Tarea 6

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Link del repositorio:

https://github.com/JesuaVAlc/Deberes-Metodos-Numericos/blob/main/Resolucion_ejercicios_Tarea6.ipynb

Determine el orden de la mejor aproximación para las siguientes funciones, usando la Serie de Taylor y el Polinomio de Lagrange:

1. $\frac{1}{25x^2+1}; x_0 = 0$

- **Series de Taylor**

i	$i!$	$f^{(i)}$	$f^{(i)}(x_0)$	P_i
0	1	$\frac{1}{25x^2+1}$	1	$1(x-0)^0$
1	1	$-\frac{50x}{(25x^2+1)^2}$	0	$1(x-0)^1$
2	2	$\frac{3750x^2-50}{(25x^2+1)^3}$	-25	$-25(x-0)^2$
3	6	$\frac{-375000x^3+15000x}{(25x^2+1)^4}$	0	$0(x-0)^3$
4	24	$\frac{4687500x^4+2200000x^2+15000}{(25x^2+1)^5}$	15000	$625(x-0)^4$

Resultado= $P(x) = 1 - 25x^2 + 625x^4$

- **Polinomio de Lagrange**

Teniendo los puntos $(-\frac{2}{10}, \frac{1}{2})(-\frac{1}{10}, \frac{4}{5})(0, 1)(\frac{1}{10}, \frac{4}{5})(\frac{2}{10}, \frac{1}{2})$

$P(x) = y_0 * L_0 + y_1 * L_1 + y_2 * L_2 + y_3 * L_3 + y_4 * L_4$ - L_0

$$\begin{aligned} L_0 &= \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} \\ &= \frac{(x+0.1)(x-0)(x-0.1)(x-0.2)}{(-0.2+0.1)(-0.2-0)(-0.2-0.1)(-0.2-0.2)} \\ &= \frac{x^4-0.2x^3-0.01x^2+0.002x}{0.0024} \end{aligned}$$

- L_1

$$
\begin{aligned}
L_1 &= \frac{(x+0.2)(x)(x-0.1)(x-0.2)}{(-0.1+0.2)(-0.1-0)(-0.1-0.1)(-0.1-0.2)} \\
&= \frac{x^4-0.1x^3-0.04x^2+0.004x}{-0.0006}
\end{aligned}
$$

- L_2

$$
\begin{aligned}
L_2 &= \frac{(x+0.2)(x+0.1)(x-0.1)(x-0.2)}{(0+0.2)(0+0.1)(0-0.1)(0-0.2)} \\
&= \frac{x^4-0.05x^2+0.0004}{0.0004}
\end{aligned}
$$

- L_3

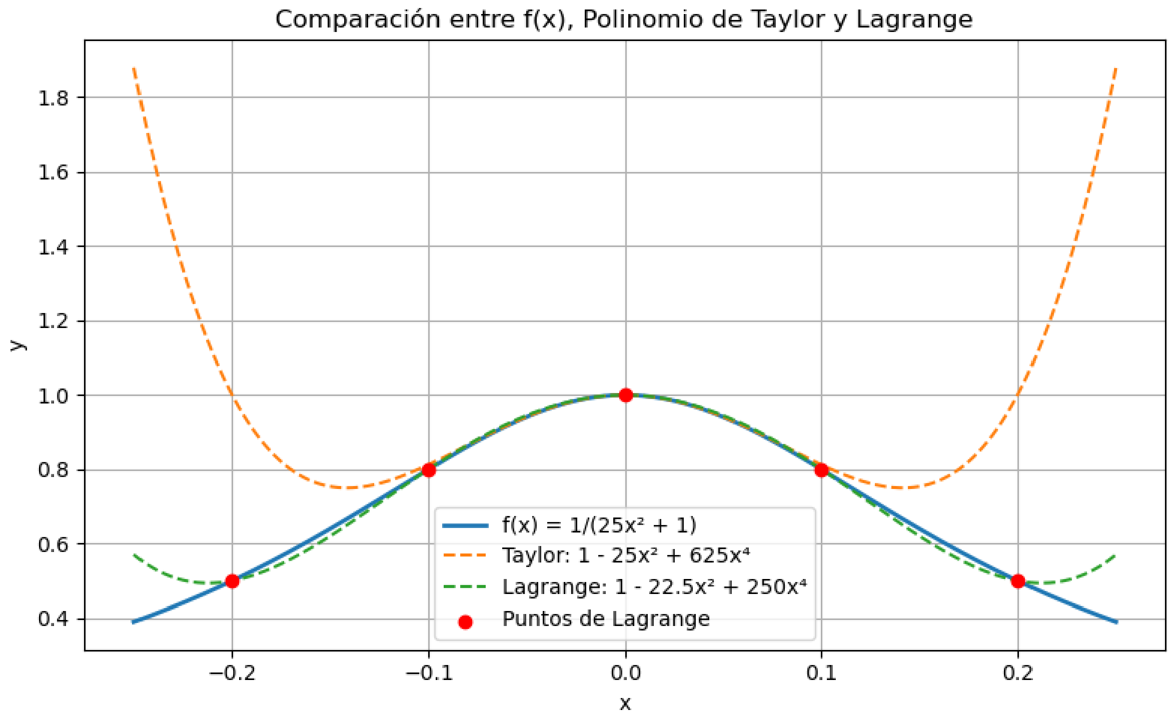
$$
\begin{aligned}
L_3 &= \frac{(x+0.2)(x+0.1)(x)(x-0.2)}{(0.1+0.2)(0.1+0.1)(0.1-0)(0.1-0.2)} \\
&= \frac{x^4+0.1x^3-0.04x^2-0.004x}{-0.0006}
\end{aligned}
$$

- L_4

$$
\begin{aligned}
L_4 &= \frac{(x+0.2)(x+0.1)(x-0)(x-0.1)}{(0.2+0.2)(0.2+0.1)(0.2-0)(0.2-0.1)} \\
&= \frac{x^4+0.2x^3-0.01x^2-0.002x}{0.0024}
\end{aligned}
$$

$$
\begin{aligned}
P(x) &= (\frac{1}{2})(\frac{x^4-0.2x^3-0.01x^2+0.002x}{0.0024}) + (\frac{4}{5})(\frac{x^4-0.1x^3-0.04x^2+0.004x}{-0.0006}) + (1)(\frac{x^4-0.05x^2+0.0004}{0.0004}) + \\
&= 250x^4-22.5x2+1
\end{aligned}
$$

Graficas



- Podemos observar como el polinomio de Lagrange se ajusta mejor en un orden de 4 que la serie de taylor para la función dada.

1. $arctan(x); x_0 = 1$

- **Series de Taylor**

i	$i!$	$f^{(i)}$	$f^{(i)}(x_0)$	P_i
0	1	$arctan(x)$	$\frac{\pi}{4}$	$\frac{\pi}{4}(x-1)^0$
1	1	$\frac{1}{1+x^2}$	$\frac{1}{2}$	$\frac{1}{2}(x-1)^1$
2	2	$\frac{-2x}{(1+x^2)^2}$	$-\frac{1}{2}$	$-\frac{1}{4}(x-1)^2$
3	6	$\frac{6x^2-2}{(1+x^2)^3}$	$\frac{1}{2}$	$\frac{1}{12}(x-1)^3$
4	24	$-\frac{24x(x^2-1)}{(x^2+1)^4}$	0	$0(x-1)^4$

Resultado= $P(x) = \frac{pi}{4} + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2 + \frac{1}{12}(x-1)^3$

- **Polinomio de Lagrange**

Teniendo los puntos $(0.5, 0.4636); (0.75, 0.6435); (1.0, 0.7854); (1.25, 0.8961); (1.5, 0.9828)$

$P(x) = y_0 * L_0 + y_1 * L_1 + y_2 * L_2 + y_3 * L_3 + y_4 * L_4$ - L_0

$$\begin{aligned} L_0 &= \frac{(x-0.75)(x-1.0)(x-1.25)(x-1.5)}{(0.5-0.75)(0.5-1.0)(0.5-1.25)(0.5-1.5)} \\ &= \frac{(x-0.75)(x-1.0)(x-1.25)(x-1.5)}{(-0.25)(-0.5)(-0.75)(-1.0)} \\ &= \frac{x^4-4.5x^3+7.1875x^2-4.6875x+0.9375}{0.09375} \\ &= 10.67x^4-48x^3+76.\overline{6}x^2-50x+10 \end{aligned}$$

- L_1

$$\begin{aligned} L_1 &= \frac{(x-0.5)(x-1.0)(x-1.25)(x-1.5)}{(0.75-0.5)(0.75-1.0)(0.75-1.25)(0.75-1.5)} \\ &= \frac{(x-0.5)(x-1.0)(x-1.25)(x-1.5)}{(0.25)(-0.25)(-0.5)(-0.75)} \\ &= \frac{x^4-4.25x^3+6.0625x^2-3.4375x+0.5625}{-0.0234375} \\ &= -42.\overline{6}x^4+181.3x^3-258.6x^2+146.6x-24 \end{aligned}$$

- L_2

$$\begin{aligned} L_2 &= \frac{(x-0.5)(x-0.75)(x-1.25)(x-1.5)}{(1.0-0.5)(1.0-0.75)(1.0-1.25)(1.0-1.5)} \\ &= \frac{(x-0.5)(x-0.75)(x-1.25)(x-1.5)}{(0.5)(0.25)(-0.25)(-0.5)} \\ &= \frac{x^4-4x^3+5.3125x^2-2.625x+0.46875}{0.015625} \\ &= 64x^4-256x^3+340x^2-168x+30 \end{aligned}$$

- L_3

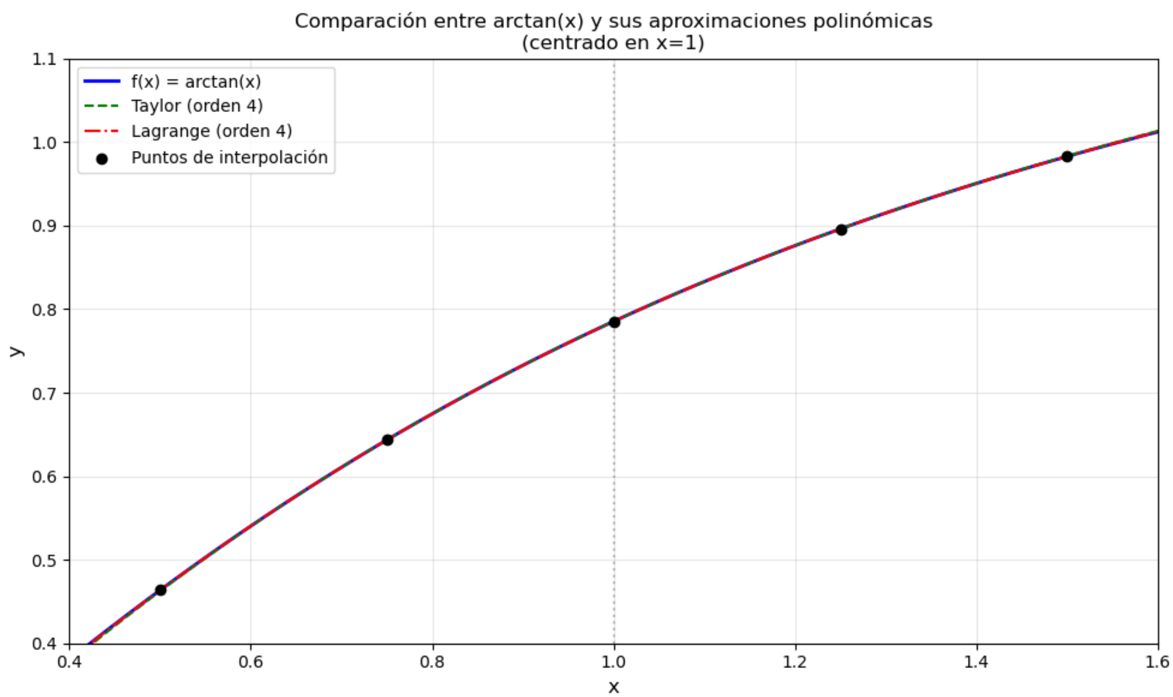
$$\begin{aligned} L_3 &= \frac{(x-0.5)(x-0.75)(x-1.0)(x-1.5)}{(1.25-0.5)(1.25-0.75)(1.25-1.0)(1.25-1.5)} \\ &= \frac{(x-0.5)(x-0.75)(x-1.0)(x-1.5)}{(0.75)(0.5)(0.25)(-0.25)} \\ &= \frac{x^4-3.75x^3+4.8125x^2-2.0625x+0.28125}{-0.0234375} \\ &= -42.6x^4+160x^3-205.3x^2+88x-12 \end{aligned}$$

- L_4

$$\begin{aligned}
L_4 &= \frac{(x-0.5)(x-0.75)(x-1.0)(x-1.25)}{(1.5-0.5)(1.5-0.75)(1.5-1.0)(1.5-1.25)} \\
&= \frac{(x-0.5)(x-0.75)(x-1.0)(x-1.25)}{(1.0)(0.75)(0.5)(0.25)} \\
&= \frac{x^4-3.5x^3+4.375x^2-1.5625x+0.1171875}{0.09375} \\
&= 10.6x^4-37.3x^3+46.6x^2-16.6x+1.25
\end{aligned}$$

$$\begin{aligned}
P(x) &= 0.4636 \cdot L_0(x) + 0.6435 \cdot L_1(x) + 0.7854 \cdot L_2(x) + 0.8961 \cdot L_3(x) + 0.9828 \cdot L_4(x) \\
&= 0.00644628x^4 + 0.04908186x^3 - 0.43624486x^2 + 1.19988835x - 0.03377348
\end{aligned}$$

Graficas



- Gracias a las graficas podemos comprobar que ambos metodos se ajustan muy bien y tienen una interpolacion casi perfecta para este caso cada una estando en el orden 4.