Module 2 - The LP Model Assignment

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1. Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a su pplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week.Management wishes to know what quantity of each type of backpack to produce per week.

```r
## Create the data frame
back_savers <- data.frame(
  "Backpack Type" = c("collegiate Backpacks", "Mini Backpacks"),
  "Material_Req(sqft)" = c(3, 2),
  "Labor_Req(min)" = c(45, 40),
  "Unit_Prof_per_Backpack($)" = c(32, 24),
  "Sales_Forecast(units/week)" = c(1000, 1200)
)

# Add additional information
back_savers$Material_Avail_Weekly <- 5000
back_savers$Total_Labor_Hours_Avail_Weekly <- 35 * 40 * 60

# Transpose the data frame
transposed_back_savers <- t(back_savers)

# Print the transposed table
print(transposed_back_savers)
```

```
                               [,1]                   [,2]
Backpack.Type                  "collegiate Backpacks" "Mini Backpacks"
Material_Req.sqft.             "3"                    "2"
Labor_Req.min.                 "45"                   "40"
Unit_Prof_per_Backpack...      "32"                   "24"
Sales_Forecast.units.week.     "1000"                 "1200"
Material_Avail_Weekly          "5000"                 "5000"
Total_Labor_Hours_Avail_Weekly "84000"                "84000"
```

(a)Clearly define the decision variables

Assume Here the decision variables are the numbers of collegiate Backpacks(X1) and Mini bagepack(X2) that are generated every week. Number of collegiate backpacks to produce per week

$$= x1$$

Number of mini backpacks to produce per week

$$= x2$$

(b)What is the objective function?

The objective function is to maximizing profit.So, X1 make a profit of $32 and X2 generate profit of $24

$$Max \quad z = 32x1 + 24x2$$

(c)What are the constraints?

Material Constraint: Back Savers cannot exceed the 5000 square feet of nylon fabric they receive each week.The material constraint can be expressed as:

$$= 3X1 + 2X2 \leq 5000$$

Sales Constraints: The maximum number of Collegiate and Mini backpacks they can sell per week is limited to 1000 and 1200, respectively. These constraints can be expressed as:

$$= x1 \leq 1000$$

$$= x2 \leq 1200$$

Labor Constraint: Back Savers has 35 laborers, each providing 40 hours of labor per week. The labor constraint can be expressed as:

$$= 45x1 + 40x2 \leq 35 * 40 * 60$$

Non-negativity Constraints: The number of backpacks produced cannot be negative:

$$= 0 \leq x1 \leq 1000$$

$$= 0 \leq x2 \leq 1200$$

Full Mathematical Formulation: The complete linear programming (LP) problem can be formulated as follows:

$$Max \quad z = 32x1 + 24x2$$

Subject to the constraints:

1. Material Constraint:

$$3X1 + 2X2 \leq 5000$$

2. Sales Constraints:

$$x1 \leq 1000$$

,

$$x2 \leq 1200$$

3. Labor Constraint:

$$45x1 + 40x2 \leq 35 * 40 * 60$$

4. Non-negativity Constraints:

$$0 \leq x1 \leq 1000$$

,

$$0 \leq x2 \leq 1200$$

(2)The Weigelt Corporation has three branch plants with excess production capacity.Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes–large, medium, and small–that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large,medium, and small sizes, respectively, would be sold per day.At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product.Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

```
# Create the data frame
lp_data <- data.frame(
  Plant = c("Plant 1", "Plant 2", "Plant 3"),
  Large_Size = c(420, 420, 420),
  Medium_Size = c(360, 360, 360),
  Small_Size = c(300, 300, 300),
  Prod_Cap = c(750, 900, 450),
  Stor_Space_Avail = c(13000, 12000, 5000),
  Large_Size_Space_Req = c(20, 20, 20),
  Medium_Size_Space_Req = c(15, 15, 15),
  Small_Size_Space_Req = c(12, 12, 12),
  Large_Size_Sales_Forecast = c(900, 900, 900),
  Medium_Size_Sales_Forecast = c(1200, 1200, 1200),
  Small_Size_Sales_Forecast = c(750, 750, 750)
)

# Transpose the data frame
transposed_lp_data <- t(lp_data)

# Print the transposed table
print(transposed_lp_data)
```

```
                           [,1]      [,2]      [,3]     
Plant                      "Plant 1" "Plant 2" "Plant 3"
Large_Size                 "420"     "420"     "420"    
Medium_Size                "360"     "360"     "360"    
Small_Size                 "300"     "300"     "300"    
Prod_Cap                   "750"     "900"     "450"    
Stor_Space_Avail           "13000"   "12000"   " 5000"  
Large_Size_Space_Req       "20"      "20"      "20"     
Medium_Size_Space_Req      "15"      "15"      "15"     
Small_Size_Space_Req       "12"      "12"      "12"     
Large_Size_Sales_Forecast  "900"     "900"     "900"    
Medium_Size_Sales_Forecast "1200"    "1200"    "1200"   
Small_Size_Sales_Forecast  "750"     "750"     "750"    
```

a. Decision Variables:

The number of large-sized units produced at Plant 1

$$= x1L$$

The number of medium-sized units produced at Plant 1.

$$= x1M$$

The number of small-sized units produced at Plant 1.

$$= x1S$$

The number of large-sized units produced at Plant 2

$$= x2L$$

The number of medium-sized units produced at Plant 2.

$$= x2M$$

The number of small-sized units produced at Plant 2.

$$= x2S$$

The number of large-sized units produced at Plant 3

$$= x3L$$

The number of medium-sized units produced at Plant 3.

$$= x3M$$

The number of small-sized units produced at Plant 3.

$$= x3S$$

b. Formulate a linear programming model for this problem.

$$MAX \quad Z = 420(x1L + x2L + x3L) + 360(x1M + x2M + x3M) + 300(x1S + x2S + x3S)$$

Production Capacity Constraints: Plant 1

$$= x1L + x1M + x1S \leq 750$$

plant 2

$$= x2L + x2M + x2S \leq 900$$

plant 3

$$= x3L + x3M + x3S \leq 450$$

In-Process Storage Space Constraints:

plant 1

$$= 20x1L + 15x1M + 12x1S \leq 13000$$

plant 2

$$= 20x2L + 15x2M + 12x2S \leq 12000$$

plant 3

$$= 20x3L + 15x3M + 12x3S \leq 5000$$

Sales Forecasts:

Large size sales forecast

$$= x1L + x2L + x3L \leq 900$$

Medium size sales forecast

$$= x1M + x2M + x3M \leq 1200$$

Small size sales forecast

$$= x1S + x2S + x3S \leq 750$$

The Plants always utilize the same % of their excess capacity to produce the new product.

$$x1L + x1M + x1S \div 750 \quad = \quad x2L + x2M + x2S \div 900 \quad = \quad x3L + x3M + x3S \div 450$$

Non-negativity Constraints:

$$= Xij \geq 0$$

$$x1L, x2L, x3L, x1M, x2M, x3M, x1S, x2S, x3S \geq 0$$