MAP562 Optimal design of structures

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Instructions: Upload your solutions as separate files to the course Moodle by February 6th. Each group will solve either Exercise 1 or Exercise 2 for one of the three configurations. We will tell you which one during the lecture. In each case there are two questions so two files to be submitted.

The objective of these exercises is to optimize the thickness h of a two-dimensional elastic body in Ω. For this homogeneous elastic body the state equation is the system of linearized elasticity: find $u \in V$ such that

$$-\nabla \cdot (\sigma(u)) = 0 \quad \text{in } \Omega,$$

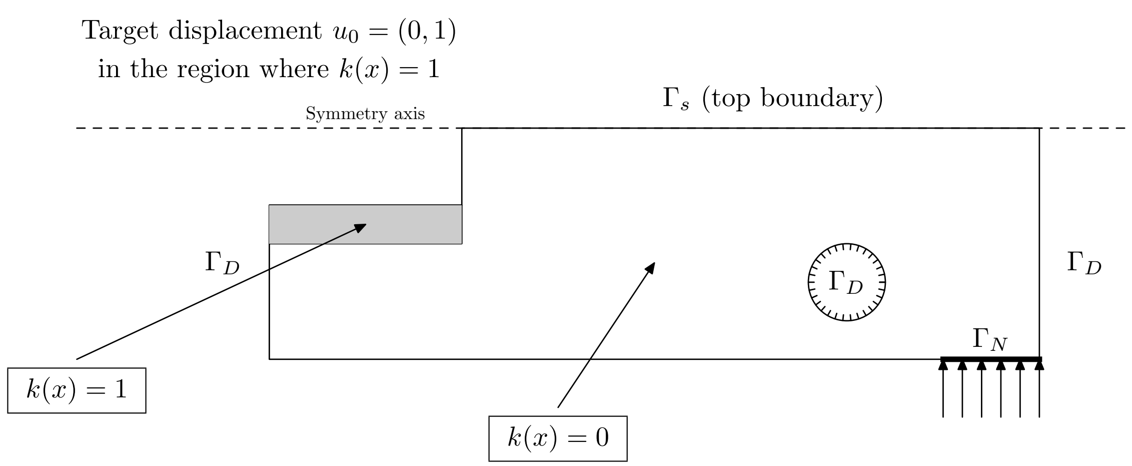
where the stress is given by the classical law:

$$\sigma(u) = h(2\mu e(u) + \lambda tr(e(u))I),$$

$e(u) = \frac{1}{2}(\nabla u + \nabla u^T)$ is the linearized stress tensor, I is the identity matrix, and μ and λ are the Lamé coefficients. As in the other exercises, we have divided the boundary $\partial\Omega$ into several (here, four) parts: Γ_D,Γ_{N_1},Γ_F and Γ_S in order to write the following boundary conditions:

$$\begin{aligned} \sigma(u) \cdot n = 0 \quad & \text{on } \Gamma_F \text{ (Free boundary)}, \\ \sigma(u) \cdot n = [0, 100]^\top \quad & \text{on } \Gamma_{N_1} \text{ (Neumann boundary)}, \\ u = (0,0)^\top \quad & \text{on } \Gamma_D \text{ (Dirichlet boundary)}, \end{aligned}$$

Symmetry boundary condition on Γ_S.



Preliminary question: What is the symmetry boundary condition in elasticity?

Exercise 1 Displacement tracking

We consider the following optimization problem

$$\inf_{h\in\mathcal{U}_{ad}} J(h), \quad J(h) = \int_\Omega k(x)|u(x)-u_0|^2\,dx,$$

where $u_0 = (0, 0.2)^T$ and the function $k(x)$ is an indicator function such that $k(x) = 1$ for $x \in \Omega_2$ (dark region displayed in the figure) and $k(x) = 0$ in the rest of Ω. The admissible set is

$$\mathcal{U}_{ad} = \{h \in L^\infty(\Omega) :\, h_{min} \le h \le h_{max} \text{ and } \int_\Omega h(x)\,dx = h_{avg}|\Omega|\}.$$

(In the numerical implementation, use: $h_{min} = 0.1$, $h_{max} = 1$ and $h_{avg} = 0.5$.)

1. Write an optimization loop for this problem.
2. **(Homework)** Implement it in Freefem++.
3. Trace the deformed mesh for the optimal thickness found and compare with the desired deformation in regions where $k(x) = 1$.

Exercise 2 Stress tracking

We consider the same problem except that we change the objective function to

$$J(h) = \frac{1}{2}\int_\Omega k(x)|\sigma(x)-\sigma_0|^2\,dx,$$

with $\sigma_0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ and $|\sigma(x)-\sigma_0|^2 = \sum_{i=1}^2\sum_{j=1}^2(\sigma(x)-\sigma_0)_{ij}^2$.

1. Write an optimization loop for this problem.
2. **(Homework)** Implement it in Freefem++.
3. Trace the final stress for the optimal thickness found by the algorithm. Compare it with the desired stress in the region where $k(x) = 1$.