**National University of Singapore**
**School of Computing**
**CS3243 Introduction to AI**

# Tutorial 7: Logical Agents I

Issued: March 14, 2022 Discussion in: Week 10

**Important Instructions**:

- **Assignment 7** consists of **Question 3** from this tutorial.
- Your solutions for this tutorial must be TYPE-WRITTEN.
- You are to submit your solutions on LumiNUS by **Week 10, Sunday, 2359 hours**.
- Refer to LumiNUS for submission guidelines

Note: you may discuss the content of the questions with your classmates, but you must work out and write up your solution individually. Solutions that are plagiarised will be heavily penaltised.

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1. Verify the following logical equivalences. Cite the equivalence law used with each step of your working (refer to Appendix B for a list of these laws).

   (a) $\neg(p \vee \neg q) \vee (\neg p \wedge \neg q) \equiv \neg p$

   (b) $(p \wedge \neg(\neg p \vee q)) \vee (p \wedge q) \equiv p$

2. Victor would like to invite three friends, Alice, Ben, and Cindy to a party, but must satisfy the following constraints:

   (a) Cindy comes only if Alice does not come.

   (b) Alice comes if either Ben or Cindy (or both) comes.

   (c) Cindy comes if Ben does not come.

   Victor would like to know who will come to the party, and who will not. Help Victor by expressing each of the above three constraints in propositional logic, and then, using these constraints, determine who will attend his party.

3. Given the following logical statements, use truth-table enumeration to show that $KB \models \alpha$. In other words, write down all possible true/false assignments to the variables, the ones for which $KB$ is true and the one for which $\alpha$ is true, and see whether one is a subset of the other.

   (a)

$$\begin{aligned} KB =& (x_1 \vee x_2) \wedge (x_1 \Rightarrow x_3) \wedge \neg x_2 \\ \alpha =& x_3 \vee x_2 \end{aligned}$$

   (b)

$$\begin{aligned} KB =& (x_1 \vee x_3) \wedge (x_1 \Rightarrow \neg x_2) \\ \alpha =& \neg x_2 \end{aligned}$$

## Appendix A: Notes on Knowledge Bases

A knowledge base $KB$ is a set of logical rules that model what the agent knows. These rules are written using a certain language (or *syntax*) and use a certain truth model (or *semantics* which say when a certain statement is true or false). In propositional logic sentences are defined as follows

1. Atomic Boolean variables are sentences.
2. If $S$ is a sentence, then so is $\neg S$.
3. If $S_1$ and $S_2$ are sentences, then so is:
   (a) $S_1 \land S_2$ "$S_1$ and $S_2$"
   (b) $S_1 \lor S_2$ "$S_1$ or $S_2$"
   (c) $S_1 \Rightarrow S_2$ "$S_1$ implies $S_2$"
   (d) $S_1 \Leftrightarrow S_2$ "$S_1$ holds if and only if $S_2$ holds"

We say that a logical statement $a$ models $b$ ($a \models b$) if $b$ holds whenever $a$ holds. In other words, if $M(q)$ is the set of all value assignments to variables in $a$ for which $a$ holds true, then $M(a) \subseteq M(b)$.

An inference algorithm $\mathcal{A}$ is one that takes as input a knowledge base $KB$ and a query $\alpha$ and decides whether $\alpha$ is derived from $KB$, written as $KB \vdash_{\mathcal{A}} \alpha$. $\mathcal{A}$ is sound if $KB \vdash_{\mathcal{A}} \alpha$ implies that $KB \models \alpha$; $\mathcal{A}$ is complete if $KB \models \alpha$ implies that $KB \vdash_{\mathcal{A}} \alpha$.

## Appendix B: Propositional Logic Laws

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|---|---|---|
| De Morgan's Laws | $\neg(p \lor q) \equiv \neg p \land \neg q$ | $\neg(p \land q) \equiv \neg p \lor \neg q$ |
| Idempotent laws | $p \lor p \equiv p$ | $p \land p \equiv p$ |
| Associative laws | $(p \lor q) \lor r \equiv p \lor (q \lor r)$ | $(p \land q) \land r \equiv p \land (q \land r)$ |
| Commutative laws | $p \lor q \equiv q \lor p$ | $p \land q \equiv q \land p$ |
| Distributive laws | $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$ | $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$ |
| Identity laws | $p \lor False \equiv p$ | $p \land True \equiv p$ |
| Domination laws | $p \land False \equiv False$ | $p \lor True \equiv True$ |
| Double negation law | $\neg\neg p \equiv p$ | |
| Complement laws | $p \land \neg p \equiv False \land \neg True \equiv False$ | $p \lor \neg p \equiv True \lor \neg False \equiv True$ |
| Absorption laws | $p \lor (p \land q) \equiv p$ | $p \land (p \lor q) \equiv p$ |
| Conditional identities | $p \Rightarrow q \equiv \neg p \lor q$ | $p \Leftrightarrow q \equiv (p \Rightarrow q) \land (q \Rightarrow p)$ |