Augmented Dickey Fuller Test (ADF)

The DF Test assumes that our time series is a AR 1:

$$\left[y_t = \mu + \phi_1 \cdot y_{t-1} + \varepsilon_t \right]$$

So, the Dickey Fuller Test is basicly a Hypothesis contrast:

$$\begin{cases} H_0: \phi_1 = 1 \leftarrow \text{Unit root} \\ H_1: \phi_1 < 1 \leftarrow \text{No unit root} \end{cases}$$

Let's substract Y_{t-1} from both sides:

$$y_t - y_{t-1} = \mu + (\phi_1 - 1) y_{t-1} + \varepsilon_t \Leftrightarrow$$

$$\Leftrightarrow \Delta Y_t = \mu + \delta \cdot y_{t-1} + \varepsilon_t \Longrightarrow$$

$$\Rightarrow \begin{cases} H_0: \phi_1 = 1 \\ H_1: \phi_1 < 1 \end{cases} \Leftrightarrow \begin{cases} H_0: \delta = 0 \\ H_0: \delta < 0 \end{cases}$$

Let's assume that we are in H_0, so:

$\Delta Y_t = \mu + \varepsilon_t$ which indicates that ΔY_t is stationary, but we cannot assume that because Y_{t-1} is non-stationary based on our first condition given by H_0 that was that y_t was non stationary.

The "special" part comes here. Because of the before explanation we cannot apply a "classic" t-test to proof or reject our hypothesis, so we'll compare the t-statistic against the dickey-Fuller distribution instead of a normal t distribution

So our statistic will be something like this:

$$t_{\hat{\delta}} = \frac{\hat{\delta}}{se(\hat{\delta})}$$

Comparison with Dickey Fuller Distrib.

$$\begin{cases} t_{\hat{\delta}} < DF_{critical} \rightarrow \text{Reject } H_0 \\ t_{\hat{\delta}} > DF_{critical} \rightarrow \text{Don't reject } H_0 \end{cases}$$

Usually our models will be more complicated than a simple AR-1 Model. That's why we consider ADF, which is useful for mor complicated models. The steps are almost the same:

$$y_t = \mu + \sum_{i=1}^{p} \phi_i \cdot y_{t-i} + \varepsilon_t$$

Let's substract Y_{t-1} from both sides:

$$\Delta y_{t-1} = \mu + \delta \cdot y_{t-1} + \sum_{i=1}^{p} \beta_i \cdot \Delta y_{t-i} + \varepsilon_t$$

$\begin{cases} H_0: \delta = 0 \\ H_1: \delta < 0 \end{cases}$, and the process is exactly the same