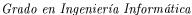
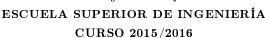


ÁLGEBRA

 $Departamento\ de\ Matem\'aticas$







Soluciones del Boletín I: MATRICES Y DETERMINANTES

$$A^{2} = \begin{pmatrix} -7 & -4 & -17 \\ -14 & -18 & -4 \\ 11 & 22 & -19 \end{pmatrix}, \quad B^{2} = \begin{pmatrix} 11 & 2 & 1 \\ 5 & 14 & 2 \\ 2 & 3 & 9 \end{pmatrix}, \quad AB = \begin{pmatrix} -2 & 16 & -7 \\ -13 & 3 & -11 \\ 22 & 18 & 9 \end{pmatrix}$$

$$AC = \begin{pmatrix} 5 & -5 & 6 \\ -6 & -27 & -17 \\ 19 & 33 & 38 \end{pmatrix}, \quad BC = \begin{pmatrix} 7 & -22 & 3 \\ 8 & 29 & 21 \\ 2 & 60 & 22 \end{pmatrix}, \quad (A - C)B = \begin{pmatrix} -36 & 8 & -21 \\ -23 & 4 & -23 \\ -27 & 8 & -16 \end{pmatrix}$$

$$A+B=\left(\begin{array}{cccc} 3 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 3 & 1 \end{array}\right), \quad A-B=\left(\begin{array}{cccc} -1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & -1 & -1 \end{array}\right), \quad A^2=\left(\begin{array}{cccc} 2 & 1 & 1 \\ 5 & 2 & 3 \\ 3 & 1 & 2 \end{array}\right)$$

$$B^{2} = \begin{pmatrix} 5 & 6 & 7 \\ 2 & 3 & 3 \\ 2 & 2 & 3 \end{pmatrix}, \quad AB = \begin{pmatrix} 2 & 3 & 3 \\ 5 & 4 & 6 \\ 3 & 1 & 3 \end{pmatrix}, \quad BA = \begin{pmatrix} 6 & 3 & 3 \\ 2 & 1 & 1 \\ 5 & 3 & 2 \end{pmatrix}$$

3.
$$A = \begin{pmatrix} 2 & 1 \\ 4 & 5 \\ 3 & 2 \end{pmatrix}$$
, $B = \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}$

4.
$$M = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 0 & 3 \end{pmatrix}, \quad N = \begin{pmatrix} 1 & 4 & 6 \\ 6 & 4 & 1 \end{pmatrix}$$

5.

6. b)
$$A = S + T$$
 con $S = \begin{pmatrix} 3 & 11/2 & 4 \\ 11/2 & 0 & 9/2 \\ 4 & 9/2 & 5 \end{pmatrix}$, $y T = \begin{pmatrix} 0 & -1/2 & -3 \\ 1/2 & 0 & -1/2 \\ 3 & 1/2 & 0 \end{pmatrix}$

7.

$$8. \ AB = \begin{pmatrix} 0 & 0 & 4 & 1 \\ 0 & 0 & 2 & 0 \\ \hline 0 & 1 & 0 & 0 \\ 2 & 2 & 0 & 0 \end{pmatrix}$$

9.

$$10. \ I = \left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

11.
$$B = \begin{pmatrix} 1 & -1 & 2 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3/2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

12.
$$A = \frac{1}{3} \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$$
, $B = \begin{pmatrix} 3 & 1 & -2 \\ 3/2 & 1 & -1 \\ 1/2 & 0 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 1/4 & -1/2 & 1/4 & -1/2 \\ -1/2 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

13. a)
$$rg(A) = 3$$
, b) $rg(B) = 3$, c)
$$\begin{cases} \text{Si } x \neq 0 \text{ y } x \neq 5 \implies rg(C) = 3 \\ \text{Si } x = 0 \text{ o } x = 5 \implies rg(C) = 2 \end{cases}$$

14.
$$x = 1, y = -\frac{1}{2}, z = \frac{4}{3}, z = -\frac{1}{3}$$

15.

16.
$$\alpha = 3$$

17. a)
$$\begin{cases} \operatorname{Si} \beta \neq 2 \Rightarrow rg(A) = 3 \\ \operatorname{Si} \beta = 2 \Rightarrow rg(A) = 2 \end{cases}, b) \begin{cases} \operatorname{Si} \beta \neq 0, \ \alpha \neq 1 \ \text{y} \ \alpha \neq -2 \Rightarrow rg(B) = 3 \\ \operatorname{Si} \alpha = 1 \Rightarrow rg(B) = 1 \ \forall \beta \\ \operatorname{Si} \alpha = -2 \Rightarrow rg(B) = 2 \ \forall \beta \\ \operatorname{Si} \beta = 0 \ \text{y} \ \alpha \neq 1 \Rightarrow rg(B) = 2 \end{cases}$$

18.
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & -2 & 0 & 0 \\ 2 & -5 & -5/2 & 0 \\ 3 & -5 & -5/2 & 0 \end{pmatrix}$$

19.
$$\alpha \neq 0$$

20.
$$\alpha = 1, \beta = 7 \Leftrightarrow rg(A) = 3$$

21.

$$22.$$
 $a)$

b)

c)

d)

e)

23. Si $\alpha \neq 0$ el rango de A es 3 y si $\alpha = 0$ el rango de A es 2.

$$24. \ a) \, rg(a) = 3 \, \forall \alpha \in \mathbb{R}, \quad b) \, B = \left(\begin{array}{cccc} 1 & 0 & -1 & 2 \\ 0 & -1 & 2 & 1 \\ 0 & 0 & 27 & 9 \end{array} \right) \, C = \left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 27 & 9 \end{array} \right)$$

25.

26.
$$|A| = x(y-x)(z-y)(t-z), \quad |B| = -10, \quad |C| = 0$$

27.
$$|A| = 3$$
, $|B| = 1$

28. a)
$$x = 2$$
 e $x = -6$, b) $x = -1$ e $x = -2$, c) $x = \frac{abc}{ab+ac+bc}$

29.
$$(b-a)(c-a)(d-a)(c-b)(d-b)(d-c)$$

30.
$$A^{-1} = \frac{1}{45} \begin{pmatrix} -42 & 55 & -31 & 1\\ -9 & 15 & 3 & -3\\ 6 & 20 & -17 & 2\\ 15 & -40 & 25 & 5 \end{pmatrix}$$

31.
$$x \neq -3, x \neq 1$$

32.

33.
$$Adj(A) = \begin{pmatrix} -16 & 0 & -8 \\ -31 & -1 & -12 \\ 38 & -2 & 16 \end{pmatrix}, \quad Adj(B) = \begin{pmatrix} 12 & 0 & 0 & 0 \\ -8 & -4 & 0 & 0 \\ -8 & -4 & -12 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}$$

34. No existen n y m números naturales.