Definition (Universal Quantification; $\forall$ is "for all")
Suppose $Q(x)$ is a predicate and D the domain of x.
Then $\forall x \in D\ Q(x)$ is a **universal statement** defined to be
true iff $Q(x)$ is true for every value of x in D and
false iff $Q(x)$ is false for one or more values of x in D.
(such an x is a **counterexample** that *disproves* $\forall x \in D\ Q(x)$).

Example: $Q(x)$ is "$x = 1 \ \leftrightarrow\ x^2 = 1$".

$D = \{0, 1, 2\}$: Is $\forall x \in D\ Q(x)$ true?

$D' = \{-1, 0, 1\}$: Is $\forall x \in D'\ Q(x)$ true?

Definition (Existential Quantification; $\exists$ is "there exists")
Suppose $Q(x)$ is a predicate and D the domain of x.
Then $\exists x \in D\ Q(x)$ is an **existential statement** defined to be true iff $Q(x)$ is true for one or more values of x in D
(such an x is an **example** that *proves* $\exists x \in D\ Q(x)$) and
false iff $Q(x)$ is false for every value of x in D.

Example: $Q(x)$ is "$(x < 1) \wedge (x^2 > 1)$".

$D = \{-1, 0, 1\}$: Is $\exists y \in D\ Q(y)$ true?

$D' = \{-2, -1, 0, 1\}$: Is $\exists y \in D'\ Q(y)$ true?

CS1231
A function $f : X \to Y$ is **one-one** if and only if
for all x_1 and x_2 in X, $x_1 = x_2$ if $f(x_1) = f(x_2)$.

CS1231
A function $f : X \to Y$ is **onto** if and only if
given any y in Y, it is possible to find an x in X such that $y = f(x)$.

CS1231
The **floor** of x (denoted $\lfloor x \rfloor$) is
the greatest integer that is less than or equal to x.

CS1231
(**Well-Ordering Principle**) Let S be a nonempty set of integers.
If there is an integer smaller than all integers in S, then there is an integer in S that is smaller or equal to all integers in S.

CS3230 (Design and Analysis of Algorithms)
$f(n)$ is $O(g(n))$ if and only if
there exists a positive real number M and an integer k such that $|f(n)| \leq M|g(n)|$ whenever integer $n > k$.

CS4232 (Theory of Computation)
Let L be an infinite regular language over Γ. Then
there is $w \in L$ such that, for some x, y and z in Γ^*,
we have $y \neq e$, $w = xyz$ and $xy^n z \in L$ for all nonnegative integers n.

CS5230 (Computational Complexity)
Suppose P $\neq$ NP. Then there exists a set in NP $-$ P that is not NP-Complete.

$\forall x \in \mathbf{R} \quad x > 1 \ \to \ x^2 > 1$

$\forall x \in \mathbf{R} \quad x^2 \leq 1 \ \to \ x \leq 1$

$\forall x \in \mathbf{R} \quad x > 1 \ \to \ x^2 > 1$

$\forall x \in \mathbf{R} \quad x^2 > 1 \ \to \ x > 1$

$\forall x \in \mathbf{R}^+ \quad x > 1 \ \to \ x^2 > 1$

$\forall x \in \mathbf{R}^+ \quad x^2 > 1 \ \to \ x > 1$

$\forall x \in \mathbf{R} \quad x > 1 \ \to \ x^2 > 1$

$\forall x \in \mathbf{R} \quad x \leq 1 \ \to \ x^2 \leq 1$

$\forall x \in \mathbf{R}^+ \quad x > 1 \ \to \ x^2 > 1$

$\forall x \in \mathbf{R}^+ \quad x \leq 1 \ \to \ x^2 \leq 1$

Symbols, Alphabets and Strings

Definition

An **alphabet** is a finite set of **symbols**.

A **string** over alphabet Γ is a finite sequence of symbols from Γ.

The **length** of a string is the length of the sequence.

Γ^* denotes the set of all strings over Γ.

A **language** over Γ is a set of strings over Γ.

Example

Let $\Gamma = \{$A,B,...,Z,$0, 1, \ldots, 9\}$ and $\Psi = \{$A,B,...,Z,a,b,...,z$\}$.
Then

A, B, 0, 1, a, b, etc. are symbols,

Γ and Ψ are alphabets,

CS1231 is a string over Γ, i.e. CS1231 $\in \Gamma^*$,

CS1231 is not a string over Ψ, i.e. CS1231 $\notin \Psi^*$,

School $\in \Psi^*$,

SoC $\in \Psi^*$.