

Higgs boson production at the Large Hadron Collider: accurate theoretical predictions at higher orders in QCD

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Outline

① Introduction to QCD

- A historical approach
- Asymptotic freedom and pQCD

② QCD and collider physics

- QCD Factorization
- Partonic cross section and perturbative QCD

③ All order perturbative resummation

- Higher order radiative corrections
- Resummation of large logarithmic corrections
- Resummed, asymptotic and fixed-order

④ Precise and fast predictions for Higgs boson physics

- Higgs production at the LHC
- HTurbo numerical code
- Preliminary results & Conclusions

Part I

QCD and collider physics

Introduction

QCD and the strong interactions

- The parton model
- QCD is the theory of the strong interactions
- Predictions in QFTs are done by computing cross sections
- Experiments for high energy physics are done at the LHC

Introduction

QCD and the strong interactions

How to explore proton's inner structure?

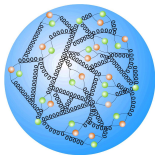


- Point-like projectile on the object \rightarrow DIS
- Smash the two objects \rightarrow LHC physics

"A way to analyze high energy collisions is to consider any hadron as a composition of point-like constituents \rightarrow **partons**" R.Feynman, 1969

QCD and collider physics

Asymptotic freedom and pQCD



- QCD and QFTs, coupling strength changes with energy
- Charge a particle feels depends on the scale
- QED

QCD is strongly coupled at short scales \longrightarrow confinement

Non-perturbative physics

QCD and collider physics

Asymptotic freedom and pQCD

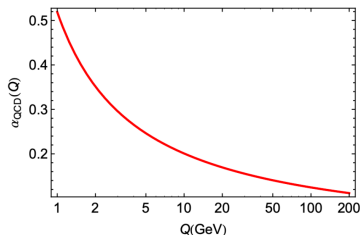
- Running coupling given by Renormalization Group Equation (RGE)

$$\mu \frac{d\alpha_s(\mu)}{d\mu} = \beta(\alpha_s(\mu)) = - \sum_{n=0}^{\infty} \beta_n \left(\frac{\alpha_s}{\pi} \right)^{n+1}$$

- Coupling α_s evolves with scale μ as given by RGE \rightarrow LO behavior driven by β_0
- $\beta_0^{\text{QED}} < 0 \implies$ strongly coupled at large energies, UV divergent
- $\beta_0^{\text{QCD}} > 0 \implies$ weakly coupled at large energies, IR divergent

QCD and collider physics

Asymptotic freedom and pQCD



- Running coupling given by Renormalization Group Equation (RGE)

$$\alpha_s(\mu) = \frac{1}{\beta_0 \log\left(\frac{\mu^2}{\Lambda_{\text{QCD}}^2}\right)}$$

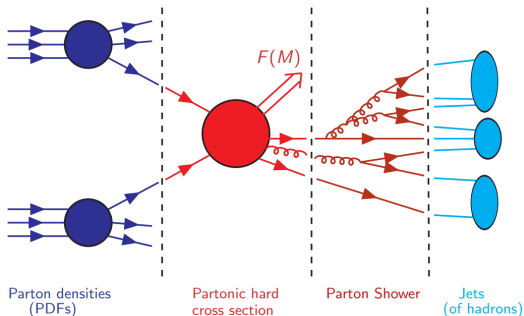
- β_0 LO of the β function, is > 0
- Λ_{QCD} , parameter that defines value of the coupling at large scales

QCD is weakly coupled for $\mu \gg \Lambda_{\text{QCD}} \rightarrow$ asymptotically free

Perturbative Quantum Chromodynamics (pQCD)

QCD and collider physics

Hadronic processes and factorization

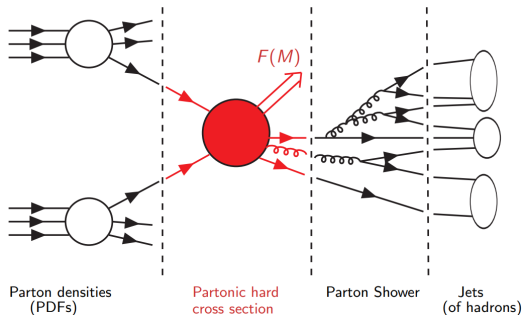


Compute hadronic cross sections is a **hard problem** \rightarrow **QCD Factorization**

$$\sigma^F(p_1, p_2) = \int_0^1 dx_1 dx_2 f_\alpha(x_1, \mu_F^2) * f_\beta(x_2, \mu_F^2) * \hat{\sigma}_{\alpha\beta}^F(x_1 p_1, x_2 p_2, \alpha_s(\mu_R^2), \mu_F^2)$$

QCD and collider physics

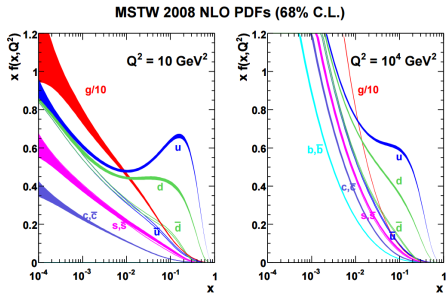
Hadronic processes and factorization



- Parton densities (PDFs) $f_a(x_i, \mu_F^2)$: non perturbative but universal
- Partonic cross section $\hat{\sigma}_{\alpha\beta}^F$: process dependent but computable as perturbative series in α_s

QCD and collider physics

Parton densities

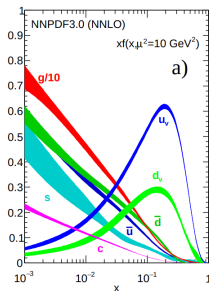


- Hadrons made of partonic objects \longrightarrow non perturbative physics
- Interactions take place only at partonic level

Parton Distribution Functions: probability distribution of finding a particular parton (u , d , ..., g) carrying a fraction x of the proton's momentum

QCD and collider physics

Parton densities

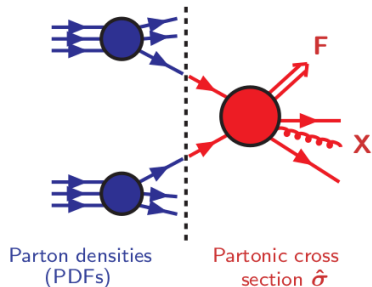


- Each parton has a different PDF $\rightarrow u(x), d(x), \dots, g(x)$
- PDFs can not predicted and yet can not measured \rightarrow extracted from data
- The N³PDF project: ML for PDFs determination

QCD and collider physics

Partonic cross section and pQCD

- Born cross section is the leading-order (LO) term of the perturbative series
- $\sigma^{(1)}, \sigma^{(2)}, \sigma^{(3)}$ are the NLO, NNLO, N3LO corrections



$$\hat{\sigma} = \sigma^{\text{Born}} \left(1 + \alpha_s \sigma^{(1)} + \alpha_s^2 \sigma^{(2)} + \alpha_s^3 \sigma^{(3)} + \dots \right)$$

Lower order predictions strongly depend on the auxiliary / unphysical scales

Need higher order corrections to increase theoretical accuracy!

QCD and collider physics

LHC phenomenology

Predictions made by the Standard Model

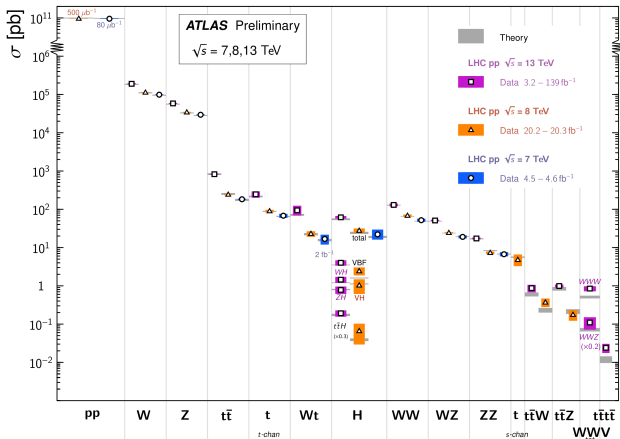
- Eight gluons and three generations of quarks, described by the $SU(3)$ gauge group, mixing angles described by CKM matrix
- Three generations of charged leptons and three generations of massive neutrinos mixing angles described by PMNS matrix
- The photon and the massive W^\pm and Z bosons
- A scalar Higgs boson, responsible for the electroweak symmetry breaking

QCD and collider physics

LHC phenomenology

Standard Model Total Production Cross Section Measurements

Status: July 2021



QCD and collider physics

LHC phenomenology

Main processes studied at the LHC

- Deep inelastic scattering
- Drell-Yan lepton pair production
- Higgs boson production through gluon fusion

Part II

All order resummation

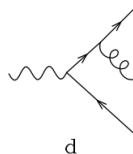
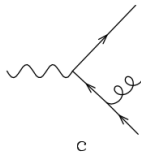
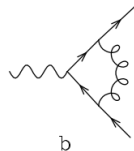
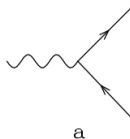
Resummation in QCD

The need for resummation

Resummation in QCD

Higher order corrections

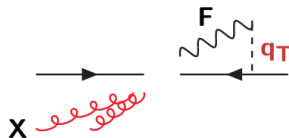
- 1 Calculation of higher order corrections is **not an easy task** due to **infrared (IR) soft and collinear singularities**
- 2 Final state singularities **cancel** by combining real and virtual contributions
- 3 Initial state collinear singularities **factorized** inside the PDFs



Resummation in QCD

q_\perp resummation

- Cancellation only works in completely inclusive final states
- Study the differential q_\perp distribution
 $h_1(p_1) + h_2(p_2) \longrightarrow F(M, \mathbf{q}_\perp) + X$



$$\int_0^{Q_\perp^2} dq_\perp^2 \frac{d\hat{\sigma}}{dq_\perp^2} \sim c_0 + \alpha_s(c_{12}L^2 + c_{11}L + c_{10}) + \dots, \quad \text{where} \quad L = \ln(M^2/q_\perp^2)$$

$\alpha_s L^2$	$\alpha_s L$	\dots	$\mathcal{O}(\alpha_s)$
$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	\dots	$\mathcal{O}(\alpha_s^2)$
\dots	\dots	\dots	\dots
$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	\dots	$\mathcal{O}(\alpha_s^n)$
dominant logs	\dots	\dots	\dots

Truncated fixed-order predictions \rightarrow enhanced $\alpha_s^n \ln^m(M^2/q_\perp^2)$ appear

Resummation in QCD

q_\perp resummation

Separate partonic q_\perp distribution as follows

$$\frac{d\hat{\sigma}_{ab}}{dq_\perp^2} = \left[\frac{d\hat{\sigma}_{ab}^{(\text{res.})}}{dq_\perp^2} \right]_{\text{l.a.}} + \left[\frac{d\hat{\sigma}_{ab}^{(\text{fin.})}}{dq_\perp^2} \right]_{\text{f.o.}}, \quad \text{such that}$$

$$\int_0^{q_\perp^2} dq_\perp^2 \frac{d\hat{\sigma}_{ab}^{(\text{res.})}}{dq_\perp^2} \sim \sum \alpha_s^n \log^m \left(\frac{M^2}{q_\perp^2} \right) \quad \text{for } q_\perp \rightarrow 0$$
$$\lim_{q_\perp \rightarrow 0} \int_0^{q_\perp^2} dq_\perp^2 \frac{d\hat{\sigma}_{ab}^{(\text{fin.})}}{dq_\perp^2} = 0$$

Resummed and finite components can be matched (LL+LO, NLL+NLO, NNLO+NNLL, ...) to have uniform accuracy in a wide range of q_\perp

Resummation in QCD

q_\perp resummation

Resummation holds in impact parameter space b

$$\frac{d\hat{\sigma}_{ab}^{(\text{res.})}}{dq_\perp^2} = \frac{M^2}{\hat{s}} \int db \frac{b}{2} J_0(bq_\perp) \mathcal{W}_{ab}(b, M)$$

with \mathcal{W}_{ab} also expressed in Mellin space (with respect to $z = M^2/\hat{s}$)

$$\mathcal{W}_N(b, M) = \mathcal{H}_N(\alpha_s) \times \exp\{\mathcal{G}_N(\alpha_s, L)\} \quad \text{being} \quad L \equiv \log(M^2 b^2)$$

- Large logarithms exponentiated in the universal Sudakov form factor $\mathcal{G}_N(\alpha_s, L)$
- Constant (b-independent) terms factorized in the process dependent hard factor $\mathcal{H}_N(\alpha_s)$

Resummation in QCD

q_\perp resummation

Sudakov factor \mathcal{G}_N and hard coefficient \mathcal{H}_N can be expanded as perturbative series in α_s

$$\mathcal{G}_N(\alpha_s, L) = L g^{(1)}(\alpha_s L) + g^{(2)}(\alpha_s L) + \frac{\alpha_s}{\pi} g^{(3)}(\alpha_s L) + \dots$$

$$\mathcal{H}_N(\alpha_s) = 1 + \alpha_s \mathcal{H}^{(1)} + \alpha_s^2 \mathcal{H}^{(2)} + \dots$$

For each new order implement a factor of \mathcal{G}_N and Hard \mathcal{H}_N

$$\text{LL}(\sim \alpha_s^n L^{n+1}) : g^{(1)}, \hat{\sigma}^{(0)}$$

$$\text{NLL}(\sim \alpha_s^n L^n) : g^{(2)}, \mathcal{H}^{(1)}$$

$$\text{NNLL}(\sim \alpha_s^n L^{n-1}) : g^{(3)}, \mathcal{H}^{(2)}$$

Each term $g^{(i)}$ and $\mathcal{H}^{(i)}$ in the series becomes increasingly complicated

Current codes able to produce only up to NNLL predictions!

Resummation in QCD

q_\perp resummation

Resummation holds in impact parameter space b

$$\frac{d\hat{\sigma}_{ab}^{(\text{res.})}}{dq_\perp^2} = \frac{M^2}{\hat{s}} \int db \frac{b}{2} J_0(bq_\perp) \mathcal{W}_{ab}(b, M)$$

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Part III

HTurbo numerical implementation

HqT and HRes

Resummation for Higgs differential distribution

- Meaningful description of exclusive final states requires resummed predictions
- Resummation is combined with the finite component

$$\sigma_{N(NLL)}^{(\text{res.})} = \hat{\sigma}_{\text{LO}}^{\text{H}} \times \mathcal{H} \times \exp \mathcal{G}$$

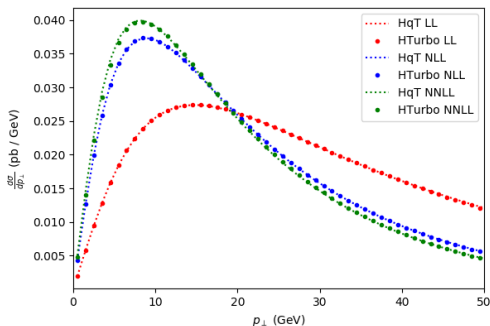
$$\sigma_{N(LO)}^{(\text{asy.})} = \hat{\sigma}_{\text{LO}}^{\text{H}} \times \mathcal{H} \times \exp \mathcal{G}$$

NNLL predictions can take up to 24h \longrightarrow need for **fast numerical implementations**

HqT and HRes

Predictions for Higgs q_{\perp} distribution

- q_{\perp} resummation implemented in numerical codes HqT and HRes [Catani, de Florian, Ferrera, Grazzini, Tommasini]
- Higher order accuracy require **high computation times**
- Codes producing fast and accurate predictions are needed for precision era of the LHC



HTurbo

Starting point DYTurbo

Numerical code **DYTurbo** [Camarda et al.] ref. at [1910.07049](#), fast and precise q_\perp resummation and several improvements for Drell-Yan ($h_1 h_2 \rightarrow V + X \rightarrow l^+ l^- + X$)

First goal: set up a numerical code for Higgs boson production starting from **DYTurbo**

- Set LO amplitude $gg \rightarrow H$
- Set Sudakov and Hard coefficients for Higgs production
- Compare with **HRes** and **HqT**

Final goal: extend theoretical accuracy up to $N^3\text{LL}+N^3\text{LO}$

HTurbo

Starting point DYTurbo

Sudakov factor \mathcal{G}_N and hard coefficient \mathcal{H}_N can be expanded as perturbative series in α_s

$$\mathcal{G}_N(\alpha_s, L) = L g^{(1)}(\alpha_s L) + g^{(2)}(\alpha_s L) + \frac{\alpha_s}{\pi} g^{(3)}(\alpha_s L) + \dots$$

$$\mathcal{H}_N(\alpha_s) = 1 + \alpha_s \mathcal{H}^{(1)} + \alpha_s^2 \mathcal{H}^{(2)} + \dots$$

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$$\text{LL}(\sim \alpha_s^n L^{n+1}) : g^{(1)}, \hat{\sigma}^{(0)}$$

$$\text{NLL}(\sim \alpha_s^n L^n) : g^{(2)}, \mathcal{H}^{(1)}$$

$$\text{NNLL}(\sim \alpha_s^n L^{n-1}) : g^{(3)}, \mathcal{H}^{(2)}$$

Start by building predictions up to NNLO+NNLL, then add **N³LO+N³LL**

HTurbo

Code optimization

Reimplementation of **HqT** and **HRes** for q_T -resummation

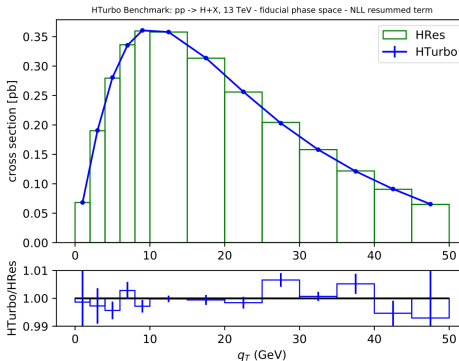
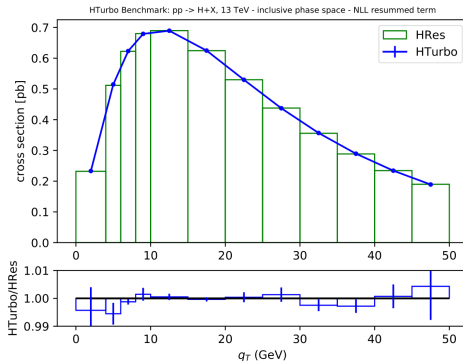
- **C++** structure with **Fortran** interfaces → Multi-threading
- Optimization in the integration routines / integral transforms
 - Factorize boson and decay kinematics
 - Gauss-Legendre quadrature rules (1-dim.)
 - Vegas/Cuhre through **Cuba** (multi-dim.)

Comparison **HRes** and **HTurbo** - speed performance

Predictions	HRes	HTurbo
resummed NNLL	10h	10'
combined NNLO+NNLL	20h	1h

Results

Comparison HTurbo and HRes - NLL resummed

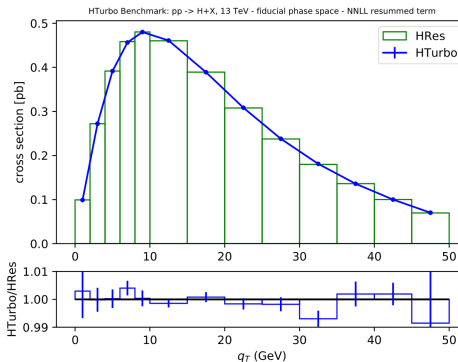
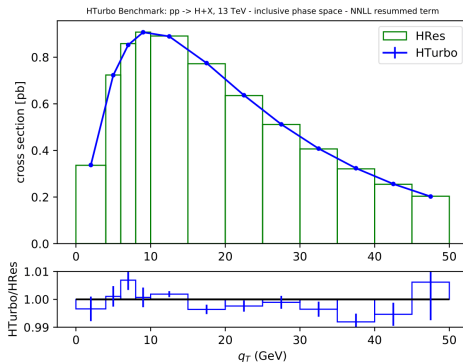


- Cross section for fully inclusive (LHS) and fiducial (RHS) phase space



Results

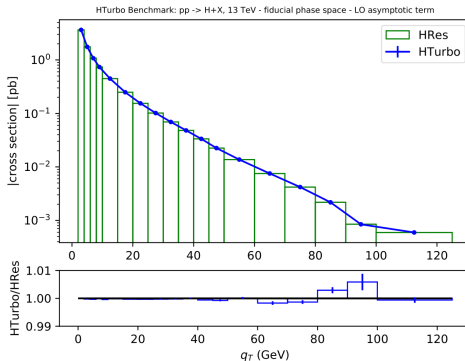
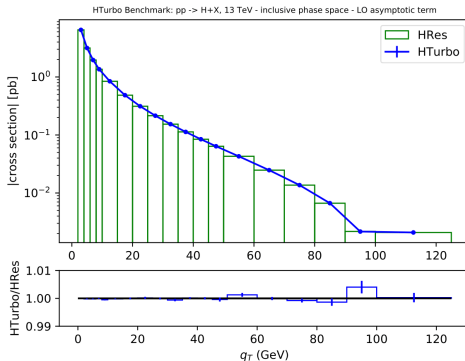
Comparison HTurbo and HRes - NNLL resummed



- Cross section for fully inclusive (LHS) and fiducial (RHS) phase space ✓
- CM energy $\sqrt{s} = 13$ GeV and PDF set NNPDF31_nnlo_as_0118 PDF set

Results

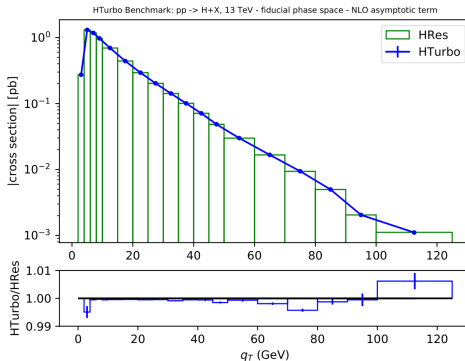
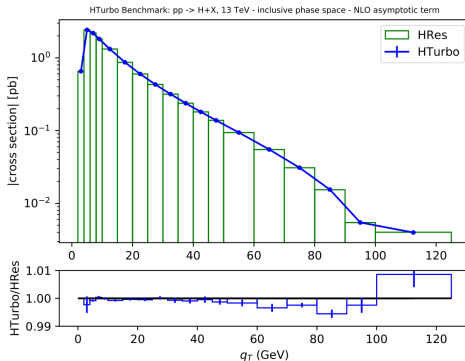
Comparison HTurbo and HRes - LO asymptotic



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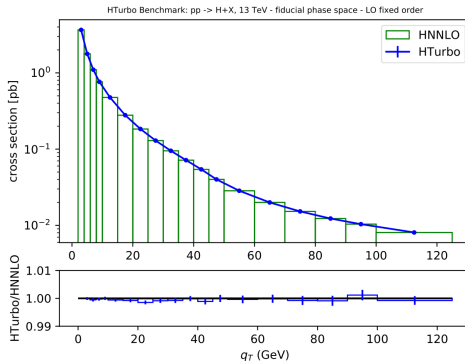
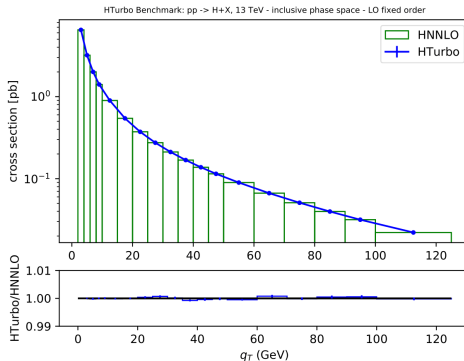
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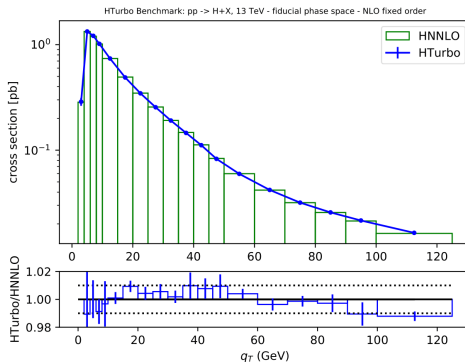
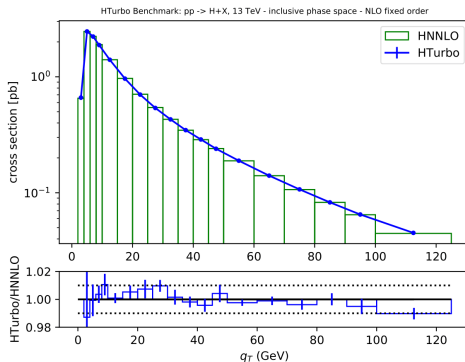
Comparison HTurbo and HRes - LO fixed-order



- Cross section for fully inclusive (LHS) and fiducial (RHS) phase space ✓
- CM energy $\sqrt{s} = 13$ GeV and PDF set NNPDF31_nlo_as_0118 PDF set

Results

Comparison HTurbo and HRes - NLO fixed-order



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- CM energy $\sqrt{s} = 13$ GeV and PDF set NNPDF31_nnlo_as_0118 PDF set

Summary & Conclusions

- ① Fast and accurate predictions are needed towards the precision era of the LHC
- ② Developing a novel numerical code, **HTurbo**, which implements q_\perp resummation for Higgs boson production
- ③ HTurbo is faster than any of the existing codes
- ④ Outlook of thesis work:
 - Add $N^3\text{LO}+N^3\text{LL}$ prediction
 - Perform phenomenological studies comparing with LHC data

Discussion & next steps

- ① Fast and accurate predictions are needed towards the precision era of the LHC
- ② Developing a novel numerical code, **HTurbo**, which implements q_{\perp} resummation for Higgs boson production
- ③ HTurbo is faster than any of the existing codes
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Thank you!



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