

Category	Student Score	Grader Score
Organization		
Basics	1/1	/1
Structure	1/1	/1
Work		
Effort	2/2	/2
Clarity	1/2	/2
Discussion	2/2	/2
Correctness	1/2	/2
Total	8/10	/10

1- Euler Practice

$$A = (3e^{-3j})(-j)(2e^{j\frac{9\pi}{4}})$$

$$= \boxed{-6e^{\left(\frac{9\pi}{4} - 3 + \frac{\pi}{2}\right)j}}$$

$$= \boxed{-4.799 + 3.601j}$$

$$B = j \frac{te^{-\frac{16\pi}{3}}}{3e^{j\frac{9\pi}{4}}}$$

=

$$\frac{t}{3} e^{\left(\frac{-16\pi}{3} - \frac{9\pi}{4} + \frac{\pi}{2}\right)j}$$

$$= \frac{\cancel{\frac{17\pi}{12}}}{\frac{t}{3} e^{\frac{85\pi}{12}j}}$$

$$\text{rect} \\ -1.298 - .345j$$

$$C = \frac{(-1+j)(j)(2-3j)}{(3-j)(2j+4)} = \frac{(-j+j^2)(-2-3j)}{6j+12-2j^2-4j}$$

$$= \frac{+2j+3j^2+2+3j}{2j+14}$$

$$= \frac{5j-1}{2j+14}$$

$$\boxed{-\frac{1}{50} + \frac{9}{25}j}$$

$$\tan^{-1}\left(\frac{5j-1}{2j+14}\right)$$

$$\frac{70}{200j} + \frac{-4}{200} = \frac{-10j^2+70j+2j-14}{192-200} = \frac{(5j-1)(-2j+14)}{-4j^2+14^2}$$

2- Matrices

$$M \begin{bmatrix} x \\ y \end{bmatrix} = B$$

$$\begin{bmatrix} 2 & -5 \\ -2 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$\frac{1}{14-10} \begin{bmatrix} 7 & 5 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & -5 \\ -2 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 7 & 5 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$\frac{1}{4} \begin{bmatrix} (14-10)(-35+35) \\ (4-4)(-10+14) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 21-20 \\ 6-8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$x = \frac{1}{4}$$

$$y = -\frac{1}{2}$$

```
disp([2,-5;-2,7]*[1/4;-1/2]);
```

```
>> test
```

```
3
```

```
-4
```

3- VECTORS

A

Handwritten calculation of the dot product $C \cdot F$ on a blackboard:

$$C \cdot F = (-3\hat{x} + 2\hat{z} - 2\hat{y}) \cdot (2\hat{x} - 4\hat{y} + 2\hat{z})$$
$$\begin{aligned} &= -6\hat{x}\hat{x} + 12\hat{x}\hat{y} - 6\hat{x}\hat{z} + 4\hat{z}\hat{x} - 8\hat{z}\hat{y} + 4\hat{z}\hat{z} - 4\hat{y}\hat{x} + 8\hat{y}\hat{y} - 4\hat{y}\hat{z} \\ &= -6 + 0 + 0 + 0 + 0 + 4 + 0 + 8 + 0 = 6 \end{aligned}$$

The final result is boxed: $C \cdot F = 6$

b)

$$\begin{aligned}
 P \times C &= (3\hat{x} - 5\hat{z}) \times (-3\hat{x} + 2\hat{z} - 2\hat{y}) \\
 &= \begin{vmatrix} 3 & 0 & -5 \\ 0 & -2 & 2 \\ -3 & 1 & 0 \end{vmatrix} \\
 &= 3(0 - 2) - 10(-6) - 10(-6) \\
 &= -6\hat{x} + 60\hat{y} + 60\hat{z} \\
 &= \boxed{-10\hat{x} + 9\hat{y} - 6\hat{z}}
 \end{aligned}$$

c)

$$\begin{aligned}
 \cos \theta &= \frac{F \cdot P}{|F||P|} = \frac{-4}{\sqrt{24}\sqrt{34}} \\
 F \cdot P &= 6 - 10 = -4 \\
 |F| &= \sqrt{4 + 16 + 4} = \sqrt{24} \\
 |P| &= \sqrt{9 + 25} = \sqrt{34} \\
 \theta &= \cos^{-1}\left(\frac{-4}{\sqrt{24}\sqrt{34}}\right) \\
 &= \\
 \theta &= 1.71 \text{ rad}
 \end{aligned}$$

d)

$$P \times C = -10\hat{x} + 9\hat{y} - 6\hat{z}$$

$$|P \times C| = \sqrt{100 + 81 + 36}$$

$$\boxed{\sqrt{217} \text{ units}^2}$$

e)

$$C \times P = \frac{1}{\Delta} \begin{bmatrix} i & j & k \\ -3 & -2 & 2 \\ -3 & 0 & -5 \end{bmatrix} = \frac{10i - 6j + 0k}{\Delta} = \frac{0i + 15j + 6k}{\Delta}$$

$$= \frac{10i - 21j + 6k}{\Delta}$$

$$= \frac{2i - 4j + 2k}{\Delta}$$

$$= \frac{20 + 84 + 12}{\Delta}$$

$$\boxed{116}$$

4- COMPLEX

a) Represent A and B in the exponential form

$$A = j e^{-j\frac{7\pi}{6}}$$

$j(a + bj) = aj - b$

$x = -y$
 $y = x$

multiply by j moves 90° CC

$$A = j e^{-j\frac{7\pi}{6}}$$

j moves $90^\circ (\frac{\pi}{2})$ to the CC direction

$$A = e^{+j(\frac{\pi}{2} - \frac{7\pi}{6})}$$

$$= e^{\frac{2\pi}{3}j}$$

$$B = (-j) (5 e^{-j\frac{5\pi}{6}})$$

j moves CC $\frac{\pi}{2}$ $(-)$ flips 180° or π

$$B = -5 e^{j(\frac{\pi}{2} - \frac{5\pi}{6} + \pi)}$$

$$= 5 e^{\frac{12\pi}{3}j}$$

b) Find $Q = AB/G$ in rectangular and exponential forms

$$G = -4 + 4j$$

$$= 4\sqrt{2} e^{\frac{3\pi}{4}j}$$

$$\frac{AB}{G} = \frac{e^{\frac{2\pi}{3}j} \times 5 e^{\frac{2\pi}{3}j}}{4\sqrt{2} e^{\frac{3\pi}{4}j}}$$

$$= \frac{5 e^{\frac{4\pi}{3}j}}{4\sqrt{2} e^{\frac{3\pi}{4}j}} = \frac{5}{4\sqrt{2}} e^{\frac{7\pi}{12}j}$$

$$= -0.228 + 0.851j$$

c) Find T so that A is equal to C

$$T = e^{\frac{35\pi j}{6}}$$

$$A = j e^{-\frac{\pi}{6}j} = T e^{-j\frac{13\pi}{2}}$$

$$= \frac{-2\pi j}{e^3} = T e^{j\frac{13\pi}{2}} e^{\frac{35\pi j}{6}}$$

$$\frac{-2\pi j}{3} = \ln(T) - j\frac{13\pi}{2}$$

$$\ln T = \frac{35\pi j}{6}$$

5- Vectors

a)

$$\cos \theta = \frac{N \cdot Q}{|N| |Q|} = \frac{6}{2\sqrt{10}}$$

$$N \cdot Q = 6$$

$$|N| = \sqrt{1+9} = \sqrt{10}$$

$$|Q| = \sqrt{4} = 2$$

$$\cos^{-1}\left(\frac{6}{2\sqrt{10}}\right) =$$

$$\boxed{18.4^\circ \text{ or } .322 \text{ rad}}$$

b)

$$\boxed{M_{||} = \frac{M \cdot N}{|N|^2} (N) = \left[\frac{2}{10} (\hat{y} - 3\hat{x}) \right]}$$

$$M \cdot N = 2$$

$$|N| = \sqrt{10}$$

$$M_{\perp} = M - M_{||} = -\hat{z} + 2\hat{y} - \frac{2}{10}\hat{y} + \frac{6}{10}\hat{x}$$

$$\boxed{M_{\perp} = \frac{6}{10}\hat{x} + \frac{18}{10}\hat{y} - \hat{z}}$$