Soluciones del Boletín II: SISTEMAS DE ECUACIONES LINEALES

1. $\begin{cases} x_1 + 2x_2 - x_3 + x_4 + x_5 = 0 \\ x_2 - \frac{3}{7}x_3 + \frac{2}{7}x_4 + \frac{4}{7}x_5 = -\frac{6}{7} \\ x_3 - \frac{13}{9}x_4 + x_5 = -\frac{59}{5} \end{cases}$

2. *a)* $(3,1,1)$, *b)* $(1,2,-2)$, *c)* $(1,2,1,0)$

3. *a)* $(\frac{7}{2}-\alpha, -\frac{9}{2}-2\alpha, \alpha, 2, \frac{7}{2})$, *b)* $(\frac{3}{2}-\alpha+\beta, -\frac{5}{2}-2\alpha-\beta, \alpha, \beta, \frac{7}{2})$

4. *a)* $(0, -\frac{3}{4}, \frac{1}{2})$, *b)* $(0, 2-1)$, *c)* $(1, \frac{5}{4}, -\frac{1}{2})$

5. a) Sistema Compatible Indeterminado (SCI) con

$$(x,y,z,t) = \left(\frac{7}{9} - \frac{2}{9}\lambda, -\frac{11}{3} + \frac{1}{3}\lambda - \alpha, -\frac{59}{9} + \frac{13}{9}\lambda - \alpha, \lambda, \alpha\right)$$

b) Sistema Compatible Determinado (SCD) con

$$(x,y,z) = \left(\frac{4}{5}, 2, \frac{16}{5}\right)$$

c) Sistema Compatible Indeterminado (SCI) con $(x,y,z,t) = (\lambda, 3\lambda-2, \lambda, 2\lambda-3)$ con $\lambda \in \mathbb{R}$

6. *a)* $\begin{cases} \text{Si } \alpha = 2,\ rg(A) = rg(A^*) = 2 \Rightarrow \text{SCI con } (x,y,z) = (\frac{5}{4} - \frac{1}{8}\lambda, \frac{1}{2} + \frac{3}{4}\lambda, \lambda) \\ \text{Si } \alpha \neq 2,\ rg(A) = rg(A^*) = 3 \Rightarrow \text{SCD con } (x,y,z) = (-5, 10\alpha+18, 15\alpha+20) \end{cases}$

b) $\begin{cases} \text{Si } \alpha = 2,\ rg(A) = rg(A^*) = 2 \Rightarrow \text{SCI con } (x,y,z) = \left(1 - \frac{3}{5}\alpha, -1 + \frac{4}{5}\alpha, \alpha\right) \\ \text{Si } \alpha = 3,\ rg(A) = 2\ rg(A^*) = 3 \Rightarrow \text{SI} \\ \text{Si } \alpha \neq 2, \text{ y } \alpha \neq 3\ rg(A) = rg(A^*) = 3 \Rightarrow \text{SCD con } (x,y,z) = \left(\frac{1-\alpha}{\alpha-3}, \frac{\alpha-1}{\alpha-3}, \frac{\alpha^2-\alpha-2}{\alpha-3}\right) \end{cases}$

c) $\begin{cases} \text{Si } \alpha = 2,\ rg(A) \neq rg(A^*) \Rightarrow \text{SI} \\ \text{Si } \alpha = 1,\ rg(A) = rg(A^*) = 1 \Rightarrow \text{SCI con } (x,y,z) = (-\lambda - \beta + 1, \lambda, \beta) \\ \text{Si } \alpha \neq 2, \text{ y } \alpha \neq 1\ rg(A) = rg(A^*) = 3 \Rightarrow \text{SCD con } (x,y,z) = \left(-\frac{2+\alpha+\alpha^2}{\alpha-2}, \frac{2+3\alpha}{\alpha-2}, -\frac{\alpha(2+\alpha)}{\alpha-2}\right) \end{cases}$

d) $\begin{cases} \text{Si } \alpha \neq -2, \text{ y } \alpha \neq 1\ rg(A) = rg(A^*) = 3 \Rightarrow \text{SCD con } (x,y,z) = \left(\frac{1}{\alpha+2}, \frac{1}{\alpha+2}, -\frac{1}{\alpha+2}\right) \\ \text{Si } \alpha = -2,\ rg(A) \neq rg(A^*) \Rightarrow \text{SI} \\ \text{Si } \alpha = 1,\ rg(A) = rg(A^*) = 1 \Rightarrow \text{SCI con } (x,y,z) = (1 - \lambda - \beta, \lambda, \beta) \end{cases}$

e) $\begin{cases} \text{Si } \alpha \neq 2,\ rg(A) = rg(A^*) = 2 \Rightarrow \text{SCI con } (x,y,z) = (\lambda, \frac{1}{2-\alpha} - \frac{1}{2-\alpha}\lambda, (\alpha-1) + (2\alpha-2)\lambda) \\ \text{Si } \alpha = 2,\ rg(A) = rg(A^*) = 2 \Rightarrow \text{SCI con } (x,y,z) = (1, \lambda, 3) \end{cases}$

7. *a)* $--$, *b)* $--$

8. *a)* Si $\alpha \neq 1,\ \alpha \neq -2 \,\mathrm{y}\, \beta \neq 0\ \Rightarrow$ SCD, Si $\beta = 0,\ \Rightarrow$ SCI

   Si $\beta \neq 0$, y $\alpha \neq 1$ $\left\{\begin{array}{l}\text{Si } \beta = 1,\ \Rightarrow \text{SCI}\\ \text{Si } \beta \neq 1,\ \Rightarrow \text{SI}\end{array}\right.$, Si $\beta \neq 0$, y $\alpha = -2$ $\left\{\begin{array}{l}\text{Si } \beta = -2,\ \Rightarrow \text{SCI}\\ \text{Si } \beta \neq -2,\ \Rightarrow \text{SI}\end{array}\right.$

   b)

   c)

9. $(x_1, x_2, x_3, x_4) = (\lambda, \lambda, -\lambda, 0)$ con $\lambda \in \mathbb{R}$

10.

11. $\alpha = 2$ y $\gamma = 1$. La solución es $(x, y, z, t) = (3 + 4\lambda, 4 + 4\lambda, \lambda, -1 - 2\lambda)$ con $\lambda \in \mathbb{R}$

12. $\alpha = -1$ y $\beta = 1$

13. $\alpha = -\frac{13}{17}$ y la solución es $(x, y, z) = (\frac{25}{17}, \frac{12}{17}, frac1417)$

14. $\left\{\begin{array}{l}\text{Si } \alpha \neq 0, \text{ y } \alpha \neq 7\ \ rg(A) = rg(A^*) = 4 \Rightarrow \text{SCD con } (x_1, x_2, x_3, x_4) = (0, 0, 0, 0)\\ \text{Si } \alpha = 0,\ \ rg(A) = rg(A^*) = 3 \Rightarrow \text{SCI con } (x_1, x_2, x_3, x_4) = (0, 0, 0, \beta)\\ \text{Si } \alpha = 7,\ \ rg(A) = rg(A^*) = 2 \Rightarrow \text{SCI con } (x_1, x_2, x_3, x_4) = (-2\beta - 7\lambda, \beta, 0, \lambda)\end{array}\right.$

15.

16. *a)* $|A| = 1 \Rightarrow A$ tiene inversa, *b)* $X_1 = \begin{pmatrix} 1 & 1 & -1 \end{pmatrix}^t,\ X_2 = \begin{pmatrix} 6 & 4 & 0 \end{pmatrix}^t$

17.

18. El sistema tiene solución cuando $\alpha = 0$ con $(x_1, x_2, x_3, x_4) = (-1 - 7\lambda, 1 + 5\lambda, -2\lambda, \lambda)$ con $\lambda \in \mathbb{R}$

19. *a)* $\alpha = 1$, *b)* $(x, y, z) = (-1, -\lambda + 2, \lambda)$

20.

21.

22. Si $\alpha = 1$ $\left\{\begin{array}{l}\text{Si } \beta \neq -1,\ \ rg(A) \neq rg(A^*) \Rightarrow \text{SI}\\ \text{Si } \beta = -1,\ \ rg(A) = rg(A^*) = 2 \Rightarrow \text{SCI}\end{array}\right.$, Si $\alpha = 0$ $\left\{\begin{array}{l}\text{Si } \beta \neq 0, \text{ y } \beta \neq 1\ \ rg(A) \neq rg(A^*) \Rightarrow \text{SI}\\ \text{Si } \beta = 0,\ \ rg(A) = rg(A^*) = 2 \Rightarrow \text{SCI}\\ \text{Si } \beta = 1,\ \ rg(A) = rg(A^*) = 2 \Rightarrow \text{SCI}\end{array}\right.$

    Si $\alpha \neq 1$, y $\alpha \neq 0$ $rg(A) = rg(A^*) = 3 \Rightarrow$ SCD

23. Si $\alpha = 0$ $\left\{\begin{array}{l}\text{Si } \beta \neq -1,\ \ rg(A) \neq rg(A^*) \Rightarrow \text{SI}\\ \text{Si } \beta = -1,\ \ rg(A) = rg(A^*) = 3 \Rightarrow \text{SCI}\end{array}\right.$, Si $\alpha \neq 0$ $\left\{\begin{array}{l}\text{Si } \beta \neq \alpha,\ \ rg(A) = rg(A^*) = 4 \Rightarrow \text{SCD}\\ \text{Si } \beta = \alpha,\ \ rg(A) \neq rg(A^*) \Rightarrow \text{SI}\end{array}\right.$

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