

# Higgs boson production at the Large Hadron Collider: accurate theoretical predictions at higher orders in QCD

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PhD thesis defense - Milan, February 25th, 2022



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This project has received funding from the European Union's Horizon 2020 research and innovation program under grant agreement No 740006.

# Outline

## ① QCD and collider physics

- The strong interactions
- Asymptotic freedom and pQCD
- Factorization in QCD
- Phenomenology at the LHC

## ② All order perturbative resummation

- Higher order radiative corrections
- Resummation of large logarithmic corrections
- Resummed component, asymptotic and fixed-order

## ③ HTurbo numerical implementation

- Higgs production at the LHC
- HTurbo numerical implementation
- $N^3LL$  implementation

## ④ Results & Conclusions

# Part I

## QCD and collider physics

# Introduction

## QCD and the strong interactions

- The Standard Model describes fundamental interactions at the TeV scale
- Particles as local excitations of fields with quantum mechanical behavior
- Lagrangian describing the fundamental objects of the theory

$$\mathcal{L} = \bar{\psi}_q^i (i\gamma^\mu)(D_\mu)_{ij}\psi_q^j - m_q \bar{\psi}_q^i \psi_{qi} - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

QCD is the theory of the strong interactions  $\longrightarrow$  interactions between **quarks and gluons**

# Introduction

## QCD and the strong interactions

How to explore proton's inner structure?

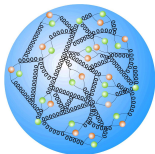


- At different scales, hadrons show different behavior
- From point-like objects to complex internal dynamics
- Scattering experiments (DIS) and hadronic physics (LHC)

"A way of describing high energy collisions is to consider any hadron as a composite object of point-like constituents → **partons**" R.Feynman, 1969

# QCD and collider physics

## Asymptotic freedom and pQCD



- Parton model as LO approximation to QCD
- QCD coupling strength  $\alpha_s$  changes with energy
- At high energies the hadron involves extremely complex internal dynamics

QCD is strongly coupled at large distances / low energies  $\longrightarrow$  confinement

Non-perturbative physics

# QCD and collider physics

## Asymptotic freedom and pQCD

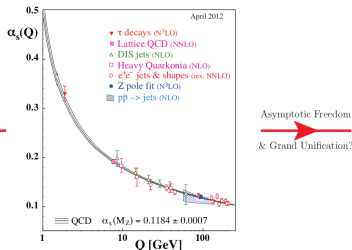
- Running coupling given by Renormalization Group Equation (RGE)

$$\mu \frac{d\alpha_s(\mu)}{d\mu} = \beta(\alpha_s(\mu)) = - \sum_{n=0}^{\infty} \beta_n \left( \frac{\alpha_s}{\pi} \right)^{n+1}$$

- Coupling  $\alpha_s$  evolves with scale  $\mu$  as given by RGE  $\rightarrow$  LO behavior driven by  $\beta_0$
- Main difference between QED and QCD
  - $\beta_0^{\text{QED}} < 0 \implies$  strongly coupled at large energies
  - $\beta_0^{\text{QCD}} > 0 \implies$  weakly coupled at large energies

# QCD and collider physics

## Asymptotic freedom and pQCD



- Running coupling given by Renormalization Group Equation (RGE)

$$\alpha_s(\mu) = \frac{1}{\beta_0 \log\left(\frac{\mu^2}{\Lambda_{\text{QCD}}^2}\right)}$$

- $\beta_0$  LO of the  $\beta$  function, is  $> 0$
- $\Lambda_{\text{QCD}}$ , parameter that defines value of the coupling at large scales

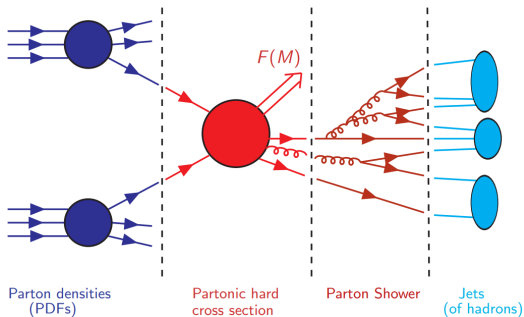
QCD is weakly coupled for  $\mu \gg \Lambda_{\text{QCD}} \rightarrow$  asymptotically free

Perturbative Quantum Chromodynamics (pQCD)



# QCD and collider physics

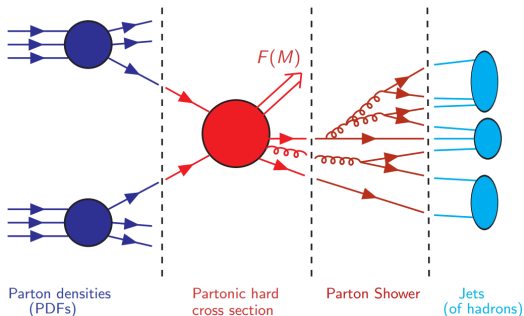
## Hadronic processes and factorization



- LHC physics rely on hadronic collisions  $\rightarrow$  pQCD
- Compute **cross section**  $\sigma^F \rightarrow$  probability for a given process

# QCD and collider physics

## Hadronic processes and factorization

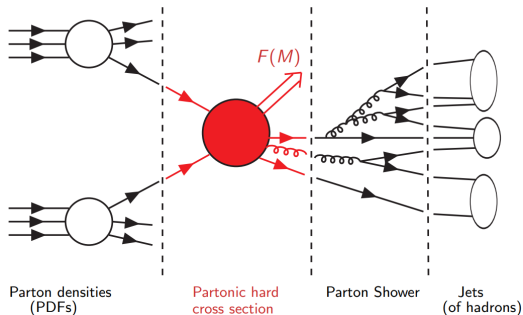


Compute hadronic cross sections is a **hard problem**  $\rightarrow$  **QCD Factorization**

$$\sigma^F(p_1, p_2) = \int_0^1 dx_1 dx_2 f_\alpha(x_1, \mu_F^2) * f_\beta(x_2, \mu_F^2) * \hat{\sigma}_{\alpha\beta}^F(x_1 p_1, x_2 p_2, \alpha_s(\mu_R^2), \mu_F^2)$$

# QCD and collider physics

## Hadronic processes and factorization

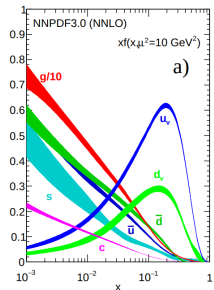


- Parton densities (PDFs)  $f_a(x_i, \mu_F^2)$ : non perturbative but universal
- Partonic cross section  $\hat{\sigma}_{\alpha\beta}^F$ : process dependent but computable as perturbative series in  $\alpha_s$

# QCD and collider physics

## Parton densities

Parton Distribution Functions: probability distribution of finding a particular parton ( $u$ ,  $d$ , ...,  $g$ ) carrying a fraction  $x$  of the proton's momentum



- Each parton has a different PDF  $\rightarrow u(x), d(x), \dots, g(x)$
- PDFs can not predicted and yet can not measured  $\rightarrow$  extracted from data (MSTW, CTEQ, NNPDF collaborations)
- The N3PDF project: Machine Learning for PDFs determination

# QCD and collider physics

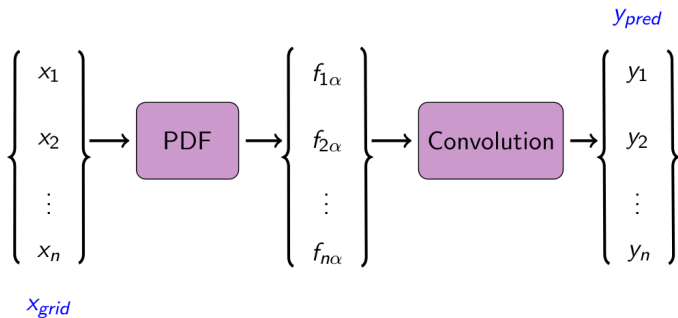
The N3PDF project



- Use TensorFlow and Keras to determine the PDFs with ML fitting models
- See paper by S.Carrazza - J.Cruz-Martinez  
"Towards a new generation of parton densities with deep learning models",  
Carrazza et al., <https://arxiv.org/abs/1907.05075>
- TensorFlow operator implementation → optimize PDF fitting  
"Towards hardware acceleration for parton densities estimation",  
Urtasun-Elizari et al., <https://arxiv.org/abs/1909.10547>

# QCD and collider physics

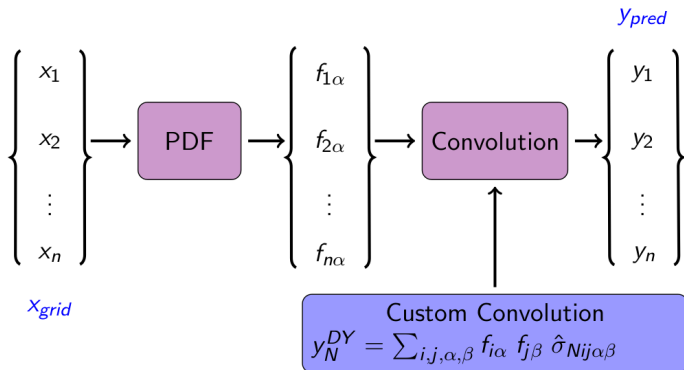
## General structure of n3fit



- 1 Build a NN to compute  $y_{pred}$  observables from a grid of momentum fractions  $x_i$
- 2 Compute  $\chi^2$  loss function by comparing with LHC data
- 3 Update values of PDF through  $\chi^2$  minimization  $\rightarrow$  **Fit**

# QCD and collider physics

## Operator implementation in TF

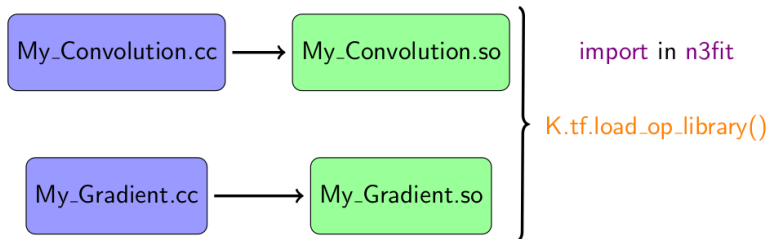


- 1 TF relies in symbolic computation  $\rightarrow$  High memory usage
- 2 Implement C++ operator replacing the convolution
- 3 [Urtasun-Elizari et al.] ref. at [1910.07049](#)

# QCD and collider physics

## Operator implementation in TF

- 1 Build `.cc` files computing the convolution and the gradient.
- 2 Compile them to build `.so` libraries.  
`g++ -std=c++11 -shared My_Convolution.cc -o My_Convolution.so $TF_FLAGS`
- 3 Load libraries in DIS/DY layers of `n3fit` with `load_op_library()` function of TF

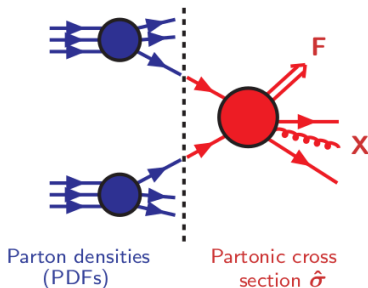




# QCD and collider physics

## Partonic cross section and pQCD

- Born cross section is the leading-order (LO) term of the perturbative series
- $\sigma^{(1)}, \sigma^{(2)}, \sigma^{(3)}$  are the NLO, NNLO, N<sup>3</sup>LO corrections



$$\hat{\sigma} = \sigma^{\text{Born}} \left( 1 + \alpha_s \sigma^{(1)} + \alpha_s^2 \sigma^{(2)} + \alpha_s^3 \sigma^{(3)} + \dots \right)$$

Lower order predictions strongly depend on the auxiliary / unphysical scales

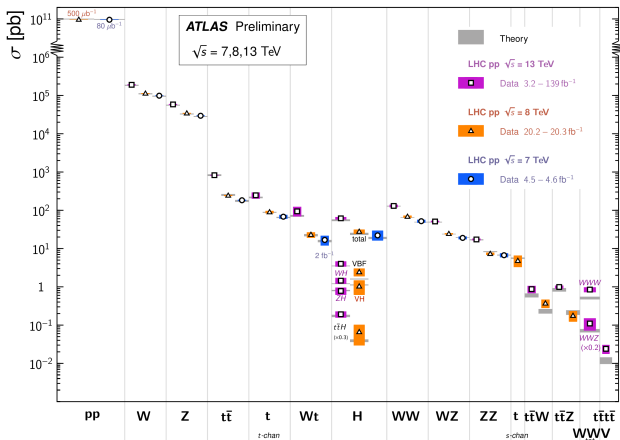
Need higher order corrections to increase theoretical accuracy!

# QCD and collider physics

## LHC phenomenology

Standard Model Total Production Cross Section Measurements

Status: July 2021



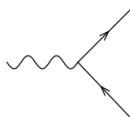
## Part II

### All order resummation

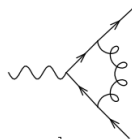
# Resummation in QCD

Higher order corrections - need for resummation

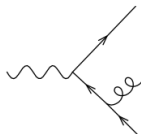
- 1 Calculation of higher order corrections is **not an easy task** due to **infrared (IR) soft and collinear singularities**
- 2 Final state singularities **cancel** by combining real and virtual contributions  $\rightarrow$  KLN theorem
- 3 Initial state collinear singularities **factorized** inside the PDFs



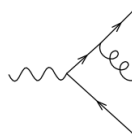
a



b



c



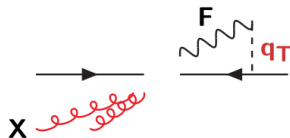
d

Complete cancellation only works in inclusive final states!

# Resummation in QCD

## $q_\perp$ resummation

- Describing exclusive final states
- Study the differential  $q_\perp$  distribution  
 $h_1(p_1) + h_2(p_2) \longrightarrow F(M, q_\perp) + X$



$$\int_0^{Q_\perp^2} dq_\perp^2 \frac{d\hat{\sigma}}{dq_\perp^2} \sim c_0 + \alpha_s (c_{12} L^2 + c_{11} L + c_{10}) + \dots, \quad \text{where} \quad L = \ln(M^2/q_\perp^2)$$

$\alpha_s L^2$	$\alpha_s L$	$\dots$	$\mathcal{O}(\alpha_s)$
$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\dots$	$\mathcal{O}(\alpha_s^2)$
$\dots$	$\dots$	$\dots$	$\dots$
$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\dots$	$\mathcal{O}(\alpha_s^n)$
dominant logs	$\dots$	$\dots$	$\dots$

Truncated fixed-order predictions  $\rightarrow$  enhanced  $\alpha_s^n \ln^m(M^2/q_\perp^2)$  appear

# Resummation in QCD

## $q_\perp$ resummation

- Catani - Bozzi - de Florian - Grazzini (CBFG) formalism (\*)  
"Transverse-momentum resummation and the spectrum of the Higgs boson at the LHC",  
Bozzi et al., <https://arxiv.org/abs/hep-ph/0508068>
- Separate partonic  $q_\perp$  distribution as follows:

$$\frac{d\hat{\sigma}_{ab}}{dq_\perp^2} = \left[ \frac{d\hat{\sigma}_{ab}^{(\text{res.})}}{dq_\perp^2} \right]_{\text{l.a.}} + \left[ \frac{d\hat{\sigma}_{ab}^{(\text{fin.})}}{dq_\perp^2} \right]_{\text{f.o.}}, \quad \text{such that}$$

$$\int_0^{q_\perp^2} dq_\perp^2 \frac{d\hat{\sigma}_{ab}^{(\text{res.})}}{dq_\perp^2} \sim \sum \alpha_s^n \log^m \left( \frac{M^2}{q_\perp^2} \right) \quad \text{for } q_\perp \rightarrow 0$$

$$\lim_{q_\perp \rightarrow 0} \int_0^{q_\perp^2} dq_\perp^2 \frac{d\hat{\sigma}_{ab}^{(\text{fin.})}}{dq_\perp^2} = 0$$

Resummed and finite components can be matched (LL+LO, NLL+NLO, NNLO+NNLL, ...) to have uniform accuracy in a wide range of  $q_\perp$

# Resummation in QCD

## Resummed component

Resummation holds in impact parameter space  $b$

$$\frac{d\hat{\sigma}_{ab}^{(\text{res.})}}{dq_{\perp}^2} = \frac{M^2}{\hat{s}} \int db \frac{b}{2} J_0(bq_{\perp}) \mathcal{W}_{ab}(b, M)$$

with  $\mathcal{W}_{ab}$  also expressed in Mellin space (with respect to  $z = M^2/\hat{s}$ )

$$\mathcal{W}_N(b, M) = \mathcal{H}_N(\alpha_s) \times \exp\{\mathcal{G}_N(\alpha_s, L)\} \quad \text{being} \quad L \equiv \log(M^2 b^2)$$

- Large logarithms exponentiated in the universal Sudakov form factor  $\mathcal{G}_N(\alpha_s, L)$
- Constant (b-independent) terms factorized in the process dependent hard factor  $\mathcal{H}_N(\alpha_s)$

# Resummation in QCD

## Extend formalism to N<sup>3</sup>LL

Sudakov factor  $\mathcal{G}_N$  and hard coefficient  $\mathcal{H}_N$  can be expanded as perturbative series in  $\alpha_s$

$$\mathcal{G}_N(\alpha_s, L) = L g^{(1)}(\alpha_s L) + g^{(2)}(\alpha_s L) + \frac{\alpha_s}{\pi} g^{(3)}(\alpha_s L) + \left(\frac{\alpha_s}{\pi}\right)^2 g^{(4)}(\alpha_s L) + \dots$$

$$\mathcal{H}_N(\alpha_s) = 1 + \alpha_s \mathcal{H}^{(1)} + \alpha_s^2 \mathcal{H}^{(2)} + \alpha_s^3 \mathcal{H}^{(3)} + \dots$$

For each new order implement a factor of  $\mathcal{G}_N$  and Hard  $\mathcal{H}_N$

$$\text{LL}(\sim \alpha_s^n L^{n+1}) : g^{(1)}, \hat{\sigma}^{(0)}$$

$$\text{NLL}(\sim \alpha_s^n L^n) : g^{(2)}, \mathcal{H}^{(1)}$$

$$\text{NNLL}(\sim \alpha_s^n L^{n-1}) : g^{(3)}, \mathcal{H}^{(2)}$$

$$\text{N}^3\text{LL}(\sim \alpha_s^n L^{n-2}) : g^{(4)}, \mathcal{H}^{(3)}$$

- Implement CBFGR resummation in C++ code
- Extend the formalism up to **N<sup>3</sup>LO+N<sup>3</sup>LL** accuracy!



## Part III

### HTurbo numerical implementation

# HTurbo

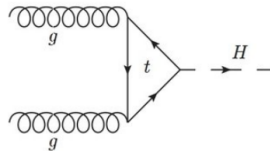
## Resummation for Higgs differential distribution

- Fast and accurate predictions for Higgs boson production cross section
- Predictions for differential cross section  $d\sigma^H/dq_\perp^2$
- Numerical implementation of resummed and finite components

$$d\sigma_{(N)\text{NLL}+(N)\text{LO}}^H = d\sigma_{(N)\text{NLL}}^{(\text{res.})} - d\sigma_{(N)\text{LO}}^{(\text{asy.})} + d\sigma_{(N)\text{LO}}^{(\text{f.o.})}$$

$$d\sigma_{(N)\text{NLL}}^{(\text{res.})} = \hat{\sigma}_{\text{LO}}^H \times \mathcal{H}_{(N)\text{LO}} \times \exp \mathcal{G}_{(N)\text{NLL}}$$

$$d\sigma_{(N)\text{LO}}^{(\text{asy.})} = \hat{\sigma}_{\text{LO}}^H \times \Sigma_{(N)\text{LO}}$$

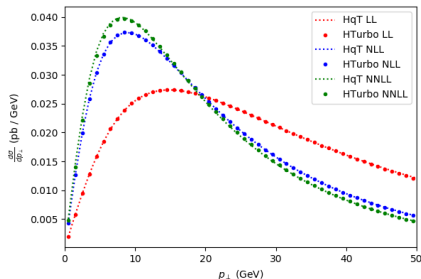


- LO process is just  $gg \rightarrow H$ , but NLO and beyond require  $gg \rightarrow H + \text{jet!}$

# HTurbo

## Predictions for Higgs $q_{\perp}$ distribution

- $q_{\perp}$  resummation implemented in numerical codes **HqT**, **HRes**, **HNNLO** [Catani, de Florian, Ferrera, Grazzini, Tommasini]
- Higher order accuracy require **high computation times**
- NNLL predictions can take more than 48h **just for 1 PDF set and 1  $\mu_R, \mu_F$  value**  $\rightarrow$  need for **fast numerical implementations**



Codes producing fast and accurate predictions are needed for precision era of the LHC (High Luminosity LHC, from 80 - 140  $\text{fb}^{-1}$  to **2000  $\text{fb}^{-1}$**  !)

# HTurbo

Starting point: DYTurbo

Numerical code **DYTurbo** [Camarda et al., <https://arxiv.org/abs/1910.07049>], fast and precise  $q_\perp$  resummation and several improvements for Drell-Yan ( $h_1 h_2 \rightarrow V + X \rightarrow l^+ l^- + X$ )

**First goal:** set up a numerical code for Higgs boson production starting from **DYTurbo**

- Set LO amplitude  $gg \rightarrow H$
- Set Sudakov and Hard coefficients for resummed component
- Set  $\Sigma$  coefficients for asymptotic term
- Implement MC producing the LO and NLO H+jet cross sections
- Compare with **HRes** and **HqT**

**Final goal:** extend theoretical accuracy up to  $N^3\text{LL}+N^3\text{LO}$

Optimized reimplementation of **HqT**, **HRes** and **HNNLO** for  $q_T$ -resummation

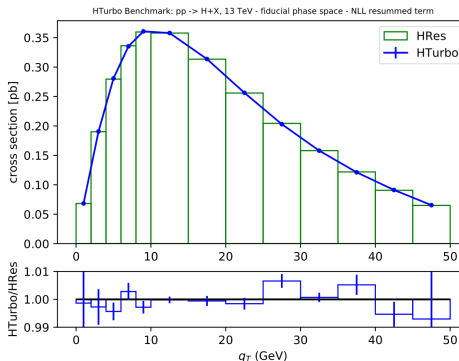
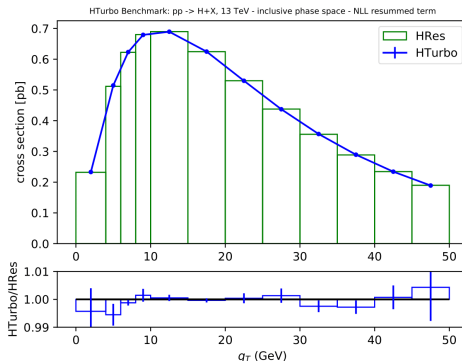
- **C++** structure with **Fortran** interfaces → Multi-threading
- Optimization in the integration routines / integral transforms
  - Factorize boson and decay kinematics
  - Gauss-Legendre quadrature rules (1-dim.)
  - Vegas/Cuhre through **Cuba** (multi-dim.)

Benchmark comparison with **HRes**, **HNNLO** - numerical and speed performance

- "Higgs boson production at the LHC: fast and precise predictions in QCD at higher orders", Urtasun-Elizari et al., <https://arxiv.org/abs/2202.10343>, submitted to EPJC

# Results

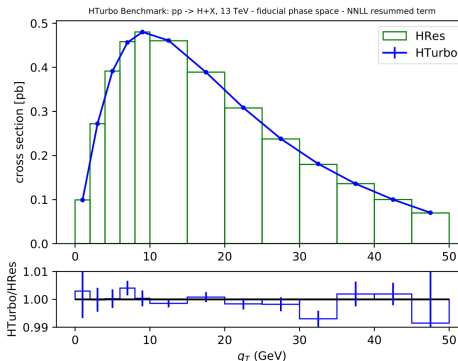
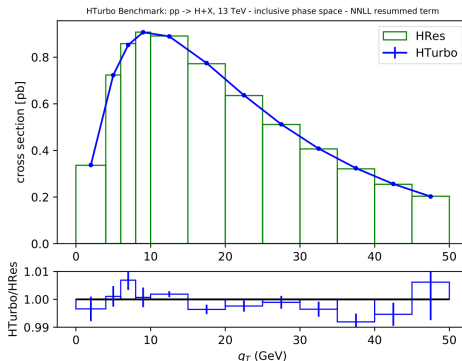
## Comparison HTurbo and HRes - NLL resummed



- Cross section for fully inclusive (LHS) and fiducial (RHS) phase space ✓
- CM energy  $\sqrt{s} = 13$  TeV and PDF set NNPDF31\_nlo\_as\_0118 PDF set

# Results

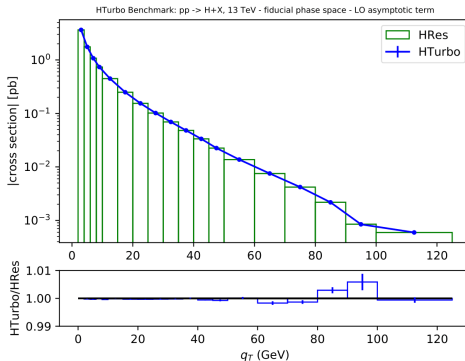
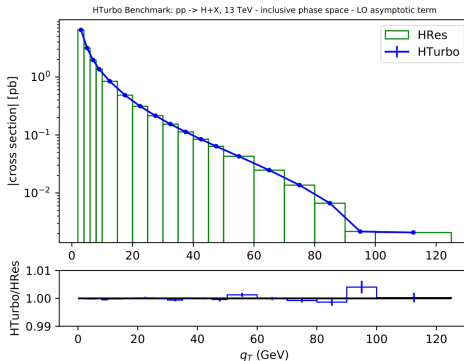
## Comparison HTurbo and HRes - NNLL resummed



- Cross section for fully inclusive (LHS) and fiducial (RHS) phase space ✓
- CM energy  $\sqrt{s} = 13$  TeV and PDF set NNPDF31\_nnlo\_as\_0118 PDF set

# Results

## Comparison HTurbo and HRes - LO asymptotic

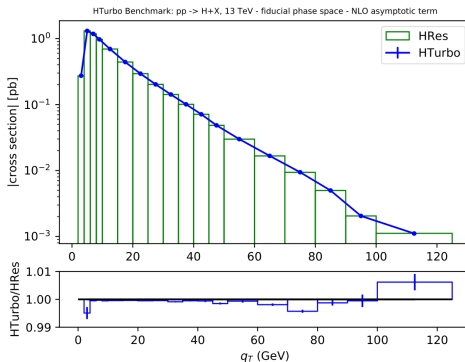
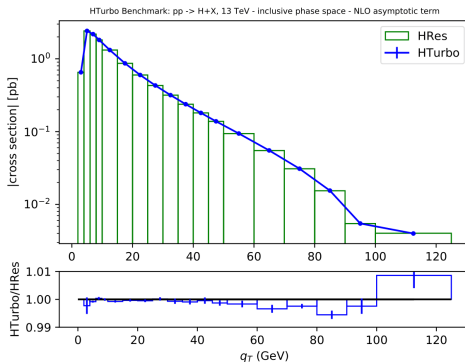


- Cross section for fully inclusive (LHS) and fiducial (RHS) phase space ✓
- CM energy  $\sqrt{s} = 13$  TeV and PDF set NNPDF31\_nlo\_as\_0118 PDF set



# Results

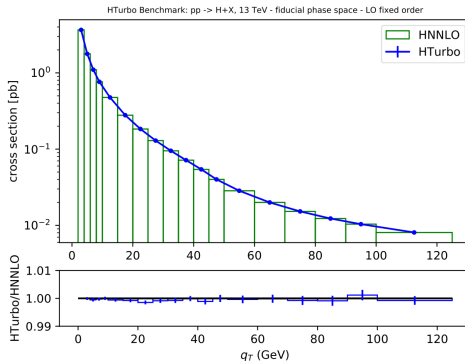
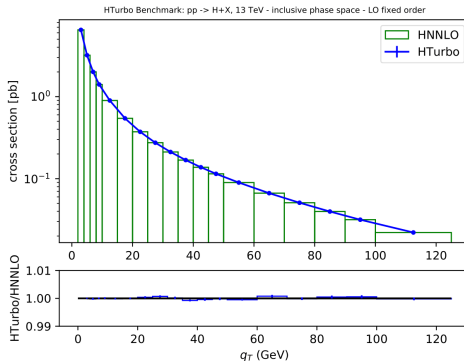
## Comparison HTurbo and HRes - NLO asymptotic



- Cross section for fully inclusive (LHS) and fiducial (RHS) phase space ✓
- CM energy  $\sqrt{s} = 13$  TeV and PDF set NNPDF31\_nnlo\_as\_0118 PDF set

# Results

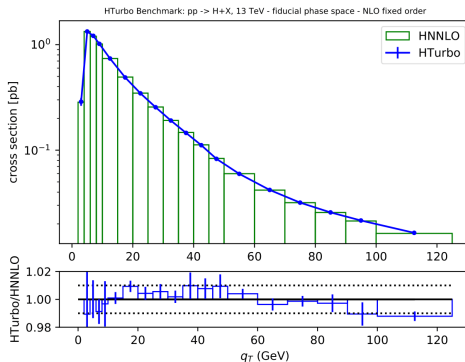
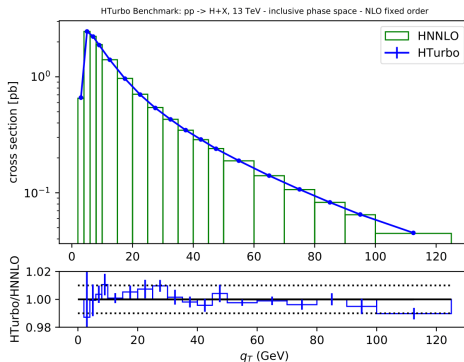
## Comparison HTurbo and HRes - LO fixed-order



- Cross section for fully inclusive (LHS) and fiducial (RHS) phase space ✓
- CM energy  $\sqrt{s} = 13$  TeV and PDF set NNPDF31\_nlo\_as\_0118 PDF set

# Results

## Comparison HTurbo and HRes - NLO fixed-order



- Cross section for fully inclusive (LHS) and fiducial (RHS) phase space ✓
- CM energy  $\sqrt{s} = 13$  TeV and PDF set NNPDF31\_nnlo\_as\_0118 PDF set

# Results

## Speed performance

Test time performance in machine with 3.50 GHz Intel Xeon CPUs:

- HRes NLL resummed  $\rightarrow$  0.5h with 1% uncertainty
- HTurbo NLL resummed (without multi-threading)  $\rightarrow$  20s with 0.001% uncertainty
- HRes NNLL resummed  $\rightarrow$  48h with 1% uncertainty
- HTurbo NNLL resummed (without multi-threading)  $\rightarrow$  5' with 0.01% uncertainty
- Improvement of two orders of magnitude in the time performance
- Improvement of three orders of magnitude in numerical precision

# Results

## N<sup>3</sup>LL implementation

Sudakov factor  $\mathcal{G}_N$  and hard coefficient  $\mathcal{H}_N$  can be expanded as perturbative series in  $\alpha_s$

$$\mathcal{G}_N(\alpha_s, L) = L g^{(1)}(\alpha_s L) + g^{(2)}(\alpha_s L) + \frac{\alpha_s}{\pi} g^{(3)}(\alpha_s L) + \left(\frac{\alpha_s}{\pi}\right)^2 g^{(4)}(\alpha_s L) + \dots$$

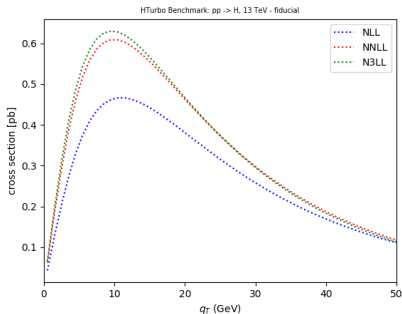
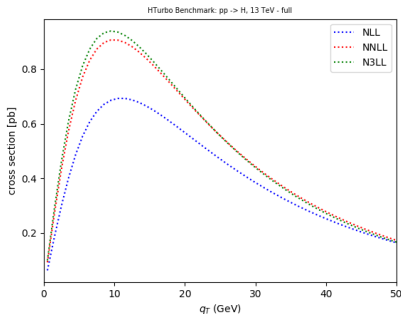
$$\mathcal{H}_N(\alpha_s) = 1 + \alpha_s \mathcal{H}^{(1)} + \alpha_s^2 \mathcal{H}^{(2)} + \alpha_s^2 \mathcal{H}^{(3)} + \dots$$

For each new order implement a new factor of  $\mathcal{G}_N$  and Hard  $\mathcal{H}_N$

- Extend the formalism up to **N<sup>3</sup>LO+N<sup>3</sup>LL** accuracy!
- Implementation of N<sup>3</sup>LL factors following
  - "Anomalous dimension for transverse-momentum resummation",  
Li - Zhu, <https://arxiv.org/abs/1604.01404>,
  - "Cusp and collinear anomalous dimensions in four-loop QCD",  
Von Manteuffel et al., <https://arxiv.org/abs/2002.04617>

# Results

## $N^3\text{LL}$ implementation



- Cross section for fully inclusive (LHS) and fiducial (RHS) phase space ✓
- Implementation of  $N^3\text{LL}$  factors following [\[Li - Zhu, 1604.01404\]](#), [\[Von Manteuffel et al., 2002.04617\]](#)
- First implementation of resummed Higgs cross section at  $N^3\text{LL}$  accuracy!

# Summary & Conclusions

- ➊ Accurate predictions are needed towards the precision era of the LHC
- ➋ Resummation is needed for describing differential distributions
- ➌ Fast numerical implementation are needed towards the precision era of the LHC
- ➍ Developing a novel numerical code, **HTurbo**, which implements  $q_{\perp}$  resummation for Higgs boson production
- ➎ HTurbo is **faster than any of the existing codes**
- ➏ HTurbo contains the **first implementation of resummation at  $N^3LL$  accuracy!**
- ➐ Next steps:
  - Add full  **$N^3LO+N^3LL$**  prediction
  - Perform phenomenological studies comparing with LHC data

Thank you!



This project has received funding from the European Union's Horizon 2020 research and innovation program under grant agreement No 740006.



# Back up

Back up

# Back up

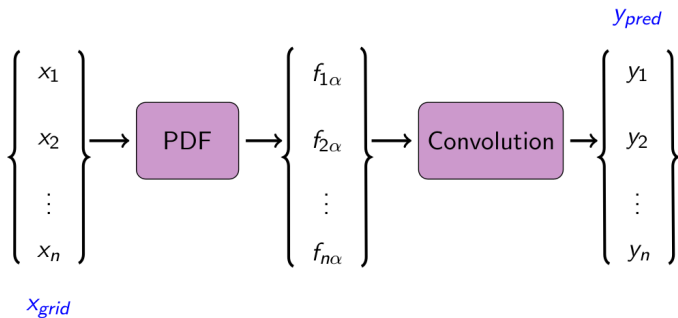
## General structure of n3fit



- Use TensorFlow and Keras to determine the PDFs
- See paper by S.Carrazza - J.Cruz-Martinez  
"Towards a new generation of parton densities  
with deep learning models",  
<https://arxiv.org/abs/1907.05075>

# Back up

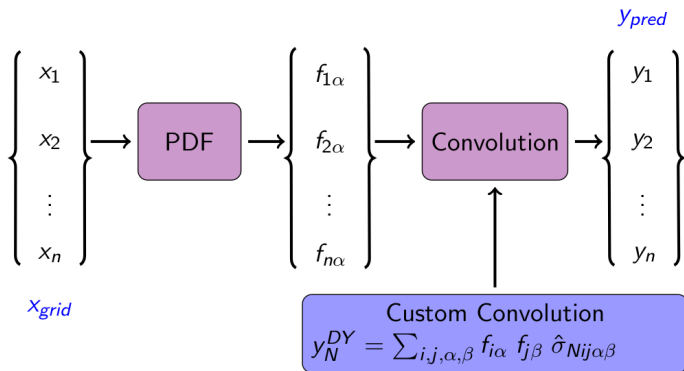
## General structure of n3fit



- 1 Build a NN to compute  $y_{pred}$  observables from a grid of momentum fractions  $x_i$
- 2 Compute loss function by comparing with LHC data
- 3 Update values of PDF  $\rightarrow$  Fit

# Back up

## Operator implementation



- 1 TF relies in symbolic computation  $\rightarrow$  High memory usage
- 2 Implement C++ operator replacing the convolution

# Back up

## Benchmark DIS

DIS only:

	TensorFlow	Custom	Ratio
Convolution	1.9207904	1.9207904	1.0000000
	2.4611666	2.4611664	0.9999999
	1.3516952	1.3516952	1.0000000
Gradient	1.8794115	1.8794115	1.0000000
	1.505316	1.505316	1.0000000
	2.866085	2.866085	1.0000000

# Back up

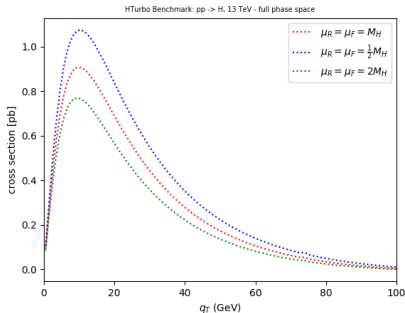
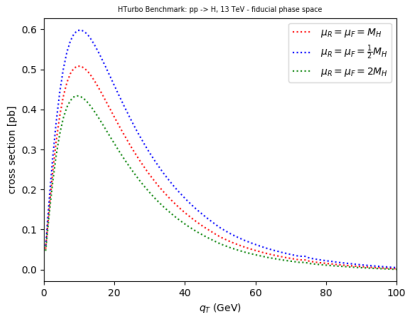
## Benchmark hadronic

DY-like only:

	TensorFlow	Custom	Ratio
Convolution	8.142365	8.142366	1.0000001
	8.947762	8.947762	1.0000000
	7.4513326	7.4513316	0.9999999
Gradient	18.525095	18.525095	1.0000000
	19.182995	19.182993	0.9999999
	19.551006	19.551004	0.9999999

# Results

## N<sup>3</sup>LL implementation



Estimate theory uncertainty:

- i) by performing scale variations on  $\mu_R$ ,  $\mu_F$
- ii) by comparing the last two contributions in the perturbative expansion