Lecture 3: Similarity

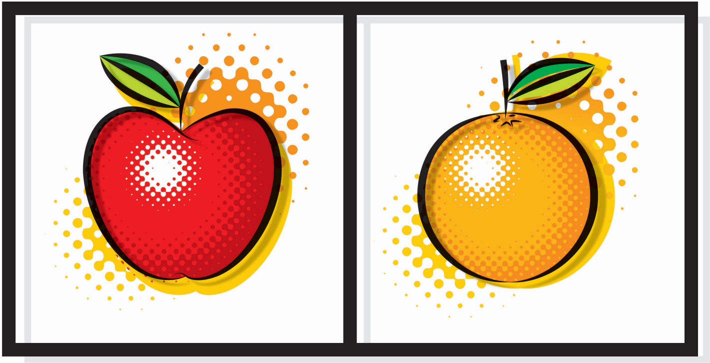
COMP90049
Knowledge Technologies

Sarah Erfani and Karin Verspoor and Jeremy Nicholson, CIS

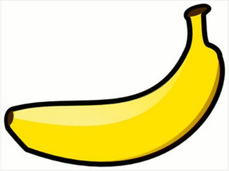
Semester 2, 2017

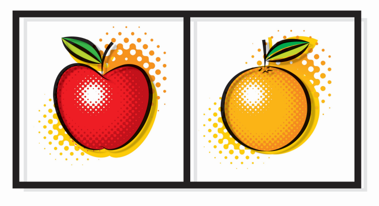
THE UNIVERSITY OF
MELBOURNE

Compare and Contrast

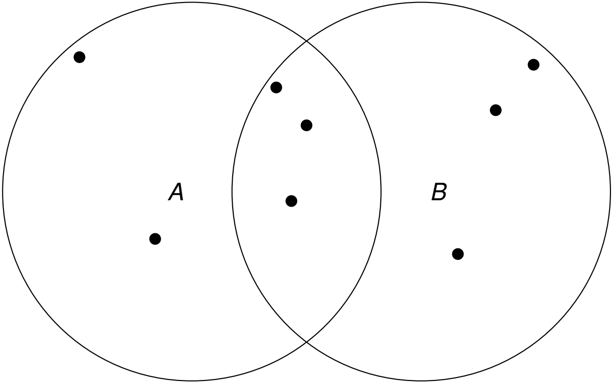


Compare and Contrast





Venn Diagram



Similarity as Set intersection

Many similarity assessments can be framed as set intersection.

- Amazon: Book purchases
- Netflix: Movies that you have watched

Refinements

- Rating sets (stars)
  - thresholding using ratings
  - different subsets for different ratings
- Categories of items
  - generalisation
  - book or movie genres

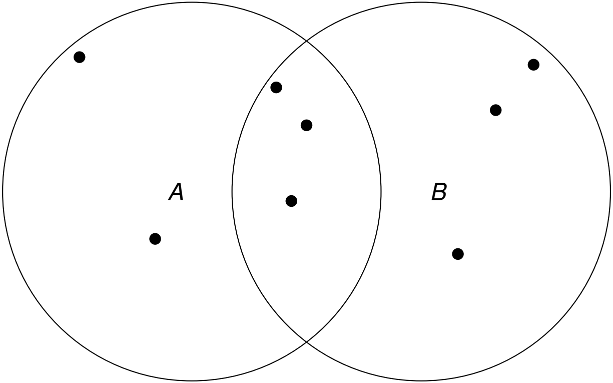
Measuring Similarity

We have discussed similarity at an intuitive level.

How do we measure similarity quantitatively?

Jaccard Similarity

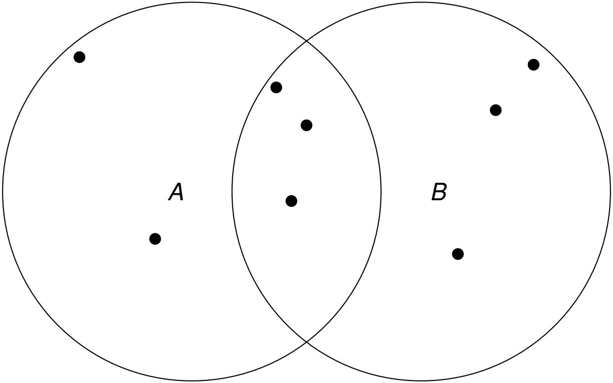
$$\frac{|A \cap B|}{|A \cup B|}$$



$$sim(A, B) = \frac{|A \cap B|}{|A \cup B|} = \frac{3}{8}$$

Dice Similarity

$$\frac{2|A \cap B|}{|A| + |B|}$$



$$sim(A, B) = \frac{2|A \cap B|}{|A| + |B|} = \frac{2 * 3}{5 + 6} = \frac{6}{11}$$

What is a model?

A model is our attempt to understand and represent the nature of reality through a particular lens, be it architectural, biological, or mathematical.

An model is an **abstraction** of the entity that we are trying to model, c.f. fruit above.

Feature vectors

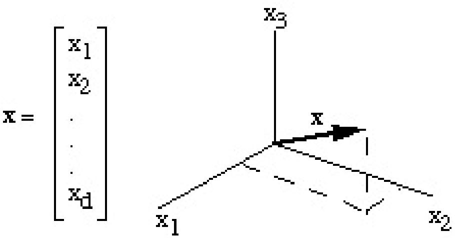
A *feature vector* is an n-dimensional vector of *features* that represent some object.

A *feature* or *attribute* is any distinct aspect, quality, or characteristic of that object

- Features may be symbolic/categorical/discrete (e.g. colour, gender)
- Features may be ordinal (e.g. cool $<$ mild $<$ hot [temperature])
- Features may be numeric/continuous (e.g., height, age)

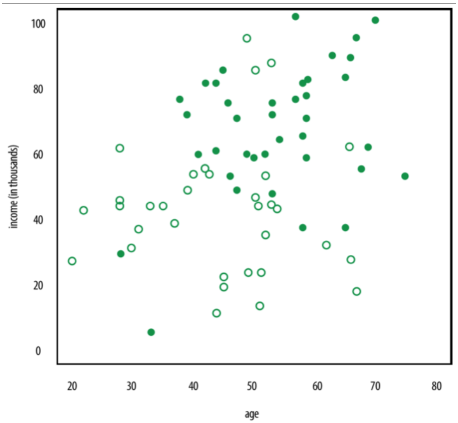
A vector locates an object (document, person, . . .) as a point in *n*-space. The angle of the vector in that space is determined by the relative weight of each term.

Feature vectors and vector space



Credit as a function of age and income

age	income	credit
33	8	low
58	42	low
49	79	low
49	17	low
58	26	high
44	71	high
. . .		



Comparing Documents

How should we compare documents to assess their similarity?

- String-level similarity (e.g., edit distance)
- Sets of common substrings (sentences, phrases, words, n-grams)
- “bag of words”

How similar are these sentences?

1. Mary is quicker than John.
2. John is quicker than Mary.
3. Mary is slower than John.
4. Jane is quicker than Mary.

Word Vectors

1. Mary is quicker than John.
2. John is quicker than Mary.
3. Mary is slower than John.
4. Jane is quicker than Mary.

Sentence	"Mary"	"John"	"Jane"	"quicker"	"slower"
1	1	1	0	1	0
2	1	1	0	1	0
3	1	1	0	0	1
4	1	0	1	1	0

Vector space model for documents

One of the earliest models proposed for retrieval of documents (information retrieval, in 1962) was the vector-space model.

Suppose there are *n* distinct indexed terms in the collection. Then each document *d* can be thought of as a vector

$$\langle w_{d,1}, w_{d,2}, \ldots, w_{d,t}, \ldots, w_{d,n} \rangle$$

where $w_{d,t}$ is a weight describing the importance of term *t* in *d*.

(Most $w_{d,t}$ values will be zero, because most documents only contain a tiny proportion of a collection's terms.)

Intuitively, if some other document d' has a vector

$$\langle w_{d',1}, w_{d',2}, \ldots, w_{d',t}, \ldots, w_{d',n} \rangle$$

where the weights are close to those of *d* – in particular, if the non-zero *w* values are for much the same set of terms – then *d* and d' are likely to be similar in topic.

Similarity vs Distance

We have discussed similarity at an intuitive and quantitative level.

$$
\begin{aligned}
sim_J(A,B) &= \frac{|A \cap B|}{|A \cup B|} = \frac{3}{8} \\
sim_D(A,B) &= \frac{2|A \cap B|}{|A| + |B|} = \frac{2 * 3}{5 + 6} = \frac{6}{11}
\end{aligned}
$$

What is the relationship between similarity and distance?

Distance measures

A distance measure on a space is a function that takes two points in a space as arguments.

1. No negative distances.

$$d(x, y) \geq 0$$

2. Distances are positive, except for the distance from a point to itself.

$$d(x, y) = 0 \text{ if and only if } x = y$$

3. Distance is symmetric.

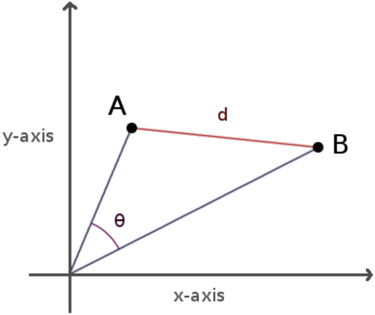
$$d(x, y) = d(y, x)$$

4. The *triangle inequality* typically holds.
(Distance measures the length of the *shortest path* between two points.)

$$d(x, y) \leq d(x, z) + d(z, y)$$

Euclidean Distance

Given two items A and B, and their corresponding feature vectors $\vec{a}$ and $\vec{b}$, respectively, we can calculate their similarity via their distance d in euclidean space:

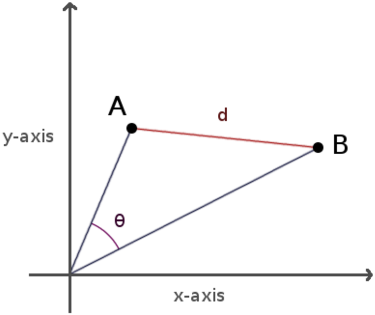


In n-dimensional space:

$$d(A, B) = \sqrt{\sum_{i=1}^{n} (a_i - b_i)^2}$$

Cosine Distance

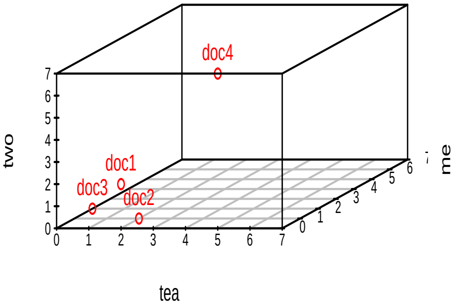
Given two items *A* and *B*, and their corresponding feature vectors $\vec{a}$ and $\vec{b}$, respectively, we can calculate their similarity via their *vector cosine* (the cosine of the angle θ between the two vectors):



$$sim(A, B) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{\sum_i a_i b_i}{\sqrt{\sum_i a_i^2}\sqrt{\sum_i b_i^2}}$$

“Long” documents & Euclidean distance

Point	tea	me	two
doc1	2	0	2
doc2	2	1	0
doc3	0	2	0
doc4	5	0	7

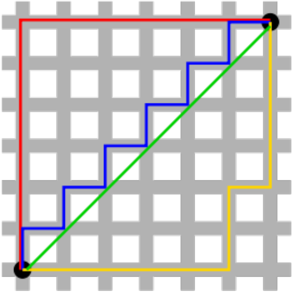


- Doc4, like Doc1, is all about “tea” and “two”.
- But because it is longer, it is in a space by itself.

Manhattan Distance

[“City block” distance or “Taxicab geometry” or “L_1 distance”]

Given two items A and B, and their corresponding feature vectors $\vec{a}$ and $\vec{b}$, respectively, we can calculate their similarity via their distance d based on the absolute differences of their cartesian coordinates:



In n-dimensional space:

$$d(A, B) = \sum_{i=1}^{n} |a_i - b_i|$$

Probabilistic measures

Relative entropy:

$$D(x \mid\mid y) = \sum_i x_i(\log_2 x_i - \log_2 y_i)$$

or alternatively *skew divergence*:

$$s_\alpha(x, y) = D(x \mid\mid \alpha y + (1-\alpha)x)$$

or *Jensen-Shannon divergence*:

$$JSD(x \mid\mid y) = \frac{1}{2}D(x \mid\mid m) + \frac{1}{2}D(y \mid\mid m)$$

where $m = \frac{1}{2}(x + y)$

NB: Probability will be reviewed next lecture!

Summary

- How can we represent a set of objects?
- What are some methods for measuring similarity between objects?

Reading

- On distance measures:

Chapter 3, especially Section 3.5
Mining of Massive Datasets
`http://infolab.stanford.edu/~ullman/mmds.html`

- On document representation:

Chapter 6
Information Retrieval, Manning *et al.*
`http://nlp.stanford.edu/IR-book/html/htmledition/scoring-term-weighting-and-the-vector-space-model-1.html`