Two-mode squeezed states in cavity optomechanics via engineering of a single reservoir

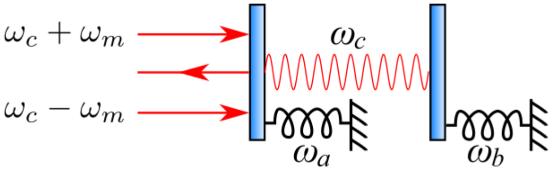
PhD course - Quantum coherent phenomena
Milan, October 2020

Outline

Introduction

System representation

- Two mechanical oscillators with resonance frequencies ω_a, ω_b
- Dispersively coupled with rates g_a, g_b to a common cavity ω_c
- Apply radiation pressure forces inside the cavity leading to generate entangled motion of the mirrors



Quantum optomechanics $\longrightarrow$ describing optical and mechanical modes with same formalism

$$\hat{\mathcal{H}} = \omega_a \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b} + \omega_c \hat{c}^\dagger \hat{c} + g_a(\hat{a} + \hat{a}^\dagger)\hat{c}^\dagger \hat{c} \\ + g_b(\hat{b} + \hat{b}^\dagger)\hat{c}^\dagger \hat{c} + \hat{H}_{\text{drive}} + \hat{H}_{\text{diss}},$$

under usual approximations, obtain the master equation

$$\dot{\rho} = -i[\hat{\mathcal{H}}', \rho] + \gamma_a(\bar{n}_a + 1)\mathcal{D}[\hat{a}]\rho + \gamma_a \bar{n}_a \mathcal{D}[\hat{a}^\dagger]\rho \\ + \gamma_b(\bar{n}_b + 1)\mathcal{D}[\hat{b}]\rho + \gamma_b \bar{n}_b \mathcal{D}[\hat{b}^\dagger]\rho + \kappa \mathcal{D}[\hat{c}]\rho,$$

being $\mathcal{H}' = \mathcal{H} - \mathcal{H}_{\text{diss}}$, and $\mathcal{D}[\hat{c}]$ the dispersive superoperator
Only dissipation term for $\hat{c} \longrightarrow$ Assuming zero thermal occupation

Reservoir engineering strategies

Bogoliubov operators

Define the Bogoliuov mechanical modes in terms of the modes $\hat{a}, \hat{b}$

$$\hat{\beta}_1 = \hat{a}\cosh r + \hat{b}^\dagger \sinh r,$$
$$\hat{\beta}_2 = \hat{b}\cosh r + \hat{a}^\dagger \sinh r.$$

Where r is the squeezing parameter.

Work in rotating frame with respect to the Hamiltonian:

$$\hat{H}_0 = (\omega_a - \Omega)\hat{a}^\dagger\hat{a} + (\omega_b + \Omega)\hat{b}^\dagger\hat{b} + \omega_c\hat{c}^\dagger\hat{c},$$

Hamiltonian in terms of the Bogoliubov modes

$$\hat{\mathcal{H}} = \Omega(\hat{\beta}_1^\dagger \hat{\beta}_1 - \hat{\beta}_2^\dagger \hat{\beta}_2) + \mathcal{G}[(\hat{\beta}_1^\dagger + \hat{\beta}_2^\dagger)\hat{c} + \mathrm{H.c.}] + \hat{H}_{\mathrm{diss}},$$

where Ω is the effective oscillation frequency and $\mathcal{G}$ an effective optomechanical coupling.

Written in terms of the original operators,

$$\hat{\mathcal{H}} = \Omega(\hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b}) + G_+[(\hat{a} + \hat{b})\hat{c} + \mathrm{H.c.}] \\ + G_-[(\hat{a} + \hat{b})\hat{c}^\dagger + \mathrm{H.c.}] + \hat{H}_{\mathrm{diss}}.$$

with couplings related by $\mathcal{G} \equiv \sqrt{G_-^2 - G_+^2}$ and $\tanh r \equiv G_+/G_-$

Define the 2-mode squeezed state as $|r\rangle_2 = \hat{S}_2(r)\,|0,0\rangle$, being the squeezing operator

$$\hat{S}_2(r) \equiv \exp[r(\hat{a}\hat{b} - \hat{a}^\dagger \hat{b}^\dagger)]$$

such that $[\hat{S}_2(r)\hat{a}\hat{S}_2^\dagger(r)]\,|r\rangle_2 = [\hat{S}_2(r)\hat{b}\hat{S}_2^\dagger(r)]\,|r\rangle_2 = 0$

Therefore, $\hat{\beta}_1 = \hat{S}_2(r)\hat{a}\hat{S}_2^\dagger(r)$, $\hat{\beta}_2 = \hat{S}_2(r)\hat{b}\hat{S}_2^\dagger(r)$ and their ground state is the two-mode squeezed state with squeezing parameter r.

Linearized Hamiltonian with 2-tone laser with amplitudes α_+ and α_-

$$\mathcal{H} = \hbar \mathrm{g}_+ \left(a^\dagger b^\dagger + ab\right) + \hbar \mathrm{g}_- \left(a^\dagger b + ab^\dagger\right)$$

being $g_\pm = g_0 \alpha_\pm$

Study different cases

- $g_- = 0 \longrightarrow$ Sideband blue $\mathcal{H} = \hbar \mathrm{g} \left(a^\dagger b^\dagger + ab\right)$ "2 - mode squeezing"
- $g_+ = 0 \longrightarrow$ Sideband red $\mathcal{H} = \hbar \mathrm{g} \left(a^\dagger b + ab^\dagger\right)$ "beam - splitter"
- $g_- = g_+ = g \longrightarrow \mathcal{H} = \hbar \mathrm{g} \left(a + a^\dagger\right)\left(b + b^\dagger\right)$ "back-action evading"

Implementation

Different cases

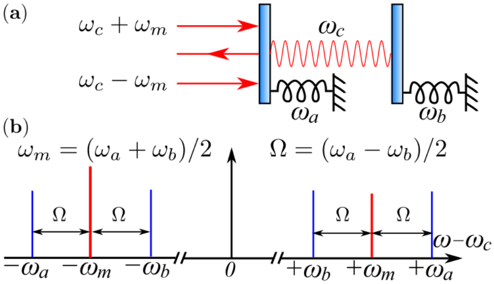
$$\hat{\mathcal{H}} = \Omega(\hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b}) + G_+[(\hat{a} + \hat{b})\hat{c} + \text{H.c.}] \\ + G_-[(\hat{a} + \hat{b})\hat{c}^\dagger + \text{H.c.}] + \hat{H}_{\text{diss}}.$$

already implemented in conventional optomechanical setups.

Different cases depending on the optomechanical couplings relation:

- Two-tone driving ($g_a = g_b$) $\longrightarrow$ two cavity drives are required
- Four-tone driving ($g_a \neq g_b$) $\longrightarrow$ four cavity drives are required
- Case similar ($g_a \sim g_b$) $\longrightarrow$ approximate with two cavity drives

Driving tones at $\omega_c \pm \omega_m$ being $\omega_m = (\omega_a + \omega_b)/2$



Apply our drive Hamiltonian

$$\hat{H}_{\text{drive}} = (\mathcal{E}_+^* e^{+i\omega_m t} + \mathcal{E}_-^* e^{-i\omega_m t}) e^{+i\omega_c t} \hat{c} + \text{H.c.}$$

Implementation

2 - tone driving ($g_a = g_b$)

Interaction picture with respect to $\hat{\mathcal{H}}_0 = \omega_m(\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b}) + \omega_c \hat{c}^\dagger \hat{c}$ leads back to out desired Hamiltonian

$$\hat{\mathcal{H}} = \Omega(\hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b}) + G_+[(\hat{a} + \hat{b})\hat{c} + \mathrm{H.c.}] + G_-[(\hat{a} + \hat{b})\hat{c}^\dagger + \mathrm{H.c.}] + \hat{H}_{\mathrm{diss}}.$$

$$\text{where} \qquad \Omega = (\omega_a - \omega_b)/2$$
$$G_\pm = (g_a + g_b)\bar{c}_\pm/2$$

and $\bar{c}_\pm$ are the steady state amplitudes at the sidebands

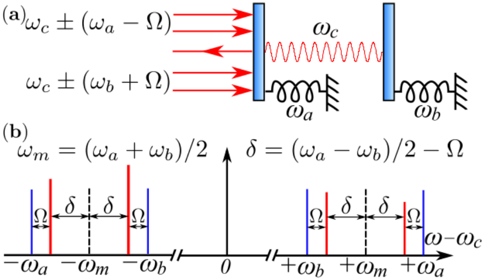
$$\bar{c}_\pm \equiv \langle \hat{c}_\pm \rangle_{\mathrm{ss}} = \frac{i\mathcal{E}_\pm}{\pm i\omega_m - \kappa/2}.$$

Implementation

4 - tone driving ($g_a \neq g_b$)

Four different sideband processes involved

Driving tones applied with detuning of Ω from the sidebands at $\omega_c \pm (\omega_a - \Omega)$ and $\omega_c \pm (\omega_b + \Omega)$



Implementation

4 - tone driving ($g_a \neq g_b$)

$$\hat{H}_{\text{drive}} = e^{+i\omega_c t}\hat{c}(\mathcal{E}^*_{1+}e^{+i(\omega_a-\Omega)t} + \mathcal{E}^*_{2+}e^{+i(\omega_b+\Omega)t} + \mathcal{E}^*_{1-}e^{-i(\omega_a-\Omega)t} + \mathcal{E}^*_{2-}e^{-i(\omega_b+\Omega)t}) + \text{H.c.}$$

steady-state amplitudes are now

$$\bar{c}_{k\pm} \equiv \langle \hat{c}_{k\pm}\rangle_{\text{ss}} = \frac{i\mathcal{E}_{k\pm}}{\pm i\omega_k - \kappa/2},$$

where we introduced notation for the drive detunings

$$\omega_1 = (\omega_a - \Omega)$$
$$\omega_2 = (\omega_b + \Omega)$$
$$G_\pm = (g_a\bar{c}_{1\pm} + g_b\bar{c}_{2\pm})/2$$

Adiabatic limit

Adiabatically eliminated master equation

- Assume the system responds fast to mechanical motion ($k > \Omega, G_{\pm}$)
- Simplify by getting rid of the cavity operator $\hat{c} = -2i\mathcal{G}(\hat{\beta}_1 + \hat{\beta}_2)/k$
- Obtain adiabatically eliminated master equation

$$\dot{\rho} = -i\Omega[\hat{\beta}_1^\dagger \hat{\beta}_1 - \hat{\beta}_2^\dagger \hat{\beta}_2, \rho] + \gamma_a(\bar{n}_a + 1)\mathcal{D}[\hat{a}]\rho + \gamma_a \bar{n}_a \mathcal{D}[\hat{a}^\dagger]\rho$$
$$+ \gamma_b (\bar{n}_b + 1) \mathcal{D}[\hat{b}]\rho + \gamma_b \bar{n}_b \mathcal{D}[\hat{b}^\dagger]\rho + \Gamma \mathcal{D}[\hat{\beta}_1 + \hat{\beta}_2]\rho,$$

with optomechanical damping rate

$$\Gamma \equiv \gamma \mathcal{C} \equiv \frac{4\mathcal{G}^2}{\kappa},$$

easy to obtain steady state, and to measure entanglement and purity.

Adiabatic limit

Entanglement

Build a way of identify entanglement on a 2-mode system
Duan criterion $\longrightarrow$ define collective quadratures

$$\hat{X}_{\pm} = (\hat{X}_a \pm \hat{X}_b)/\sqrt{2},$$
$$\hat{P}_{\pm} = (\hat{P}_a \pm \hat{P}_b)/\sqrt{2},$$

as combination of the usual quadrature modes

$$\hat{X}_s = (\hat{s} + \hat{s}^\dagger)/\sqrt{2}, \quad \hat{P}_s = -i(\hat{s} - \hat{s}^\dagger)/\sqrt{2}.$$

Duan inequality states that a state for which

$$\langle \hat{X}_+^2 \rangle + \langle \hat{P}_-^2 \rangle < 1$$

is inseparable $\longrightarrow$ entangled!

Adiabatic limit

Entanglement

Quadratures can be written as function of the drive asymmetry

$$
\begin{aligned}
\langle \hat{X}_{\pm}^2 \rangle = \langle \hat{P}_{\mp}^2 \rangle &= \frac{\gamma}{\gamma + \Gamma}(\bar{n} + 1/2) + \frac{\Gamma}{\gamma + \Gamma}\frac{e^{\mp 2r}}{2} \\
&= \frac{\gamma\kappa}{\gamma\kappa + 4(G_-^2 - G_+^2)}(\bar{n} + 1/2) \\
&\quad + \frac{2(G_- \mp G_+)^2}{\gamma\kappa + 4(G_-^2 - G_+^2)}.
\end{aligned}
$$

Use also logarithmic negativity $E_{\mathcal{N}} = \max\{0, -\ln 2\eta\}$, with η factor in terms of the covariance matrix *

Purity defined as trace of the density matrix

$$\mu \equiv \mathrm{tr}[\rho^2]$$

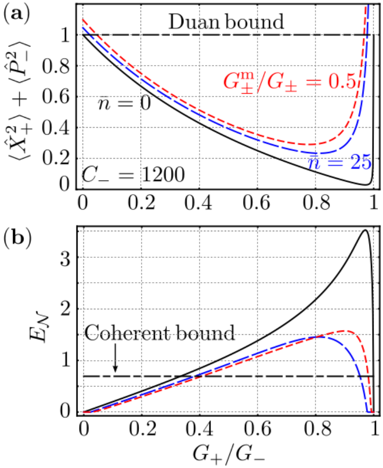
and as function of the covariance matrix *

$$\mu = \frac{1}{4\sqrt{\det \mathbf{V}}}$$

Again, demanding from experimental point of view.

Adiabatic limit

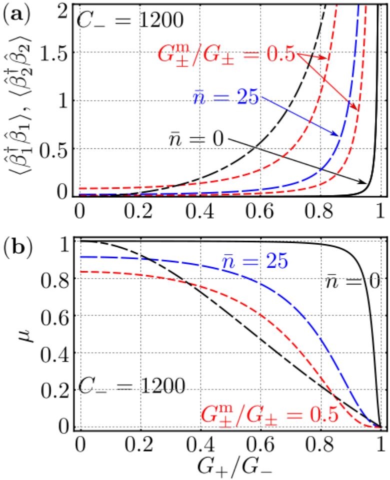
Entanglement



- Duan quantity and logarithmic negativity as function of the drive asymmetry
- Solid curve with mechanical thermal occupation $\bar{n} = 0$ and no imperfections on effective coupling
- Thermal occupation and imperfections in the effective coupling lead to less entanglement

Adiabatic limit

Entanglement



- Steady state occupations and purity as function of the drive asymmetry
- Solid curve with mechanical thermal occupation $\bar{n} = 0$ and no imperfections on effective coupling
- Thermal occupation and imperfections in the effective coupling lead to degradation of purity

Experimental observability

Output spectrum

- Reconstructing covariance matrix is experimentally demanding
- Directly measuring quadratures is a hard problem
- Seek signature of entanglement in output spectrum

Spectrum as Fourier transform of expected value

$$S[\omega] = \int dt\, e^{i\omega t} \langle \delta\hat{c}^{\dagger}_{\text{out}}(t)\delta\hat{c}_{\text{out}}(0)\rangle,$$

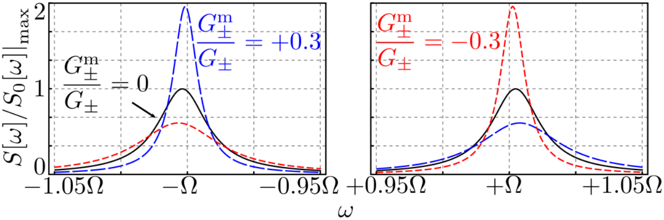
being $\delta\hat{c}_{\text{out}} = \hat{c}_{\text{out}} - < \hat{c}_{\text{out}} >$

Spectrum can be related to the occupation of modes

$$\int_{-\infty}^{0} S[\omega]d\omega = \int_{0}^{+\infty} S[\omega]d\omega$$
$$= 8\pi\kappa \frac{\mathcal{G}^2}{4\mathcal{G}^2 + \kappa(\kappa+\gamma)} \langle \hat{\beta}^{\dagger}_i \hat{\beta}_i\rangle,$$

Experimental observability

Output spectrum



- Output spectrum entered around the detunings from the cavity resonance frequency
- Solid black curve without imperfections
- Steady-state mechanical entanglement can be bounded based on a measurement of the output spectrum
- Experimental work realized in "Stabilized entanglement of massive mechanical oscillators", Nature, 2018.

Conclusions

1. Three-mode optomechanical system such as the steady state includes highly pure and highly entangled two-mode squeezed state is built.
2. Ways of describing both entanglement and purity are described as function of the drive asymmetry.
3. Problem of unequal single-photon optomechanical couplings solved by using four-tone driving scheme.
4. Proposal implementable for existing technology.

Thank you!

Back up

Bogoliubov operators

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Work in rotating frame with respect to the Hamiltonian:

$$\hat{H}_0 = (\omega_a - \Omega)\hat{a}^\dagger \hat{a} + (\omega_b + \Omega)\hat{b}^\dagger \hat{b} + \omega_c \hat{c}^\dagger \hat{c},$$

where choice of detuning Ω is such that collective mechanical quadratures $\hat{X}_\pm$, $\hat{P}_\pm$ (defined later) rotate in a non-trivial way.

Back up

Generate the 2-mode squeezed state

- i) Two cavity modes to independently cool the Bogoliubov modes (beam splitter $\hat{\beta}_i^\dagger \hat{c}_i$)
- ii) Couple the cavity to one Bogoliubov mode ($\hat{\beta}_1$), and then this one to $\hat{\beta}_2$ through ($\hat{\beta}_1^\dagger \hat{\beta}_2$)
- iii) Couple the cavity to sum of the Bogoliubov modes , then the sum to the difference (swap interaction $\hat{\beta}_{\mathrm{sum}}^\dagger \hat{\beta}_{\mathrm{diff}}$ allows diff to cool).

$$\hat{\beta}_{\mathrm{sum}} = \frac{1}{\sqrt{2}}(\hat{\beta}_1 + \hat{\beta}_2)$$
$$\hat{\beta}_{\mathrm{diff}} = \frac{1}{\sqrt{2}}(\hat{\beta}_1 - \hat{\beta}_2)$$

Cooling $\hat{\beta}_{\mathrm{sum}}$ and $\hat{\beta}_{\mathrm{diff}}$ is equivalent to cool $\hat{\beta}_1$ and $\hat{\beta}_2$ ✓
Direct coupling not needed, just difference in their resonance frequencies ✓

Back up

4-tone driving

Where we demand the driving strengths of the 4-tone driving are "matched" as

$$\frac{\bar{c}_{1\pm}}{\bar{c}_{2\pm}} = \frac{g_b}{g_a}$$

meaning, asymmetry in steady-state amplitudes is set by the asymmetry in the optomechanical couplings

Back up

Effective coupling imperfections

Imperfections in the optomechanical couplings

$$G_{\pm}^{m} = \pm(g_a - g_b)\bar{c}_{\pm}/2 \qquad \text{2-tone driving}$$
$$G_{\pm}^{m} = \pm(g_a\bar{c}_{1\pm} - g_b\bar{c}_{2\pm})/2 \qquad \text{4-tone driving}$$

In the 2-tone driving case, imperfections coming from mismatch in the optomechanical couplings

In th 4-tone driving case, imperfection arises from the drives not being weighted precisely according to the matching condition