EJERCICIO 5

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Calcular $E[ZN]$ y $V[ZN]$

Si definimos:

$X_1 + X_2 + \cdots + X_N = numero\ de\ dardos\ que\ cayeron\ dentro$

Tomamos una variable llamada Z_N

$$Z_N = 4\frac{X_1 + X_2 + \cdots + X_N}{N}$$

Ahora bien, la esperanza de $Z_N\ es$

$$E[Z_N] = 4\frac{E[X_1 + X_2 + \cdots + X_N]}{N}$$

$$E[Z_N] = 4\frac{E[X_1] + E[X]_2 + \cdots + E[X_N]}{N}$$

$$E[Z_N] = 4\frac{N_P}{N} = 4p = 4 * \frac{\pi}{4}$$

$$E[Z_N] = \pi$$

Para la varianza tenemos

$$V[Z\_N] = \left(\frac{4}{\mathrm{N}}\right)^2 V[X_1 + X_2 + \cdots + X_N]$$

$$V[Z_N] = \left(\frac{4}{\mathrm{N}}\right)^2 (V[X1] + V[X2] + \cdots + V[XN])$$

$$V[Z_N] = \frac{16}{\mathrm{N}^2} Np(1-p) = \frac{16}{\mathrm{N}} p(1-p)$$

$$V[Z_N] = \frac{16}{\mathrm{N}} p(1-p)$$

¿Qué valor debe tomar N para que el error sea de 0.01?

Usando la desigualdad de Chebyshev

$$P(Z_N{}^{-\pi} \mid \leq \varepsilon) < \frac{16p(1 - p)}{N\varepsilon^2}$$

$$P(Z_N{}^{-\pi} \mid \leq 0.01) < \frac{16p(1 - p)}{N(0.01)^2}$$

$$< \frac{16p(1/4)}{N(0.01)^2}$$

$$\frac{16p(1/4)}{N10^{-4}} < 0.001$$

Despejamos N

$$N > \frac{16\left(\frac{1}{4}\right)}{10^{-4}(0.001)}$$

$$N > \frac{4}{10^{-4} * 10^{-3}}$$

$$N > 4 * 10^{7}$$

$$N > 40{,}000{,}000$$