Backshift Operator (LAG)

ARMA (3,3) :

$$y_t = \phi_1 \cdot y_{t-1} + \phi_2 \cdot y_{t-2} + \phi_3 \cdot y_{t-3} +$$

$$\theta_1 \cdot \varepsilon_{t-1} + \theta_2 \cdot \varepsilon_{t-2} + \theta_3 \cdot \varepsilon_{t-3} + \varepsilon_t \Rightarrow$$

$$\Rightarrow y_t - \phi_1 \cdot y_{t-1} - \phi_2 \cdot y_{t-2} - \phi_3 \cdot y_{t-3} =$$

$$= \varepsilon_t + \theta_1 \cdot \varepsilon_{t-1} + \theta_2 \cdot \varepsilon_{t-2} + \theta_3 \cdot \varepsilon_{t-3} \quad (*)$$

$$\left(\begin{array}{l} \text{Backshift Operator} = L \\ \boxed{L \cdot y_t = y_{t-1}} \Rightarrow L^2 \cdot y_t = y_{t-2} \quad \ldots \end{array} \right)$$

$$(*) \Leftrightarrow 1 \cdot y_t - \phi_1 \cdot L \cdot y_t - \phi_2 \cdot L^2 \cdot y_t - \phi_3 \cdot L^3 \cdot y_t =$$

$$= 1 \cdot \varepsilon_t + \theta_1 \cdot L \cdot \varepsilon_t + \theta_2 \cdot L^2 \cdot \varepsilon_t + \theta_3 \cdot L^3 \cdot \varepsilon_t \Leftrightarrow$$

$$\Leftrightarrow \overbrace{(1 - \phi_1 L - \phi_2 L^2 - \phi_3 L^3)}^{\Phi(L)} y_t =$$

$$= \underbrace{(1 + \theta_1 L + \theta_2 L^2 + \theta_3 L^3)}_{\Theta(L)} \varepsilon_t \Rightarrow$$

$$\Rightarrow \boxed{\Phi(L) \cdot y_t = \Theta(L) \cdot \varepsilon(t)}$$

It is a notation shortcut basicly.