National University of Singapore
School of Computing
CS3243 Introduction to AI

Tutorial 2: Informed Search

Issued: January 24, 2022 Discussion in: Week 4

Important Instructions:

- **Assignment 2** consists of **Question 5** from this tutorial.
- Your solutions for this tutorial must be TYPE-WRITTEN.
- You are to submit your solutions on LumiNUS by **Week 3, Sunday, 2359 hours**.
- Refer to LumiNUS for submission guidelines

Note: you may discuss the content of the questions with your classmates, but you must work out and write up your solution individually. Solutions that are plagiarised will be heavily penaltised.

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1. (a) Provide a counter-example to show that the tree-based implementation for the **Greedy Best-First Search** algorithm is incomplete.
   (b) Briefly explain why the limited-graph-based implementation of the **Greedy Best-First Search** algorithm is complete.
   (c) Provide a counter-example to show that neither the tree-based nor the limited-graph-based implementations of the **Greedy Best-First Search** algorithm are optimal.

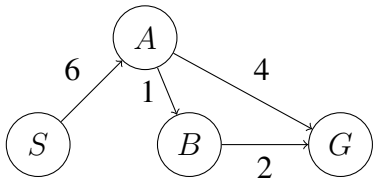
2. (a) Prove that the tree-based variant of the **A* Search** algorithm is optimal when an **admissible heuristic** is utilised.
   (b) Prove that the limited-graph-based variant of the **A* Search** algorithm is optimal when a **consistent heuristic** is utilised.

3. (a) Given that a heuristic h is such that $h(t) = 0$, where t is any goal state, prove that if h is consistent, then it must be admissible.
   (b) Give an example of an admissible heuristic that is not consistent.

4. We have seen various search strategies in class, and analyzed their worst-case running time. Prove that *any deterministic search algorithm* will, in the worst case, search the entire state space. More formally, prove the following theorem

   Theorem 1. *Let $\mathcal{A}$ be some complete, deterministic search algorithm. Then for any search problem defined by a finite connected graph $G = \langle V, E \rangle$ (where V is the set of possible states and E are the transition edges between them), there exists a choice of start node s_0 and goal node g so that $\mathcal{A}$ searches through the entire graph G.*

5. (a) In the search problem below, we have listed 5 heuristics. Indicate whether each heuristic is admissible and/or consistent in the table below.



	S	A	B	G	Admissible	Consistent
h_1	0	0	0	0		
h_2	8	1	1	0		
h_3	9	3	2	0		
h_4	6	3	1	0		
h_5	8	4	2	0		

   (b) Write out the order of the nodes that are explored by the **A* Search** algorithm. Assume a limited-graph-based implementation that utilises heuristic h_4.
   You should express your answer in the form S–B–A–G (i.e., no spaces, all uppercase letters, delimited by the dash (–) character), which, for example, corresponds to the order S, B, A, and G.
   (c) Which heuristic would you use? Explain why.
   (d) Prove or disprove the following statement:

   The heuristic $h(n) = max\{h_3(n), h_5(n)\}$ is admissible.