QCD and Monte Carlo event generators

Monte Carlo course seminar - Milan, February 2021

UNIVERSITÀ
DEGLI STUDI
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Istituto Nazionale
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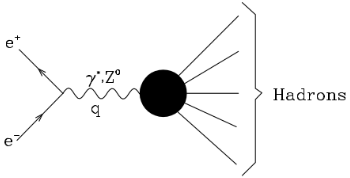
Outline

1. Hadron collisions and strong interactions
   - Hadron collisions and strong interactions
   - Renormalization group
   - IR divergences
2. MC and Parton Showers
   - Factorization theorem
   - Final state radiation
   - Initial state radiation
3. Hadronization: some basics

Strong interactions

QCD from e^+e^- annihilation

Quantum Chromodynamics (QCD) $\rightarrow$ theory describing the interaction between quarks and gluons (strong interactions)



QCD arises already from e^+e^- annihilation $\rightarrow$ R_0 ratio

$$R_0 = \frac{\sigma(\gamma^* \rightarrow \text{hadrons})}{\sigma(\gamma^* \rightarrow \mu^+\mu^-)} = 3\sum_f c_f^2$$

1. Color factor (3 color for each quark)
2. Sum over charges of different flavors
3. Threshold and higher order corrections

Questions for a field theory

1. Can we go to arbitrarily large energies? $\rightarrow$ divergences arise, renormalization / factorization needed
2. Can we compute R_0 for every process? $\rightarrow$ IR observables

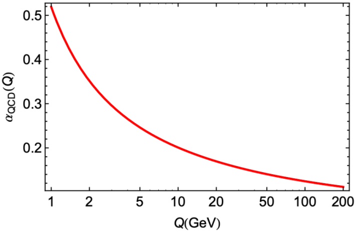
- Running coupling given by Renormalization Group Equation (RGE)

$$\mu \frac{d\alpha_s(\mu)}{d\mu} = \beta(\alpha_s(\mu)) = -\sum_{n=0}^{\infty} \beta_n \left(\frac{\alpha_s}{\pi}\right)^{n+1}$$

- Coupling α_s evolves with scale μ as given by RGE $\rightarrow$ LO behavior driven by β_0
- $\beta_0^{\mathrm{QCD}} > 0 \implies$ weakly coupled at large energies, asymptotic freedom
- $\beta_0^{\mathrm{QED}} < 0 \implies$ strongly coupled at large energies, UV divergent!

Strong interactions

Renormalization group



- Running coupling given by Renormalization Group Equation (RGE)

$$\alpha_s(\mu) = \frac{1}{b_0 \log\left(\frac{\mu^2}{\Lambda_s^2}\right)}$$

- β_0
- Λ_s

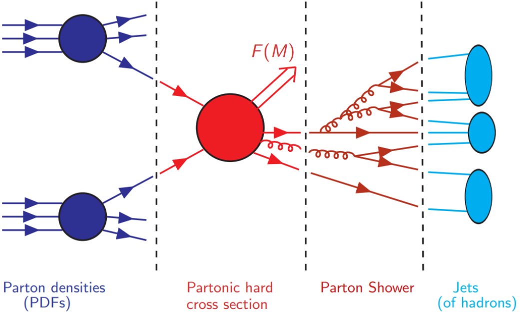
QCD is weakly coupled for $\mu >> \Lambda_s \longrightarrow$ asymptotically free

Perturbative Quantum Chromodynamics (pQCD)

Factorization theorem

QCD factorization

LHC processes $H_1 + H_2 \to \mathrm{F}$



Separate process PDFs and partonic (hard) interaction

$$\sigma^{\mathrm{F}}(p_1, p_2) = \int_0^1 dx_1 dx_2 \; f_\alpha(x_1, \mu_F^2) * f_\beta(x_2, \mu_F^2) \; * \; \hat{\sigma}^{\mathrm{F}}_{\alpha\beta}(x_1 p_1, x_2 p_2, \alpha_s(\mu_R^2), \mu_F^2)$$

Partons in the initial and final state emit radiation. State Radiation (ISR) and Final State Radiation (FSR)

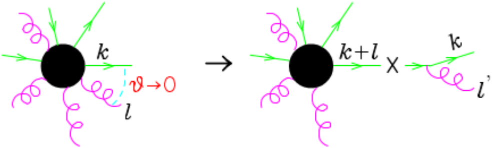
Shower Monte Carlo programs (HERWIG, PYTHIA)

- Libraries for computing SM and BSM cross sections
- Shower algorithms produce the parton shower from final state or initial state partons
- Hadronization models, underlying event, decays of unstable hadrons, etc

Parton showers

Collinear limit

- An emitted parton is collinear to an incoming or outgoing parton (θ small)
- Measurement not sensitive to such small scales
- σ dominated by collinear emission $q \to qg, g \to gg, g \to q\bar{q}$



Collinear factorization $\longrightarrow$ Factor out tree level amplitude and splitting

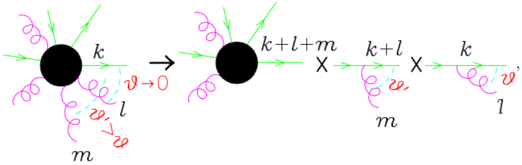
$$|M_{n+1}|^2 d\Phi_{n+1} \Rightarrow |M_n|^2 d\Phi_n \quad \frac{\alpha_S}{2\pi} \frac{dt}{t} P_{q,qg}(z)\, dz\, \frac{d\phi}{2\pi}.$$

Parton showers

Kinematics of splitting

- Kinematics of splitting (t, z, ϕ)
  - t has dimensions of energy (virtuality, $p_\perp$, angular variable)
  - z represents the fraction of momentum of radiated parton
  - ϕ represents azimuth of the k, l plane

- Factorization holds for small angles. Applied recursively



Parton showers

AP splitting functions

Altarelli-Parisi splitting functions

$$
\begin{aligned}
P_{\mathrm{q,qg}}(z) &= C_{\mathrm{F}} \frac{1+z^2}{1-z} \\
P_{\mathrm{g,gg}}(z) &= C_{\mathrm{A}} \left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right) \\
P_{\mathrm{g,q\bar{q}}}(z) &= T_{\mathrm{f}} (z^2 + (1-z)^2)
\end{aligned}
$$

We can proceed in an iterative way

$$
|M_{n+2}|^2 d\Phi_{n+2} = |M_n|^2 d\Phi_n \frac{\alpha_s(t')}{2\pi} P_{\mathrm{q,qg}}(z') \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi} \frac{\alpha_s(t)}{2\pi} P_{\mathrm{q,qg}}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi}
$$

Exclusive final state: limit to the most singular terms, in ordered sequence of angles
Collinear approximation $\longrightarrow$ Leading log approximation

Final state radiation MC

General structure

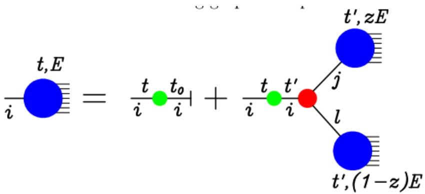
Approximated description of a hadronic final state. Model a given hard scattering with arbitrary number of enhanced radiations

- Choose hard interaction with specified Born kinematics.
- Consider all possible splittings for each coloured parton.
- Assign the variables t, z, ϕ at each splitting vertex, t ordered in decreasing way.
- At each splitting vertex assign the weight (...)
- Each line has a weight known as Sudakov factor (...)

Final state radiation MC

Formal representation of a shower

Approximated description of a hadronic final state. Model a given hard scattering with arbitrary number of enhanced radiations



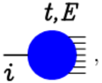
Forward evolution equation

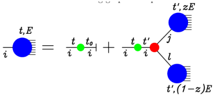
$$S_i(t,E) = \Delta_i(t,t_0)S_i(t_0,E) + \sum_{jl}\int_{t_0}^{t}\frac{dt'}{t'}\int_0^1 dz\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{a_S(t')}{2\pi}\Delta_i(t',t_0)S_j(t',zE)S_l(t',(1-z)E)$$

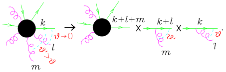
Final state radiation MC

Probabilistic interpretation

$$S_i(t,E) = \Delta_i(t,t_0)S_i(t_0,E) + \sum_{jl}\int_{t_0}^{t}\frac{dt'}{t'}\int_0^1 dz\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{a_S(t')}{2\pi}\Delta_i(t',t_0)S_j(t',zE)S_l(t',(1-z)E)$$

$\mathcal{S}_i(t,E) =$ 





FSR IV - MC programs

Final state radiation MC

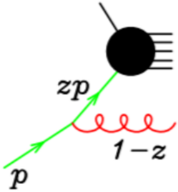
Shower algorithm

Generate hard process with probability proportional to its parton level cross section. For each final state colored parton:

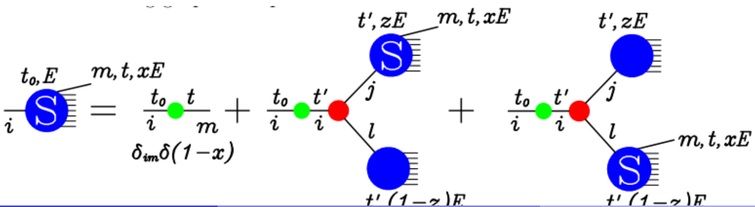
1. Set scale $t = Q$, hard scale of the process
2. Generate random number $0 < r < 1$
3. Solve $r = \Delta_i(t, t')$ for t'
4. i) if $t' < t_0$, no further branching and stop shower
5. ii) if $t' \geq t_0$, one branching into partons j, l with energies $E_j = zE_i$ and $E_l = (1 - z)E_i$, z following the $P_{i,jl}(z)$ distribution and ϕ uniform in the interval $[0, 2\pi]$ (variables, ...)
6. For each branched partons set $t = t'$ and start from (2)

Initial state radiation MC

General structure

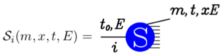


- Lines between t_1 and t_2 (consecutive radiations) are spacelike (*)
- Difference in Sudakov factors and Splitting functions start at NLO



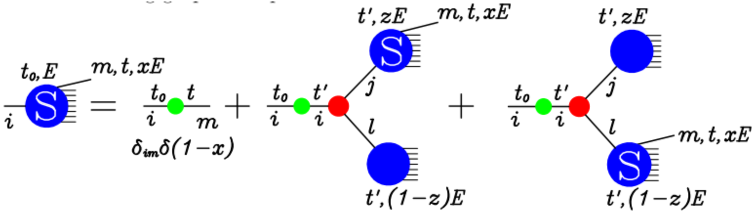
Initial state radiation MC

Formal representation



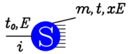
- Lines between t_1 and t_2 (consecutive radiations) are spacelike (*)
- Difference in Sudakov factors and Splitting functions start at NLO

Forward evolution equation. Great amount of computation time to generate configurations -¿ the scattering that we want



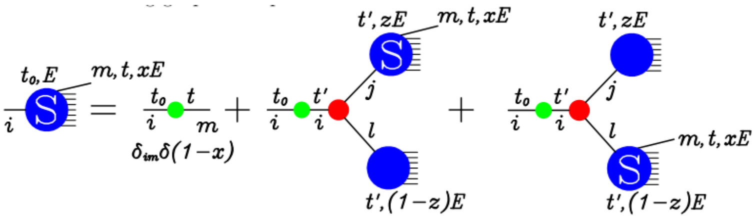
Initial state radiation MC

Formal representation

$\mathcal{S}_i(m, x, t, E) =$ 

- Lines between t_1 and t_2 (consecutive radiations) are spacelike (*)
- Difference in Sudakov factors and Splitting functions start at NLO

Backward evolution equation



Initial state radiation MC

Shower algorithm

Generate hard process with probability proportional to its parton level cross section. For each final state colored parton:

1. Set scale t to Q, hard scale of the process
2. Generate random number $0 < r < 1$
3. Solve (...) for t'
4. i) if $t' < t_0$, no further branching and stop shower
5. ii) if $t' \geq t_0$, one branching into partons j, l with energies $E_j = zE_i$ and $E_l = (1-z)E_i$, z following the $P_{i,jl}(z)$ distribution and ϕ uniform in the interval $[0, 2\pi]$
6. For parton j (...), for parton l generate a timelike parton shower according to the algorithm shown previously

Hadronization

Basics

Hadronization

Lund string model

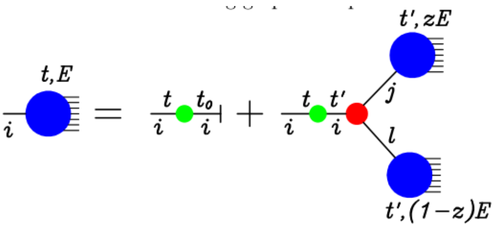
Hadronization

Clustering models

Parton showers and MC generators

Formal representation of a shower

Ensemble of all possible radiations as the sum of no radiation, with radiation and shower from radiated partons



- Ansatz for Sudakov

$$\Delta_i(t,t') = \exp\left\{ -\int_{t'}^{t} \frac{dt''}{t''} \int dz \sum_{jl} P_{i,jl}(z) \frac{\alpha_s(t')}{2\pi} \right\}$$

- Therefore $\partial\Delta(t,t')/\partial t \propto \Delta(t,t') \longrightarrow$ apply shower recursively

Generate hard process with probability proportional to its parton level cross section. For each final state colored parton:

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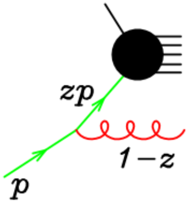
Parton showers and MC generators

Initial state radiation

ISR already important in QED $\longrightarrow$ Used to determine the Z peak at LEP



- QCD coupling much larger $\longrightarrow$ QCD ISR even more important
- Specially large for small momentum transfer
- Same as final state partons *always* manifest as jets, initial state ones *always* lead to ISR

Parton showers and MC generators

Ordering variables

HERWIG

- Ordering variable $t = E^2\theta^2/2$
- Order of transverse momentum as "angular ordering"
- IR cut-off needed

PYTHIA

- There is not angular ordering
- More natural kinematics
- Unphysical increase of number of partons $\longrightarrow$ solve by imposing veto to branchings that violate angular ordering