Soluciones del Boletín I: MATRICES Y DETERMINANTES

1.
$$A^2=\begin{pmatrix}-7&-4&-17\\-14&-18&-4\\11&22&-19\end{pmatrix},\quad B^2=\begin{pmatrix}11&2&1\\5&14&2\\2&3&9\end{pmatrix},\quad AB=\begin{pmatrix}-2&16&-7\\-13&3&-11\\22&18&9\end{pmatrix}$$
$$AC=\begin{pmatrix}5&-5&6\\-6&-27&-17\\19&33&38\end{pmatrix},\quad BC=\begin{pmatrix}7&-22&3\\8&29&21\\2&60&22\end{pmatrix},\quad (A-C)B=\begin{pmatrix}-36&8&-21\\-23&4&-23\\-27&8&-16\end{pmatrix}$$

2.
$$A+B=\begin{pmatrix}3&1&3\\3&1&2\\1&3&1\end{pmatrix},\quad A-B=\begin{pmatrix}-1&-1&-1\\1&1&0\\1&-1&-1\end{pmatrix},\quad A^2=\begin{pmatrix}2&1&1\\5&2&3\\3&1&2\end{pmatrix}$$
$$B^2=\begin{pmatrix}5&6&7\\2&3&3\\2&2&3\end{pmatrix},\quad AB=\begin{pmatrix}2&3&3\\5&4&6\\3&1&3\end{pmatrix},\quad BA=\begin{pmatrix}6&3&3\\2&1&1\\5&3&2\end{pmatrix}$$

3. $A=\begin{pmatrix}2&1\\4&5\\3&2\end{pmatrix},\quad B=\begin{pmatrix}1&1\\2&1\\1&3\end{pmatrix}$

4. $M=\begin{pmatrix}1&0&1\\2&0&3\end{pmatrix},\quad N=\begin{pmatrix}1&4&6\\6&4&1\end{pmatrix}$

5.

6. *b)* $A=S+T$ con $S=\begin{pmatrix}3&11/2&4\\11/2&0&9/2\\4&9/2&5\end{pmatrix}$, y $T=\begin{pmatrix}0&-1/2&-3\\1/2&0&-1/2\\3&1/2&0\end{pmatrix}$

7.

8. $AB=\left(\begin{array}{cc|cc}0&0&4&1\\0&0&2&0\\\hline0&1&0&0\\2&2&0&0\end{array}\right)$

9.

10. $I=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$

11. $B = \begin{pmatrix} 1 & -1 & 2 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3/2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

12. $A = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 1 & -2 \\ 3/2 & 1 & -1 \\ 1/2 & 0 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1/4 & -1/2 & 1/4 & -1/2 \\ -1/2 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

13. $a)\, rg(A) = 3, \quad b)\, rg(B) = 3, \quad c) \begin{cases} \text{Si } x \neq 0 \text{ y } x \neq 5 \;\Rightarrow\; rg(C) = 3 \\ \text{Si } x = 0 \text{ o } x = 5 \;\Rightarrow\; rg(C) = 2 \end{cases}$

14. $x = 1$, $y = -\frac{1}{2}$, $z = \frac{4}{3}$, $z = -\frac{1}{3}$

15.

16. $\alpha = 3$

17. $a) \begin{cases} \text{Si } \beta \neq 2 \;\Rightarrow\; rg(A) = 3 \\ \text{Si } \beta = 2 \;\Rightarrow\; rg(A) = 2 \end{cases}, \quad b) \begin{cases} \text{Si } \beta \neq 0,\ \alpha \neq 1 \text{ y } \alpha \neq -2 \;\Rightarrow\; rg(B) = 3 \\ \text{Si } \alpha = 1 \;\Rightarrow\; rg(B) = 1\ \forall \beta \\ \text{Si } \alpha = -2 \;\Rightarrow\; rg(B) = 2\ \forall \beta \\ \text{Si } \beta = 0 \text{ y } \alpha \neq 1 \;\Rightarrow\; rg(B) = 2 \end{cases}$

18. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & -2 & 0 & 0 \\ 2 & -5 & -5/2 & 0 \\ 3 & -5 & -5/2 & 0 \end{pmatrix}$

19. $\alpha \neq 0$

20. $\alpha = 1$, $\beta = 7 \Leftrightarrow rg(A) = 3$

21.

22. $a)$

    $b)$

    $c)$

    $d)$

    $e)$

23. Si $\alpha \neq 0$ el rango de A es 3 y si $\alpha = 0$ el rango de A es 2.

24. $a)\, rg(a) = 3\ \forall \alpha \in \mathbb{R}, \quad b)\, B = \begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & -1 & 2 & 1 \\ 0 & 0 & 27 & 9 \end{pmatrix}\ C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 27 & 9 \end{pmatrix}$

25.

26. $|A| = x(y-x)(z-y)(t-z), \quad |B| = -10, \quad |C| = 0$

27. $|A| = 3, \quad |B| = 1$

28. $a)\, x = 2$ e $x = -6$, $\quad b)\, x = -1$ e $x = -2$, $\quad c)\, x = \frac{abc}{ab+ac+bc}$

29. $(b-a)(c-a)(d-a)(c-b)(d-b)(d-c)$

30. $A^{-1} = \frac{1}{45}\begin{pmatrix} -42 & 55 & -31 & 1 \\ -9 & 15 & 3 & -3 \\ 6 & 20 & -17 & 2 \\ 15 & -40 & 25 & 5 \end{pmatrix}$

31. $x \neq -3,\ x \neq 1$

32.

33. $Adj(A) = \begin{pmatrix} -16 & 0 & -8 \\ -31 & -1 & -12 \\ 38 & -2 & 16 \end{pmatrix}, \quad Adj(B) = \begin{pmatrix} 12 & 0 & 0 & 0 \\ -8 & -4 & 0 & 0 \\ -8 & -4 & -12 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}$

34. No existen n y m números naturales.