Two-mode squeezed states in cavity optomechanics via engineering of a single reservoir

Quantum coherent phenomena course seminar - Milan, October 2020

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Outline

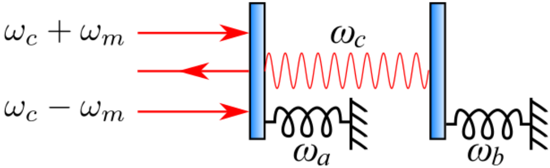
1. Introduction, system and Hamiltonian
2. Reservoir engineering strategies
3. Implementation
4. Full system
5. Experimental observability
6. Conclusions

1. Generation and detection of entangled states of macroscopic M.O
2. Reservoir engineering $\longrightarrow$ Two-mode squeezed states
3. Easy to implement in existing experimental configurations
4. Quantum optomechanics $\longrightarrow$ couple Bogoliubov modes to a single reservoir (damped cavity)

Introduction

System representation

- Two mechanical oscillators, resonance frequencies ω_a, ω_b
- Dispersively coupled g_a, g_b to a common cavity ω_c
- Dispersively coupled g_a, g_b to a common cavity ω_c



Quantum optomechanics Hamiltonian

$$\hat{\mathcal{H}} = \omega_a \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b} + \omega_c \hat{c}^\dagger \hat{c} + g_a(\hat{a} + \hat{a}^\dagger)\hat{c}^\dagger \hat{c} + g_b(\hat{b} + \hat{b}^\dagger)\hat{c}^\dagger \hat{c} + \hat{H}_{\text{drive}} + \hat{H}_{\text{diss}},$$

Under usual approximations, obtain the master formula

$$\dot{\rho} = -i[\hat{\mathcal{H}}', \rho] + \gamma_a(\bar{n}_a + 1)\mathcal{D}[\hat{a}]\rho + \gamma_a \bar{n}_a \mathcal{D}[\hat{a}^\dagger]\rho + \gamma_b(\bar{n}_b + 1)\mathcal{D}[\hat{b}]\rho + \gamma_b \bar{n}_b \mathcal{D}[\hat{b}^\dagger]\rho + \kappa \mathcal{D}[\hat{c}]\rho,$$

Being $\mathcal{H}' = \mathcal{H} - \mathcal{H}_{\text{diss}}$, and $\mathcal{D}[\hat{c}]$ the dispersive superoperator

Reservoir engineering strategies

Bogoliubov operators

Define the Bogoliuov modes in terms of the modes $\hat{a}, \hat{b} \longrightarrow$

$$\hat{\beta}_1 = \hat{a}\cosh r + \hat{b}^\dagger \sinh r,$$
$$\hat{\beta}_2 = \hat{b}\cosh r + \hat{a}^\dagger \sinh r.$$

Rotation with respect to a frame, being r the squeezing parameter

$$\hat{H}_0 = (\omega_a - \Omega)\hat{a}^\dagger \hat{a} + (\omega_b + \Omega)\hat{b}^\dagger \hat{b} + \omega_c \hat{c}^\dagger \hat{c},$$

Choice of Ω for non-trivial rotation of collective mechanical quadratures $\hat{X}_{\pm}$, $hatP_{\pm}$

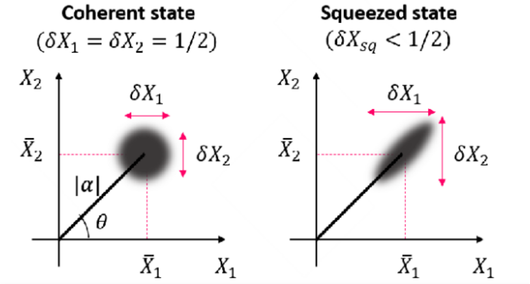
Reservoir engineering strategies

Squeezed modes

2-mode squeezed state defined by $|r >= S_2(r)|00 >$

$$\hat{S}_2(r) \equiv \exp[r(\hat{a}\hat{b} - \hat{a}^\dagger \hat{b}^\dagger)]$$

Such that $\hat{\beta}_1$ and $\hat{\beta}_2$ are the two-mode squeezed state with squeezing parameter r



Reservoir engineering strategies

Squeezed modes

- i) Two cavity modes to independently cool the Bogoliubov modes (beam splitter $\hat{\beta}_i^\dagger \hat{c}_i$)
- ii) Couple the cavity to one Bogoliubov mode, and then this to the other via $\hat{\beta}_1^\dagger \hat{\beta}_2$ and the this to the other one
- iii) Couple the cavity to sum of the Bogoliubov modes , then the sum to the difference . Again, beam splitter interaction $\hat{\beta}_{\mathrm{sum}}^\dagger \hat{\beta}_{\mathrm{diff}}$ allows diff to cool.

$$\hat{\beta}_{\mathrm{sum}} = \frac{1}{\sqrt{2}}(\hat{\beta}_1 + \hat{\beta}_2)$$
$$\hat{\beta}_{\mathrm{diff}} = \frac{1}{\sqrt{2}}(\hat{\beta}_1 - \hat{\beta}_2)$$

Cooling $\hat{\beta}_{\mathrm{sum}}$ and $\hat{\beta}_{\mathrm{diff}}$ is equivalent to cool $\hat{\beta}_1$ and $\hat{\beta}_2$ given $< \hat{\beta}_{\mathrm{sum}}^\dagger \hat{\beta}_{\mathrm{sum}} > + < \hat{\beta}_{\mathrm{diff}}^\dagger \hat{\beta}_{\mathrm{diff}} >=< \hat{\beta}_1^\dagger \hat{\beta}_1 > + \hat{\beta}_2^\dagger \hat{\beta}_2$

Reservoir engineering strategies

Hamiltonian

Hamiltonian in terms of the Bogoliubov modes

$$\hat{\mathcal{H}} = \Omega(\hat{\beta}_1^\dagger \hat{\beta}_1 - \hat{\beta}_2^\dagger \hat{\beta}_2) + \mathcal{G}[(\hat{\beta}_1^\dagger + \hat{\beta}_2^\dagger)\hat{c} + \mathrm{H.c.}] + \hat{H}_{\mathrm{diss}},$$

where Ω is the effective frequency and $\mathcal{G}$ an effective coupling. Written in terms of the original operators,

$$\begin{aligned}\hat{\mathcal{H}} = \Omega(\hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b}) + G_+[(\hat{a} + \hat{b})\hat{c} + \mathrm{H.c.}] \\ + G_-[(\hat{a} + \hat{b})\hat{c}^\dagger + \mathrm{H.c.}] + \hat{H}_{\mathrm{diss}}.\end{aligned}$$

with couplings related by $\mathcal{G} \equiv \sqrt{G_-^2 - G_+^2}$ and $\tanh r \equiv= G_+/G_-$

Implementation

Different cases

Hamiltonian is already implemented in conventional optomechanical setups. Focus on regime $|G_+| < |G_-|$

- Two-tone driving ($g_a = g_b$)
- Four-tone driving ($g_a = g_b$)
- Case similar ($g_a \sim g_b$)

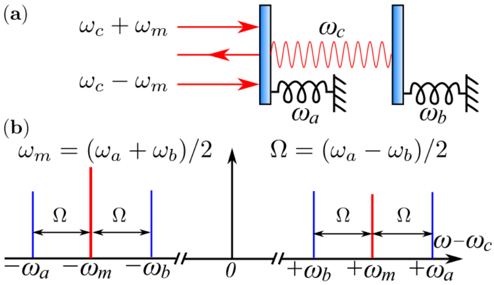
(...)

Implementation

2 - tone driving

Single photon coupling rates equal $\longrightarrow$ cavity drive tones at $\omega_c \pm \omega_m$

$$\hat{H}_{\text{drive}} = (\mathcal{E}_+^* e^{+i\omega_m t} + \mathcal{E}_-^* e^{-i\omega_m t}) e^{+i\omega_c t} \hat{c} + \text{H.c.}$$

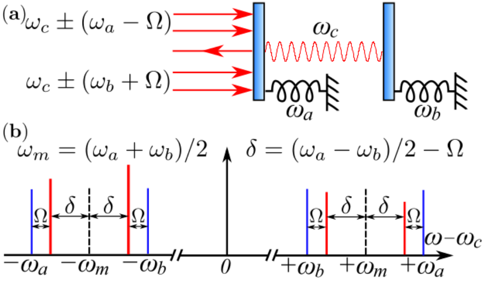


- Interaction picture with respect to H_0
- Find the steady state amplitudes at the sidebands

$$\bar{c}_\pm \equiv \langle \hat{c}_\pm \rangle_{\text{ss}} = \frac{i\mathcal{E}_\pm}{\pm i\omega_m - \kappa/2}.$$

- Assumptions used (...)

Couplings unequal $\longrightarrow$ driving tones applied with detuning of Ω from the sidebands $\omega_c \pm (\omega_a - \Omega)$ and $\omega_c \pm (\omega_b + \Omega)$



$$\hat{H}_{\text{drive}} = e^{+i\omega_c t}\hat{c}(\mathcal{E}_{1+}^* e^{+i(\omega_a - \Omega)t} + \mathcal{E}_{2+}^* e^{+i(\omega_b + \Omega)t} + \mathcal{E}_{1-}^* e^{-i(\omega_a - \Omega)t} + \mathcal{E}_{2-}^* e^{-i(\omega_b + \Omega)t}) + \text{H.c.}$$

Implementation

4 - tone driving

Couplings unequal $\longrightarrow$ driving tones applied with detuning of Ω from the sidebands $\omega_c \pm (\omega_a - \Omega)$ and $\omega_c \pm (\omega_b + \Omega)$

$$\bar{c}_{k\pm} \equiv \langle \hat{c}_{k\pm} \rangle_{\text{ss}} = \frac{i\mathcal{E}_{k\pm}}{\pm i\omega_k - \kappa/2},$$

- Where we demand the strengths match as $\bar{c}_{1\pm}/\bar{c}_{2\pm} = g_b/g_a$
- Working in interaction picture with respect to Hamiltonian (4)
- Imprecision in the matching lead to add contributions as in Hamiltonian (14)

Condition $\gamma \ll \Omega \ll (\omega_a - \omega_b)/2 - \gamma$, sufficiently coupled Bogoliubov modes and unwanted sideband processes have no effect.

Adiabatic limit

Our system

- Assume the system responds rapidly to mechanical motion $k > \Omega$, $G_{\pm}$, but still in $\omega_a, \omega_b \gg k$
- Get rid of the cavity operator $\hat{c} = -2i\mathcal{G}(\hat{\beta}_1 + \hat{\beta}_2)/k$
- Obtain adiabatically eliminated master equation

$$\dot{\rho} = -i\Omega[\hat{\beta}_1^\dagger\hat{\beta}_1 - \hat{\beta}_2^\dagger\hat{\beta}_2, \rho] + \gamma_a(\bar{n}_a + 1)\mathcal{D}[\hat{a}]\rho + \gamma_a\bar{n}_a\mathcal{D}[\hat{a}^\dagger]\rho \\ + \gamma_b(\bar{n}_b + 1)\mathcal{D}[\hat{b}]\rho + \gamma_b\bar{n}_b\mathcal{D}[\hat{b}^\dagger]\rho + \Gamma\mathcal{D}[\hat{\beta}_1 + \hat{\beta}_2]\rho,$$

with optomechanical damping rate

$$\Gamma \equiv \gamma\mathcal{C} \equiv \frac{4\mathcal{G}^2}{\kappa},$$

Easy to obtain steady state, and to measure entanglement and purity.

Adiabatic limit

Entangled systems

Adiabatic limit

Entanglement

Entanglement criterion using Duan inequality

$$\hat{X}_{\pm} = (\hat{X}_a \pm \hat{X}_b)/\sqrt{2},$$
$$\hat{P}_{\pm} = (\hat{P}_a \pm \hat{P}_b)/\sqrt{2},$$

Where we introduced the quadrature modes as

$$\hat{X}_s = (\hat{s} + \hat{s}^\dagger)/\sqrt{2}, \quad \hat{P}_s = -i(\hat{s} - \hat{s}^\dagger)/\sqrt{2}.$$

Where we introduced the quadrature modes as

$$\langle \hat{X}_+^2 \rangle + \langle \hat{P}_-^2 \rangle < 1$$

Adiabatic limit

Purity

Entanglement criterion using Duan inequality

$$\hat{X}_\pm = (\hat{X}_a \pm \hat{X}_b)/\sqrt{2},$$
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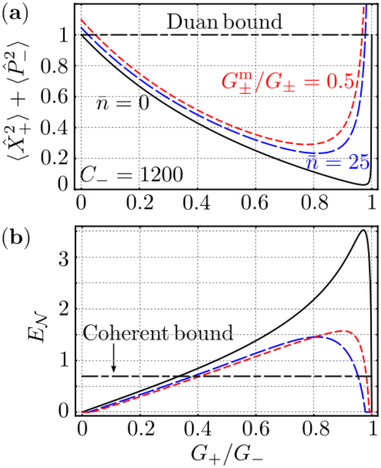
Where we introduced the quadrature modes as

$$\langle \hat{X}_+^2 \rangle + \langle \hat{P}_-^2 \rangle < 1$$

Adiabatic limit

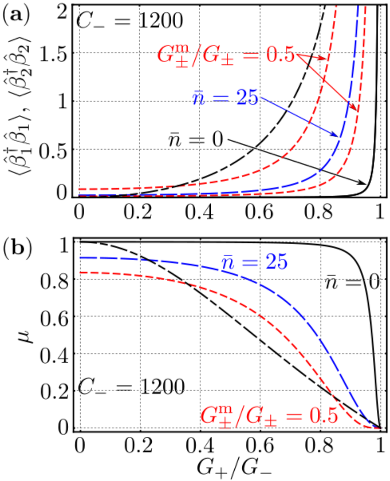
Entanglement



- 1
- 2
- 3

Adiabatic limit

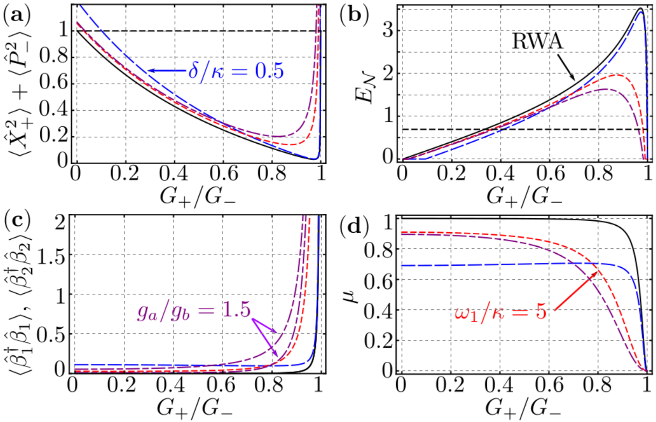
Entanglement



- 1
- 2
- 3

Time dependence

time dependence



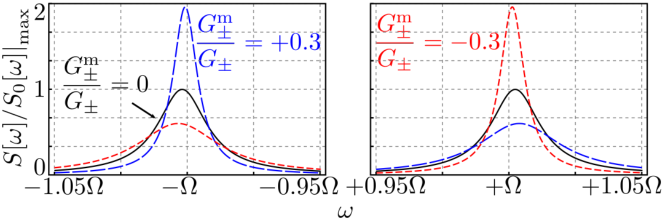
Experimental observability

Output spectrum

$$\int_{-\infty}^{0} S[\omega]d\omega = \int_{0}^{+\infty} S[\omega]d\omega$$
$$= 8\pi\kappa \frac{\mathcal{G}^2}{4\mathcal{G}^2 + \kappa(\kappa + \gamma)} \langle \hat{\beta}_i^\dagger \hat{\beta}_i \rangle,$$

Experimental observability

Output spectrum



Conclusions

1. Configuring a three-mode optomechanical system such as the steady state includes highly pure and highly entangled two-mode squeezed state.
2. Symmetry on the steady-state makes it attractive for implementation of continuous-variable teleportation protocols
3. Problem of unequal single-photon optomechanical couplings solved by using four-tone driving scheme
4. Proposal implementable for existing technology