Two-mode squeezed states in cavity optomechanics via engineering of a single reservoir

PhD course - Quantum coherent phenomena
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UNIVERSITÀ DEGLI STUDI DI MILANO

Istituto Nazionale di Fisica Nucleare

European Research Council

Outline

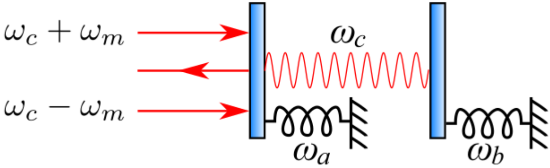
Introduction

1. Generation and detection of entangled states of macroscopic mechanical oscillators
2. Reservoir engineering $\longrightarrow$ Two-mode squeezed states
3. Quantum optomechanics $\longrightarrow$ describe mesoscopic systems

Introduction

System representation

- Two mechanical oscillators with resonance frequencies ω_a, ω_b
- Dispersively coupled g_a, g_b to a common cavity ω_c
- Radiation pressure forces inside the cavity lead motion of the mirrors become highly entangled



Quantum optomechanics $\longrightarrow$ Hamiltonian describing optical and mechanical modes with same formalism

$$\hat{\mathcal{H}} = \omega_a \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b} + \omega_c \hat{c}^\dagger \hat{c} + g_a(\hat{a} + \hat{a}^\dagger)\hat{c}^\dagger \hat{c} \\ + g_b(\hat{b} + \hat{b}^\dagger)\hat{c}^\dagger \hat{c} + \hat{H}_{\text{drive}} + \hat{H}_{\text{diss}},$$

Under usual approximations , obtain the master equation

$$\dot{\rho} = -i[\hat{\mathcal{H}}', \rho] + \gamma_a(\bar{n}_a + 1)\mathcal{D}[\hat{a}]\rho + \gamma_a \bar{n}_a \mathcal{D}[\hat{a}^\dagger]\rho \\ + \gamma_b(\bar{n}_b + 1)\mathcal{D}[\hat{b}]\rho + \gamma_b \bar{n}_b \mathcal{D}[\hat{b}^\dagger]\rho + \kappa \mathcal{D}[\hat{c}]\rho,$$

Being $\mathcal{H}' = \mathcal{H} - \mathcal{H}_{\text{diss}}$, and $\mathcal{D}[\hat{c}]$ the dispersive superoperator
Only dissipation term for $\hat{c}$ $\longrightarrow$ Assuming cavity is at T $= 0$

Define the Bogoliuov mechanical modes in terms of the modes $\hat{a}, \hat{b}$

$$\hat{\beta}_1 = \hat{a}\cosh r + \hat{b}^\dagger \sinh r,$$
$$\hat{\beta}_2 = \hat{b}\cosh r + \hat{a}^\dagger \sinh r.$$

being r the squeezing parameter
Work in rotating frame with respect to the Hamiltonian

$$\hat{H}_0 = (\omega_a - \Omega)\hat{a}^\dagger \hat{a} + (\omega_b + \Omega)\hat{b}^\dagger \hat{b} + \omega_c \hat{c}^\dagger \hat{c},$$

where choice of detuning Ω is such that collective mechanical quadratures $\hat{X}_\pm$, $\hat{P}_\pm$ rotate in a non-trivial way

Reservoir engineering strategies

2-mode squeezed state

Define the 2-mode squeezed as $|r\rangle_2 = \hat{S}_2(r)|0,0\rangle$, being the squeezing operator

$$\hat{S}_2(r) \equiv \exp[r(\hat{a}\hat{b} - \hat{a}^\dagger \hat{b}^\dagger)]$$

such that $[\hat{S}_2(r)\hat{a}\hat{S}_2^\dagger(r)]|r\rangle_2 = [\hat{S}_2(r)\hat{b}\hat{S}_2^\dagger(r)]|r\rangle_2 = 0$

Therefore, $\hat{\beta}_1 = \hat{S}_2(r)\hat{a}\hat{S}_2^\dagger(r)$, $\hat{\beta}_2 = \hat{S}_2(r)\hat{b}\hat{S}_2^\dagger(r)$ and their ground state is the two-mode squeezed state with squeezing parameter r

Reservoir engineering strategies

Note on Quantum Optomechanics

Linearized Hamiltonian with 2-tone laser with amplitudes $\alpha_{\pm}$

$$\mathcal{H} = \hbar \mathrm{g}_{+} \left(a^{\dagger} b^{\dagger} + ab\right) + \hbar \mathrm{g}_{-} \left(a^{\dagger} b + ab^{\dagger}\right)$$

being $g_{\pm} = g_0 \alpha_{\pm}$

Study different cases

- $g_{-} = 0 \longrightarrow$ Sideband blue $\mathcal{H} = \hbar \mathrm{g} \left(a^{\dagger} b^{\dagger} + ab\right)$ "2 - mode squeezing"
- $g_{+} = 0 \longrightarrow$ Sideband red $\mathcal{H} = \hbar \mathrm{g} \left(a^{\dagger} b + ab^{\dagger}\right)$ "beam - splitter"
- $g_{-} = g_{+} = g \longrightarrow \mathcal{H} = \hbar \mathrm{g} \, (a + a^{\dagger})(b + b^{\dagger})$ "back-action evading"

Reservoir engineering strategies

Generate the 2-mode squeezed state

- i) Two cavity modes to independently cool the Bogoliubov modes (beam splitter $\hat{\beta}_i^\dagger \hat{c}_i$)
- ii) Couple the cavity to one Bogoliubov mode ($\hat{\beta}_1$), and then this one to $\hat{\beta}_2$ through ($\hat{\beta}_1^\dagger \hat{\beta}_2$)
- iii) Couple the cavity to sum of the Bogoliubov modes , then the sum to the difference (swap interaction $\hat{\beta}_{\text{sum}}^\dagger \hat{\beta}_{\text{diff}}$ allows diff to cool).

$$\hat{\beta}_{\text{sum}} = \frac{1}{\sqrt{2}}(\hat{\beta}_1 + \hat{\beta}_2)$$
$$\hat{\beta}_{\text{diff}} = \frac{1}{\sqrt{2}}(\hat{\beta}_1 - \hat{\beta}_2)$$

Cooling $\hat{\beta}_{\text{sum}}$ and $\hat{\beta}_{\text{diff}}$ is equivalent to cool $\hat{\beta}_1$ and $\hat{\beta}_2$ ✓
Direct coupling not needed, just difference in their resonance frequencies ✓

Reservoir engineering strategies

Hamiltonian

Hamiltonian in terms of the Bogoliubov modes

$$\hat{\mathcal{H}} = \Omega(\hat{\beta}_1^\dagger \hat{\beta}_1 - \hat{\beta}_2^\dagger \hat{\beta}_2) + \mathcal{G}[(\hat{\beta}_1^\dagger + \hat{\beta}_2^\dagger)\hat{c} + \mathrm{H.c.}] + \hat{H}_{\mathrm{diss}},$$

where Ω is the effective oscillation frequency and $\mathcal{G}$ an effective optomechanical coupling.

Written in terms of the original operators,

$$\hat{\mathcal{H}} = \Omega(\hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b}) + G_+[(\hat{a} + \hat{b})\hat{c} + \mathrm{H.c.}] + G_-[(\hat{a} + \hat{b})\hat{c}^\dagger + \mathrm{H.c.}] + \hat{H}_{\mathrm{diss}}.$$

with couplings related by $\mathcal{G} \equiv \sqrt{G_-^2 - G_+^2}$ and $\tanh r \equiv G_+/G_-$

Implementation

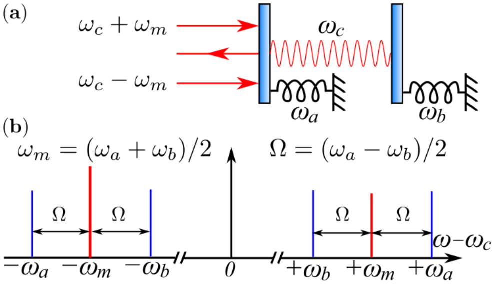
Different cases

$\hat{\mathcal{H}}$ is already implemented in conventional optomechanical setups.

Different cases depending on the optomechanical couplings relation

- Two-tone driving ($g_a = g_b$) $\longrightarrow$ two cavity drives are required
- Four-tone driving ($g_a \neq g_b$) $\longrightarrow$ four cavity drives are required
- Case similar ($g_a \sim g_b$) $\longrightarrow$ approximate with two cavity drives

Driving tones at $\omega_c \pm \omega_m$ being $\omega_m = (\omega_a + \omega_b)/2$



Apply our drive Hamiltonian

$$\hat{H}_{\text{drive}} = (\mathcal{E}_+^* e^{+i\omega_m t} + \mathcal{E}_-^* e^{-i\omega_m t}) e^{+i\omega_c t} \hat{c} + \text{H.c.}$$

Implementation

2 - tone driving ($g_a = g_b$)

Interaction picture with respect to $\hat{\mathcal{H}}_0 = \omega_m(\hat{a}^\dagger\hat{a} + \hat{b}^\dagger\hat{b}) + \omega_c\hat{c}^\dagger\hat{c}$ leads back to out desired Hamiltonian

$$\hat{\mathcal{H}} = \Omega(\hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b}) + G_+[(\hat{a} + \hat{b})\hat{c} + \text{H.c.}] \\ + G_-[(\hat{a} + \hat{b})\hat{c}^\dagger + \text{H.c.}] + \hat{H}_{\text{diss}}.$$

$$\text{where} \qquad \Omega = (\omega_a - \omega_b)/2$$
$$G_\pm = (g_a + g_b)\bar{c}_\pm/2$$

and $\bar{c}_\pm$ are the steady state amplitudes at the sidebands

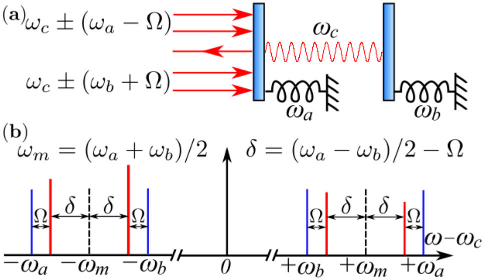
$$\bar{c}_\pm \equiv \langle\hat{c}_\pm\rangle_{\text{ss}} = \frac{i\mathcal{E}_\pm}{\pm i\omega_m - \kappa/2}.$$

Implementation

4 - tone driving ($g_a \neq g_b$)

Four different sideband processes involved

Driving tones applied with detuning of Ω from the sidebands ar $\omega_c \pm (\omega_a - \Omega)$ and $\omega_c \pm (\omega_b + \Omega)$



Implementation

4 - tone driving ($g_a \neq g_b$)

$$\hat{H}_{\text{drive}} = e^{+i\omega_c t}\hat{c}(\mathcal{E}_{1+}^* e^{+i(\omega_a-\Omega)t} + \mathcal{E}_{2+}^* e^{+i(\omega_b+\Omega)t} + \mathcal{E}_{1-}^* e^{-i(\omega_a-\Omega)t} + \mathcal{E}_{2-}^* e^{-i(\omega_b+\Omega)t}) + \text{H.c.}$$

steady-state amplitudes are now

$$\bar{c}_{k\pm} \equiv \langle \hat{c}_{k\pm} \rangle_{\text{ss}} = \frac{i\mathcal{E}_{k\pm}}{\pm i\omega_k - \kappa/2},$$

where we introduced notation for the drive detunings

$$\omega_1 = (\omega_a - \Omega)$$
$$\omega_2 = (\omega_b + \Omega)$$

Where we demand the strengths are "matched" as $\bar{c}_{1\pm}/\bar{c}_{2\pm} = g_b/g_a$

Adiabatic limit

Adiabatically eliminated master equation

- Assume the system responds rapidly to mechanical motion $k > \Omega, G_{\pm}$, but still in the regime $\omega_a, \omega_b \gg k$
- Simplify by getting rid of the cavity operator $\hat{c} = -2i\mathcal{G}(\hat{\beta}_1 + \hat{\beta}_2)/k$
- Obtain adiabatically eliminated master equation

$$\dot{\rho} = -i\Omega[\hat{\beta}_1^\dagger \hat{\beta}_1 - \hat{\beta}_2^\dagger \hat{\beta}_2, \rho] + \gamma_a(\bar{n}_a + 1)\mathcal{D}[\hat{a}]\rho + \gamma_a \bar{n}_a \mathcal{D}[\hat{a}^\dagger]\rho$$
$$+ \gamma_b (\bar{n}_b + 1) \mathcal{D}[\hat{b}]\rho + \gamma_b \bar{n}_b \mathcal{D}[\hat{b}^\dagger]\rho + \Gamma \mathcal{D}[\hat{\beta}_1 + \hat{\beta}_2]\rho,$$

with optomechanical damping rate

$$\Gamma \equiv \gamma \mathcal{C} \equiv \frac{4\mathcal{G}^2}{\kappa},$$

Easy to obtain steady state, and to measure entanglement and purity.

Adiabatic limit

Entanglement

Build a way of identify entanglement on a 2-mode system
Duan criterion $\longrightarrow$ define collective quadratures

$$\hat{X}_{\pm} = (\hat{X}_a \pm \hat{X}_b)/\sqrt{2},$$
$$\hat{P}_{\pm} = (\hat{P}_a \pm \hat{P}_b)/\sqrt{2},$$

as combination of the usual quadrature modes

$$\hat{X}_s = (\hat{s} + \hat{s}^{\dagger})/\sqrt{2}, \quad \hat{P}_s = -i(\hat{s} - \hat{s}^{\dagger})/\sqrt{2}.$$

Duan inequality states that a state for which

$$\langle \hat{X}_+^2 \rangle + \langle \hat{P}_-^2 \rangle < 1$$

is inseparable $\longrightarrow$ entangled!

Adiabatic limit
Entanglement

Quadratures can be written as function of the drive asymmetry

$$\begin{aligned}\langle \hat{X}_{\pm}^2\rangle = \langle \hat{P}_{\mp}^2\rangle &= \frac{\gamma}{\gamma+\Gamma}(\bar{n}+1/2) + \frac{\Gamma}{\gamma+\Gamma}\frac{e^{\mp 2r}}{2} \\ &= \frac{\gamma\kappa}{\gamma\kappa+4(G_-^2-G_+^2)}(\bar{n}+1/2) \\ &\quad + \frac{2(G_-\mp G_+)^2}{\gamma\kappa+4(G_-^2-G_+^2)}.\end{aligned}$$

Use also logarithmic negativity

$$E_{\mathcal{N}} = \max\{0, -\ln 2\eta\}$$

with η factor in terms of the covariance matrix *

Adiabatic limit

Purity

Purity defined as trace of the density matrix

$$\mu \equiv \mathrm{tr}[\rho^2]$$

As function of the covariance matrix *

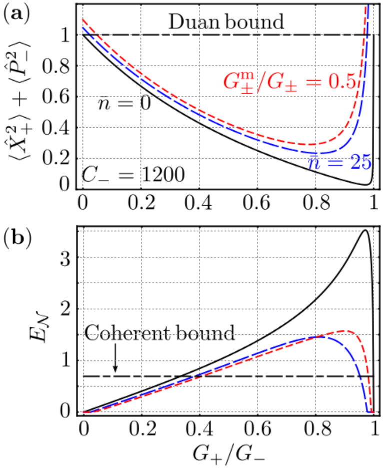
$$\mu = \frac{1}{4\sqrt{\det \mathbf{V}}}$$

Purity can be written as function of the drive asymmetry

$$\mu = \frac{(\gamma + \Gamma)^2}{[\gamma(1+2\bar{n}) + \Gamma]^2 + 4(1+2\bar{n})\gamma\Gamma \sinh^2 r}.$$

Adiabatic limit

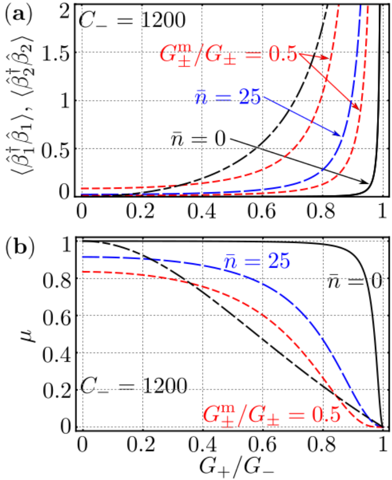
Entanglement



- Duan quantity and logarithmic negativity as function of the drive asymmetry (case $\gamma_a = \gamma_b$, $\bar{n}_a = \bar{n}_b$ and RWA)
- Solid curve with mechanical thermal occupation $\bar{n} = 0$ and no imperfections on effective coupling $G^m_\pm = 0$
- Thermal occupation and imperfections in the effective coupling lead to less entanglement

Adiabatic limit

Entanglement



- Steady state occupations and purity as function of the drive asymmetry
- Solid curve with mechanical thermal occupation $\bar{n} = 0$ and no imperfections on effective coupling $G^m_{\pm} = 0$
- Thermal occupation and imperfections in the effective coupling lead to less pure state

Experimental observability

Output spectrum

- Reconstructing covariance matrix is experimentally demanding
- Directly measuring quadratures is a hard problem
- Seek signature of entanglement in output spectrum

Spectrum as Fourier transform of expected value

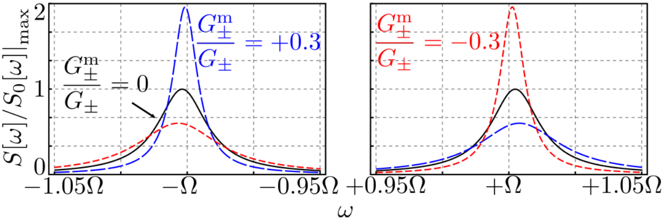
$$S[\omega] = \int dt\, e^{i\omega t} \langle \delta \hat{c}^{\dagger}_{\text{out}}(t) \delta \hat{c}_{\text{out}}(0) \rangle,$$

being $\delta \hat{c}_{\text{out}} = \hat{c}_{\text{out}} - < \hat{c}_{\text{out}} >$
Spectrum can be related to the occupation of modes

$$\int_{-\infty}^{0} S[\omega] d\omega = \int_{0}^{+\infty} S[\omega] d\omega$$
$$= 8\pi\kappa \frac{\mathcal{G}^2}{4\mathcal{G}^2 + \kappa(\kappa + \gamma)} \langle \hat{\beta}_i^{\dagger} \hat{\beta}_i \rangle,$$

Experimental observability

Output spectrum



- Centered around the detunings from the cavity resonance frequency
- Solid black curve without imperfections $G^m_{\pm}/G_{\pm} = 0$
- Imperfections on the effective couplings described by

$$S[\pm\Omega] = \gamma\kappa \frac{(G_- \pm G^{\mathrm{m}}_-)^2 \bar{n} + (G_+ \pm G^{\mathrm{m}}_+)^2 (1+\bar{n})}{[G_-^2 - (G^{\mathrm{m}}_-)^2 - G_+^2 + (G^{\mathrm{m}}_+)^2]^2}$$

Experimental observability

Output spectrum

- Experimental work realized in "Stabilized entanglement of massive mechanical oscillators", Nature, 2018.
- Measure output spectrum and reconstruct quadratures to identify entanglement

Conclusions

1. Configuring a three-mode optomechanical system such as the steady state includes highly pure and highly entangled two-mode squeezed state.
2. Symmetry on the steady-state makes it attractive for implementation of continuous-variable teleportation protocols
3. Problem of unequal single-photon optomechanical couplings solved by using four-tone driving scheme
4. Proposal implementable for existing technology

Back up

Thermal occupation

1. Occupation (photons) at $\omega_c \sim 10^{10}\mathrm{Hz}$ and $T \sim 300K$

$$\bar{n}(T) = \frac{1}{e^{\frac{\hbar\omega_c}{K_B T}} - 1} \simeq 0$$

2. Occupation (phonons) $\longrightarrow \omega_m \sim 10\ \mathrm{KHz}, 1\mathrm{GHz}$ and $T \sim 300K$

$$\bar{n}_{\mathrm{phonons}}(T) = \frac{1}{e^{\frac{\hbar\omega_c}{K_B T}} - 1} \gg 1$$