

Spell analysis in multistate Markov models

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Abstract

Multistate matrix population models are typically used to compute statistics such as state occupancies or the moments thereof. Recently formulas have been derived to compute the average number of spells for a given state, which when combined with the average state occupancy yields the average spell duration. I've since wondered what the age pattern to average spell durations is. Here I demonstrate how to use simulations to estimate a variety of age and other patterns of different state episodes.

1 Introduction

The amount of population-level measures that one might devise for a particular population-level process is dizzying, so it may beg the question from the outset why we might desire to have more. Incidence-based matrix models are rather undeveloped with respect to tenure-statistics, and these might be of interest for a variety of substantive reasons. By tenure statistics I refer to the statistics of particular spells or episodes of a state. Namely, in incidence-based models with bidirectional flows (i.e. allowing for recovery and then re-onset, and so on), a hypothetical individual may pass through a state, such as sickness, many times before death. Typically a transition matrix manipulation would only give us the average time spent sick or moment statistics thereof. Recently matrix calculations have been described for how to calculate the average number of episodes of a given state (Dudel and Myrskylä 2017a). Combined with the average state occupancy, this information yields the average duration of episodes.

One may wonder how the average spell duration changes with age, and for this there is no ready matrix expression. Such analytic expressions are surely possible, but in the following I intend to propose a suite of operations (including the aforementioned) so broad and flexible as to fill the lifetime of an imaginary matrix-lover. I will proceed using simulations rather than matrix calculations because it will save the work of deriving and checking dozens of formulas. In this way, we have the liberty to change definitions without incurring methodological setbacks. Since we simulate, we get stochastic stationary distributions of each measurement for free, which I'll represent using fan-chart visualizations. This approach is not all that different from that proposed by Laditka and Wolf (1998), but I expand on their approach by proposing a set of count statistics called *clocks* and age *realignments*, resulting in a large set of age-like patterns of state episodes. Simulation is not strictly required if an empirical population of individual sequences is on hand, but these would pertain to a non-stationary population.

2 Data

The point of departure for all calculations is a transition matrix. To demonstrate, I use a published transition matrix from a recent study of working life expectancy in the United States (Dudel and Myrskylä 2017b). This matrix refers to black females aged 50-100 in 1994. The same sorts of things can be done with any age-stage matrix. One can indeed also do the same with an ageless transition matrix of the life course, if the simulated sequence steps are interpretable as age steps.

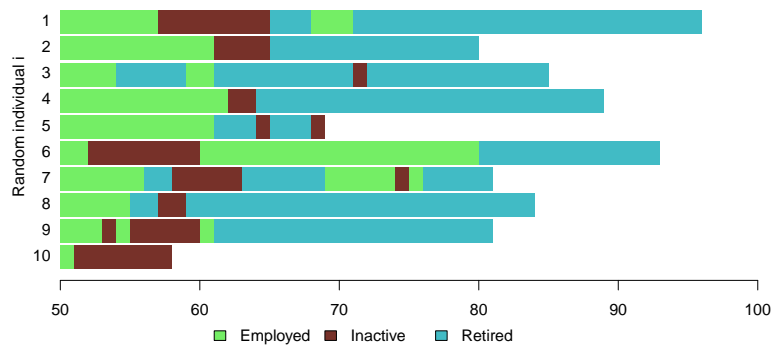
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3 Simulation

I take advantage of the recently published R package `markovchain` (Spedicato 2017), which includes a random state sequence generator function `rmarkovchain()` that merely requires a transition matrix to do its work.¹ I generate a large number, say 10k or more trajectories to operate on as the stationary population. Each trajectory consists in a `character` vector of states [`Employed`,`Inactive`,`Retired`,`Dead`], where dead is of course an absorbing state, but the other states can be switched on and off annually from ages 50 and higher.²

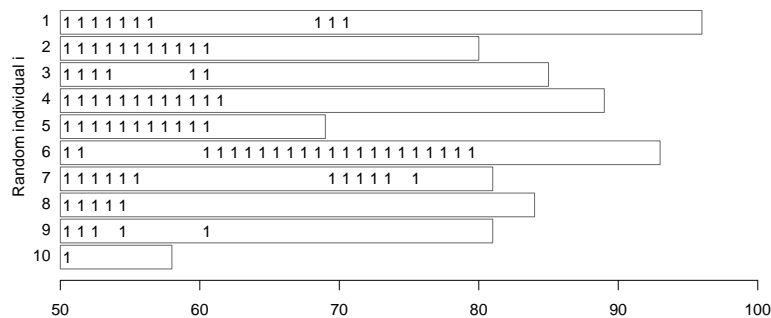
A glimpse of the first 10 randomly generated individuals is shown in Figure 1. These ten individuals will be recycled in all of the following data manipulations used to demonstrate concepts. All aggregate calculations of age patterns (and so on) are done on the full simulated population. In this case, I simulated assuming that one starts in a state of employment at age 50, but the starting state could also easily be a mixture of states.

Figure 1: Ten randomly generated state sequences from the 1994 transition matrix of black females (Dudel and Myrskylä 2017b)



Standard calculations of prevalence typically proceed by imputing reference states with 1s (with 0s elsewhere) and taking column means over survivors in each age. Figure 2 shows such a data construct, where the state sequence matrix has been converted to a binary matrix, with 1s for employment episodes, 0s for other living states (shown blank). Typically one might impute NA values in dead states for this sort of calculation. Operations on objects such as this can yield age patterns of prevalence or expectancies, for example.

Figure 2: Binary imputation of employment spells



¹There are some trivial object definition steps to convert the employment matrix into a conformable markov object before feeding it to the random generator.

²It would also be possible to further graduate transition rates to standardized month or week units to produce higher resolution sequences, but this is unnecessary for the present treatment.

4 Running clocks and alignment

Beyond counting episodes, one may wish to aggregate statistics on each episode in novel ways. For instance, conditional on being in state s in age x , what is the average duration of spell that one finds oneself in? Or time spent or left in the state?

4.1 Clocks

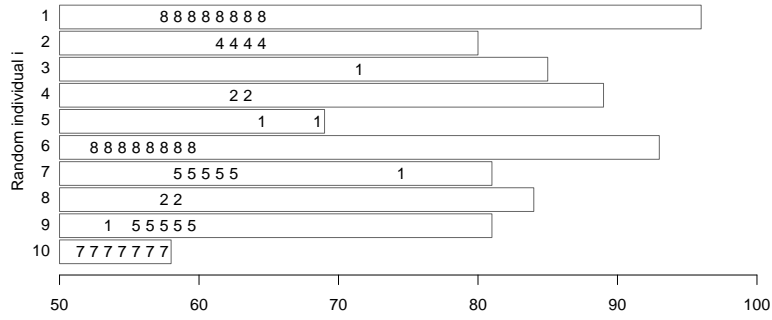
To calculate the average spell duration by age, first convert our state sequences to a data object something like Figure 3a. Using chronological age as the reference, one may also wish to calculate time spent or left in the state episode, per Figure 3b or 3c³. In either of these cases, value alignment is with respect to episode entry or exit, but aggregation alignment remains pegged to age. Statistics across individuals in an array will therefore produce age patterns.

One may fill episodes with other markers, such as episode order, as in Figure 4 for the case of employment spells, or episode fractions. One could further condition age patterns of total duration, time spent, or time left on episode order. If spells are filled with 1s, then aggregation results in prevalence. Note, time *left* in the episode has no left-truncation problem, also not in the aggregate. These series of values, that I call clocks, are then aggregated in some way. Aggregation proceeds within some external structuring classes defined in the following section.

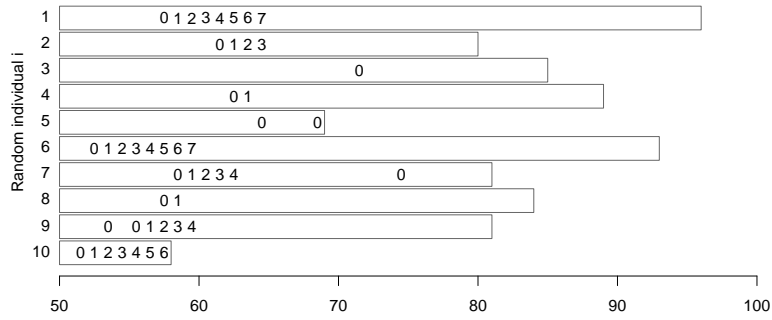
³Actually, I'd increment values by $\frac{1}{2}$ for mid-state clocking, but decimals would squeeze the figure too much.

Figure 3: Inactivity spells from Figure 1 are imputed with different duration count variables. It's probably better to add $\frac{1}{2}$ to the displayed *running* values.

(a) Static; Total episode duration of inactivity.



(b) Running; Time spent in episode of inactivity.



(c) Running; Time left in episode of inactivity

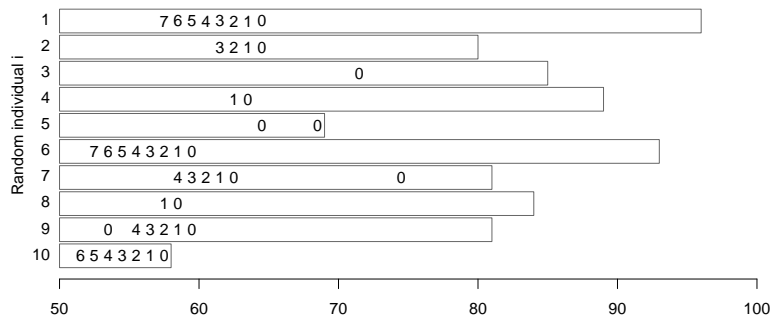
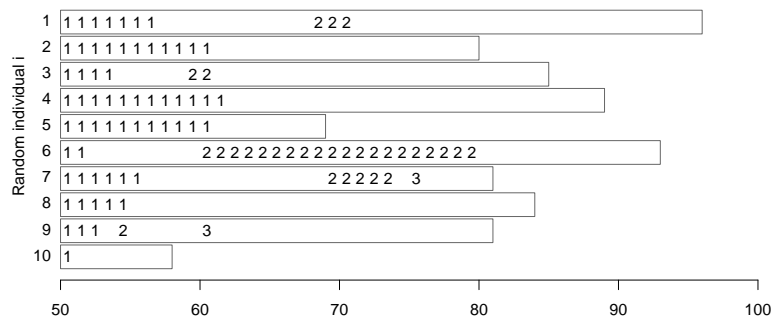
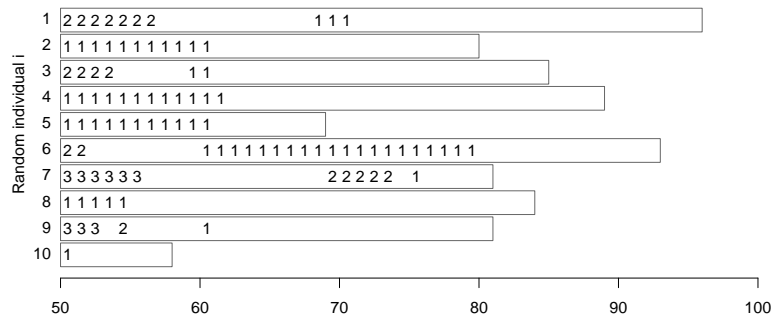


Figure 4: Employment episodes from Figure 1 are imputed with order count variables.

(a) Employment episode order, increasing.



(b) Employment episode order, decreasing.



4.2 Alignment

Episodic *clock* values are aggregated according to some structuring criteria. In all previous figures, the structuring criteria was chronological age, which is how data were generated in the first instance. To introduce a term, the sequences in these figures are *left-aligned* on the event of birth. This is the most common default alignment in social and medical sciences, but other choices may be more compelling for particular questions.

For late-life processes, birth is usually decades away from the events and states of interest, and sharper empirical regularity may be found with respect to other alignment criteria. Aligning lifelines requires two choices: 1) a reference moment or anchoring *event* must be selected, and 2) the alignment direction must be chosen. A reference event could be any instance of entry, exit, or other compelling anchor point, such as a spell midpoint—ergo such events may relate to episodes themselves. For repeated events, the choice of anchoring episode could itself follow a regular criterion, such as first, last, or longest episode. The *direction* of alignment could be left, right, center, or perhaps something else.

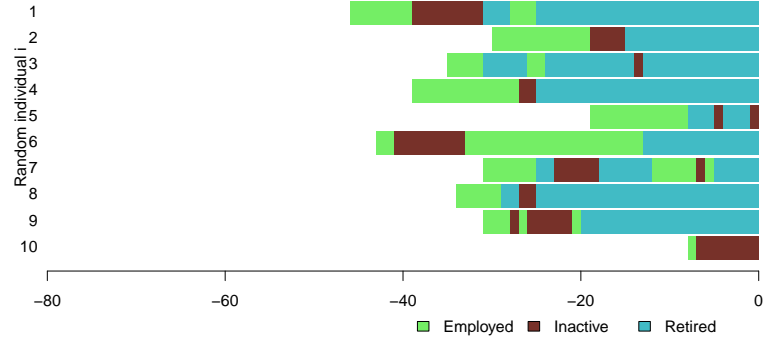
Aggregated patterns would certainly turn out different if we were to right-align on the moment of death, per Figure 5a. This particular realignment doesn't seem so compelling for the demonstrated process, but I suppose it would be illuminating for health states and the sequence of events leading up to death.

Figure 5 shows a set of four alignment selections out of the many possible choices. Figure 5b left-aligns on entry to *first* retirement (if any). One could also choose last, longest, or some other episode of retirement, or of course right-align on exit. Figure 5c left-aligns on entry into each individual's longest spell of inactivity, whereas Figure 5d right-aligns on exit from the same spell.

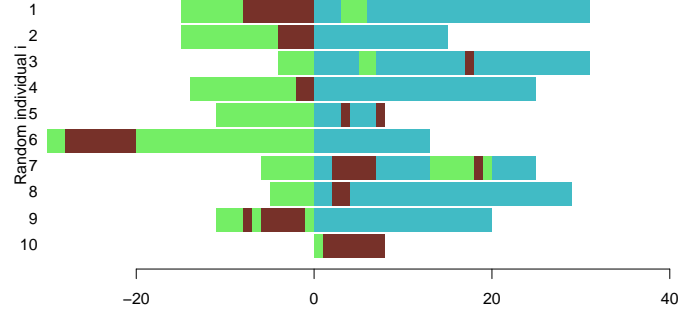
The purpose of realigning is to bring transitions into focus. We do not showcase *sorting* in this treatment, as this is another can of worms, likely in need of sophisticated clustering and sequence analysis. For researchers doing sequence analysis, count and alignment operations would precede standard analysis, and one would need to be judicious in choosing a clustering algorithm that is less sensitive to realignment. For the present, I would rather like to operate on aggregations of individual sequences, in which case between-individual sorting is unimportant. We'd still like to make statements about population-level characteristics.

Figure 5: The sequences from Figure 1 under a variety of alignment types.

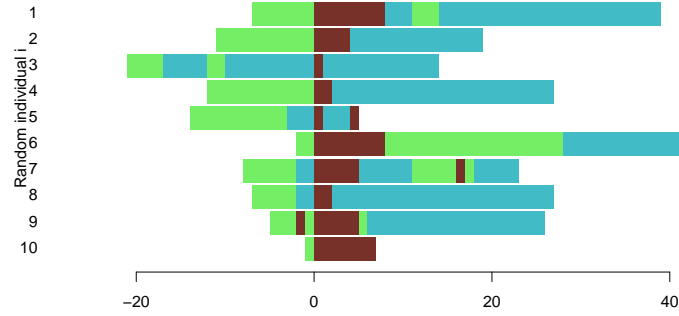
(a) Right-aligned on death.



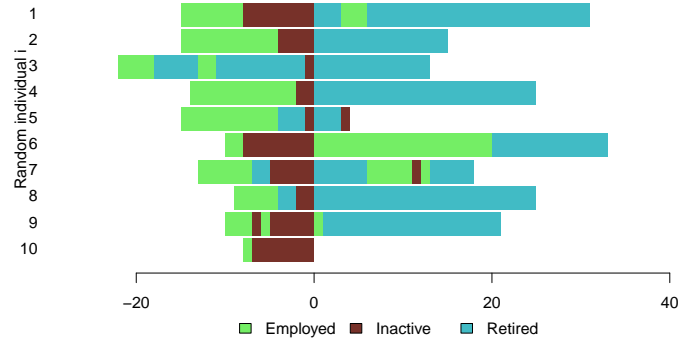
(b) Left-aligned on *first* retirement.



(c) Left-aligned on entrance to *longest* spell of inactivity



(d) Right-aligned on exit from *longest* spell of inactivity



5 Aggregate patterns

Include a big set of patterns from the example matrix. Somehow. Systematically. Like a 5x5 panel plot, where five clocks in columns and five alignments in rows. Some combinatorics to see the researcher degrees of freedom too.

6 Discussion

As stated the purpose of episode clocks and sequencec realignment is to detect important patterns in data that are likely to otherwise go unnoticed. Some reasonable priors might include that i) life course events condition each other; ii) temporal proximity to life course transitions is likely to be an important predictor of other transitions; iii) within-episode patterns of other characteristics might be monotonically increasing or decreasing, or concave or convex. Therefore, it may be the case that aggregate patterns derived from such operations are simply sharper and of more obvious interpretation and consequence than are age patterns themselves.

7 Conclusions

References

- C. Dudel and M. Myrskylä. Estimating the expected number and length of episodes using markov chains with rewards. (under review), 2017a.
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