Two-mode squeezed states in cavity optomechanics via engineering of a single reservoir

PhD course - Quantum coherent phenomena
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Outline

1. Introduction, system and Hamiltonian
2. Reservoir engineering strategies
3. Implementation and observable quantities
4. Experimental observability
5. Conclusions

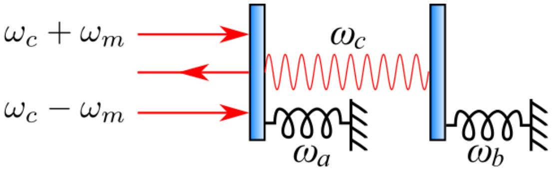
Introduction

1. Generation and detection of entangled states of macroscopic mechanical oscillators
2. Reservoir engineering $\longrightarrow$ Two-mode squeezed states
3. Easy to implement in existing experimental configurations
4. Quantum optomechanics $\longrightarrow$ describe mesoscopic systems

Introduction

System representation

- Two mechanical oscillators with resonance frequencies ω_a, ω_b
- Dispersively coupled g_a, g_b to a common cavity ω_c
- Radiation pressure forces inside the cavity lead motion of the mirrors become highly entangled



Introduction

System and Hamiltonian

Quantum optomechanics $\longrightarrow$ Hamiltonian describing optical and mechanical modes with same formalism

$$\hat{\mathcal{H}} = \omega_a \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b} + \omega_c \hat{c}^\dagger \hat{c} + g_a(\hat{a} + \hat{a}^\dagger)\hat{c}^\dagger \hat{c}$$
$$+ g_b(\hat{b} + \hat{b}^\dagger)\hat{c}^\dagger \hat{c} + \hat{H}_{\text{drive}} + \hat{H}_{\text{diss}},$$

Under usual approximations , obtain the master formula

$$\dot{\rho} = -i[\hat{\mathcal{H}}', \rho] + \gamma_a(\bar{n}_a + 1)\mathcal{D}[\hat{a}]\rho + \gamma_a \bar{n}_a \mathcal{D}[\hat{a}^\dagger]\rho$$
$$+ \gamma_b(\bar{n}_b + 1)\mathcal{D}[\hat{b}]\rho + \gamma_b \bar{n}_b \mathcal{D}[\hat{b}^\dagger]\rho + \kappa \mathcal{D}[\hat{c}]\rho,$$

Being $\mathcal{H}' = \mathcal{H} - \mathcal{H}_{\text{diss}}$, and $\mathcal{D}[\hat{c}]$ the dispersive superoperator
Only dissipation term for $\hat{c} \longrightarrow$ Assuming cavity is at $\mathrm{T} = 0$

Reservoir engineering strategies

Bogoliubov operators

Define the Bogoliuov mechanical modes in terms of the modes $\hat{a}, \hat{b}$

$$\hat{\beta}_1 = \hat{a}\cosh r + \hat{b}^\dagger \sinh r,$$
$$\hat{\beta}_2 = \hat{b}\cosh r + \hat{a}^\dagger \sinh r.$$

being r the squeezing parameter
Work in rotating frame with respect to the Hamiltonian

$$\hat{H}_0 = (\omega_a - \Omega)\hat{a}^\dagger \hat{a} + (\omega_b + \Omega)\hat{b}^\dagger \hat{b} + \omega_c \hat{c}^\dagger \hat{c},$$

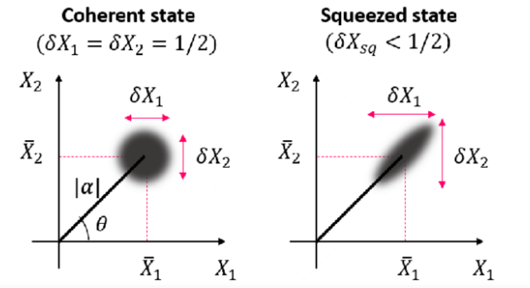
where choice of detuning Ω is such that collective mechanical quadratures $\hat{X}_\pm$, $\hat{P}_\pm$ rotate in a non-trivial way

Reservoir engineering strategies

Note on squeezed modes

Squeezed modes minimize the variance of quadrature operators

$$\hat{S}_2(r) \equiv \exp[r(\hat{a}\hat{b} - \hat{a}^\dagger \hat{b}^\dagger)]$$



Define the 2-mode squeezed as $|r\rangle_2 = \hat{S}_2(r)\,|0,0\rangle$

$$\hat{S}_2(r) \equiv \exp[r(\hat{a}\hat{b} - \hat{a}^\dagger\hat{b}^\dagger)]$$

such that $[\hat{S}_2(r)\hat{a}\hat{S}_2^\dagger(r)]\,|r\rangle_2 = [\hat{S}_2(r)\hat{b}\hat{S}_2^\dagger(r)]\,|r\rangle_2 = 0$

Therefore, $\hat{\beta}_1 = \hat{S}_2(r)\hat{a}\hat{S}_2^\dagger(r)$ and $\hat{\beta}_2 = \hat{S}_2(r)\hat{b}\hat{S}_2^\dagger(r)$ and their ground state is the two-mode squeezed state with squeezing parameter r

Linearized Hamiltonian with 2-tone laser with amplitudes $\alpha_\pm$)

$$\mathcal{H} = \hbar \mathrm{g}_+ \left(a^\dagger b^\dagger + ab\right) + \hbar \mathrm{g}_- \left(a^\dagger b + ab^\dagger\right)$$

being $g_\pm = g_0 \alpha_\pm$
Study different cases

- $g_- = 0 \longrightarrow$ Sideband blue $\mathcal{H} = \hbar \mathrm{g} \left(a^\dagger b^\dagger + ab\right)$ "2 - mode squeezing"
- $g_+ = 0 \longrightarrow$ Sideband red $\mathcal{H} = \hbar \mathrm{g} \left(a^\dagger b + ab^\dagger\right)$ "beam - splitter"
- $g_- = g_+ = g \longrightarrow \mathcal{H} = \hbar \mathrm{g} \left(a^\dagger b + ab^\dagger\right)$ "backaction evading"

Reservoir engineering strategies

Squeezed modes

Study different ways of producing our 2-mode squeezed state

- i) Two cavity modes to independently cool the Bogoliubov modes (beam splitter $\hat{\beta}_i^\dagger \hat{c}_i$)
- ii) Couple the cavity to one Bogoliubov mode, and then this to the other via $\hat{\beta}_1^\dagger \hat{\beta}_2$ and the this to the other one
- iii) Couple the cavity to sum of the Bogoliubov modes , then the sum to the difference . Again, beam splitter interaction $\hat{\beta}_{\mathrm{sum}}^\dagger \hat{\beta}_{\mathrm{diff}}$ allows diff to cool.

$$\hat{\beta}_{\mathrm{sum}} = \frac{1}{\sqrt{2}}(\hat{\beta}_1 + \hat{\beta}_2)$$
$$\hat{\beta}_{\mathrm{diff}} = \frac{1}{\sqrt{2}}(\hat{\beta}_1 - \hat{\beta}_2)$$

Cooling $\hat{\beta}_{\mathrm{sum}}$ and $\hat{\beta}_{\mathrm{diff}}$ is equivalent to cool $\hat{\beta}_1$ and $\hat{\beta}_2$ given $< \hat{\beta}_{\mathrm{sum}}^\dagger \hat{\beta}_{\mathrm{sum}} > + < \hat{\beta}_{\mathrm{diff}}^\dagger \hat{\beta}_{\mathrm{diff}} > = < \hat{\beta}_1^\dagger \hat{\beta}_1 > + \hat{\beta}_2^\dagger \hat{\beta}_2$

Reservoir engineering strategies
Hamiltonian

Hamiltonian in terms of the Bogoliubov modes

$$\hat{\mathcal{H}} = \Omega(\hat{\beta}_1^\dagger \hat{\beta}_1 - \hat{\beta}_2^\dagger \hat{\beta}_2) + \mathcal{G}[(\hat{\beta}_1^\dagger + \hat{\beta}_2^\dagger)\hat{c} + \mathrm{H.c.}] + \hat{H}_{\mathrm{diss}},$$

where Ω is the effective frequency and $\mathcal{G}$ an effective coupling. Written in terms of the original operators,

$$\begin{aligned}\hat{\mathcal{H}} = {} & \Omega(\hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b}) + G_+[(\hat{a} + \hat{b})\hat{c} + \mathrm{H.c.}] \\ & + G_-[(\hat{a} + \hat{b})\hat{c}^\dagger + \mathrm{H.c.}] + \hat{H}_{\mathrm{diss}}.\end{aligned}$$

with couplings related by $\mathcal{G} \equiv \sqrt{G_-^2 - G_+^2}$ and $\tanh r \equiv= G_+/G_-$

$\mathcal{H}$ is already implemented in conventional optomechanical setups. Focus on regime $|G_+| < |G_-|$

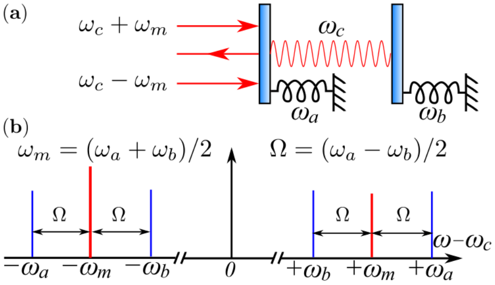
Quantum optomechanics Hamiltonian in terms of β_1 and β_2 modes

- Two-tone driving ($g_a = g_b$) $\longrightarrow$ cavity drive tones at $\omega_c \pm \omega_m$
- Four-tone driving ($g_a = g_b$)
- Case similar ($g_a \sim g_b$)

Implementation

2 - tone driving

Single photon coupling rates equal $\longrightarrow$ cavity drive tones at $\omega_c \pm \omega_m$



Apply our drive Hamiltonian

$$\hat{H}_{\text{drive}} = (\mathcal{E}_+^* e^{+i\omega_m t} + \mathcal{E}_-^* e^{-i\omega_m t}) e^{+i\omega_c t} \hat{c} + \text{H.c.}$$

Implementation

2 - tone driving

Couplings equal $\longrightarrow$ driving tones $\omega_c \pm \omega_m$ being $\omega_m = (\omega_a + \omega_b)/2$
Single photon coupling rates equal $\longrightarrow$ cavity drive tones at $\omega_c \pm \omega_m$
Driving tones applied with single relative phase
Interaction picture with respect to $\mathcal{H}_0$ leads to H (8) ✓

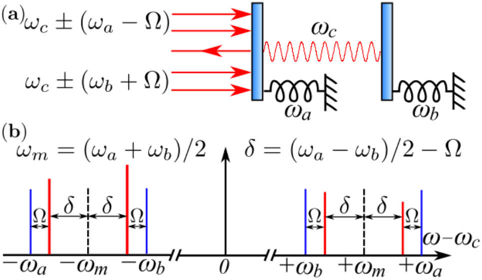
$$\bar{c}_\pm \equiv \langle \hat{c}_\pm \rangle_{\mathrm{ss}} = \frac{i\mathcal{E}_\pm}{\pm i\omega_m - \kappa/2}.$$

- Interaction picture with respect to $\mathcal{H}_0$
- Find the steady state amplitudes at the sidebands

$$\bar{c}_\pm \equiv \langle \hat{c}_\pm \rangle_{\mathrm{ss}} = \frac{i\mathcal{E}_\pm}{\pm i\omega_m - \kappa/2}.$$

- Assumptions used (...)

Driving tones applied with detuning of Ω from the sidebands $\omega_c \pm (\omega_a - \Omega)$ and $\omega_c \pm (\omega_b + \Omega)$



$$\hat{H}_{\text{drive}} = e^{+i\omega_c t}\hat{c}(\mathcal{E}_{1+}^* e^{+i(\omega_a - \Omega)t} + \mathcal{E}_{2+}^* e^{+i(\omega_b + \Omega)t} + \mathcal{E}_{1-}^* e^{-i(\omega_a - \Omega)t} + \mathcal{E}_{2-}^* e^{-i(\omega_b + \Omega)t}) + \text{H.c.}$$

Implementation

4 - tone driving

Couplings unequal $\longrightarrow$ driving tones applied with detuning of Ω from the sidebands $\omega_c \pm (\omega_a - \Omega)$ and $\omega_c \pm (\omega_b + \Omega)$ Interaction picture with respect to $\mathcal{H}_0$ leads to H (14) ✓

$$\bar{c}_{k\pm} \equiv \langle \hat{c}_{k\pm} \rangle_{\rm ss} = \frac{i\mathcal{E}_{k\pm}}{\pm i\omega_k - \kappa/2},$$

- Where we demand the strengths match as $\bar{c}_{1\pm}/\bar{c}_{2\pm} = g_b/g_a$
- Working in interaction picture with respect to Hamiltonian (4)
- Imprecision in the matching lead to add contributions as in Hamiltonian (14)

Condition $\gamma \ll \Omega \ll (\omega_a - \omega_b)/2 - \gamma$, sufficiently coupled Bogoliubov modes and unwanted sideband processes have no effect.

Adiabatic limit

Our system

- Assume the system responds rapidly to mechanical motion $k > \Omega$, $G_{\pm}$, but still in the regime $\omega_a, \omega_b \gg k$
- Simplify by getting rid of the cavity operator $\hat{c} = -2i\mathcal{G}(\hat{\beta}_1 + \hat{\beta}_2)/k$
- Obtain adiabatically eliminated master equation

$$\dot{\rho} = -i\Omega[\hat{\beta}_1^\dagger \hat{\beta}_1 - \hat{\beta}_2^\dagger \hat{\beta}_2, \rho] + \gamma_a(\bar{n}_a + 1)\mathcal{D}[\hat{a}]\rho + \gamma_a \bar{n}_a \mathcal{D}[\hat{a}^\dagger]\rho \\ + \gamma_b(\bar{n}_b + 1)\mathcal{D}[\hat{b}]\rho + \gamma_b \bar{n}_b \mathcal{D}[\hat{b}^\dagger]\rho + \Gamma\mathcal{D}[\hat{\beta}_1 + \hat{\beta}_2]\rho,$$

with optomechanical damping rate

$$\Gamma \equiv \gamma\mathcal{C} \equiv \frac{4\mathcal{G}^2}{\kappa},$$

Being $\mathcal{G}$ effective optomechanical coupling

Easy to obtain steady state, and to measure entanglement and purity.

Adiabatic limit

Entanglement

Build a way of identify entanglement on a 2-mode system
Duan criterion $\longrightarrow$ define collective quadratures

$$\hat{X}_{\pm} = (\hat{X}_a \pm \hat{X}_b)/\sqrt{2},$$
$$\hat{P}_{\pm} = (\hat{P}_a \pm \hat{P}_b)/\sqrt{2},$$

as combination of the usual quadrature modes

$$\hat{X}_s = (\hat{s} + \hat{s}^{\dagger})/\sqrt{2}, \quad \hat{P}_s = -i(\hat{s} - \hat{s}^{\dagger})/\sqrt{2}.$$

Duan inequality states that a state for which

$$\langle \hat{X}_{+}^2 \rangle + \langle \hat{P}_{-}^2 \rangle < 1$$

is inseparable.

Adiabatic limit

Entanglement

Quadratures can be written as function of the drive asymmetry

$$
\begin{aligned}
\langle \hat{X}_{\pm}^2 \rangle = \langle \hat{P}_{\mp}^2 \rangle &= \frac{\gamma}{\gamma + \Gamma}(\bar{n} + 1/2) + \frac{\Gamma}{\gamma + \Gamma}\frac{e^{\mp 2r}}{2} \\
&= \frac{\gamma\kappa}{\gamma\kappa + 4(G_-^2 - G_+^2)}(\bar{n} + 1/2) \\
&\quad + \frac{2(G_- \mp G_+)^2}{\gamma\kappa + 4(G_-^2 - G_+^2)}.
\end{aligned}
$$

Use also logarithmic negativity

Adiabatic limit
Purity

Study purity of the steady state
Entanglement does not imply pure state
Purity defined as trace of the density matrix

$$\mu \equiv \mathrm{tr}[\rho^2]$$

As function of the covariance matrix

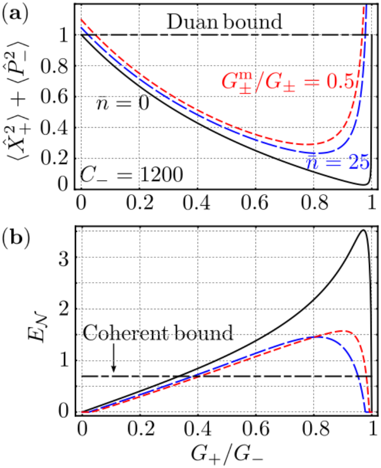
$$\mu = \frac{1}{4\sqrt{\det \mathbf{V}}}$$

Purity can be written as function of the drive asymmetry

$$\mu = \frac{(\gamma + \Gamma)^2}{[\gamma(1 + 2\bar{n}) + \Gamma]^2 + 4(1 + 2\bar{n})\gamma\Gamma \sinh^2 r}.$$

Adiabatic limit

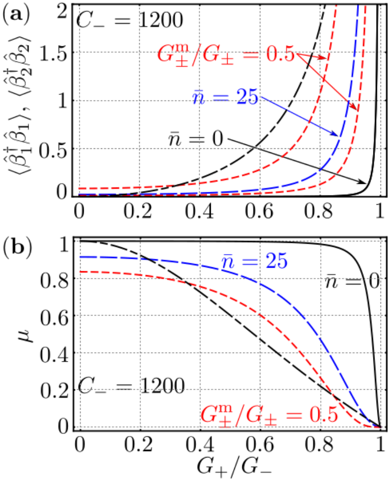
Entanglement



- Case $\gamma_a = \gamma_b$ and $\bar{n}_a = \bar{n}_b$
- Solid curve with mechanical thermal occupation $\bar{n} = 0$ and no imperfection effective coupling $G^m_{\pm} = 0$
- Add thermal occupation leads to less entanglement
- Add drive asymmetry leads to less entanglement

Adiabatic limit

Entanglement



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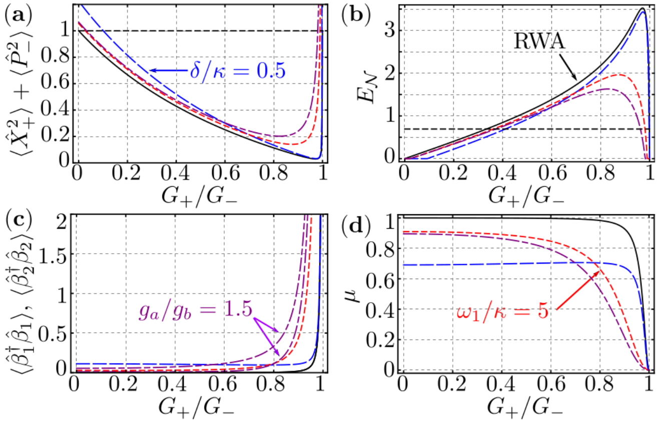
Time dependence

Counter rotating terms and time dependence



Counter-rotating effects lead to degradation of entanglement and purity
RWA recovers recovers the behavior of time-independent Hamiltonian

Experimental observability

Output spectrum

- Extremely demanding reconstruct covariance matrix
- Directly measure quadratures is a hard problem
- Seek signature of entanglement in output spectrum

Spectrum as Fourier transform of expected value

$$S[\omega] = \int dt\, e^{i\omega t} \langle \delta\hat{c}^{\dagger}_{\text{out}}(t)\delta\hat{c}_{\text{out}}(0)\rangle,$$

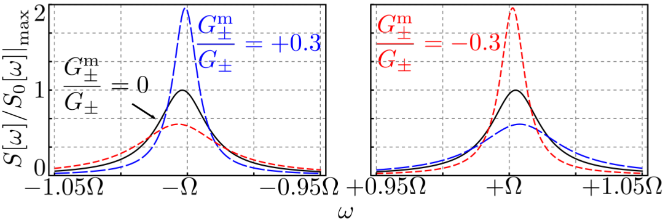
being $\delta\hat{c}_{\text{out}} = \hat{c}_{\text{out}} - < \hat{c}_{\text{out}} >$

Spectrum can be related to the occupation of modes

$$\begin{aligned}\int_{-\infty}^{0} S[\omega]d\omega &= \int_{0}^{+\infty} S[\omega]d\omega \\ &= 8\pi\kappa \frac{\mathcal{G}^2}{4\mathcal{G}^2 + \kappa(\kappa+\gamma)} \langle \hat{\beta}_i^{\dagger}\hat{\beta}_i\rangle,\end{aligned}$$

Experimental observability

Output spectrum



- Centered around the detunings from the cavity resonance frequency
- Solid black curve without imperfections $G^m_\pm / G_\pm = 0$
- Imperfections on the effective couplings described by

$$S[\pm\Omega] = \gamma\kappa \frac{(G_- \pm G^{\rm m}_-)^2 \bar{n} + (G_+ \pm G^{\rm m}_+)^2 (1+\bar{n})}{[G_-^2 - (G^{\rm m}_-)^2 - G_+^2 + (G^{\rm m}_+)^2]^2}$$

Experimental observability

Output spectrum

- Experimental work realized in "Stabilized entanglement of massive mechanical oscillators", Nature, 2018.
- Measure output spectrum and reconstruct quadratures to identify entanglement

Conclusions

1. Configuring a three-mode optomechanical system such as the steady state includes highly pure and highly entangled two-mode squeezed state.
2. Symmetry on the steady-state makes it attractive for implementation of continuous-variable teleportation protocols
3. Problem of unequal single-photon optomechanical couplings solved by using four-tone driving scheme
4. Proposal implementable for existing technology

Back up

Thermal occupation

1. Occupation (photons) at $\omega_c \sim 10^{10}\mathrm{Hz}$

$$\bar{n}_{\mathrm{photons}} = \frac{1}{e^{\frac{\hbar\omega_c}{K_B T}} - 1} \simeq 0$$

2. Occupation (phonons) $\longrightarrow \omega_m \sim 10\ \mathrm{KHz} - 1\ \mathrm{GHz}$

$$\bar{n}_{\mathrm{photons}} = \frac{1}{e^{\hbar\omega_c/K_B T} - 1} \gg 1$$

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