UNIT ROOTS

It is so important to know what is a unit root because if we have a time series with a unit root then it is directly non-stationary and we cannot apply a model like AR, MA, ARMA...

We would have to do some transformations to remove that unit root remove it from the time series.

We'll use a AR 1 model (simple model) to go deeper on this topic:

AR 1 model ≡ ∞ MA model

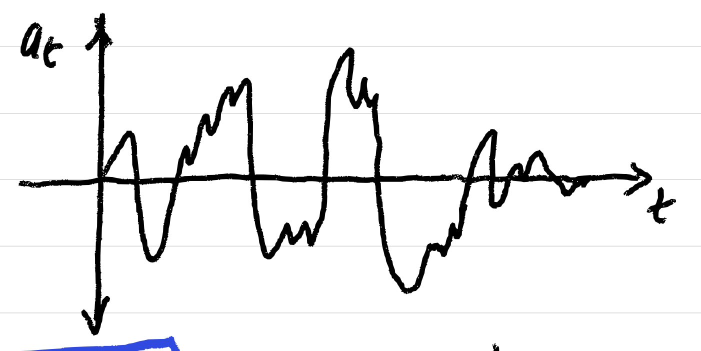
$$a_t = \phi \cdot a_{t-1} + \varepsilon_t \;=\; \text{[redacted]}$$

$$= \phi^t \cdot a_0 + \sum_{k=0}^{t-1} \phi^k \cdot \varepsilon_{t-k}$$

$$\begin{cases} Var(a_t) = \sigma^2 \cdot [\phi^0 + \phi^2 + \phi^4 + \dots + \phi^{2(t-1)}] \\ \mathbb{E}(a_t) = \phi \cdot \mathbb{E}(a_{t-1}) = \phi^2 \cdot \mathbb{E}(a_{t-2}) = \dots = \phi^t \cdot a_0 \end{cases}$$

* Let's see three examples a demonstrate if the function is stationary or not.

1.

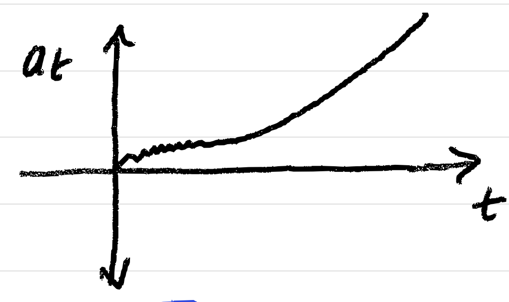


$\boxed{|\phi| < 1} \Rightarrow \begin{cases} \mathbb{E}(a_t) = \phi^t \cdot a_0 \to 0 \Rightarrow \mathbb{E}(a_t) \to 0 \checkmark \\ Var(a_t) = \sigma^2 \cdot [\phi^0 + \phi^2 + \dots + \phi^{2(t-1)}] \Rightarrow \end{cases}$

$\Rightarrow Var(a_t) \to \dfrac{\sigma^2}{1-\phi^2} \Rightarrow Var(a_t) = cte \checkmark$

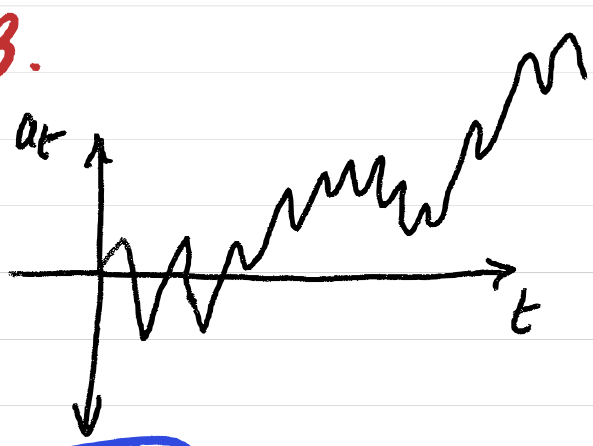
So we can conclude that this time series is Stationary

2.



$\boxed{|\phi| > 1} \Rightarrow \begin{cases} \mathbb{E}(a_t) = \phi^t \cdot a_0 \Rightarrow \mathbb{E}(a_t) \to \pm\infty \\ \text{So it is non-stationary} \checkmark \end{cases}$

3.



$\boxed{|\phi| = 1}$

A function has a Unit root if its $|\phi| = 1$.

All types of function can have one or some unit roots and all of them cause problems on time series for applying the models.

$\phi = 1 \begin{cases} \mathbb{E}(a_t) = \phi^t \cdot a_0 = a_0 \Rightarrow \mathbb{E}(a_t) = cte. \checkmark \\ Var(a_t) = \dots = t \cdot \sigma^2 \leftarrow \end{cases}$ For this reason time series with a unit root are non-stationary although they seem to be graphicall

• Trick on the AR 1 model to make it stationary.

We can make the first difference ($a'(t)$)

$a'(t) = d(t) = a(t) - a(t-1) \begin{cases} d(t) = \varepsilon(t) \Rightarrow \\ \Rightarrow \begin{cases} \mathbb{E}(d_t) = 0 \\ Var(d_t) = \sigma^2 \end{cases} \Rightarrow \end{cases}$

⇒ This new function is clearly Stationary