Name:______________________________ **Tutorial Group:**__________________ **(day/time)**

1 Determine whether $((p \vee q) \wedge (q \to r)) \to r$ is a tautology. [2 marks]

2 Let $A = \{-2, -1, 0, 1, 2\}$, $B = \{0, 1, 4\}$ and $C = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$. Which of the following are true? Justify your answer.

(i) $\forall x \in C\ (x \in A) \leftrightarrow (x^2 \in B)$. [2 marks]

(ii) $\forall x \in C\ (\forall y \in B\ xy \in B) \to (x^2 = x)$. [2 marks]

(iii) $\exists x \in A\ \forall y \in A\ (x \neq 0) \wedge (xy \in B)$. [2 marks]

(iv) $\sim (\forall x \in A\ \exists y \in A\ (x \neq 0) \wedge (xy \in B))$. [2 marks]

3 Consider the claim:

"For any integers m and n, if $m+n$ is even, then either both m and n are even or both are odd."

(i) State the claim symbolically, using predicates $Even(x)$ and $Odd(y)$. [2 marks]

The following is a proof:

"We prove by contradiction. Suppose one of them is odd and the other is even. Without loss of generality, we may assume m is even and n is odd. Then, $m = 2h$ and $n = 2k + 1$ for some integers h and k, so $m + n = 2(h + k) + 1$. Since $h + k$ is an integer, $m + n$ is therefore odd, so we get a contradiction."

(ii) Let p be the claim in (i). Why does the the proof for p start by assuming that one integer is odd and the other is even? [2 marks]

(iii) Point out one example of universal instantiation in this proof. [1 mark]

(iv) Point out one example of modus ponens in this proof. [1 mark]

(v) Explain what is meant by "Without loss of generality" in this proof. [1 mark]

4 Two sequences β and γ are said to **span** a space S over field F if and only if

"every sequence α in S can be expressed as $\alpha = b\beta + c\gamma$ for some b and c in F".

(i) State the condition (in "...") symbolically. [1 mark]

A student writes: "ψ and η span S because $\omega \in S$, $0 \in F$ and $\omega = 0\psi + 0\eta$."
(Note: $\omega \in S$, $0 \in F$ and $\omega = 0\psi + 0\eta$ are all correct.)

(ii) Explain why this argument might be wrong. [1 mark]

(iii) Explain why this argument might be correct. [1 mark]