c_0 + 3d_0^2 = b_1$$

[Ecuacion 6]

$S_i'' = 2c_i + 6d_i(x - x_i)$

$$S_0''(x_1) = S_1''(x_1)$$
$$2c_0 + 6d_0x = 2c_1 + 6d_1(x-1)$$
$$2c_0 + 6d_0 = 2c_1$$

[Ecuacion 7] Frontera Natural

$$S_0''(x_0) = 0$$
$$2c_0 = 0$$
$$c_0 = 0$$

[Ecuacion 7] Frontera Natural

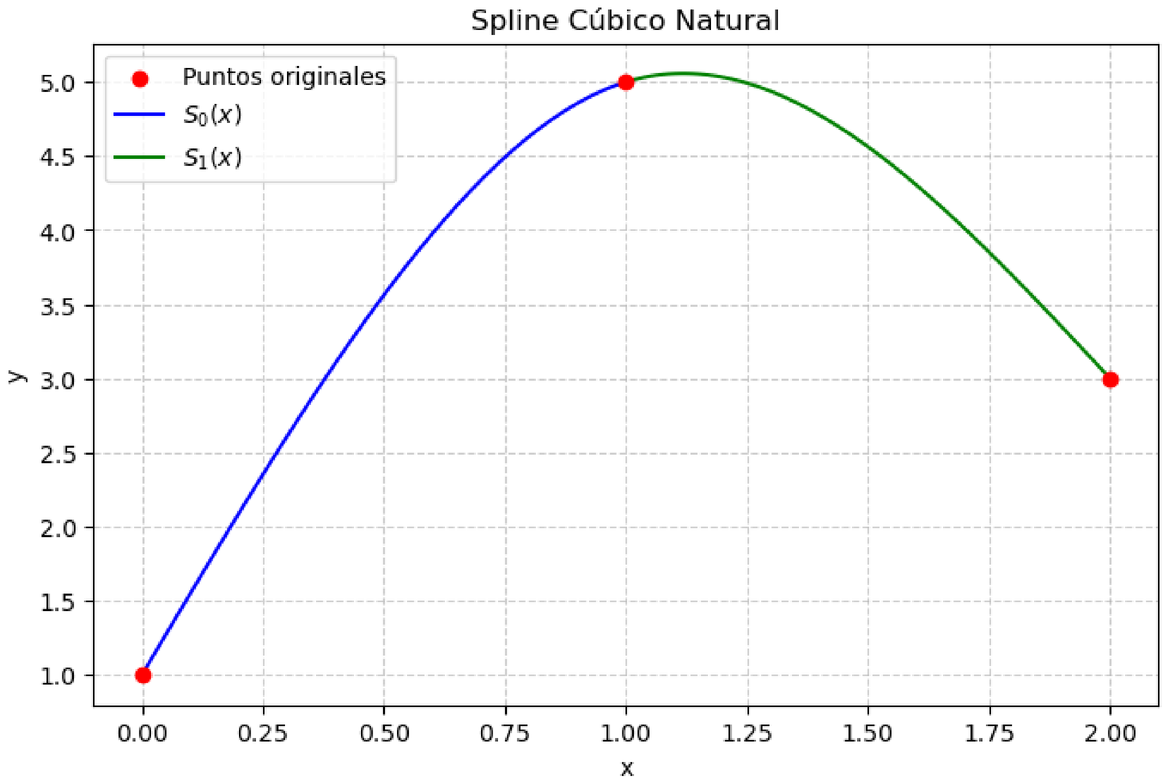
$$S_1''(x_2) = 0$$
$$2c_1 + 6d_1(x_2 - x_1) = 0$$
$$2c_1 + 6d_1 = 0$$

Sistema de ecuaciones

$$\begin{cases} b_0 + d_0 = 4 \\ b_1 + c_1 + d_1 = -2 \\ b_0 + 3d_0 = b_1 \\ 3d_0 = c_1 \\ c_1 + 3d_1 = 0 \end{cases}$$

Splines Completos

$$\begin{cases} S_0(x) = 1 + \frac{11}{2}x - \frac{3}{2}x^3 \\ S_1(x) = 5 + (x-1) - \frac{9}{2}(x-1)^2 - \frac{3}{2}(x-1)^3 \end{cases}$$



2) Dados los puntos $(-1{,}1)$, $(1{,}3)$, determine el spline cúbico sabiendo que $f'(x_0) = 1 \quad y \quad f'(x_n) = 2$.

$$\begin{aligned} S_0(x) &= a_0 + b_0(x - x_0) + c_0(x - x_0)^2 + d_0(x - x_0)^3 \\ &= a_0 + b_0(x + 1) + c_0(x + 1)^2 + d_0(x + 1)^3 \end{aligned}$$

[Ecuacion 1]

$$S_0(x_0) = y_0$$
$$a_0 = y_0 \Rightarrow a_0 = 1$$

[Ecuacion 2]

$$S_0(x_1) = y_1 \Rightarrow 1 + b_0(x+1) + c_0(x+1)^2 + d_0(x+1)^3$$
$$1 + b_0(2) + c_0(2)^2 + d_0(2)^3 = 3$$
$$2b_0 + 4c_0 + 8d_0 = 2$$

[Ecuacion 3] Frontera Condicionada

$$S_0'(x_0) = 1$$
$$b_0 + 2c_0(x+1) + 3d_0(x+1)^2 = 1$$
$$b_0 = 1$$

[Ecuacion 4] Frontera Condicionada

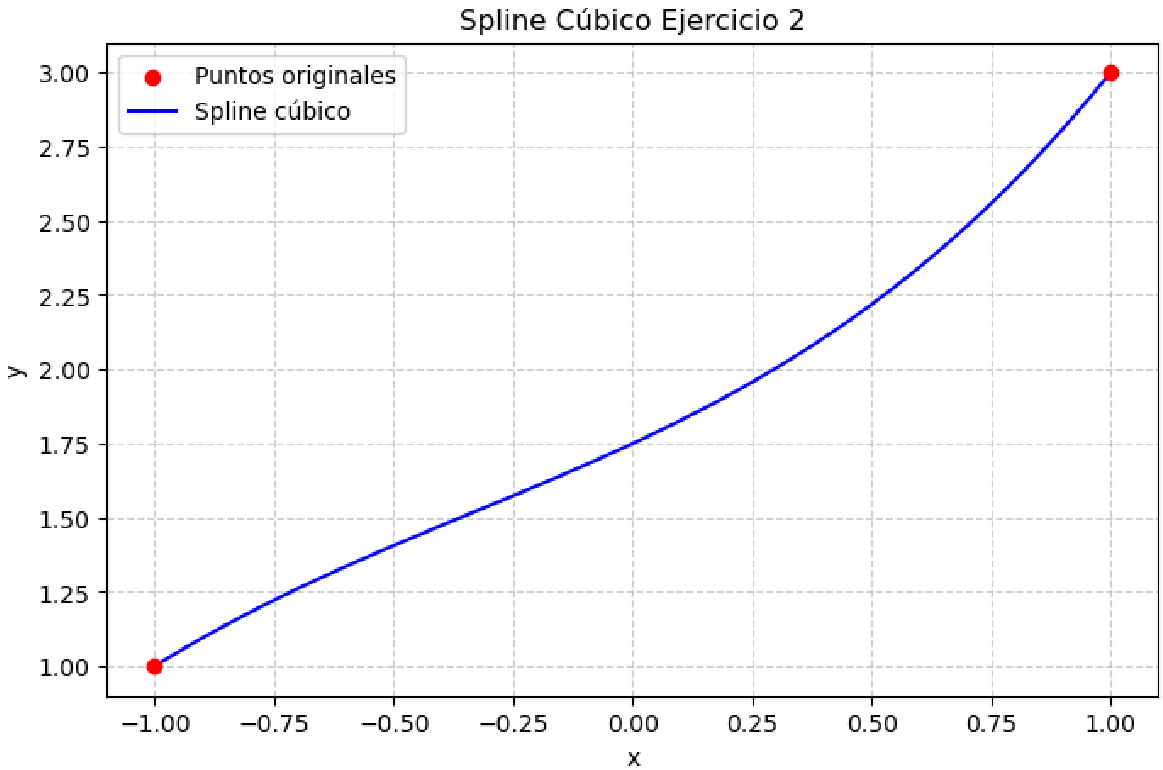
$$S_0'(x_1) = 2$$
$$b_0 + 2c_0(x+1) + 3d_0(x+1)^2 = 2$$
$$b_0 + 4c_0 + 12d_0 = 1$$

Sistema de ecuaciones

$$\begin{cases} b_0 = 1 \\ 2b_0 + 4c_0 + 8d_0 = 2 \\ b_0 + 4c_0 + 12d_0 = 2 \end{cases}$$

Spline Completos

$S_0(x) = 1 + (x+1) - \frac{1}{2}(x+1)^2 + \frac{1}{4}(x+1)^3$



3)) Diríjase al pseudocódigo del spline cúbico con frontera natural provisto en clase, en base a ese pseudocódigo complete la siguiente función: https://github.com/ztjona/EPN-numerical-analysis/blob/main/cubic_splines.ipynb

4) Usando la función anterior, encuentre el spline cúbico para:

```
xs = [1, 2, 3]
ys = [2, 3, 5]
```

```
1 3 1.5 0.75 -0.25
0 2 0.75 0.0 0.25
```

$0.75x + 0.25\left(x - 1\right)^{3} + 1.25$

$1.5x - 0.25\left(x - 2\right)^{3} + 0.75\left(x - 2\right)^{2}$

$0.25x^3 - 0.75x^2 + 1.5x + 1.0$

$-0.25x^3 + 2.25x^2 - 4.5x + 5.0$

5) Usando la función anterior, encuentre el spline cúbico para:

```
xs = [0, 1, 2, 3]
ys = [-1 ,1, 5, 2]
```

```
2 5 1.0 -6.0 2.0
1 1 4.0 3.0 -3.0
0 -1 1.0 0.0 1.0
```

$1.0x^3 + 1.0x - 1$

$4.0x - 3.0\left(x-1\right)^3 + 3.0\left(x-1\right)^2 - 3.0$

$1.0x + 2.0\left(x-2\right)^3 - 6.0\left(x-2\right)^2 + 3.0$

```
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```

$1.0x^3 + 1.0x - 1$

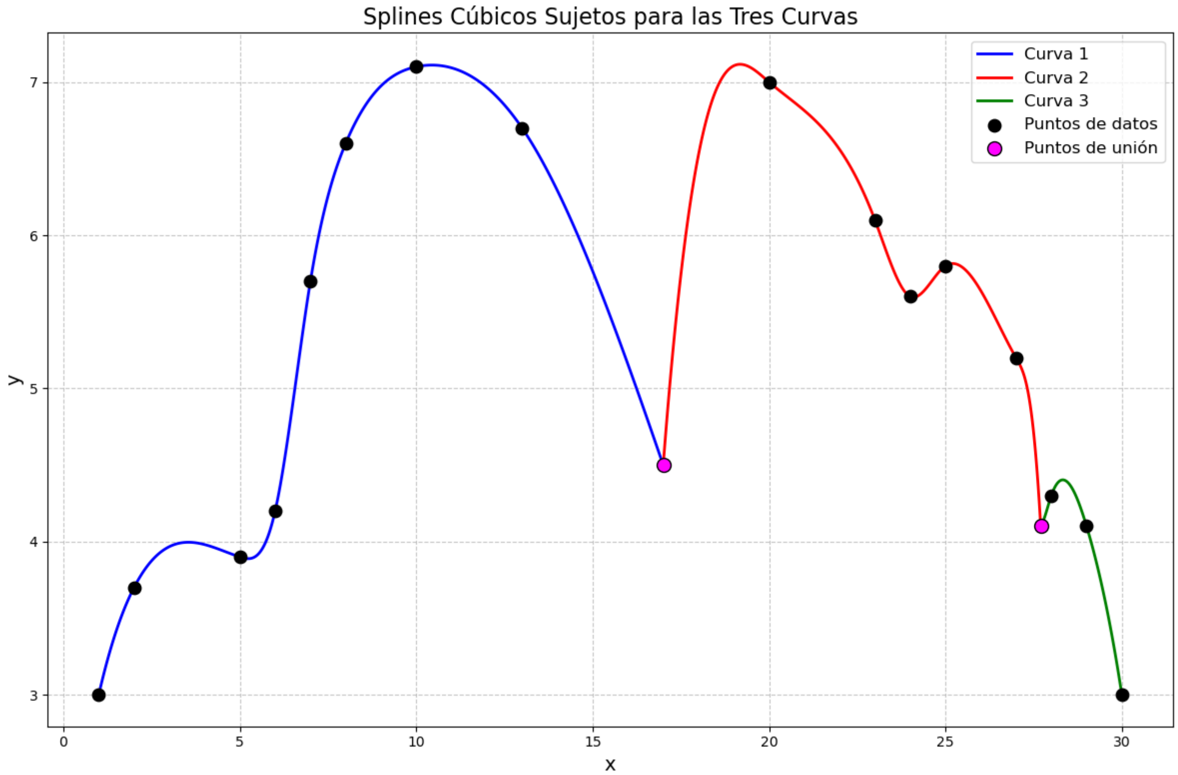
$-3.0x^3 + 12.0x^2 - 11.0x + 3.0$

$2.0x^3 - 18.0x^2 + 49.0x - 37.0$

6) Use la función cubic_spline_clamped, provista en el enlace de Github, para graficar los datos de la siguiente tabla.

`Curva 1` `Curva 2` `Curva 3`

i	x	f(x)	f (x)	i	x	f(x)	f (x)	i	x	f(x)	f (x)
0	1	3.0	1.0	0	17	4.5	3.0	0	27.7	4.1	0.33
1	2	3.7		1	20	7.0		1	28	4.3	
2	5	3.9		2	23	6.1		2	29	4.1	
3	6	4.2		3	24	5.6		3	30	3.0	–1.5
4	7	5.7		4	25	5.8					
5	8	6.6		5	27	5.2					
6	10	7.1		6	27.7	4.1	–4.0				
7	13	6.7									
8	17	4.5	–0.67								


Splines Cúbicos Sujetos para las Tres Curvas
Curva 1
Curva 2
Curva 3
Puntos de datos
Puntos de unión
y
x
0
5
10
15
20
25
30
3
4
5
6
7