MAP562 Optimal design of structures

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Instructions: Upload your solutions as separate `FreeFem++` files to the course Moodle by March 20th. If additional files are needed (photos containing the final results), use a zip archive.

Exercise 1

The SIMP method was presented in class and a model problem is contained in the Freefem++ script `simp_elas.edp` available on the course Moodle. The state equation is the two-dimensional linear elasticity system. The functional that is minimized is the compliance under a volume constraint.

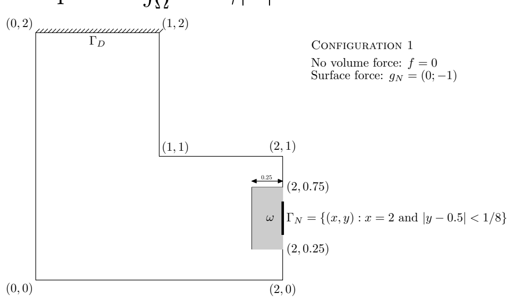
Important:

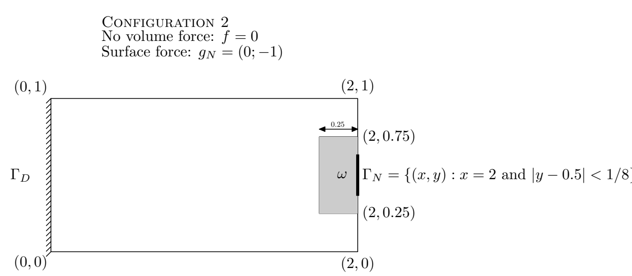
(a) You should solve the problem and refine the mesh two times in order to observe the dependence of the solution on the mesh.

(b) You may also investigate what happens when the volume fraction γ varies (don't make a loop, just try a few different values).

(c) Save **one photo of the final result** and **the convergence curve** for each computation you make (using the command `ps="filename.png"` in the `plot` command or by typing W on the final view of the solution). Put these photos in a zip file along with the `FreeFem++` codes when you submit your solution on Moodle.

1. Solve the same problem for one of the geometries shown below. Recall that the state equation is the linearized elasticity with Dirichlet boundary conditions on Γ_D, surface loads applied on Γ_N and the rest of the boundary is traction-free. The objective function is the compliance and a volume constraint is imposed: $\int_\Omega \theta = \gamma|\Omega|$.





2. Consider a small domain ω included in the design space. Change the objective function to the following:

$$J(u) = \int_\omega |u|^2.$$

The objective is to minimize the displacements in the region ω. Computations should be made again under a volume constraint: $\int_\Omega \theta = \gamma|\Omega|$.

Hint: Note that for this choice of J, the problem is no longer self-adjoint. You need to define and solve an adjoint problem.