

# Higgs boson production at the Large Hadron Collider: accurate theoretical predictions at higher orders in QCD

Jesús Urtasun Elizari

PhD presentation - Milan, February 25th, 2022



UNIVERSITÀ  
DEGLI STUDI  
DI MILANO



European  
Research  
Council

This project has received funding from the European Union's Horizon 2020 research and innovation program under grant agreement No 740006.

# Outline

- ① Introduction to QCD
  - A historical approach
  - Asymptotic freedom and pQCD
- ② QCD and collider physics
  - QCD Factorization
  - Partonic cross section and perturbative QCD
- ③ All order perturbative resummation
  - Higher order radiative corrections
  - Resummation of large logarithmic corrections
  - Resummed, asymptotic and fixed-order
- ④ Precise and fast predictions for Higgs boson physics
  - Higgs production at the LHC
  - HTurbo numerical code
  - Preliminary results & Conclusions

# Introduction

# Introduction

## QCD - A historical approach

- The Standard Model
- QCD and the strong interactions
- Higgs boson physics and the LHC

# Part I

## QCD and collider physics

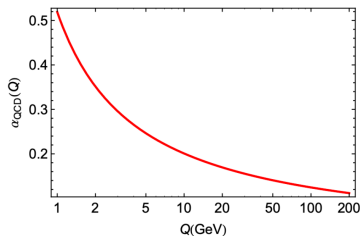
# Introduction

## QCD and the strong interactions

- The Standard Model
- QCD and the strong interactions
- Higgs boson physics and the LHC

# QCD and collider physics

## Asymptotic freedom and pQCD



- Running coupling given by Renormalization Group Equation (RGE)

$$\alpha_s(\mu) = \frac{1}{\beta_0 \log\left(\frac{\mu^2}{\Lambda_{\text{QCD}}^2}\right)}$$

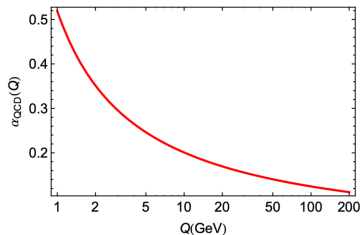
- $\beta_0$  LO of the  $\beta$  function, is  $> 0$
- $\Lambda_{\text{QCD}}$ , parameter that defines value of the coupling at large scales

QCD is weakly coupled for  $\mu \gg \Lambda_{\text{QCD}} \rightarrow$  asymptotically free

Perturbative Quantum Chromodynamics (pQCD)

# QCD and collider physics

## Asymptotic freedom and pQCD



- Running coupling given by Renormalization Group Equation (RGE)

$$\alpha_s(\mu) = \frac{1}{\beta_0 \log\left(\frac{\mu^2}{\Lambda_{\text{QCD}}^2}\right)}$$

- $\beta_0$  LO of the  $\beta$  function, is  $> 0$
- $\Lambda_{\text{QCD}}$ , parameter that defines value of the coupling at large scales

QCD is weakly coupled for  $\mu \gg \Lambda_{\text{QCD}} \rightarrow$  asymptotically free

Perturbative Quantum Chromodynamics (pQCD)



# QCD and collider physics

## Hadronic processes and factorization

- The Standard Model
- QCD and the strong interactions
- Higgs boson physics and the LHC

# QCD and collider physics

## The parton model

- The Standard Model
- QCD and the strong interactions
- Higgs boson physics and the LHC

# QCD and collider physics

## Parton densities

- The Standard Model
- QCD and the strong interactions
- Higgs boson physics and the LHC

# QCD and collider physics

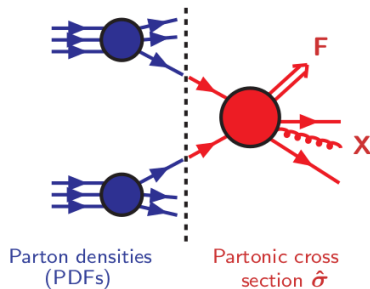
## Fixed-order QCD

- The Standard Model
- QCD and the strong interactions
- Higgs boson physics and the LHC

# QCD and collider physics

## Perturbative QCD

- Born cross section is the leading-order (LO) term of the perturbative series
- $\sigma^{(1)}, \sigma^{(2)}, \sigma^{(3)}$  are the NLO, NNLO, N3LO corrections

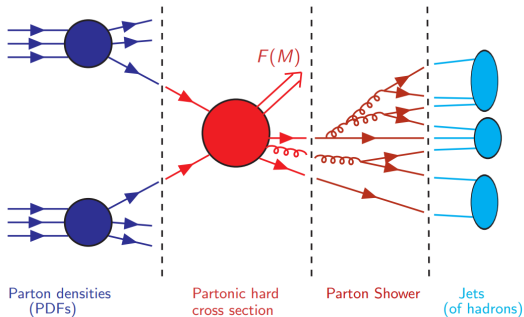


$$\hat{\sigma} = \sigma^{\text{Born}} \left( 1 + \alpha_s \sigma^{(1)} + \alpha_s^2 \sigma^{(2)} + \alpha_s^3 \sigma^{(3)} + \dots \right)$$

Lower order predictions strongly depend on the auxiliary and unphysical renormalization and factorization scales  $\rightarrow$  **Need higher order corrections to increase theoretical accuracy!**

# QCD and collider physics

## Factorization theorem

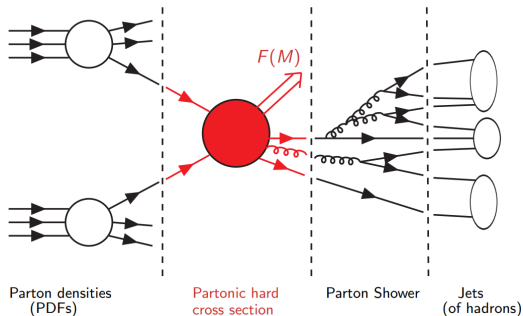


Compute hadronic cross sections is a **hard problem**  $\longrightarrow$  QCD Factorization

$$\sigma^F(p_1, p_2) = \int_0^1 dx_1 dx_2 f_\alpha(x_1, \mu_F^2) * f_\beta(x_2, \mu_F^2) * \hat{\sigma}_{\alpha\beta}^F(x_1 p_1, x_2 p_2, \alpha_s(\mu_R^2), \mu_F^2)$$

# QCD and collider physics

## Partonic cross section



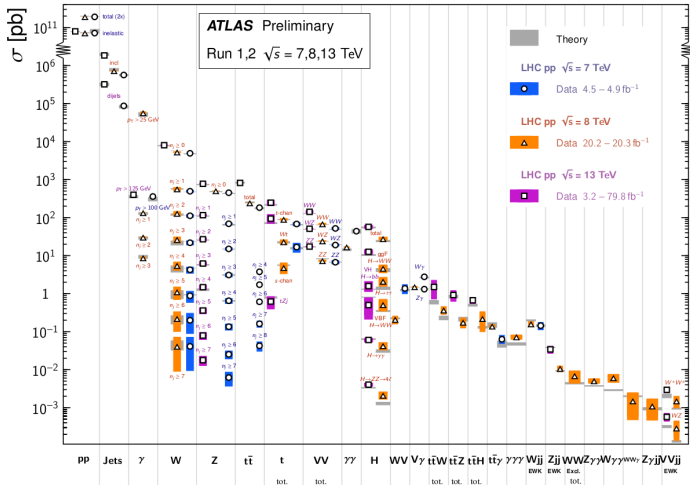
- Parton densities (PDFs)  $f_{\alpha}(x_i, \mu_F^2)$ : non perturbative but universal
- Partonic cross section  $\hat{\sigma}_{\alpha\beta}^F$ : process dependent but computable as perturbative series in  $\alpha_s$

# QCD and collider physics

## LHC phenomenology

### Standard Model Production Cross Section Measurements

Status: July 2018





## Part II

### All order resummation

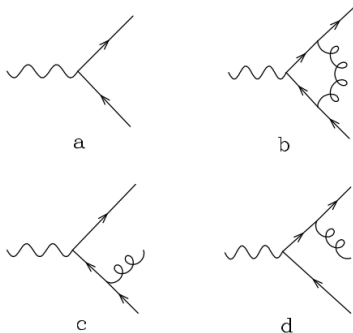
# Resummation in QCD

The need for resummation

# Resummation in QCD

## Higher order corrections

- 1 Calculation of higher order corrections is **not an easy task** due to **infrared (IR) soft and collinear singularities**
- 2 Final state singularities **cancel** by combining real and virtual contributions
- 3 Initial state collinear singularities **factorized** inside the PDFs

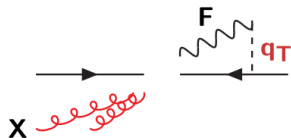


# Resummation in QCD

## $q_\perp$ resummation

Study the differential  $q_\perp$  distribution

$$h_1(p_1) + h_2(p_2) \longrightarrow F(M, \mathbf{q}_\perp) + X$$



$$\int_0^{Q_\perp^2} dq_\perp^2 \frac{d\hat{\sigma}}{dq_\perp^2} \sim c_0 + \alpha_s (c_{12} L^2 + c_{11} L + c_{10}) + \dots, \quad \text{where} \quad L = \ln(q_\perp/M^2)$$

|                     |                       |         |                           |
|---------------------|-----------------------|---------|---------------------------|
| $\alpha_S L^2$      | $\alpha_S L$          | $\dots$ | $\mathcal{O}(\alpha_S)$   |
| $\alpha_S^2 L^4$    | $\alpha_S^2 L^3$      | $\dots$ | $\mathcal{O}(\alpha_S^2)$ |
| $\dots$             | $\dots$               | $\dots$ | $\dots$                   |
| $\alpha_S^n L^{2n}$ | $\alpha_S^n L^{2n-1}$ | $\dots$ | $\mathcal{O}(\alpha_S^n)$ |
| dominant logs       | $\dots$               | $\dots$ | $\dots$                   |

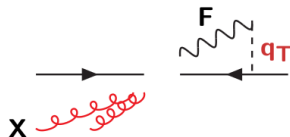
Truncated fixed order predictions  $\rightarrow$  enhanced  $\alpha_s^n \ln^m(M^2/q_\perp^2)$  appear

# Resummation in QCD

## $q_\perp$ resummation

Study the differential  $q_\perp$  distribution

$$h_1(p_1) + h_2(p_2) \longrightarrow F(M, \mathbf{q}_\perp) + X$$



$$\int_0^{Q_\perp^2} dq_\perp^2 \frac{d\hat{\sigma}}{dq_\perp^2} \sim c_0 + \alpha_s (c_{12} L^2 + c_{11} L + c_{10}) + \dots, \quad \text{where} \quad L = \ln(q_\perp / M^2)$$

|                     |                       |         |                           |
|---------------------|-----------------------|---------|---------------------------|
| $\alpha_S L^2$      | $\alpha_S L$          | $\dots$ | $\mathcal{O}(\alpha_S)$   |
| $\alpha_S^2 L^4$    | $\alpha_S^2 L^3$      | $\dots$ | $\mathcal{O}(\alpha_S^2)$ |
| $\dots$             | $\dots$               | $\dots$ | $\dots$                   |
| $\alpha_S^n L^{2n}$ | $\alpha_S^n L^{2n-1}$ | $\dots$ | $\mathcal{O}(\alpha_S^n)$ |
| dominant logs       | $\dots$               | $\dots$ | $\dots$                   |

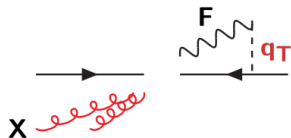
Truncated fixed order predictions  $\rightarrow$  enhanced  $\alpha_S^n \ln^m(M^2/q_\perp^2)$  appear

# Resummation in QCD

## $q_\perp$ resummation

Study the differential  $q_\perp$  distribution

$$h_1(p_1) + h_2(p_2) \longrightarrow F(M, \mathbf{q}_\perp) + X$$



$$\int_0^{Q_\perp^2} dq_\perp^2 \frac{d\hat{\sigma}}{dq_\perp^2} \sim c_0 + \alpha_s (c_{12} L^2 + c_{11} L + c_{10}) + \dots, \quad \text{where} \quad L = \ln(q_\perp/M^2)$$

|                     |                       |         |                           |
|---------------------|-----------------------|---------|---------------------------|
| $\alpha_S L^2$      | $\alpha_S L$          | $\dots$ | $\mathcal{O}(\alpha_S)$   |
| $\alpha_S^2 L^4$    | $\alpha_S^2 L^3$      | $\dots$ | $\mathcal{O}(\alpha_S^2)$ |
| $\dots$             | $\dots$               | $\dots$ | $\dots$                   |
| $\alpha_S^n L^{2n}$ | $\alpha_S^n L^{2n-1}$ | $\dots$ | $\mathcal{O}(\alpha_S^n)$ |
| dominant logs       | $\dots$               | $\dots$ | $\dots$                   |

Truncated fixed order predictions  $\rightarrow$  enhanced  $\alpha_S^n \ln^m(M^2/q_\perp^2)$  appear

# Resummation in QCD

## $q_\perp$ resummation

Separate partonic  $q_\perp$  distribution as follows

$$\frac{d\hat{\sigma}_{ab}}{dq_\perp^2} = \left[ \frac{d\hat{\sigma}_{ab}^{(\text{res.})}}{dq_\perp^2} \right]_{\text{l.a.}} + \left[ \frac{d\hat{\sigma}_{ab}^{(\text{fin.})}}{dq_\perp^2} \right]_{\text{f.o.}}, \quad \text{such that}$$

$$\int_0^{q_\perp^2} dq_\perp^2 \frac{d\hat{\sigma}_{ab}^{(\text{res.})}}{dq_\perp^2} \sim \sum \alpha_s^n \log^m \frac{M^2}{q_\perp^2} \quad \text{for } q_\perp \rightarrow 0$$
$$\lim_{q_\perp \rightarrow 0} \int_0^{q_\perp^2} dq_\perp^2 \frac{d\hat{\sigma}_{ab}^{(\text{fin.})}}{dq_\perp^2} = 0$$

Resummed and finite components can be matched (LL+LO, NLL+NLO, NNLO+NNLL, ...) to have uniform accuracy in a wide range of  $q_\perp$

# Resummation in QCD

## $q_\perp$ resummation

Resummation holds in impact parameter space  $b$

$$\frac{d\hat{\sigma}_{ab}^{(\text{res.})}}{dq_\perp^2} = \frac{M^2}{\hat{s}} \int db \frac{b}{2} J_0(bq_\perp) \mathcal{W}_{ab}(b, M)$$

with  $\mathcal{W}_{ab}$  also expressed in Mellin space (with respect to  $z = M^2/\hat{s}$ )

$$\mathcal{W}_N(b, M) = \mathcal{H}_N(\alpha_s) \times \exp\{\mathcal{G}_N(\alpha_s, L)\} \quad \text{being} \quad L \equiv \log(M^2 b^2)$$

- Large logarithms exponentiated in the universal Sudakov form factor  $\mathcal{G}_N(\alpha_s, L)$
- Constant ( $b$ -independent) terms factorized in the process dependent hard factor  $\mathcal{H}_N(\alpha_s)$



## Part III

### HTurbo numerical implementation

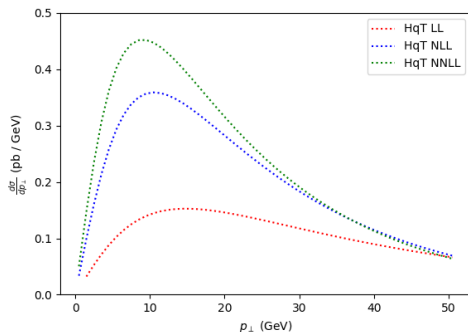
# HqT and HRes

Need for fast numerical implementations

# HqT and HRes

## Predictions for Higgs $q_{\perp}$ distribution

- $q_{\perp}$  resummation implemented in numerical codes HqT and HRes [Catani, de Florian, Ferrera, Grazzini, Tommasini]
- Higher order accuracy require **high computation times**
- Codes producing fast and accurate predictions are needed for precision era of the LHC



# HTurbo

Starting point DYTurbo

Numerical code **DYTurbo** [Camarda et al.] ref. at [1910.07049](#), fast and precise  $q_{\perp}$  resummation and several improvements for Drell-Yan ( $h_1 h_2 \rightarrow V + X \rightarrow l^+ l^- + X$ )

- **First goal**: set up a numerical code for Higgs boson production starting from **DYTurbo**
- Set LO amplitude  $gg \rightarrow H$
- Set Sudakov and Hard coefficients for Higgs production
- Compare with **HRes** and **HqT**

**Final goal**: extend theoretical accuracy up to  $N^3\text{LL}+N^3\text{LO}$

# HTurbo

## Starting point DYTurbo

Both Sudakov factor  $\mathcal{G}_N$  and hard coefficient  $\mathcal{H}_N$  can be expanded as perturbative series in  $\alpha_s$

$$\mathcal{G}_N(\alpha_s, L) = L g^{(1)}(\alpha_s L) + g^{(2)}(\alpha_s L) + \frac{\alpha_s}{\pi} g^{(3)}(\alpha_s L) + \dots$$

$$\mathcal{H}_N(\alpha_s) = 1 + \alpha_s \mathcal{H}^{(1)} + \alpha_s^2 \mathcal{H}^{(2)} + \dots$$

For each new order implement a factor of  $\mathcal{G}_N$  and Hard  $\mathcal{H}_N$

$$\text{LL}(\sim \alpha_s^n L^{n+1}) : g^{(1)}, \hat{\sigma}^{(0)}$$

$$\text{NLL}(\sim \alpha_s^n L^n) : g^{(2)}, \mathcal{H}^{(1)}$$

$$\text{NNLL}(\sim \alpha_s^n L^{n-1}) : g^{(3)}, \mathcal{H}^{(2)}$$

Start by building predictions up to NNLO+NNLL, then add  
**N<sup>3</sup>LO+N<sup>3</sup>LL**

Reimplementation of **HqT** and **HRes** for  $q_T$ -resummation

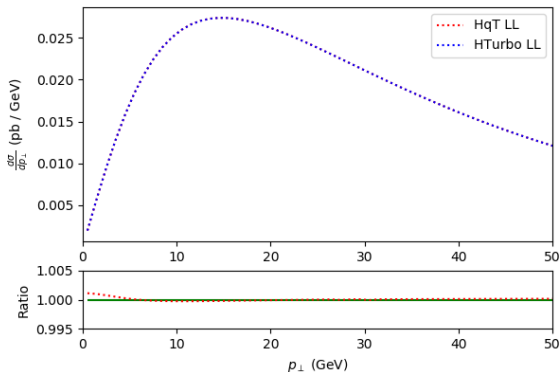
- **C++** structure with **Fortran** interfaces → Multi-threading
- Optimization in the integration routines / integral transforms
  - Factorize boson and decay kinematics
  - Gauss-Legendre quadrature rules (1-dim.)
  - Vegas/Cuhre through **Cuba** (multi-dim.)

Comparison **HRes** and **HTurbo** - speed performance

| Predictions        | <b>HRes</b> | <b>HTurbo</b> |
|--------------------|-------------|---------------|
| resummed NNLL      | 10h         | 10'           |
| combined NNLO+NNLL | 20h         | 1h            |

# Results

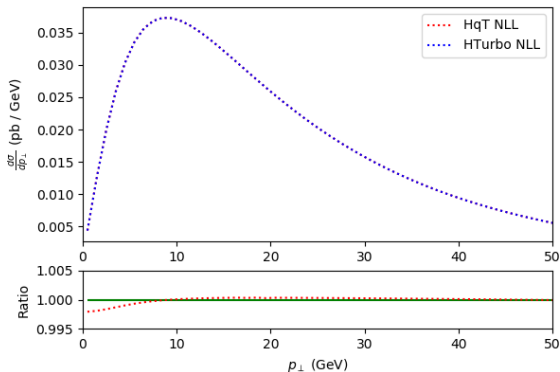
## Comparison HTurbo and HqT - LL



- HTurbo  $q_{\perp}$  distribution vs HRes and HqT at LL
- Excellent numerical agreement up to the 0.1% level

# Results

## Comparison HTurbo and HqT - NLL

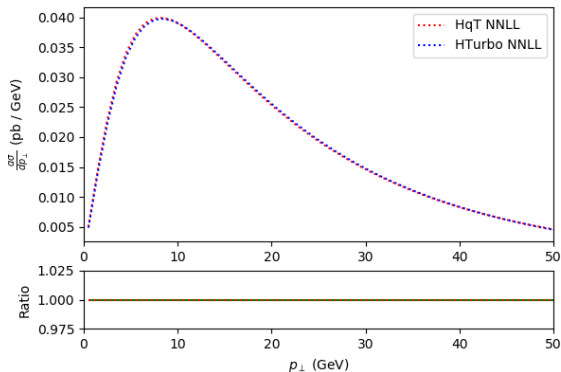


- HTurbo  $q_{\perp}$  distribution vs HRes and HqT at NLL
- Excellent numerical agreement up to the 0.1% level



# Results

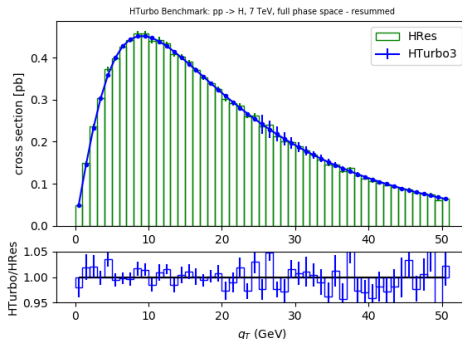
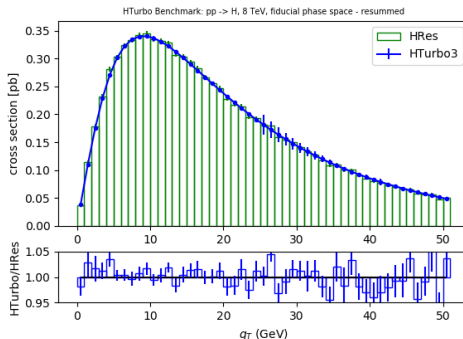
## Comparison HTurbo and HqT - NNLL



- HTurbo  $q_{\perp}$  distribution vs HRes and HqT at NNLL
- Excellent numerical agreement up to the 0.1% level

# Results

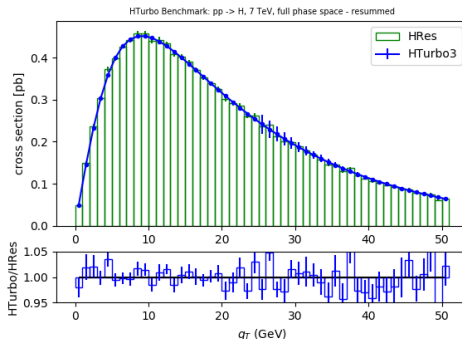
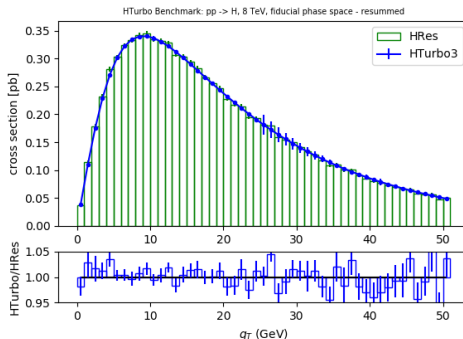
## Comparison HTurbo and HRes - resummed



- Represent full (LHS) and fiducial (RHS) phase space ✓
- Agreement up to resummed NNLL  $\rightarrow$  ready for  $N^3$ LL

# Results

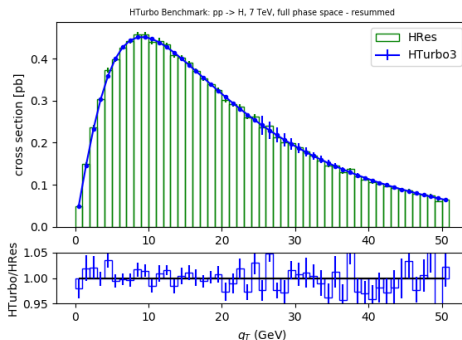
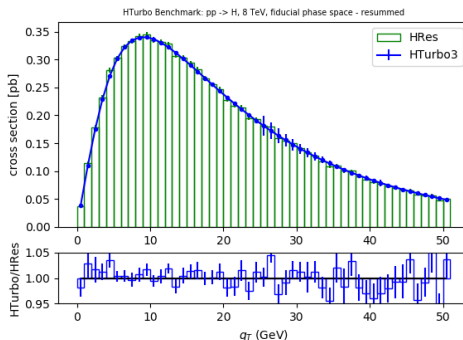
## Comparison HTurbo and HRes - asymptotic



- Represent full (LHS) and fiducial (RHS) phase space ✓
- Agreement up to resummed NNLL  $\rightarrow$  ready for  $N^3$ LL

# Results

## Comparison HTurbo and HRes - fixed-order



- Represent full (LHS) and fiducial (RHS) phase space ✓
- Agreement up to resummed NNLL  $\rightarrow$  ready for  $N^3$ LL

# Summary & Conclusions

- ① Fast and accurate predictions are needed towards the precision era of the LHC
- ② Developing a novel numerical code, **HTurbo**, which implements  $q_{\perp}$  resummation for Higgs boson production
- ③ HTurbo is faster than any of the existing codes
- ④ Outlook of thesis work:
  - Add  $N^3\text{LO}+N^3\text{LL}$  prediction
  - Perform phenomenological studies comparing with LHC data

## Discussion & next steps

- ① Fast and accurate predictions are needed towards the precision era of the LHC
- ② Developing a novel numerical code, **HTurbo**, which implements  $q_{\perp}$  resummation for Higgs boson production
- ③ HTurbo is faster than any of the existing codes
- ④ Outlook of thesis work:
  - Add  $N^3\text{LO}+N^3\text{LL}$  prediction
  - Perform phenomenological studies comparing with LHC data

Thank you!



This project has received funding from the European Union's Horizon 2020 research and innovation program under grant agreement No 740006.