QCD and Monte Carlo event generators

Monte Carlo course seminar - Milan, February 2021

UNIVERSITÀ DEGLI STUDI DI MILANO

INFN Istituto Nazionale di Fisica Nucleare

erc European Research Council

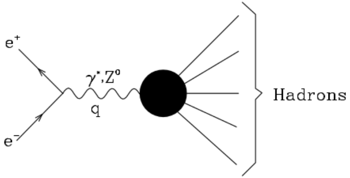
Outline

1. Hadron collisions and strong interactions
   - Hadron collisions and strong interactions
   - Renormalization group
   - QCD factorization
2. MC and Parton Showers
   - Collinear factorization
   - Final state radiation
   - Initial state radiation
3. Hadronization: some basics
   - Large number of colors approximation
   - Hadronization models

Strong interactions

QCD from e^+e^- annihilation

Quantum Chromodynamics (QCD) $\rightarrow$ theory describing the interaction between quarks and gluons (strong interactions)



QCD arises already from e^+e^- annihilation $\rightarrow$ R_0 ratio

$$R_0 = \frac{\sigma(\gamma^* \rightarrow \text{hadrons})}{\sigma(\gamma^* \rightarrow \mu^+\mu^-)} = 3\sum_f c_f^2$$

1. Color factor (3 color for each quark)
2. Sum over charges of different flavors
3. Threshold and higher order corrections

Strong interactions

Renormalization group

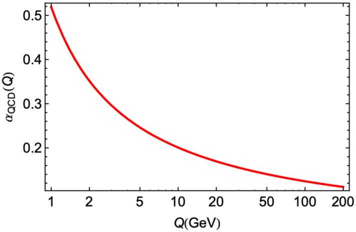
- Running coupling given by Renormalization Group Equation (RGE)

$$\mu \frac{d\alpha_s(\mu)}{d\mu} = \beta(\alpha_s(\mu)) = -\sum_{n=0}^{\infty} \beta_n \left(\frac{\alpha_s}{\pi}\right)^{n+1}$$

- Coupling α_s evolves with scale μ as given by RGE $\rightarrow$ LO behavior driven by β_0
- $\beta_0^{\mathrm{QCD}} > 0 \implies$ weakly coupled at large energies, asymptotic freedom
- $\beta_0^{\mathrm{QED}} < 0 \implies$ strongly coupled at large energies, UV divergent!

Strong interactions

Renormalization group



- Running coupling given by Renormalization Group Equation (RGE)

$$\alpha_s(\mu) = \frac{1}{\beta_0 \log\left(\frac{\mu^2}{\Lambda_s^2}\right)}$$

- β_0 LO of the β function, is > 0
- Λ_s, parameter that defines value of the coupling at large scales

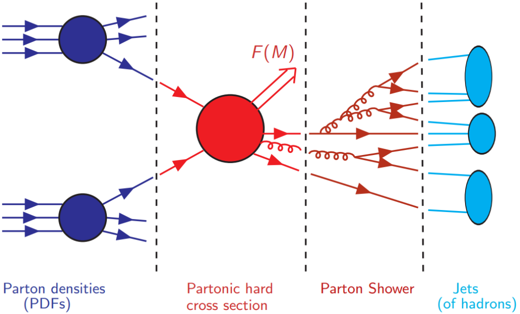
QCD is weakly coupled for $\mu >> \Lambda_s \longrightarrow$ asymptotically free

Perturbative Quantum Chromodynamics (pQCD)

Factorization theorem

QCD factorization

LHC processes $H_1 + H_2 \to \mathrm{F}$



Separate process PDFs and partonic (hard) interaction

$$\sigma^{\mathrm{F}}(p_1, p_2) = \sum_{\alpha,\beta} \int_0^1 dx_1 dx_2 \; f_\alpha(x_1, \mu_F^2) * f_\beta(x_2, \mu_F^2) \; * \; \hat{\sigma}^{\mathrm{F}}_{\alpha\beta}(x_1 p_1, x_2 p_2, \alpha_s(\mu_R^2), \mu_F^2)$$

Parton showers

MC Parton showers

Partons in the initial and final state emit radiation. Initial state Radiation (ISR) and Final State Radiation (FSR) model by Monte Carlo (MC) shower algorithms

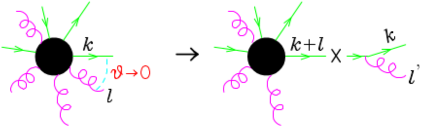
Shower Monte Carlo programs (HERWIG, PYTHIA)

- Libraries for computing SM and BSM cross sections
- Shower algorithms produce a number of enhanced coloured parton emissions to be added to the hard process
- Hadronization models, underlying event, decays of unstable hadrons, etc

Parton showers

Collinear limit

- QCD emission processes are enhanced in the collinear limit (θ small)
- σ dominated by collinear splittings $q \to qg, g \to gg, g \to q\bar{q}$ (measurement not sensitive to such small scales)



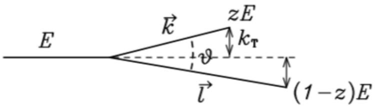
Collinear factorization $\longrightarrow$ The cross section factorizes into the product of a tree-level cross section and a splitting factor out tree level amplitude and splitting

$$|M_{n+1}|^2 d\Phi_{n+1} \Rightarrow |M_n|^2 d\Phi_n \quad \frac{\alpha_S}{2\pi} \frac{dt}{t} P_{q,qg}(z)\, dz\, \frac{d\phi}{2\pi}.$$

$$d\Phi_n = (2\pi)^4 \delta^4 \left(\sum_i^n k_i - q \right) \prod_i^n \frac{d^3 k_i}{(2\pi)^3 2k_i^0}$$

Parton showers

Kinematics of splitting



Kinematics of splitting given by (t, z, ϕ)

- t: parameter with dimensions of energy that vanish in the collinear limit
  - Virtuality $t = (k+l)^2 = k^0 l^0 4 \sin^2\left(\frac{\theta}{2}\right) \approx k^0 l^0 \theta^2 \approx z(1-z)E^2\theta^2$
  - Transverse momentum $t = k_\perp^2 = l_\perp^2 = z^2(1-z)^2E^2\theta^2$
  - Hardness $E^2\theta^2$
- z: fraction of energy of radiated parton $z = \frac{k^0}{k^0+l^0}$
- ϕ represents azimuth of the k, l plane

Parton showers

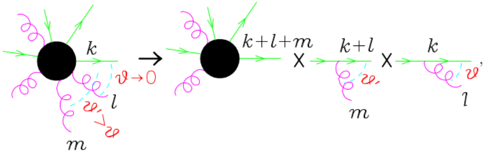
AP splitting functions

Factorization holds for small angles $\rightarrow$ small t variable
Difference in the splitting $\rightarrow$ Altarelli-Parisi splitting functions (singular in $z \rightarrow 0, 1$)

$$P_{q,qg}(z) = C_F \frac{1+z^2}{1-z}$$

$$P_{g,gg}(z) = C_A \left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right)$$

$$P_{g,q\bar{q}}(z) = T_f(z^2 + (1-z)^2)$$



We can proceed in an iterative way

$$|M_{n+2}|^2 d\Phi_{n+2} = |M_n|^2 d\Phi_n \frac{\alpha_s(t')}{2\pi} P_{q,qg}(z') \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi} \frac{\alpha_s(t)}{2\pi} P_{q,qg}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi}$$

Angles become small, maintaining a strong ordering relation $\theta' >> \theta \rightarrow 0$

Parton showers

Exclusive final state

To describe exclusive final state $\rightarrow$ sum perturbative expansion to all orders in α_s

$$\sigma_0 \alpha_s^n \int \frac{dt_1}{t_1} \ldots \frac{dt_n}{t_n} \theta(Q^2 > t_1^2 > \ldots > t_n^2 > \Lambda_S^2) = \sigma_0 \frac{\alpha_s^n}{n!} \log^n \left(\frac{Q^2}{\Lambda_S^2} \right)$$

Possible if we limit to the most singular terms, in ordered sequence of angles
Collinear approximation $\longrightarrow$ Leading log approximation

Final state radiation MC

General structure

Approximated description of a hadronic final state
Model a given hard scattering with arbitrary number of enhanced radiations

- Choose hard interaction with specified Born kinematics
- Consider all possible tree-level splittings for each coloured parton
- Assign the variables (t, z, ϕ) at each splitting vertex, t ordered in decreasing way
- At each splitting vertex assign the weight

$$\frac{\alpha_S(t)}{2\pi} P_{i,jl}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi}$$

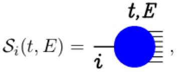
- Each line has a weight known as Sudakov factor

$$\Delta_i(t', t'') = \exp\left[- \sum_{(jl)} \int_{t''}^{t'} \frac{dt}{t} \int_0^1 dz \; \frac{\alpha_S(t)}{2\pi} \; P_{i,jl}(z) \right]$$

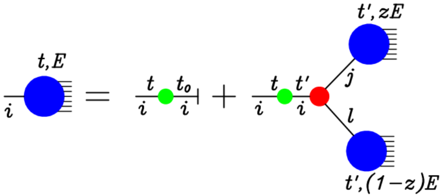
Final state radiation MC

Formal representation of a shower

Graphical notation for the representation of a shower



Ensemble of all possible branchings from parton i at scale t



Forward evolution equation $\rightarrow$ recursive structure

$$S_i(t,E) = \Delta_i(t,t_0)S_i(t_0,E) + \sum_{jl}\int_{t_0}^{t}\frac{dt'}{t'}\int_0^1 dz\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{a_S(t')}{2\pi}\Delta_i(t',t_0)S_j(t',zE)S_l(t',(1-z)E)$$

Final state radiation MC

Probabilistic interpretation

$$\frac{\alpha_s(t')}{2\pi} P_{\mathrm{i,jl}}(z') \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi}$$

Probability of branching in the infinitesimal volume $dt'\ dz\ d\phi$

$$dP_{br} = \frac{\alpha_s(t')}{2\pi} \frac{dt'}{t'} \int_0^1 dz' P_{\mathrm{i,jl}}(z') \int_0^{2\pi} \frac{d\phi'}{2\pi}$$

Probability of branching in the interval dt'

$$dP_{nobr} = 1 - dP_{br} = 1 - \frac{\alpha_s(t')}{2\pi} \frac{dt'}{t'} \int_0^1 dz' P_{\mathrm{i,jl}}(z') \int_0^{2\pi} \frac{d\phi'}{2\pi}$$

Probability of first branching in the infinitesimal volume dt'

$$\Delta_i(t,t') = 1 - dP_{br} = \prod_i^n \left(1 - \frac{\alpha_s(t_i)}{2\pi} \frac{\delta t}{t_i} \int_0^1 dz' \int_0^{2\pi} P_{\mathrm{i,jl}}(z') \frac{d\phi'}{2\pi} \right)$$

Sudakov form factor

$$dP_{fbr} = \Delta_i(t,t') \frac{\alpha_s(t')}{2\pi} P_{\mathrm{i,jl}}(z') \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi}$$

Probability of no-branching in the infinitesimal volume $dt'\ dz\ d\phi$

Final state radiation MC

Shower algorithm

Generate hard process with probability proportional to its parton level cross section.
For each final state colored parton:

1. Set scale $t = Q$, hard scale of the process.
2. Generate random number $0 < r < 1$.
3. Solve $r = \Delta_i(t, t')$ for t'.
4. i) if $t' < t_0$, no further branching and stop shower.
5. ii) if $t' \geq t_0$, generate j, l with energies

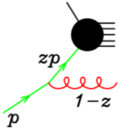
$$E_j = zE_i \quad \text{and} \quad E_l = (1 - z)E_i,$$

   following the $P_{i,jl}(z)$ distribution and with azimuth ϕ uniform in the interval $[0, 2\pi]$.
   the angle of between their momenta is fixed by t'.
6. For each branched partons set $t = t'$ and start from (2).

Initial state radiation MC

General structure

ISR showers are spacelike



$$p_{\pm} = E \pm p_z \qquad p_{\mathrm{T}} = \sqrt{p_{\mathrm{x}}^2 + p_{\mathrm{y}}^2}$$

$$E^2 - p_x^2 - p_y^2 - p_z^2 = m^2 \Rightarrow p_+ p_- = m^2 + p_{\mathrm{T}}^2$$

Consider the splitting between a particle a that splits into b and c

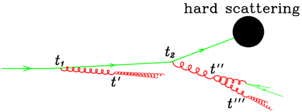
$$p_{-a} = p_{-b} + p_{-c} \Leftrightarrow \frac{m_{\mathrm{a}}^2}{p_{+a}} = \frac{m_b^2 + p_{b\mathrm{T}}^2}{p_{+b}} + \frac{m_c^2 + p_{c\mathrm{T}}^2}{p_{+c}} \Leftrightarrow m_a^2 = \frac{m_b^2}{z} + \frac{m_c^2}{1-z} + \frac{p_{\mathrm{T}}^2}{z(1-z)}$$

$$\texttt{ISR} \quad m_a \approx 0, m_c \approx 0 \ \Rightarrow m_b^2 \approx -\frac{p_{\mathrm{T}}^2}{1-z}$$

$$\texttt{FSR} \quad m_b \approx 0, m_c \approx 0 \ \Rightarrow m_a^2 \approx \frac{p_{\mathrm{T}}^2}{z(1-z)}$$

Initial state radiation MC

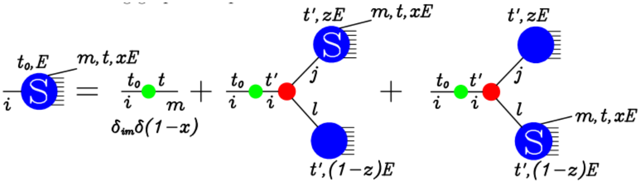
Formal representation



$S_i(m,x,t,E) =$ 

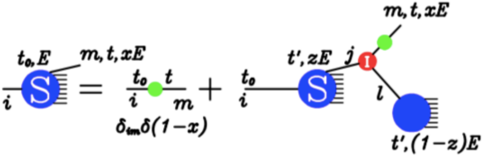
- Lines between t_1 and t_2 (consecutive radiations) are spacelike (*)
- Difference in Sudakov factors and Splitting functions start at NLO

Forward evolution equation. Great amount of computation time to generate configurations leading to the scattering that we want



Initial state radiation MC

Backwards evolution equation



$$\sum_{\mathscr{F}} S_i(m,t,x,E) = f_m^{(i)}(x,t)$$

- Moderns MC programs $\rightarrow$ recursive procedure starting at the large scale rather than the small one
- The blob I at the splitting vertex given by the inclusive splitting kernel P_{jm}, instead of the exclusive one $P_{j,ml}$
- Backwards evolution equation (scale dependent parton density)

$$f_m^{(i)}(x,t) = \delta_{im}\delta(1-x)\Delta_{\mathrm{m}}(t,t_0) + \int_{t_0}^{t}\frac{dt'}{t'}\int_x^1\frac{dz}{z}\sum_j f_j^{(i)}(z,t')\frac{\alpha_{\mathrm{S}}(t')}{2\pi}\hat{P}_{jm}\left(\frac{x}{z}\right)\Delta_m(t,t')$$

Initial state radiation MC

Shower algorithm

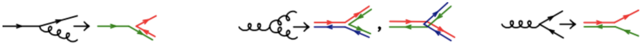
Generate hard process with probability proportional to its parton level cross section. For each final state colored parton:

1. Set scale $t = Q$, hard scale of the process.
2. Generate random number $0 < r < 1$
3. Solve
$$r = \frac{f_m^{(i)} \Delta_m(t, t')}{f_m^{(i)}(t, x)} \quad \text{for } t'.$$
4. i) if $t' < t_0$, no further branching and stop shower.
5. ii) if $t' \geq t_0$, generate j, l with energies
$$E_j = zE_i \quad \text{and} \quad E_l = (1 - z)E_i,$$
following the $P_{i,jl}(z)$ distribution and with azimuth ϕ uniform in the interval $[0, 2\pi]$.
the angle of between their momenta is fixed by t'.
6. For parton j set $t = t'$ and start from (2). For parton l generate a timelike parton shower according to the algorithm shown previously.

Hadronization

Some basics

A parton becomes a measurable hadron through the emission of a partonic shower
Large number of color approximation $\rightarrow$ each parton identified by a unique label



Hadronization models

- Lund string model
    - non perturbative production of quarks and antiquarks
    - intermediate gluons are transverse kicks of a continuum medium
- Cluster models
    - preconfinement, assuming subsystems of color singlet partons with universal invariant mass distribution (power suppressed at high masses)
    - gluons are forced to split in quark-antiquark pair

Summary

- LHC processes require factorization in perturbative and non perturbative part
- pQCD applied at high energies
- Monte Carlo shower programs describe non perturbative physics in hadron physics
- Agreement and precision Monte Carlo shower programs