National University of Singapore
School of Computing
CS3243 Introduction to AI

Final Examination Revision (Solutions)

1. Ancient Lore in the World of Adventure tells us that:

   - Every dragon sleeps in some lair.
   - Every wyvern is a dragon, and every wyvern is poisonous.
   - Every lair in which a poisonous dragon sleeps is toxic.
   - Anything that sleeps in anything that is toxic has slime minions.

   The above are to be taken as facts in the World of Adventure.

   A wizard now claims that every wyvern has slime minions.

   Using resolution, prove the wizard's claim. Note that you should NOT use first-order logic (FOL).

   Solution: We denote the following:

   - W: Wyvern
   - D: Dragon
   - P: Poisonous
   - T: Toxic lair
   - S: Slime minions

   Thus, we have knowledge base (KB):

   R_1: $W \Rightarrow D \equiv \neg W \vee D$

   R_2: $W \Rightarrow P \equiv \neg W \vee P$

   R_3: $P \Rightarrow T \equiv \neg P \vee T$

   R_4: $T \Rightarrow S \equiv \neg T \vee S$

   Taking the negation of the query, we have:

   $$\neg(W \Rightarrow S) \equiv \neg(\neg W \vee S) \equiv W \wedge \neg S$$

   Or rather:

R_5: W

R_6: $\neg S$

We then resolve as follows:

R_7: P (from R2 and R5)

R_8: T (from R3 and R7)

R_9: S (from R4 and R8)

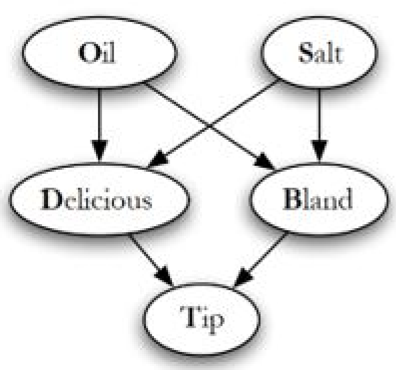
R_{10}: $\{\}$ (from R6 and R9)

Since the negation of the query is unsatisfiable, we can infer that $W \Rightarrow S$.

2. An ambitious restaurant waiter wants to maximise the number of tips he earns. He notices that whether patrons tip or not depends on the quality of the chef's cooking. When the chef uses more Oil and more $Salt$, more people comment that the food is $Delicious$ and fewer people say that the food is $Bland$. When people comment that the food is $Delicious$, the waiter is much more likely to receive a Tip.

   (a) If the waiter models this by writing down the joint probability distribution table, what is the minimum number of table entries required to fully specify the scenario? Write the answer as a decimal number.

   Suppose that the employee does not want to write down a table that large. Instead, he uses his domain knowledge to represent the scenario as the following Bayesian Network.



(b) Recall that a Bayesian Network represents the full joint distribution in a compact form by exploiting knowledge of which variables are conditionally independent. Write down the formula that this Bayesian Network expresses for the joint probability distribution, $Pr[O, S, B, D, T]$.

(c) Which of the following statements is true for the given Bayesian Network?

i. $Delicious$ and $Bland$ are conditionally independent causes for the effect, Tip.

ii. Oil and $Salt$ are independent events.

iii. Given the values of Oil and $Salt$, $Delicious$ is conditionally independent of every other value.

iv. Given the values of $Delicious$ and $Bland$, Tip is conditionally independent of every other value.

(d) Suppose that we are given that $Pr[O] = 0.4$, $Pr[S] = 0.2$, and the following.

O	S	$Pr[D\|O, S]$
F	F	0.2
F	T	0.4
T	F	0.3
T	T	0.6

O	S	$Pr[B\|O, S]$
F	F	0.7
F	T	0.4
T	F	0.4
T	T	0.1

D	B	$Pr[T\|D, B]$
F	F	0.2
F	T	0.1
T	F	0.6
T	T	0.3

(Note: $Delicious$ and $Bland$ are not opposites of each other. Some people at the table might comment that the food is $Delicious$, whereas others might comment that it is $Bland$. The Tip probability is expressed in terms of whether at least one D/B comment was heard.)

Compute the following probabilities (give the answers as decimal values, e.g., 0.3; express your answer to 3 significant figures):

i. Probability that a Tip was offered, given that someone said the food is $Bland$.

ii. Probability that someone said the food was $Delicious$.

iii. Probability that the chef added Oil, given that someone said the food was $Delicious$.

Solution:

(a) $2^5 - 1 = 31$

(b) $Pr[O, S, D, B, T] = Pr[T|D, B].Pr[D|O, S].Pr[B|O, S].Pr[O].Pr[S]$

(c) i. True

ii. True

iii. False, D is dependent on T, i.e., our knowledge of whether the food was $Delicious$ is affected by whether a Tip was given.

iv. True

(d) i. First, we determine that $P[D|B] = 0.249$ (see below), then, we have:

$$\begin{aligned}
Pr[T|B] &= Pr[T,B]/Pr[B] && \text{(Product Rule)}\\
&= (Pr[T,D,B]+Pr[T,\neg D,B])/Pr[B] && \text{(Marginalisation)}\\
&= (Pr[T|D,B] \times Pr[D|B] \times Pr[B]+\\
&\quad Pr[T|\neg D,B]) \times Pr[\neg D|B] \times Pr[B])/Pr[B] && \text{(Chain Rule)}\\
&= Pr[T|D,B] \times Pr[D|B]+\\
&\quad Pr[T|\neg D,B] \times Pr[\neg D|B]\\
&= 0.3 \times 0.249 + 0.1 \times 0.751\\
&= 0.1498
\end{aligned}$$

Note $P[D|B] = (\sum_{o,s} Pr[B,D|o,s] \times Pr[o] \times Pr[s])/Pr[B]$
$= (\sum_{o,s} Pr[B|o,s] \times Pr[D|o,s] \times Pr[o] \times Pr[s])/Pr[B]$

ii. Following the first table:

$$\begin{aligned}
Pr[D] &= 0.6 \times 0.8 \times 0.2 + 0.6 \times 0.2 \times 0.4 + 0.4 \times 0.8 \times 0.3 + 0.4 \times 0.2 \times 0.6\\
&= 0.4 \times (0.12 + 0.24 + 0.12) + 0.12 \times 0.8\\
&= 0.4 \times 0.48 + 0.12 \times 0.8\\
&= 0.288
\end{aligned}$$

iii.
$$\begin{aligned}
Pr[O|D] &= Pr[O,D]/Pr[D]\\
&= (0.4 * (0.8 * 0.3 + 0.2 * 0.6))/0.288\\
&= (0.4 * 0.36)/0.288\\
&= 0.5
\end{aligned}$$