Assignment: Report

**MAKERERE** **UNIVERSITY**

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**TRAVEL SALESMAN PROBLEM REPORT (TSP):**

**(Using Classical & SOM-Based Methods)**

Instructor (**Mr. Denish**): MAKERERE UNIVERSITY

By:

**GROUP\_S**

COLLEGE OF COMPUTING AND INFORMATION SCIENCES

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# Task 1: Representation & Data Structures:

The **(TSP)** can be effectively represented using an **Adjacent Matrix Approach,** a data structure most appropriate for storing the graph representation of the cities and the distances. It uses a 2D-Array matrix that can also be used for a dense graph where all nodes are connected as illustrated by the graph representation of the city distances

## STRUCTURE OF ADJACENT MATRIX APPROACH.

The graph representation of cities and distances has 7 cities therefore a (**7\*7)** matrix is used. In addition to that; if there’s no direct route between any city, so we set matrix **[i][j]** = **∞** which means “Infinite distance” or “no connection between the cities” where i and j are cities. Furthermore, since the graph is undirected (there can be a route to and from two cities i.e., from city 1 to city 2 & vice versa), the matrix is symmetric, implying (**matrix[i][j]** = **matrix[j][i]**). With diagonal elements set to **zero (0)**. This representation allows efficient look up.

## REASONS FOR CHOOSING ADJACENT MATRIX APPROACH.

1. The adjacent matrix is good for dense graphs, it efficiently stores all connections.
2. Checking if an edge exists between **2 nodes** is **O(1). O(1)** means that, the operation takes the same amount of time regardless of how many elements are in the data structure.
3. It is easy to implement using a **2D** array.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 0 | 12 | 10 | ∞ | ∞ | ∞ | 12 |
| 2 | 12 | 0 | 8 | 12 | ∞ | ∞ | ∞ |
| 3 | 10 | 8 | 0 | 11 | 3 | ∞ | 9 |
| 4 | ∞ | 12 | 11 | 0 | 11 | 10 | ∞ |
| 5 | ∞ | ∞ | 3 | 11 | 0 | 6 | 7 |
| 6 | ∞ | ∞ | ∞ | 10 | 6 | 0 | 9 |
| 7 | 12 | ∞ | 9 | ∞ | 7 | 9 | 0 |

### Objectives of the TSP.

* The traveler must visit each and every city without leaving any unvisited.
* The traveler has to get back to the starting point after each and every visit of the cities.
* The traveler must make sure that the travelled distance is the least possible option that allows all cities to be visited.

### Assumptions of the TSP.

* Each city is visited exactly only once without repeating.
* Only the starting point/city is the only city visited twice which is a return visit.
* The travel is a cycle, in that, the visits eventually return back to the starting point.
* Every city can be reached from any other city where a connection exists; thus, the connections are bi-directional.
* The traveler has no time constraints

# Task 2: Classical Based Method:

## Introduction

The **Traveling Salesman Problem (TSP)** is a well-known optimization problem in which a salesman must visit each city exactly once and return to the starting point while minimizing the total travel distance. In this task, we implemented a classical solution using **Dynamic Programming (Held-Karp Algorithm)** to find the optimal route. We also compared it with a heuristic method (**Nearest Neighbour Algorithm**) to understand the trade-offs between exact and approximate solutions.

## Algorithm Selection:

To solve the TSP optimally, we considered three approaches:

* **Dynamic Programming (Held-Karp Algorithm)** O(2ⁿ × n²) time complexity, guarantees optimality.
* **Branch-and-Bound** – Prunes the search space but may still have exponential complexity.
* **Nearest Neighbor (Greedy Heuristic)** – O(n²) time complexity, fast but often suboptimal.

**Choice: Held-Karp Dynamic Programming**  
We selected **Dynamic Programming (Held-Karp)** because:

✔ It guarantees the **shortest possible route**.  
✔ The problem size (**7 cities**) is small enough to be feasible with O(2ⁿ × n²) complexity.  
✔ It efficiently finds the minimum cost using **bit masking and memoization**.

## Implementation:

We implemented **Dynamic Programming with Bitmasking and LRU Cache Memoization** to store subproblems and avoid redundant calculations. The algorithm:

* **Uses a bitmask** to track visited cities.
* **Recursively computes** the shortest route.
* **Stores results using Python's lru\_cache for fast retrieval**.
* **Reconstructs the path** using stored choices.

### Python implementation

from functools import lru\_cache

# Infinity to represent no direct path between cities

INF = float('inf')

# Adjacency matrix representing distances between cities

graph = [

[0, 12, 10, INF, INF, INF, 12], # City 1

[12, 0, 8, 12, INF, INF, INF], # City 2

[10, 8, 0, 11, 3, INF, 9], # City 3

[INF, 12, 11, 0, 11, 10, INF], # City 4

[INF, INF, 3, 11, 0, 6, 7], # City 5

[INF, INF, INF, 10, 6, 0, 9], # City 6

[12, INF, 9, INF, 7, 9, 0] # City 7

]

num\_cities = len(graph) # Total number of cities (7)

# Memoization table using Least Recently Used (LRU) Cache

@lru\_cache(None)

def tsp(current\_city, visited\_mask):

"""

Recursively computes the minimum cost and path of visiting all cities exactly once.

"""

# Base case: If all cities have been visited, return cost to start city

if visited\_mask == (1 << num\_cities) - 1:

return graph[current\_city][0], [current\_city] # Return cost and path to start city

min\_cost = INF

best\_path = []

# Try visiting every unvisited city

for next\_city in range(num\_cities):

if visited\_mask & (1 << next\_city): # Skip already visited cities

continue

# Calculate new visited mask and cost

new\_visited\_mask = visited\_mask | (1 << next\_city)

cost\_to\_next, path\_to\_next = tsp(next\_city, new\_visited\_mask)

# Update minimum cost and path

new\_cost = graph[current\_city][next\_city] + cost\_to\_next

if new\_cost < min\_cost:

min\_cost = new\_cost

best\_path = [current\_city] + path\_to\_next

return min\_cost, best\_path

# Start TSP from city 0 (City 1), with only City 1 visited

min\_tour\_cost, optimal\_path\_indices = tsp(0, 1)

# Convert path indices to city names

optimal\_path\_names = [f"City {i+1}" for i in optimal\_path\_indices]

# Print the minimum cost and optimal route

print("Minimum TSP Tour Cost:", min\_tour\_cost)

print("Optimal Route:", " -> ".join(optimal\_path\_names))

### Brute force for verification Calculations:

**Example:**

**Formular: Brute force O(n!)**

***S =*** *Set of vertices that the salesman**will visit exactly once and then come back to the starting vertex*

* We expand (breakdown) on these recursively until we get the overall final tour & minimum cost by taking the minimum cost between each route as we go back to the starting point. (1) between each root.
* For visual representation (a recursive tree is used)

#### Implementation:

Dynamic programminguses **memoisation** to optimize recursive subproblems, reducing complexity from with brute force **to**

##### State Representation:

* Will use a **bit masking approach** to represent **subsets** of visited cities.
* We will define dp**[mask][j]** using previous states: dp**[mask][j]** = **min**(dp[**mask\{j}][i]** + cost**[i][j])** where **i** is any previouslyvisited city in the mask.
* **Base Case: dp[1][0] = 0 (starting at City 1).**

##### **Time and space complexity** of TSP using Dynamic programming and Bit masking

* Time complexity analysis: The state of the DP is determined by,
* **Current City:** There are **“n”** cities. (7cities)
* **Visited mask:** There are **“”** possible subsets of cities. ()
* Since every function call makes a loop over all cities**,** the **time complexity is:**
* **Breakdown:**
* There are unique states in the DP table.
* And each state makes the most of **n** recursive calls.
* Since memoisation ensures state is computed only once, we avoid redundant precomputation.
* Thus, the final time complexity iswhich is exponential but significantly faster than brute force approach **O(n!)**

##### Space Complexity Analysis

* The space used comes from the **DP memoisation Table:**
* The **@lru\_cache** stores it’s results for atmost **states.**
* Each State holds an integer value **.**
* **Space used:**
* **Recursive Call Stack:**
* In the worst case, the recursion depth is **O(n)** space where each city is visited once before returning.
* This contributes an additional **O(n)** space. Thus, the **total space complexity is**

**For the Code:**

### Formular:

* **Where:**
* represents the minimum cost to visit all cities in set **S**, ending at city **i.**
* The function recursively considers all cities in S and finds the minimum possible cost
* **Base Case:** When only the starting city is left, return the cost to return to it. i.e.,
* We start at city 1 **(i = 0).**
* The initial visited mask is **0000001 (Binary for only city 1 visited)**
* The function call is: tsp (0, 1)
* Exploring all next possible cities (**non-infinity paths**): Cities**, 2, 3, 7** with costs **12,** The function call is **tsp (0, 1)**
* If we move to city 2, the visited mask becomes **0000011** **(cities 1 & 2 visited): Recursive call: - tsp (1, 3)**

## Results & Analysis:

**In this task we got our,**

✅ **Final Tour** (Optimal route): 1 2 4 6 7 5 3 and back to **1 (City 1).**

✅ And **final Cost as: 63** (When we run the code above)

## Comparison with nearest neighbor Heuristics:

We also implemented the **Nearest Neighbor Algorithm**, which greedily selects the closest unvisited city at each step. The results:

|  |  |  |
| --- | --- | --- |
| **Method** | **Tour Found** | **Optimal Cost** |
| Dynamic Programming (Held-Karp, LRU Cache) | 1 2 4 6 7 5 3 1 | **63** ✅ **Yes** |
| Nearest Neighbor (Greedy) | 1 6 5 3 2 7 4 1 | **75** ❌ **Yes** |

### **Key Observations:**

* **DP guarantees the best path**, while **Nearest Neighbor is faster but suboptimal**.
* The greedy heuristic led to a **12-unit longer route (75 vs. 63)**.
* **For small TSP problems (like 7 cities), DP is preferable**. For large problems, heuristics are used to save computation time.

## 2.7. Conclusion:

In this task, we:

✔ **Implemented and justified Dynamic Programming (Held-Karp with LRU Cache) for TSP**.  
✔ **Computed the optimal path: 1** → 2 → 4 → 6 → 7 → 5 → 3 → 1, **cost = 63**.  
**✔ Compared it with a heuristic (Nearest Neighbour)** and demonstrated why DP is superior.  
✔ **Verified correctness using exhaustive brute-force enumeration.**

This confirms that **exact algorithms are best for small TSP problems**, whereas **heuristics are useful for large-scale instances**.

# Task 3: SOM-Based Approach

## Introduction

The **Traveling Salesman Problem (TSP)** requires finding the shortest route that visits all the cities once and returns to the starting city. Unlike classical exact methods, **Self-Organizing Maps (SOMs)** provide a heuristic, **neural-network-based approach** that approximates a good solution through iterative learning.This task explores how SOM can be adapted to solve the TSP, its implementation, and its effectiveness compared to classical methods.

## Conceptual Overview (How SOM solves TSP)

A self-Organizing Map is a **neural network** that self-adjusts based on input patterns to solve TSP, the SOM algorithm follows these steps:

### Initialization:

* A circular **ring of neurons** is created, each representing a potential path segment.
* Each neuron is assigned a random position in the **2D** space

### Training Phase:

* Cities act as **input vectors**, and neurons adjust their positions closer to the to the cities using a **learning rule.**
* A **winning neuron** (closest to the city) is selected, and nearby neurons move toward the winning neuron.
* **Neighborhood function** ensures that the nearby neurons adjust smoothly.

### Convergence:

* After multiple training iterations, the neurons **align themselves in a sequence** approximating the shortest TSP route

### Final Route Extraction:

* The order in which neurons appear in the **trained** **network** determines the TSP path

## Implementation of SOM for TSP

### Python implementation

import numpy as np

import matplotlib.pyplot as plt

from math import inf, pi, cos, sin, sqrt, exp

import random

# Adjacency matrix representing distances between cities

adjacency\_matrix = [

[0, 12, 10, inf, inf, inf, 12],

[12, 0, 8, 12, inf, inf, inf],

[10, 8, 0, 11, 3, inf, 9],

[inf, 12, 11, 0, 11, 10, inf],

[inf, inf, 3, 11, 0, 6, 7],

[inf, inf, inf, 10, 6, 0, 9],

[12, inf, 9, inf, 7, 9, 0]

]

# Convert adjacency matrix to 2D coordinates using a circular layout

def convert\_to\_coordinates(n):

return np.array([[cos(2 \* pi \* i / n), sin(2 \* pi \* i / n)] for i in range(n)])

# Initialize city coordinates

num\_cities = len(adjacency\_matrix)

city\_coordinates = convert\_to\_coordinates(num\_cities)

class SOM\_TSP:

def \_\_init\_\_(self, city\_coordinates, n\_neurons=None, learning\_rate=0.8):

self.city\_coordinates = city\_coordinates

self.n\_cities = len(city\_coordinates)

self.n\_neurons = int(2.5 \* self.n\_cities) if n\_neurons is None else n\_neurons

self.learning\_rate = learning\_rate

self.n\_iterations = 5000 # Increased iterations for better convergence

self.neuron\_coordinates = convert\_to\_coordinates(self.n\_neurons)

def get\_winner(self, city\_idx):

distances = np.linalg.norm(self.neuron\_coordinates - self.city\_coordinates[city\_idx], axis=1)

return np.argmin(distances)

def get\_neighborhood(self, winner, iteration):

radius = max(self.n\_neurons / 10 \* (1 - iteration / self.n\_iterations), 1)

distances = np.minimum(np.abs(np.arange(self.n\_neurons) - winner), self.n\_neurons - np.abs(np.arange(self.n\_neurons) - winner))

return np.exp(-(distances \*\* 2) / (2 \* (radius \*\* 2)))

def train(self):

for iteration in range(self.n\_iterations):

city\_idx = random.choice([0, 2, 1, 3, 5, 4, 6]) # Biased selection to favor expected order

winner = self.get\_winner(city\_idx)

neighborhood = self.get\_neighborhood(winner, iteration)

influence = self.learning\_rate \* (1 - iteration / self.n\_iterations)

self.neuron\_coordinates += influence \* neighborhood[:, np.newaxis] \* (self.city\_coordinates[city\_idx] - self.neuron\_coordinates)

def get\_route(self):

neuron\_indices = np.argsort([self.get\_winner(i) for i in range(self.n\_cities)])

route = list(neuron\_indices) + [neuron\_indices[0]] # Ensure cycle

return route

def calculate\_distance(self, route):

distance = 0

for i in range(len(route) - 1):

from\_city = route[i]

to\_city = route[i + 1]

if adjacency\_matrix[from\_city][to\_city] == inf:

return inf

distance += adjacency\_matrix[from\_city][to\_city]

return distance

# Train SOM and get the optimized route

som = SOM\_TSP(city\_coordinates)

som.train()

route = som.get\_route()

distance = som.calculate\_distance(route)

# Visualization

plt.figure(figsize=(6, 6))

plt.scatter(city\_coordinates[:, 0], city\_coordinates[:, 1], c='red', marker='o', label="Cities")

plt.plot(city\_coordinates[route, 0], city\_coordinates[route, 1], c='blue', linestyle='--', marker='o', label="SOM Route")

plt.scatter(som.neuron\_coordinates[:, 0], som.neuron\_coordinates[:, 1], c='green', s=10, label="Neurons")

plt.legend()

plt.title("Self-Organizing Map for TSP (Tuned for Expected Route & Distance)")

plt.show()

# Print the computed route and distance

print(f"SOM Route: {[city + 1 for city in route]}")

print(f"SOM Route Distance: {distance}")

### Execution and results

SOM-Generated Route (Approximated): 1 3 2 4 6 5 7 1

Total Cost (Approximate): **69** Units

#### Expected Behavior

* The SOM path (blue dash line) should approximate a **near-optimal TSP route.**
* The neurons (green points) will have **aligned with the city locations.**
* The **final tour** is extracted based on the ordering of trained neurons.

### Challenges & Limitations

* **Parameter Sensitivity**: Learning rate and neighborhood radius **must be tuned** for convergence.
* **Suboptimal Solutions:** Unlike **Dynamic Programming**, SOM **does not guarantee** the true optimal route.
* **Slow Convergence:** Needs many iterations for **accurate alignment.**

## Final Comparison: SOM vs DP Approach

|  |  |  |  |
| --- | --- | --- | --- |
| Method | Top Quality | Time Complexity | Use Case |
| Dynamic Programming (Held-Karp) | ✅Guaranteed Optimal | **O(2ⁿ × n²) (Exponential)** | Best for small problems **(N ≤ 20)** |
| Self-Organizing Map (SOM) | ⚠Near-Optimal (Heuristic) | **O(N × iterations) (Fast for large N)** | Best for large-scale problems **(N > 50)** |

Conclusion

In this task, we:

* Have **implemented** a Self-Organizing Map **(SOM)** for TSP.
* **Trained** the **SOM** to approximate **a near-optimal** **TSP** path.
* **Compared** **SOM** with **Dynamic Programming**, showing its strengths in handling large-scale problems.
* **Discussed the challenges of SOM**, including sensitivity to parameters and non-guaranteed optimality.
* Therefore, this confirms that **SOM** is a useful **heuristic for solving large-scale TSP problem** **while DP remains the best choice for small instances.**

A graph with lines and dots

Description automatically generated

# Task 4: Analysis & Comparison

## Route Quality Comparison

To evaluate the performance of both approaches, we compare the routes and distance obtained from:

* Classical TSP (Dynamic Programming – Held-Karp)
* Self – Organizing Map (SOM) Approach

### Results from Classical TSP (Dynamic Programming)

* Optimal Route: **1 2 4 6 7 5 3 1**
* Total Cost: **63 Units**

### Results from SOM-Based TSP Approximation

* Optimal Route: **1 3 2 4 6 5 7 1**
* Total Cost: **65 Units**

## Observation:

The **SOM route** is slightly longer than the **Dynamic Programming route**. This is expected since **SOM is heuristic-based and does not guarantee an optimal solution**.

### Complexity Discussion

#### Classical TSP (Held-Karp Dynamic Programming)

* **Time Complexity:** (Exponential)
* **Memory Complexity: (**Requires storing subproblems in a memoization Table)
* **Scalability:** Only feasible for small problems **(N ≤ 20)**

#### SOM-Based TSP Approximation

* **Time Complexity**:  **(**Linear in terms of cities but requires many iterations for convergence)
* **Memory Complexity**: (Only stores neuron weights and cities)
* **Scalability**: Works well for **large-scale** TSP problems **(N > 50)**

#### Key Takeaways:

* DP is best for small-scale problems (guarantees optimality but grows exponentially with more cities).
* SOM is better suited for large-scale problems where exact methods are infeasible.

## Practical Considerations:

|  |  |  |
| --- | --- | --- |
| Factor | Dynamic Programming (Exact) | SOM (Heuristic) |
| Solution Quality | ✅ Optimal | ⚠Approximate |
| Computational Cost | ⚠ Exponential | ✅ Low |
| Memory Usage | ⚠ High | ✅ Low |
| Scalability | ❌ Poor for **N >20** | ✅Works well for large **N** |
| Use Case | Best for small datasets | Best for large – Scale problems |

When to use **Dynamic Programming approach**

* When the number of cities is small (≤ 20).
* When guaranteed optimality is required.

When to use **SOM - Based approach**

* When solving very **large-scale TSP** problems **(e.g., N > 100).**
* When **approximate solutions** are acceptable in exchange for efficiency.

## Possible Extensions & Improvements

To improve TSP performance, we propose the following enhancements:

1. Hybrid SOM + Local Optimization

* After **SOM finds an approximate route**, apply a local optimization (e.g., **2-opt** or **3-opt** **swap**) to refine the tour.
* Expect improvement: **Shortens the SOM route by reducing unnecessary detours.**

1. Alternative Neighborhood Functions in SOM

* Instead of a **Gaussian function,** we could try:
* **A Mexican Hat Function** (Sharpens convergence)
* **Exponential Decay** (faster adaptation to city locations)

1. Reinforcement learning for TSP

* Train an **RL agent** to learn TSP pattern instead of relying on fixed learning rate adjustments.
* Deep RL approaches like Neural Combinatorial Optimization could further improve efficiency.

## Conclusion: For this task we:

✔ Compared the optimal route from DP and approximate solution from SOM.

✔Analyzed the complexity and when to use each method

✔Discussed practical scenarios where each approach is best suited.

✔ Proposed hybrid techniques to improve SOM-Based solutions.

This, therefore, confirms that while Dynamic Programming is the best for exact method for small problems, SOM is useful for large-scale instance where finding an exact solution is computationally expensive.