

Perturbative QCD and Monte Carlo event generators

Monte Carlo course seminar - Milan, September 2020



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Outline

① Hadron collisions

- Hadron collisions and strong interactions
- Renormalization group
- Jets and IR divergences

② Collinear factorization

- Factorization theorem
- Kinematics of splitting
- Fixed Order calculations

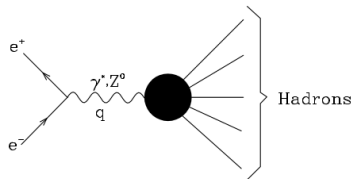
③ Parton showers

- Final state radiation
- Initial state radiation
- Ordering variables (PYTHIA and HERWIG)

Hadron collisions

Hadron collisions

QCD from e^+e^- annihilation



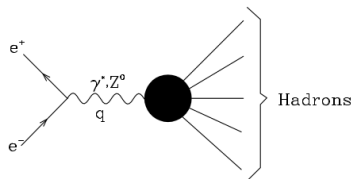
QCD arise already from e^+e^- annihilation $\rightarrow R_0$ ratio

$$R_0 = \frac{\sigma(\gamma^* \rightarrow \text{hadrons})}{\sigma(\gamma^* \rightarrow \mu^+\mu^-)} = 3 \sum_f c_f^2$$

- ① Color factor (3 color for each quark)
- ② Sum over charges of different flavour quarks
- ③ Corrections near the threshold and at higher orders

Hadron collisions

QCD from e^+e^- annihilation



Consider corrections to R_0 from gluon radiation. Renormalize coupling.

$$R = R_0 \left(1 + \frac{\alpha_S(\mu)}{\pi} + \left[c + \pi b_0 \log \frac{\mu^2}{Q^2} \right] \left(\frac{\alpha_S(\mu)}{\pi} \right)^2 \right) + \mathcal{O}(\alpha_S(\mu)^3).$$

Define renormalized coupling, running with the scale
UV logarithmic divergence in $\log(\mu^2/Q^2)$

Hadron collisions

QCD from e^+e^- annihilation

Questions for a field theory

- ① Can we go to arbitrarily large energies? \rightarrow divergences arise, renormalization / factorization needed
- ② Can we compute R_0 for every process? \rightarrow IR observables

Hadron collisions

Renormalization group

- UV divergences are encountered in field theories
- Take a physical quantity G depending on a scale M , a coupling α and some invariants s_1, \dots, s_n
- Define a "renormalized" coupling $\alpha_{\text{Ren}} = \alpha + c_1\alpha^2 + c_2\alpha^3 + \dots$

The physical quantity in terms of $\{\alpha, M\}$ and $\{\alpha_{\text{Ren}}, \mu\}$

$$G(\alpha, M, s_1 \dots s_n) = \tilde{G}(\alpha_{\text{ren}}, \mu, s_1 \dots s_n).$$

Physics must be invariant under change of $\{\alpha_{\text{Ren}}, \mu\}$

$$\frac{\partial \alpha(\alpha_{\text{ren}}, M/\mu)}{\partial \alpha_{\text{ren}}} d\alpha_{\text{ren}} + \frac{\partial \alpha(\alpha_{\text{ren}}, M/\mu)}{\partial \mu^2} d\mu^2 = 0$$

Hadron collisions

Renormalization group

- Running coupling given by Renormalization Group Equation (RGE)

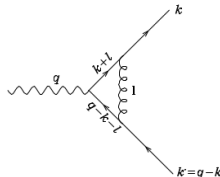
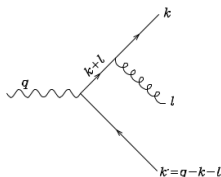
$$\mu \frac{d\alpha_s(\mu)}{d\mu} = \beta(\alpha_s(\mu)) = - \sum_{n=0}^{\infty} \beta_n \left(\frac{\alpha_s}{\pi} \right)^{n+1}$$

- Coupling α_s evolves with scale μ as given by RGE \rightarrow LO behavior driven by β_0
- $\beta_0^{\text{QCD}} > 0 \implies$ weakly coupled at large energies, asymptotic freedom
- $\beta_0^{\text{QED}} < 0 \implies$ strongly coupled at large energies, UV divergent!

Hadron collisions

Jets in e^+e^-

Consider α_s corrections to born level amplitude



$$\mathcal{M}_{\text{Born}} = \bar{u}(k)\epsilon^\mu\gamma_\mu v(k')$$

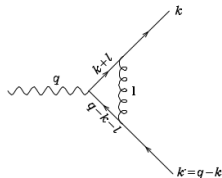
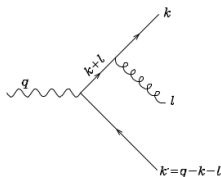
$$\mathcal{M}_1 = \mathcal{M} \frac{k_\alpha}{k \cdot l} \longrightarrow \text{Real radiation}$$

$$\mathcal{M}_1 = -\mathcal{M} \frac{k'_\alpha}{k' \cdot l} \longrightarrow \text{Virtual contribution}$$

Hadron collisions

Jets in e^+e^-

Consider α_s corrections to born level amplitude



Sum real and virtual contributions to the Born matrix element, square and integrate with phase space element $d^3l = (l^0)^2 dl^0 d\cos\theta d\phi$

$$\sigma_{q\bar{q}g} = C_F \frac{\alpha_s}{2\pi} \sigma_{q\bar{q}}^{\text{Born}} \int d\cos\theta \frac{dl^0}{l^0} \frac{4}{(1 - \cos\theta)(1 + \cos\theta)}$$

Hadron collisions

Jets in e^+e^-

$$\sigma_{q\bar{q}g} = C_F \frac{\alpha_S}{2\pi} \sigma_{q\bar{q}}^{\text{Born}} \int d\cos\theta \frac{dl^0}{l^0} \frac{4}{(1 - \cos\theta)(1 + \cos\theta)}$$

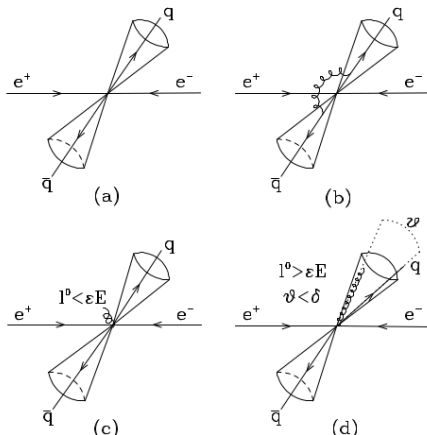
- Soft ($l^0 \rightarrow 0$) and collinear ($\theta \rightarrow 0, \pi$) divergences
- No renormalization procedure to apply \rightarrow divergences coming from long distance effects (fermion masses, hadronization, etc)
- Kinoshita-Lee-Nauenberg theorem (*)

Understand Born cross section as the LO term in a well defined perturbative expansion

Hadron collisions

Sterman-Weinberg jets

Sterman - Weinberg jets. "In a hadronic event with CM energy E , 2 cones can be found with opening δ containing $(1 - \epsilon)$ fraction of E ."



Hadron collisions

Sterman-Weinberg jets

Sterman - Weinberg jets. "In a hadronic event with CM energy E , 2 cones can be found with opening δ containing $(1 - \epsilon)$ fraction of E ."

$$\begin{aligned}\text{Born} + \text{Virtual} + \text{Real (a)} + \text{Real (b)} &= \sigma_0 - \sigma_0 \frac{4\alpha_s C_F}{2\pi} \int_{\epsilon E}^E \frac{dl^0}{l^0} \int_{\theta=\delta}^{\pi-\delta} \frac{d\cos\theta}{1 - \cos^2\theta} \\ &= \sigma_0 \left(1 - \frac{4\alpha_s C_F}{2\pi} \log \epsilon \log \delta^2 \right)\end{aligned}$$

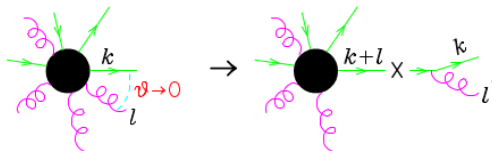
When all contributions summed, the cross section is no longer singular (*)
Computed in terms of partons, but representing hadronic final state
Jets as IR finite final state (*)

Collinear factorization

Collinear factorization

QCD from e^+e^- annihilation

- When computing partonic cross section, collinear partons can be emitted from incoming/outgoing parton
- σ dominated by collinear decay of parton with small virtuality



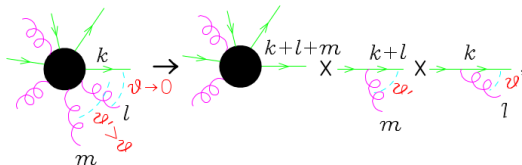
Factorization theorem \rightarrow Factor out tree level amplitude and splitting

$$|M_{n+1}|^2 d\Phi_{n+1} \Rightarrow |M_n|^2 d\Phi_n \frac{\alpha_S}{2\pi} \frac{dt}{t} P_{q,qg}(z) dz \frac{d\phi}{2\pi}.$$

Collinear factorization

QCD from e^+e^- annihilation

- Kinematics of splitting (t, z, ϕ)
 - t has dimensions of energy (virtuality, p_\perp , angular variable)
 - z represents the fraction of momentum of radiated parton
 - ϕ represents azimuth of the k, l plane
- Factorization holds for small angles. Applied recursively



Parton showers and MC generators

Parton showers and MC generators

Formal representation of a shower

Approximated description of a hadronic final state. Model a given hard scattering with arbitrary number of enhanced radiations

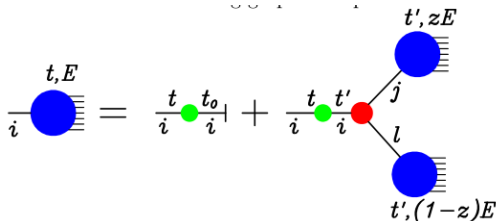
$$S_i(t, E) = \text{---} \underset{i}{\bullet} \overset{t, E}{\text{---}},$$

- Ensemble of all possible showers from a parton i at scale t
- Sudakov form factor $\Delta_i(t, t_0)$ such that $\Delta_i(t_0, t_0) = 1$
- Shower $S_i(t, E)$ such that $S_i^{\text{inc}} = \sum_{\mathcal{F}} S_i(t, E) = 1$

Parton showers and MC generators

Formal representation of a shower

Ensemble of all possible radiations as the sum of no radiation, with radiation and shower from radiated partons



- Ansatz for Sudakov

$$\Delta_i(t, t') = \exp \left\{ - \int_{t'}^t \frac{dt''}{t''} \int dz \sum_{jl} P_{i,jl}(z) \frac{\alpha_s(t'')}{2\pi} \right\}$$

- Therefore $\partial \Delta(t, t') / \partial t \propto \Delta(t, t') \rightarrow$ apply shower recursively

Parton showers and MC generators

Shower algorithm

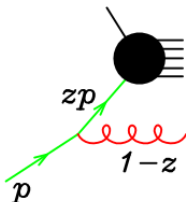
Generate hard process with probability proportional to its parton level cross section. For each final state colored parton:

- ① Set scale t to Q , hard scale of the process
- ② Generate random number $0 < r < 1$
- ③ Solve $r = \Delta_i(t, t')$ for t'
- ④ i) if $t' < t_0$, no further branching and stop shower
- ⑤ ii) if $t' \geq t_0$, one branching into partons j, l with energies $E_j = zE_i$ and $E_l = (1 - z)E_i$, z following the $P_{i,jl}(z)$ distribution and ϕ uniform in the interval $[0, 2\pi]$
- ⑥ For each branched partons set $t = t'$ and start from (2)

Parton showers and MC generators

Initial state radiation

ISR already important in QED \rightarrow Used to determine the Z peak at LEP



- QCD coupling much larger \rightarrow QCD ISR even more important
- Specially large for small momentum transfer
- Same as final state partons *always* manifest as jets, initial state ones *always* lead to ISR

Parton showers and MC generators

Initial state radiation

$$S_i(m, x, t, E) = \frac{t_o, E}{i} \text{S} \begin{array}{c} m, t, xE \\ \text{radiation lines} \end{array}$$

- Lines between t_1 and t_2 (consecutive radiations) are spacelike (*)
- Difference in Sudakov factors and Splitting functions start at NLO

$$\begin{array}{c} t_o, E \\ \text{S} \end{array} \begin{array}{c} m, t, xE \\ \text{radiation lines} \end{array} = \frac{t_o}{i} \text{green dot} \frac{t}{m} + \frac{t_o}{i} \text{green dot} \frac{t'}{i} \text{red dot} \begin{array}{c} j \\ \text{S} \\ t', zE \\ m, t, xE \end{array} \begin{array}{c} l \\ \text{S} \\ t', (1-z)E \end{array} + \frac{t_o}{i} \text{green dot} \frac{t'}{i} \text{red dot} \begin{array}{c} j \\ \text{S} \\ t', zE \end{array} \begin{array}{c} l \\ \text{S} \\ t', (1-z)E \\ m, t, xE \end{array}$$

$\delta_{im} \delta(1-x)$

Parton showers and MC generators

Ordering variables

HERWIG

- Ordering variable $t = E^2\theta^2/2$
- Order of transverse momentum as "angular ordering"
- IR cut-off needed

PYTHIA

- There is not angular ordering. Virtualies strictly ordered in the shower.
- More natural kinematics
- Unphysical increase of number of partons \longrightarrow solve by veto branchings that violate angular ordering.