Gaussian Process for the estimation of theory uncertainties

Jesús Urtasun Elizari

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N3PDF
Machine Learning • PDFs • QCD

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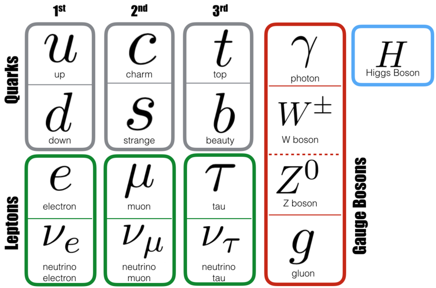
Outline

1. QCD in a nutshell
   - The Standard Model & strong interactions
   - Parton Distribution Functions
   - Factorization theorem
2. Machine Learning
   - Motivation for Machine Learning
   - Neural Networks & general strategy
3. Gaussian Process and theory uncertainties
   - The NNPDF methodology
   - Operator implementation in TensorFlow
   - Results & Conclusions

Quantum Chromodynamics in a nutshell

QCD in a nutshell

The Standard Model



Quantum Field Theory describing physics at the TeV scale

1. Fermions composing matter
2. Bosons mediating interactions
3. Scalar Higgs generating mass

QCD in a nutshell

Explore the strong interactions

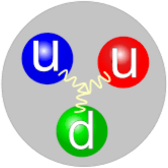
How to explore proton's inner structure?

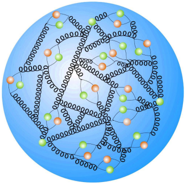
- Point-like projectile on the object $\longrightarrow$ DIS
- Smash the two objects $\longrightarrow$ LHC physics

"A way to analyze high energy collisions is to consider any hadron as a composition of point-like constituents $\longrightarrow$ **partons"** R.Feynman, 1969

QCD in a nutshell

Parton Distribution Functions





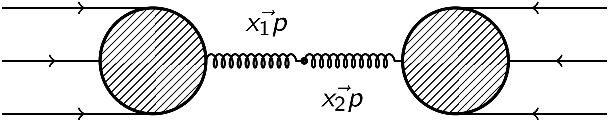
- Hadrons made of partonic objects $\longrightarrow$ non perturbative physics
- Interactions take place only at partonic level

Parton Distribution Functions: probability distribution of finding a particular parton (u, d, ..., g) carrying a fraction x of the proton's momentum

QCD in a nutshell

Factorization theorem

Observables in hadronic events $\longrightarrow$ σ is hard to compute

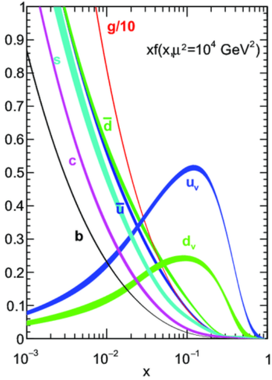


Factorize the problem $\longrightarrow$ Convolute the PDFs with the partonic $\hat{\sigma}_{ij}$

$$\sigma = \int_0^1 dx_1 \, dx_2 \, f_\alpha(x_1, \mu_F) * f_\beta(x_2, \mu_F) * \hat{\sigma}_{\alpha\beta}(\alpha_s(\mu_R), \mu_F)$$

- Partonic $\hat{\sigma}$ can be computed as perturbative series in α_s
- PDFs absorb the non perturbative effects, evaluated at μ_F

- Each parton has a different PDF $u(x), d(x), ..., g(x)$
- PDFs are not predicted, and can not be measured
- PDFs are extracted from data



Machine Learning

Machine learning

What is Machine Learning?

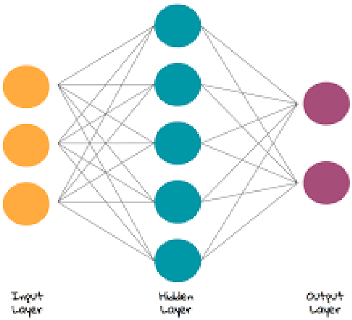
1. A subset of Artificial Intelligence (AI) algorithms
2. Used to solve *complex* tasks like classification and regression
3. Rely on comparison with data $\longrightarrow$ Learning

Machine learning

Building a ML model

Machine learning

Building a Ml model

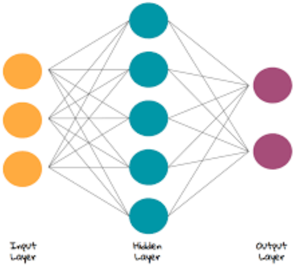


1. Turn the input set into an array and build a random prediction
2. Compare with truth and compute a Loss function
3. Update the parameters in a specific way

Machine learning

Building a ML model

$\longrightarrow$

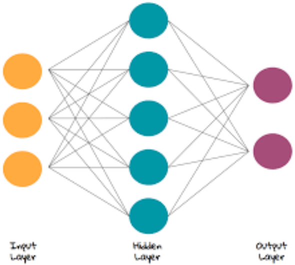


$\longrightarrow$ Tree ✓

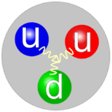
Machine learning

Building a ML model

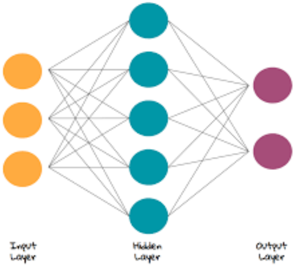
$\longrightarrow$



$\longrightarrow$ Tree ✓



$\longrightarrow$

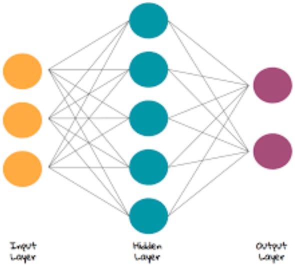


$\longrightarrow$ Not tree X

Machine learning

Building a ML model

$\longrightarrow$



$\longrightarrow$ Tree ✓

$$Loss = \begin{Bmatrix} y_1^{true} \\ y_2^{true} \\ \vdots \\ y_n^{true} \end{Bmatrix} - \begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{Bmatrix}$$

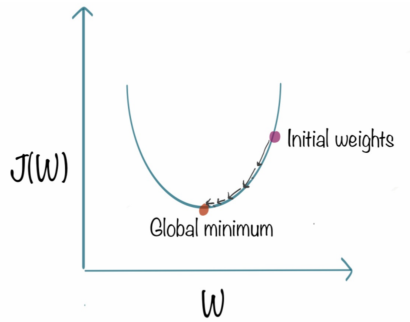
$$\underset{y_{true}}{} \qquad\qquad \underset{y_{pred}}{}$$

1. Compute a loss function

$$L = \sum_{i=1}^{N} \left(y_i^{true} - y_i^{pred}\right)^2$$

2. Perfect prediction will mean $L = 0$

1. Loss is a function of weights and bias $L = L(w, b)$
2. Compute gradient $\nabla_{w_{ij}} L$ to look for the minimum of L

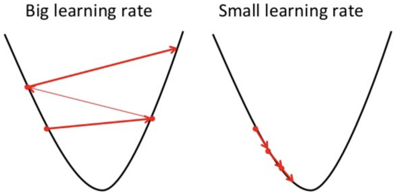


Update rule

Update the parameters of the network following the gradient descend

$$w_{ij} \longrightarrow w_{ij} - \alpha \nabla L$$
$$b_i \longrightarrow b_i - \alpha \nabla L$$

Where α is the learning rate



The N3PDF project

Machine Learning for the precision determination of PDFs

Factorize the problem $\longrightarrow$ Convolute the PDFs with the partonic $\hat{\sigma}_{ij}$

$$\sigma = \int_0^1 dx_1\, dx_2\, f_\alpha(x_1, \mu_F) * f_\beta(x_2, \mu_F) * \hat{\sigma}_{\alpha\beta}(\alpha_s(\mu_R), \mu_F)$$

- Partonic $\hat{\sigma}_{\alpha\beta}$ is computed perturbatively. Hadronic σ is measured.
- Use a Neural Networks to generate (*fit*) the PDFs
- Generate a vector of observables σ_N to be compared with data

$$\sigma_N = \sum_{i,j,\alpha,\beta} f_\alpha(x_i)\, f_\beta(x_j)\, \hat{\sigma}_{Nij\alpha\beta}$$

The N3PDF project

General structure of n3fit

- Use TensorFlow and Keras to determine the PDFs
- See paper by S.Carraza - J.Cruz-Martinez
  "Towards a new generation of parton densities
  with deep learning models",
  https://arxiv.org/abs/1907.05075

The N3PDF project

General structure of n3fit

σ_{pred}

$$\left\{\begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{matrix}\right\} \longrightarrow \boxed{\text{PDF}} \longrightarrow \left\{\begin{matrix} f_{1\alpha} \\ f_{2\alpha} \\ \vdots \\ f_{n\alpha} \end{matrix}\right\} \longrightarrow \boxed{\text{Convolution}} \longrightarrow \left\{\begin{matrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_n \end{matrix}\right\}$$

x_{grid}

1. Build a NN to compute σ_{pred} observables from a grid x_i
2. Compute χ^2 loss function by comparing with data
3. Update values of PDF $\longrightarrow$ Fit

Summary & Conclusions

1. PDFs are required to have accurate predictions in high energy physics
2. ML provides a new way of determine the PDFs
3. Operator implementation leads to memory saving by taking full control on the computation

Thank you!

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