

Tarea 10

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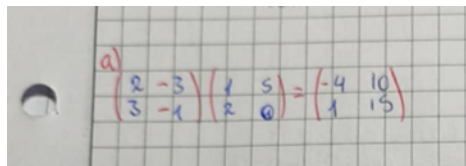
Link al notebook con las celdas de python para la resolucio de los ejercicios

https://github.com/JesuaVAlc/Deberes-Metodos-Numericos/blob/main/Tarea10_JesuaVillacis.ipynb

1. Realice las siguientes multiplicaciones matriz-matriz:

a)

$$\begin{bmatrix} 2 & -3 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 2 & 0 \end{bmatrix}$$

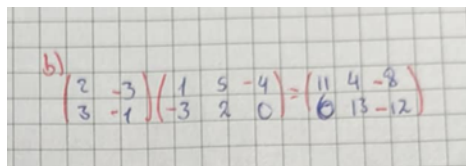


a) $\begin{pmatrix} 2 & -3 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 1 & 5 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} -4 & 10 \\ 1 & 15 \end{pmatrix}$

Figura 1: image.png

b)

$$\begin{bmatrix} 2 & -3 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 5 & -4 \\ -3 & 2 & 0 \end{bmatrix}$$



b) $\begin{pmatrix} 2 & -3 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 1 & 5 & -4 \\ -3 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 11 & 4 & -8 \\ 6 & 13 & -12 \end{pmatrix}$

Figura 2: image.png

c)

$$\begin{bmatrix} 2 & -3 & 1 \\ 4 & 3 & 0 \\ 5 & 2 & -4 \end{bmatrix} \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & -1 \\ 2 & 3 & -2 \end{bmatrix}$$

$$\begin{pmatrix} 2 & -3 & 1 \\ 4 & 3 & 0 \\ 5 & 2 & -4 \end{pmatrix} \begin{pmatrix} 0 & 1 & -2 \\ 1 & 0 & -1 \\ 2 & 3 & -2 \end{pmatrix} = \begin{pmatrix} -1 & 5 & -3 \\ 3 & 4 & -11 \\ -6 & -7 & -4 \end{pmatrix}$$

Figura 3: image.png

d)

$$\begin{bmatrix} 2 & 1 & 2 \\ -2 & 3 & 0 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ -4 & 1 \\ 0 & 2 \end{bmatrix}$$

$$\begin{pmatrix} 2 & 1 & 2 \\ -2 & 3 & 0 \\ 2 & -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ -4 & 1 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 3 \\ -14 & 7 \\ 6 & 1 \end{pmatrix} //$$

Figura 4: image.png

2. Determine cuáles de las siguientes matrices son no singulares y calcule la inversa de esas matrices:

a)

$$\begin{bmatrix} 4 & 2 & 6 \\ 3 & 0 & 7 \\ -2 & -1 & -3 \end{bmatrix}$$

$$\begin{pmatrix} 4 & 2 & 6 \\ 3 & 0 & 7 \\ -2 & -1 & -3 \end{pmatrix} \begin{matrix} 4F_2 - 3F_1 \rightarrow F_2 \\ F_2 + \frac{1}{2}F_1 \rightarrow F_3 \end{matrix} \begin{pmatrix} 4 & 2 & 6 \\ 0 & -6 & -11 \\ 0 & 0 & 0 \end{pmatrix} \begin{matrix} 1 & 0 & 0 \\ -3 & 4 & 0 \\ 0 & 0 & 1 \end{matrix}$$

$\text{Det} = 4(6(0)) = 0$
 \neq Matriz singular

Figura 5: image.png

b)

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -1 \\ 3 & 1 & -1 \end{bmatrix}$$

Handwritten solution for part b) showing row reduction of a 3x3 matrix. The matrix is augmented with the identity matrix. Row operations include $R_2 - R_1$, $R_3 - R_1$, and $R_3 - 5R_2$. The final result shows the determinant is -2 and the inverse matrix is calculated.

Figura 6: image.png

c)

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & 2 & -4 & -2 \\ 2 & 1 & 1 & 5 \\ -1 & 0 & -2 & -4 \end{bmatrix}$$

Handwritten solution for part c) showing row reduction of a 4x4 matrix. The matrix is augmented with the identity matrix. Row operations include $R_2 - R_1$, $R_3 - 2R_1$, $R_4 + R_1$, and $R_3 - R_2$. The final result shows the determinant is 0 and the inverse matrix is calculated.

Figura 7: image.png

d)

$$\begin{bmatrix} 4 & 0 & 0 & 0 \\ 6 & 7 & 0 & 0 \\ 9 & 11 & 1 & 0 \\ 5 & 4 & 1 & 1 \end{bmatrix}$$

d)

$$A = \begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 5 & 4 & 1 & 0 \end{pmatrix} \quad \text{Det } A = (4)(2)(1)(1) = 28$$

$$A^{-1} = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 \\ -\frac{3}{4} & \frac{1}{2} & 1 & 0 \\ \frac{3}{28} & -\frac{1}{28} & \frac{1}{28} & 0 \\ \frac{1}{28} & \frac{1}{28} & -\frac{1}{28} & \frac{1}{28} \end{pmatrix}$$

Figura 8: image.png

3. Resuelva los sistemas lineales 4 x 4 que tienen la misma matriz de coeficientes:

$$\begin{aligned} x_1 - x_2 + 2x_3 - x_4 &= 6, & x_1 - x_2 + 2x_3 - x_4 &= 1 \\ x_1 - x_3 + x_4 &= 4, & x_1 - x_3 + x_4 &= 1 \\ 2x_1 + x_2 + 3x_3 - 4x_4 &= -2, & 2x_1 + x_2 + 3x_3 - 4x_4 &= 2 \\ -x_2 + x_3 - x_4 &= 5, & -x_2 + x_3 - x_4 &= -1 \end{aligned}$$

4. Encuentre los valores de A que hacen que la siguiente matriz sea singular.

$$A = \begin{bmatrix} 1 & -1 & \alpha \\ 2 & 1 & 1 \\ 0 & \alpha & -\frac{3}{2} \end{bmatrix}$$

a)

$$A = \begin{pmatrix} 1 & -1 & \alpha \\ 2 & 1 & 1 \\ 0 & \alpha & -\frac{3}{2} \end{pmatrix} \quad |A| = \begin{vmatrix} 1 & -1 & \alpha \\ 2 & 1 & 1 \\ 0 & \alpha & -\frac{3}{2} \end{vmatrix} = (-3-\alpha) - (-1)(3) + \alpha(2\alpha) = 2\alpha^2 - \alpha - 6 = 0$$

$$(2\alpha + 3)(\alpha - 2) = 0$$

$$\alpha_1 = -\frac{3}{2}$$

$$\alpha_2 = 2$$

* la matriz será singular si $\alpha = -\frac{3}{2}$ o $\alpha = 2$

Figura 9: image.png

5. Resuelva los siguientes sistemas lineales:

a)

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Solución: [-3.75 3.5 1.]

b)

$$\begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$$

Solución: [-4. 4. -1.]

6. Factorice las siguientes matrices en la descomposición LU mediante el algoritmo de factorización LU con $l_{ii} = 1$ para todas las i .

a)

$$\begin{bmatrix} 2 & -1 & 1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{bmatrix}$$

L:

```
[[1. 0. 0.]  
 [1.5 1. 0.]  
 [1.5 1. 1.]]
```

U:

```
[[ 2. -1. 1.]  
 [ 0. 4.5 7.5]  
 [ 0. 0. -4.]]
```

b)

$$\begin{bmatrix} 1.012 & -2.132 & 3.104 \\ -2.132 & 4.096 & -7.013 \\ 3.104 & -7.013 & 0.014 \end{bmatrix}$$

L:

```
[[ 1.          0.          0.          ]  
 [-2.10671937  1.          0.          ]  
 [ 3.06719368  1.19775553  1.          ]]
```

U:

```
[[ 1.012      -2.132      3.104      ]  
 [ 0.         -0.39552569 -0.47374308]  
 [ 0.          0.         -8.93914077]]
```

c)

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 1.5 & 0 & 0 \\ 0 & -3 & 0.5 & 0 \\ 2 & -2 & 1 & 1 \end{bmatrix}$$

L:

$$\begin{bmatrix} 1. & 0. & 0. & 0. &] \\ 0.5 & 1. & 0. & 0. &] \\ 0. & -2. & 1. & 0. &] \\ 1. & -1.33333333 & 2. & 1. &] \end{bmatrix}$$

U:

$$\begin{bmatrix} 2. & 0. & 0. & 0. &] \\ 0. & 1.5 & 0. & 0. &] \\ 0. & 0. & 0.5 & 0. &] \\ 0. & 0. & 0. & 1. &] \end{bmatrix}$$

d)

$$\begin{bmatrix} 2.1756 & 4.0231 & -2.1732 & 5.1967 \\ -4.0231 & 6.0000 & 0 & 1.1973 \\ -1.0000 & -5.2107 & 1.1111 & 0 \\ 6.0235 & 7.0000 & 0 & -4.1561 \end{bmatrix}$$

L:

$$\begin{bmatrix} 1. & 0. & 0. & 0. &] \\ -1.84919103 & 1. & 0. & 0. &] \\ -0.45964332 & -0.25012194 & 1. & 0. &] \\ 2.76866152 & -0.30794361 & -5.35228302 & 1. &] \end{bmatrix}$$

U:

$$\begin{bmatrix} 2.17560000e+00 & 4.02310000e+00 & -2.17320000e+00 & 5.19670000e+00 \\ 0.00000000e+00 & 1.34394804e+01 & -4.01866194e+00 & 1.08069910e+01 \\ 0.00000000e+00 & 4.44089210e-16 & -8.92952394e-01 & 5.09169403e+00 \\ 0.00000000e+00 & 2.37689114e-15 & 0.00000000e+00 & 1.20361280e+01 \end{bmatrix}$$

7. Modifique el algoritmo de eliminación gaussiana de tal forma que se pueda utilizar para resolver un sistema lineal usando la descomposición LU y, a continuación, resuelva los siguientes sistemas lineales.

a)

$$\begin{aligned} 2x_1 - x_2 + x_3 &= -1, \\ 3x_1 + 3x_2 + 9x_3 &= 0, \\ 3x_1 + 3x_2 + 5x_3 &= 4. \end{aligned}$$

L:

```
[[1.  0.  0. ]  
 [1.5 1.  0. ]  
 [1.5 1.  1. ]]
```

U:

```
[[ 2.  -1.  1. ]  
 [ 0.   4.5  7.5]  
 [ 0.   0. -4. ]]
```

Solución: [1. 2. -1.]

b)

$$\begin{aligned}1.012x_1 - 2.132x_2 + 3.104x_3 &= 1.984, \\ -2.132x_1 + 4.096x_2 - 7.013x_3 &= -5.049, \\ 3.104x_1 - 7.013x_2 + 0.014x_3 &= -3.895.\end{aligned}$$

L:

```
[[ 1.          0.          0.          ]  
 [-2.10671937  1.          0.          ]  
 [ 3.06719368  1.19775553  1.          ]]
```

U:

```
[[ 1.012      -2.132      3.104      ]  
 [ 0.         -0.39552569 -0.47374308]  
 [ 0.         0.         -8.93914077]]
```

Solución: [1. 1. 1.]

c)

$$\begin{aligned}2x_1 &= 3, \\ x_1 + 1.5x_3 &= 4.5, \\ -3x_2 + 0.5x_3 &= -6.6, \\ 2x_1 - 2x_2 + x_3 + x_4 &= 0.8.\end{aligned}$$

L:

```
[[ 1.          0.          0.          0.          ]  
 [ 0.5         1.          0.          0.          ]  
 [ 0.         -2.          1.          0.          ]  
 [ 1.         -1.33333333  2.          1.          ]]
```

U:

```
[[2.  0.  0.  0. ]  
 [0.  1.5 0.  0. ]]
```

```

[0.  0.  0.5 0. ]
[0.  0.  0.  1. ]]
Solución: [ 1.5  2.  -1.2  3. ]

```

d)

$$\begin{aligned}
2.1756x_1 + 4.0231x_2 - 2.1732x_3 + 5.1967x_4 &= 17.102, \\
-4.0231x_1 + 6.0000x_2 + 1.1973x_4 &= -6.1593, \\
-1.0000x_1 - 5.2107x_2 + 1.1111x_3 &= 3.0004, \\
6.0235x_1 + 7.0000x_2 - 4.1561x_4 &= 0.0000.
\end{aligned}$$

L:

```

[[ 1.          0.          0.          0.          ]
 [-1.84919103  1.          0.          0.          ]
 [-0.45964332 -0.25012194  1.          0.          ]
 [ 2.76866152 -0.30794361 -5.35228302  1.          ]]

```

U:

```

[[ 2.17560000e+00  4.02310000e+00 -2.17320000e+00  5.19670000e+00]
 [ 0.00000000e+00  1.34394804e+01 -4.01866194e+00  1.08069910e+01]
 [ 0.00000000e+00  4.44089210e-16 -8.92952394e-01  5.09169403e+00]
 [ 0.00000000e+00  2.37689114e-15  0.00000000e+00  1.20361280e+01]]

```

Solución: [2.9398512 0.0706777 5.67773512 4.37981223]