Perturbative QCD and Monte Carlo event generators

Monte Carlo course seminar - Milan, September 2020

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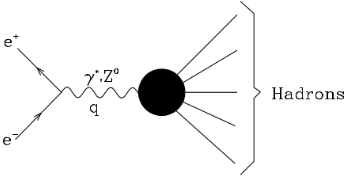
Outline

1. Hadron collisions
   - Hadron collisions and strong interactions
   - Renormalization group
   - Jets and IR divergences
2. Collinear factorization
   - Factorization theorem
   - Kinematics of splitting
   - Recursive factorization
3. Parton showers
   - Final state radiation
   - Initial state radiation
   - Ordering variables (PYTHIA and HERWIG)

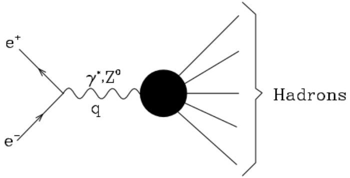
Hadron collisions



QCD arise already from e^+e^- annihilation $\rightarrow R_0$ ratio

$$R_0 = \frac{\sigma(\gamma^* \rightarrow \text{hadrons})}{\sigma(\gamma^* \rightarrow \mu^+\mu^-)} = 3\sum_f c_f^2$$

1. Color factor (3 color for each quark)
2. Sum over charges of different flavour quarks
3. Corrections near the threshold and at higher orders



Consider corrections to R_0 from gluon radiation. Renormalize coupling.

$$R = R_0 \left(1 + \frac{\alpha_S(\mu)}{\pi} + \left[c + \pi b_0 \log \frac{\mu^2}{Q^2}\right] \left(\frac{\alpha_S(\mu}{\pi}\right)^2\right) + \mathcal{O}\left(\alpha_S(\mu)^3\right).$$

Define renormalized coupling, running with the scale
UV logarithmic divergence in $\log(\mu^2/Q^2)$

Questions for a field theory

1. Can we go to arbitrarily large energies? $\rightarrow$ divergences arise, renormalization / factorization needed
2. Can we compute R_0 for every process? $\rightarrow$ IR observables

Hadron collisions

Renormalization group

- UV divergences are encountered in field theories
- Take a physical quantity G depending on a scale M, a coupling α and some invariants $s_1, \ldots, s_n$
- Define a "renormalized" coupling $\alpha_{\mathrm{Ren}} = \alpha + c_1\alpha^2 + c_2\alpha^3 + \ldots$

The physical quantity in terms of $\{\alpha, M\}$ and $\{\alpha_{\mathrm{Ren}}, \mu\}$

$$G(\alpha, M, s_1 \ldots s_n) = \tilde{G}(\alpha_{\mathrm{ren}}, \mu, s_1 \ldots s_n)\,.$$

Physics must be invariant under change of $\{\alpha_{\mathrm{Ren}}, \mu\}$

$$\frac{\partial\alpha(\alpha_{\mathrm{ren}}, M/\mu)}{\partial\alpha_{\mathrm{ren}}} d\alpha_{\mathrm{ren}} + \frac{\partial\alpha(\alpha_{\mathrm{ren}}, M/\mu)}{\partial\mu^2} d\mu^2 = 0$$

Hadron collisions

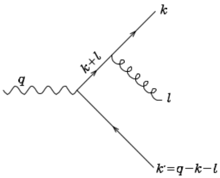
Renormalization group

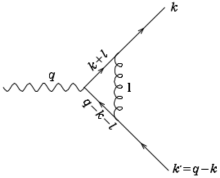
- Running coupling given by Renormalization Group Equation (RGE)

$$\mu \frac{d\alpha_s(\mu)}{d\mu} = \beta(\alpha_s(\mu)) = -\sum_{n=0}^{\infty} \beta_n \left(\frac{\alpha_s}{\pi}\right)^{n+1}$$

- Coupling α_s evolves with scale μ as given by RGE $\rightarrow$ LO behavior driven by β_0
- $\beta_0^{\mathrm{QCD}} > 0 \implies$ weakly coupled at large energies, asymptotic freedom
- $\beta_0^{\mathrm{QED}} < 0 \implies$ strongly coupled at large energies, UV divergent!

Consider α_s corrections to born level amplitude



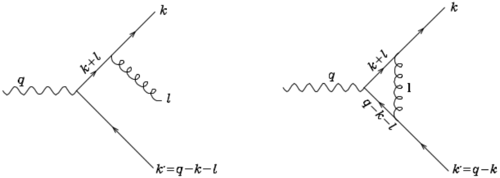


$$\mathcal{M}_{\mathrm{Born}} = \bar{u}(k)\epsilon^{\mu}\gamma_{\mu}v(k')$$

$$\mathcal{M}_1 = \mathcal{M}\frac{k_\alpha}{k \cdot l} \longrightarrow \text{Real radiation}$$

$$\mathcal{M}_1 = -\mathcal{M}\frac{k'_\alpha}{k' \cdot l} \longrightarrow \text{Virtual contribution}$$

Consider α_s corrections to born level amplitude



Sum real and virtual contributions to the Born matrix element, square and integrate with phase space element $d^3l = (l^0)^2 dl^0 d\cos\theta d\phi$

$$\sigma_{q\bar{q}g} = C_F \frac{\alpha_S}{2\pi}\sigma_{q\bar{q}}^{\text{Born}} \int d\cos\theta \frac{dl^0}{l^0}\frac{4}{(1-\cos\theta)(1+\cos\theta)}$$

$$\sigma_{q\bar{q}g} = C_F \frac{\alpha_S}{2\pi} \sigma_{q\bar{q}}^{\mathrm{Born}} \int d\cos\theta \frac{dl^0}{l^0} \frac{4}{(1-\cos\theta)(1+\cos\theta)}$$

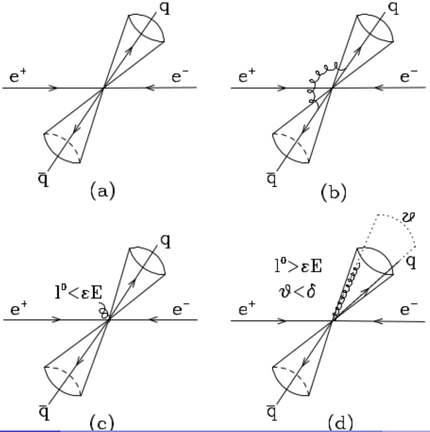
- Soft ($l^0 \to 0$) and collinear ($\theta \to 0, \pi$) divergences
- No renormalization procedure to apply $\to$ divergences coming from long distance effects (fermion masses, hadronization, etc)
- Kinoshita-Lee-Nauemberg theorem (*)

Understand Born cross section as the LO term in a well defined perturbative expansion

Hadron collisions

Sterman-Weinberg jets

Sterman - Weinberg jets. "In a hadronic event with CM energy E, 2 cones can be found with opening δ containing $(1-\epsilon)$ fraction of E."



Sterman - Weinberg jets. "In a hadronic event with CM energy E, 2 cones can be found with opening δ containing $(1-\epsilon)$ fraction of E."

$$\begin{aligned}\text{Born} + \text{Virtual} + \text{Real (a)} + \text{Real (b)} &= \sigma_0 - \sigma_0 \frac{4\alpha_S C_F}{2\pi} \int_{\epsilon E}^{E} \frac{dl^0}{l^0} \int_{\theta=\delta}^{\pi-\delta} \frac{d\cos\theta}{1-\cos^2\theta} \\ &= \sigma_0 \left(1 - \frac{4\alpha_S C_F}{2\pi} \log\epsilon \log\delta^2\right)\end{aligned}$$

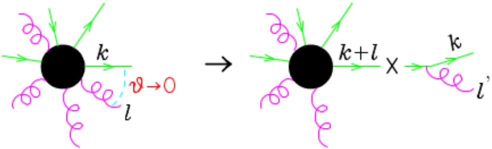
When all contributions summed, the cross section is no longer singular (*)
Computed in terms of partons, but representing hadronic final state
Jets as IR finite final state (*)

Collinear factorization

Collinear factorization

QCD from e^+e^- annihilation

- When computing partonic cross section, collinear partons can be emitted from incoming/outgoing parton
- σ dominated by collinear decay of parton with small virtuality



Factorization theorem $\longrightarrow$ Factor out tree level amplitude and splitting

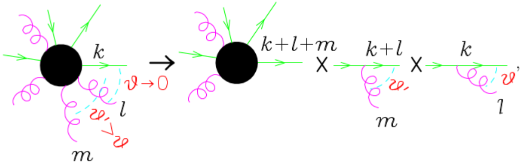
$$|M_{n+1}|^2 d\Phi_{n+1} \Rightarrow |M_n|^2 d\Phi_n \quad \frac{\alpha_S}{2\pi}\frac{dt}{t} P_{q,qg}(z)\, dz\, \frac{d\phi}{2\pi}.$$

Collinear factorization

QCD from e^+e^- annihilation

- Kinematics of splitting (t, z, ϕ)
  - t has dimensions of energy (virtuality, $p_\perp$, angular variable)
  - z represents the fraction of momentum of radiated parton
  - ϕ represents azimuth of the k, l plane

- Factorization holds for small angles. Applied recursively

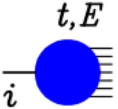


Parton showers and MC generators

Parton showers and MC generators

Formal representation of a shower

Approximated description of a hadronic final state. Model a given hard scattering with arbitrary number of enhanced radiations

$\mathcal{S}_i(t, E) =$ 

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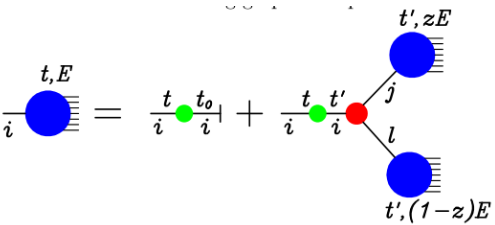
- Enseble of all possible showers from a parton i at scale t
- Sudakov form factor $\Delta_i(t, t_0)$ such that $\Delta_i(t_0, t_0) = 1$
- Shower $S_i(t, E)$ such that $S_i^{\text{inc}} = \sum_{\mathcal{F}} S_i(t, E) = 1$

Parton showers and MC generators

Formal representation of a shower

Ensemble of all possible radiations as the sum of no radiation, with radiation and shower from radiated partons



- Ansatz for Sudakov

$$\Delta_i(t,t') = \exp\left\{ -\int_{t'}^{t} \frac{dt''}{t''} \int dz \sum_{jl} P_{i,jl}(z) \frac{\alpha_s(t')}{2\pi} \right\}$$

- Therefore $\partial\Delta(t,t')/\partial t \propto \Delta(t,t') \longrightarrow$ apply shower recursively

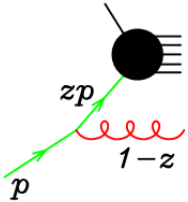
Generate hard process with probability proportional to its parton level cross section. For each final state colored parton:

1. Set scale t to Q, hard scale of the process
2. Generate random number $0 < r < 1$
3. Solve $r = \Delta_i(t, t')$ for t'
4. i) if $t' < t_0$, no further branching and stop shower
5. ii) if $t' \geq t_0$, one branching into partons j, l with energies $E_j = zE_i$ and $E_l = (1 - z)E_i$, z following the $P_{i,jl}(z)$ distribution and ϕ uniform in the interval $[0, 2\pi]$
6. For each branched partons set $t = t'$ and start from (2)

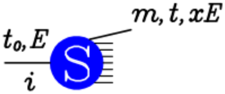
Parton showers and MC generators

Initial state radiation

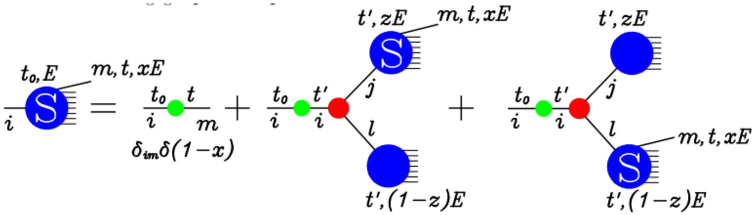
ISR already important in QED $\longrightarrow$ Used to determine the Z peak at LEP



- QCD coupling much larger $\longrightarrow$ QCD ISR even more important
- Specially large for small momentum transfer
- Same as final state partons *always* manifest as jets, initial state ones *always* lead to ISR

$$S_i(m, x, t, E) =$$ 

- Lines between t_1 and t_2 (consecutive radiations) are spacelike (*)
- Difference in Sudakov factors and Splitting functions start at NLO



HERWIG

- Ordering variable $t = E^2\theta^2/2$
- Order of transverse momentum as "angular ordering"
- IR cut-off needed

PYTHIA

- There is not angular ordering
- More natural kinematics
- Unphysical increase of number of partons $\longrightarrow$ solve by imposing veto to branchings that violate angular ordering