

# MAP562 Optimal design of structures

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## Homework Sheet 6, Feb 13th, 2019

**Instructions** Upload your solutions as **FreeFem++** files to the course Moodle by February 20th.

### Exercise 4 Optimization of an inclusion

This exercise is related to the exercise 4 in class. Question 1 is particularly useful, as well as the **FreeFem++** example shown on Moodle.

1. **(Homework)** Consider the following types of parametric curves:

1. **Stadiums:** rectangles of dimensions  $a \times b$  with a half disk of diameter  $b$  glued to the two opposite sides of length  $b$ . Optimization variables:  $a, b$  and the center.
2. **Superellipse:** parametric curve defined by  $\frac{|x|^p}{a^p} + \frac{|y|^p}{b^p} = 1$  where  $p = 4$ . Optimization variables:  $a, b$  and the center.
3. **Generic parametric curve:** curve with given radial parametrization  $\rho(\beta) = 1 + a \cos(2\beta) + b \cos(3\beta)$ . Recall that given a radial function  $[0, 2\pi] \ni \beta \mapsto \rho(\beta) \in (0, \infty)$  the corresponding parametric curve is defined by  $(x(\beta), y(\beta)) = (\rho(\beta) \cos \beta, \rho(\beta) \sin \beta)$ . (notice that this is one of the standard ways to define a curve in **FreeFem++**) You may suppose that  $|a| \leq 0.5, |b| \leq 0.5$  and the bounding box is the square  $(-2.5, 2.5)^2$ . For this type of curve you may assume that the center for the radial parametrization is fixed. The optimization variables are:  $a, b$ . Note that in the computation of the derivative you may need the value of the angle  $\beta$  on the boundary of the curve  $\partial\omega$ . You may use the **FreeFem++** function **atan2** to do this (an example will be posted on Moodle).

Choose one of the categories of curves defined above and compute the partial derivatives of  $J(\omega)$  with respect to the relevant parameters. As above, it is useful to find what is the corresponding vector field  $\theta$  when varying the desired parameter and then use the general shape derivative formula found at Question 1.