Adversarial Search: Playing Games

CS3243: Introduction to Artificial Intelligence – Lecture 7

7 March 2022

Contents

Reference: AIMA 4th Edition, Section 6.1-6.3

Administrative Matters

Graded Assessments

- Marks on Gradebook
  - 7 diagnostic quizzes (up to 7%)
    - DQ0 – DQ6
  - 4 tutorial assignments (4%)
    - TA1 – TA4
    - Requires tutorial attendance
  - 1 project (10%)
    - Project 1
- Issues with marks
  - Check with your tutor

Midterm & Project 2

- Midterm Examination
  - Midterm Examination Review in this week's tutorial
    - No tutorial assignment due this week
    - Refer to LumiNUS > Module Details > Schedule
- Project 2 FAQ session
  - Today (7 March)
  - 1600-1700 hrs
  - LumiNUS > Conferencing > CS3243 Project 2 Consultation
    - Consultation will include discussion on general problem formulation

Upcoming…

- Deadlines
  - DQ7 (released today)
    - *Two attempts*
    - *Due this Sunday (13 March), 2359 hrs*
  - TA6 (released today)
    - *Due next Sunday (20 March), 2359 hrs*

  - Project 2
    - *Due next Sunday (20 March), 2359 hrs*

Reviewing Search Problems

So Far...

- Path search (path planning)
  - Search for a path from start to goal
    - Complete: finds a solution or says when there isn't one
    - Optimal: path cost of path found is minimal
  - Uninformed
    - Systematically search all paths via general search problem formulation
  - Informed
    - Uses a heuristic to estimate cost from any state to state goal
- Goal search
  - Focus on goal and ignore path
    - Completeness consideration only
  - Local search
    - Uses heuristic to guide search to goal (uses restarts; many variants)
  - Constraint satisfaction problem
    - Uses specific search problem formulation and shrinks search space via inference

Games

Games and Search

- Can we solve games using existing methods?
  - In our searching thus far, we control all actions
    - All actions taken are determined by our agent
  - With games, your opponent decides actions too...
    - Multi-agent problem
    - Conventional planning ⇒ wasted computation since opponent can spoil your plans

Games and Search

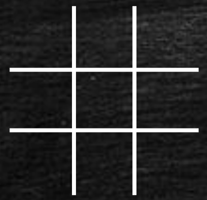
- What is a game anyway?
  - Assume two players
  - Zero-sum game
    - Winner gets paid, and loser pays
  - We define
    - MAX player – player 1, who wants to maximise value (agent)
    - MIN player – player 2, who want to minimise value (opponent who wants agent to lose)
- General idea behind the search problem
  - Simulate play against utility maximising opponent
  - Find a strategy – i.e., define a move for every possible opponent response

Search Problem Formulation for Games

Formulating Games

- State representation
  - As per general formulation

s_0



s_1

- TO-MOVE(s)
  - Returns p, the player to move in state s
- ACTIONS(s)
  - Legal moves in state s
- RESULT(s, a)
  - The transition model; returns resultant state when taking action a at state s
- IS-TERMINAL(s)
  - Returns TRUE when game is over and FALSE otherwise
    - States where game has ended are called terminal states
- UTILITY(s, p)
  - Defines the final numeric value to player p when the game ends in terminal state s

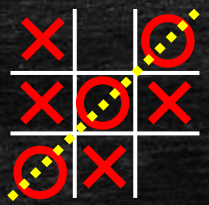
Note on Utility

- Given zero-sum games
  - At terminal state s
    - UTILITY(MAX,s) + UTILITY(MIN,s) = 0
- Tic-Tac-Toe example
  - X (agent) wins
    - UTILITY(s_i, MAX) = 1
    - UTILITY(s_i, MIN) = -1

s_i :

  - O (opponent) wins
    - UTILITY(s_j, MAX) = -1
    - UTILITY(s_j, MIN) = 1

s_j :

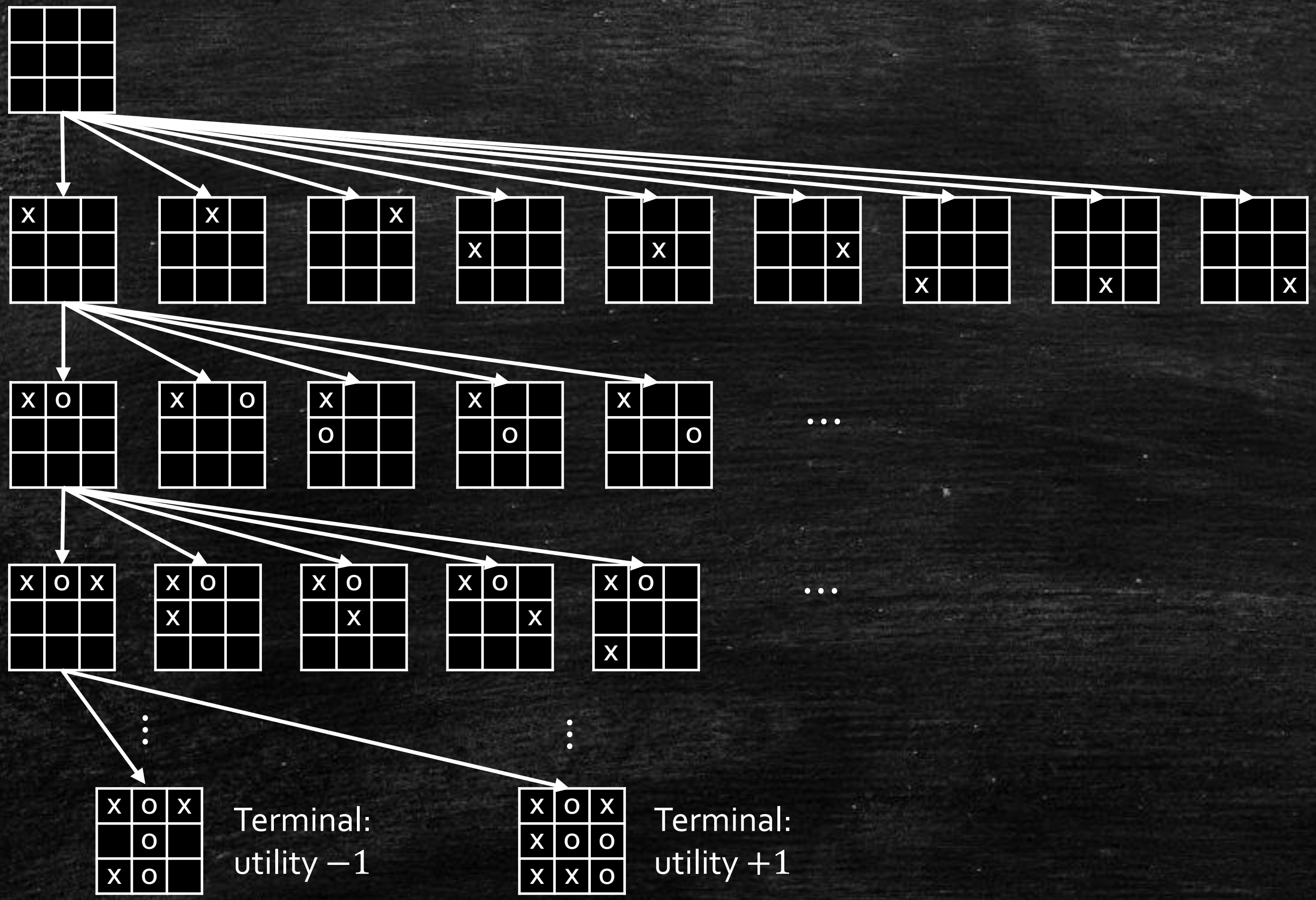


  - Draw
    - UTILITY(s_k, MAX) = 0
    - UTILITY(s_k, MIN) = 0

s_k :

Game Trees

Example Game Tree



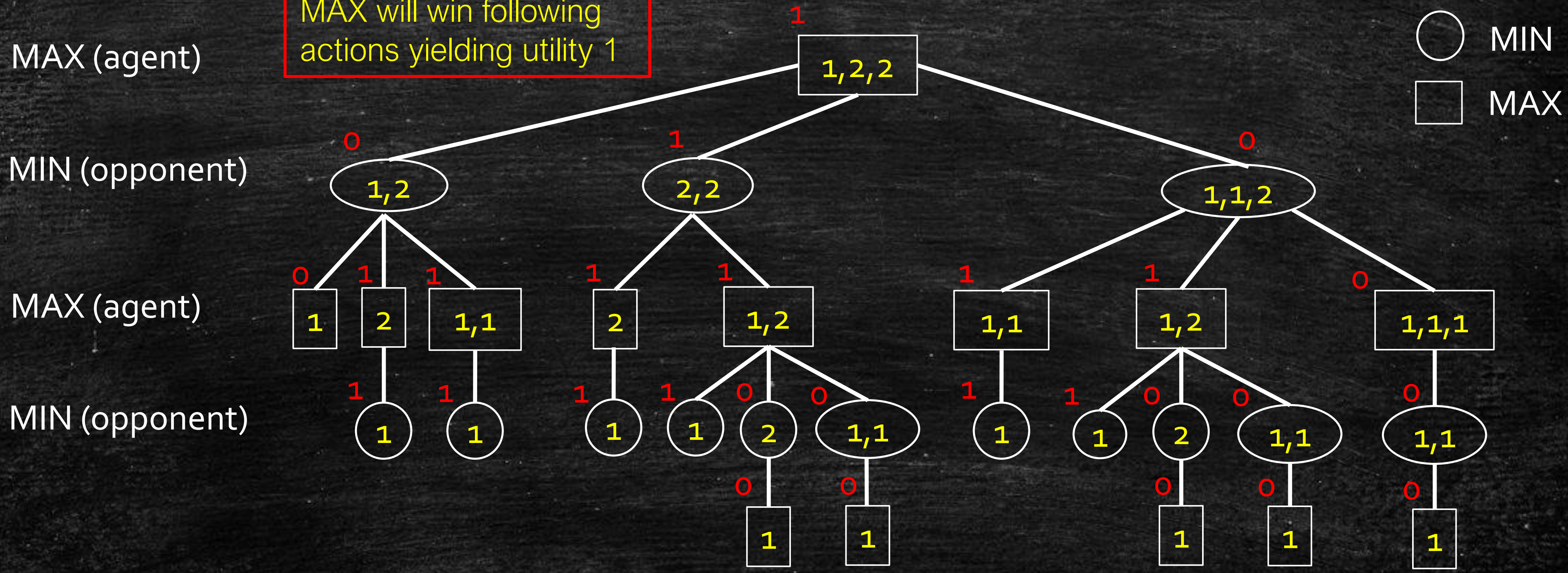
- More on environment characteristics
  - 2-player
  - Deterministic
  - Turn-taking
- Zero-sum implications
  - Loser for every winner
  - Agent utilises sum to zero
  - Also considered constant-sum game
  - Completely adversarial game

Another Example: Game of NIM

- Several piles of sticks are given
  - Represent the configuration of piles by a monotone sequence of integers
    - Example: (1,3,5)
  - With each turn, a player may remove any number of sticks from ONE pile
    - Example:
      - Remove 4 sticks from last pile (of 5 sticks) ⇒ (1,3,5) becomes (1,1,3)
  - The player who takes the last stick loses
- Let's try...
  - Represent the NIM game (1,2,2) as a game tree

Game of NIM: (1,2,2) Game Tree

MAX will win following actions yielding utility 1



DFS Traversal with Backwards induction on utility

Strategies

Player Strategies

- A strategy ***s*** for ***player i***:
  - What will ***player i*** do at every node of the game tree that they make a move in?
    - Need to specify behaviour in states that may never be reached!
- Winning strategy

A strategy s_1^* for Player 1 is called winning if
for any strategy s_2 by Player 2,
the game ends with Player 1 as the winner.

- Non-losing strategy

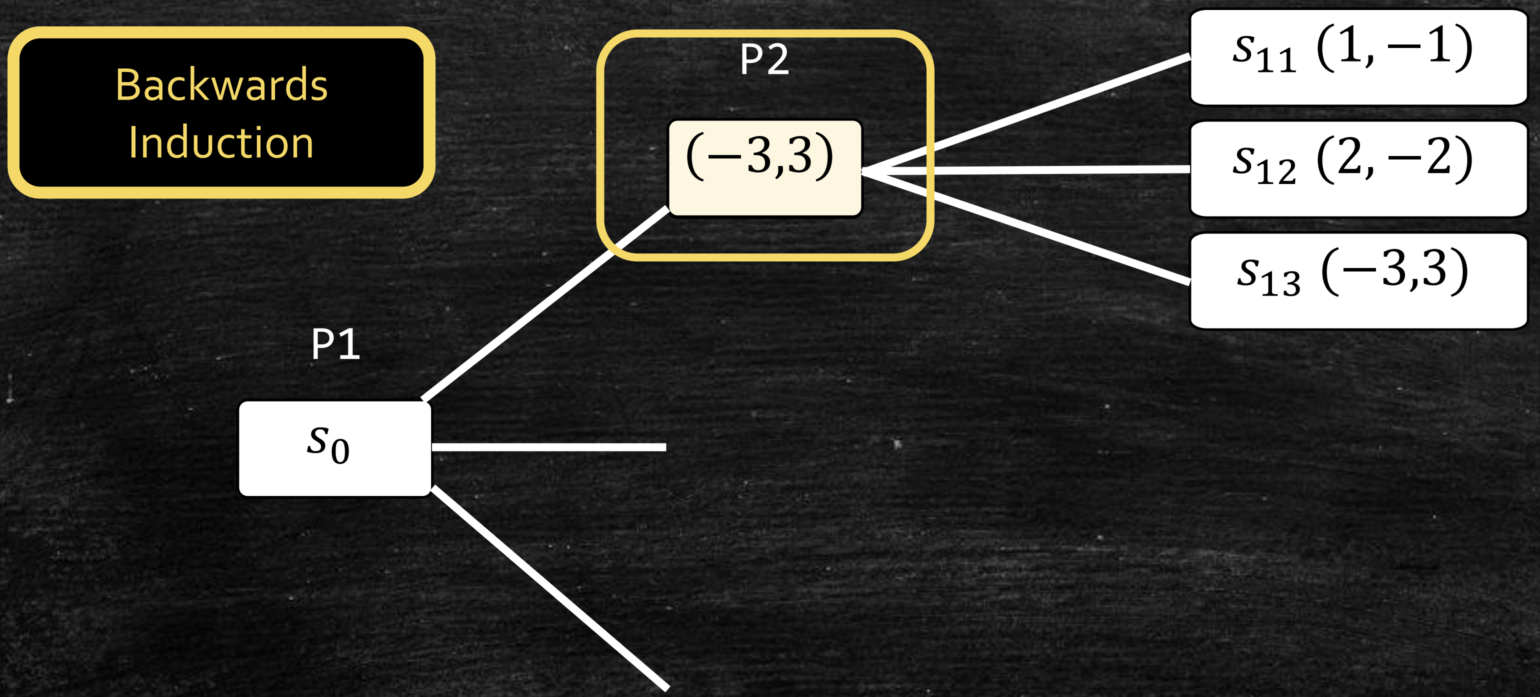
A strategy t_1^* for Player 1 is called non-losing if
for any strategy s_2 by Player 2,
the game ends in either a tie or a win for Player 1.

Optimal Strategy at Node - Minimax

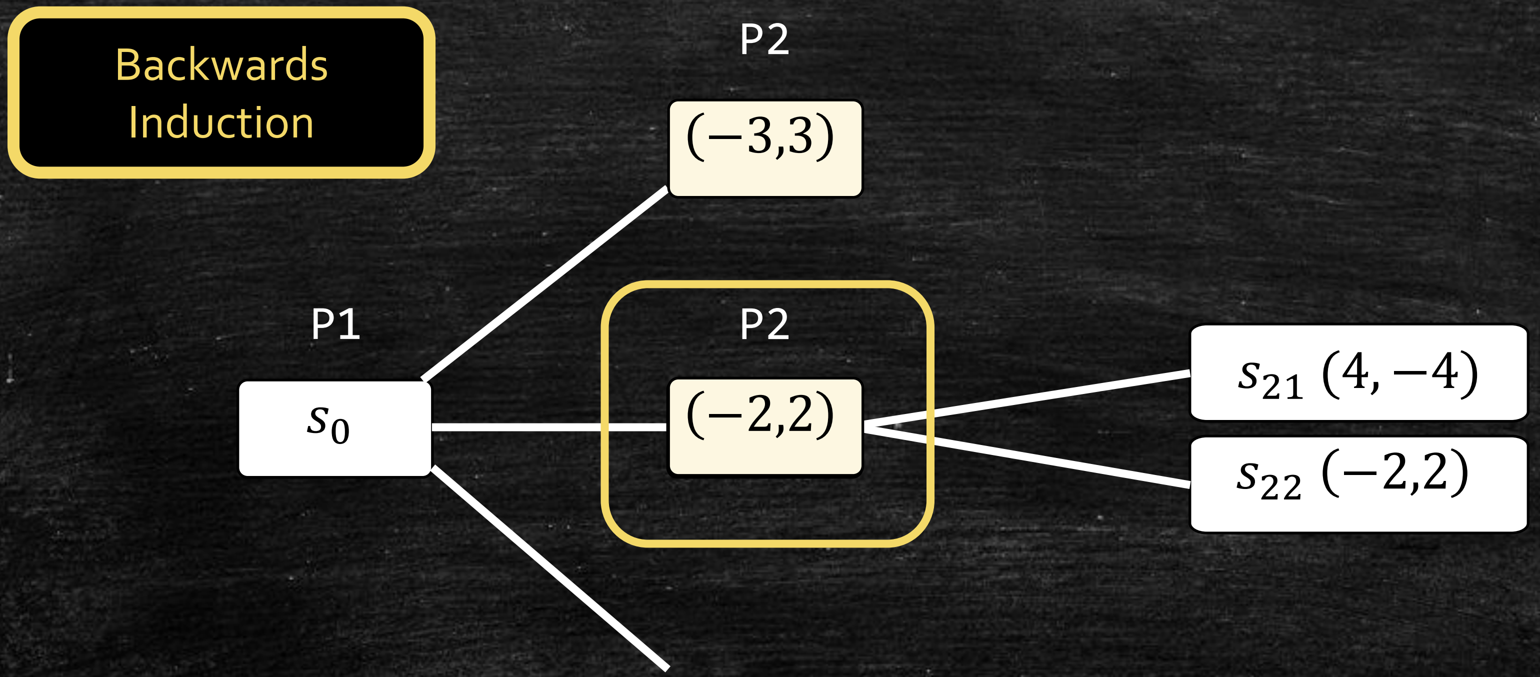
$$Minimax(s) = \begin{cases} Utility\big(s, \mathrm{To{-}Move}(s)\big) \text{ if Is}{-}\text{Terminal}(s) \\ \max_{a \in \mathrm{Actions}(s)} \mathrm{Minimax}\big(\mathrm{Result}(s,a)\big) \text{ if To}{-}\text{Move}(s) = \mathrm{MAX} \\ \min_{a \in \mathrm{Actions}(s)} \mathrm{Minimax}\big(\mathrm{Result}(s,a)\big) \text{ if To}{-}\text{Move}(s) = \mathrm{MIN} \end{cases}$$

- Intuitively
  - MAX chooses move to maximise the minimum payoff
    - MIN chooses at successors
  - MIN chooses move to minimise the maximum payoff
    - MAX chooses at successors

Minimax Play – Subgame Perfect Nash Equilibrium



Minimax Play – Subgame Perfect Nash Equilibrium



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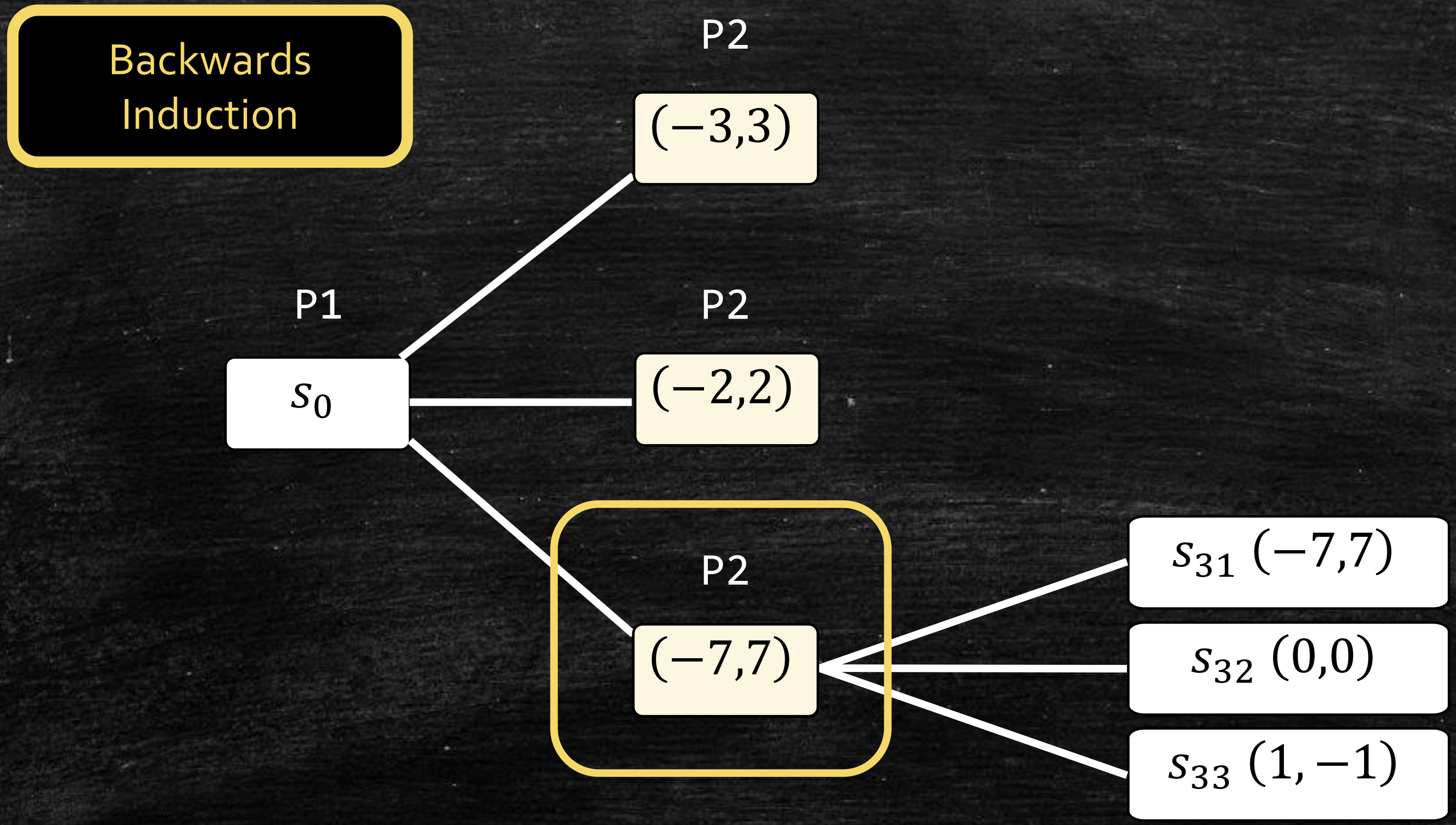
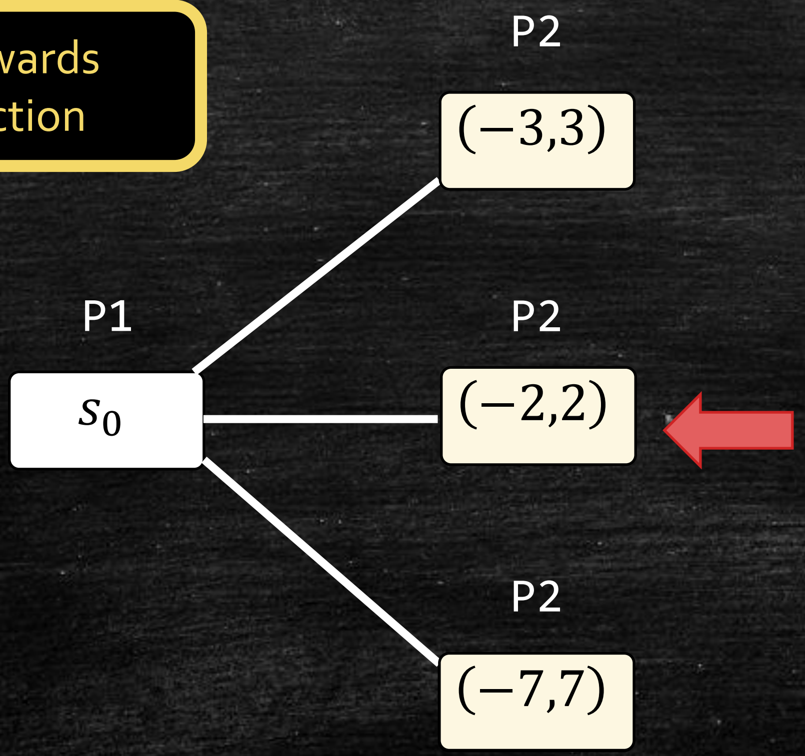


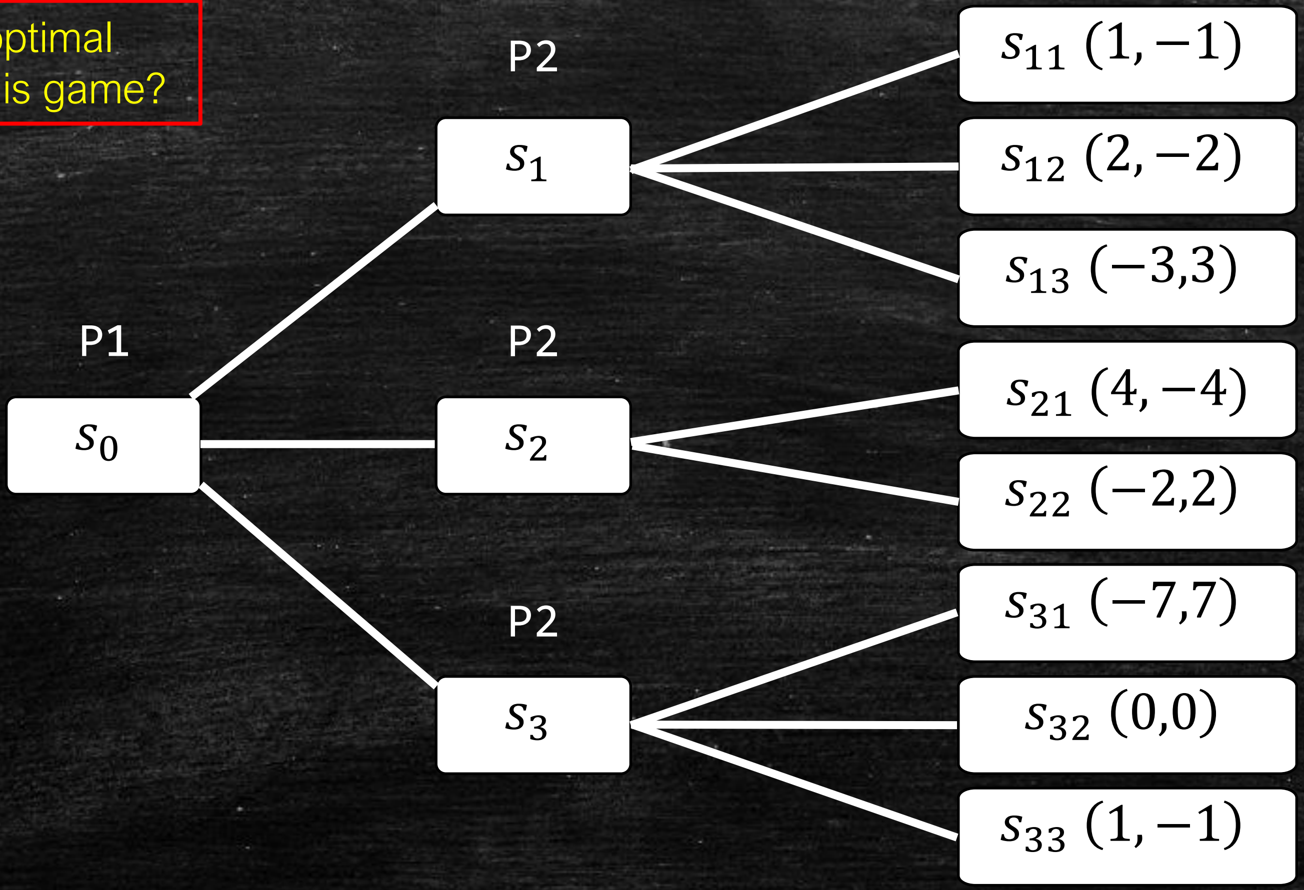
Minimax Play – Subgame Perfect Nash Equilibrium

Backwards Induction



Minimax Play – Subgame Perfect Nash Equilibrium

What are the optimal strategies in this game?



Questions on the Lecture so far?

- Was anything unclear?
- Do you need to clarify anything?

- Channels
  - Verbally on Zoom
  - On Archipelago
  - Via Zoom Chat

OR https://archipelago.rocks/app/resend-invite/29374922712

Minimax Algorithm Properties

- Complete?
  - Yes (if game tree is finite)
- Optimal?
  - Yes (optimal gameplay)
- Time
  - $O(b^m)$
- Space
  - $O(bm)$

- Minimax runs in time polynomial in tree size
- Returns a subgame perfect Nash Equilibrium
  - I.e., the best action at every node

Are we done?

Backwards Induction

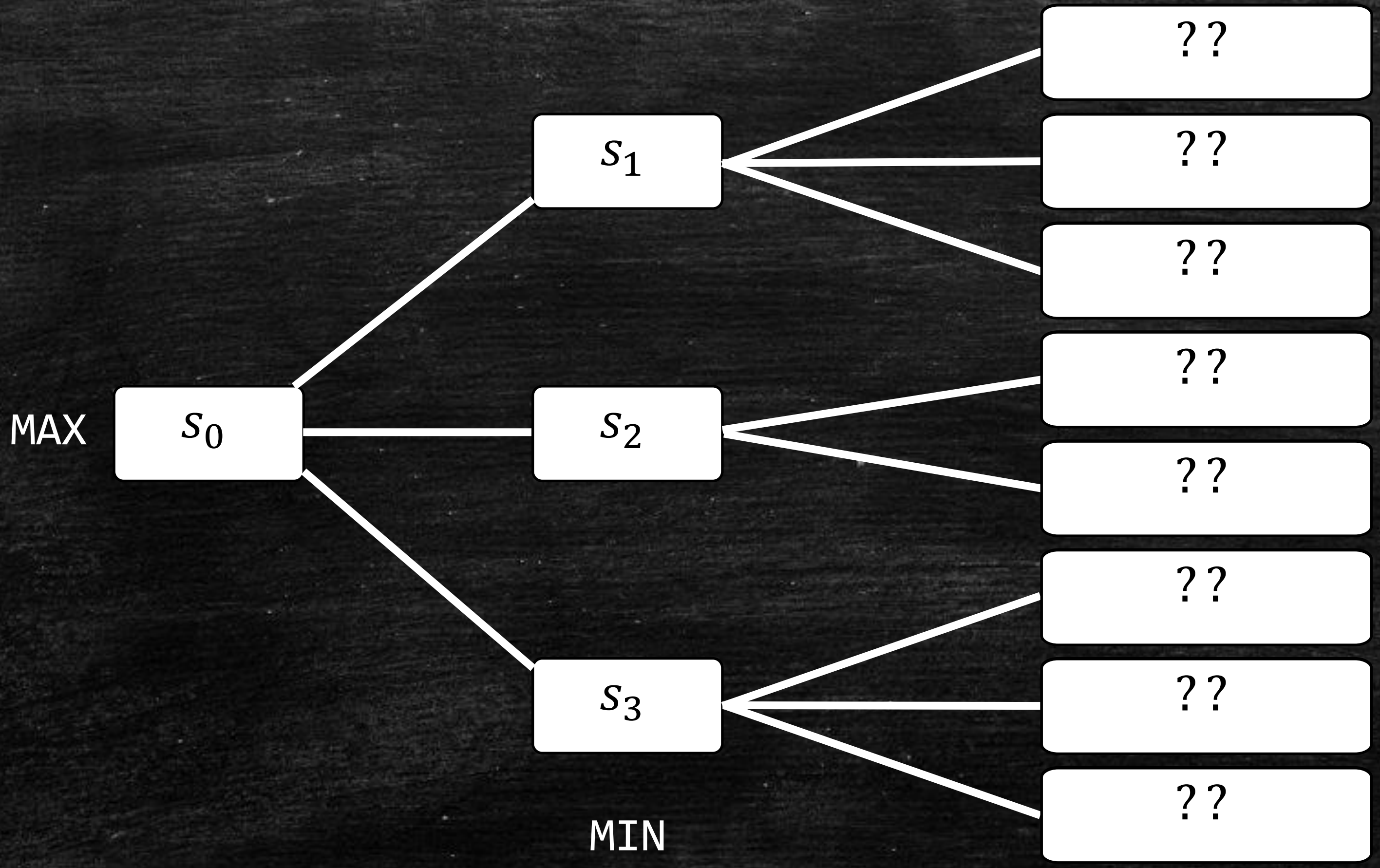
- Game trees are massive
  - Chess has a massive game tree
    - 10^{123} nodes
  - In comparison, planet Earth has about 10^{50} atoms …
- Impossible to expand the entire tree
- Have to find ways to shrink the search tree
  - We've seen this before
  - Common theme in search

α-β Pruning

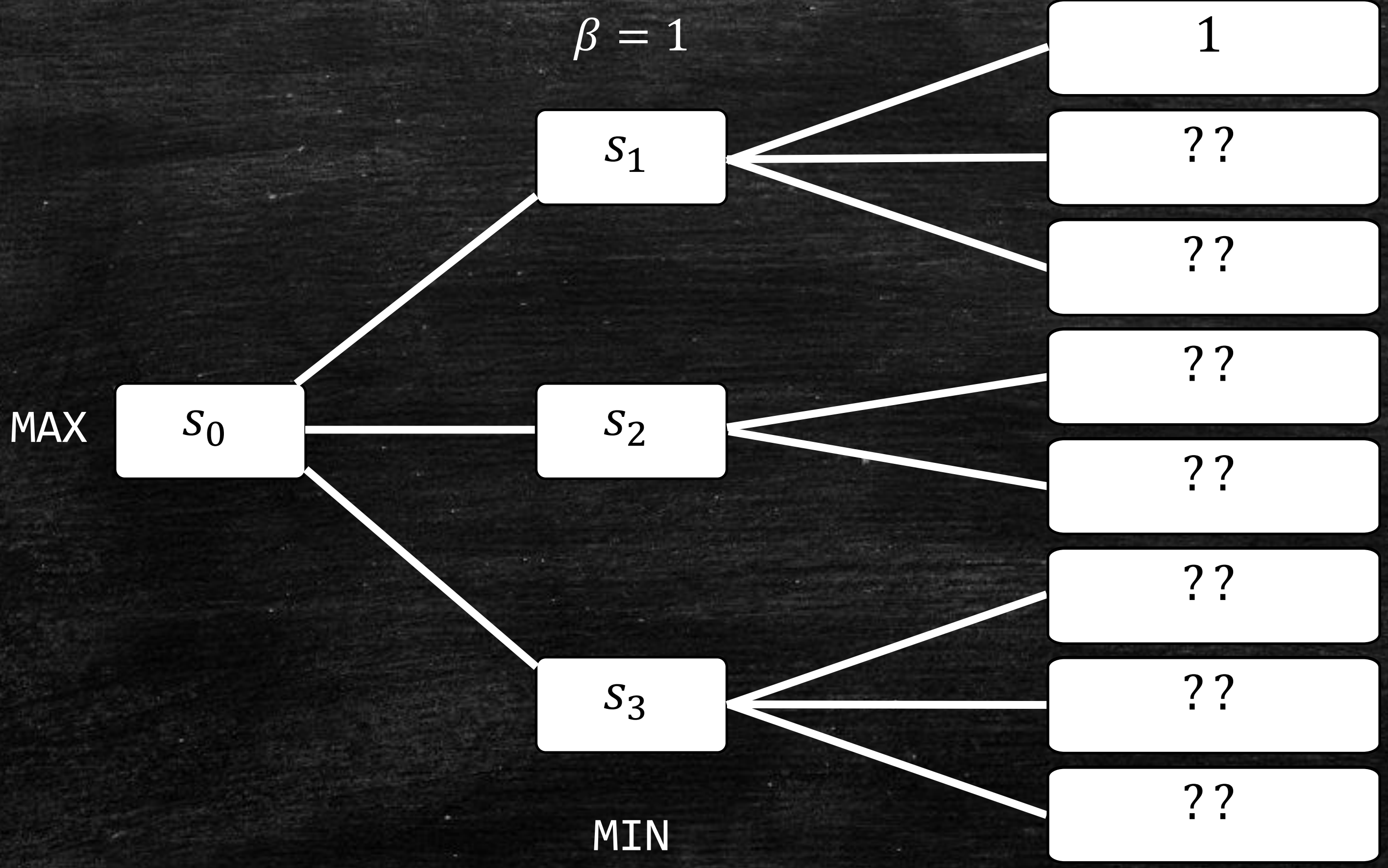
α-β Pruning - General Idea

- Basic idea
  - Don't explore moves that would never be considered
- Maintain bounds on values seen thus far while searching
  - Lower bound α of MAX's values
  - Upper bound β of MIN's values
- Prune subtrees that will never affect Minimax decision

α-β Pruning – Example Trace



α-β Pruning – Example Trace



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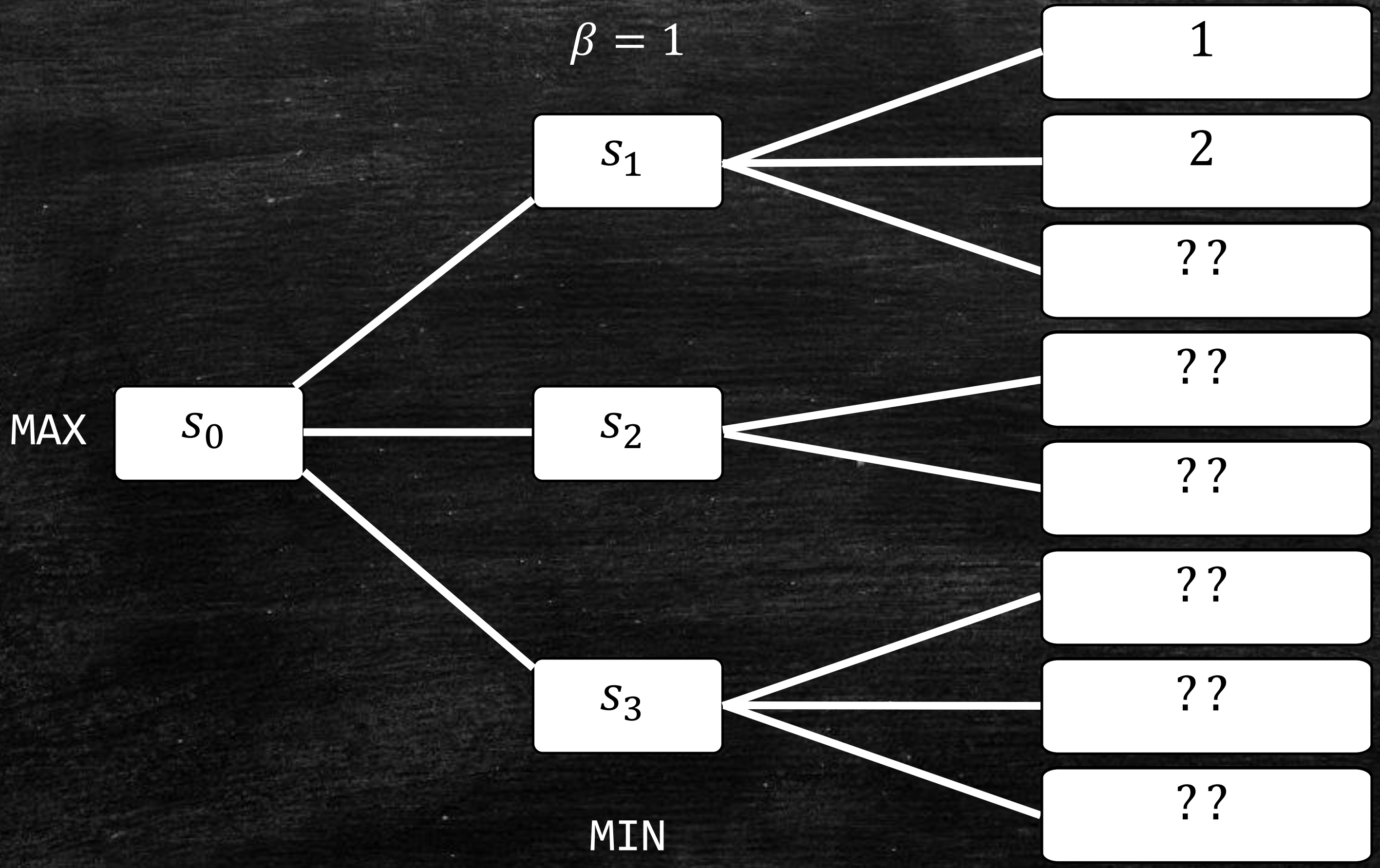

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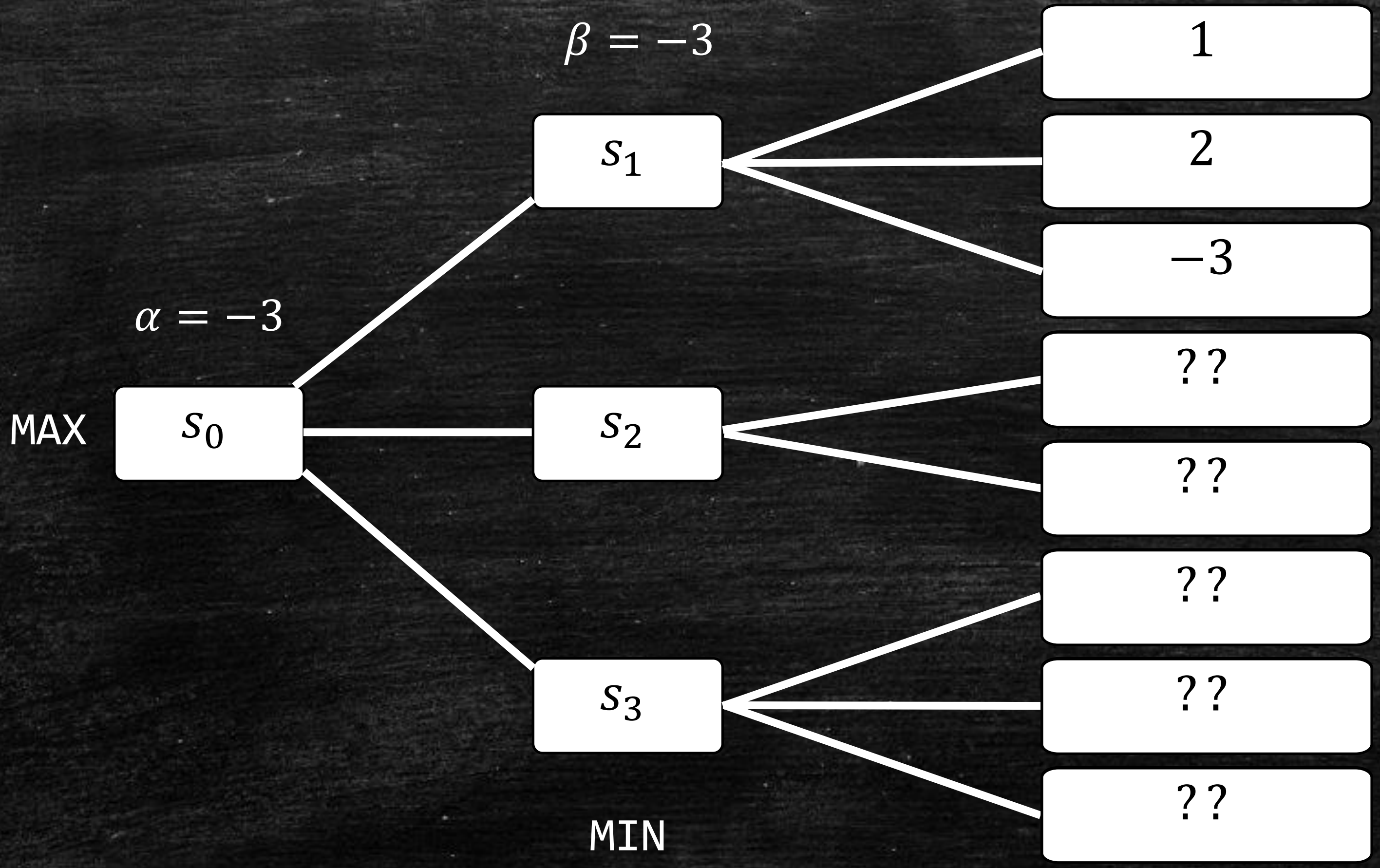

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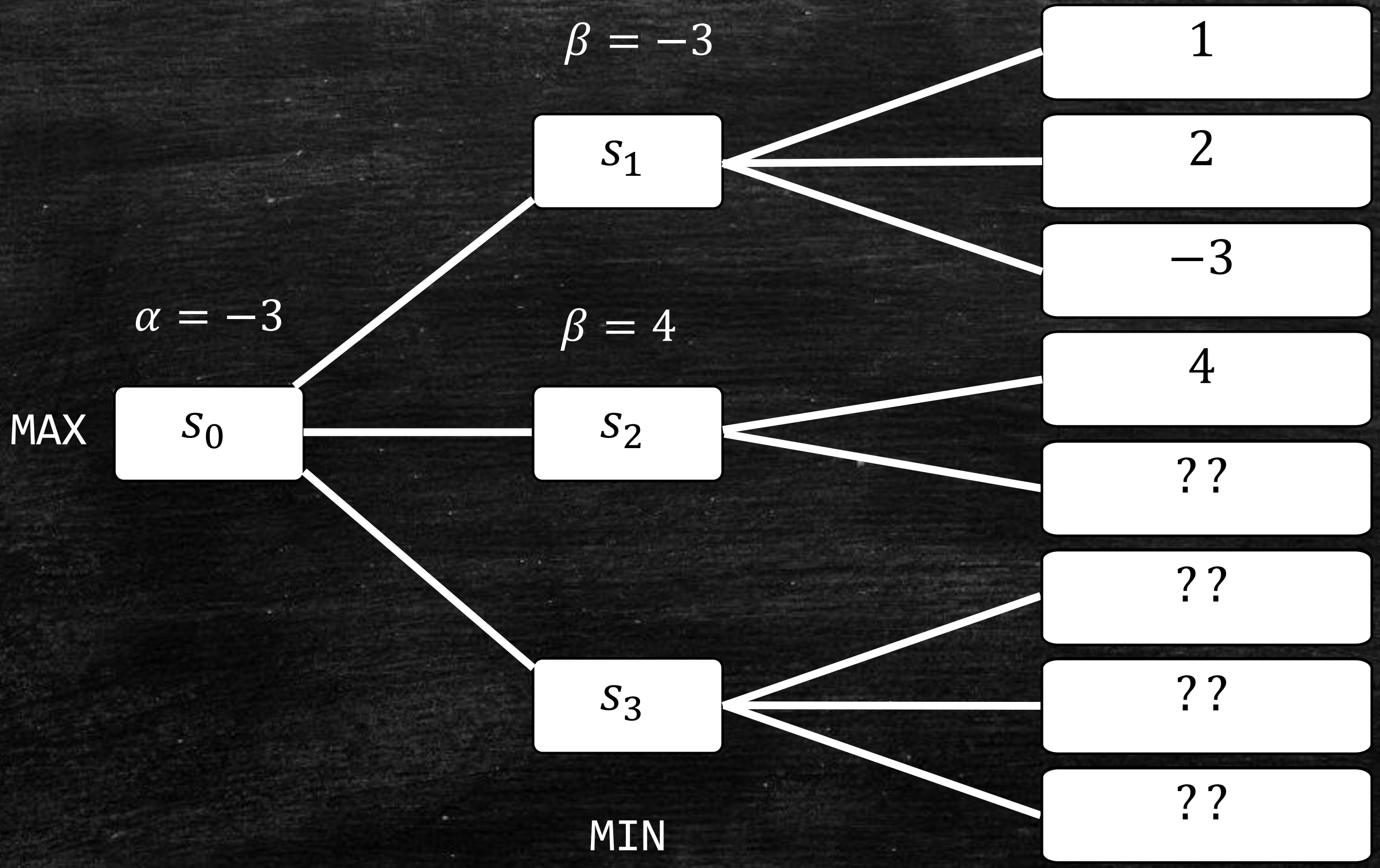

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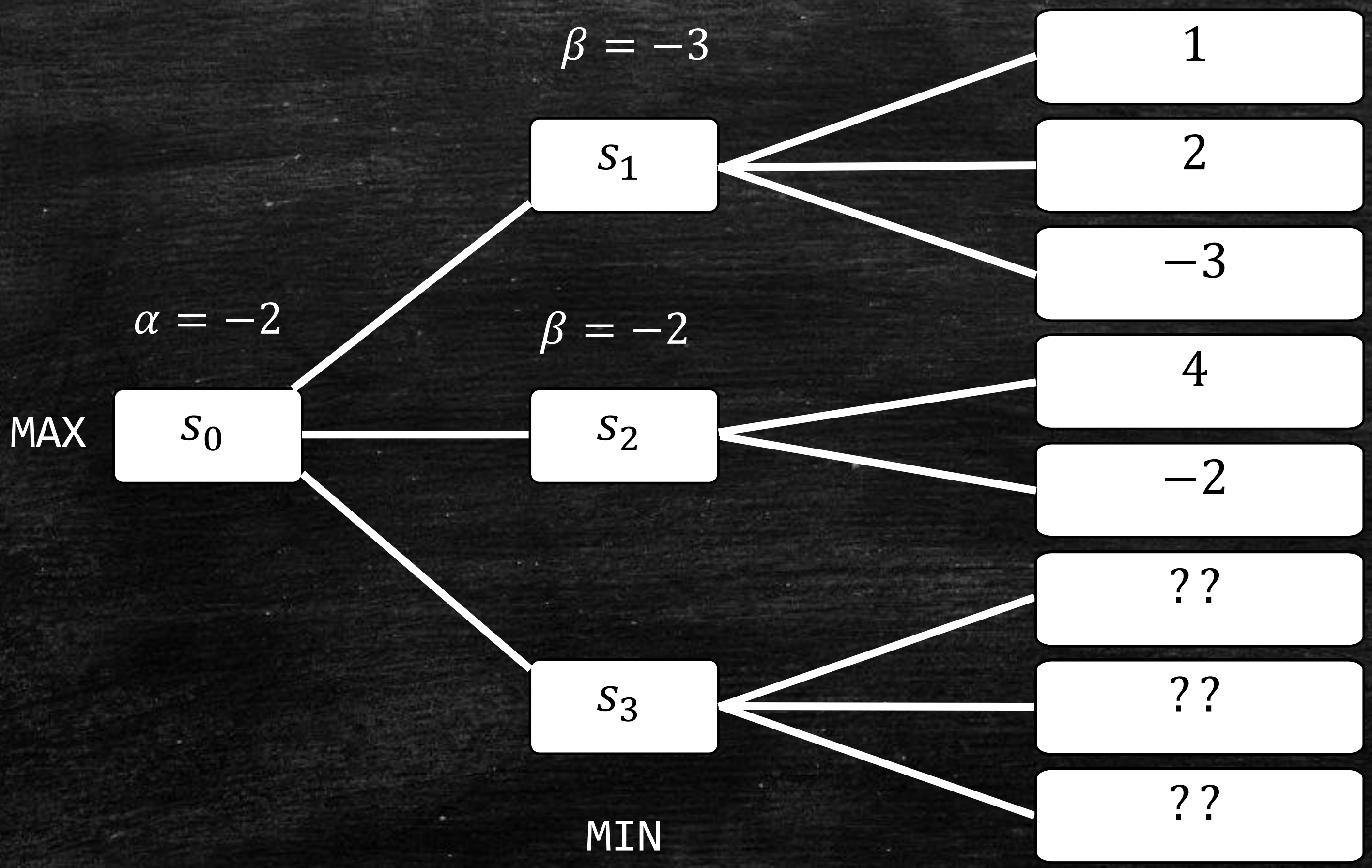

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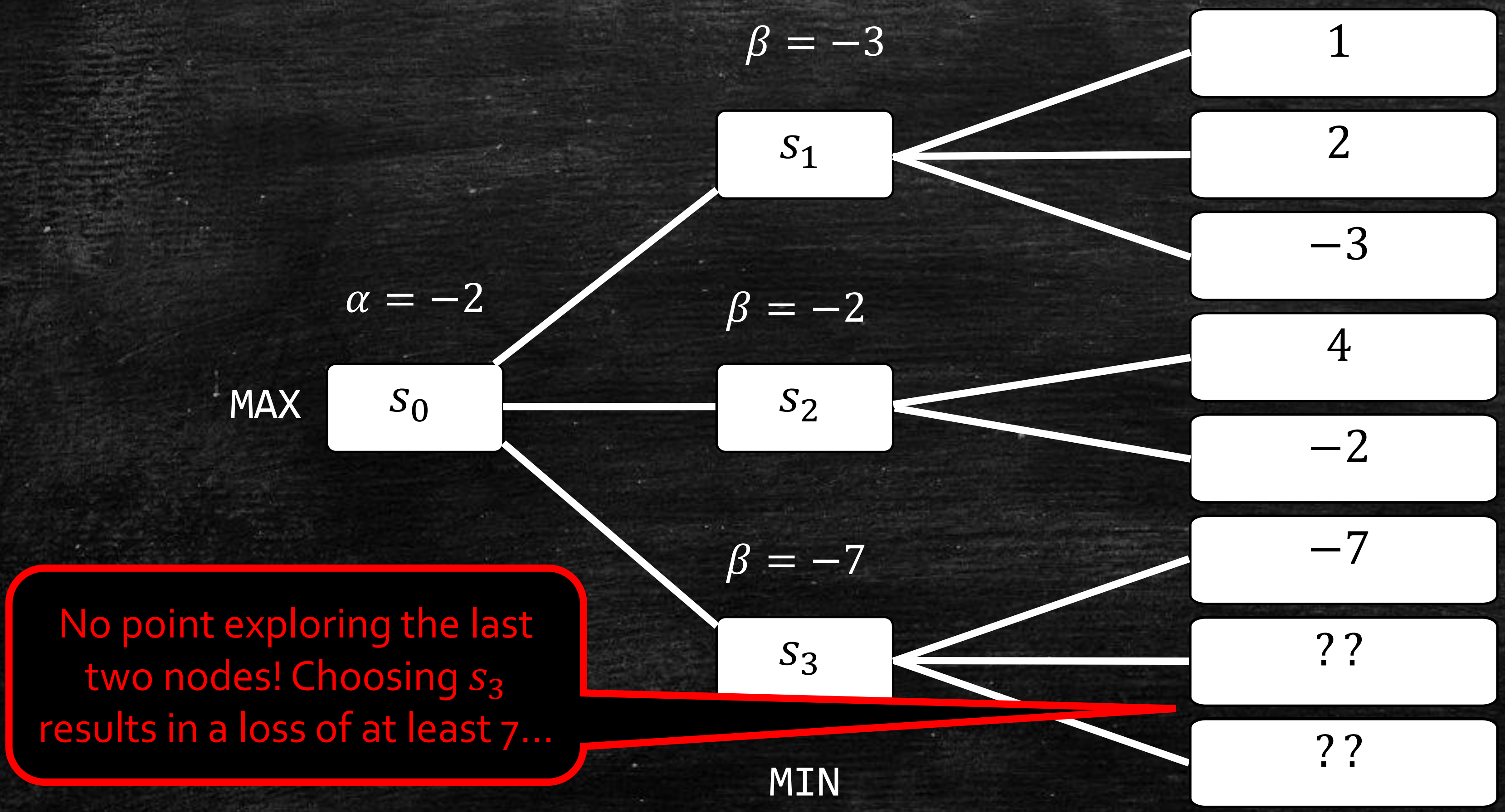
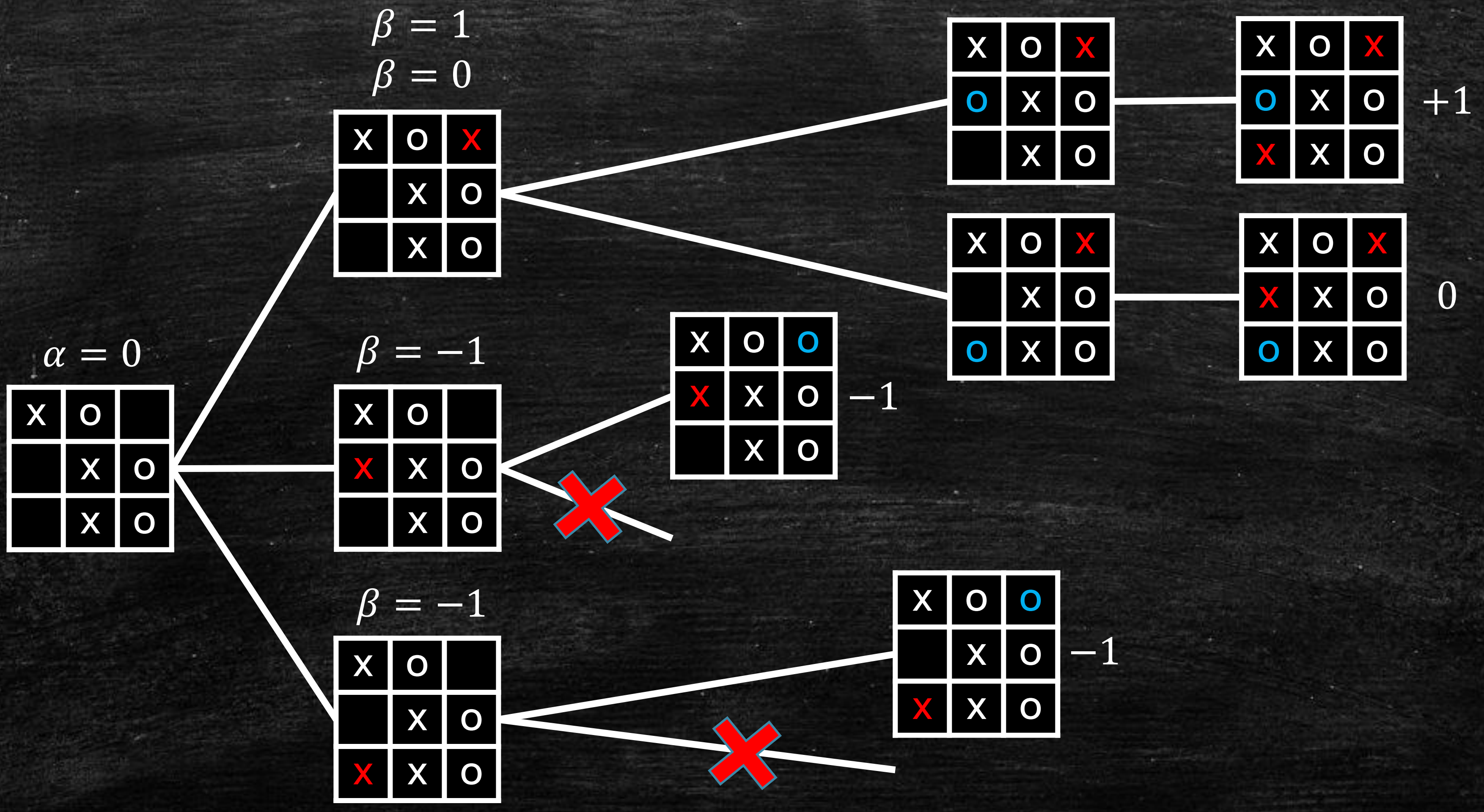


α-β Pruning – Tic-Tac-Toe Example



α-β Pruning

- MAX node n
  - α(n) = highest observed value found on path from n
  - Initially α(n) = -∞
- MIN node n
  - β(n) = lowest observed value found on path from n
  - Initially β(n) = +∞
- Pruning rules
  - Given a MIN node n, stop searching below n if
    - Some MAX ancestor i (of n) with $\alpha(i) \geq \beta(n)$
  - Given a MAX node n, stop searching below n if
    - Some MIN ancestor i (of n) with $\beta(i) \leq \alpha(n)$

MIN will choose β(n) or lower at n, but ancestor MAX will NEVER choose the subtree at n since at i, there is a better option with higher value α(i)

MAX will choose α(n) or higher at n, but ancestor MIN will NEVER choose the subtree at n since at i, there is a better option with lower value β(i)

α-β Pruning Analysis

- Pruning a branch never affects the final outcome
- Good move ordering improves effectiveness of pruning
  - "Perfect" ordering
    - Time complexity $O(b^{m/2})$
    - Good pruning strategies allow us to search twice as deep!
  - Example: Chess
    - Simple ordering gets you close to best-case result
      - Checks
      - Take pieces
      - Forward moves
      - Backwards move
- Expansion-order heuristics will improve the search
- Random ordering gives complexity $O(b^{3m/4})$ for $b < 1000$

Issue with α-β Pruning

- Original Problem
  - Most games have very large game trees
- Solution
  - α-β pruning can remove large parts of search trees
- Unresolved Issue
  - Maximum depth of tree
    - Backwards induction works backwards from terminal states
    - Still have to traverse to a terminal states
  - Standard solution – Heuristic Minimax
    - Cutoff test – e.g., depth limit (DLS)
    - Evaluation function – estimates expected utility of state

Heuristic Minimax

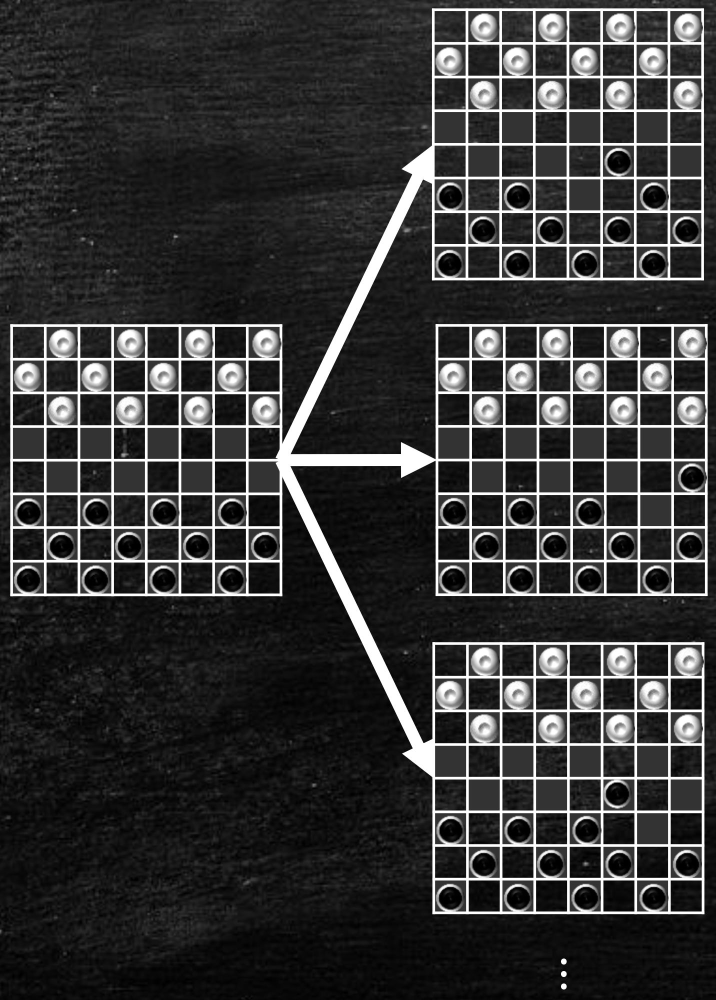
Heuristic Minimax Value

$$Minimax(s) = \begin{cases} Utility(s, \text{To}-\text{Move}(s)) \text{ if Is}-\text{Terminal}(s) \\ \max_{a \in \text{Actions}(s)} \text{Minimax}(\text{Result}(s,a)) \text{ if To}-\text{Move}(s) = \text{MAX} \\ \min_{a \in \text{Actions}(s)} \text{Minimax}(\text{Result}(s,a)) \text{ if To}-\text{Move}(s) = \text{MIN} \end{cases}$$

$$H-Minimax(s,d) = \begin{cases} Eval(s, \text{To}-\text{Move}(s)) \text{ if Cutoff}-\text{Test}(s,d) \\ \max_{a \in \text{Actions}(s)} \text{H}-\text{Minimax}(\text{Result}(s,a), d+1) \text{ if To}-\text{Move}(s) = \text{MAX} \\ \min_{a \in \text{Actions}(s)} \text{H}-\text{Minimax}(\text{Result}(s,a), d+1) \text{ if To}-\text{Move}(s) = \text{MIN} \end{cases}$$

Run Minimax until depth d; then start using the evaluation function to choose nodes

Evaluation Functions – Checkers Example



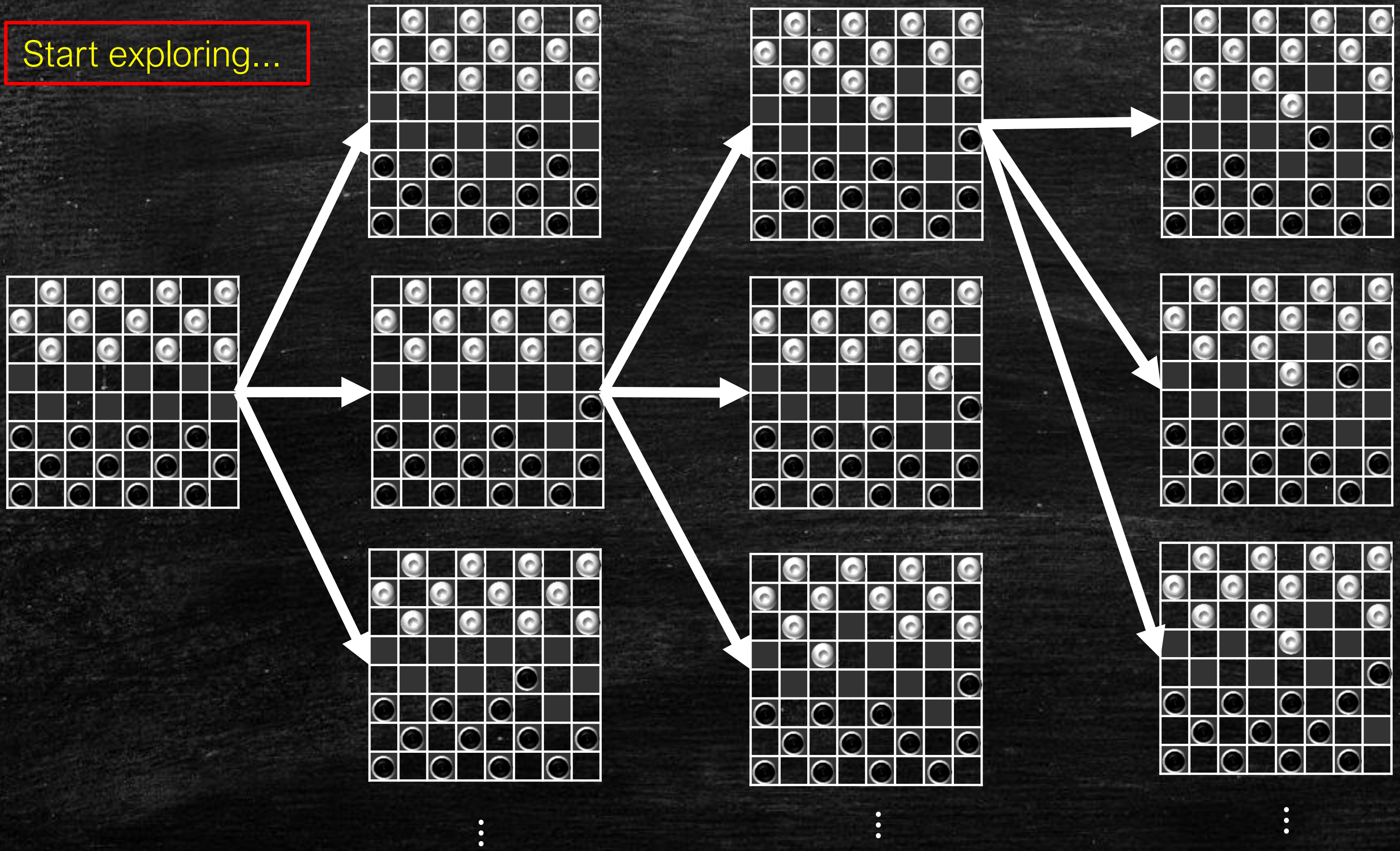
How good is this move?

To know for sure, we need to explore entire subtree up to terminal states.

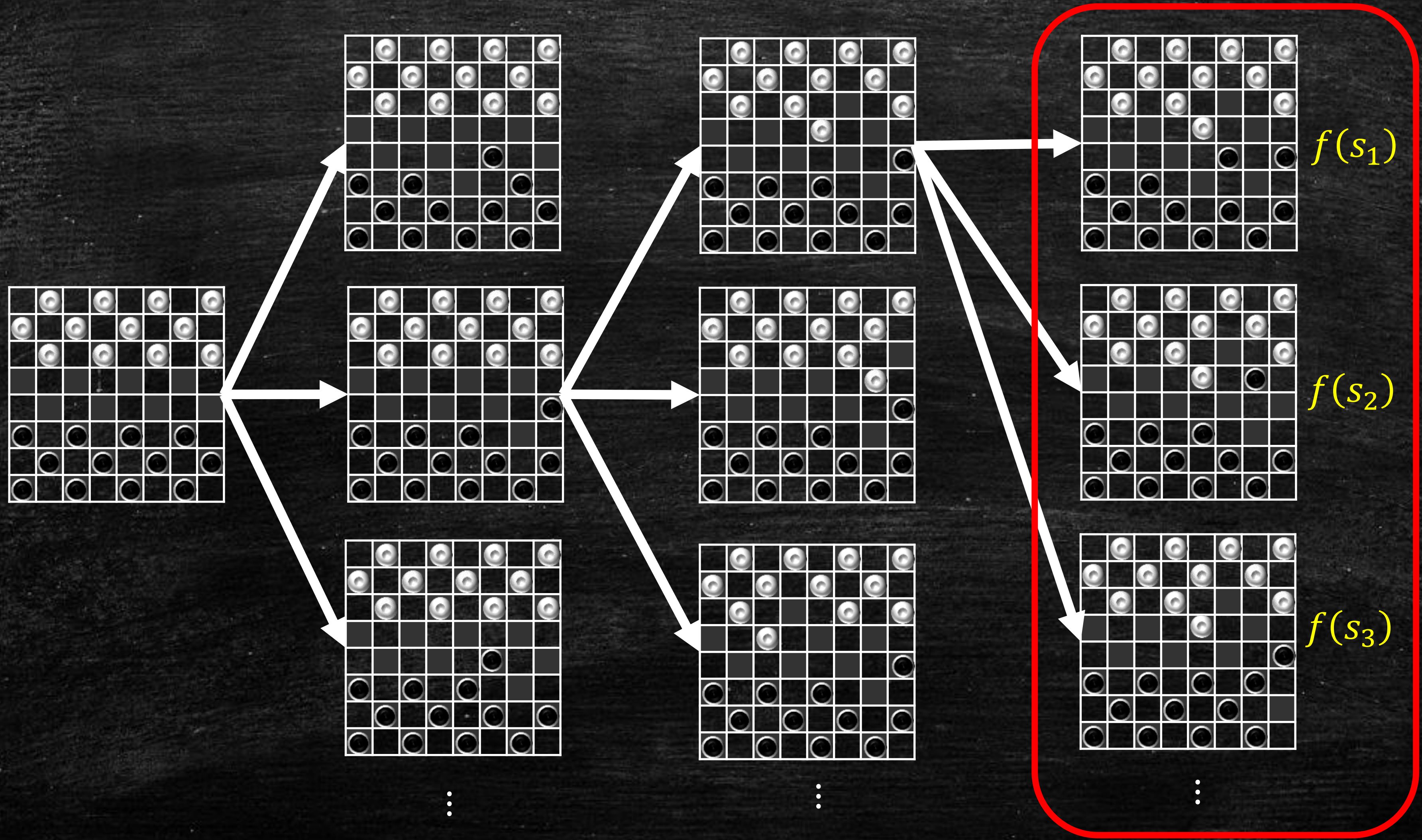
Unrealistic (even for 'simple' game of checkers)

Evaluation Functions – Checkers Example

Start exploring...



Evaluation Functions – Checkers Example



Pretend these are terminal nodes with values given by $f(.)$

Evaluation Functions

- An evaluation function is a mapping from game states to real values
  - $f : \mathcal{S} \rightarrow \mathbb{R}$
- Default evaluation function:

$$f(s) = \begin{cases} Utility(s, MAX) \text{ if } Is\text{—}Terminal(s) \\ 0 \; otherwise \end{cases}$$

No information on quality of non-terminal nodes

- Determine a function to estimate value that is strongly correlated to actual chances of winning
  - Modelling problem (similar to heuristic design problem from informed/local search)

Evaluation Functions

- Determine important features/variables
- Chess example
  - # of pieces ($NPcs$)
  - # of queens ($NQns$)
  - # of controlled squares ($CtlSqs$)
  - # of threatened opponent pieces ($ThrPcs$)
  - ...
- $f(n) = w_1 \times (NPcs) + w_2 \times (NQns) + w_3 \times (CtlSqs) + w_4 \times (ThrPcs)$

Determine values for $w_1, \ldots, w_4$

Cutting Off Search

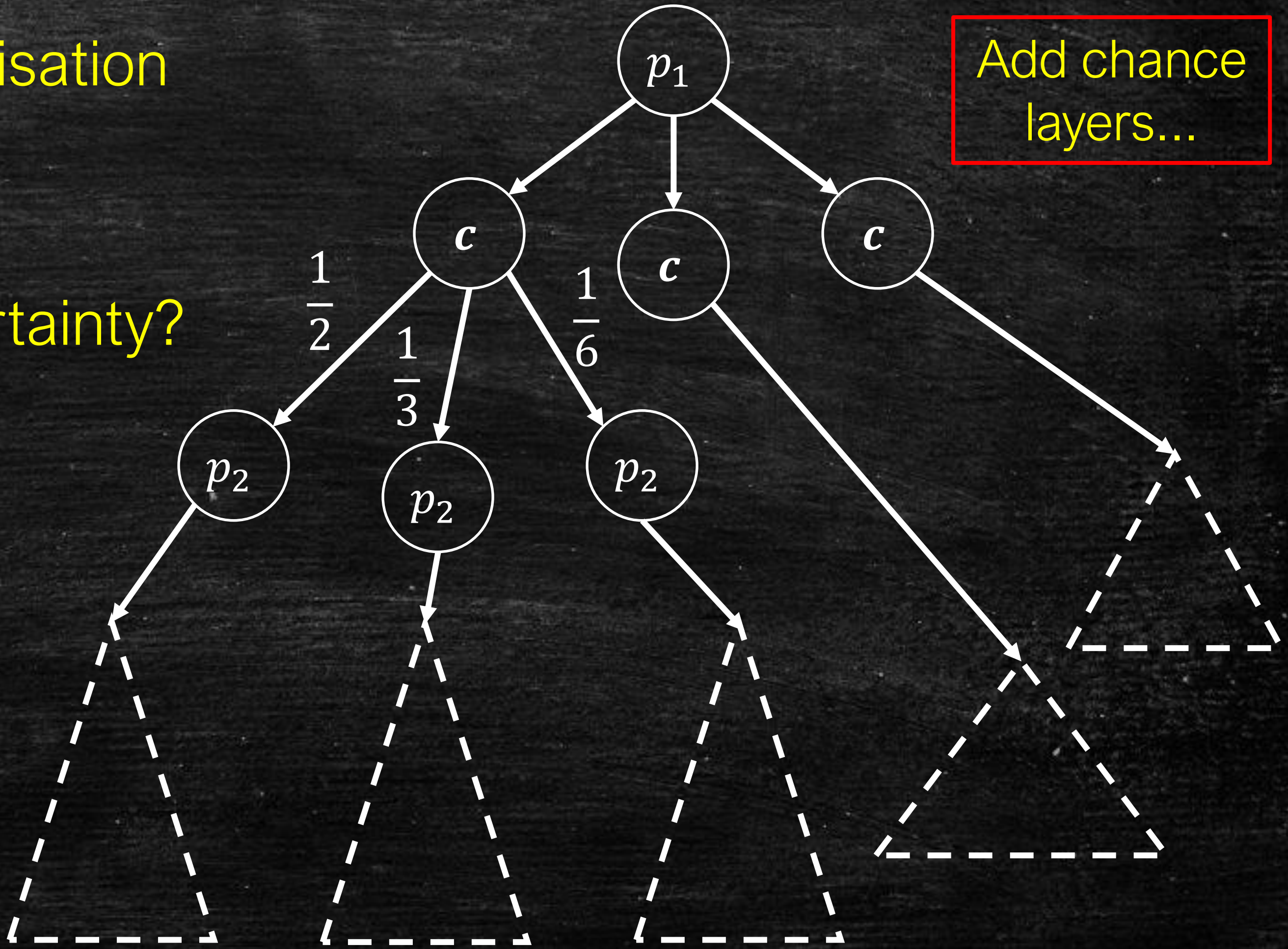
- Modify Minimax or α-β Pruning algorithms by replacing
  - Is-Terminal(s) with Cutoff-Test(s, d)
  - Utility(s, p) with Eval(s, p)

- Can replace DLS strategy with IDS

Stochastic Games

- Many games have randomisation
  - Settlers of Catan
  - Poker
- How do we deal with uncertainty?
  - Can still use Minimax
  - Search tree is larger

Calculate expected value of a state (MUCH harder than deterministic games)



Add chance layers...

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OR https://archipelago.rocks/app/resend-invite/29374922712