

Question 1 (a).

Since $ABC \cap CD = C$ and C is a superkey of R_2 (due to the FD $C \rightarrow D$), the decomposition is lossless.

Question 1 (b).

Since $AC \cap ABD = A$ and A is a superkey of R_1 (due to the FD $A \rightarrow BCD$), the decomposition is lossless.

Question 1 (c).

A decomposition is lossless join if there exists a sequence of binary lossless-join decompositions that generates that decomposition. It turns out that there is one such decomposition. First, decompose R into $R_3(A,C,D)$ and $R_4(A,B,C,E)$. This is lossless because $R_3 \cap R_4 = AC$, and AC is a superkey of R_3 . Next, decompose R_4 into $R_1(A,B,C)$ and $R_2(A,B,E)$. This is also lossless because $R_1 \cap R_2 = AB$, and AB is a superkey of R_1 . Therefore, $\{R_1(A,B,C), R_2(A,B,E), R_3(A,C,D)\}$ is a lossless-join decomposition.

Question 2 (a).

R is not in BCNF because $A \rightarrow E$ violates BCNF: A is not a superkey of R as $\{A\}^+ = ABDE$.

Question 2 (b).

The decomposition is lossless-join because $R_1 \cap R_2 = \{E\}$ and $E \rightarrow BDE$.

Question 2 (c).

Note that $\{A\}^+ = ABDE$. Therefore, on R_2 , we have $\{A\}^+ = AE$. This indicates that R_2 is not in BCNF.

Question 2 (d).

1. $\{E\}^+ = \{EDB\}$, which indicates that R is not in BCNF. Accordingly, we decompose R into $R_1(E, B, D)$ and $R_2(E, A, C)$.
2. The only non-trivial FD on R_1 is $E \rightarrow DB$, and E is a key of R_1 . Thus, R_1 is in BCNF.
3. $\{A\}^+ = \{AEDB\}$, and hence, we have $\{A\}^+ = \{AE\}$ on R_2 . This indicates that R_2 is not in BCNF. Accordingly, we decompose R_2 into $R_3(A, E)$ and $R_4(A, C)$.
4. R_3 and R_4 are in BCNF since each of them contains only two attributes.
5. Therefore, $\{R_1(E, D, B), R_3(A, E), R_4(A, C)\}$ is a BCNF decomposition of R .

Question 3.

1. $\{B\}^+ = BCDE$, which indicates that R is not in BCNF. Accordingly, we decompose R into $R_1(B, C, D, E)$ and $R_2(B, A)$.
2. Since R_2 has only two attributes, it is in BCNF.
3. For R_1 , let's examine the closure of each attribute subset:
 - $\{B\}^+ = \{BCDE\}$, which indicates that B is a key of R_1 .
 - $\{C\}^+ = \{BCDE\}$, which indicates that C is a key of R_1 .
 - $\{D\}^+ = \{D\}$
 - $\{E\}^+ = \{E\}$
 - $\{DE\}^+ = \{DE\}$
 - All other attribute subsets are superkeys since they are supersets of either B or C.

Therefore, R_1 is in BCNF.

4. As a consequence, $\{R_1(B, C, D, E), R_2(A, B)\}$ is a BCNF decomposition of R.