Two-mode squeezed states in cavity optomechanics via engineering of a single reservoir

Quantum coherent phenomena course seminar - Milan, October 2020

UNIVERSITÀ DEGLI STUDI DI MILANO

Istituto Nazionale di Fisica Nucleare

European Research Council

Outline

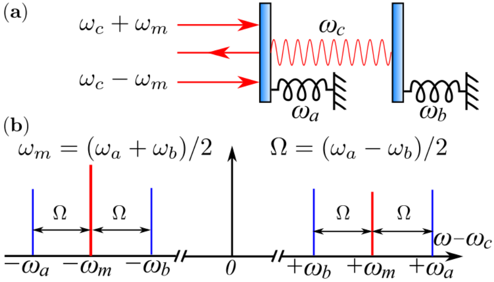
Introduction

Entangled states

1. Generation and detection of entangled states of macroscopic M.O
2. Reservoir engineering $\longrightarrow$ Two-mode squeezed states. Bogoliuov modes
3. Coherent feedback, steady state as EPR channel
4. Reservoir engineering studied in optomechanical systems. Krauter demonstrations (*)

Introduction

System representation



- Two mechanical oscillators, resonance frequencies ω_a, ω_b
- Dispersively coupled g_a, g_b to a common cavity ω_c

Hamiltonian of the system

$$\hat{\mathcal{H}} = \omega_a \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b} + \omega_c \hat{c}^\dagger \hat{c} + g_a(\hat{a} + \hat{a}^\dagger)\hat{c}^\dagger \hat{c} + g_b(\hat{b} + \hat{b}^\dagger)\hat{c}^\dagger \hat{c} + \hat{H}_{\text{drive}} + \hat{H}_{\text{diss}},$$

Under usual approximations, obtain the master formula

$$\dot{\rho} = -i[\hat{\mathcal{H}}', \rho] + \gamma_a(\bar{n}_a + 1)\mathcal{D}[\hat{a}]\rho + \gamma_a \bar{n}_a \mathcal{D}[\hat{a}^\dagger]\rho + \gamma_b(\bar{n}_b + 1)\mathcal{D}[\hat{b}]\rho + \gamma_b \bar{n}_b \mathcal{D}[\hat{b}^\dagger]\rho + \kappa \mathcal{D}[\hat{c}]\rho,$$

Being $\mathcal{D}$ dispersive superoperator, dampings and $\mathcal{H}'$

2 mechanical oscillators. Modes $\hat{a}, \hat{b} \longrightarrow$ Bogoliuov operators

$$\hat{\beta}_1 = \hat{a} \cosh r + \hat{b}^\dagger \sinh r,$$
$$\hat{\beta}_2 = \hat{b} \cosh r + \hat{a}^\dagger \sinh r.$$

Rotation with respect to some frame (*)

$$\hat{H}_0 = (\omega_a - \Omega)\hat{a}^\dagger \hat{a} + (\omega_b + \Omega)\hat{b}^\dagger \hat{b} + \omega_c \hat{c}^\dagger \hat{c},$$

Choice of Ω
Collective mechanical quadratures (...)

Reservoir engineering strategies

Ground state

2-mode squeezed state defined by $|r >= S_2(r)|00 >$

$$\hat{S}_2(r) \equiv \exp[r(\hat{a}\hat{b} - \hat{a}^\dagger \hat{b}^\dagger)]$$

(...)

Adding freq. difference between the mechanic oscillators, we break degeneracy of the Bogoliubov modes $\longrightarrow$ they couple to different frequency components of the reservoir

$$\hat{\mathcal{H}} = \Omega(\hat{\beta}_1^\dagger \hat{\beta}_1 - \hat{\beta}_2^\dagger \hat{\beta}_2) + \mathcal{G}[(\hat{\beta}_1^\dagger + \hat{\beta}_2^\dagger)\hat{c} + \text{H.c.}] + \hat{H}_{\text{diss}},$$

where Ω is the effective frequency and $\mathcal{G}$ an effective coupling.

In terms of the original operators

$$\hat{\mathcal{H}} = \Omega(\hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b}) + G_+[(\hat{a} + \hat{b})\hat{c} + \text{H.c.}] + G_-[(\hat{a} + \hat{b})\hat{c}^\dagger + \text{H.c.}] + \hat{H}_{\text{diss}}.$$

Optomechanical couplings related by

$$\mathcal{G} \equiv \sqrt{G_-^2 - G_+^2}, \quad \tanh r \equiv G_+/G_-,$$

Reservoir engineering strategies

Goal

Engineer the driving Hamiltonian such that the squeezed-system results in $\hat{b}eta_i$ cooled to their ground state.

- Experimental advantage
- Second method
- Third approach (*)

Implementation

Different cases

Hamiltonian is already implemented in conventional optomechanical setups. Focus on regime $|G_+| < |G_-|$

- Two-tone driving ($g_a = g_b$)
- Four-tone driving ($g_a = g_b$)
- Case similar ($g_a \sim g_b$)

(...)

Adiabatic limit

Our system

Operator $\hat{c} = -2i\mathcal{G}(\hat{\beta}_1 + \hat{\beta}_2)/k$
Substitute into the dissipative terms of master equation $\longrightarrow$ adiabatically eliminated master equation

$$\dot{\rho} = -i\Omega[\hat{\beta}_1^\dagger \hat{\beta}_1 - \hat{\beta}_2^\dagger \hat{\beta}_2, \rho] + \gamma_a(\bar{n}_a + 1)\mathcal{D}[\hat{a}]\rho + \gamma_a \bar{n}_a \mathcal{D}[\hat{a}^\dagger]\rho$$
$$+ \gamma_b (\bar{n}_b + 1)\mathcal{D}[\hat{b}]\rho + \gamma_b \bar{n}_b \mathcal{D}[\hat{b}^\dagger]\rho + \Gamma \mathcal{D}[\hat{\beta}_1 + \hat{\beta}_2]\rho,$$

with optomechanical damping rate

$$\Gamma \equiv \gamma \mathcal{C} \equiv \frac{4\mathcal{G}^2}{\kappa},$$

Alternative view of the cooling of the Bogoliuov modes is possible (*)

Entanglement criterion using Duan inequality

$$\hat{X}_{\pm} = (\hat{X}_a \pm \hat{X}_b)/\sqrt{2},$$
$$\hat{P}_{\pm} = (\hat{P}_a \pm \hat{P}_b)/\sqrt{2},$$

Where we introduced the quadrature modes as

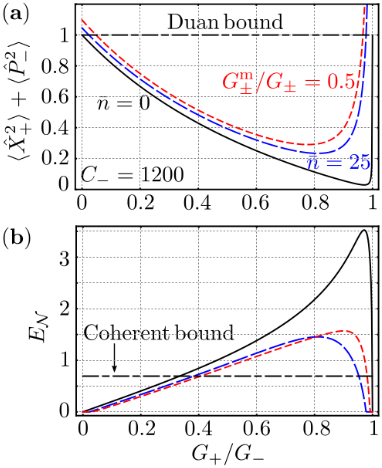
$$\hat{X}_s = (\hat{s} + \hat{s}^{\dagger})/\sqrt{2}, \quad \hat{P}_s = -i(\hat{s} - \hat{s}^{\dagger})/\sqrt{2}.$$

Where we introduced the quadrature modes as

$$\langle \hat{X}_{+}^{2} \rangle + \langle \hat{P}_{-}^{2} \rangle < 1$$

Adiabatic limit

Entanglement



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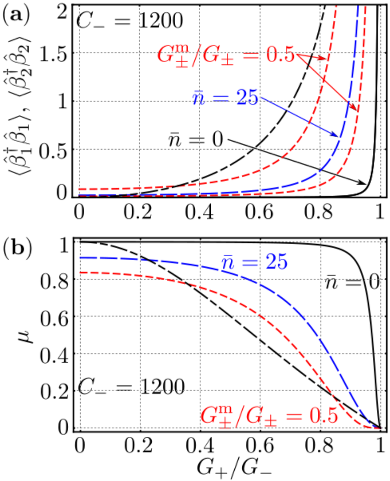
Adiabatic limit

Entanglement



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Conclusions

1. Configuring a three-mode optomechanical system such as the steady state includes highly pure and highly entangled two-mode squeezed state.
2. Symmetry on the steady-state makes it attractive for implementation of continuous-variable teleportation protocols
3. Problem of unequal single-photon optomechanical couplings solved by using four-tone driving scheme
4. Proposal implementable for existing technology