Tarea 10

Jesua Villacis

Link al notebook con las celdas de python para la resolucion de los ejercicios

https://github.com/JesuaVAlc/Deberes-Metodos-Numericos/blob/main/Tarea10_JesuaVillacis.ipynb

1. Realice las siguientes multiplicaciones matriz-matriz:

a)

$$\begin{bmatrix} 2 & -3 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 2 & 0 \end{bmatrix}$$

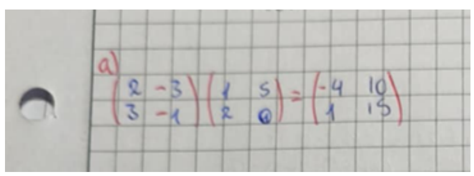

Figura 1: image.png

b)

$$\begin{bmatrix} 2 & -3 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 5 & -4 \\ -3 & 2 & 0 \end{bmatrix}$$

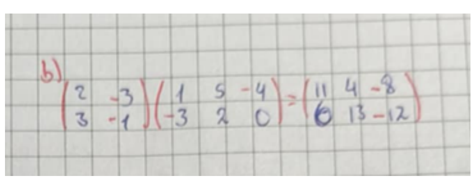

Figura 2: image.png

c)

$$\begin{bmatrix} 2 & -3 & 1 \\ 4 & 3 & 0 \\ 5 & 2 & -4 \end{bmatrix} \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & -1 \\ 2 & 3 & -2 \end{bmatrix}$$

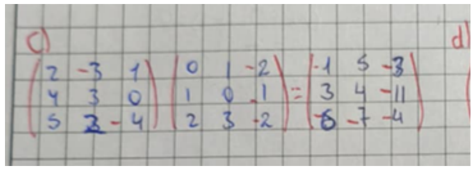

Figura 3: image.png

d)

$$\begin{bmatrix} 2 & 1 & 2 \\ -2 & 3 & 0 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ -4 & 1 \\ 0 & 2 \end{bmatrix}$$

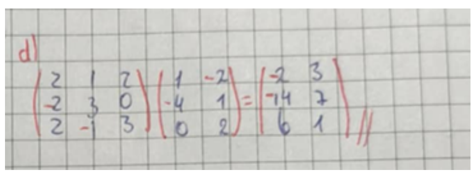

Figura 4: image.png

2. Determine cuáles de las siguientes matrices son no singulares y calcule la inversa de esas matrices:

a)

$$\begin{bmatrix} 4 & 2 & 6 \\ 3 & 0 & 7 \\ -2 & -1 & -3 \end{bmatrix}$$

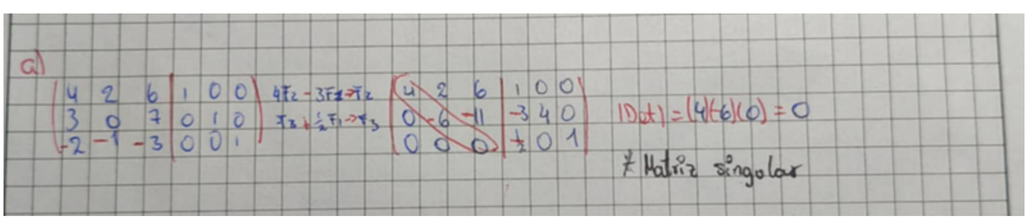

Figura 5: image.png

b)

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -1 \\ 3 & 1 & -1 \end{bmatrix}$$

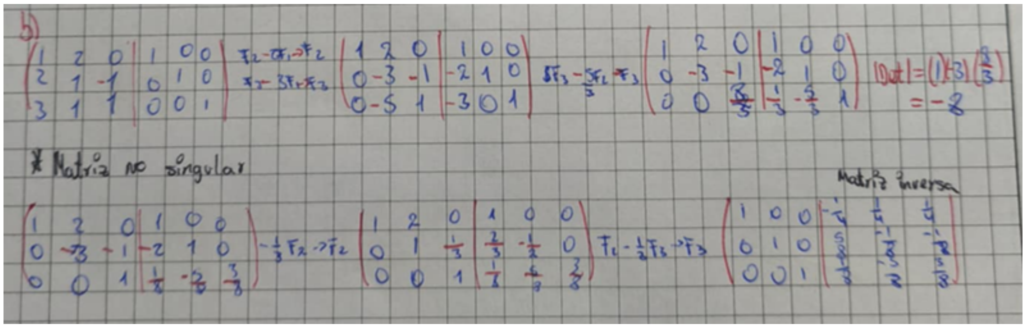

Figura 6: image.png

c)

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & 2 & -4 & -2 \\ 2 & 1 & 1 & 5 \\ -1 & 0 & -2 & -4 \end{bmatrix}$$

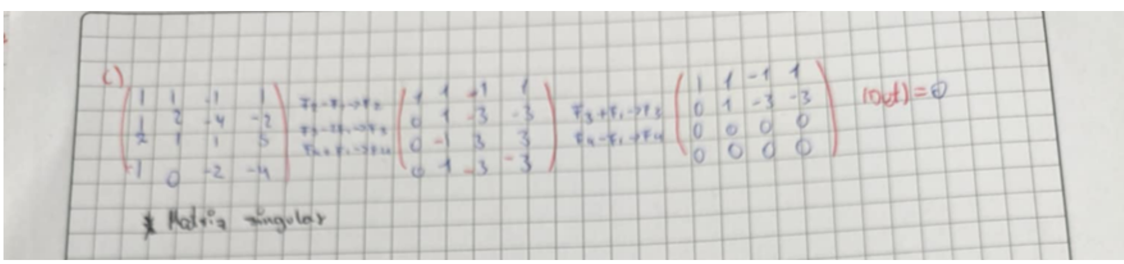

Figura 7: image.png

d)

$$\begin{bmatrix} 4 & 0 & 0 & 0 \\ 6 & 7 & 0 & 0 \\ 9 & 11 & 1 & 0 \\ 5 & 4 & 1 & 1 \end{bmatrix}$$

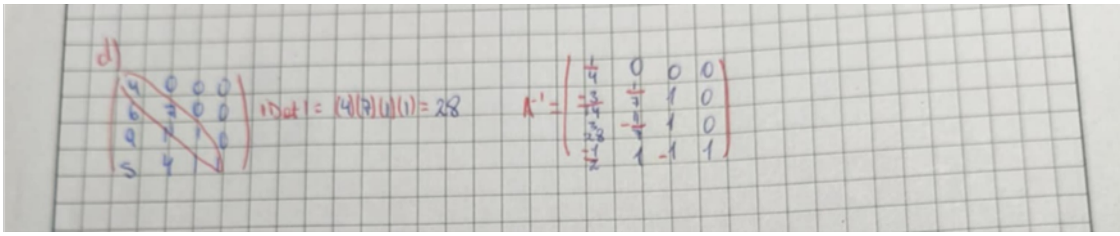

Figura 8: image.png

3. Resuelva los sistemas lineales 4 x 4 que tienen la misma matriz de coeficientes:

$$
\begin{aligned}
x_1 - x_2 + 2x_3 - x_4 &= 6, \quad x_1 - x_2 + 2x_3 - x_4 = 1 \\
x_1 - x_3 + x_4 &= 4, \quad x_1 - x_3 + x_4 = 1 \\
2x_1 + x_2 + 3x_3 - 4x_4 &= -2, \quad 2x_1 + x_2 + 3x_3 - 4x_4 = 2 \\
-x_2 + x_3 - x_4 &= 5, \quad -x_2 + x_3 - x_4 = -1
\end{aligned}
$$

4. Encuentre los valores de A que hacen que la siguiente matriz sea singular.

$$
A = \begin{bmatrix} 1 & -1 & \alpha \\ 2 & 1 & 1 \\ 0 & \alpha & -\frac{3}{2} \end{bmatrix}
$$

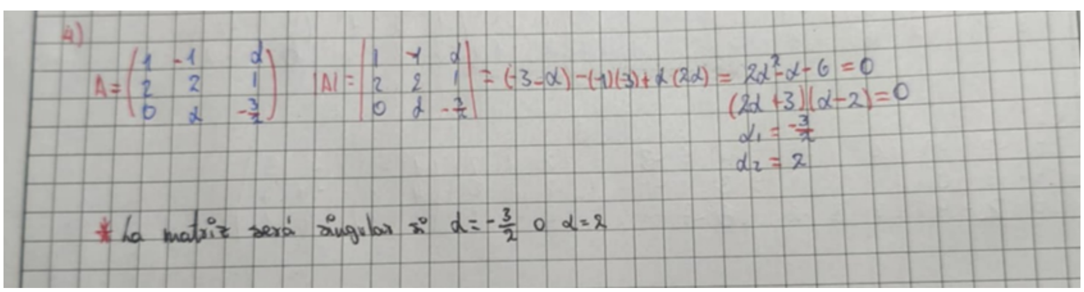

Figura 9: image.png

5. Resuelva los siguientes sistemas lineales:

a)

$$
\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}
$$

```
Solución:  [-3.75  3.5   1.  ]
```

b)

$$\begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$$

```
Solución:  [-4.  4. -1.]
```

6. Factorice las siguientes matrices en la descomposición LU mediante el algoritmo de factorización LU con lii = 1 para todas las i.

a)

$$\begin{bmatrix} 2 & -1 & 1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{bmatrix}$$

```
L:
 [[1.  0.  0. ]
 [1.5 1.  0. ]
 [1.5 1.  1. ]]
U:
 [[ 2.  -1.   1. ]
 [ 0.   4.5  7.5]
 [ 0.   0.  -4. ]]
```

b)

$$\begin{bmatrix} 1.012 & -2.132 & 3.104 \\ -2.132 & 4.096 & -7.013 \\ 3.104 & -7.013 & 0.014 \end{bmatrix}$$

```
L:
 [[ 1.          0.          0.        ]
 [-2.10671937  1.          0.        ]
 [ 3.06719368  1.19775553  1.        ]]
U:
 [[ 1.012      -2.132       3.104     ]
 [ 0.         -0.39552569 -0.47374308]
 [ 0.          0.         -8.93914077]]
```

c)

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 1.5 & 0 & 0 \\ 0 & -3 & 0.5 & 0 \\ 2 & -2 & 1 & 1 \end{bmatrix}$$

```
L:
 [[ 1.          0.          0.          0.        ]
 [ 0.5         1.          0.          0.        ]
 [ 0.         -2.          1.          0.        ]
 [ 1.         -1.33333333  2.          1.        ]]
U:
 [[2.  0.  0.  0. ]
 [0.  1.5 0.  0. ]
 [0.  0.  0.5 0. ]
 [0.  0.  0.  1. ]]
```

d)

$$\begin{bmatrix} 2.1756 & 4.0231 & -2.1732 & 5.1967 \\ -4.0231 & 6.0000 & 0 & 1.1973 \\ -1.0000 & -5.2107 & 1.1111 & 0 \\ 6.0235 & 7.0000 & 0 & -4.1561 \end{bmatrix}$$

```
L:
 [[ 1.          0.          0.          0.        ]
 [-1.84919103  1.          0.          0.        ]
 [-0.45964332 -0.25012194  1.          0.        ]
 [ 2.76866152 -0.30794361 -5.35228302  1.        ]]
U:
 [[ 2.17560000e+00  4.02310000e+00 -2.17320000e+00  5.19670000e+00]
 [ 0.00000000e+00  1.34394804e+01 -4.01866194e+00  1.08069910e+01]
 [ 0.00000000e+00  4.44089210e-16 -8.92952394e-01  5.09169403e+00]
 [ 0.00000000e+00  2.37689114e-15  0.00000000e+00  1.20361280e+01]]
```

7. Modifique el algoritmo de eliminación gaussiana de tal forma que se pueda utilizar para resolver un sistema lineal usando la descomposición LU y, a continuación, resuelva los siguientes sistemas lineales.

a)

$$\begin{aligned} 2x_1 - x_2 + x_3 &= -1, \\ 3x_1 + 3x_2 + 9x_3 &= 0, \\ 3x_1 + 3x_2 + 5x_3 &= 4. \end{aligned}$$

```
L:
 [[1.  0.  0. ]
 [1.5 1.  0. ]
 [1.5 1.  1. ]]
U:
 [[ 2.  -1.   1. ]
 [ 0.   4.5  7.5]
 [ 0.   0.  -4. ]]
Solución:  [ 1.  2. -1.]
```

b)

$$
\begin{aligned}
1.012x_1 - 2.132x_2 + 3.104x_3 &= 1.984,\\
-2.132x_1 + 4.096x_2 - 7.013x_3 &= -5.049,\\
3.104x_1 - 7.013x_2 + 0.014x_3 &= -3.895.
\end{aligned}
$$

```
L:
 [[ 1.          0.          0.        ]
 [-2.10671937  1.          0.        ]
 [ 3.06719368  1.19775553  1.        ]]
U:
 [[ 1.012      -2.132       3.104     ]
 [ 0.         -0.39552569 -0.47374308]
 [ 0.          0.         -8.93914077]]
Solución:  [1. 1. 1.]
```

c)

$$
\begin{aligned}
2x_1 &= 3,\\
x_1 + 1.5x_3 &= 4.5,\\
-3x_2 + 0.5x_3 &= -6.6,\\
2x_1 - 2x_2 + x_3 + x_4 &= 0.8.
\end{aligned}
$$

```
L:
 [[ 1.          0.          0.          0.        ]
 [ 0.5         1.          0.          0.        ]
 [ 0.         -2.          1.          0.        ]
 [ 1.         -1.33333333  2.          1.        ]]
U:
 [[2.  0.  0.  0. ]
 [0.  1.5 0.  0. ]
```

```
 [0.  0.  0.5 0. ]
 [0.  0.  0.  1. ]]
Solución:  [ 1.5  2.  -1.2  3. ]
```

d)

$$
\begin{gathered}
2.1756x_1 + 4.0231x_2 - 2.1732x_3 + 5.1967x_4 = 17.102, \\
-4.0231x_1 + 6.0000x_2 + 1.1973x_4 = -6.1593, \\
-1.0000x_1 - 5.2107x_2 + 1.1111x_3 = 3.0004, \\
6.0235x_1 + 7.0000x_2 - 4.1561x_4 = 0.0000.
\end{gathered}
$$

```
L:
 [[ 1.          0.          0.          0.        ]
 [-1.84919103  1.          0.          0.        ]
 [-0.45964332 -0.25012194  1.          0.        ]
 [ 2.76866152 -0.30794361 -5.35228302  1.        ]]
U:
 [[ 2.17560000e+00  4.02310000e+00 -2.17320000e+00  5.19670000e+00]
 [ 0.00000000e+00  1.34394804e+01 -4.01866194e+00  1.08069910e+01]
 [ 0.00000000e+00  4.44089210e-16 -8.92952394e-01  5.09169403e+00]
 [ 0.00000000e+00  2.37689114e-15  0.00000000e+00  1.20361280e+01]]
Solución:  [2.9398512  0.0706777  5.67773512 4.37981223]
```