CS2100 COMPUTER ORGANISATION

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Lecture #19

Sequential Logic

NUS National University of Singapore | School of Computing

Lecture #19: Sequential Logic (1/2)

Lecture #19: Sequential Logic (2/2)

5. Asynchronous Inputs
6. Synchronous Sequential Circuit
   - 6.1 Flip-flop Characteristic Tables
   - 6.2 Analysis
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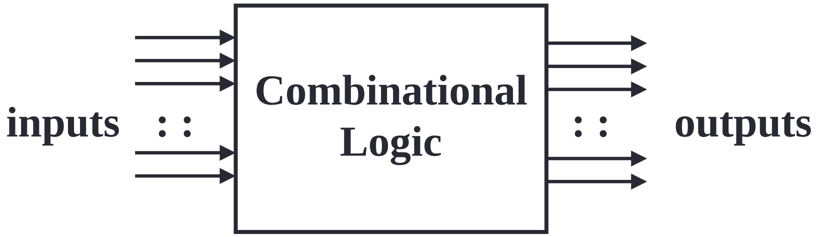
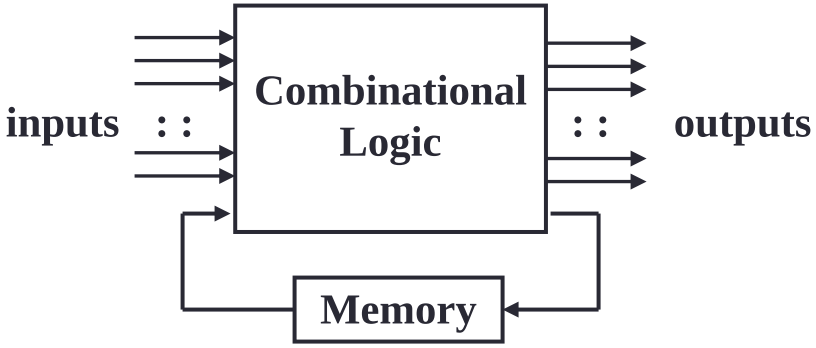
1. Introduction (1/2)

- Two classes of logic circuits
  - Combinational
  - Sequential

- Combinational Circuit
  - Each output depends entirely on the immediate (present) inputs.



- Sequential Circuit
  - Each output depends on both present inputs and state.



1. Introduction (2/2)

- Two types of sequential circuits:
  - Synchronous: outputs change only at specific time
  - Asynchronous: outputs change at any time
- Multivibrator: a class of sequential circuits
  - Bistable (2 stable states)
  - Monostable or one-shot (1 stable state)
  - Astable (no stable state)
- Bistable logic devices
  - Latches and flip-flops.
  - They differ in the methods used for changing their state.

2. Memory Elements (1/3)

- Memory element: a device which can remember value indefinitely, or change value on command from its inputs.

- Characteristic table:

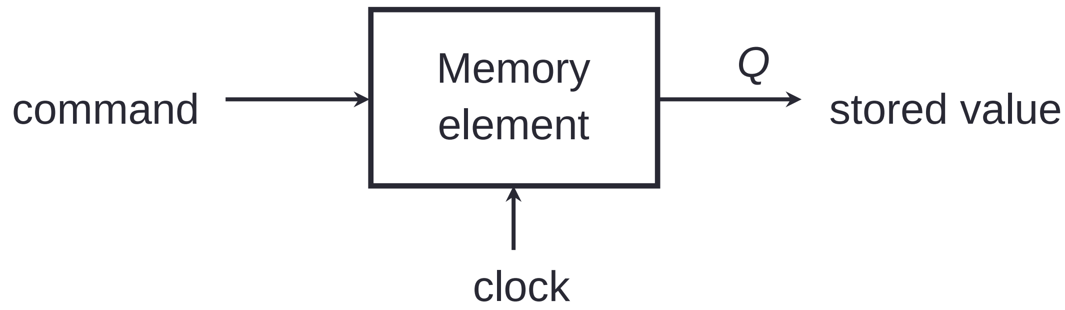
Command (at time *t*)	*Q(t)*	*Q(t+1)*
Set	X	1
Reset	X	0
Memorise / No Change	0	0
	1	1

Q(t) or ***Q***: current state

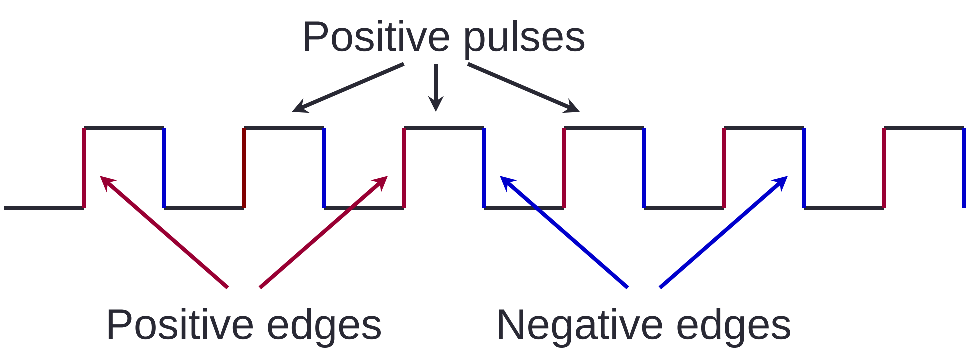
Q(t+1) or Q^+: next state

2. Memory Elements (2/3)

- Memory element with clock.

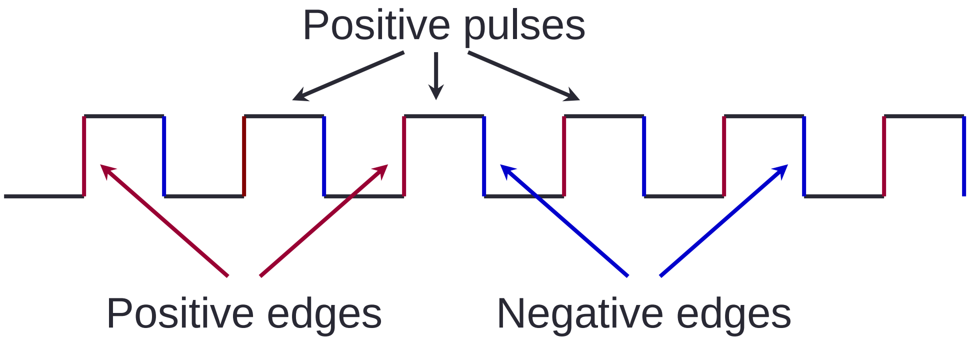


- Clock is usually a square wave.



2. Memory Elements (3/3)

- Two types of triggering/activation
  - Pulse-triggered
  - Edge-triggered



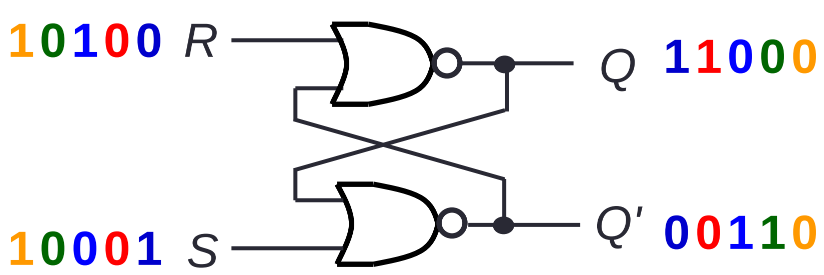
- Pulse-triggered
  - Latches
  - ON = 1, OFF = 0
- Edge-triggered
  - Flip-flops
  - Positive edge-triggered (ON = from 0 to 1; OFF = other time)
  - Negative edge-triggered (ON = from 1 to 0; OFF = other time)

3.1 *S*-*R* Latch (1/3)

- Two inputs: *S* and *R*.
- Two complementary outputs: *Q* and *Q'*.
  - When *Q* = HIGH, we say latch is in SET state.
  - When *Q* = LOW, we say latch is in RESET state.
- For active-high input *S*-*R* latch (also known as NOR gate latch)
  - *R* = HIGH and *S* = LOW → *Q* becomes LOW (RESET state)
  - *S* = HIGH and *R* = LOW → *Q* becomes HIGH (SET state)
  - Both *R* and *S* are LOW →No change in output *Q*
  - Both *R* and *S* are HIGH →Outputs *Q* and *Q'* are both LOW (invalid!)
- Drawback: invalid condition exists and must be avoided.

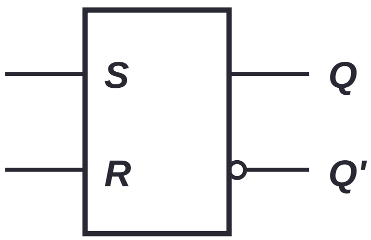
3.1 *S*-*R* Latch (2/3)

- Active-high input *S*-*R* latch:



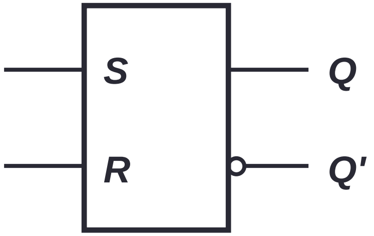
S	*R*	*Q*	*Q'*	
1	0	1	0	initial
0	0	1	0	(afer *S*=1, *R*=0)
0	1	0	1	
0	0	0	1	(after *S*=0, *R*=1)
1	1	0	0	invalid!

- Block diagram:



3.1 *S-R* Latch (3/3)

- Characteristic table for active-high input *S-R* latch:



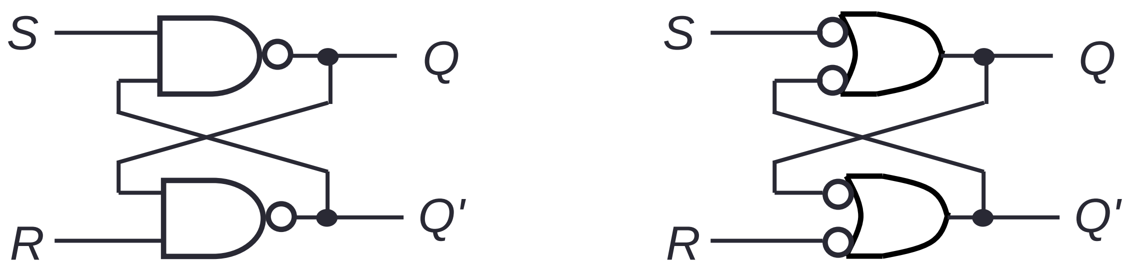
S	*R*	*Q*	*Q'*	
0	**0**	**NC**	**NC**	**No change. Latch remained in present state.**
1	**0**	**1**	**0**	**Latch SET.**
0	**1**	**0**	**1**	**Latch RESET.**
1	**1**	**0**	**0**	**Invalid condition.**

S	*R*	*Q(t+1)*	
0	0	*Q(t)*	No change
0	1	0	Reset
1	0	1	Set
1	1	indeterminate	

$Q(t+1)$ = **?**

3.1 Active-Low *S*-*R* Latch

- (You may skip this slide.)
- What we have seen is active-high input *S*-*R* latch.
- There are active-low input *S*-*R* latches, where NAND gates are used instead. See diagram on the left below.



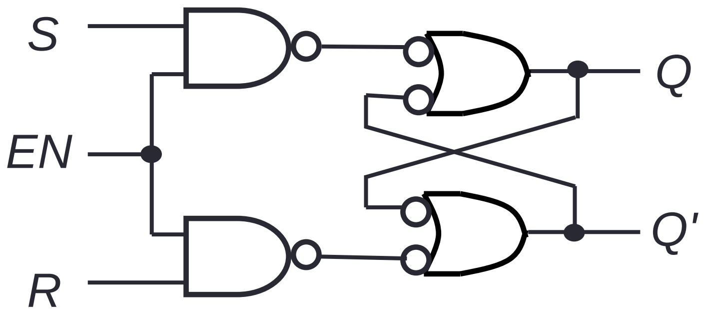
- In this case,
  - when *R*=0 and *S*=1, the latch is reset (i.e. Q becomes 0)
  - when *R*=1 and *S*=0, the latch is set (i.e. Q becomes 1)
  - when *S*=*R*=1, it is a no-change command.
  - when *S*=*R*=0, it is an invalid command.

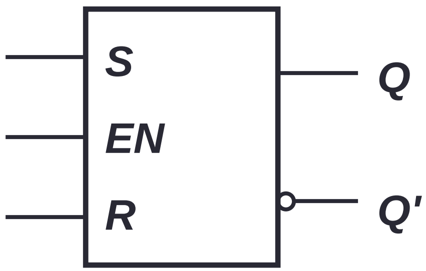
(Sometimes, the inputs are labelled as *S'* and *R'*.)

- Sometimes, we use the alternative gate diagram for the NAND gate. See diagram on the right above. (This appears in more complex latches/flip-flops in the later slides.)

3.1 Gated *S*-*R* Latch

- *S*-*R* latch + *enable input* (*EN*) and 2 NAND gates
  → a gated *S*-*R* latch.

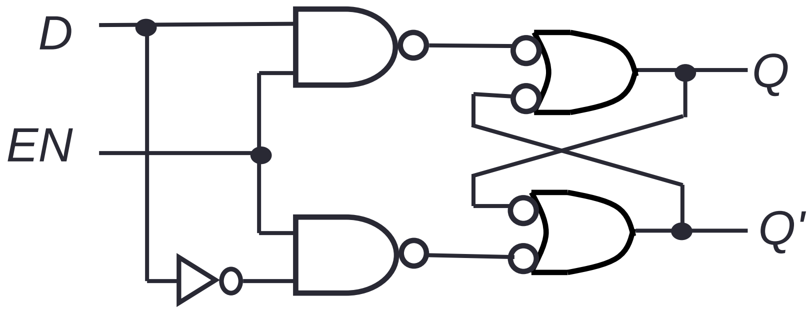


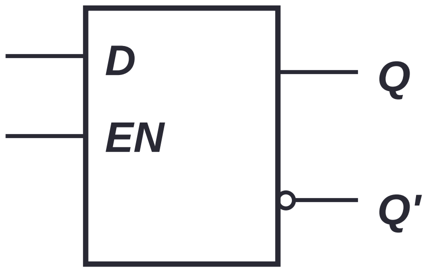


- Outputs change (if necessary) only when *EN* is high.

3.2 Gated *D* Latch (1/2)

- Make input *R* equal to *S'* → gated *D* latch.
- *D* latch eliminates the undesirable condition of invalid state in the *S*-*R* latch.





3.2 Gated *D* Latch (2/2)

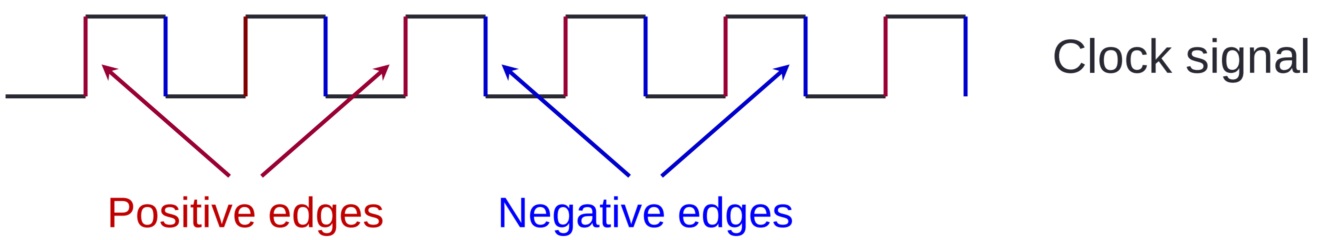
- When *EN* is high,
  - *D* = HIGH → latch is SET
  - *D* = LOW → latch is RESET
- Hence when EN is high, *Q* "follows" the *D* (data) input.
- Characteristic table:

EN	***D***	***Q(t+1)***	
1	**0**	**0**	**Reset**
1	**1**	**1**	**Set**
0	**X**	***Q(t)***	**No change**

When *EN*=1, ***Q(t+1)* = ?**

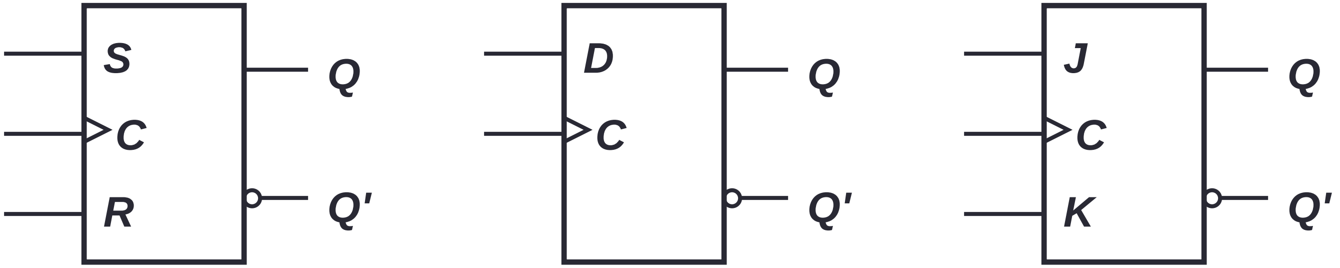
4. Flip-flops (1/2)

- Flip-flops are synchronous bistable devices.
- Output changes state at a specified point on a triggering input called the clock.
- Change state either at the positive (rising) edge, or at the negative (falling) edge of the clock signal.

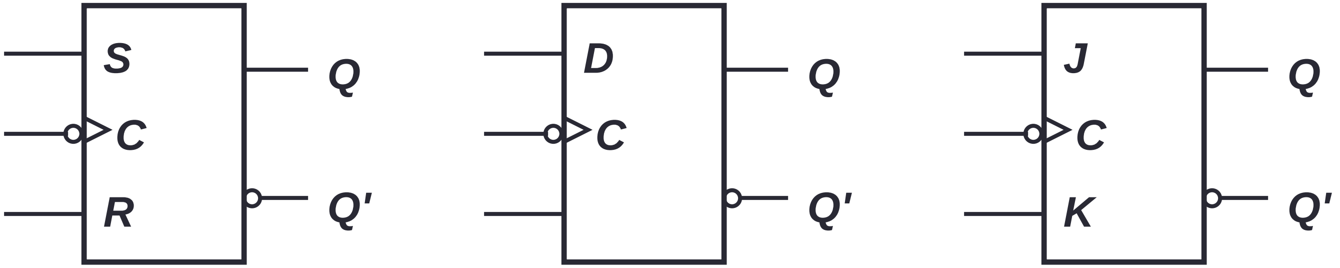


4. Flip-flops (2/2)

- *S-R* flip-flop, *D* flip-flop, and *J-K* flip-flop.
- Note the ">" symbol at the clock input.



Positive edge-triggered flip-flops

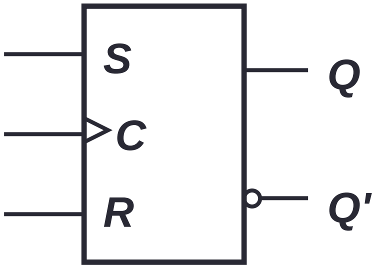


Negative edge-triggered flip-flops

4.1 *S*-*R* Flip-flop

- *S*-*R* flip-flop: On the triggering edge of the clock pulse,
  - *R* = HIGH and *S* = LOW ➔ *Q* becomes LOW (RESET state)
  - *S* = HIGH and *R* = LOW ➔ *Q* becomes HIGH (SET state)
  - Both *R* and *S* are LOW ➔No change in output *Q*
  - Both *R* and *S* are HIGH ➔Invalid!

- Characteristic table of positive edge-triggered *S*-*R* flip-flop:

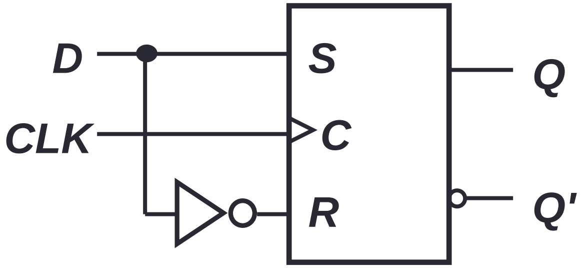


S	*R*	*CLK*	*Q(t+1)*	Comments
0	0	X	*Q(t)*	No change
0	1	↑	0	Reset
1	0	↑	1	Set
1	1	↑	?	Invalid

X = irrelevant (“don’t care”)
↑ = clock transition LOW to HIGH

4.2 *D* Flip-flop (1/2)

- *D* flip-flop: Single input *D* (data). On the triggering edge of the clock pulse,
  - *D* = HIGH ➔ *Q* becomes HIGH (SET state)
  - *D* = LOW ➔ *Q* becomes LOW (RESET state)
- Hence, *Q* “follows” *D* at the clock edge.
- Convert *S-R* flip-flop into a *D* flip-flop: add an inverter.



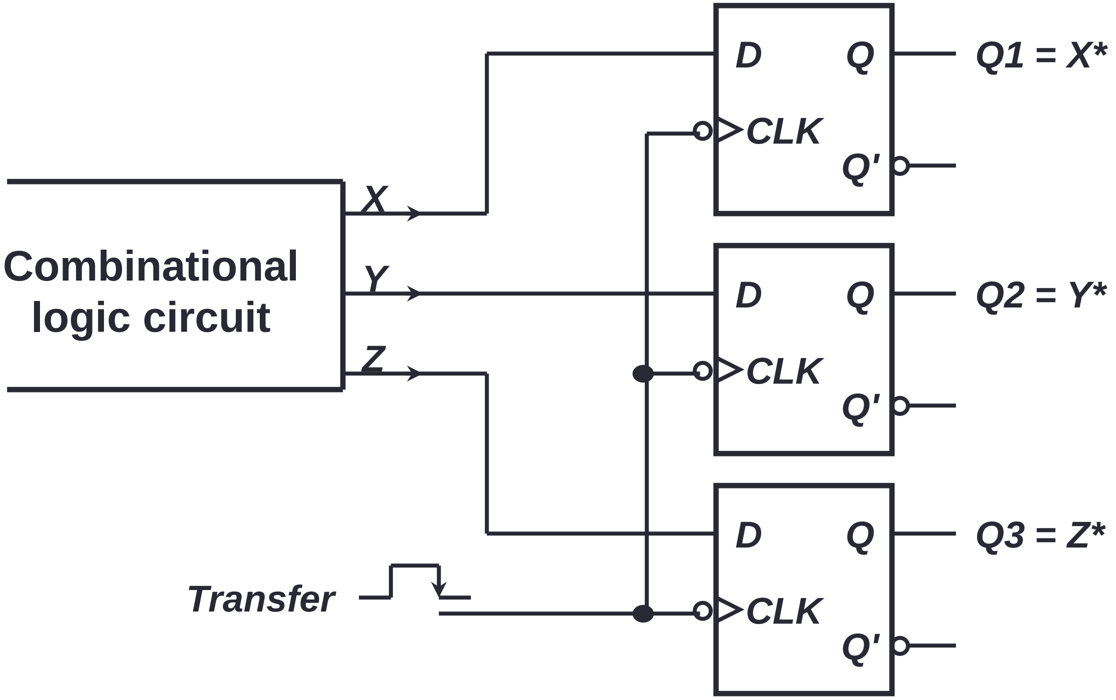
A positive edge-triggered D flip-flop formed with an S-R flip-flop.

D	*CLK*	*Q(t+1)*	Comments
1	↑	1	Set
0	↑	0	Reset

↑ = clock transition LOW to HIGH

4.2 *D* Flip-flop (2/2)

- Application: Parallel data transfer.
  - To transfer logic-circuit outputs *X*, *Y*, *Z* to flip-flops *Q*1, *Q*2 and *Q*3 for storage.



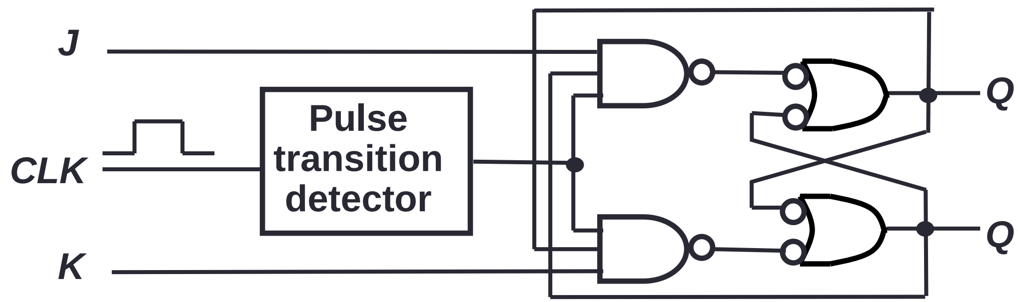
*** After occurrence of negative-going transition**

4.3 *J-K* Flip-flop (1/2)

- *J-K* flip-flop: *Q* and *Q'* are fed back to the pulse-steering NAND gates.
- No invalid state.
- Include a toggle state
  - *J* = HIGH and *K* = LOW ➔ *Q* becomes HIGH (SET state)
  - *K* = HIGH and *J* = LOW ➔ *Q* becomes LOW (RESET state)
  - Both *J* and *K* are LOW ➔No change in output *Q*
  - Both *J* and *K* are HIGH ➔Toggle

4.3 *J-K* Flip-flop (2/2)

- *J-K* flip-flop circuit:



- Characteristic table:

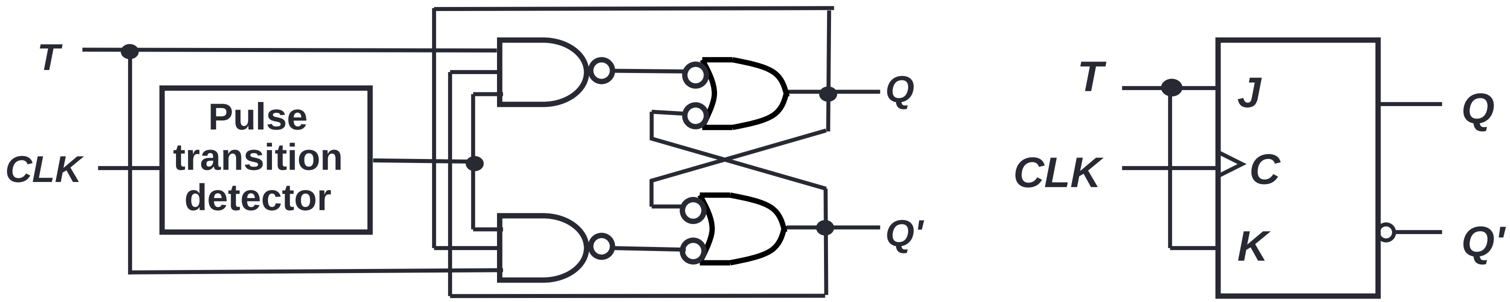
J	*K*	*CLK*	*Q(t+1)*	Comments
0	0	↑	*Q(t)*	No change
0	1	↑	0	Reset
1	0	↑	1	Set
1	1	↑	*Q(t)'*	Toggle

Q	*J*	*K*	*Q(t+1)*
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Q(t+1) = **?**

4.4 *T* Flip-flop

- *T* flip-flop: Single input version of the *J-K* flip-flop, formed by tying both inputs together.



- Characteristic table:

T	*CLK*	*Q(t+1)*	Comments
0	↑	*Q(t)*	No change
1	↑	*Q(t)'*	Toggle

Q	*T*	*Q(t+1)*
0	0	0
0	1	1
1	0	1
1	1	0

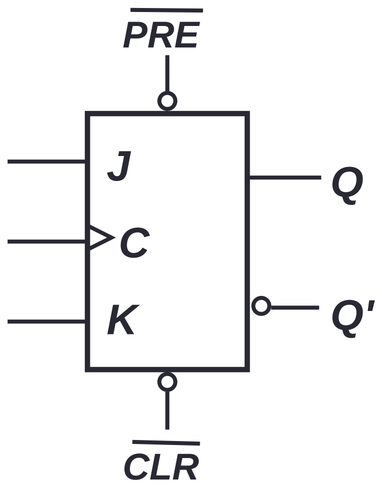
***Q(t+1)* = ?**

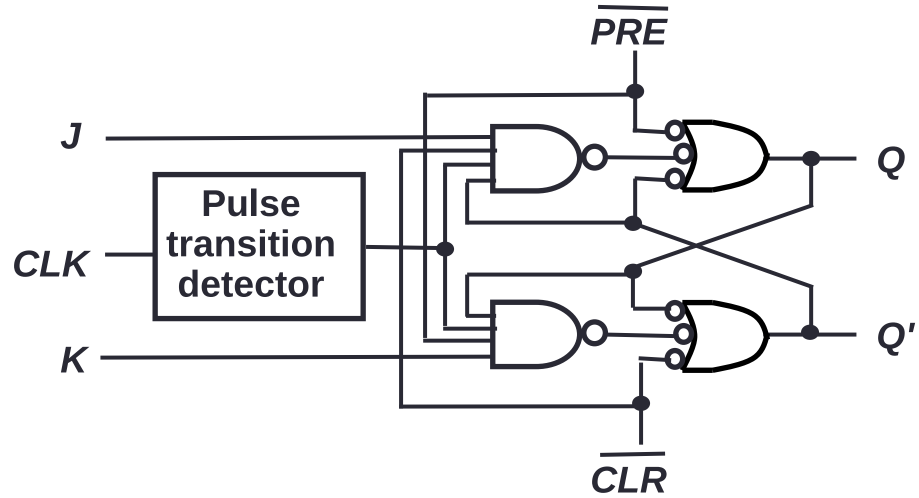
5. Asynchronous Inputs (1/2)

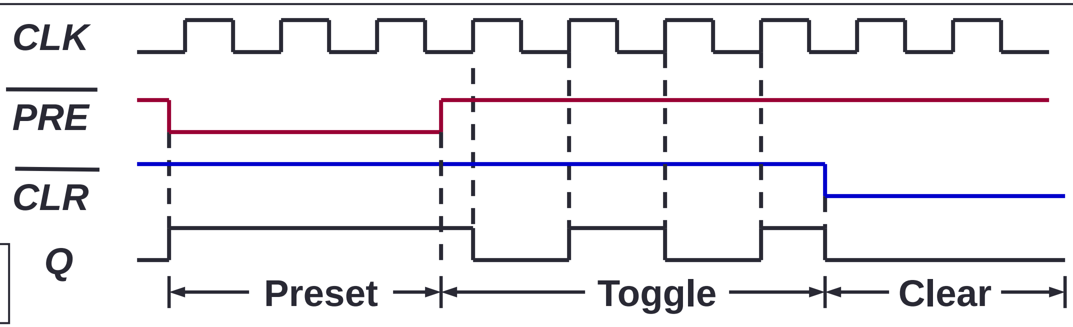
- *S-R*, *D* and *J-K* inputs are synchronous inputs, as data on these inputs are transferred to the flip-flop's output only on the triggered edge of the clock pulse.
- Asynchronous inputs affect the state of the flip-flop independent of the clock; example: *preset* (*PRE*) and *clear* (*CLR*) [or *direct set* (*SD*) and *direct reset* (*RD*)].
- When *PRE*=HIGH, *Q* is <u>immediately</u> set to HIGH.
- When *CLR*=HIGH, *Q* is <u>immediately</u> cleared to LOW.
- Flip-flop in normal operation mode when both *PRE* and *CLR* are LOW.

5. Asynchronous Inputs (2/2)

- A *J-K* flip-flop with active-low PRESET and CLEAR asynchronous inputs.







J = *K* = HIGH

6. Synchronous Sequential Circuits

- Building blocks: logic gates and flip-flops.
- Flip-flops make up the memory while the gates form one or more combinational sub-circuits.
- We have discussed *S-R* flip-flop, *J-K* flip-flop, *D* flip-flop and *T* flip-flop.

6.1 Flip-flop Characteristic Tables

- Each type of flip-flop has its own behaviour, shown by its characteristic table.

J	*K*	*Q(t+1)*	Comments
0	0	*Q(t)*	No change
0	1	0	Reset
1	0	1	Set
1	1	*Q(t)'*	Toggle

S	*R*	*Q(t+1)*	Comments
0	0	*Q(t)*	No change
0	1	0	Reset
1	0	1	Set
1	1	?	Unpredictable

D	*Q(t+1)*	
0	0	Reset
1	1	Set

T	*Q(t+1)*	
0	*Q(t)*	No change
1	*Q(t)'*	Toggle

6.2 Sequential Circuits: Analysis (1/7)

- Given a sequential circuit diagram, we can analyze its behaviour by deriving its *state table* and hence its *state diagram*.
- Requires *state equations* to be derived for the flip-flop inputs, as well as *output functions* for the circuit outputs other than the flip-flops (if any).
- We use ***A(t)*** and ***A(t+1)*** (or simply ***A*** and $\boldsymbol{A^+}$) to represent the present state and next state, respectively, of a flip-flop represented by *A*.

6.2 Sequential Circuits: Analysis (2/7)

- Example using *D* flip-flops

State equations:

$A^+ = A \cdot x + B \cdot x$

$B^+ = A' \cdot x$

Output function:

$y = (A + B) \cdot x'$

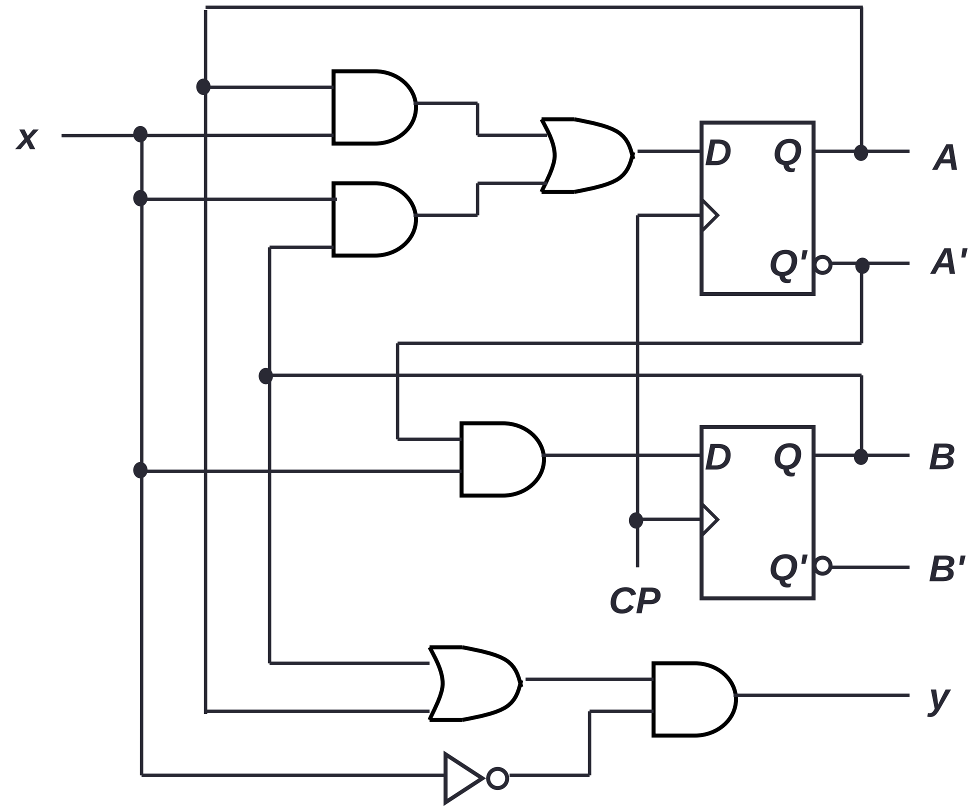


Figure 1

6.2 Sequential Circuits: Analysis (3/7)

- From the *state equations* and *output function*, we derive the ***state table***, consisting of all possible binary combinations of present states and inputs.
- State table
  - Similar to truth table.
  - Inputs and present state on the left side.
  - Outputs and next state on the right side.
- *m* flip-flops and *n* inputs $\rightarrow 2^{m+n}$ rows.

6.2 Sequential Circuits: Analysis (4/7)

- ***State table*** for circuit of Figure 1:

State equations:

$A^+ = A{\cdot}x + B{\cdot}x$

$B^+ = A'{\cdot}x$

Output function:

$y = (A + B){\cdot}x'$

Present State		Input	Next State		Output
A	***B***	***x***	A^+	B^+	***y***
0	**0**	**0**	**0**	**0**	**0**
0	**0**	**1**	**0**	**1**	**0**
0	**1**	**0**	**0**	**0**	**1**
0	**1**	**1**	**1**	**1**	**0**
1	**0**	**0**	**0**	**0**	**1**
1	**0**	**1**	**1**	**0**	**0**
1	**1**	**0**	**0**	**0**	**1**
1	**1**	**1**	**1**	**0**	**0**

6.2 Sequential Circuits: Analysis (5/7)

- Alternative form of state table:

Full table

Present State		Input	Next State		Output
A	B	x	A^+	B^+	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

Compact table

Present State	Next State		Output	
	x=0	x=1	x=0	x=1
AB	A^+B^+	A^+B^+	y	y
00	00	01	0	0
01	00	11	1	0
10	00	10	1	0
11	00	10	1	0

6.2 Sequential Circuits: Analysis (6/7)

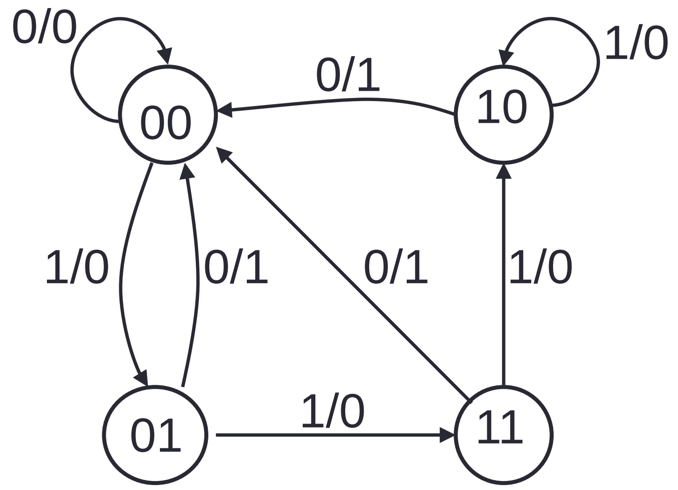
- From the *state table*, we can draw the ***state diagram***.
- State diagram
  - Each state is denoted by a circle.
  - Each arrow (between two circles) denotes a transition of the sequential circuit (a row in state table).
  - A label of the form *a*/*b* is attached to each arrow where *a* (if there is one) denotes the inputs while *b* (if there is one) denotes the outputs of the circuit in that transition.
- Each combination of the flip-flop values represents a state. Hence, m flip-flops $\rightarrow$ up to 2^m states.

6.2 Sequential Circuits: Analysis (7/7)

- **State diagram** of the circuit of Figure 1:

Present State	Next State		Output	
	x=0	x=1	x=0	x=1
AB	A^+B^+	A^+B^+	y	y
00	00	01	0	0
01	00	11	1	0
10	00	10	1	0
11	00	10	1	0

DONE!



6.2 Flip-flop Input Functions (1/3)

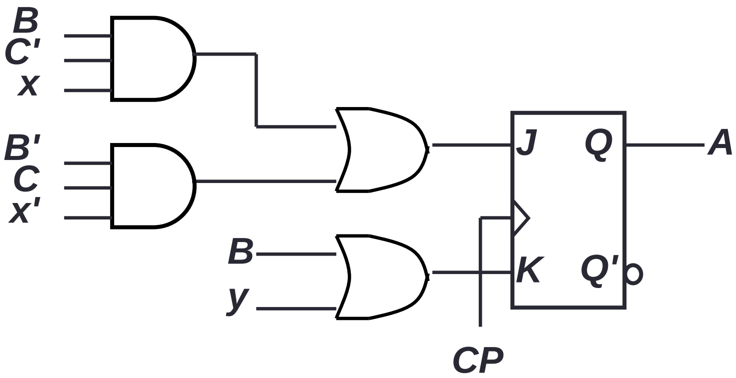
- The outputs of a sequential circuit are functions of the present states of the flip-flops and the inputs. These are described algebraically by the *circuit output functions*.
  - In Figure 1: $y = (A + B) \cdot x'$
- The part of the circuit that generates inputs to the flip-flops are described algebraically by the *flip-flop input functions* (or *flip-flop input equations*).
- The flip-flop input functions determine the next state generation.
- From the flip-flop input functions and the characteristic tables of the flip-flops, we obtain the next states of the flip-flops.

6.2 Flip-flop Input Functions (2/3)

- Example: circuit with a *JK* flip-flop.
- We use 2 letters to denote each flip-flop input: the first letter denotes the input of the flip-flop (*J* or *K* for *J-K* flip-flop, *S* or *R* for *S-R* flip-flop, *D* for *D* flip-flop, *T* for *T* flip-flop) and the second letter denotes the name of the flip-flop.

$JA = B \cdot C' \cdot x + B' \cdot C \cdot x'$

$KA = B + y$



6.2 Flip-flop Input Functions (3/3)

- In Figure 1, we obtain the following state equations by observing that $Q^+ = DQ$ for a D flip-flop:

  $\boldsymbol{A^+ = A \cdot x + B \cdot x}$ (since $DA = A \cdot x + B \cdot x$)

  $\boldsymbol{B^+ = A' \cdot x}$ (since $DB = A' \cdot x$)

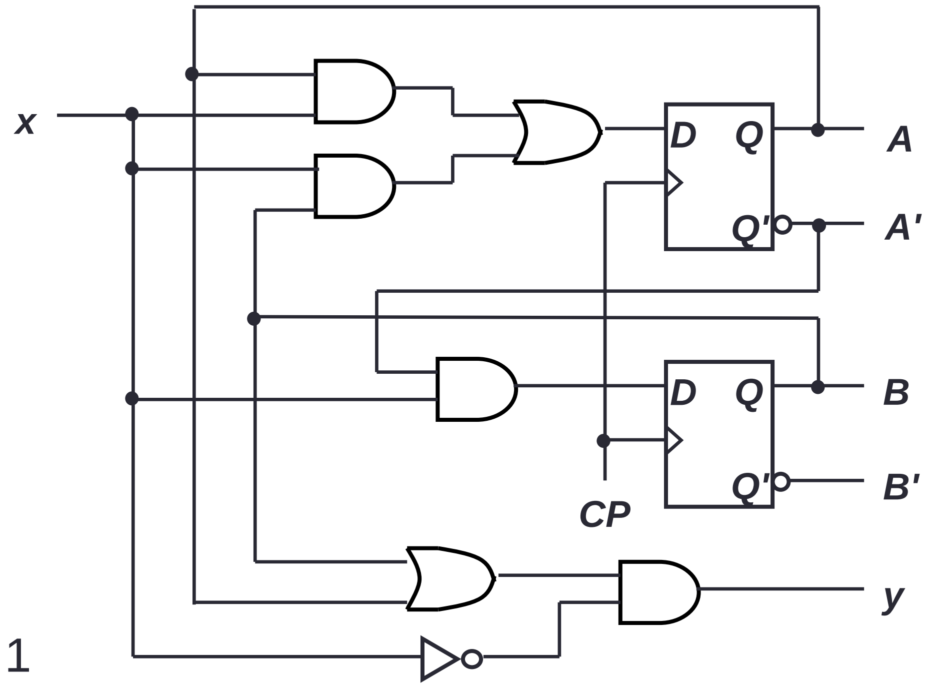


Figure 1

6.2 Analysis: Example #2 (1/3)

- Given Figure 2, a sequential circuit with two *J-K* flip-flops *A* and *B*, and one input *x*.

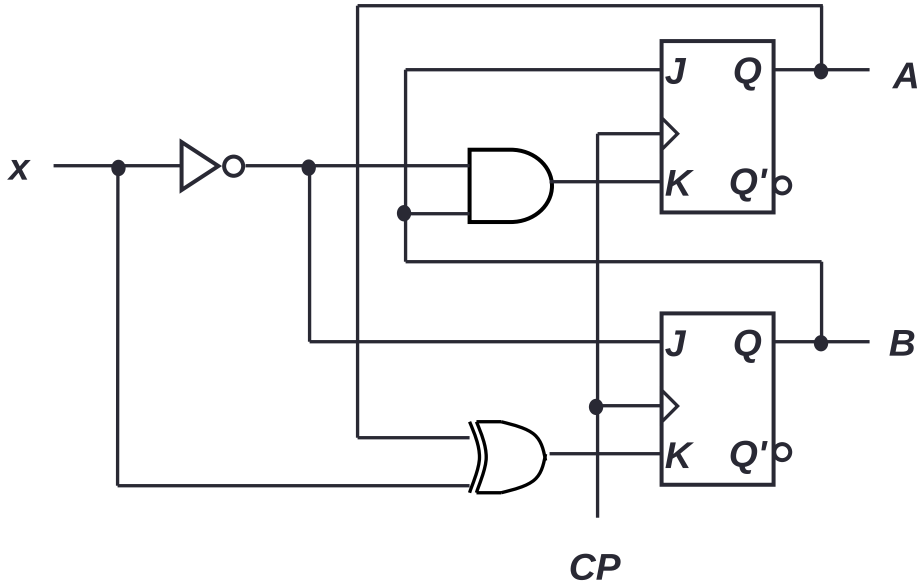


Figure 2

- Obtain the flip-flop input functions from the circuit:

$JA = B$ $JB = x'$

$KA = B \cdot x'$ $KB = A' \cdot x + A \cdot x' = A \oplus x$

6.2 Analysis: Example #2 (2/3)

$JA = B \qquad JB = x'$

$KA = B \cdot x' \qquad KB = A' \cdot x + A \cdot x' = A \oplus x$

- Fill the state table using the above functions, knowing the characteristics of the flip-flops used.

J	*K*	*Q(t+1)*	Comments
0	0	*Q(t)*	No change
0	1	0	Reset
1	0	1	Set
1	1	*Q(t)'*	Toggle

Present state		Input	Next state		Flip-flop inputs			
A	*B*	*x*	A^+	B^+	*JA*	*KA*	*JB*	*KB*
0	0	0			0	0	1	0
0	0	1			0	0	0	1
0	1	0			1	1	1	0
0	1	1			1	0	0	1
1	0	0			0	0	1	1
1	0	1			0	0	0	0
1	1	0			1	1	1	1
1	1	1			1	0	0	0

6.2 Analysis: Example #2 (3/3)

- Draw the state diagram from the state table.

Present state		Input	Next state		Flip-flop inputs			
A	*B*	*x*	A^+	B^+	*JA*	*KA*	*JB*	*KB*
0	0	0			0	0	1	0
0	0	1			0	0	0	1
0	1	0			1	1	1	0
0	1	1			1	0	0	1
1	0	0			0	0	1	1
1	0	1			0	0	0	0
1	1	0			1	1	1	1
1	1	1			1	0	0	0

6.2 Analysis: Example #3 (1/3)

- Derive the state table and state diagram of this circuit.

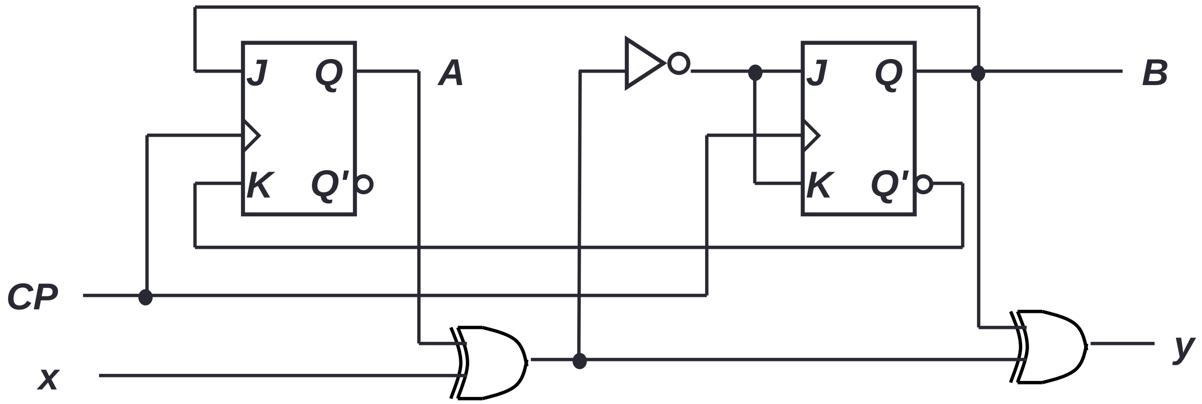


Figure 3

- Flip-flop input functions:

  JA = B ***JB = KB = (A ⊕ x)' = A·x + A'·x'***

  KA = B'

6.2 Analysis: Example #3 (2/3)

- Flip-flop input functions:

  $JA = B$ $JB = KB = (A \oplus x)' = A{\cdot}x + A'{\cdot}x'$

  $KA = B'$

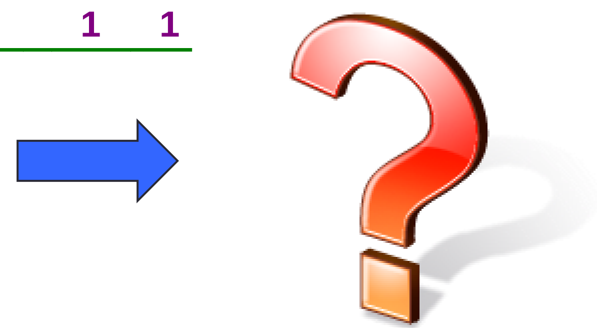
- State table:

Present state		Input	Next state		Output	Flip-flop inputs			
A	B	x	A^+	B^+	y	JA	KA	JB	KB
0	0	0			0	0	1	1	1
0	0	1			1	0	1	0	0
0	1	0			1	1	0	1	1
0	1	1			0	1	0	0	0
1	0	0			1	0	1	0	0
1	0	1			0	0	1	1	1
1	1	0			0	1	0	0	0
1	1	1			1	1	0	1	1

6.2 Analysis: Example #3 (3/3)

- State diagram:

Present state		Input	Next state		Output	Flip-flop inputs			
A	*B*	*x*	A^+	B^+	*y*	*JA*	*KA*	*JB*	*KB*
0	0	0			0	0	1	1	1
0	0	1			1	0	1	0	0
0	1	0			1	1	0	1	1
0	1	1			0	1	0	0	0
1	0	0			1	0	1	0	0
1	0	1			0	0	1	1	1
1	1	0			0	1	0	0	0
1	1	1			1	1	0	1	1



6.3 Flip-flop Excitation Tables (1/2)

- *Analysis*: Starting from a circuit diagram, derive the state table or state diagram.
- *Design*: Starting from a set of specifications (in the form of state equations, state table, or state diagram), derive the logic circuit.
- *Characteristic tables* are used in analysis.
- *Excitation tables* are used in design.

6.3 Flip-flop Excitation Tables (1/2)

- *Excitation tables*: given the required transition from present state to next state, determine the flip-flop input(s).

Q	Q^+	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

JK Flip-flop

Q	Q^+	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

SR Flip-flop

Q	Q^+	D
0	0	0
0	1	1
1	0	0
1	1	1

D Flip-flop

Q	Q^+	T
0	0	0
0	1	1
1	0	1
1	1	0

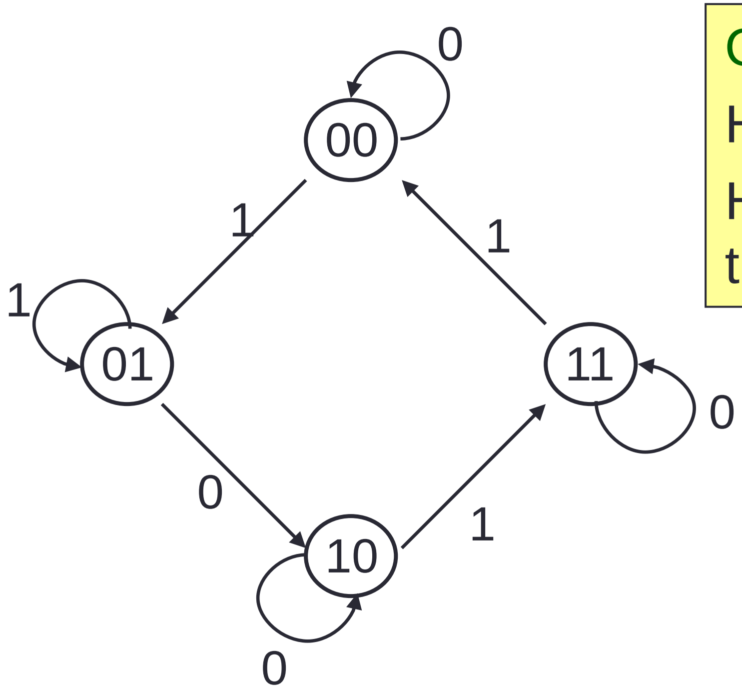
T Flip-flop

6.4 Sequential Circuits: Design

- Design procedure:
  - Start with circuit specifications – description of circuit behaviour, usually a state diagram or state table.
  - Derive the state table.
  - Perform state reduction if necessary.
  - Perform state assignment.
  - Determine number of flip-flops and label them.
  - Choose the type of flip-flop to be used.
  - Derive circuit excitation and output tables from the state table.
  - Derive circuit output functions and flip-flop input functions.
  - Draw the logic diagram.

6.4 Design: Example #1 (1/5)

- Given the following state diagram, design the sequential circuit using *JK* flip-flops.



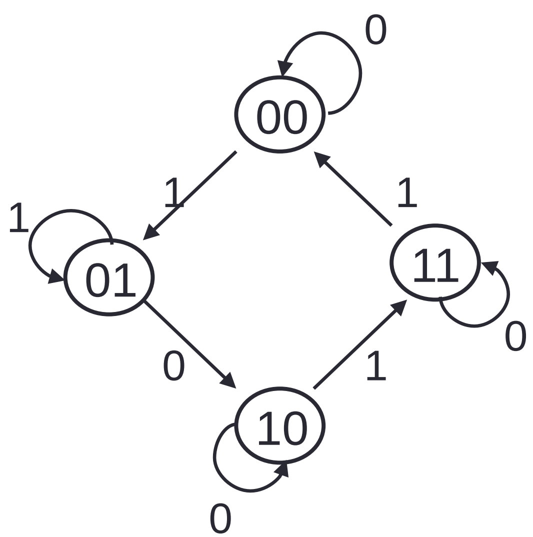
Questions:

How many flip-flops are needed?

How many input variable are there?

6.4 Design: Example #1 (2/5)

- Circuit state/excitation table, using *JK* flip-flops.



Present State	Next State	
	x=0	x=1
AB	A^+B^+	A^+B^+
00	00	01
01	10	01
10	10	11
11	11	00

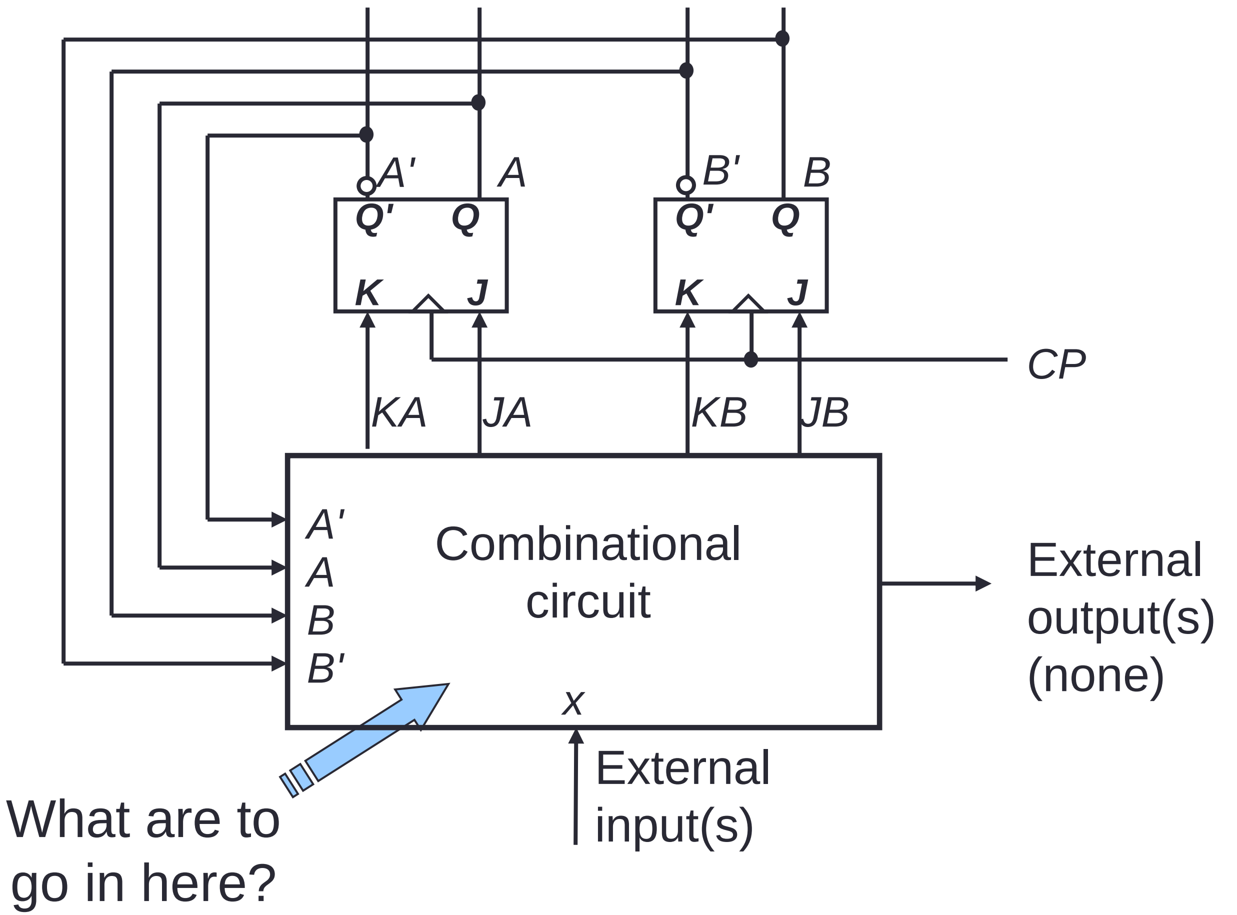
Q	Q^+	*J*	*K*
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

JK Flip-flop's excitation table.

Present state		Input	Next state		Flip-flop inputs			
A	*B*	*x*	A^+	B^+	*JA*	*KA*	*JB*	*KB*
0	0	0	0	0				
0	0	1	0	1				
0	1	0	1	0				
0	1	1	0	1				
1	0	0	1	0				
1	0	1	1	1				
1	1	0	1	1				
1	1	1	0	0				

6.4 Design: Example #1 (3/5)

- Block diagram.



6.4 Design: Example #1 (4/5)

- From state table, get flip-flop input functions.

Present state		Input	Next state		Flip-flop inputs			
A	*B*	*x*	A^+	B^+	*JA*	*KA*	*JB*	*KB*
0	0	0	0	0	0	X	0	X
0	0	1	0	1	0	X	1	X
0	1	0	1	0	1	X	X	1
0	1	1	0	1	0	X	X	0
1	0	0	1	0	X	0	0	X
1	0	1	1	1	X	0	1	X
1	1	0	1	1	X	0	X	0
1	1	1	0	0	X	1	X	1

A \ Bx	00	01	11	10
0				1
1	X	X	X	X

JA = B·x'

A \ Bx	00	01	11	10
0	X	X	X	X
1			1	

KA = B·x

A \ Bx	00	01	11	10
0		1	X	X
1		1	X	X

JB = x

A \ Bx	00	01	11	10
0	X	X		1
1	X	X	1	

KB = (A ⊕ x)'

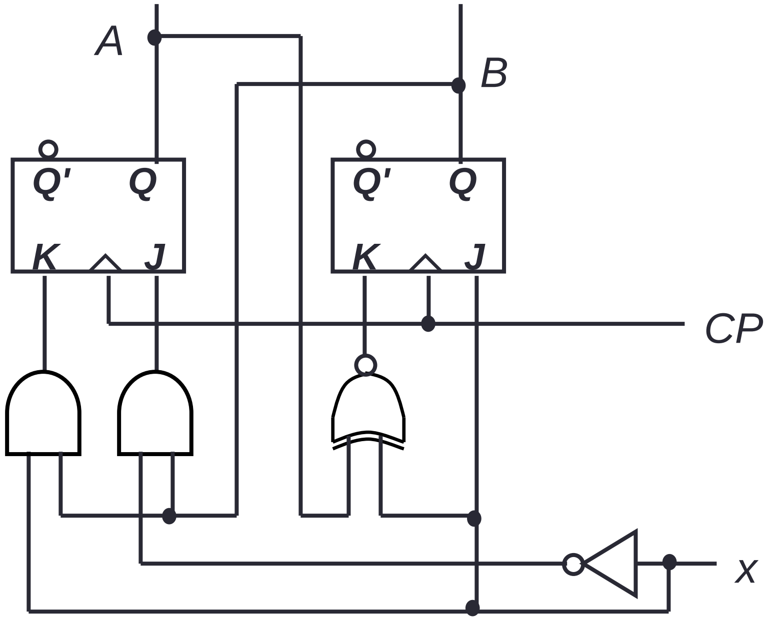
6.4 Design: Example #1 (5/5)

- Flip-flop input functions:

  JA = B·x' ***JB = x***

  KA = B·x ***KB = (A ⊕ x)'***

- Logic diagram:



6.4 Design: Example #2 (1/3)

- Using *D* flip-flops, design the circuit based on the state table below. (Exercise: Design it using *JK* flip-flops.)

Present state		Input	Next state		Output
A	*B*	*x*	A^+	B^+	*y*
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	1	0	0
0	1	1	0	1	0
1	0	0	1	0	0
1	0	1	1	1	1
1	1	0	1	1	0
1	1	1	0	0	0

6.4 Design: Example #2 (2/3)

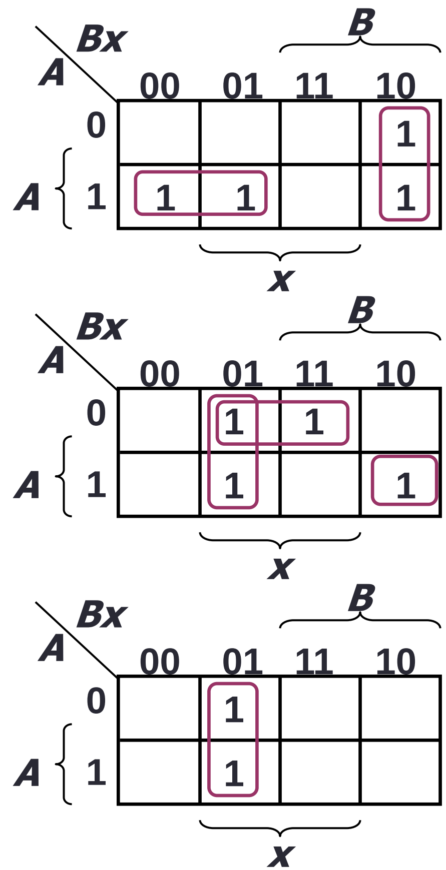
- Determine expressions for flip-flop inputs and the circuit output *y*.

Present state		Input	Next state		Output
A	B	x	A^+	B^+	y
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	1	0	0
0	1	1	0	1	0
1	0	0	1	0	0
1	0	1	1	1	1
1	1	0	1	1	0
1	1	1	0	0	0

$DA(A,B,x) = \Sigma\ m(2,4,5,6)$

$DB(A,B,x) = \Sigma\ m(1,3,5,6)$

$y(A,B,x) = \Sigma\ m(1,5)$



$DA = A{\cdot}B' + B{\cdot}x'$

$DB = A'{\cdot}x + B'{\cdot}x + A{\cdot}B{\cdot}x'$

$y = B'{\cdot}x$

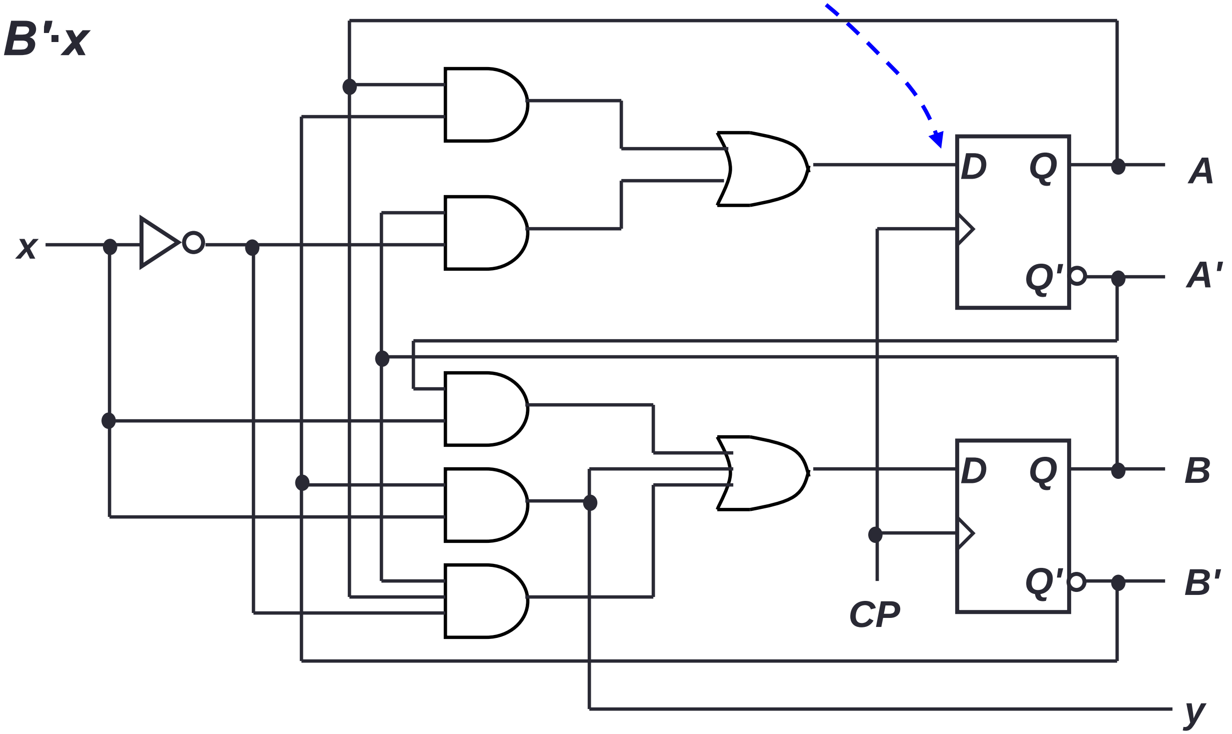
6.4 Design: Example #2 (3/3)

- From derived expressions, draw logic diagram:

$DA = A \cdot B' + B \cdot x'$

$DB = A' \cdot x + B' \cdot x + A.B \cdot x'$

$y = B' \cdot x$



6.4 Design: Example #3 (1/4)

- Design involving unused states.

Present state			Input	Next state			Flip-flop inputs						Output
A	*B*	*C*	*x*	A^+	B^+	C^+	*SA*	*RA*	*SB*	*RB*	*SC*	*RC*	*y*
0	0	1	0	0	0	1	0	X	0	X	X	0	0
0	0	1	1	0	1	0	0	X	1	0	0	1	0
0	1	0	0	0	1	1	0	X	X	0	1	0	0
0	1	0	1	1	0	0	1	0	0	1	0	X	0
0	1	1	0	0	0	1	0	X	0	1	X	0	0
0	1	1	1	1	0	0	1	0	0	1	0	1	0
1	0	0	0	1	0	1	X	0	0	X	1	0	0
1	0	0	1	1	0	0	X	0	0	X	0	X	1
1	0	1	0	0	0	1	0	1	0	X	X	0	0
1	0	1	1	1	0	0	X	0	0	X	0	1	1

Given these

Derive these

Are there other unused states?

Unused state 000:

0	0	0	0	X	X	X	X	X	X	X	X	X	X
0	0	0	1	X	X	X	X	X	X	X	X	X	X

6.4 Design: Example #3 (2/4)

- From state table, obtain expressions for flip-flop inputs.

AB \ Cx	00	01	11	10
00	X	X		
01		1	1	
11	X	X	X	X
10	X	X	X	

SA = ?

AB \ Cx	00	01	11	10
00	X	X	X	X
01	X			X
11	X	X	X	X
10				1

RA = ?

AB \ Cx	00	01	11	10
00	X	X	1	
01	X			
11	X	X	X	X
10				

SB = ?

AB \ Cx	00	01	11	10
00	X	X		X
01		1	1	1
11	X	X	X	X
10	X	X	X	X

RB = ?

6.4 Design: Example #3 (3/4)

- From state table, obtain expressions for flip-flop inputs (cont'd).

AB \ Cx	00	01	11	10
00	X	X		X
01	1			X
11	X	X	X	X
10	1			X

SC = ?

AB \ Cx	00	01	11	10
00	X	X	1	
01		X	1	
11	X	X	X	X
10		X	1	

RC = ?

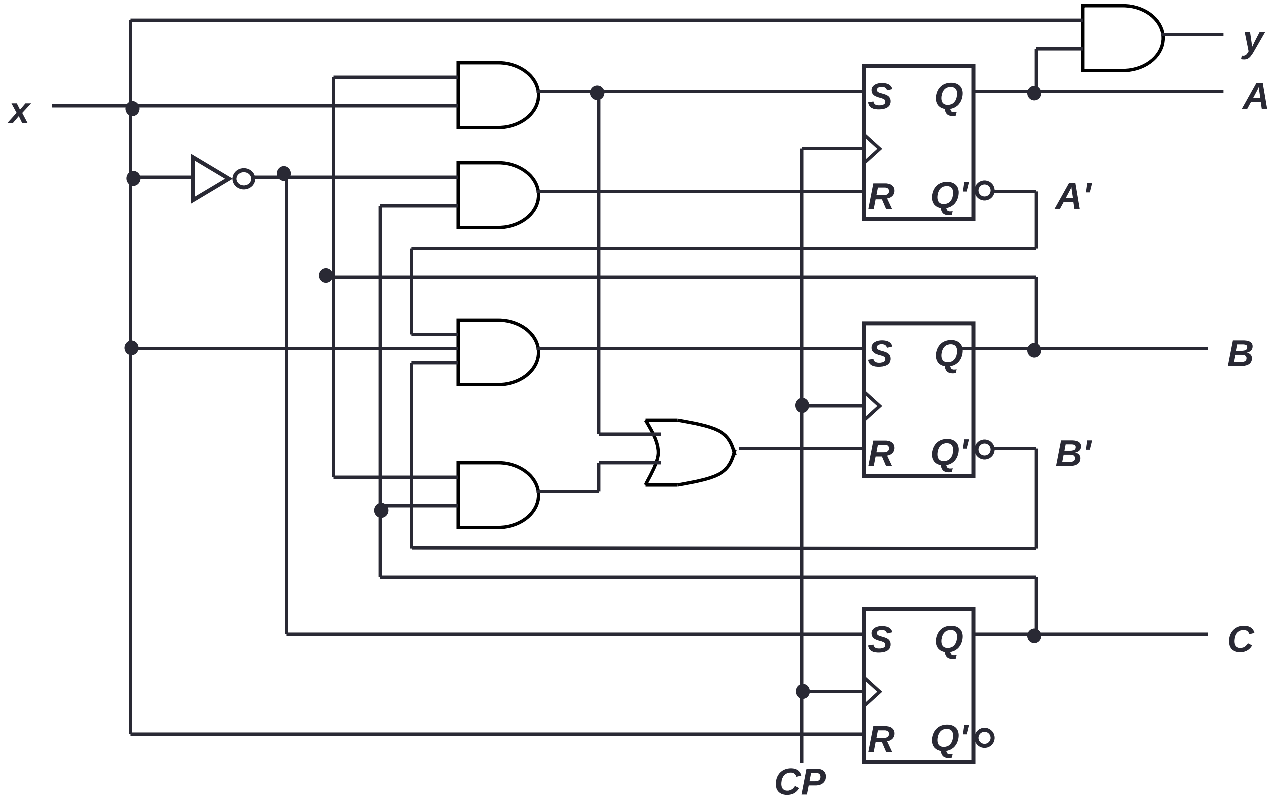
AB \ Cx	00	01	11	10
00	X	X		
01				
11	X	X	X	X
10		1	1	

y = **?**

6.4 Design: Example #3 (4/4)

- From derived expressions, draw the logic diagram:

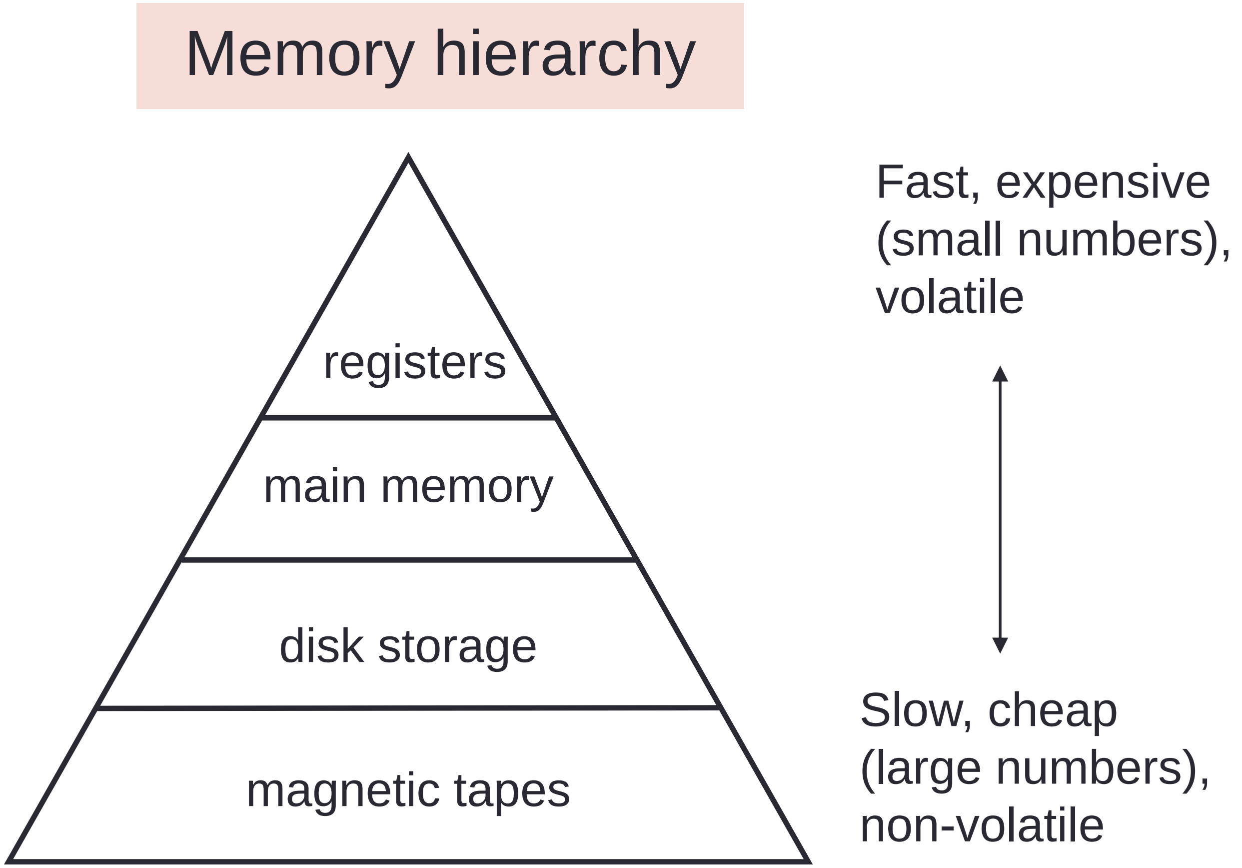
$SA = B \cdot x$	$SB = A' \cdot B' \cdot x$	$SC = x'$	$y = A \cdot x$
$RA = C \cdot x'$	$RB = B \cdot C + B \cdot x$	$RC = x$	



7. Memory (1/4)

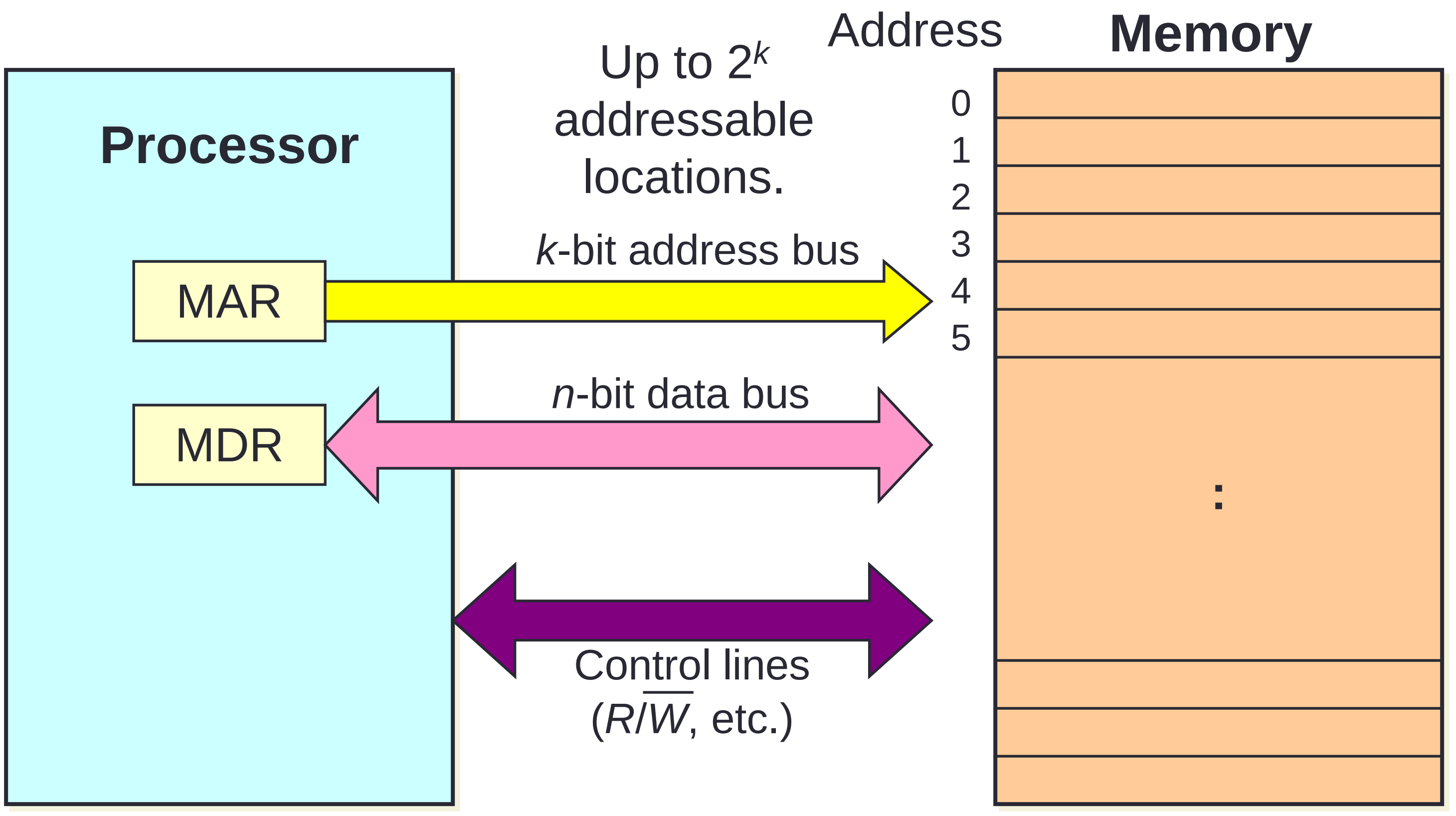
- Memory stores programs and data.
- Definitions:
  - 1 byte = 8 bits
  - 1 word: in multiple of bytes, a unit of transfer between main memory and registers, usually size of register.
  - 1 KB (kilo-bytes) = 2^{10} bytes; 1 MB (mega-bytes) = 2^{20} bytes; 1 GB (giga-bytes) = 2^{30} bytes; 1 TB (tera-bytes) = 2^{40} bytes.
- Desirable properties: fast access, large capacity, economical cost, non-volatile.
- However, most memory devices do not possess all these properties.

7. Memory (2/4)


Memory hierarchy
registers
main memory
disk storage
magnetic tapes
Fast, expensive
(small numbers),
volatile
Slow, cheap
(large numbers),
non-volatile

7. Memory (3/4)

Data transfer

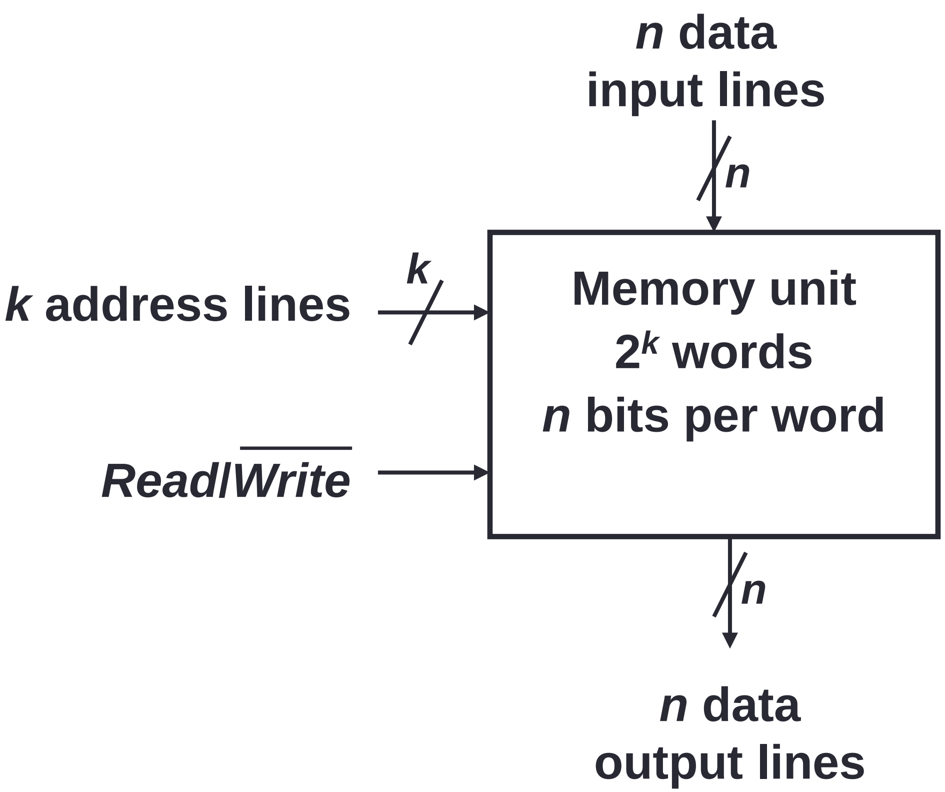


7. Memory (4/4)

- A memory unit stores binary information in groups of bits called *words*.
- The data consists of *n* lines (for *n*-bit words). Data input lines provide the information to be stored (*written*) into the memory, while data output lines carry the information out (*read*) from the memory.
- The address consists of *k* lines which specify which word (among the 2^k words available) to be selected for reading or writing.
- The control lines *Read* and *Write* (usually combined into a single control line *Read/Write*) specifies the direction of transfer of the data.

7.1 Memory Unit

- Block diagram of a memory unit:



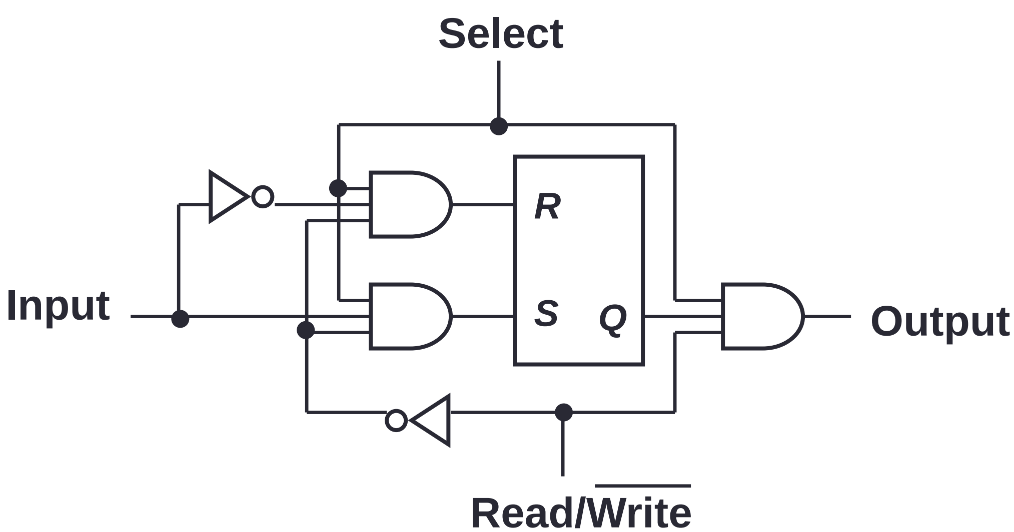
7.2 Read/Write Operations

- Write operation:
  - Transfers the address of the desired word to the address lines.
  - Transfers the data bits (the word) to be stored in memory to the data input lines.
  - Activates the *Write* control line (set *Read/Write* to 0).
- Read operation:
  - Transfers the address of the desired word to the address lines.
  - Activates the *Read* control line (set *Read/Write* to 1).

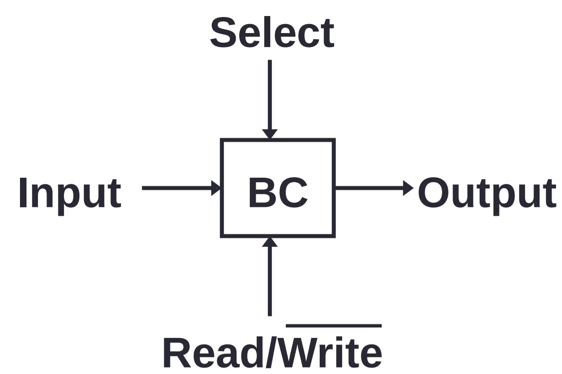
Memory Enable	*Read/$\overline{\text{Write}}$*	Memory Operation
0	X	None
1	0	Write to selected word
1	1	Read from selected word

7.3 Memory Cell

- Two types of RAM
  - Static RAMs use flip-flops as the memory cells.
  - Dynamic RAMs use capacitor charges to represent data. Though simpler in circuitry, they have to be constantly refreshed.
- A single memory cell of the static RAM has the following logic and block diagrams:



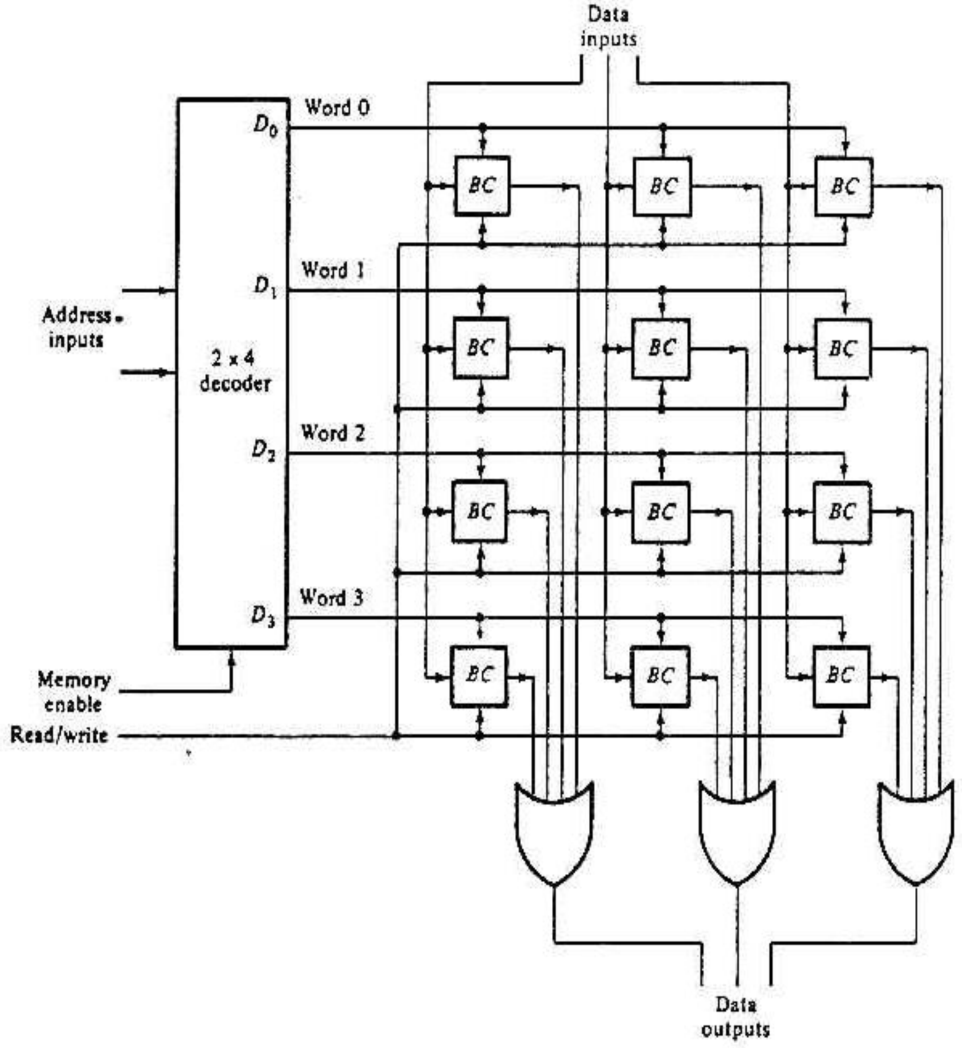
Logic diagram



Block diagram

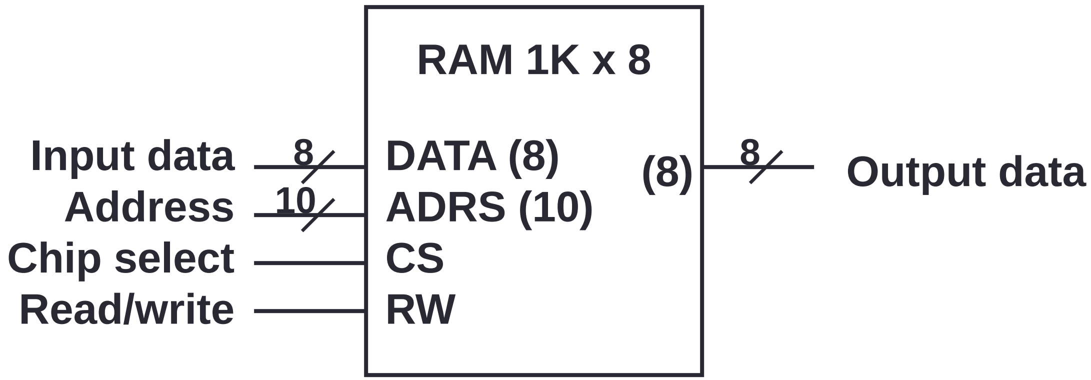
7.4 Memory Arrays (1/4)

- Logic construction of a 4×3 RAM (with decoder and OR gates):



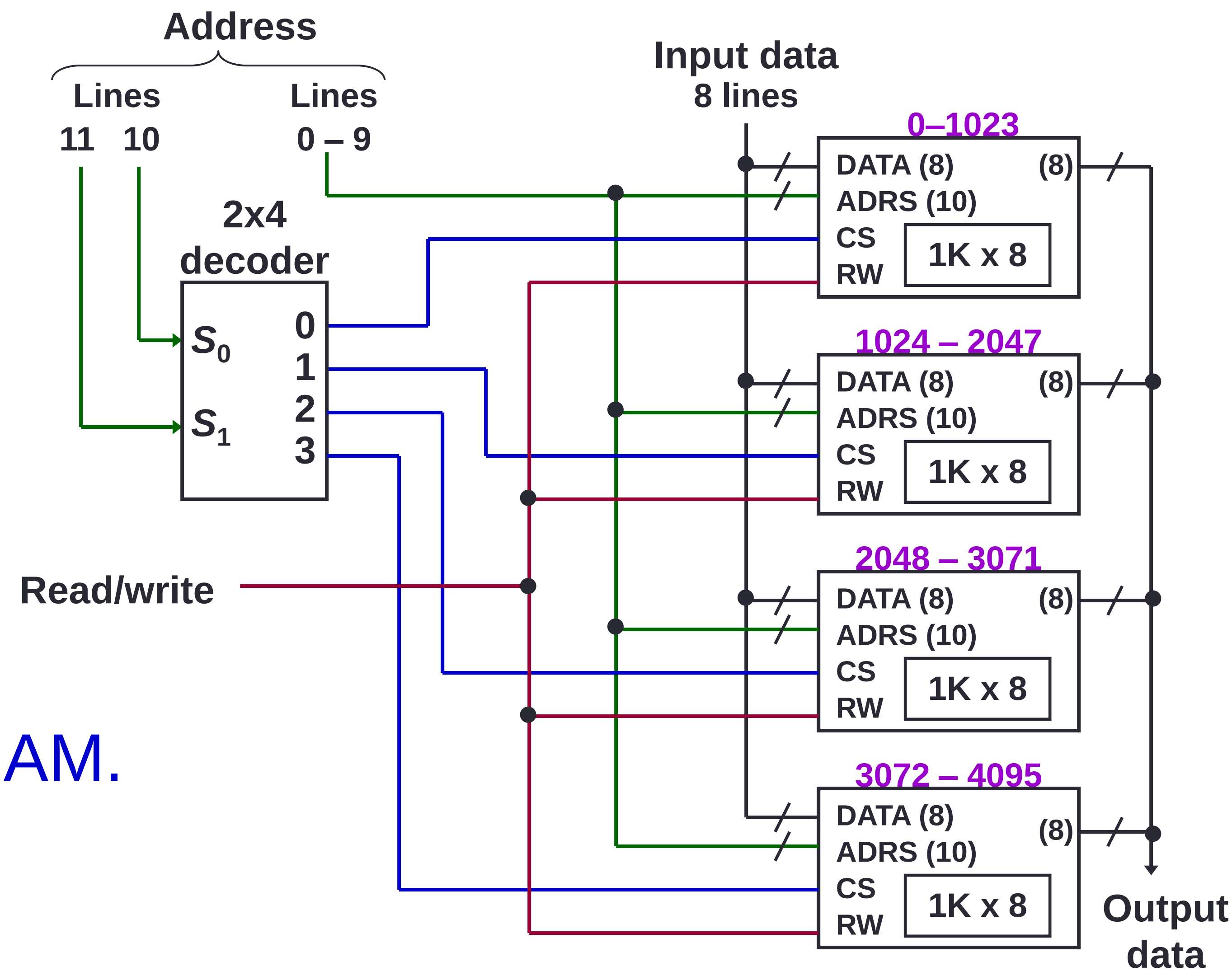
7.4 Memory Arrays (2/4)

- An array of RAM chips: memory chips are combined to form larger memory.
- A 1K × 8-bit RAM chip:



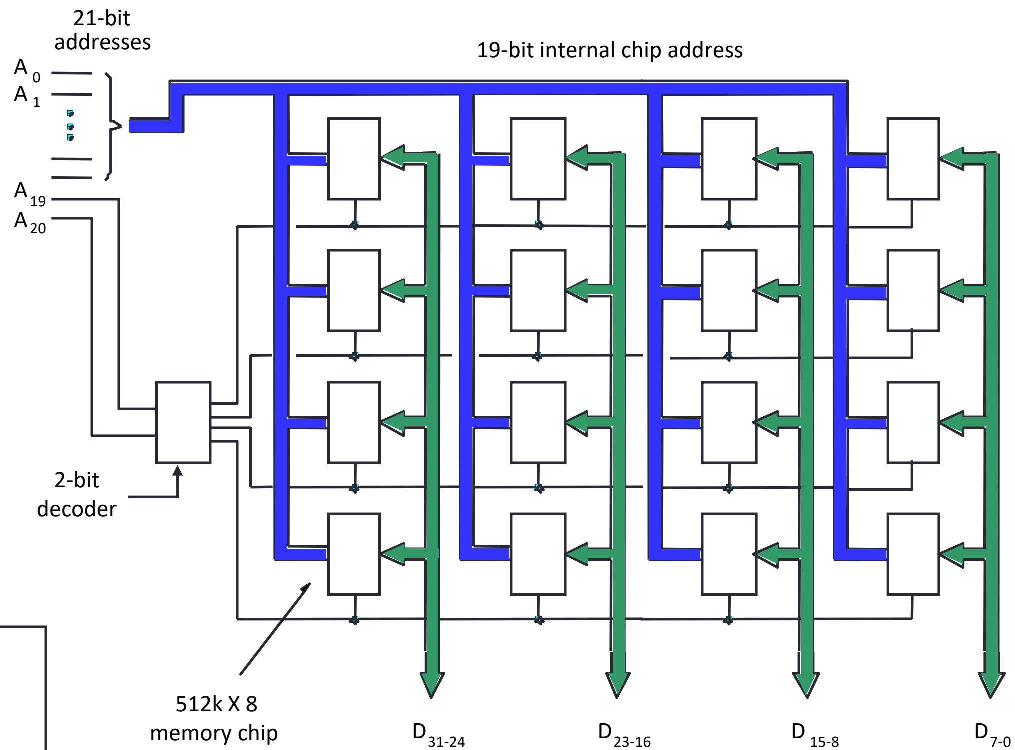
Block diagram of a 1K x 8 RAM chip

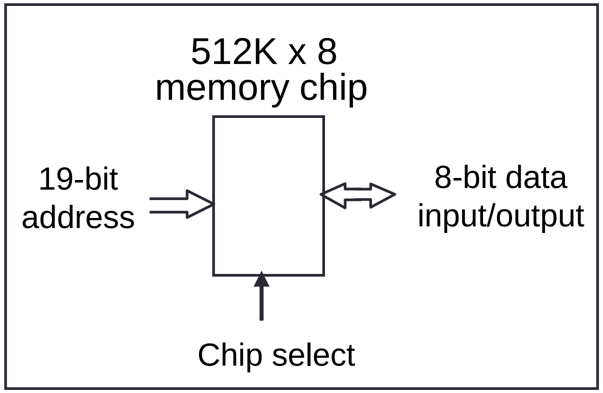
7.4 Memory Arrays (3/4)



- 4K × 8 RAM.

7.4 Memory Arrays (4/4)





- $2M \times 32$ memory module
  - Using $512K \times 8$ memory chips.

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