Automatic method for extracting depletion voltage from CV measurements

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Abstract

A method for automatic analysis of capacitance-voltage measurements is presented. The work takes an alternate approach to a method implemented by last year's summer student. The method is evaluated on two datasets and is shown to be an improvement of the previous work.

1 Introduction

The Large Hadron Collider (LHC) is a particle accelerator which studies the fundamental particles by colliding them and measuring the products of collision. As detectors, different devices are used, among most common are silicon diodes. These detectors decay in performance as they are exposed to radiation created in collisions and thus need to be well characterized and studied in terms of radiation hardness. A common technique for this is looking at the effective doping concentration as a function of fluence.

2 Silicon diodes

A diode is a two terminal electronic component which passes current when voltage is applied from anode to cathode (forward bias), but not in the opposite direction (reverse bias). It is often made out of silicon, which by itself is not very conductive as there are no free (unbound) charge carriers. However, by introducing impurities - elements which introduce an excess or a lack of electrons in the lattice, it becomes conductive. We thus differentiate between a p-type silicon, which is doped such that it has positive charge carriers called holes (lack of electrons), whereas the n-type has an excess of electrons. If the two are adjacent they form a p-n junction, where electrons and holes diffuse toward lower concentrations and upon encountering each other, they recombine. This process leaves behind ions in the lattice, creating an electric field which counteracts diffusion. For this reason, the region in the vicinity of the p-n junction is called space charge region. It is also called depletion region, as any free charges are forced out by the electric field. In Figure 1, this process is briefly summarized.

The width of the depletion region is dependent on voltage and can be calculated as:

$$w(V) = \sqrt{\frac{2\,\varepsilon}{e\,|N_{\text{eff}}|}V} \tag{1}$$

where ε is the permittivity of the material, e is the elementary charge, V is voltage and N_{eff} is effective doping of the material. As the diode's depletion region expands with the voltage, it acts like a parallel plate capacitor with capacitance, where A denotes diode area:

$$C(V) = \frac{\varepsilon A}{w(V)} = A\sqrt{\frac{\varepsilon e|N_{\text{eff}}|}{2\,V}} \tag{2}$$

When the depletion region reaches metalization, the capacitance remains constant and is called geometric or end capacitance, C_{end}. The voltage when this occurs is called depletion voltage, V_{dep}.

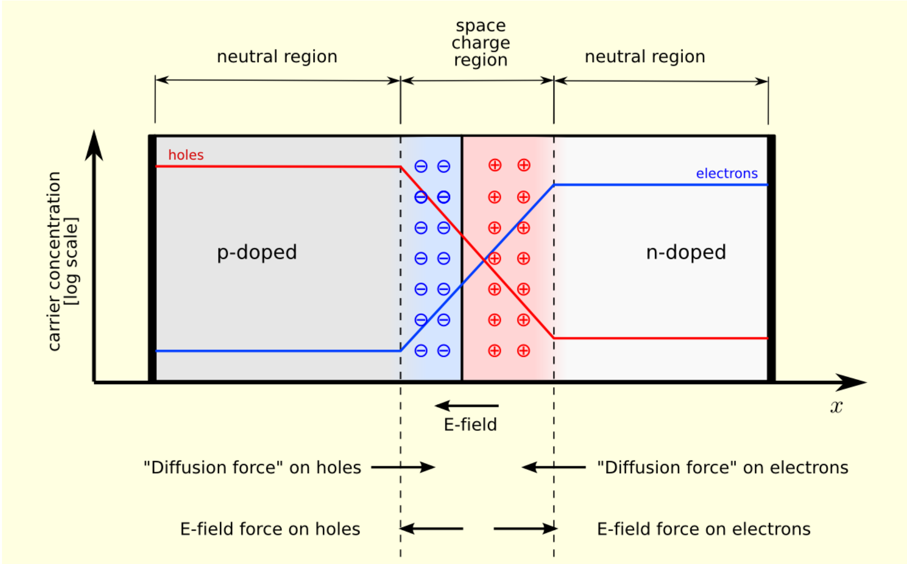
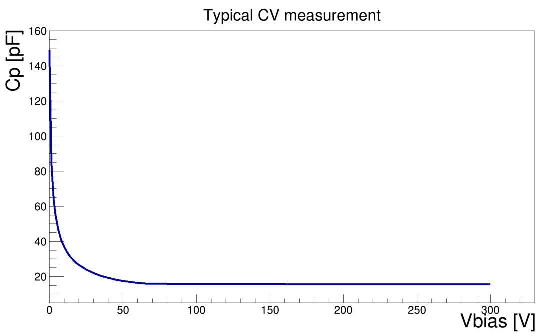


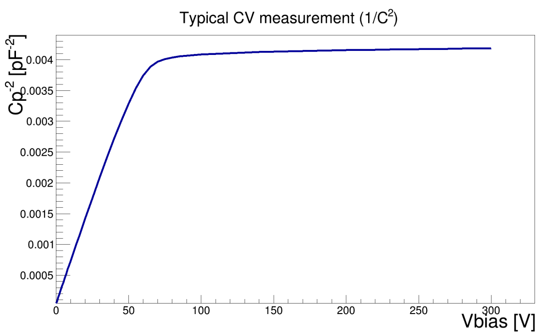
Figure 1: An ideal p-n junction, with all the relevant parameters marked. Taken from Wikipedia [1].

3 Capacitance-voltage measurement

A very common technique for measuring radiation damage is called capacitance-voltage (CV) measurement. The sensor's capacitance is measured as a function of voltage. A typical curve obtained can be seen on Figure 2a and closely follows equation (2). It is common that depletion voltage is obtained by taking the inverse squared, such as on Figure 2b, fitting two lines and finding the intersection. If the area of the sensor is known, it is possible to extract effective doping from eq. (2) and depletion thickness from the eq. (1).



(a) Capacitance versus voltage



(b) Inverse capacitance squared versus voltage

Figure 2: Typical CV curves. The depletion voltage is clearly seen on 2b as the point where the slope suddenly changes and is about 60 V.

4 CVIV_tree program

In order to analyze the CV-IV measurements, a program for analysis was created. The program is given a list of measurements and creates a .root file containing a tree with three branches called cv, iv and Type. Each entry in a tree contains one either CV or IV class, containing measurements and analysis', which get filled in their respective branches whereas the other is set to default value. The branch Type and has the value of 1 or 2, for IV and CV measurement respectively. The members of the class and their meaning is summarized in Table 1.

The program is available through its git repository on `https://gitlab.cern.ch/SSD/CVIV`.

Table 1: Members of the CV class. Square brackets indicate arrays of length specified in the brackets- NCV for number of CV measurement points and NFit for number of different analysis methods.

Class member	Description	Type
sensor	sensor name	TString
fnm	filename (with location)	TString
who	who took it	TString
bulk	material bulk	TString
comment	comment	TString
utc	time at measurement	TDatime
tann	annealing time (min)	Double_t
Fluence	fluence	Double_t
irrad	type of irradiation	TString
Temp	temperature	Double_t
Cp	capacitance (parallel)	Double_t[NCV]
Cs	capacitance (series)	Double_t[NCV]
areaSensor	area of sensor	Double_t
rho	resistivity	Double_t
iC2p	1/Cp^2	Double_t[NCV]
iC2s	1/Cs^2	Double_t [NCV]
Neffp	effective doping (parallel)	Double_t[NFit]
Neffs	effective doping (series)	Double_t[NFit]
ds	depletion thickness (series)	Double_t[NFit]
dp	depletion thickness (parallel)	Double_t[NFit]
Itot	total current	Double_t[NCV]
Ipad	pad current	Double_t[NCV]
Vdepp	depletion voltage (parallel)	Double_t[NFit]
Vdeps	depletion voltage (series)	Double_t[NFit]
Confidencep	tells if fit of Vdepp good	Int_t
Confidences	tells if fit of Vdeps good	Int_t

5 Automatic depletion voltage extraction

For a quick analysis of CV measurements, an automatic method for determining depletion voltage is needed. In 2017, a summer student implemented a method which finds ranges for automatic fitting and then calculates depletion voltage from the intersection of the two lines [2]. For the purposes of this document it shall be referd to as Pablo's method. This year, a different approach was taken. First, an estimate is obtained which is then used for determining the fit ranges. In the following discussion, it is assumed that the bias voltage is positive, but the program works with negative bias voltages as well.

5.1 Radius of curvature method

In the first method, the depletion voltage is found by looking at the radius of curvature, which is calculated with the equation (3), where y' and y'' denote first and second derivative of the data, respectively.

$$R_\mathrm{c} = \left| \frac{(1+y'^2)^{1.5}}{y''} \right| \tag{3}$$

The position of the kink should correspond to its minimum, as a kink implies a rapid change in the graph. However, many measurements contain an appreciable amount of noise, which is then further amplified when taking the derivatives. To combat this, smoothing is used. Unfortunately, strong smoothing of data was found to have adverse effects for data which contains little noise. Therefore, the algorithm first determines the strength of smoothing by counting the times first derivative changes value. Then, it calculates the derivatives from the smoothed data and further smooths them. Furthermore, in finding the minimum only the points satisfying the following criteria are considered:

1. The first derivative decreases, to avoid convex parts of the plot
2. The first derivative is not outside of 1.5 times RMS value, to filter out noise
3. The first derivative is not negative, to avoid peaks due to noise

5.1.1 Automatic fitting method

The second method uses the voltage obtained in the first method to determine fit ranges. The procedure for the first line is as follows:

1. The initial range is from 20% of V_dep to 85% of V_dep
2. Then, increase the value of the lower bound until there are three consecutive positive derivatives
3. If the first estimate of V_dep is above 25 V, increase the lower bound by 1.8 times
4. If this results in zero degrees freedom fit, refit from minimum of bias voltage to V_dep

For the second line the procedure is as follows:

1. The initial range is from 130% of V_dep to 90% of the maximum bias voltage
2. If the first estimate of V_dep is below 50V, set the lower bound to 60% of maximum bias voltage
3. If this results in zero degree of freedom in fit, refit using maximum of bias voltage as the upper bound

Then, the depletion voltage is found according to the formula:

$$V_\mathrm{dep} = \frac{n_2 - n_1}{k_1 - k_2} \tag{4}$$

where n and k are the offset and slope of the fitted line, respectively. After obtaining depletion voltage, the effective doping concentration is calculated using the following equation, derived from eq. (2):

$$|N_\mathrm{eff}| = \frac{2\, C_\mathrm{end}^2 V_\mathrm{dep}}{A^2 \varepsilon e} \tag{5}$$

The parameter C^2_{end} is obtained from the second line in the middle of the fit. Additionally, by inserting depletion voltage in eq. (1) thickness of the sensor is obtained.

Finally, the result is evaluated and deemed bad under the following criteria:

1. The second fit's slope is not larger than 60 % of slope
2. The depletion voltage falls within the bias voltage range
3. The fits have number of degrees of freedom larger than 0

If one of these conditions is met, the appropriate "Confidence" parameter is set to zero.

5.2 Results

Upon running the script, a pdf with fits is created with fitted measurements. A typical example can be seen on Figure 3. The black line is the measurement, whereas the green line is the smoothed data. The green triangle corresponds to automatically obtained depletion voltage and the yellow triangle with the automatic fitting method, which is obtained using the thick yellow and red fits. The blue line represents automatically obtained depletion voltage using Pablo's method [2] and the thin red lines his fits.

In order to develop the algorithm, a dataset of 40 sensors has been used. The depletion voltage was obtained manually by a skilled user [3] and the results of both methods can be seen on Figures 4, 5 and 6. It is observed by comparing Figures 5a and 6a with 5b and 6b that the radius of curvature method appears to have a higher relative and absolute error than the automatic fitting method. For the latter, about 90 % of results fall within ±10 V and about 70 % within ±10 % of the correct value.

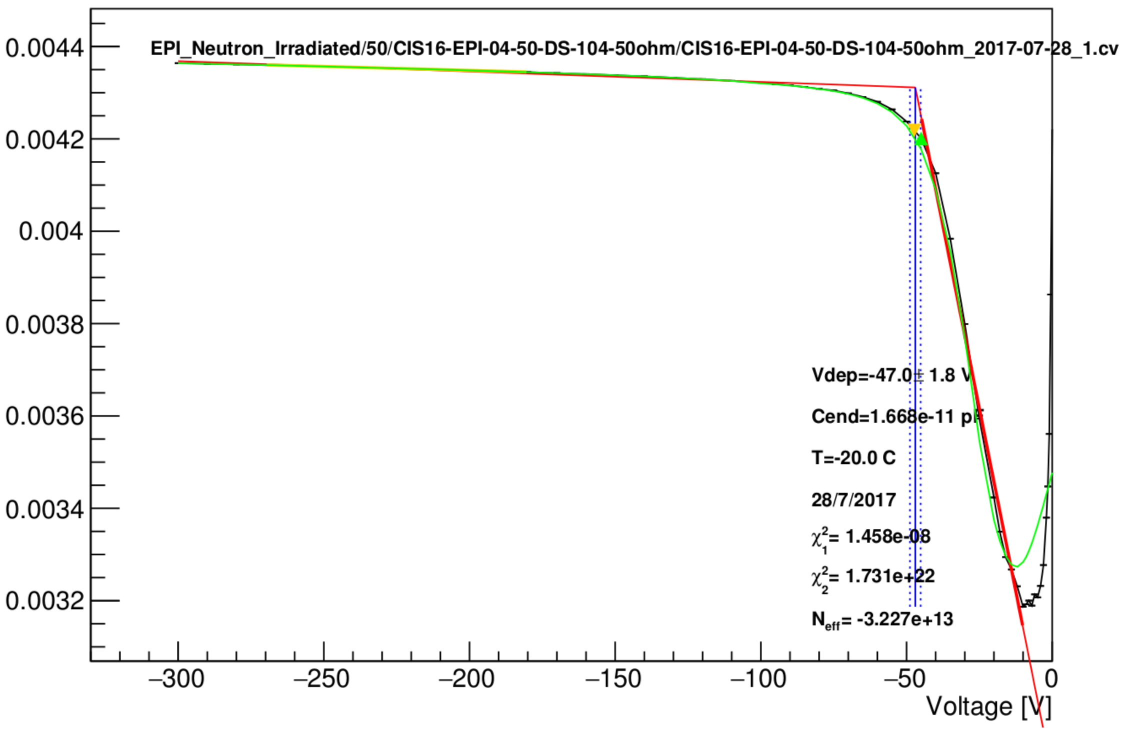
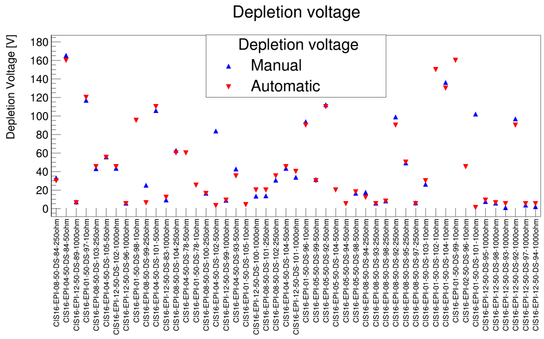
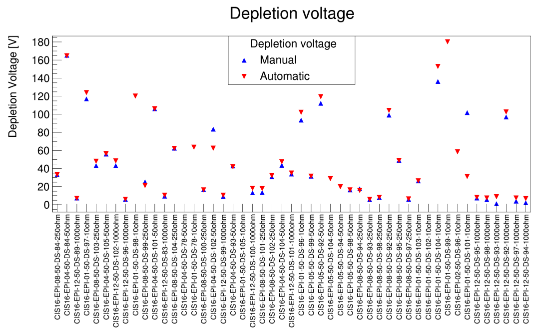


Figure 3: Typical CV curve with the automatic fitting fitting methods. Taken from the first dataset.

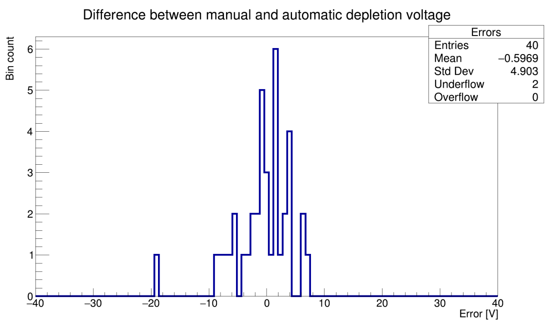


(a) Results using radius of curvature.

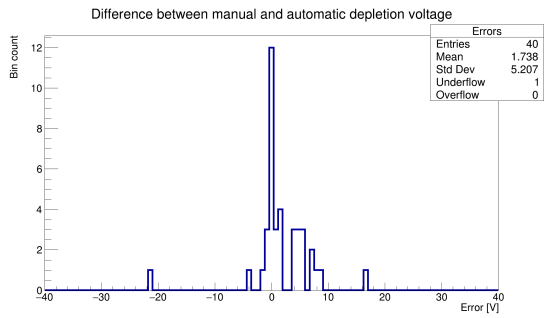


(b) Results using automatic methods

Figure 4: Depletion voltages for different sensors. The red inverse triangles were automatically obtained and the blue triangles were manually obtained. From the first dataset.

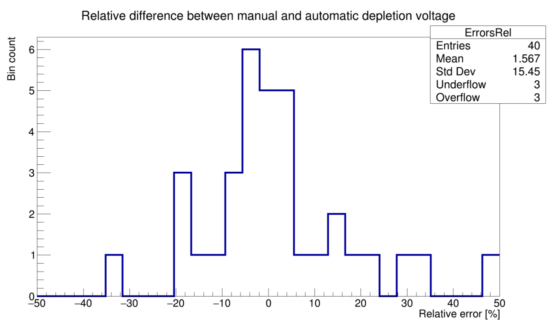


(a) Radius of curvature method

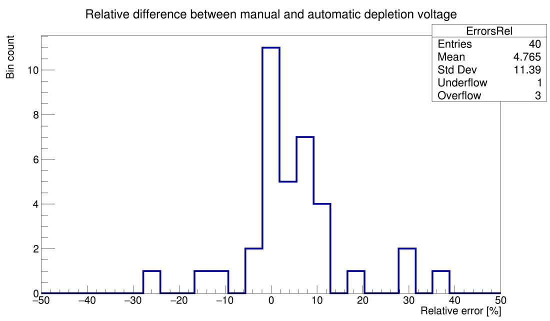


(b) Automatic fitting method

Figure 5: Absolute error of the methods. From the first dataset.



(a) Radius of curvature method

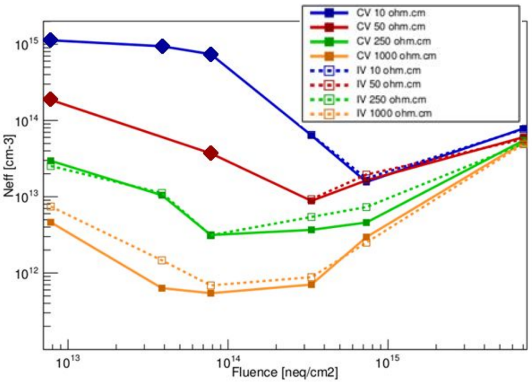


(b) Automatic fitting method

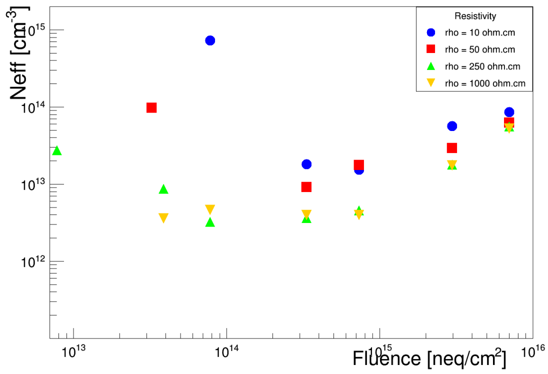
Figure 6: Relative error of the methods. From the first dataset.

After running the program, to plot N_{eff} versus fluence for all desired sensors can be easily obtained by using the ROOT Draw function with proper selection of parameters in Table 1. An example can be seen on Figure 7. It is worth noting that there are several missing points on Figure 7b filtered out with "Confidencep" parameter. In all of the cases, the depletion is never reached as the the sensors go into breakdown before reaching depletion. A different method, specific to those types of sensors, was used when obtaining those points manually.

In order to further validate the method, every 53rd and 100th entry on EOS was taken and manually fitted. The resulting absolute and relative errors can be seen on Figures 8 and 9. Here, a comparison between the second method and Pablo's method is made. The new algorithm shows a performance increase both in terms of overflow and underflow bins and in terms of hits within $\pm10\%$ and ±10 V of the correct values.

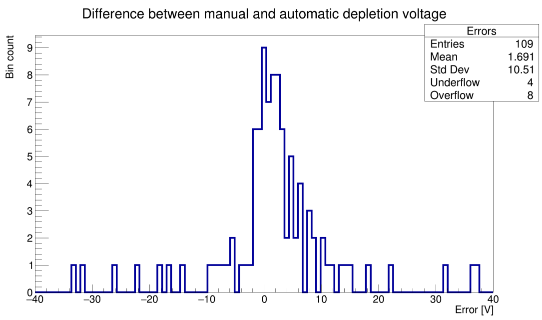


(a) Manually obtained N_{eff} vs fluence [3]

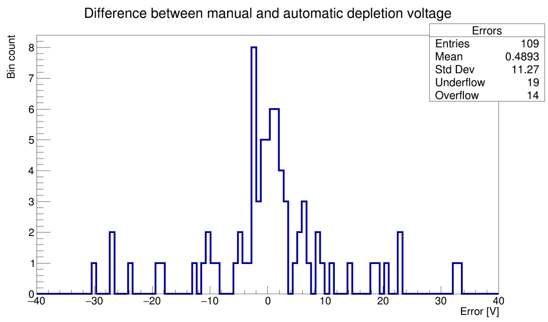


(b) Automatically obtained N_{eff} vs fluence

Figure 7: Comparison between manually obtained effective doping concentration (left) [3] and using the second automatic method (right). The error is largest for lower fluences with resistivity of 1000 ohm-cm, as the depletion voltage is very low. From the first dataset.

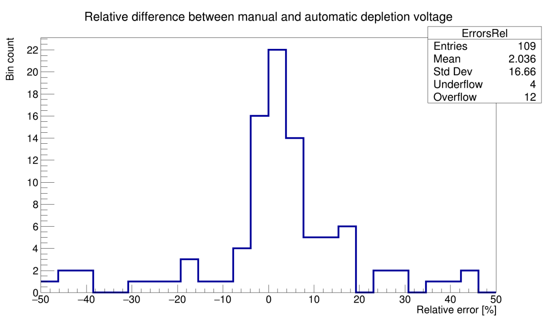


(a) Automatic fitting method

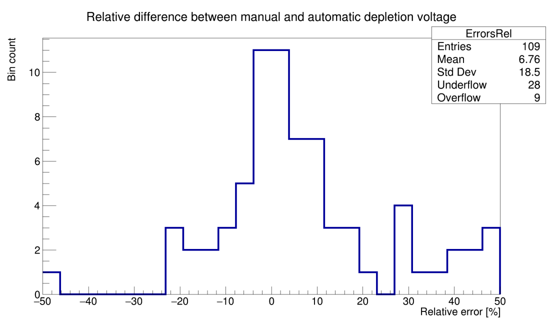


(b) Pablo's method

Figure 8: Comparison of absolute errors of automatic fitting method and Pablo's method. From the second dataset.



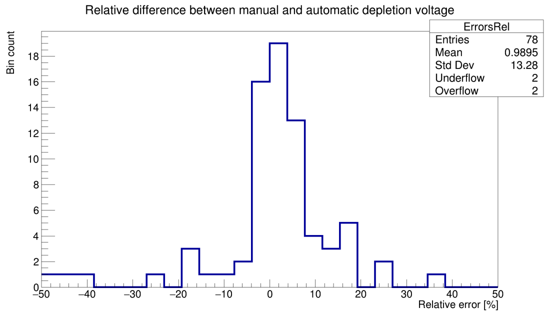
(a) Automatic fitting method



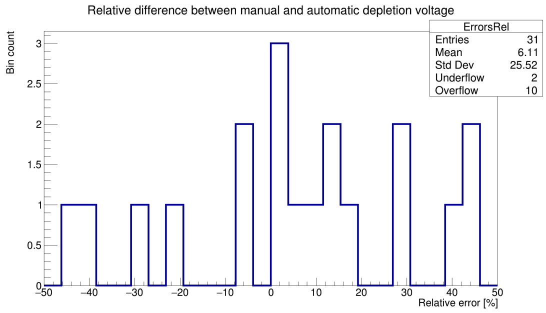
(b) Pablo's method

Figure 9: Comparison of relative errors of automatic fitting method and Pablo's method. From the second dataset.

The main contributor to variance and overflow in relative error seems to be that the method tends to perform worse for depletion voltages lower than 25 V. For depletion voltages with true value larger than 25 V, it works fairly well, with about 70% of values within ±10% error.



(a) $V_{\mathrm{dep}} > 25$ V



(b) $V_{\mathrm{dep}} < 25$ V

Figure 10: Histogram on Figure 9a split into two histograms based on the manual depletion voltage. It can be seen that the method is less accurate for lower values of V_{dep}. From the second dataset.

6 Conclusion

The presented new methods provide the user with a fairly reliable estimate for depletion voltage and effective doping concentration. Furthermore, main parameters of detector are calculated, drastically increasing the speed of analysis. The methods can be improved, since the choice of some parameters is rather arbitrary, for example the second method's choice of fitting ranges.

References

[1] TheNoise, Pn-junction-equilibrium, accessed July 30, 2018
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[2] Pablo Matorras Cuevas: An automatic method to analyze the Capacity-Voltage and Current-Voltage curves of a sensor, accessed July 30, 2018
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