QCD and Monte Carlo event generators

Monte Carlo course seminar - Milan, February 2021

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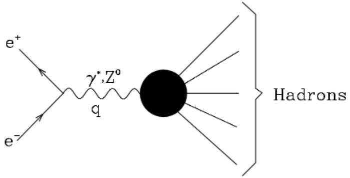
Outline

1. Hadron collisions and strong interactions
   - Hadron collisions and strong interactions
   - Renormalization group
   - IR divergences
2. MC and Parton Showers
   - Factorization theorem
   - Final state radiation
   - Initial state radiation
3. Hadronization: some basics

Strong interactions

QCD from e^+e^- annihilation

Quantum Chromodynamics (QCD) $\rightarrow$ theory describing the interaction between quarks and gluons (strong interactions)



QCD arises already from e^+e^- annihilation $\rightarrow$ R_0 ratio

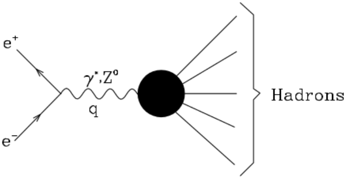
$$R_0 = \frac{\sigma(\gamma^* \rightarrow \text{hadrons})}{\sigma(\gamma^* \rightarrow \mu^+\mu^-)} = 3\sum_f c_f^2$$

1. Color factor (3 color for each quark)
2. Sum over charges of different flavors
3. Threshold and higher order corrections

Strong interactions

QCD from e^+e^- annihilation

Higher order corrections to R_0



QCD arises already from e^+e^- annihilation $\rightarrow$ R_0 ratio

$$R = R_0 \left(1 + \frac{\alpha_S(\mu)}{\pi} + \left[c + \pi b_0 \log \frac{\mu^2}{Q^2} \right] \left(\frac{\alpha_S(\mu}{\pi} \right)^2 \right) + \mathcal{O}\left(\alpha_S(\mu)^3\right).$$

1. Color factor (3 color for each quark)
2. Sum over charges of different flavors
3. Threshold and higher order corrections

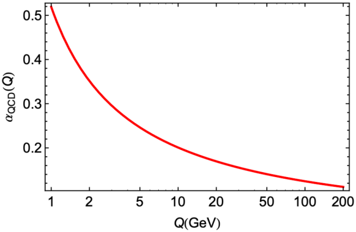
- Running coupling given by Renormalization Group Equation (RGE)

$$\mu \frac{d\alpha_s(\mu)}{d\mu} = \beta(\alpha_s(\mu)) = -\sum_{n=0}^{\infty} \beta_n \left(\frac{\alpha_s}{\pi}\right)^{n+1}$$

- Coupling α_s evolves with scale μ as given by RGE $\rightarrow$ LO behavior driven by β_0
- $\beta_0^{\mathrm{QCD}} > 0 \implies$ weakly coupled at large energies, asymptotic freedom
- $\beta_0^{\mathrm{QED}} < 0 \implies$ strongly coupled at large energies, UV divergent!

Strong interactions

Renormalization group



- Running coupling given by Renormalization Group Equation (RGE)

$$\alpha_s(\mu) = \frac{1}{\beta_0 \log\left(\frac{\mu^2}{\Lambda_s^2}\right)}$$

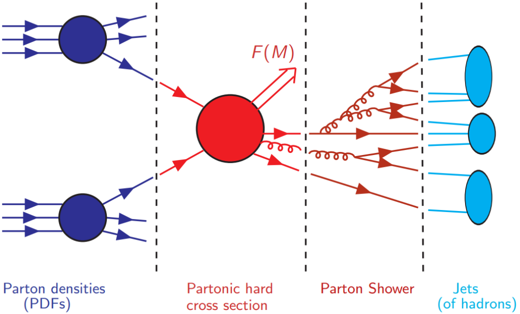
- β_0 LO of the β function, is > 0
- Λ_s, parameter that defines value of the coupling at large scales

QCD is weakly coupled for $\mu >> \Lambda_s \longrightarrow$ asymptotically free

Perturbative Quantum Chromodynamics (pQCD)

Factorization theorem
QCD factorization

LHC processes $H_1 + H_2 \to \mathrm{F}$



Separate process PDFs and partonic (hard) interaction

$$\sigma^{\mathrm{F}}(p_1, p_2) = \sum_{\alpha,\beta} \int_0^1 dx_1 dx_2 \; f_\alpha(x_1, \mu_F^2) * f_\beta(x_2, \mu_F^2) \; * \; \hat{\sigma}^{\mathrm{F}}_{\alpha\beta}(x_1 p_1, x_2 p_2, \alpha_s(\mu_R^2), \mu_F^2)$$

Parton showers

MC Parton showers

Partons in the initial and final state emit radiation. Initial state Radiation (ISR) and Final State Radiation (FSR) model by Monte Carlo (MC) shower algorithms.

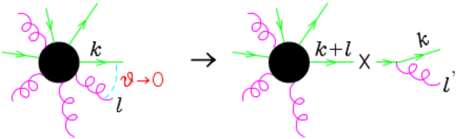
Shower Monte Carlo programs (HERWIG, PYTHIA)

- Libraries for computing SM and BSM cross sections
- Shower algorithms produce the parton shower from final state or initial state partons (accurate only at LO?...)
- Hadronization models, underlying event, decays of unstable hadrons, etc

Parton showers

Collinear limit

- An emitted parton is collinear to an incoming or outgoing parton (θ small)
- σ dominated by collinear emission $q \to qg, g \to gg, g \to q\bar{q}$ (measurement not sensitive to such small scales)



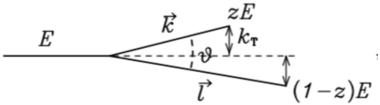
Collinear factorization $\longrightarrow$ Factor out tree level amplitude and splitting

$$|M_{n+1}|^2 d\Phi_{n+1} \Rightarrow |M_n|^2 d\Phi_n \;\; \frac{\alpha_S}{2\pi}\, \frac{dt}{t}\, P_{q,qg}(z)\, dz\, \frac{d\phi}{2\pi}.$$

$$d\Phi_n = (2\pi)^4 \delta^4 \left(\sum_i^n k_i - q \right) \prod_i^n \frac{d^3 k_i}{(2\pi)^3 2k_i^0}$$

Parton showers

Kinematics of splitting



Kinematics of splitting given by (t, z, ϕ)

- ϕ represents azimuth of the k, l plane
- z is the fraction of energy of radiated parton

$$z = \frac{k^0}{k^0 + l^0}$$

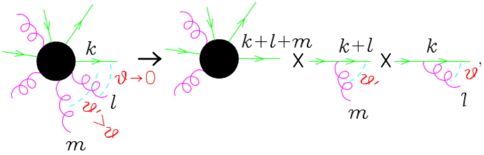
- t has dimensions of energy
  - virtuality

$$t = (k+l)^2 = k^0 l^0 4 \sin^2\left(\frac{\theta}{2}\right) \approx k^0 l^0 \theta^2 \approx z(1-z)E^2\theta^2$$

  - transverse momentum $t = k_\perp^2 = l_\perp^2 = z^2(1-z)^2 E^2 \theta^2$
  - hardness $E^2\theta^2$

Altarelli-Parisi splitting functions

$$P_{q,qg}(z) = C_F \frac{1+z^2}{1-z}$$
$$P_{g,gg}(z) = C_A \left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right)$$
$$P_{g,q\bar{q}}(z) = T_f(z^2 + (1-z)^2)$$



We can proceed in an iterative way

$$|M_{n+2}|^2 d\Phi_{n+2} = |M_n|^2 d\Phi_n \frac{\alpha_s(t')}{2\pi} P_{q,qg}(z') \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi} \frac{\alpha_s(t)}{2\pi} P_{q,qg}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi}$$

Exclusive final state: limit to the most singular terms, in ordered sequence of angles
Collinear approximation $\longrightarrow$ Leading log approximation

Parton showers

Exclusive final state

Exclusive final state: sum the perturbative expansions to all orders in *alpha*$_s$
Limit to the most singular terms in ordered sequence of angles *alpha*$_s$

$$\sigma_0 \alpha_s^n \int \frac{dt_1}{t_1} \dots \frac{dt_n}{t_n} \theta(Q^2 > t_1^2 > \dots > t_n^2 > \Lambda_S^2) = \sigma_0 \frac{\alpha_s^n}{n!} \log^n \left(\frac{Q^2}{\Lambda_S^2} \right)$$

Exclusive final state: limit to the most singular terms, in ordered sequence of angles
Collinear approximation $\longrightarrow$ Leading log approximation

Final state radiation MC

General structure

Approximated description of a hadronic final state
Model a given hard scattering with arbitrary number of enhanced radiations

- Choose hard interaction with specified Born kinematics.
- Consider all possible splittings for each coloured parton.
- Assign the variables t, z, ϕ at each splitting vertex, t ordered in decreasing way.
- At each splitting vertex assign the weight

$$\frac{\alpha_S(t)}{2\pi} P_{i,jl}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi}$$

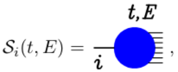
- Each line has a weight known as Sudakov factor

$$\Delta_i(t', t'') = \exp\left(- \sum_{ij} \int_{t''}^{t'} \frac{dt}{t} \frac{\alpha(t)_S}{2\pi} \int_0^1 dz P_{i,jl}(z) \right)$$

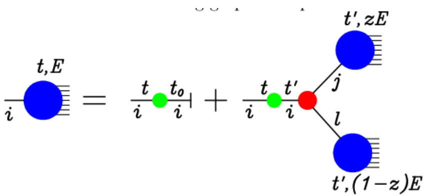
Final state radiation MC

Formal representation of a shower

Approximated description of a hadronic final state
Model a given hard scattering with arbitrary number of enhanced radiations



Ensemble of all possible branchings at scale t (...)



Forward evolution equation

$$S_i(t,E) = \Delta_i(t,t_0) S_i(t_0,E) + \sum_{jl} \int_{t_0}^{t} \frac{dt'}{t'} \int_0^1 dz \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{a_S(t')}{2\pi} \Delta_i(t',t_0) S_j(t',zE) S_l(t',(1-z)E)$$

Final state radiation MC

Probabilistic interpretation

$$\frac{\alpha_s(t')}{2\pi} P_{i,jl}(z') \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi}$$

Probability of branching in the infinitesimal volume $dt'\ dz\ d\phi$

$$dP_{br} = \frac{\alpha_s(t')}{2\pi} \frac{dt'}{t'} \int_0^1 dz' P_{i,jl}(z') \int_0^{2\pi} \frac{d\phi'}{2\pi}$$

Probability of branching in the interval dt'

$$dP_{nobr} = 1 - dP_{br} = 1 - \frac{\alpha_s(t')}{2\pi} \frac{dt'}{t'} \int_0^1 dz' P_{i,jl}(z') \int_0^{2\pi} \frac{d\phi'}{2\pi}$$

Probability of first branching in the infinitesimal volume dt'

$$\Delta_i(t,t') = 1 - dP_{br} = \prod_i^n \left(1 - \frac{\alpha_s(t_i)}{2\pi} \frac{\delta t}{t_i} \int_0^1 dz' \int_0^{2\pi} P_{i,jl}(z') \frac{d\phi'}{2\pi} \right)$$

Sudakov form factor from unitarity

$$dP_{fbr} = \Delta_i(t,t') \frac{\alpha_s(t')}{2\pi} P_{i,jl}(z') \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi}$$

Probability of no-branching in the infinitesimal volume $dt'\ dz\ d\phi$

Final state radiation MC

Shower algorithm

Generate hard process with probability proportional to its parton level cross section.
For each final state colored parton:

1. Set scale $t = Q$, hard scale of the process.
2. Generate random number $0 < r < 1$.
3. Solve $r = \Delta_i(t, t')$ for t'.
4. i) if $t' < t_0$, no further branching and stop shower.
5. ii) if $t' \geq t_0$, generate j, l with energies

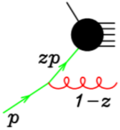
   $$E_j = zE_i \quad \text{and} \quad E_l = (1-z)E_i,$$

   following the $P_{i,jl}(z)$ distribution and with azimuth ϕ uniform in the interval $[0, 2\pi]$.
   the angle of between their momenta is fixed by t'.
6. For each branched partons set $t = t'$ and start from (2).

Initial state radiation MC

General structure

ISR showers are spacelike



$$p_\pm = E \pm p_z \qquad p_\mathrm{T} = \sqrt{p_\mathrm{x}^2 + p_\mathrm{y}^2}$$

$$E^2 - p_x^2 - p_y^2 - p_z^2 = m^2 \Rightarrow p_+ p_- = m^2 + p_\mathrm{T}^2$$

Consider the splitting between a particle a that splits into b and c

$$p_{-a} = p_{-b} + p_{-c} \Leftrightarrow \frac{m_\mathrm{a}^2}{p_{+a}} = \frac{m_b^2 + p_{b\mathrm{T}}^2}{p_{+b}} + \frac{m_c^2 + p_{c\mathrm{T}}^2}{p_{+c}} \Leftrightarrow m_a^2 = \frac{m_b^2}{z} + \frac{m_c^2}{1-z} + \frac{p_\mathrm{T}^2}{z(1-z)}$$

ISR $\quad m_a \approx 0, m_c \approx 0 \ \Rightarrow m_b^2 \approx -\dfrac{p_\mathrm{T}^2}{1-z}$

FSR $\quad m_b \approx 0, m_c \approx 0 \ \Rightarrow m_a^2 \approx \dfrac{p_\mathrm{T}^2}{z(1-z)}$

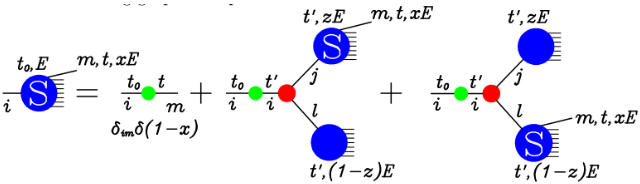
Initial state radiation MC

Formal representation

$\mathcal{S}_i(m,x,t,E) =$ 

- Lines between t_1 and t_2 (consecutive radiations) are spacelike (*)
- Difference in Sudakov factors and Splitting functions start at NLO

Forward evolution equation. Great amount of computation time to generate configurations leading to the scattering that we want



Initial state radiation MC

Shower algorithm

Generate hard process with probability proportional to its parton level cross section. For each final state colored parton:

1. Set scale $t = Q$, hard scale of the process.
2. Generate random number $0 < r < 1$
3. Solve
$$r = \frac{f_m^{(i)} \Delta_m(t, t')}{f_m^{(i)}(t, x)} \quad \text{for } t'.$$
4. i) if $t' < t_0$, no further branching and stop shower.
5. ii) if $t' \geq t_0$, generate j, l with energies
$$E_j = zE_i \quad \text{and} \quad E_l = (1 - z)E_i,$$
following the $P_{i,jl}(z)$ distribution and with azimuth ϕ uniform in the interval $[0, 2\pi]$.
the angle of between their momenta is fixed by t'.
6. For parton j set $t = t'$ and start from (2). For parton l generate a timelike parton shower according to the algorithm shown previously.

Hadronization

Basics

A parton becoming a measurable hadron through the emission of a partonic shower
Large number of color approximation $\rightarrow$ each parton identified by a unique label

Hadronization models

- Lund string model
    - non perturbative production of quarks and antiquarks
    - intermediate gluons are transverse kicks of a continuum medium
- Cluster models
    - preconfinement, assuming subsystems of color singlet partons with universal invariant mass distribution (power suppressed at high masses)
    - gluons are forced to split in quark-antiquark pair

Summary

- LHC processes require factorization in perturbative and non perturbative part
- pQCD applied at high energies
- Monte Carlo shower programs describe non perturbative physics in hadron physics
- Agreement and precision Monte Carlo shower programs