National University of Singapore
Department of Computer Science

2021/22 (Sem.1) **CS1231 Discrete Structures** **Tutorial 4**

1. Let $A = \{0, 1\}$, $B = \{a, b, c\}$ and $C = \{01, 10\}$. Determine the following:
   (a) $B \times C$; (b) $A \times B \times C$; (c) $\varnothing \times A$; (d) $\mathcal{P}(\{\varnothing\}) \times A$.

2.* Consider a probabilistic experiment like the following: first toss a coin; if the toss is head, then pick a ball from a box with 2 black balls and 3 white balls; if the toss is tail, then pick a ball from a box with 4 red balls and 5 white balls. An **outcome** of such an experiment may be, say, a tail followed by a red ball.

   (a) Use an ordered pair to represent each possible outcome of the experiment; the set of all such ordered pairs is called the **sample space**.

   (b) An **event** for this experiment may be "the toss is tail" or "a white ball is picked". How can these events be represented as subsets of the sample space?

   [This is an example of mathematical **modeling**, which is ubiquitous in Computer Science; here, sets and ordered pairs are used to model the experiment.]

3. Let $A = \{1, 2, 3, 4\}$, $B = \{-1, 0, 1\}$, $C = \{2, 3, 5, 7\}$, $R = \{(a, b) \in A \times B : ab \text{ is even}\}$ and $S = \{(b, c) \in B \times C : b + 2c \text{ is odd}\}$.

   (a) Draw arrow diagrams for R, S, $S \circ R$ and their inverses R^{-1}, S^{-1} and $(S \circ R)^{-1}$.

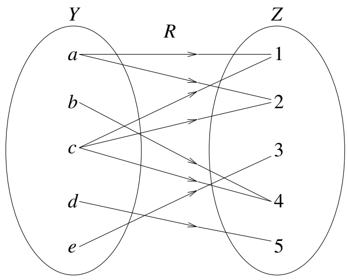
   (b) Verify that $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$.

4. Recall the database relations in Figure 5.1 of the lecture notes. Draw arrow diagrams for SN, SM and $SM \circ (SN)^{-1}$.

5. Let $Y = \{a, b, c, d, e\}$ and $Z = \{1, 2, 3, 4, 5\}$, and the arrow diagram of the relation R from Y to Z be as shown on the right.

   (a) Determine R^{-1}.

   (b) Determine $R^{-1} \circ R$.



6. Consider the relation $S = \{(m, n) \in \mathbb{Z}^2 : m^3 + n^3 \text{ is even}\}$ on $\mathbb{Z}$. Determine S^{-1}, $S \circ S$ and $S \circ S^{-1}$.

7.* A predicate $P(x, y)$ can be represented by a relation R, so that $P(x, y)$ is true if and only if $(x, y) \in R$. For example, if the domains of x and y are $B = \{2, 3, 5, 7, 11, 13\}$ and $C = \{0, 2, 4, 6, 8\}$ respectively, then one can represent the predicate $x = y + 1$ by the relation $\{(3, 2), (5, 4), (7, 6)\}$ over B and C.

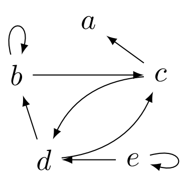
   (a) For the domains B for x and C for y above, determine the relations that represent the predicates
   (i) $x < y$; (ii) x divides y (see Tutorial 2, Problem 2); (iii) $x - y \in C$.

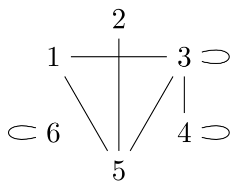
   (b) In general, what can you say about the relation R over X and Y that represents a predicate $P(x, y)$ if
   (i) $\forall x \in X\ \forall y \in Y\ P(x, y)$ is true? (ii) $\exists x \in X\ \exists y \in Y\ P(x, y)$ is true?

   (c) In (b), viewing $X \times Y$ as the universal set, and what is $P(x, y)$ when $(x, y) \in \overline{R}$?

8. Let A, B, C, D be sets and $R \subseteq A \times B$, $S \subseteq B \times C$ and $T \subseteq C \times D$. Prove that $T \circ (S \circ R) = (T \circ S) \circ R$ (i.e. composition is associative for relations).

9. The directed graph (A, D) and the undirected graph (B, E) are shown below:





Determine A, D, B and E.

10.* Let $A = \{a, b, c, d, e\}$ and $R = \{(b, b), (b, e), (c, c), (c, d), (d, d), (d, e), (e, a), (e, e)\}$, considered as a relation on A.

(a) Draw an arrow diagram for R.

(b) Determine R^{-1}.

(c) Determine $R \circ R$.

11. Draw the following directed graphs:

(a) $(\mathcal{P}(\{a, b, c\}), \subseteq)$ where $\subseteq$ is the "subset" relation;

(b) $(\{2, 3, 4, 5, 6, 12, 13, 14, 15, 16, 17, 18, 19, 20\}, |)$, where $|$ is the "divides" relation (from Tutorial 2, Problem 2).

12.* Let $C = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$ and $R = \{(x, y) \in C^2 : y = x^2 - 1\}$, considered as a relation on C.

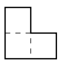
(a) Draw an arrow diagram for R.

(b) Determine $R \circ R$.

13. A $C \times C$ *chessboard* is a square divided into C rows of C unit squares, where $C \in \mathbb{Z}^+$. For example, the usual chessboard is a $2^3 \times 2^3$ chessboard. An *L-tile* is a $2^1 \times 2^1$ chessboard with one unit square missing (as shown).

Given a $C \times C$ chessboard and any one of its unit squares singled out (like the black one below), can the rest of the chessboard can be covered by non-overlapping L-tiles? (See the example below.)

Investigate into the cases $C = 4$, $C = 5$ and $C = 6$. (To be continued in Tutorial 5.)



L-tile