─────────────────────── MODULE *tictactoe* ───────────────────────

EXTENDS *Sequences*, *Integers*

This module serves to emulate a tic-tac-toe-game

VARIABLES *field*, *current_player*
$vars \triangleq \langle field, \ current\_player \rangle$

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$initial\_field\_state \triangleq \text{``\_''}$
$field\_states \triangleq \{initial\_field\_state, \ \text{``X''}, \ \text{``O''}\}$

$field\_size \triangleq \{1, 2, 3\}$

$supported\_field \triangleq field\_size \times field\_size \times field\_states$

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$players \triangleq \{\text{``player\_one''}, \ \text{``player\_two''}\}$
$player\_symbol\_mapping \triangleq [player\_one \mapsto \text{``X''}, \ player\_two \mapsto \text{``O''}]$

$NextPlayer \triangleq \text{CHOOSE } n \in players : n \neq current\_player$

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$TypeOK \triangleq$
    $\wedge$ $current\_player \in players$
    $\wedge$ $field \subseteq supported\_field$

$Init \triangleq$
    $\wedge current\_player = \text{``player\_one''}$
    $\wedge field = field\_size \times field\_size \times \{initial\_field\_state\}$

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$PlayerSymbol \triangleq player\_symbol\_mapping[current\_player]$
$PlayerSymbolOfPlayer(p) \triangleq player\_symbol\_mapping[p]$

$PristineField(x, y) \triangleq \langle x, \ y, \ initial\_field\_state \rangle$
$MarkedField(x, y) \triangleq \langle x, \ y, \ PlayerSymbol \rangle$

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$AllowedToMark(x, y) \triangleq \exists f \in field : f = PristineField(x, y)$

$SetMark \triangleq$
    $\exists x, y \in field\_size :$
        $\wedge AllowedToMark(x, y)$
        $\wedge field' = (field \setminus \{PristineField(x, y)\}) \cup \{MarkedField(x, y)\}$

$Next \triangleq$
$\quad \land current\_player' = NextPlayer$
$\quad \land SetMark$

$Spec \triangleq Init \land \quad \Box[Next]_{vars}$

THEOREM $Spec \Rightarrow \Box TypeOK$

$XCoordinate(fieldx) \triangleq fieldx[1]$
$YCoordinate(fieldy) \triangleq fieldy[2]$
$Value(fieldv) \triangleq fieldv[3]$

$HorizontalWin \triangleq$
$\quad \exists\, x \in field\_size,\, p \in players :$
$\qquad$ LET $row \triangleq \{f \in field : XCoordinate(f) = x\}$
$\qquad$ IN $\quad \forall\, el \in row : Value(el) = PlayerSymbolOfPlayer(p)$

$VerticalWin \triangleq$
$\quad \exists\, y \in field\_size,\, p \in players :$
$\qquad$ LET $column \triangleq \{f \in field : YCoordinate(f) = y\}$
$\qquad$ IN $\quad \forall\, el \in column : Value(el) = PlayerSymbolOfPlayer(p)$

$AllowedDiagonalWinLeftToRightFields(field\_left\_to\_right) \triangleq$
$\quad \lor\, (XCoordinate(field\_left\_to\_right) = 1 \land YCoordinate(field\_left\_to\_right) = 1)$
$\quad \lor\, (XCoordinate(field\_left\_to\_right) = 2 \land YCoordinate(field\_left\_to\_right) = 2)$
$\quad \lor\, (XCoordinate(field\_left\_to\_right) = 3 \land YCoordinate(field\_left\_to\_right) = 3)$

$DiagonalWinLeftToRight \triangleq$
$\quad \exists\, p \in players :$
$\qquad$ LET $diagonal\_left\_to\_right \triangleq \{f \in field : AllowedDiagonalWinLeftToRightFields(f)\}$
$\qquad$ IN $\quad \forall\, el \in diagonal\_left\_to\_right : Value(el) = PlayerSymbolOfPlayer(p)$

$AllowedDiagonalWinRightToLeftFields(field\_right\_to\_left) \triangleq$
$\quad \lor\, (XCoordinate(field\_right\_to\_left) = 1 \land YCoordinate(field\_right\_to\_left) = 3)$
$\quad \lor\, (XCoordinate(field\_right\_to\_left) = 2 \land YCoordinate(field\_right\_to\_left) = 2)$
$\quad \lor\, (XCoordinate(field\_right\_to\_left) = 3 \land YCoordinate(field\_right\_to\_left) = 1)$

$DiagonalWinRightToLeft \triangleq$
$\quad \exists\, p \in players :$
$\qquad$ LET $diagonal\_right\_to\_left \triangleq \{f \in field : AllowedDiagonalWinRightToLeftFields(f)\}$
$\qquad$ IN $\quad \forall\, el \in diagonal\_right\_to\_left : Value(el) = PlayerSymbolOfPlayer(p)$

$DiagonalWin \triangleq$

$\lor$ *DiagonalWinLeftToRight*
$\lor$ *DiagonalWinRightToLeft*

set this as an invariant to get informed about game winning moves + player on model checking

GameEnded $\triangleq$
    $\land$ *HorizontalWin* $\neq$ TRUE
    $\land$ *VerticalWin* $\neq$ TRUE
    $\land$ *DiagonalWin* $\neq$ TRUE

\ * Modification History
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