EE 435 Homework 2 Spring 2024

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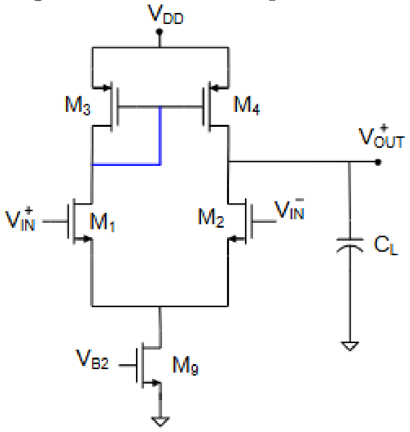
GitHub Page

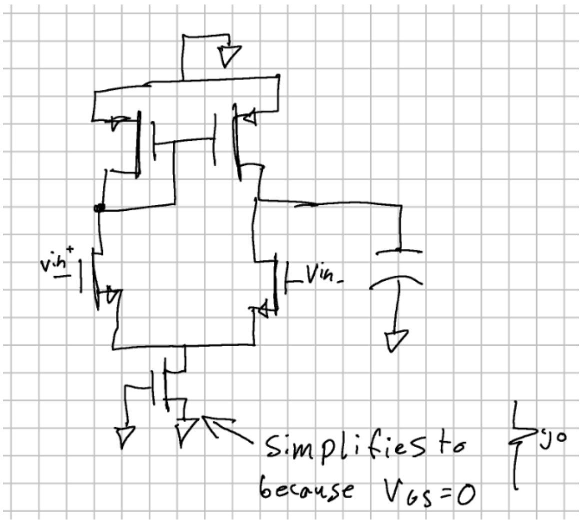
Problem 1 and 2

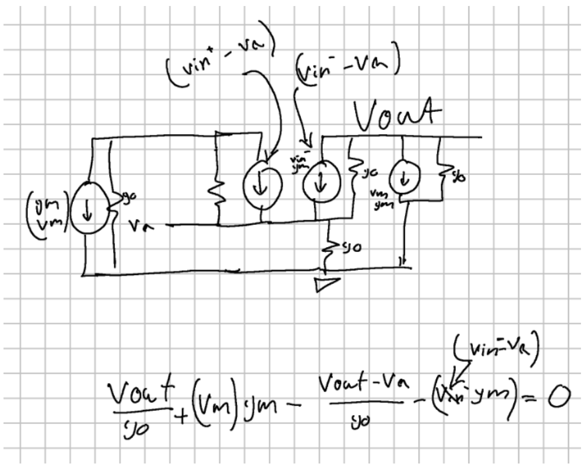
Consider the following operational amplifier. The goal is to obtain an expression for the small-signal output voltage in terms of the input variables INV+ and INV-.



a)

Write a complete set of small-signal equations that can be solved to obtain V_{OUT}. Assume the small-signal parameter g_o is present in all MOS devices.





Three equations are created via KCL analysis of the small signal model. They are as follows:

$$(V_{\text{out}} - V_a) \cdot g_0 + V_{\text{out}} \cdot g_0 + (V_{\text{inN}} - V_a) \cdot g_m + V_m \cdot g_m = 0$$

$$(V_m - V_a) \cdot g_0 + V_m \cdot g_0 + (V_{\text{inP}} - V_a) \cdot g_m + V_m \cdot g_m = 0$$

$$V_a \cdot g_0 + (V_a - V_m) \cdot g_0 + (V_a - V_{\text{out}}) \cdot g_0 - (V_{\text{inP}} - V_a) \cdot g_m - (V_{\text{inN}} - V_a) \cdot g_m = 0$$

b)

Solve these equations by hand for V_{OUT}. If you do not have a solution at the end of $\frac{1}{2}$ hour, stop, and comment on your progress and the amount of effort that you believe would be required to finish the solution.

INCORRECT

NOTE: this is attempt was incorrect because I used go as resistance rather than conductance

$$\boxed{1}\quad \frac{V_{out}}{g_0} + V_m g_m = \left(\frac{V_{out}-V_a}{g_0} + (V_{in}^- - V_a)g_m\right)$$

$$\boxed{2}\quad \frac{V_m}{g_0} + V_m g_m = \left(\frac{V_m - V_a}{g_0} + (V_{in}^+ - V_a)g_m\right)$$

$$\frac{V_a}{g_0} = (V_{in}^+ - V_a)g_m + (V_{in}^- - V_a)g_m + \frac{V_m - V_a}{g_0} + \frac{V_{out}-V_a}{g_0}$$

$$\frac{V_a}{g_0} = \frac{V_m}{g_0} + V_m g_m + \frac{V_{out}}{g_0} + V_m g_m$$

$$\frac{V_a}{g_0} = \frac{V_m + V_{out}}{g_0} + 2(V_m)(g_m)$$

$$\boxed{3}\quad \boxed{V_a = V_m + V_{out} + 2(g_m)(V_m)(g_0)}$$

2 unknowns and 3 equations

$$\frac{V_{out}}{g_0} + (V_m)gm = \frac{V_{out} - V_a}{g_0} + V_{in-}g_m \tag{1}$$

$$\frac{V_{out}}{g_0}\frac{V_{out} - V_a}{g_0} = (V_m)gm + V_{in-}g_m \tag{2}$$

$$V_{out} * \frac{2}{g_0} = (V_{in-} + V_m - V_a)g_m + \frac{V_a}{g_0} \tag{3}$$

$$V_{out} = \frac{(V_{in-} + V_m - V_a)g_m}{2} + \frac{V_a}{2} \tag{4}$$

$$V_m = (\frac{-V_{out}}{g_0} + V_{in-}g_m) * \frac{1}{\frac{1}{g_0} - g_m + 2g_m^2 g_0} \tag{5}$$

$$V_a = V_m + V_{out} + 2(g_m)(V_m)(g_0) \tag{6}$$

$$V_a = V_{out} + V_m(1 + 2(g_m)(g_0)) \tag{7}$$

Now replace all V_a in the VOUT equation.

$$V_{out} = \frac{(V_{in-} + V_{out} - 2(g_m)(V_m)(g_0))g_m}{2} + \frac{V_m + V_{out} + 2(g_m)(V_m)(g_0)}{2} \tag{8}$$

Now replace all V_m in the equation.

$$\begin{aligned} V_{out} = {} & \frac{(V_{in-} + V_{out} - 2(g_m)((\frac{-V_{out}}{g_0} + V_{in-}g_m) * \frac{1}{\frac{1}{g_0} - g_m + 2g_m^2 g_0})(g_0))g_m}{2} \\ & + \frac{(\frac{-V_{out}}{g_0} + V_{in-}g_m) * \frac{1}{\frac{1}{g_0} - g_m + 2g_m^2 g_0} + V_{out} + 2(g_m)((\frac{-V_{out}}{g_0} + V_{in-}g_m) * \frac{1}{\frac{1}{g_0} - g_m + 2g_m^2 g_0})(g_0)}{2} \end{aligned} \tag{9}$$

At this point I spent way to much time on this problem. So I moved onto part c. I then realized that I was using g0 as a resistance rather than conductance.
CORRECTED

c)

Obtain a parametric (symbolic) solution for the transfer function V_{OUT}/V_{IN} from this set of equations with MATLAB. How many total product terms appear in this solution? In this part, $V_{IN} = V_{IN+} - V_{IN-}$.

```
syms Vm Va Vout go gm Vin VinN VinP

eq1 = (Vout-Va)*go + Vout*go + (VinN-Va)*gm +Vm*gm == 0;
eq2 = (Vm-Va)*go + Vm*go + (VinP-Va)*gm +Vm*gm == 0;
eq3 = Va*go + (Va-Vm)*go +(Va-Vout)*go - (VinP-Va)*gm - (VinN-Va)*gm == 0;
eq4 = Vin == VinP-VinN;

out = solve([eq1,eq2,eq3,eq4], [Vout,Vm,VinP,VinN,Va])
out.Vout
```

```
ans =

(2*Vin*gm^2 + Vin*go*gm)/(4*go*(gm + go))
```

$$\frac{V_{out}}{V_{in}} = \frac{(2 * g_m^2) + (g_m g_0)}{4g_0(g_m + g_0)} \tag{10}$$

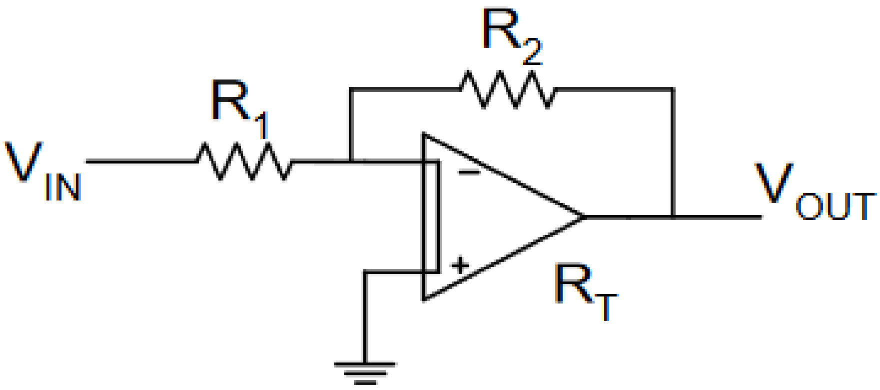
d)

Simplify the solution obtained with MATLAB under the assumption that all g_o terms are small compared to g_m terms.

$$\frac{V_{out}}{V_{in}} = \frac{g_m}{2g_0} \tag{11}$$

Problem 3

A transresistance amplifier with a gain R_T is shown. Derive an expression for the voltage gain of the amplifier as a function of the transresistance gain R_T and determine what that reduces to if R_T is very large.



In this circuit we are using a transresistance amplifier which has a gain of $V_{out} = R_f * i_{in}$. The i_{in} for this circuit is $\frac{V_{in}}{R_1}$ because V_{neg} is a virtual ground. We can then do a KVL.

$$\frac{V_{in}}{R_1} + \frac{V_{out}}{R_2} - i_{in} = 0 \tag{1}$$

$$\frac{V_{in}}{R_1} + \frac{V_{out}}{R_2} - \frac{V_{out}}{(}R_f) = 0 \tag{2}$$

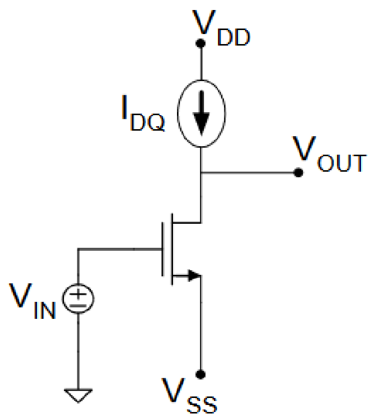
$$\frac{V_{in}}{R_1} = V_{out} * \frac{R_2 - R_f}{R_2 * R_f} \tag{3}$$

$$\frac{V_{out}}{V_{in}} = \frac{R_2 * R_f}{R_1(R_2 - R_f)} \tag{4}$$

$$lim_{R_f \to \infty} \frac{R_2 * R_f}{R_1(R_2 - R_f)} = \frac{R_2}{R_1} \tag{5}$$

Problem 4

Assume the amplifier shown below is designed in a 0.18μ CMOS process. Assume also that $V_{DD} = 1V$, $V_{SS} = -1V$, and $I_{DQ} = 4mA$.



a)

Analytically determine the W and L needed to establish a quiescent output voltage of 0.5V when $V_{INQ} = 0.5V$.

Since we are using a mosfet in saturation region, we can use the following equations:

$$i_d = K * (V_{GS} - V_{TN})^2 \tag{1}$$

Where

$$K = \frac{1}{2}(\mu_n C_{ox})\frac{W}{L} \tag{2}$$

$$V_{out} = V_{DS} + V_{SS} = 0.5V \tag{3}$$

$$V_{DS} = 1.5V \tag{4}$$

$$V_{in} = V_{GS} - V_{SS} = 0.5V \tag{5}$$

$$V_{GS} = 1.5V \tag{6}$$

To get the W/L we can use the current and set K:

$$i_d = K * (V_{GS} - V_{TN})^2 \tag{7}$$

To get the VTN, I used the reference sheet for the process.

$$V_{TN} = 0.5V \tag{8}$$

The output current of the amplifier is:

$$i_d = 4mA = K * (1.5 - 0.5)^2 \tag{9}$$

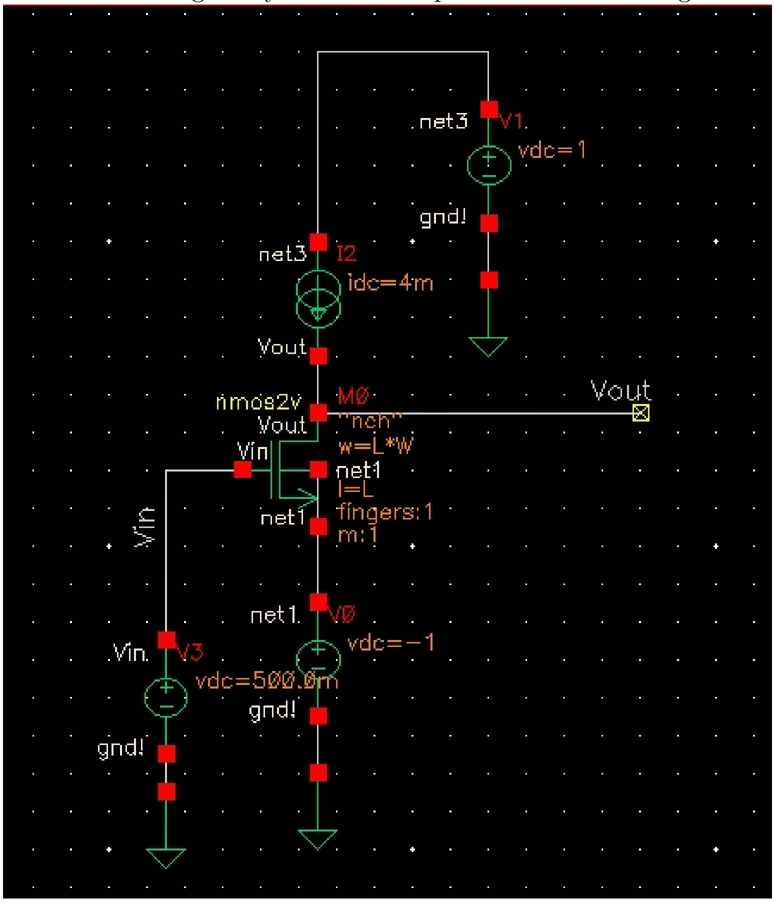
$$4mA = K = 171.8\frac{uA}{V^2}\frac{W}{L} \tag{10}$$

$$\frac{W}{L} = \frac{20000}{859} \approx 23.28 \tag{11}$$

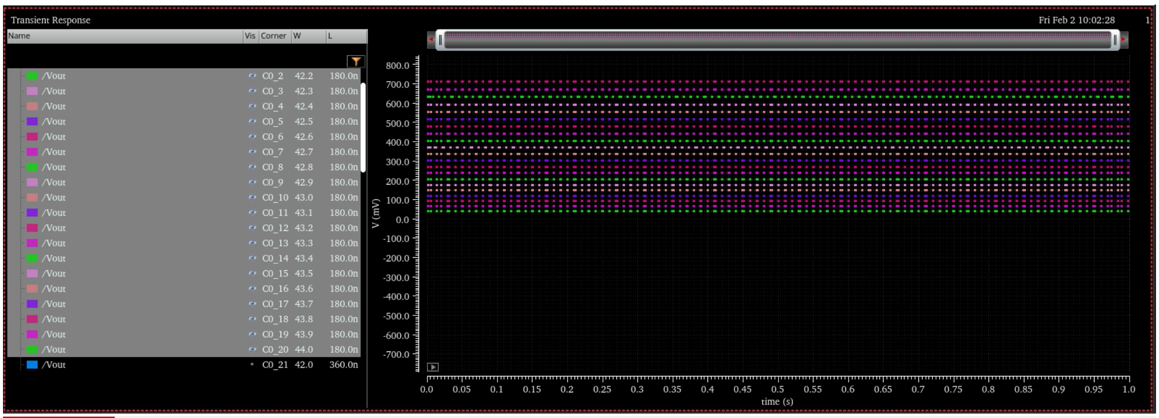
b)

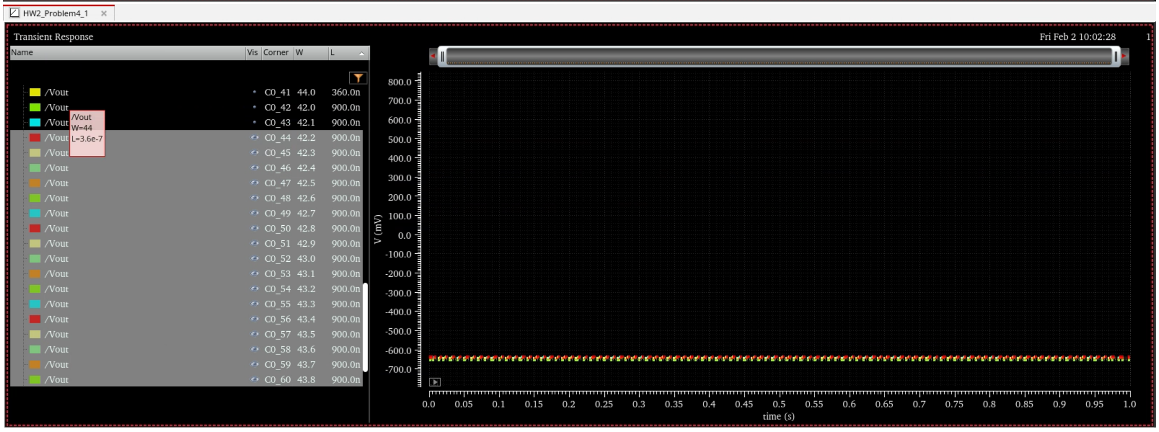
Verify the transfer characteristics by Spice simulation.

After running many sims on LTspice I moved to using Cadence with the provided spetcre models.

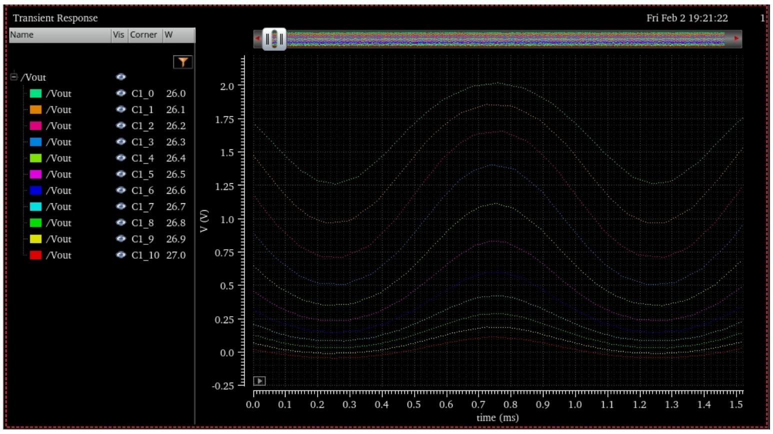


One issue found was how much the DC point changed with length regardless of the W/L ratio.





After testing many ratios at different lengths and although the Q point lowered with length (regardless of width ratio), the ratio of 23.23 would not work. I found that the ratio of 26.5 with a length of 900n would work well.



c)

Analytically determine the dc voltage gain at the Q-point established in a).

$$g_m = \frac{\partial i_d}{\partial v_{sig}} \tag{12}$$

With the equation:

$$i_d = K * (V_{GS} - V_{TN})^2 (1 + \lambda V_{DS}) \tag{13}$$

We can add the small signal to the equation:

$$i_d = K * (V_{GS} + V_{sig} - V_{TN})^2 (1 + \lambda V_{DS}) \tag{14}$$

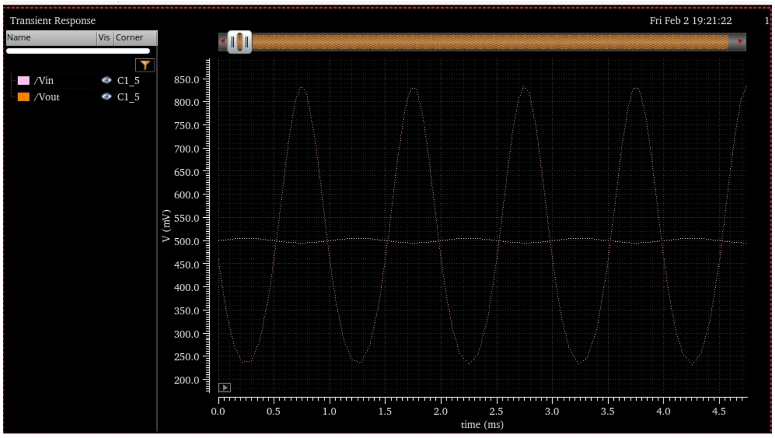
$$\frac{\partial i_d}{\partial v_{sig}} = 2K * (V_{GS} + V_{sig} - V_{TN})(1 + \lambda V_{DS}) \tag{15}$$

$$\frac{\partial i_d}{\partial v_{sig}} = 2 * 171.8 \frac{uA}{V^2} * (1 + V_{sig}) * \frac{W}{L} (1 + \lambda 1.5) \tag{16}$$

$$\frac{\partial i_d}{\partial v_{sig}} = 2 * 0.004 * (1 + V_{sig})(1 + \lambda 1.5) \tag{17}$$

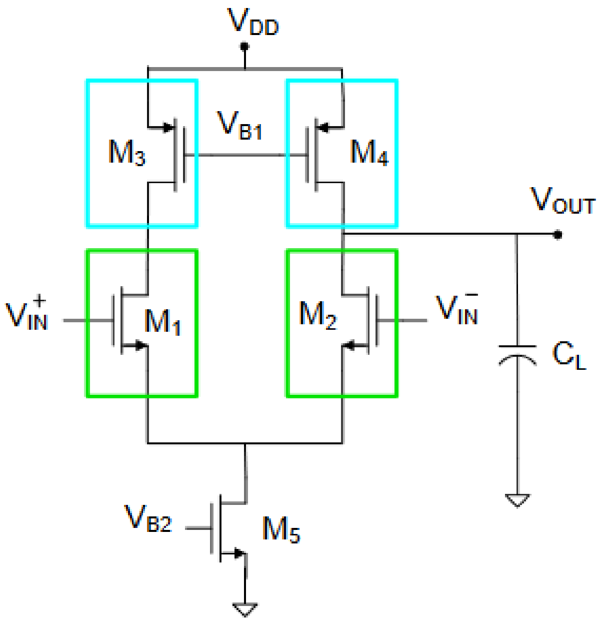
d)

Using SPICE, obtain a plot of the small signal voltage gain versus the quiescent output voltage.



Problem 5 and 6

Design a 5T op amp to have a dc gain of 50dB and a GB of 2MHz in the ON 0.5μm CMOS process. Assume $V_{DD} = 3.5V$ and $C_L = 1pF$. Assume also that the bias voltages V_{B1} and V_{B2} can be precisely set so that a CMFB circuit is not needed. Verify the gain and the GB of your design with a SPICE simulation.



First convert the $50dB = 10log(\frac{V_{out}}{V_{in}})$. We then get that $A = 10^5$. From the data sheet we find that:

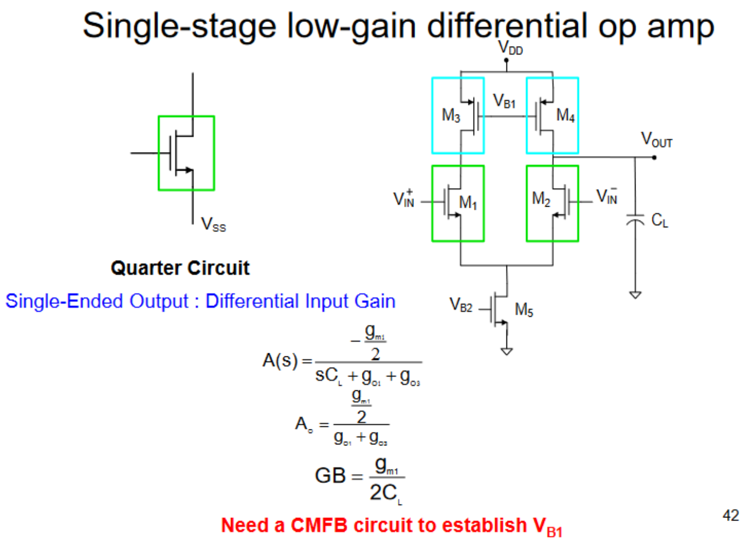
$$\lambda_1 = 0.30 \tag{1}$$

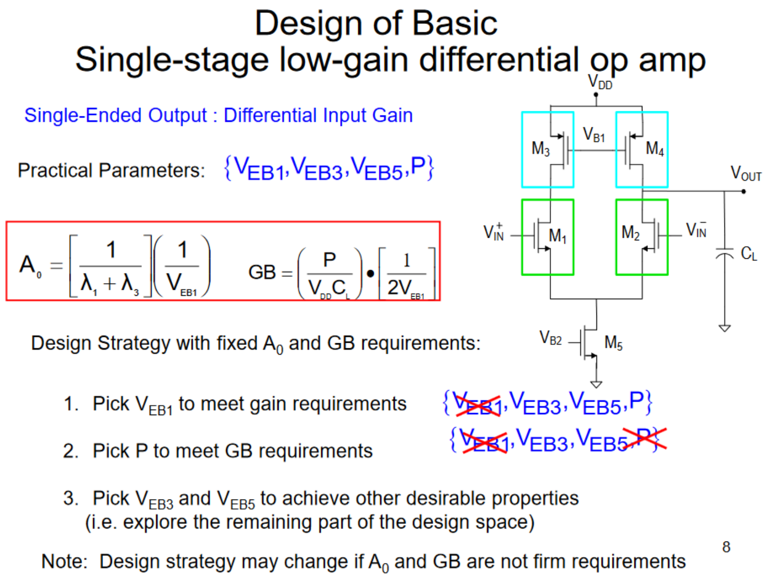
$$\lambda_3 = 0.35 \tag{2}$$

$$V_{THP} = -0.90 \tag{3}$$

$$V_{THN} = 0.76 \tag{4}$$

We can use the equations given in the lecture slides.





$$A = \frac{1}{\lambda_1 + \lambda_3} \frac{1}{V_{EB1}} = 10^5 = \frac{20}{13} \frac{1}{V_{EB1}} \tag{5}$$

$$V_{EB1} = 1.538 * 10^{-5} V \tag{6}$$

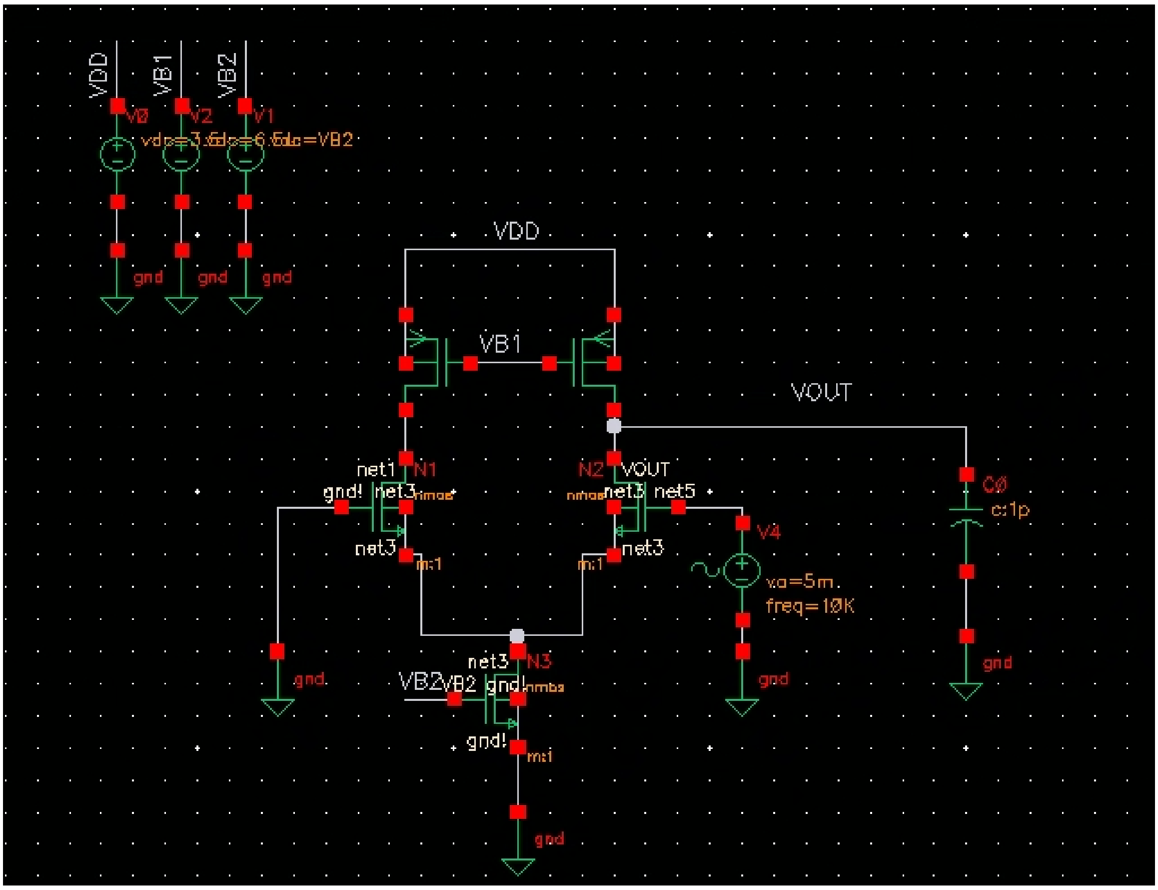
$$GB = 2Mhz = \frac{P}{V_{DD} C_L} \frac{1}{V_{EB1}} \tag{7}$$

$$P = 1.077 * 10^{-}10W \tag{8}$$

$$g_m = 2K * (V_{GS} - V_{TH}) \tag{9}$$

$$g_{m3} = 2K * (V_{B1} + 0.94) \tag{10}$$

$$g_{m3} = 2K * .94 \tag{11}$$



Problem 7

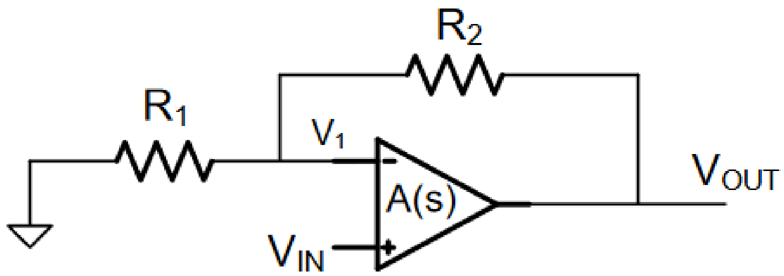
Determine the common-mode input range and the output signal swing of the amplifier you designed in the previous problem.

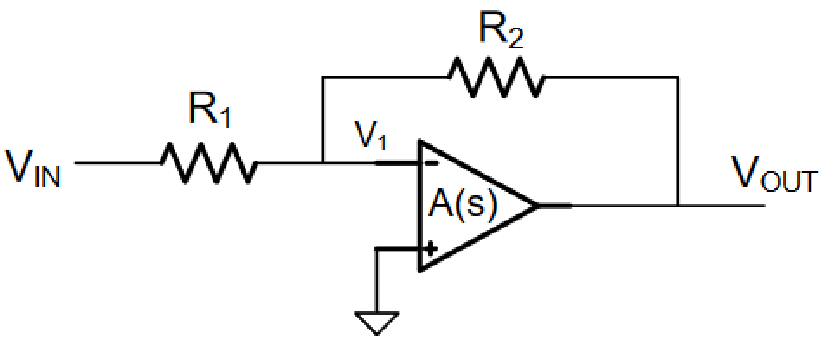
Problem 8

Assume that the op amp is a single-pole amplifier with gain given by the expression $A(s) = \frac{A_0 \omega_A}{s + \omega_A} \approx \frac{GB}{s}$, where the gain-bandwidth product of the op amp is $GB = A_0 \omega_A$. The right-most approximation to the gain is almost always justifiable whenused to characterize the operational amplifier since the gain of the op amp is so large at dc and since at frequencies even modestly above ω_A, there is little difference between the middle term and the right term in this expression.

a)

Assuming that the frequency-dependent gain of the op amp can be modeled as $A(s) \approx \frac{A_{GB}}{s}$, determine the transfer function $\frac{V_{OUT}}{V_{IN}}$ for the following two amplifiers.





$$\frac{V_{OUT}}{V_{IN}} = \frac{A_{GB}}{s} \times \frac{1}{1 + \frac{1}{2}s}$$

First for the first circuit we start with the following equations.

$$V_{out} = A(s)(V_{in} - V_N) \tag{1}$$

$$V_N = \frac{V_{OUT} * R1}{R_1 + R_2} \tag{2}$$

Then we can solve using the VN equation to replace VN in the original.

$$V_{out} = A(s)(V_{in} - \frac{V_{OUT} * R1}{R_1 + R_2}) \tag{3}$$

$$V_{out}(1 + \frac{R_1 * A(s)}{R_2 + R_1} = A(s)V_{in} \tag{4}$$

$$\frac{V_{out}}{V_{in}} = \frac{A(s)}{1 + \frac{R_1 * A(s)}{R_2 + R_1}} \tag{5}$$

$$\frac{V_{out}}{V_{in}} = \frac{A(s)(R_1 + R_2)}{(R_1 + R_2) + R_1 * A(s)} \tag{6}$$

Now replace A(s)!

$$\frac{V_{out}}{V_{in}} = \frac{\frac{A_{GB}}{s}(R_1 + R_2)}{(R_1 + R_2) + R_1 * \frac{A_{GB}}{s}} \tag{7}$$

$$\frac{V_{out}}{V_{in}} = \frac{A_{GB}(R_1 + R_2)}{s(R_1 + R_2) + R_1 * A_{GB}} \tag{8}$$

Now for the second circuit we do a similar analysis.

$$V_{out} = A(s)(V_P - V_1) \tag{9}$$

$$V_P = 0 \tag{10}$$

$$\frac{(V_{in} - V_1)}{R_1} = \frac{(V_1 - V_{out})}{R_2} \tag{11}$$

$$\frac{V_{in}}{R_1} = -(\frac{\frac{V_{out}}{A} - 1}{R_2} + \frac{\frac{V_{out}}{A}}{R_1}) \tag{12}$$

$$\frac{V_{out}}{V_{in}} = \frac{-(R_2)(A)}{R_1(A) + R_1 + R_2} \tag{13}$$

$$\frac{V_{out}}{V_{in}} = \frac{-(R_2)(\frac{A_{GB}}{s})}{R_1(\frac{A_{GB}}{s}) + R_1 + R_2} \tag{14}$$

b)

With the same op amp model used in part a), analytically determine the 3dB bandwidth of the following two amplifiers.

We need to find the magnitude of the transfer function while using $s = j\omega$ and set it to $\frac{1}{\sqrt{2}}$.

$$|H(\omega)| = \frac{|A_{GB}(R_1 + R_2)|}{\sqrt{\omega(R_1 + R_2)^2 + (R_1 * A_{GB})^2}} \tag{15}$$

$$\frac{1}{\sqrt{2}} = \frac{|A_{GB}(R_1 + R_2)|}{\sqrt{\omega(R_1 + R_2)^2 + (R_1 * A_{GB})^2}} \tag{16}$$

And for the second circuit.

$$\frac{V_{out}}{V_{in}} = \frac{-(R_2)(\frac{A_{GB}}{s})}{R_1(\frac{A_{GB}}{s}) + R_1 + R_2} \tag{17}$$

$$|H(\omega)| = \frac{|(R_2)A_{GB}|}{\sqrt{(R_1 A_{GB})^2 + (\omega(R_1 + R_2))^2}} \tag{18}$$

Problem 9

It was stated in class that all even-ordered distortion terms introduced by the amplifier vanish in symmetric fully differential amplifiers. Prove this fact.

Because of the symmetry of the op amp the gain of each output is considered to be the same. We can represent distortion as:

$$V_d = V_{in+} - Vin- \tag{1}$$

The outputs can be represented as a taylor series.

Because Vin+ can be assumed to be positive (in respect to Vin-) all the time we can get a positive derivative every .

$$V_{out+} = a_1 V_{in+} + a_2 V_{in+}^2 + a_3 V_{in+}^3 + ... \tag{2}$$

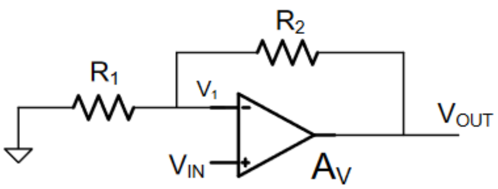
But because Vin- is assumed negative (in respect to Vin+) we can assume that the every other derivative will be negative.

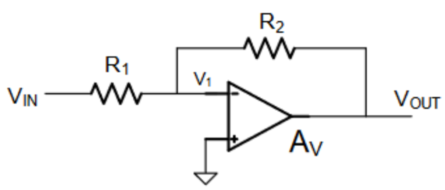
$$V_{out-} = a_1 V_{in-} + a_2 V_{in-}^2 + a_3 V_{in-}^3 + ... \tag{3}$$

When we get our final VOUT we can see that all the even terms of these series will cancel out.

Problem 10 (Extra Credit)

The "dead network" of a circuit is obtained by setting all small-signal inputs to 0. That is, by replacing all ac voltage sources with short circuits and all ac current sources with open circuits. The β of a feedback amplifier is a characteristic of the "dead network". Consider the basic inverting and noninverting feedback amplifiers shown below. These are widely used as small-signal voltage amplifiers.

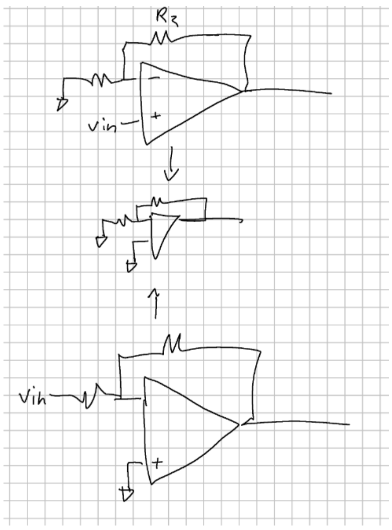




a)

Show that they both have the same "dead network".

If Vin is set to 0 then it is the same as ground. Replacing Vin with ground in the first circuit yields the same as replacing Vin with ground in the second circuit.



b)

The β of the two amplifiers shown is $\frac{R_1}{R_1+R_2}$. Show that the gain of the noninverting feedback amplifier can be expressed by the standard feedback equation

$$A = \frac{V_{OUT}}{V_{IN}} = \frac{A_v}{1 + A_v\beta}$$

This can be seen by using the equation 5 from problem 8.

$$\frac{V_{out}}{V_{in}} = \frac{A(s)}{1 + \frac{R_1 * A(s)}{R_2 + R_1}} \tag{1}$$

c)

Take the limit as A_V goes to ∞ for the gain derived in part b) and compare with that derived in EE 230 for the gain of the noninverting feedback amplifier.

$$\frac{V_{OUT}}{V_{IN}} = \lim_{A_V \to \infty} \frac{A_v(1 + \frac{R_1}{R_2})}{(1 + \frac{R_1}{R_2}) + A_v} = 1 + \frac{R_1}{R_2} \tag{2}$$

It is the same!

d)

Derive the gain of the inverting feedback amplifier in terms of A_V and β and comment on why it does not look like the standard feedback equation.

In problem 8 we derived this to be:

$$\frac{V_{out}}{V_{in}} = \frac{-(R_2)(A)}{R_1(A) + R_1 + R_2} \tag{3}$$

$$\frac{V_{out}}{V_{in}} = \frac{-(A)}{\frac{R_1}{R_2}(A) + \frac{R_1}{R_2} + 1} \tag{4}$$

It seems that we have an extra term in the bottom of the equation.
http://class.ece.iastate.edu/ee435/miscHandouts/TSMC
http://class.ece.iastate.edu/ee330/lectures/EE