Assignment

CS1231 Discrete Structures

2021/22 Semester 1

Instructions

1. Make sure your writing is clear and big enough. Write your answers tidily. Do not submit rough work.

2. Write your name, your Student Number, and your tutorial group at the top of the first page of your submission. There is no need to copy the questions, but the question numbers should be clearly specified.

3. Scan or take pictures of your work. Put all the images into a single `pdf` file for submission. Instructions on how to do this with Office Lens are available on Wiki.nus at `https://wiki.nus.edu.sg/display/THES/Useful+Resources`. You may also use other softwares for this. Make sure your file is legible before submitting.

4. Name your submission as ⟨*your Student Number*⟩`.pdf`, for example, `A123456R.pdf`.

5. The deadline for submission is **Sunday 10 October, 2021** at **23:59**.

6. Submit your work on LumiNUS > Files > Assignment > Submission > ⟨*your tutorial group*⟩. Do not wait until the last minutes, as LumiNUS may slow down when there is a large amount of traffic.

7. Late submission may not be accepted without a valid reason.

8. If you have a valid reason for late submission, then e-mail the module coordinator Prof. Tay Yong Chiang at dcstayyc@nus.edu.sg as early as possible. The e-mail should include evidence of your situation, for example, a medical certificate, if applicable.

9. If you want any clarification about the questions, then please post your queries on LumiNUS > Forum > Assignment. Queries made via other channels will not be answered, because we want all the information to be in one place, and all students get the same information.

10. You may discuss the questions with your classmates, but do not copy from each other. Those who are found involved in copying or being copied will receive no mark for the assignment.

Assignment problems

1. Let $A = \{3, 5, 7\}$ and $B = \{2, 4, 6\}$. Write down the sets C, D, and E in roster notation, where

   $$C = \{a+b : a \in A \text{ and } b \in B\}, \quad D = \{a_1 \times a_2 : a_1, a_2 \in A\}, \quad E = \{X \in \mathcal{P}(A) : 5 \in X\}.$$
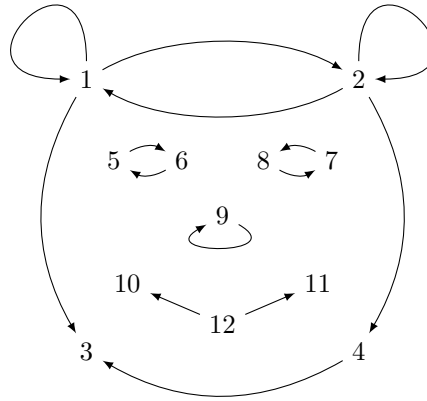
   No justification is needed. [3 marks]

2. Let U be a set and $S \subseteq U$. Prove that, if $S \cap A = A$ for all $A \subseteq U$, then $S = U$, i.e.,

   $$(\forall A \subseteq U \ \ S \cap A = A) \quad \Rightarrow \quad S = U.$$

   [5 marks]

3. The following is an arrow diagram representing a relation R on a set A.

   (a) Write down A and R in roster notation.

   (b) Write down $R^{-1} \setminus R$ in roster notation.

   (c) Is R reflexive? Justify your answer.

   (d) Is R symmetric? Justify your answer.

   (e) Is R antisymmetric? Justify your answer.

   (f) Is R transitive? Justify your answer.

   (g) Write down in roster notation the largest subset $B \subseteq A$ such that $R \cap B^2$ is an equivalence relation on B, i.e.,

     (i) $R \cap B^2$ is an equivalence relation on B; and

     (ii) if $C \subseteq A$ such that $R \cap C^2$ is an equivalence relation on C, then $C \subseteq B$.

   Except for (c), (d), (e) and (f), no justification is needed. [12 marks]