# Higgs boson production at the Large Hadron Collider: accurate theoretical predictions at higher orders in QCD

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N3PDF Machine Learning • PDFs • QCD

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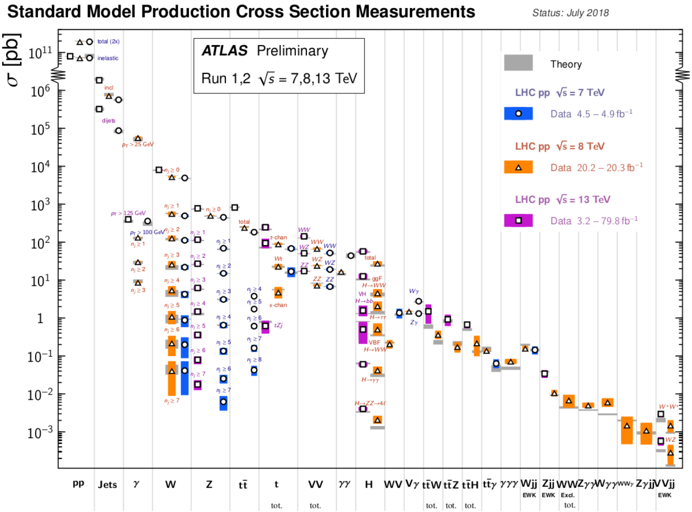
# Outline

1. QCD and collider physics
   - QCD Factorization
   - Partonic cross section and perturbative QCD
2. All order perturbative resummation
   - Higher order radiative corrections
   - Resummation of large logarithmic corrections
3. Precise and fast predictions for Higgs boson physics
   - Higgs production at the LHC
   - HTurbo numerical code
   - Preliminary results & Conclusions

# QCD and collider physics

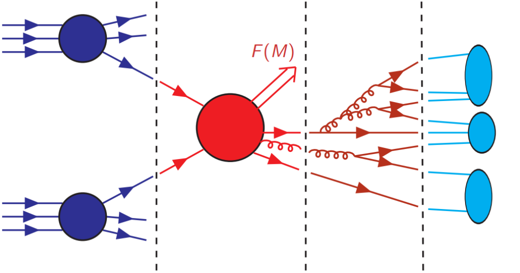
# QCD and collider physics

## LHC physics



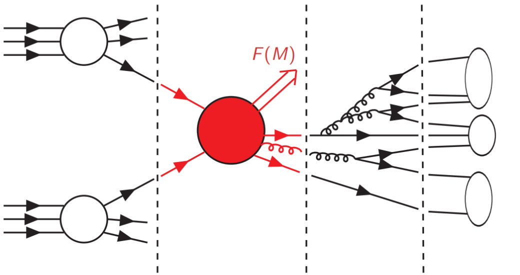
# QCD and collider physics

## Factorization theorem

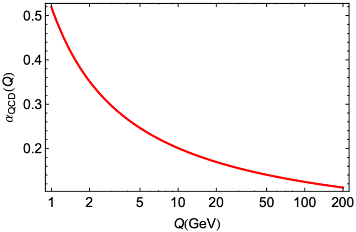



Compute hadronic cross sections is a hard problem $\longrightarrow$ QCD Factorization

$$\sigma^{\mathrm{F}}(p_1, p_2) = \int_0^1 dx_1 dx_2 \; f_\alpha(x_1, \mu_F^2) * f_\beta(x_2, \mu_F^2) * \hat{\sigma}^{\mathrm{F}}_{\alpha\beta}(x_1 p_1, x_2 p_2, \alpha_s(\mu_R^2), \mu_F^2)$$




- Parton densities (PDFs) $f_\alpha(x_i, \mu_F^2)$: non perturbative but universal
- Partonic cross section $\hat{\sigma}^{\mathrm{F}}_{\alpha\beta}$: process dependent but computable as perturbative series in $\alpha_s$




- Running coupling given by Renormalization Group Equation (RGE)

$$\alpha_s(\mu) = \frac{1}{\beta_0 \log\left(\frac{\mu^2}{\Lambda^2_{\rm QCD}}\right)}$$

- $\beta_0$ LO of the $\beta$ function, is $> 0$
- $\Lambda_{\rm QCD}$, parameter that defines value of the coupling at large scales

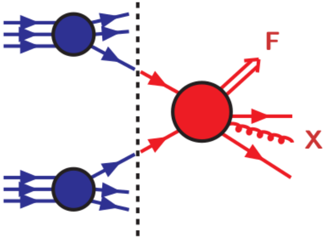
QCD is weakly coupled for $\mu >> \Lambda_{\rm QCD} \longrightarrow$ asymptotically free

Perturbative Quantum Chromodynamics (pQCD)

# QCD and collider physics

## Perturbative QCD

- Born cross section is the leading-order (LO) term of the perturbative series
- $\sigma^{(1)}, \sigma^{(2)}, \sigma^{(3)}$ are the NLO, NNLO, N3LO corrections




$$\hat{\sigma} = \sigma^{\mathtt{Born}} \left( 1 + \alpha_s \sigma^{(1)} + \alpha_s^2 \sigma^{(2)} + \alpha_s^3 \sigma^{(3)} + \ldots \right)$$

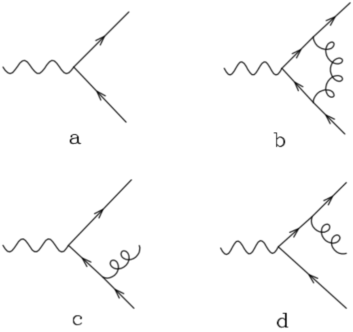
Lower order predictions strongly depend on the auxiliary and unphysical renormalization and factorization scales $\longrightarrow$ Need higher order corrections to increase theoretical accuracy!

## All order perturbative resummation

# Resummation in QCD

## Higher order corrections

1. Calculation of higher order corrections is not an easy task due to infrared (IR) soft and collinear singularities
2. Final state singularities cancel by combining real and virtual contributions
3. Initial state collinear singularities factorized inside the PDFs

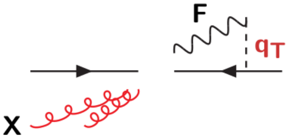


# Resummation in QCD

## $q_\perp$ resummation

Study the differential $q_\perp$ distribution

$$h_1(p_1) + h_2(p_2) \longrightarrow F(M, {\color{red} q_\perp}) + X$$




$$\int_0^{Q_\perp^2} dq_\perp^2 \frac{d\hat{\sigma}}{dq_\perp^2} \sim c_0 + \alpha_s(c_{12}L^2 + c_{11}L + c_{10}) + \ldots, \quad \text{where} \quad L = \ln(q_\perp/M^2)$$

| | | | |
|---|---|---|---|
| $\alpha_S L^2$ | $\alpha_S L$ | $\cdots$ | $\mathcal{O}(\alpha_S)$ |
| $\alpha_S^2 L^4$ | $\alpha_S^2 L^3$ | $\cdots$ | $\mathcal{O}(\alpha_S^2)$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $\alpha_S^n L^{2n}$ | $\alpha_S^n L^{2n-1}$ | $\cdots$ | $\mathcal{O}(\alpha_S^n)$ |
| dominant logs | $\cdots$ | $\cdots$ | $\cdots$ |

Truncated fixed order predictions $\rightarrow$ enhanced $\alpha_s^n \ln^m(M^2/q_\perp^2)$ appear

Separate partonic $q_\perp$ distribution as follows

$$\frac{d\hat{\sigma}_{ab}}{dq_\perp^2} = \left[\frac{d\hat{\sigma}_{ab}^{(\text{res.})}}{dq_\perp^2}\right]_{\text{l.a.}} + \left[\frac{d\hat{\sigma}_{ab}^{(\text{fin.})}}{dq_\perp^2}\right]_{\text{f.o.}} \quad , \quad \text{such that}$$

$$\int_0^{q_\perp^2} dq_\perp^2 \, \frac{d\hat{\sigma}_{ab}^{(\text{res.})}}{dq_\perp^2} \sim \sum \alpha_s^n \log^m \frac{M^2}{q_\perp^2} \quad \text{for} \quad q_\perp \to 0$$

$$\lim_{q_\perp \to 0} \int_0^{q_\perp^2} dq_\perp^2 \, \frac{d\hat{\sigma}_{ab}^{(\text{fin.})}}{dq_\perp^2} = 0$$

Resummed and finite components can be matched (LL+LO, NLL+NLO, NNLO+NNLL, ...) to have uniform accuracy in a wide range of $q_\perp$

# Resummation in QCD

## $q_\perp$ resummation

Resummation holds in impact parameter space $b$

$$\frac{d\hat{\sigma}_{ab}^{(\text{res.})}}{dq_\perp^2} = \frac{M^2}{\hat{s}} \int db \; \frac{b}{2} \; J_0(bq_\perp) \; \mathcal{W}_{ab}(b, M)$$

with $\mathcal{W}_{ab}$ also expressed in Mellin space (with respect to $z = M^2/\hat{s}$)

$$\mathcal{W}_N(b, M) = \mathcal{H}_N(\alpha_s) \times \exp\{\mathcal{G}_N(\alpha_s, L)\} \quad \text{being} \quad L \equiv \log(M^2 b^2)$$

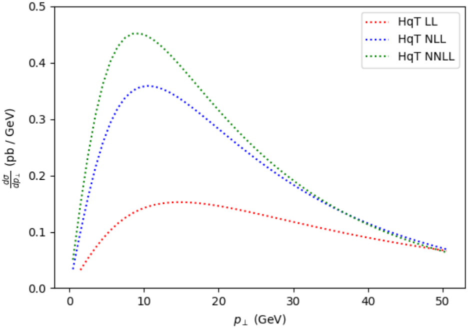
- Large logarithms exponentiated in the universal Sudakov form factor $\mathcal{G}_N(\alpha_s, L)$
- Constant (b-independent) terms factorized in the process dependent hard factor $\mathcal{H}_N(\alpha_s)$

## Accurate predictions for Higgs boson at the LHC

# HqT and HRes

## Predictions for Higgs $q_\perp$ distribution

- $q_\perp$ resummation implemented in numerical codes HqT and HRes [Catani, de Florian, Ferrera, Grazzini, Tommasini]
- Higher order accuracy require high computation times
- Codes producing fast and accurate predictions are needed for precision era of the LHC



Numerical code **DYTurbo** [Camarda et al.] ref. at 1910.07049, fast and precise $q_\perp$ resummation and several improvements for Drell-Yan ($h_1 h_2 \to V + X \to l^+ l^- + X$)

- First goal: set up a numerical code for Higgs boson production starting from **DYTurbo**
- Set LO amplitude $gg \to H$
- Set Sudakov and Hard coefficients for Higgs production
- Compare with **HRes** and **HqT**

Final goal: extend theoretical accuracy up to N$^3$LL+N$^3$LO

# HTurbo

## Starting point DYTurbo

Both Sudakov factor $\mathcal{G}_N$ and hard coefficient $\mathcal{H}_N$ can be expanded as perturbative series in $\alpha_s$

$$\mathcal{G}_N(\alpha_s, L) = L\, g^{(1)}(\alpha_s L) + g^{(2)}(\alpha_s L) + \frac{\alpha_s}{\pi} g^{(3)}(\alpha_s L) + \ldots$$

$$\mathcal{H}_N(\alpha_s) = 1 + \alpha_s \mathcal{H}^{(1)} + \alpha_s^2 \mathcal{H}^{(2)} + \ldots$$

For each new order implement a factor of $\mathcal{G}_N$ and Hard $\mathcal{H}_N$

$$\mathrm{LL}(\sim \alpha_s^n L^{n+1}) : g^{(1)}, \hat{\sigma}^{(0)}$$

$$\mathrm{NLL}(\sim \alpha_s^n L^{n}) : g^{(2)}, \mathcal{H}^{(1)}$$

$$\mathrm{NNLL}(\sim \alpha_s^n L^{n-1}) : g^{(3)}, \mathcal{H}^{(2)}$$

Start by building predictions up to NNLO+NNLL, then add $\mathrm{N^3LO+N^3LL}$

# HTurbo

## Code optimization

Reimplementation of **HqT** and **HRes** for $q_T$-resummation

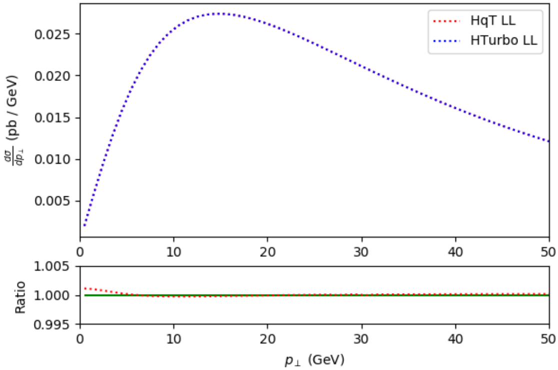
- **C++** structure with **Fortran** interfaces $\rightarrow$ Multi-threading
- Optimization in the integration routines / integral transforms
  - Factorize boson and decay kinematics
  - Gauss-Legendre quadrature rules (1-dim.)
  - Vegas/Cuhre through **Cuba** (multi-dim.)

Comparison **HRes** and **HTurbo** - speed performance

| Predictions | **HRes** | **HTurbo** |
|---|---|---|
| resummed NNLL | 10h | 10' |
| combined NNLO+NNLL | 20h | 1h |

# Results

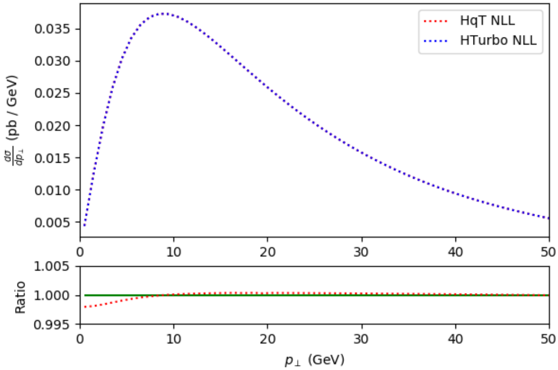
## Comparison HTurbo and HqT - LL




- HTurbo $q_\perp$ distribution vs HRes and HqT at LL
- Excellent numerical agreement up to the 0.1% level

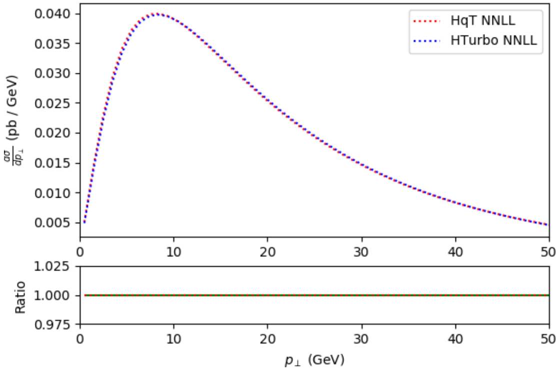
# Results

## Comparison HTurbo and HqT - NLL




- HTurbo $q_\perp$ distribution vs HRes and HqT at NLL
- Excellent numerical agreement up to the 0.1% level

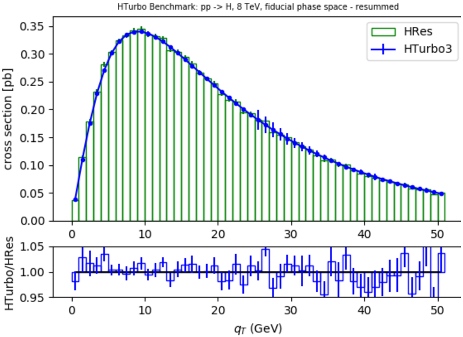
## Comparison HTurbo and HqT - NNLL

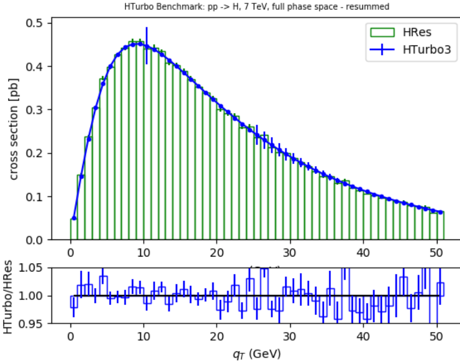



- HTurbo $q_\perp$ distribution vs HRes and HqT at NNLL
- Excellent numerical agreement up to the 0.1% level

# Results

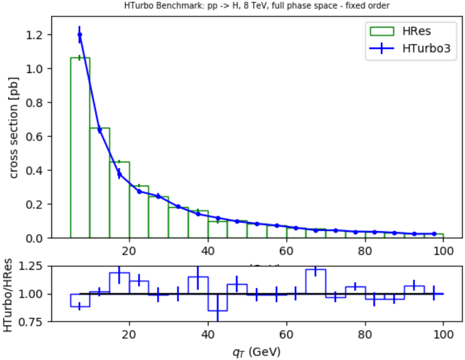
## Comparison HTurbo and HRes - resummed




- Represent full (LHS) and fiducial (RHS) phase space ✓
- Agreement up to resummed NNLL $\longrightarrow$ ready for N$^3$LL

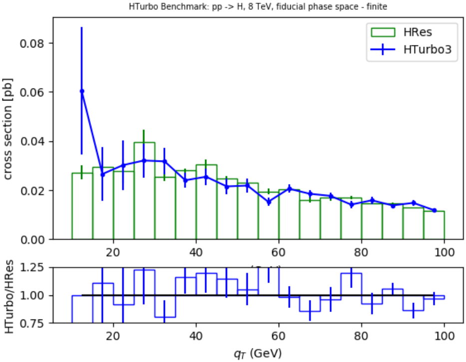
## Comparison HTurbo and HRes - fixed order




- Represent fiducial (RHS) phase space - HRes vs HTurbo ✓
- Agreement up to fixed order NL0 $\longrightarrow$ ready for N$^3$LL

# Results

## Comparison HTurbo and HRes - finite




- Represent fiducial phase space - HRes vs HTurbo ✓
- Agreement up to finite NNLL+NLO $\longrightarrow$ ready for N$^3$LL

# Summary & Conclusions

1. Fast and accurate predictions are needed towards the precision era of the LHC
2. Developing a novel numerical code, **HTurbo**, which implements $q_\perp$ resummation for Higgs boson production
3. HTurbo is faster than any of the existing codes
4. Outlook of thesis work:
   - Add $\mathrm{N^3LO{+}N^3LL}$ prediction
   - Perform phenomenological studies comparing with LHC data

Thank you!

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