QCD and Monte Carlo event generators

Monte Carlo course seminar - Milan, February 2021

UNIVERSITÀ
DEGLI STUDI
DI MILANO

INFN
Istituto Nazionale
di Fisica Nucleare

erc
European
Research
Council

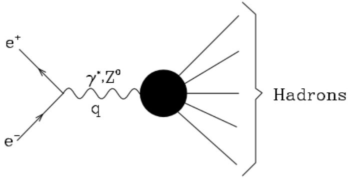
Outline

1. Hadron collisions and strong interactions
   - Hadron collisions and strong interactions
   - Renormalization group
   - QCD factorization
2. MC and Parton Showers
   - Collinear factorization
   - Final state radiation
   - Initial state radiation
3. Hadronization: some basics
   - Large number of colors approximation
   - Hadronization models

Strong interactions

QCD from e^+e^- annihilation

Quantum Chromodynamics (QCD) $\rightarrow$ theory describing the interaction between quarks and gluons (strong interactions)



QCD arises already from e^+e^- annihilation $\rightarrow$ R_0 ratio

$$R_0 = \frac{\sigma(\gamma^* \rightarrow \text{hadrons})}{\sigma(\gamma^* \rightarrow \mu^+\mu^-)} = 3\sum_f c_f^2$$

1. Color factor (3 color for each quark)
2. Sum over charges of different flavors
3. Threshold and higher order corrections

Strong interactions

Renormalization group

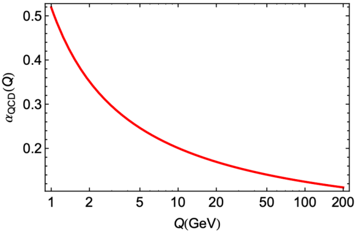
- Running coupling given by Renormalization Group Equation (RGE)

$$\mu \frac{d\alpha_s(\mu)}{d\mu} = \beta(\alpha_s(\mu)) = -\sum_{n=0}^{\infty} \beta_n \left(\frac{\alpha_s}{\pi}\right)^{n+1}$$

- Coupling α_s evolves with scale μ as given by RGE $\rightarrow$ LO behavior driven by β_0
- $\beta_0^{\mathrm{QED}} < 0 \implies$ strongly coupled at large energies, UV divergent
- $\beta_0^{\mathrm{QCD}} > 0 \implies$ weakly coupled at large energies, IR divergent

Strong interactions

Renormalization group



- Running coupling given by Renormalization Group Equation (RGE)

$$\alpha_s(\mu) = \frac{1}{\beta_0 \log\left(\frac{\mu^2}{\Lambda^2_{\mathrm{QCD}}}\right)}$$

- β_0 LO of the β function, is > 0
- Λ_{QCD}, parameter that defines value of the coupling at large scales

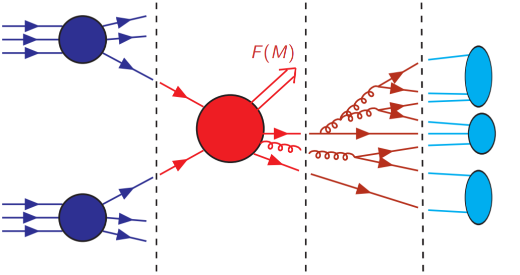
QCD is weakly coupled for $\mu >> \Lambda_{\mathrm{QCD}} \longrightarrow$ asymptotically free

Perturbative Quantum Chromodynamics (pQCD)

Factorization theorem

QCD factorization

LHC processes $H_1 + H_2 \to \mathrm{F}$



Separate process PDFs and partonic (hard) interaction

$$\sigma^{\mathrm{F}}(p_1, p_2) = \sum_{\alpha,\beta} \int_0^1 dx_1 dx_2 \; f_\alpha(x_1, \mu_F^2) * f_\beta(x_2, \mu_F^2) \; * \; \hat{\sigma}^{\mathrm{F}}_{\alpha\beta}(x_1 p_1, x_2 p_2, \alpha_s(\mu_R^2), \mu_F^2)$$

Parton showers

MC Parton showers

Partons in the initial and final state emit radiation. Initial state Radiation (ISR) and Final State Radiation (FSR) model by Monte Carlo (MC) shower algorithms

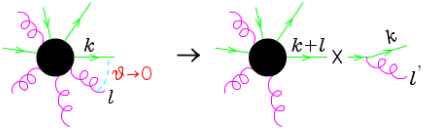
Shower Monte Carlo programs (HERWIG, PYTHIA)

- Libraries for computing SM and BSM cross sections
- Shower algorithms produce a number of enhanced coloured parton emissions to be added to the hard process
- Hadronization models, underlying event, decays of unstable hadrons, etc

Parton showers

Collinear limit

- QCD emission processes are enhanced in the collinear limit (θ small)
- σ dominated by collinear splittings $q \to qg, g \to gg, g \to q\bar{q}$



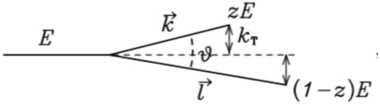
Collinear factorization $\longrightarrow$ The cross section factorizes into the product of a tree-level cross section and a splitting probability

$$|M_{n+1}|^2 d\Phi_{n+1} \Rightarrow |M_n|^2 d\Phi_n \quad \frac{\alpha_S}{2\pi} \frac{dt}{t} P_{q,qg}(z)\, dz\, \frac{d\phi}{2\pi}.$$

$$d\Phi_n = (2\pi)^4 \delta^4 \left(\sum_i^n k_i - q \right) \prod_i^n \frac{d^3 k_i}{(2\pi)^3 2k_i^0}$$

Parton showers

Kinematics of splitting



Kinematics of splitting given by (t, z, ϕ)

- t: parameter with dimensions of energy that vanish in the collinear limit
  - Virtuality $t = (k + l)^2 \approx z(1 - z)E^2\theta^2$
  - Transverse momentum $t = k_\perp^2 = l_\perp^2 \approx z^2(1 - z)^2E^2\theta^2$
  - Hardness $t = E^2\theta^2$
- z: fraction of energy of radiated parton $z = \frac{k^0}{k^0+l^0}$
- ϕ represents azimuth of the k, l plane

Parton showers

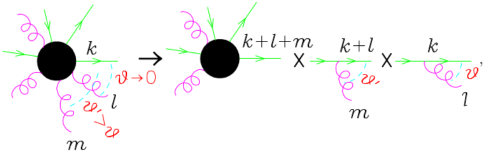
AP splitting functions

Factorization holds for small angles $\rightarrow$ small t variable
Difference in the splitting $\rightarrow$ Altarelli-Parisi splitting functions (singular in $z \rightarrow 0, 1$)

$$P_{\mathrm{q,qg}}(z) = C_{\mathrm{F}} \frac{1+z^2}{1-z}$$

$$P_{\mathrm{g,gg}}(z) = C_{\mathrm{A}} \left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right)$$

$$P_{\mathrm{g,q\bar{q}}}(z) = T_{\mathrm{f}}(z^2 + (1-z)^2)$$



We can proceed in an iterative way

$$|M_{n+2}|^2 d\Phi_{n+2} = |M_n|^2 d\Phi_n \frac{\alpha_{\mathrm{s}}(t')}{2\pi} P_{\mathrm{q,qg}}(z') \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi} \frac{\alpha_{\mathrm{s}}(t)}{2\pi} P_{\mathrm{q,qg}}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi}$$

Angles become small, maintaining a strong ordering relation $\theta' >> \theta \rightarrow 0$

Parton showers

Exclusive final state

To describe exclusive final state → sum perturbative expansion to all orders in α_s

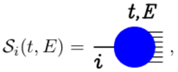
$$\sigma_0 \alpha_s^n \int \frac{dt_1}{t_1} \ldots \frac{dt_n}{t_n} \theta(Q^2 > t_1^2 > \ldots > t_n^2 > \Lambda_S^2) = \sigma_0 \frac{\alpha_s^n}{n!} \log^n\left(\frac{Q^2}{\Lambda_S^2}\right)$$

Limit to the most singular terms, in ordered sequence of angles

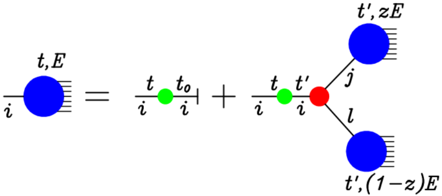
Collinear approximation $\longrightarrow$ Leading log approximation

Final state radiation MC

Formal representation of a shower



- Ensemble of all possible branchings from parton i at scale t
- Shower Unitarity $\sum_{\mathcal{F}} S_i(t, E) = 1$



- Splitting weighted by AP

$$\frac{\alpha_S(t)}{2\pi} P_{\mathrm{i,jl}}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi}$$

- Each line weighted by Sudakov factor

$$\Delta_i(t', t'') = \exp\left[-\sum_{(jl)} \int_{t''}^{t'} \frac{dt}{t} \int_0^1 dz \; \frac{\alpha_S(t)}{2\pi} \; P_{i,jl}(z) \right]$$

Forward evolution equation

$$S_i(t, E) = \Delta_i(t, t_0) S_i(t_0, E) + \sum_{jl} \int_{t_0}^{t} \frac{dt'}{t'} \int_0^1 dz \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{a_S(t')}{2\pi} \Delta_i(t', t_0) S_j(t', zE) S_l(t', (1-z)E)$$

Final state radiation MC

Probabilistic interpretation

$$\frac{\alpha_s(t')}{2\pi} P_{i,jl}(z') \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi}$$

Probability of branching in the infinitesimal volume $dt'\ dz'\ d\phi'$

$$dP_{br} = \frac{\alpha_s(t')}{2\pi} \frac{dt'}{t'} \int_0^1 dz' P_{i,jl}(z') \int_0^{2\pi} \frac{d\phi'}{2\pi}$$

Probability of branching in the interval dt'

$$dP_{nobr} = 1 - dP_{br} = 1 - \frac{\alpha_s(t')}{2\pi} \frac{dt'}{t'} \int_0^1 dz' P_{i,jl}(z') \int_0^{2\pi} \frac{d\phi'}{2\pi}$$

Probability of no branching in the interval dt'

$$\Delta_i(t, t') = 1 - dP_{br} = \prod_i^n \left(1 - \frac{\alpha_s(t_i)}{2\pi} \frac{\delta t}{t_i} \int_0^1 dz' \int_0^{2\pi} P_{i,jl}(z') \frac{d\phi'}{2\pi} \right)$$

Sudakov form factor in $[t, t']$

$$dP_{fbr} = \Delta_i(t, t') \frac{\alpha_s(t')}{2\pi} P_{i,jl}(z') \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi}$$

Probability of, starting at t, first branching in the phase space element $dt'\ dz'\ d\phi'$

Final state radiation MC

Shower algorithm

Generate hard process with probability proportional to its parton level cross section
For each final state colored parton:

1. Set scale $t = Q$, hard scale of the process
2. Generate random number $0 < r < 1$
3. Solve $r = \Delta_i(t, t')$ for t'
4. i) if $t' < t_0$, no further branching and stop shower
5. ii) if $t' \geq t_0$, generate j, l with energies

$$E_j = zE_i \quad \text{and} \quad E_l = (1-z)E_i,$$

   with z following a distribution given by $P_{i,jl}(z)$ and with azimuth ϕ uniformly distributed in the interval $[0, 2\pi]$.
6. For each branched partons set $t = t'$ and start from (2)

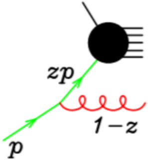
Initial state radiation MC

General structure

- QCD coupling much larger than QED
- Coupling grows for small momentum transfer

We can never neglect QCD ISR

Radiation from some initial state leading to some hard collision



- Incoming parton is on shell $p^2 = 0$
- ISR showers are spacelike $q^2 = (k - k')^2 < 0$
- Parton with scaled momentum zp acquires negative virtuality

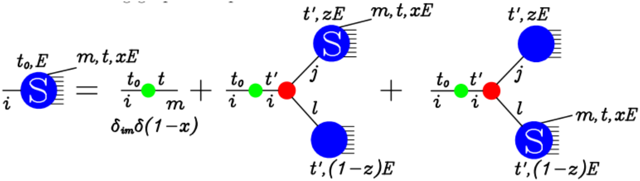
Factorization formula

$$d\sigma_j^{\mathrm{ISR}}(p, \ldots) = \frac{\alpha_S}{2\pi}\frac{dt}{t}dz P_{ij}(z) d\sigma_i(zp, \ldots)$$

$S_i(m, x, t, E) =$ 

$\delta x\ S_i(m, x, t, E)$ represents ensemble of all possible states containing a spacelike parton m with energy between xE and $(x + \delta x)E$ and scale t

- Procedure for evolution equation as in FSR (with spacelike showers)
- Forward evolution equation

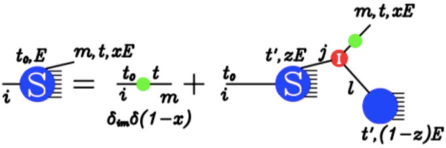


Difference from FSR in Sudakov factors and Splitting functions arise at NLO

Initial state radiation MC

Backwards evolution equation

Great amount of computation time to hard scattering that we want
Moderns MC programs → recursive procedure starting at the large scale



$$\sum_{\mathscr{F}} S_i(m,t,x,E) = f_m^{(i)}(x,t)$$

- The blob I at the splitting vertex is given by the inclusive splitting kernel P_{jm}
- Backwards evolution equation (scale dependent parton density)
  Sum over all final states (yields 1 for timelike blobs, $f_m^{(i)}(x,t)$ for spacelike)

$$f_m^{(i)}(x,t) = \delta_{im}\delta(1-x)\Delta_{\mathrm{m}}(t,t_0) + \int_{t_0}^{t}\frac{dt'}{t'}\int_x^1\frac{dz}{z}\sum_j f_j^{(i)}(z,t')\frac{\alpha_{\mathrm{S}}(t')}{2\pi}\hat{P}_{jm}\left(\frac{x}{z}\right)\Delta_m(t,t')$$

Initial state radiation MC

Shower algorithm

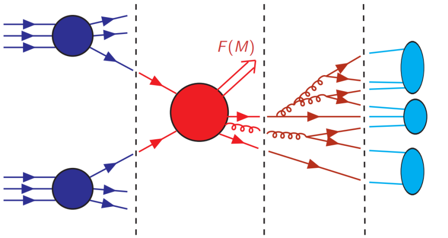
Generate hard process with probability proportional to its parton level cross section
For each initial state colored parton:

1. Set scale $t = Q$, hard scale of the process
2. Generate random number $0 < r < 1$
3. Solve
$$r = \frac{f_i^{(h)}(t', x)\Delta_i(t, t')}{f_i^{(h)}(t, x)} \quad \text{for } t'$$
4. i) if $t' < t_0$, no further branching and stop shower
5. ii) if $t' \geq t_0$, j, l generate j and z following a distribution given by $P_{ij}(z)$, and azimuth ϕ uniformly distributed in the interval $[0, 2\pi]$
Call l radiated parton, assign energies
$$E_j = zE_i \quad \text{and} \quad E_l = (1 - z)E_i,$$
6. For parton j, set $t = t'$ and start from (2)
7. For parton l, set $t = t'$ and proceed with timelike shower as described in FSR

Hadronization

Some basics

Emit QCD radiation until becoming a measurable hadron → non-perturbative physics



Hadronization

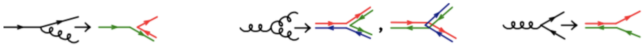
Some basics

Hadronization models

- Independent fragmentation model
  - Each final state particle treated independently from all the others
  - Gluon forced to split in $q\bar{q}$ pair
  - Introduce diquarks for formation of baryons
- Cluster models
- Lund string model

Large number of color approximation (planar limit)

- Color indices ranging from 1 to N_c, replace $(N_c^2 - 1) \to N_c^2$
- Represent color flow $\to$ t'Hooft double line notation



Hadronization models have unavoidably a large number of parameters
They are one of the most complex aspects of Shower Monte Carlo algorithms

- LHC processes factorization in perturbative and non-perturbative components
- Perturbative QCD applied only at high energies
- Monte Carlo shower algorithms describe the enhanced emissions produced by initial and final state partons
- Hadronization models describing the non-perturbative physics

Thank you!