Perturbative QCD and Monte Carlo event generators

Monte Carlo course seminar - Milan, September 2020

UNIVERSITÀ DEGLI STUDI DI MILANO

INFN Istituto Nazionale di Fisica Nucleare

erc European Research Council

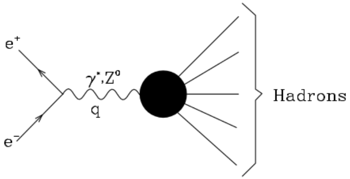
Outline

1. Hadron collisions
   - Hadron collisions and strong interactions
   - Renormalization group
   - Jets and IR divergences
2. Collinear factorization
   - Factorization theorem
   - Kinematics of splitting
   - Fixed Order calculations
3. Parton showers
   - Final state radiation
   - Initial state radiation
   - Ordering variables (PYTHIA and HERWIG)

Hadron collisions

Hadron collisions

QCD from e^+e^- annihilation



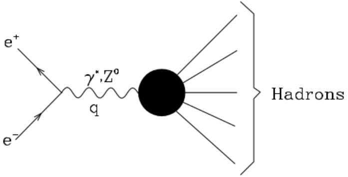
QCD arise already from e^+e^- annihilation $\rightarrow$ R_0 ratio

$$R_0 = \frac{\sigma(\gamma^* \rightarrow \text{hadrons})}{\sigma(\gamma^* \rightarrow \mu^+\mu^-)} = 3\sum_f c_f^2$$

1. Color factor (3 color for each quark)
2. Sum over charges of different flavour quarks

Hadron collisions

QCD from e^+e^- annihilation



Consider corrections to R_0 from gluon radiation. Renormalize coupling.

$$R = R_0 \left(1 + \frac{\alpha_S(\mu)}{\pi} + \left[c + \pi b_0 \log \frac{\mu^2}{Q^2}\right] \left(\frac{\alpha_S(\mu}{\pi}\right)^2\right) + \mathcal{O}\left(\alpha_S(\mu)^3\right).$$

1. Can we go to arbitrarily large energies? $\to$ divergences arise, renormalization / factorization needed
2. Can we compute R_0 for every process? $\to$ IR observables

Hadron collisions

Renormalization group

1. UV divergences are encountered in field theories
2. Take a physical quantity G depending on a scale M, a coupling α and some invariants $s_1, ..., s_n$
3. Define a "renormalized" $\alpha_{\mathrm{Ren}} = \alpha + c_1\alpha^2 + c_2\alpha^3 + ...$

The physical quantity in terms of $\{\alpha, M\}$ and $\{\alpha_{\mathrm{Ren}}, \mu\}$

$$G(\alpha, M, s_1 \ldots s_n) = \tilde{G}(\alpha_{\mathrm{ren}}, \mu, s_1 \ldots s_n)\,.$$

Physics must be invariant under change of $\{\alpha_{\mathrm{Ren}}, \mu\}$

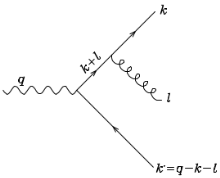
$$\frac{\partial\alpha(\alpha_{\mathrm{ren}}, M/\mu)}{\partial\alpha_{\mathrm{ren}}} d\alpha_{\mathrm{ren}} + \frac{\partial\alpha(\alpha_{\mathrm{ren}}, M/\mu)}{\partial\mu^2} d\mu^2 = 0\,.$$

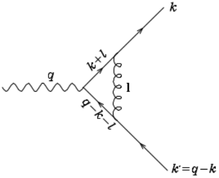
- Running coupling given by Renormalization Group Equation (RGE)

$$\mu \frac{d\alpha_s(\mu)}{d\mu} = \beta(\alpha_s(\mu)) = -\sum_{n=0}^{\infty} \beta_n \left(\frac{\alpha_s}{\pi}\right)^{n+1}$$

- Coupling α_s evolves with scale μ as given by RGE $\rightarrow$ LO behavior driven by β_0
- $\beta_0^{\mathrm{QCD}} < 0 \implies$ weakly coupled at large energies, asymptotic freedom
- $\beta_0^{\mathrm{QED}} > 0 \implies$ strongly coupled at large energies, UV unsafe

Consider α_s corrections to born level amplitude



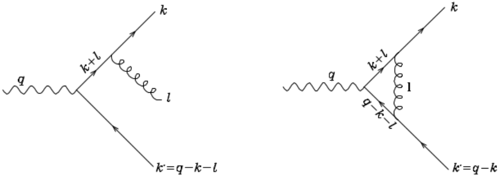


$$\mathcal{M}_{\text{Born}} = \bar{u}(k)\epsilon^{\mu}\gamma_{\mu}v(k')$$

$$\mathcal{M}_1 = \mathcal{M}\frac{k_\alpha}{k\cdot l}$$

$$\mathcal{M}_1 = -\mathcal{M}\frac{k'_\alpha}{k'\cdot l}$$

Consider α_s corrections to born level amplitude



Sum real and virtual contributions to the Born matrix element, with phase space element $d^3l = (l^0)^2 dl^0 d\cos\theta d\phi$

$$\sigma_{q\bar{q}g} = C_F \frac{\alpha_S}{2\pi}\sigma^{\rm Born}_{q\bar{q}} \int d\cos\theta \frac{dl^0}{l^0} \frac{4}{(1-\cos\theta)(1+\cos\theta)}.$$

$$\sigma_{q\bar{q}g} = C_F \frac{\alpha_S}{2\pi} \sigma_{q\bar{q}}^{\text{Born}} \int d\cos\theta \frac{dl^0}{l^0} \frac{4}{(1-\cos\theta)(1+\cos\theta)}.$$

- Soft ($l^0 \to 0$) and collinear ($\theta \to 0, \pi$) divergences
- No renormalization procedure to apply $\to$ divergences coming from long distance effects
- Kinoshita-Lee-Nauemberg theorem (*)

Hadron collisions

Jets in e^+e^-

Sterman - Weinberg jets. "In a hadronic event with CM energy E, 2 cones can be found with opening δ containing $(1-\epsilon)$ fraction of E."

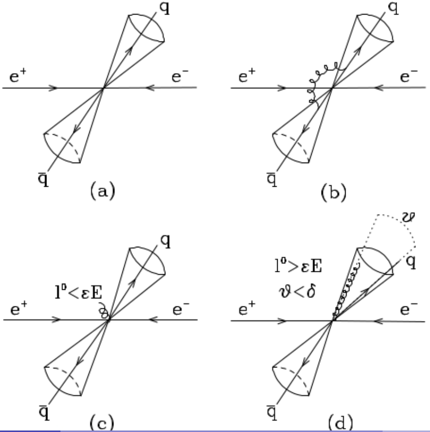
$$\begin{aligned}\text{Born} + \text{Virtual} + \text{Real (a)} + \text{Real (b)} &= \sigma_0 - \sigma_0 \frac{4\alpha_S C_F}{2\pi} \int_{\epsilon E}^{E} \frac{dl^0}{l^0} \int_{\theta=\delta}^{\pi-\delta} \frac{d\cos\theta}{1-\cos^2\theta} \\ &= \sigma_0 \left(1 - \frac{4\alpha_S C_F}{2\pi} \log\epsilon \log\delta^2\right)\end{aligned}$$

When all contributions summed, the cross section is no longer singular (*)

Hadron collisions

Jets in e^+e^-

Sterman - Weinberg jets. "In a hadronic event with CM energy E, 2 cones can be found with opening δ containing $(1-\epsilon)$ fraction of E."

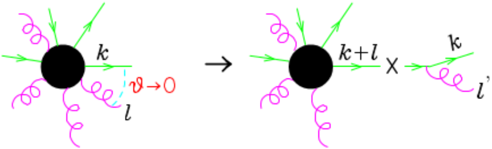


Collinear factorization

Collinear factorization

QCD from e^+e^- annihilation

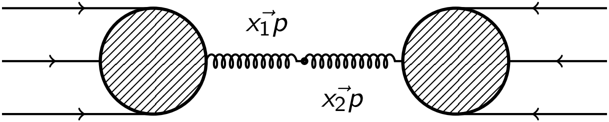
- When computing partonic cross section, collinear partons can be emitted from incoming/outgoing parton.
- Cross section dominated by collinear decay



Factorization theorem

$$|M_{n+1}|^2 d\Phi_{n+1} \Rightarrow |M_n|^2 d\Phi_n \quad \frac{\alpha_S}{2\pi}\,\frac{dt}{t}\,P_{q,qg}(z)\,dz\,\frac{d\phi}{2\pi}.$$

Observables in hadronic events $\longrightarrow$ σ is hard to compute



Factorize the problem $\longrightarrow$ Convolute the PDFs with the partonic $\hat{\sigma}_{ij}$

$$\sigma = \int_0^1 dx_1 \, dx_2 \, f_\alpha(x_1, \mu_F) * f_\beta(x_2, \mu_F) * \hat{\sigma}_{\alpha\beta}(\alpha_s(\mu_R), \mu_F)$$

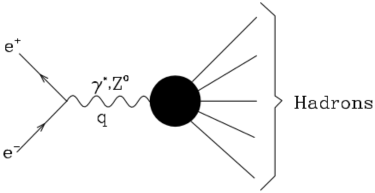
- Partonic $\hat{\sigma}$ can be computed as perturbative series in α_s
- PDFs absorb the non perturbative effects, evaluated at μ_F

Dealing with divergences

Partonic cross section and pQCD

Why do we need series expansion?

1. QCD in $e+e-$ collisions
2. Measure only hadrons in the final state
3. Factorization theorem helps us to understand short range interactions



Write the cross section as a perturbative series

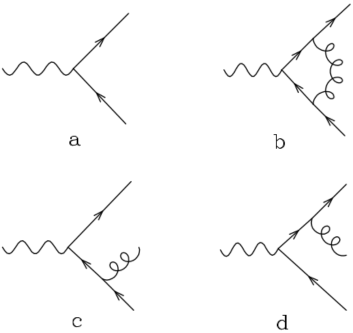
$$\hat{\sigma} = \sigma^{\mathrm{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \ldots\right)$$

Leading order predictions can strongly depend on the renormalization and factorization scales $\rightarrow$ Go for higher order corrections!

Perturbative QCD

Higher order corrections

1. Higher order corrections as virtual and real contributions
2. Large number of diagrams to consider $\rightarrow$ harder to compute



$$\sigma_{q\bar{q}g} = C_F \frac{\alpha_s}{2\pi} \sigma_{q\bar{q}^{\mathrm{Born}}} \int d\cos\theta \frac{dl^0}{l} \frac{4}{(1-cos\theta)(1+cos\theta)}$$

Fixed Order computations diverge!

Truncated fixed order predictions $\rightarrow$ divergent $\ln^m(M^2/q_\perp^2)$ appear.
Then the $q_\perp$ distribution need to be evaluated by replacing the partonic cross section as follows

$$\frac{d\hat{\sigma}_{ab}}{dq_\perp^2} \rightarrow \left[\frac{d\hat{\sigma}_{ab}^{\mathrm{res.}}}{dq_\perp^2}\right]_{\mathrm{l.a.}} + \left[\frac{d\hat{\sigma}_{ab}^{\mathrm{fin.}}}{dq_\perp^2}\right]_{\mathrm{f.o.}}$$

Resummed expression

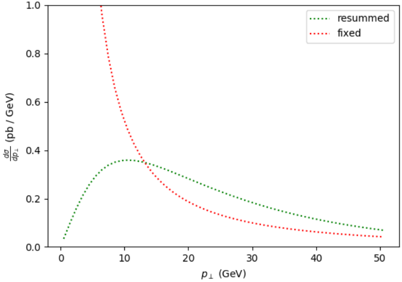
$$\frac{d\hat{\sigma}_{ab}^{\mathrm{res.}}}{dq_\perp^2} = \frac{M^2}{\hat{s}} \int db\, \frac{b}{2}\, J_0(bq_\perp)\, \mathcal{W}_{ab}(b, M, \hat{s}; \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)$$

Being

$$\mathcal{W}_N(b, M, \hat{s}; \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2) = \mathcal{H} \times exp\{\mathcal{G}\}$$

Resummation in QCD

Resumming large logs

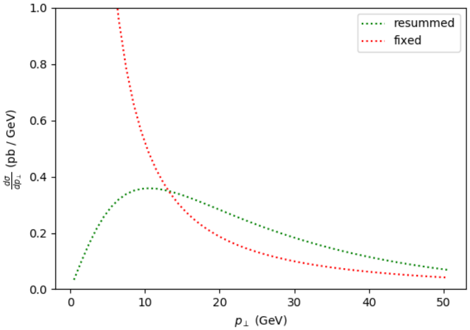


1. FO distribution diverges at small $q_\perp$
2. Sudakov factor kills the divergence
3. Matched at some intermediate accuracy

HTurbo: Fast predictions for Higgs production

HqT and HRes

Predictions for Higgs $q_\perp$ distribution



HTurbo

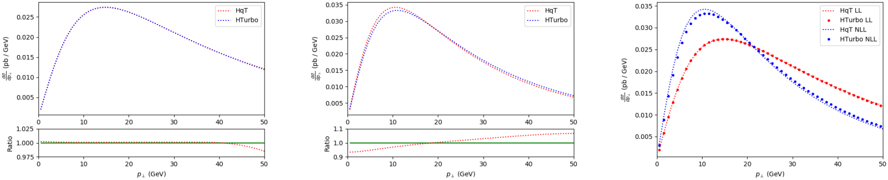
Higgs distribution from Drell Yan

Take DYTurbo from DYTurbo
Ref. at 1910.07049

1. Matrix element
2. Sudakov factors
3. Hard coefficients
4. LO integration

Results

Comparison HTurbo and HqT



- HTurbo produces qt distributions that match HRes and HqT
- Excellent numerical agreement up to NNLO

Summary & Conclusions

1. Fast predictions are required towards the precision era of the LHC
2. HTurbo produces qt distributions that perfectly match HRes and HqT
3. Predictions by HTurbo are much faster than any of the existing codes
4. Next steps: Implement PDF evolution N3LO distributions

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