Spell analysis in multistate Markov models

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Abstract

Multistate matrix population models are typically used to compute statistics such as the state occupancies or the moments thereof. Recently formulas have been derived to compute the average number of spells for a given state, which combined with the average state occupancy yields the average spell duration. I've since wondered what the age pattern to average spell durations is. Here I demonstrate how to use simulations to estimate a variety of age and other patterns of different state spells.

1 Introduction

The amount of population-level measures that one might devise for a particular population-level process is dizzying, so it may beg the question from the outset why we might desire to have more. Matrix-based manipulations of incidence-based models are rather undeveloped with respect to tenure-statistics, and these might be of interest for a variety of substantive reasons. By tenure statistics I refer to the statistics of particular spells or episodes of a state. Namely, in incidence-based models with bidirectional flows (i.e. allowing for recovery and then re-onset, and so on), a hypothetical individual may pass through a state (say sickness) many times before death. Typically a transition matrix manipulation would only give us the average time spent sick (or moment statistics thereof). Recently matrix calculations have been described for how to calculate the average number of episodes of a given state (Dudel and Myrskylä 2017a). Combined with the average state occupancy, this information yields the average duration of episodes.

One may wonder how the average spell duration changes with age, and for this there is no ready matrix expression (although it would of course be possible). I will procede using simulations rather than matrix calculations because it will save the work of deriving and checking dozens of formulas. In this way, we have the liberty to change definitions without incuring methodological setbacks. Since we simulate, we get stochastic stationary distributions of each measurement, which I'll represent using fan-chart visualizations. This approach is not all that different from that proposed by Laditka and Wolf (1998), but I take things a bit farther by proposing a suite of age realignments and resulting age-like patterns of episode statistics.

2 Data

The point of departure for all calculationsis a transition matrix. To demonstrate, I use a published transition matrix from a recent study of working life expectancy in the United States (Dudel and Myrskylä 2017b). This matrix refers to black females aged 50-100 in 1994. The same sorts of things can be done with any age-stage matrix. Actually technically you could do the same with an ageless matrix, if the simulated sequence steps are interpretable as age.

3 Simulation

I take advantage of the recently published `R` package `markovchain` (Spedicato 2017), which includes a random state sequence generator function `rmarkovchain()` that mereley requires a transition matrix to

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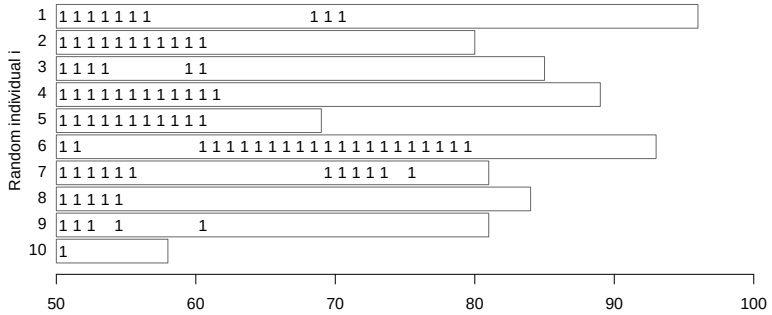
do its work.[1] I generate a large number, say 10k or more trajectories to operate on as the stationary population. Each trajectory consists in a `character` vector of states [`Employed`,`Inactive`,`Retired`,`Dead`], where dead is of course an absorbing state, but the other states can be switched on and off annually from ages 50 and higher.[2]

A glimpse of the first 10 randomly generated individuals is shown in Figure 1. These ten individuals will be recycled in all of the following data manipulations used to demonstrate concepts. All aggregate calculations of age patterns (and so on) are done on the full simulated population. In this case, I simulated assuming that one starts in a state of employment at age 50, but the starting state could also easily be a mixture of states.

Figure 1: Ten randomly generated state sequences from the 1994 transition matrix of black females (Dudel and Myrskylä 2017b)

Standard calculations of prevalence typically procede by imputing reference states with 1s (with 0s elsewhere) and taking column means over survivors in each age. Figure 2 shows such a data construct, where the state sequence matrix has been converted to a binary matrix, with 1s for employment episodes, 0s for other living states (shown blank). Typically one might impute NA values in dead states for this sort of calculation. Operations on objects such as this can yield age patterns of prevalence or expectancies, for example.

Figure 2: Binary imputation of employment spells



4 Running clocks and alignment

Beyond counting episodes, one may wish to aggregate statistics on each episode in novel ways. For instance, conditional on being in state s in age x, what is the average duration of spell that one finds oneself in?

[1] There are some trivial object definition steps to convert the employment matrix into a conformable markov object before feeding it to the random generator.

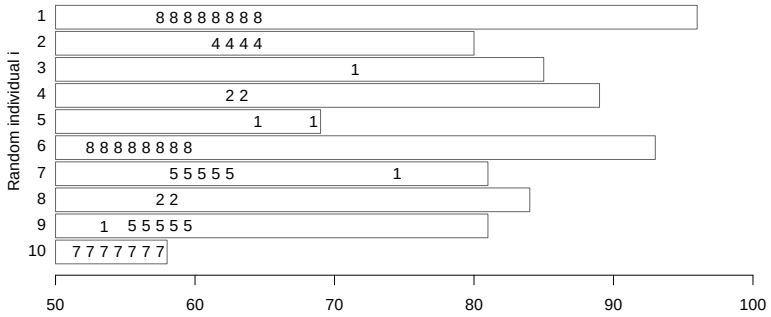
[2] It would also be possible to further graduate transition rates to standardized month or week units to produce higher resolution sequences, but this is unnecessary for the present treatment.

4.1 Clocks

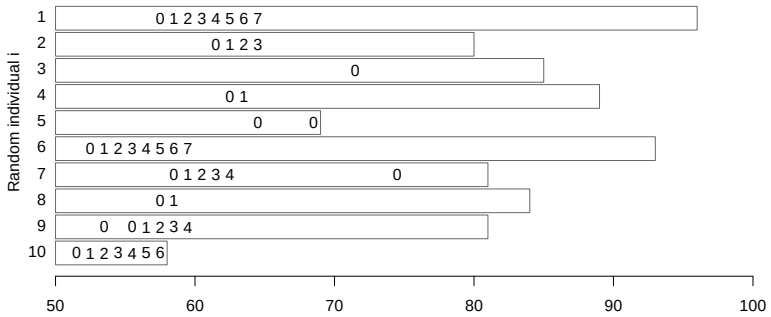
To calculate the average spell duration by age, first convert our state sequences to a data object something like Figure 3a. Using chronological age as the reference, one may also wish to calculate time spent or left in the state episode, per Figure 3b or 3c [3]. In either of these cases, value alignment is with respect to episode entry or exit, but aggregation alignment remains pegged to age. Statistics across individuals in an array with therefore produce age patterns.

Figure 3: Inactivity spells from Figure 1 are imputed with different duration count variables. It's probably better to add $\frac{1}{2}$ to the displayed *running* values.

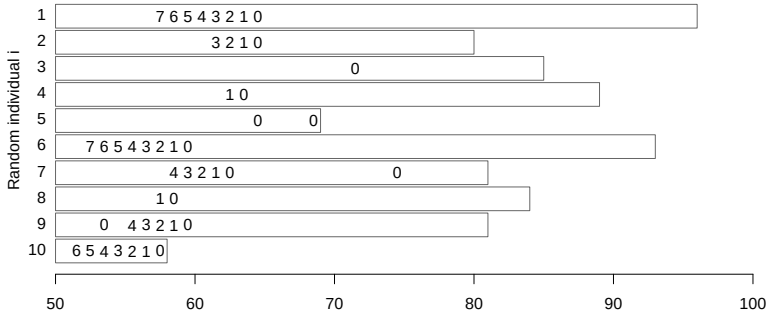
(a) Static; Total episode duration of inactivity.



(b) Running; Time spent in episode of inactivity.



(c) Running; Time left in episode of inactivity



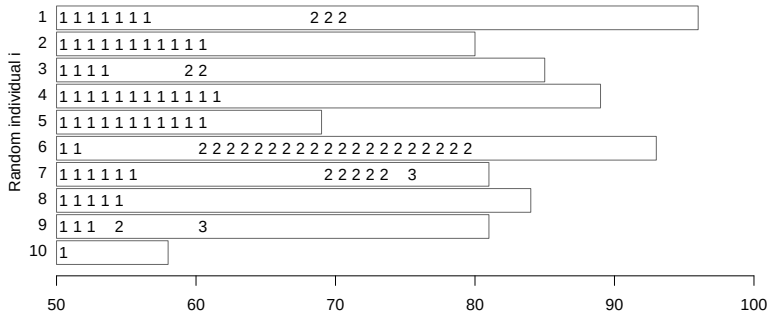
One may fill episodes with other markers, such as episode order, as in Figure 4 for the case of employment spells, or episode fractions. One could further condition age patterns of total duration, time spent, or time left on episode order. If spells are filled with 1s, then aggregation results in prevalence. Note, time *left* in the episode has no left-truncation problem, also not in the aggregate. These series of values, that I call clocks, are then aggregated in some way, and aggregation is always within some

[3] Actually, I'd increment values by $\frac{1}{2}$ for mid-state clocking, but decimals would squeeze the figure too much.

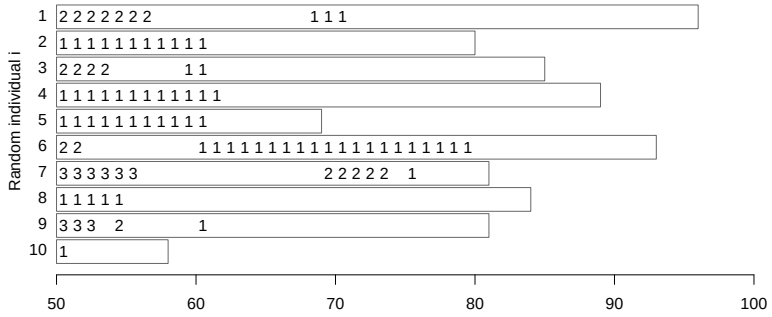
external structuring classes that remain to be defined.

Figure 4: Employment episodes from Figure 1 are imputed with order count variables.

(a) Employment episode order, increasing.



(b) Employment episode order, decreasing.



4.2 Alignment

Episodic clock values are aggregated according to some structuring criteria. In all previous figures, the structuring criteria was chronological age, which is how data were generated in the first instance. To introduce a term, these figures are *left-aligned* on the event of birth. This is the most common default alignment in social and medical sciences, but other choices may be more compelling for particular questions.

For late-life processes, birth is usually a long ways off, and empirical regularity may be be found with respect to other alignment criteria. Aligning lifelines requires two choices: 1) a reference moment or anchoring *event* must be selected, and 2) the alignment direction must be chosen. A reference event could be any instance of entry, exit, or other compelling anchor point, such as a spell midpoint– ergo such events likely relate to episodes. For repeated events, the choice of anchoring episode could itself follow a regular criterion, such as first, last, or longest episode. The *direction* of alignment could be left, right, center, or perhaps something else.

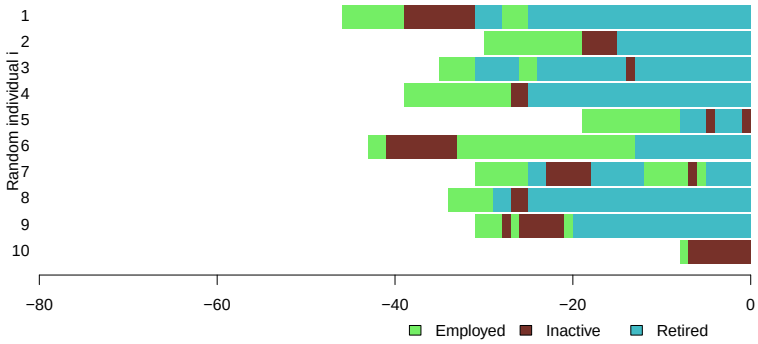
Aggregated patterns would certainly turn out different if we were to right-align on the moment of death, per Figure 5a. This particular realignment doesn't seem so compelling for the demonstrated process, but I suppose it would be illuminating for health states and the sequence of events leading up to death.

Figure 5 shows a set of four alignment selections out of the many possible choices. Figure 5b left-aligns on entry to *first* retirement (if any). One could also choose last, longest, or some other episode of retirement, or of course right-align on exit. Figure 5c left-aligns on entry into each individual's longest spell of inactivity, whereas Figure 5d right-aligns on exit from the same spell.

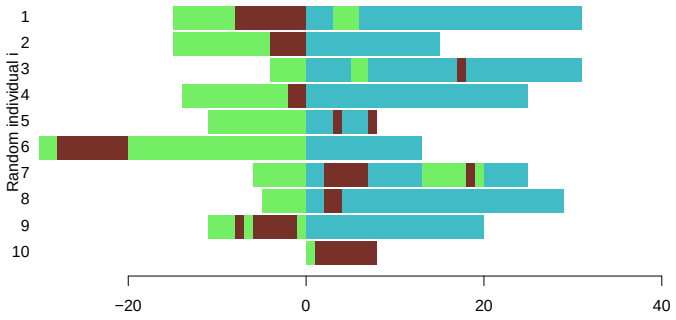
The purpose of realigning is to bring transitions into focus. We do not showcase *sorting* in this treatment, as this is another can of worms. Rather, here we would like to operate on aggregations of individual sequences, in which case between-individual sorting is unimportant. We'd still like to make statements about population-level characteristics.

Figure 5: The sequences from Figure 1 under a variety of alignment types.

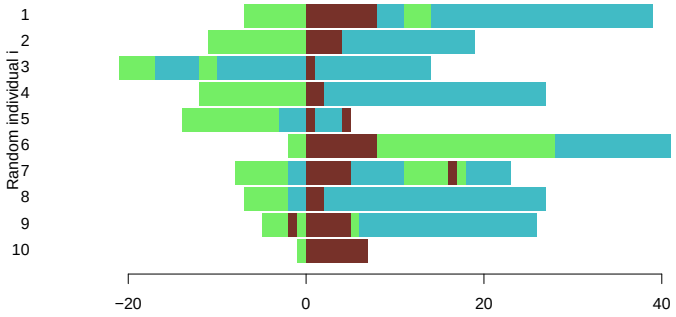
(a) Right-aligned on death.



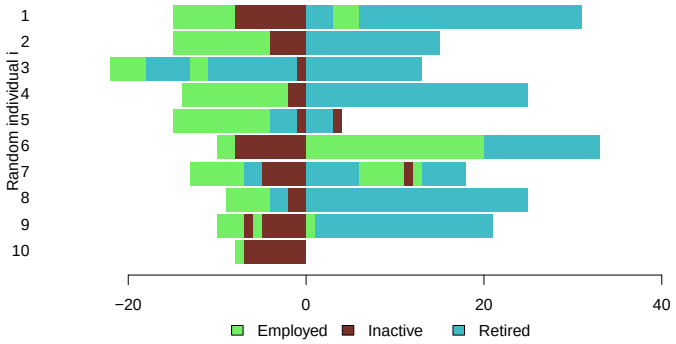
(b) Left-aligned on *first* retirement.



(c) Left-aligned on entrance to *longest* spell of inactivity



(d) Right-aligned on exit from *longest* spell of inactivity



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