National University of Singapore
School of Computing
CS3243 Introduction to AI

Tutorial 9: Uncertainty and Bayesian Networks

Issued: March 28, 2022 Discussion in: Week 12

Important Instructions:

- **Assignment 9** consists of **Question 4** from this tutorial.
- Your solutions for this tutorial must be TYPE-WRITTEN.
- You are to submit your solutions on LumiNUS by **Week 12, Sunday, 2359 hours**.
- Refer to LumiNUS for submission guidelines

Note: you may discuss the content of the questions with your classmates, but you must work out and write up your solution individually. Solutions that are plagiarised will be heavily penaltised.

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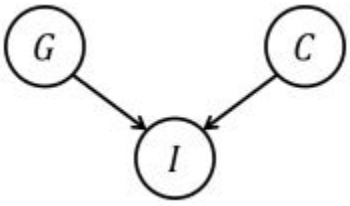
1. Suppose that having both $good\ grades$ (G) and $good\ communication\ skills$ (C) will increase your chances of $performing\ well\ in\ your\ interview$ (I). We thus have the following belief network.



Suppose further that $\Pr[G = 1] = 0.7$, $\Pr[C = 1] = 0.2$, and the probability table of I given G and C is as follows.

G	C	$\Pr[I = 1 \mid G, C]$
1	1	0.9
1	0	0.5
0	1	0.5
0	0	0.1

(a) What is the probability that Alice, who has poor grades and communication skills, performs well on her interview?

(b) What is the probability that Bob is a student with great communication skills, assuming we do not know anything about him?

(c) What is the probability that a student has good communication skills, given that he or she has performed well in an interview? Are good communication skills independent of good performance in an interview?

2. Assume that 2% of the population in a country carry a particular virus. A test kit developed by a pharmaceutical firm is able to detect the presence of the virus from a patient's blood sample. The firm claims that the test kit has a high accuracy of detection in terms of the following conditional probabilities obtained from their quality control testing:

$$\Pr[\text{the kit shows positive}|\text{the patient is a carrier}] = 0.998$$
$$\Pr[\text{the kit shows negative}|\text{the patient is not a carrier}] = 0.996$$

(a) Given that a patient is tested to be positive using this kit, what is the posterior belief that he is not a carrier? Give your answer to 3 decimal places.

(b) Suppose that the patient doesn't entirely trust the result offered by the first kit (perhaps because it has expired) and decides to use another test kit. If the patient is again tested to be positive using this second kit, what is the (updated) likelihood that he is not a carrier? You can assume conditional independence between results of different test kits given the patient's state of virus contraction. Give your answer to 4 decimal places.

3. Assume that the following conditional probabilities are available:

P(Wet_Grass | Sprinkler $\wedge$ Rain)	0.95
P(Wet_Grass | Sprinkler $\wedge \neg$ Rain)	0.9
P(Wet_Grass | $\neg$ Sprinkler $\wedge$ Rain)	0.8
P(Wet_Grass | $\neg$ Sprinkler $\wedge \neg$ Rain)	0.1
P(Sprinkler | Rainy_Season)	0.0
P(Sprinkler | $\neg$ Rainy_Season)	1.0
P(Rain | Rainy_Season)	0.9
P(Rain | $\neg$ Rainy_Season)	0.1
P(Rainy_Season)	0.7

Construct a Bayesian network and determine the probability

$$P(\text{Wet\_Grass} \wedge \text{Rainy\_Season} \wedge \neg \text{Rain} \wedge \neg \text{Sprinkler}).$$

4. An expert system called PROSPECTOR for use in geological exploration makes use of an inference mechanism similar to a Bayesian Network. The following are two modified versions of its rule patterns:

   If E1
   Then H1 (P(H1 | E1), P(H1 | ¬E1))

   If E2
   and E3
   Then H2 (P(H2 | E2 ∧ E3), P(H2 | E2 ∧ ¬E3), P(H2 | ¬E2 ∧ E3), P(H2 | ¬E2 ∧ ¬E3))

   The following is a hypothetical set of PROSPECTOR's rules (where we also use two letters to represent propositions for your easy working later)

   > If the igneous rocks in the region have a fine to medium grain size (Gr)
   > Then they have a porphyritic texture (Tx) (0.6, 0.2)
   >
   > If the igneous rocks in the region have a fine to medium grain size (Gr)
   > and they have a porphyritic texture (Tx)
   > Then the region is a hypabyssal environment (Hy) (0.88, 0.76, 0.52, 0.02)
   >
   > If the region is a hypabyssal environment (Hy)
   > Then the region has a favourable level of erosion (Er) (0.75, 0.12)
   >
   > If the region has a favourable level of erosion (Er)
   > Then the region is favourable for copper deposits (Cu) (0.92, 0.03)

   Assume that a geologist could only ascertain with probability 0.15 that a region's igneous rocks have a fine to medium grain size.

   (a) Construct a Bayesian network based on the above rules.

   (b) Determine the probability that this region is favourable for copper deposits and has a favourable level of erosion, given that the region (1) has large grain size igneous rocks, (2) has non-porphyritic texture rocks, and (3) is a hypabyssal environment.