Lecture #16

Quine-McCluskey

NUS | School of
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Lecture #16: Quine-McCluskey

This topic is only for your own reading only.

- A tabulation method similar in concept to K-map

- Applicable for functions with any number of variables

  - K-maps are useful for functions with at most 5 or 6 variables

- Tedious on paper, but can be automated (programmed)

- Non-examinable

  - But knowing it may enhance your understanding of K-maps

PIs and EPIs

- To find the simplest SOP expression from a K-map, you need to obtain:
  - Minimum number of literals per product term; and
  - Minimum number of product terms.

- Achieved through K-map using
  - *Biggest groupings* of minterms (prime implicants) where possible; and
  - *No redundant groupings* (look for essential prime implicants)

Eg: $F(A,B,C,D) = \Sigma m(2,3,4,5,7,8,10,13,15)$

Step 1: List out all minterms in groups with same number of 1s in their binary codes.

1st column

2: 0010
4: 0100
8: 1000

⎱ Codes with one 1

3: 0011
5: 0101
10: 1010

⎱ Codes with two 1s

7: 0111
13: 1101

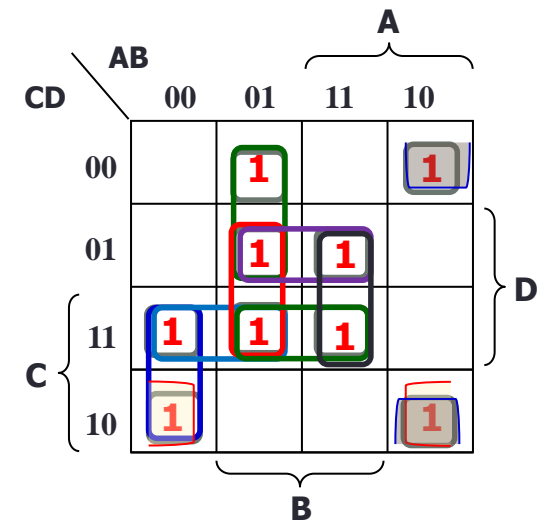⎱ Codes with three 1s

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15: 1111

⎱ Codes with four 1s

Eg: F(A,B,C,D) = Σm(2,3,4,5,7,8,10,13,15)

Step 2: Combine codes that differ by 1 bit into bigger group, write the combined code in next column.

1ˢᵗ column
✓ 2: 0010
✓ 4: 0100
✓ 8: 1000

✓ 3: 0011
✓ 5: 0101
✓ 10: 1010

✓ 7: 0111
✓ 13: 1101

✓ 15: 1111

2ⁿᵈ column
  2,3: 001-
  2,10: -010
  4,5: 010-
  8,10: 10-0 ⎫ Codes with one 1

  3,7: 0-11
  5,7: 01-1 ⎬ Codes with two 1s
  5,13: -101

  7,15: -111
  13,15: 11-1 ⎬ Codes with three 1s

Eg: F(A,B,C,D) = Σm(2,3,4,5,7,8,10,13,15)

Step 3:   Repeat step 2 – Combine codes that differ by 1 bit into bigger group, write the combined code in next column.

1st column

✓ 2: 0010
✓ 4: 0100
✓ 8: 1000

✓ 3: 0011
✓ 5: 0101
✓10: 1010

✓ 7: 0111
✓13: 1101

✓15: 1111

2nd column

2,3: 001-
2,10: -010
4,5: 010-
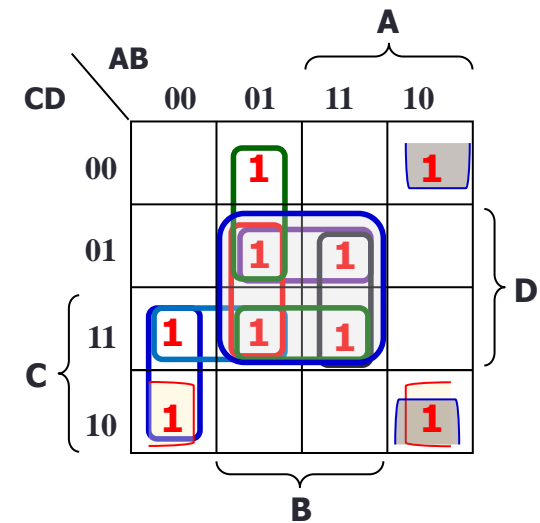8,10: 10-0

3,7: 0-11
✓ 5,7: 01-1
✓5,13: -101

✓ 7,15: -111
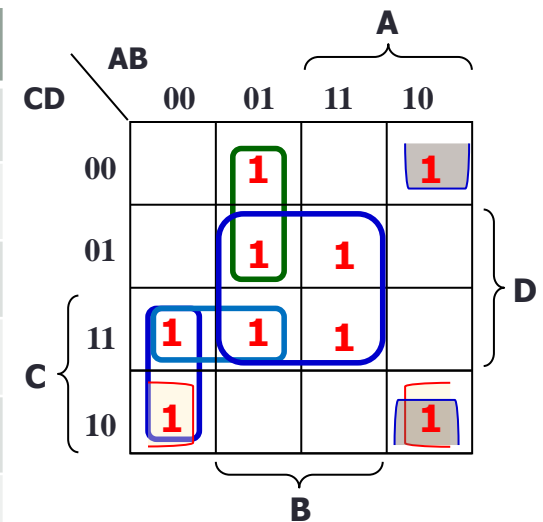✓13,15: 11-1

3rd column

5,7,13,15: -1-1

~~5,7,13,15: -1-1~~

We have completed
Phase 1: Identifying all the
Prime Implicants (PIs)!

Eg: F(A,B,C,D) = Σm(2,3,4,5,7,8,10,13,15)

Phase 2: Identify the Essential Prime Implicants (EPIs)

- Draw the PI chart

	2	3	4	5	7	8	10	13	15
2,3: 001- (A'.B'.C)	✓	✓							
2,10: -101 (B.C'.D)	✓						✓		
EPI 4,5: 010- (A'.B.C')			✓	✓					
EPI 8,10: 10-0 (A.B'.D')						✓	✓		
3,7: 0-11 (A'.C.D)		✓			✓				
EPI 5,7,13,15: -1-1 (B.D)				✓	✓			✓	✓

Where are the EPIs? Look for columns containing a single tick.
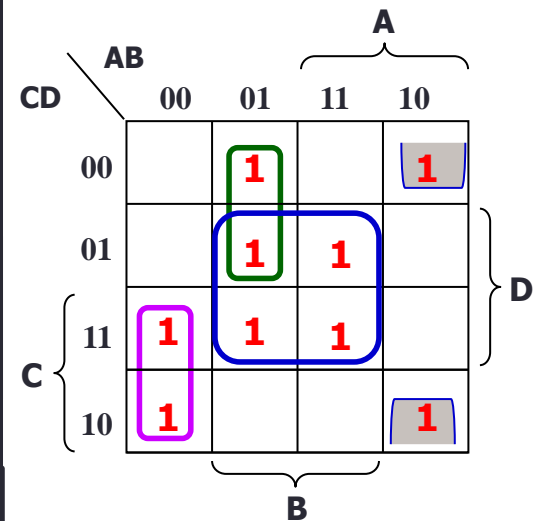
EPIs are: A'.B.C', A.B'.D', and B.D

But we are not done yet. There are still minterms not covered by the EPIs!

Eg: $F(A,B,C,D) = \Sigma m(2,3,4,5,7,8,10,13,15)$

Phase 2: After identifying the EPIs

- Draw the reduced PI chart if there are minterms not covered

	2	3	4	5	7	8	10	13	15
2,3: 001- (A'.B'.C)	✓	✓							
2,10: -101 (B.C'.D)	✓						✓		
EPI 4,5: 010- (A'.B.C')			✓	✓					
EPI 8,10: 10-0 (A.B'.D')						✓	✓		
3,7: 0-11 (A'.C.D)		✓			✓				
EPI 5,7,13,15: -1-1 (B.D)				✓	✓			✓	✓

1. Find out what are the minterms covered by the EPIs.

2. Remove the EPIs and minterms they cover from the chart → reduced PI chart.

3. Find the minimum number of remaining PIs to cover the remaining minterms.

Answer:
B.D + A'.B.C' + A.B'.D'
+ A'.B'.C

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