National University of Singapore
Department of Computer Science

2021/22 (Sem.1) **CS1231 Discrete Structures** **Tutorial 1**

The tutors will discuss the problems that do not have a $*$; if there is sufficient time remaining, they will discuss the problems with a $*$ as well.

1. Restate the following symbolically (you can introduce your own statement variables.)

   (i) "An undirected graph (V, E) that is connected and acyclic must have $|E| = |V| - 1$."

   (ii) "If $A \setminus B$ is countable, then A is countable or B is uncountable."

   (iii)* "$\text{Prob}(B) \leq \text{Prob}(A)$ whenever $B \subseteq A$."

Solution:

(i) ("(V, E) is an undirected graph" $\wedge$ "(V, E) is connected" $\wedge$ "(V, E) is acyclic")
$\rightarrow |E| = |V| - 1$

(ii) "$A \setminus B$ is countable" $\rightarrow$ ("A is countable" $\vee$ "B is uncountable")
or
"$A \setminus B$ is countable" $\rightarrow$ ("A is countable" $\vee$ $\sim$("B is countable"))

(iii)* $B \subseteq A \;\rightarrow\; \text{Prob}(B) \leq \text{Prob}(A)$

2. Use truth tables to prove the following equivalences (you may skip the ones that have already been proved in class):
   For any statement variables p, q and r, a tautology t and a contradiction c,

(a)	Commutativity	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
(b)*	Associativity	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
(c)	Distributivity	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
(d)*	Idempotence	$p \wedge p \equiv p$	$p \vee p \equiv p$
(e)*	Absorption	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
(f)	De Morgan's Law	$\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$	$\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$
(g)*	Identities	$p \wedge t \equiv p, \quad p \vee t \equiv t$	$p \vee c \equiv p, \quad p \wedge c \equiv c$
(h)	Negation	$p \vee \sim p \equiv t, \quad \sim(\sim p) \equiv p$	$p \wedge \sim p \equiv c, \quad \sim t \equiv c$

Solution:

(a)

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

The last two columns show $p \wedge q \equiv q \wedge p$. Similarly for $p \vee q \equiv q \vee p$.

(b)*

p	q	r	$p \wedge q$	$q \wedge r$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

The last two columns show $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$. Similarly for $\vee$.

(c)

p	q	r	$q \wedge r$	$p \vee q$	$p \vee r$	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

The last two columns show $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.
Similarly for $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.

(d)*

p	$p \wedge p$
T	T
F	F

p	$p \vee p$
T	T
F	F

(e)*

p	q	$p \wedge q$	$p \vee (p \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

p	q	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	F

The first and last columns in each table show equivalence.

(f)

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$(\sim p) \vee (\sim q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

The last two columns show $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$. Similarly for $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$.

(g)*

p	t	$p \wedge t$
T	T	T
F	T	F

p	t	$p \vee t$
T	T	T
F	T	T

p	c	$p \vee c$
T	F	T
F	F	F

p	c	$p \wedge c$
T	F	F
F	F	F

(h)

p	$\sim p$	$p \vee \sim p$	t
T	F	T	T
F	T	T	T

p	$\sim p$	$\sim(\sim p)$
T	F	T
F	T	F

p	$\sim p$	$p \wedge \sim p$	c
T	F	F	F
F	T	F	F

t	$\sim t$	c
T	F	F

3. Prove that

(a) $p \leftrightarrow q \equiv q \leftrightarrow p$ (b)* $p \leftrightarrow q \equiv (\sim p) \leftrightarrow (\sim q)$
(c) $p \rightarrow q \not\equiv q \rightarrow p$ (d)* $p \rightarrow q \not\equiv (\sim p) \rightarrow (\sim q)$

Many students make mistakes by confusing (a) with (c) and (b) with (d) in their mathematical arguments. Here are two examples.

(i) "Since $xy = xz$, therefore $y = z$". Why would a student make such a claim?

(ii)* Later in the course, we will prove the following result:
"If $|X| < |Y|$, then there is no bijection from X to Y."
A student invokes this result with the sentence:
"... Since $|A| = 7$ and $|B| = 6$, there is a bijection from A to B ..."
What is wrong with the argument?
(The logical issue here does not depend on the meaning of "bijection".)

Solution:

(a) $p \leftrightarrow q \equiv q \leftrightarrow p$: Exchanging columns of truth table has no effect.

(b)* The last two columns below show $p \leftrightarrow q \equiv (\sim p) \leftrightarrow (\sim q)$:

p	q	$\sim p$	$\sim q$	$p \leftrightarrow q$	$(\sim p) \leftrightarrow (\sim q)$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

(c) The 5th and 6th columns below show $p \to q \not\equiv q \to p$:

p	q	$\sim p$	$\sim q$	$p \to q$	$q \to p$	$(\sim p) \to (\sim q)$
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

(d)* The 5th and 7th columns above show $p \to q \not\equiv (\sim p) \to (\sim q)$.

(i) $y = z \to xy = xz$ is true (quantification omitted).
The student's claim is $xy = xz \to y = z$, thus confusing $p \to q$ with $q \to p$ when they are in fact not equivalent (converse error).

(ii)* **Theorem** $|X| < |Y| \ \to\ \sim$(there is a bijection from X to Y).
The student's claim is
$\sim(|A| < |B|) \ \to\ $ (there is a bijection from A to B),
thus confusing $p \to q$ with $(\sim p) \to (\sim q)$ (inverse error).

4. Which of the following pairs are equivalent?

(i) $p \vee (q \to r)$ and $(p \vee q) \to (p \vee r)$ (ii)* $(p \wedge q) \to r$ and $(p \to r) \wedge (q \to r)$

[The point of this problem is to check if $\to$, $\wedge$ and $\vee$ obey distributivity (see Problem 2(c)).]

Solution:

(i) Use truth table, or:

$(p \vee q) \to (p \vee r) \equiv (\sim(p \vee q)) \vee (p \vee r) \equiv ((\sim p) \wedge (\sim q)) \vee (p \vee r)$

$$\equiv ((\sim p) \vee (p \vee r)) \wedge ((\sim q) \vee (p \vee r)) \equiv (\sim q) \vee (p \vee r) \equiv p \vee (\sim q \vee r) \equiv p \vee (q \to r)$$

(ii)* The last two columns below show that $(p \wedge q) \to r \not\equiv (p \to r) \wedge (q \to r)$:

p	q	r	$p \wedge q$	$p \to r$	$q \to r$	$(p \wedge q) \to r$	$(p \to r) \wedge (q \to r)$
$\vdots$							
T	F	F	F	F	T	T	F
$\vdots$							

E.g. $(x \geq 0) \wedge (x \leq 0) \ \to x = 0$ is not the same as $(x \geq 0 \to x = 0) \wedge (x \leq 0 \to x = 0)$.

5. Use the equivalences in Problem 2 to prove that the following are tautologies.

(a)* $(p \wedge q) \to p$ (b)* $((p \vee q) \wedge \sim p) \to q$
(c) $((p \to q) \wedge p) \to q$ (d)* $((p \to q) \wedge \sim q) \to \sim p$
(e) $(\sim p \to (q \wedge \sim q)) \to p$ (f) $((p \to q) \wedge (q \to r)) \to (p \to r)$

[Mathematical arguments are often constructed by using one implication after another. Logically speaking, such an argument is constructed by using implications that are tautologies, like the ones above. For example, (c) is *modus ponens* and (e) is proof by contradiction.]

Solution:
Note: $p \to q \;\equiv\; (\sim p) \vee q$.

(a)* $(p \wedge q) \to p \equiv \sim(p \wedge q) \vee p \equiv (\sim p) \vee (\sim q) \vee p \equiv ((\sim p) \vee p) \vee (\sim q) \equiv T$.

(b)* $((p \vee q) \wedge \sim p) \to q \equiv \sim((p \vee q) \wedge \sim p) \vee q \equiv (\sim(p \vee q)) \vee (p \vee q) \equiv T$.

(c) $((p \to q) \wedge p) \to q \equiv \sim((p \to q) \wedge p) \vee q \equiv (\sim(p \to q)) \vee (\sim p) \vee q \equiv (\sim(p \to q)) \vee (p \to q) \equiv T$.

(d)* $((p \to q) \wedge \sim q) \to \sim p \equiv \sim((p \to q) \wedge \sim q) \vee \sim p \equiv (\sim(p \to q)) \vee q \vee \sim p$
$\equiv (\sim(p \to q)) \vee (\sim p \vee q) \equiv (\sim(p \to q)) \vee (p \to q) \equiv T$.

(e) $(\sim p \to (q \wedge \sim q)) \to p \equiv (\sim(\sim p \to (q \wedge \sim q))) \vee p \equiv (\sim(p \vee (q \wedge \sim q)) \vee p \equiv (\sim p) \vee p \equiv T$.

(f) $((p \to q) \wedge (q \to r)) \to (p \to r) \equiv \sim((p \to q) \wedge (q \to r)) \vee (p \to r)$
$\equiv (\sim(p \to q) \vee (\sim(q \to r)) \vee (p \to r) \equiv (\sim(\sim p \vee q)) \vee (\sim(\sim q \vee r)) \vee (\sim p \vee r)$
$\equiv (p \wedge \sim q) \vee (q \wedge \sim r) \vee (\sim p) \vee r \equiv ((p \wedge \sim q) \vee (\sim p) \vee ((q \wedge \sim r) \vee r)$
$\equiv ((p \vee \sim p) \wedge ((\sim q) \vee (\sim p))) \vee ((q \vee r) \wedge ((\sim r) \vee r) \equiv (\sim q) \vee (\sim p) \vee q \vee r$
$\equiv ((\sim q) \vee q) \vee (\sim p) \vee r \equiv T$.

6. Let P and Q be compound statements. Prove that

(i) $P \leftrightarrow Q$ is a tautology if and only if $P \equiv Q$;
(ii)* if $P \equiv Q$, then $\sim P \equiv\ \sim Q$;
(iii)* if $P \equiv Q$, then $P \wedge R \equiv Q \wedge R$ for any statement R.

[(i) establishes an intuitive relationship between tautology and equivalence.
(ii) and (iii) are useful for deriving equivalences.]

Solution:

(i) $(\Leftarrow)$ Assume $P \equiv Q$. Then

P	Q	$P \leftrightarrow Q$
T	T	T
F	F	T

so $P \leftrightarrow Q$ is a tautology.

$(\Rightarrow)$ Assume $P \leftrightarrow Q$ is a tautology. Then $P \to Q$ is true. Also $Q \to P$ is true, so $\sim P \to \sim Q$ is true. Therefore, if P is true, then $P \to Q$ implies Q is true; also, if P is false, then $\sim P \to \sim Q$ implies Q is false. Thus,

P	Q
T	T
F	F

so $P \equiv Q$.

(ii)* Assume $P \equiv Q$. Then

P	Q	$\sim P$	$\sim Q$
T	T	F	F
F	F	T	T

so $\sim P \equiv \sim Q$.

(iii)* Assume $P \equiv Q$. Then

P	Q	R	$P \wedge R$	$Q \wedge R$
T	T	T	T	T
T	T	F	F	F
F	F	T	F	F
F	F	F	F	F

so $P \wedge R \equiv Q \wedge R$.

7. Recall that $p \to q$ is defined by the truth table

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

The first two rows look reasonable, but the last two rows look disturbing. However, this way of defining $p \to q$ gives us the nice intuitive transitive property in Problem 5(f). Now, suppose we try an alternative for the last two rows, and define $\hookrightarrow_1$, $\hookrightarrow_2$ and $\hookrightarrow_3$ as follows:

p	q	$p \hookrightarrow_1 q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \hookrightarrow_2 q$
T	T	T
T	F	F
F	T	T
F	F	F

p	q	$p \hookrightarrow_3 q$
T	T	T
T	F	F
F	T	F
F	F	T

In each case, the tautology in Problem 5(f) no longer holds.
Prove this for $\hookrightarrow_3$, i.e. $((p \hookrightarrow_3 q) \wedge (q \hookrightarrow_3 r)) \hookrightarrow_3 (p \hookrightarrow_3 r)$ is not always true.

Solution:

p	q	r	$p \hookrightarrow_1 q$	$q \hookrightarrow_1 r$	$p \hookrightarrow_1 r$	$(p \hookrightarrow_1 q) \wedge (q \hookrightarrow_1 r)$	$((p \hookrightarrow_1 q) \wedge (q \hookrightarrow_1 r)) \hookrightarrow_1 (p \hookrightarrow_1 r)$
$\vdots$							
T	T	F	T	F	F	F	F
$\vdots$							($\Rightarrow$ not a tautology)

p	q	r	$p \hookrightarrow_2 q$	$q \hookrightarrow_2 r$	$p \hookrightarrow_2 r$	$(p \hookrightarrow_2 q) \wedge (q \hookrightarrow_2 r)$	$((p \hookrightarrow_2 q) \wedge (q \hookrightarrow_2 r)) \hookrightarrow_2 (p \hookrightarrow_2 r)$
$\vdots$							
T	T	F	T	F	F	F	F
$\vdots$							($\Rightarrow$ not a tautology)

p	q	r	$p \hookrightarrow_3 q$	$q \hookrightarrow_3 r$	$p \hookrightarrow_3 r$	$(p \hookrightarrow_3 q) \wedge (q \hookrightarrow_3 r)$	$((p \hookrightarrow_3 q) \wedge (q \hookrightarrow_3 r)) \hookrightarrow_3 (p \hookrightarrow_3 r)$
$\vdots$							
T	F	T	F	F	T	F	F
$\vdots$							($\Rightarrow$ not a tautology)

8. Let $B = \{1, 3, 5, 7, 11, 13\}$ and $C = \{0, 2, 4, 6\}$.

(i) Which of the following are true?

(a)	$\forall x \in B$	x is odd	(b)	$\exists x \in C$	x is odd
(c)	$\forall x \in C$	$x + 1 \in B$	(d)	$\forall x \in B$	$x - 1 \in C$
(e)*	$\forall x \in C$	$x + 2 \in C$	(f)*	$\exists x \in C$	$x + 2 \notin C$
(g)	$\forall x \in B$	$x \leq 10 \to x - 1 \in C$	(h)	$\forall x \in C$	$x \geq 10 \to x - 1 \in B$

Which of the above statements are negations of one another?

(ii) Negate each of the following statements and "push in" the negation as far as possible.

(a)	$\forall x \in C \ \ \forall y \in C \ x - y \in C$	(b)	$\forall x \in B \ \ \forall y \in B \ ((x < y) \wedge (y < 10)) \to y - x \in C$
(c)*	$\forall x \in B \ \ \forall y \in C \ x = y + 1$	(d)*	$\exists x \in B \ \ \exists y \in C \ x = y + 1$
(e)*	$\exists y \in C \ \ \exists x \in B \ x = y + 1$	(f)*	$\forall x \in B \ \ \forall y \in C \ x \neq y + 1$
(g)	$\exists x \in B \ \ \exists y \in C \ x \neq y + 1$	(h)	$\forall x \in B \ \ \exists y \in C \ x > y$
(i)*	$\forall x \in B \ \ \exists y \in C \ x \leq y$	(j)*	$\exists y \in C \ \ \forall x \in B \ x > y$
(k)*	$\forall x \in B \ \ \exists y \in C \ x = y + 1$	(l)*	$\forall y \in C \ \ \exists x \in B \ x = y + 1$
(m)	$\exists x \in B \ \ \forall y \in C \ x = y + 1$	(n)	$\exists x \in B \ \ \forall y \in C \ x \neq y + 1$
(o)*	$\forall y \in C \ \ \exists x \in B \ x \neq y + 1$	(p)*	$\forall x \in B \ \ (x < 10) \to (\exists y \in C \ x = y + 1)$
(q)*	$\exists x \in B \ \ (x < 10) \to (\forall y \in C \ x = y + 1)$	(r)*	$\exists y \in C \ \ (x < 10) \to (\forall x \in B \ x = y + 1)$
(s)	$\forall x \in B \ \ (x < 10) \to (\forall y \in C \ x = y + 1)$	(t)	$\forall x \in B \ \ (x < 10) \to (\forall y \in C \ x \neq y + 1)$

Which of the above statements are true? Which of them are equivalent (i.e. one is true if and only if the other is true)? Which of them are negations of each other?

Solution: Let $B = \{1, 3, 5, 7, 11, 13\}$ and $C = \{0, 2, 4, 6\}$.

(i) (a) $\forall x \in B \ x$ is odd : $1, 3, 5, 7, 11, 13$ are all odd, so (a) is **True**

(b) $\exists x \in C \ x$ is odd : $0, 2, 4, 6$ are all even, so (b) is **False**
$\sim(\exists x \in C \ x \text{ is odd}) \equiv \forall x \in C \sim(x \text{ is odd}) \equiv \forall x \in C (x \text{ is not odd})$ is **True**

(c) $\forall x \in C \ x + 1 \in B$: for $x = 0$, $x + 1 = 1 \in B$; for $x = 2$, $x + 1 = 3 \in B$;
for $x = 4$, $x + 1 = 5 \in B$; and for $x = 6$, $x + 1 = 7 \in B$; so (c) is **True**

(d) $\forall x \in B \ x - 1 \in C$: **False** (counterexample: $x = 11$)
$\sim(\forall x \in B \ x - 1 \in C) \equiv \exists x \in B \ x - 1 \notin C$

(e)* $\forall x \in C \ x + 2 \in C$: **False** (counterexample: $x = 6$)
$\sim(\forall x \in C \ x + 2 \in C) \equiv \exists x \in C \ x + 2 \notin C$ is **True**

(f)* $\exists x \in C \ x + 2 \notin C$: **True** (example: $x = 6$; $\sim$(e)$\equiv$(f))

(g) $\forall x \in B \ x \leq 10 \to x - 1 \in C$:
for $x = 1, 3, 5, 7$, $x \leq 10$ and $x - 1 \in C$ are both true;
for $x = 11, 13$, $x \leq 10$ is false, so $x \leq 10 \to x - 1 \in C$ is also true; thus (g) is **True**.

(h) $\forall x \in C \ x \geq 10 \to x - 1 \in B$: vacuously **True**
($x \geq 10$ is False for all $x \in C$).

(ii) (a) $\sim(\forall x \in C \ \forall y \in C \ x - y \in C) \equiv \exists x \in C \ \exists y \in C \ x - y \notin C$
$\forall x \in C \ \forall y \in C \ x - y \in C$: **False** (counterexample: $x = 2$, $y = 4$)

(b) $\sim(\forall x \in B \ \forall y \in B \ ((x < y) \wedge (y < 10)) \to y - x \in C)$
$\equiv \exists x \in B \ \exists y \in B \ (x < y) \wedge (y < 10) \wedge (y - x \notin C)$
$\forall x \in B \ \forall y \in B \ (x < y) \wedge (y < 10) \to y - x \in C)$:
$y - x \in \ C$ is true for $x = 1$ and $y = 3, 5, 7$, for $x = 3$ and $y = 5, 7$,
and for $x = 5$ and $y = 7$, so (b) is **True**

(c)* $\sim(\forall x \in B \ \forall y \in C \ x = y + 1) \equiv \exists x \in B \ \exists y \in C \ x \neq y + 1$
$\forall x \in B \ \forall y \in C \ x = y + 1$: **False** (counterexample: $x = 1$, $y = 2$)

(ii) (d)* $\sim(\exists x \in B\ \exists y \in C\ x = y+1) \equiv \forall x \in B\ \forall y \in C\ x \neq y+1$
$\exists x \in B\ \exists y \in C\ x = y+1$: **True** (example: $x = 3,\ y = 2$)

(e)* $\sim(\exists y \in C\ \exists x \in B\ x = y+1) \equiv \forall y \in C\ \forall x \in B\ x \neq y+1$
$\exists y \in C\ \exists x \in B\ x = y+1$: **True** (Note: (e)≡(d).)

(f)* $\sim(\forall x \in B\ \forall y \in C\ x \neq y+1) \equiv \exists x \in B\ \exists y \in C\ x = y+1$
$\forall x \in B\ \forall y \in C\ x \neq y+1$: **False** (counterexample: $x = 3,\ y = 2$;
note: ∼(d)≡(f)≡ ∼(e))

(g) $\sim(\exists x \in B\ \exists y \in C\ x \neq y+1) \equiv \forall x \in B\ \forall y \in C\ x = y+1$
$\exists x \in B\ \exists y \in C\ x \neq y+1$: **True** (example: $x = 1,\ y = 2$; note: (g)≡ ∼(c))

(h) $\sim(\forall x \in B\ \exists y \in C\ x > y) \equiv \exists x \in B\ \forall y \in C\ x \leq y$
$\forall x \in B\ \exists y \in C\ x > y$: for $x = 1, 3, 5, 7, 11, 13$, we can pick $y = 0$ as example,
so (h) is **True**

(i)* $\sim(\forall x \in B\ \exists y \in C\ x \leq y) \equiv \exists x \in B\ \forall y \in C\ x > y$
$\forall x \in B\ \exists y \in C\ x \leq y$: **False** (counterexample: $x = 11$)

(j)* $\sim(\exists y \in C\ \forall x \in B\ x > y) \equiv \forall y \in C\ \exists x \in B\ x \leq y$
$\exists y \in C\ \forall x \in B\ x > y$: **True** (example: $y = 0$)
note that (h)≢(j) in general: consider $B = C = \mathbb{Z}$)

(k)* $\sim(\forall x \in B\ \exists y \in C\ x = y+1) \equiv \exists x \in B\ \forall y \in C\ x \neq y+1$
$\forall x \in B\ \exists y \in C\ x = y+1$: **False** (counterexample: $x = 11$)

(l)* $\sim(\forall y \in C\ \exists x \in B\ x = y+1) \equiv \exists y \in C\ \forall x \in B\ x \neq y+1$
$\forall y \in C\ \exists x \in B\ x = y+1$: for $y = 0$, the example is $x = 1$; for $y = 2$, pick $x = 3$;
for $y = 4$, pick $x = 5$; for $y = 6$, pick $x = 7$; so (l) is **True**.

(m) $\sim(\exists x \in B\ \forall y \in C\ x = y+1) \equiv \forall x \in B\ \exists y \in C\ x \neq y+1$
$\exists x \in B\ \forall y \in C\ x = y+1$: **False**

(n) $\sim(\exists x \in B\ \forall y \in C\ x \neq y+1) \equiv \forall x \in B\ \exists y \in C\ x = y+1$
$\exists x \in B\ \forall y \in C\ x \neq y+1$: **True** (example: $x = 11$; note: (n)≡ ∼(k))

(o)* $\sim(\forall y \in C\ \exists x \in B\ x \neq y+1) \equiv \exists y \in C\ \forall x \in B\ x = y+1$
$\forall y \in C\ \exists x \in B\ x \neq y+1$: **True**

(p)* $\sim(\forall x \in B\ (x < 10) \to (\exists y \in C\ x = y+1))$
$\equiv \exists x \in B((x < 10) \wedge \sim(\exists y \in C\ x = y+1))$
$\equiv \exists x \in B((x < 10) \wedge (\forall y \in C\ x \neq y+1))$
$\forall x \in B\ (x < 10) \to (\exists y \in C\ x = y+1)$: for $x = 1$, pick $y = 0$; for $x = 3$, pick $y = 2$;
for $x = 5$, pick $y = 4$; for $x = 7$, pick $y = 6$; so (p) is **True**

(q)* $\sim(\exists x \in B\ (x < 10) \to (\forall y \in C\ x = y+1))$
$\equiv \forall x \in B\ ((x < 10) \wedge \sim(\forall y \in C\ x = y+1))$
$\equiv \forall x \in B\ ((x < 10) \wedge (\exists y \in C\ x \neq y+1))$
$\exists x \in B\ (x < 10) \to (\forall y \in C\ x = y+1)$: **True** (example: $x = 11$)

(r)* $\sim(\exists y \in C\ (x < 10) \to (\forall x \in B\ x = y+1))$ is not a statement
(x not quantified)

(ii) (s) $\sim(\forall x \in B\ (x < 10) \to (\forall y \in C\ x = y + 1)) \equiv \exists x \in B\ ((x < 10) \wedge \sim(\forall y \in C\ x = y + 1))$

$\equiv \exists x \in B\ ((x < 10) \wedge (\exists y \in C\ x \neq y + 1))$

$\forall x \in B\ (x < 10) \to (\forall y \in C\ x = y + 1)$: **False** (counterexample: $x = 3$, $y = 0$)

(t) $\sim(\forall x \in B\ (x < 10) \to (\forall y \in C\ x \neq y + 1)) \equiv \exists x \in B\ ((x < 10) \wedge \sim(\forall y \in C\ x \neq y + 1))$

$\equiv \exists x \in B\ ((x < 10) \wedge (\exists y \in C\ x = y + 1))$

$\forall x \in B\ (x < 10) \to (\forall y \in C\ x \neq y + 1)$: **False** (counterexample: $x = 3$, $y = 2$)