EE2023 Signals & Systems Tutorial 5

Section I : Exercises that are straightforward applications of the concepts covered in class. Please attempt these problems on your own.

1. Solve the following Laplace Transform questions :

(a) $\mathcal{L}\left\{\cos^2 \omega t\right\}$
ANSWER : $\dfrac{1}{2}\left[\dfrac{s}{s^2 + 4\omega^2} + \dfrac{1}{s}\right]$

(b) $\mathcal{L}^{-1}\left\{\dfrac{1}{(s-1)(s+2)(s+4)}\right\}$
ANSWER : $\dfrac{1}{15}e^t - \dfrac{1}{6}e^{-2t} + \dfrac{1}{10}e^{-4t}$

Application of the shift in the s-domain function rule : $\mathcal{L}\{e^{-\alpha t}f(t)\} = F(s+\alpha)$

(c) $\mathcal{L}^{-1}\left\{\dfrac{1}{(s+1)^2}\right\}$
ANSWER : te^{-t}

(d) $\mathcal{L}^{-1}\left\{\dfrac{s+9}{s^2 + 6s + 13}\right\}$
ANSWER : $e^{-3t}\left[\cos 2t + 3\sin 2t\right]$

(e) $\mathcal{L}\left\{\dfrac{3}{5} - \dfrac{\sqrt{45}}{5}e^{-2t}\sin(t + \tan^{-1} 0.5)\right\}$
ANSWER : $\dfrac{3}{s(s^2 + 4s + 5)}$

Application of the shift in the time-domain function rule : $\mathcal{L}\{f(t-t_0)U(t-t_0)\} = e^{-st_0}F(s)$

(f) $\mathcal{L}\left\{(t-1)^2 U(t-1)\right\}$
ANSWER : $\dfrac{2}{s^3}e^{-s}$

(g) $\mathcal{L}\left\{t^2 U(t-1)\right\}$
ANSWER : $\dfrac{2}{s^3}e^{-s} + \dfrac{2}{s^2}e^{-s} + \dfrac{1}{s}e^{-s}$

(h) $\mathcal{L}^{-1}\left\{\dfrac{se^{-2s}}{s^2 + \pi^2}\right\}$
ANSWER : $\cos(\pi t)U(t-2)$

Application of the derivative of transforms rule : $F'(s) = \mathcal{L}\{-tf(t)\}$

(i) $\mathcal{L}\left\{te^{-t}\sin t\right\}$
ANSWER : $\dfrac{2(s+1)}{(s^2 + 2s + 2)^2}$

(j) $\mathcal{L}^{-1}\left\{\dfrac{s}{(s^2+9)^2}\right\}$
ANSWER : $\dfrac{1}{6}t\sin 3t$

2. Solve the following linear second order differential equation using Laplace Transform :

$$\ddot{y}(t) + 4\dot{y}(t) + 3y(t) = 2r(t) \text{ assuming that } r(t) = 1 \text{ when } t \geq 0, y(0) = 1 \text{ and } \dot{y}(0) = 0$$

ANSWER : $y(t) = \frac{1}{2}e^{-t} - \frac{1}{6}e^{-3t} + \frac{2}{3}$

Section II : Problems that will be discussed in class.

1. The circuit shown in Figure 1 is operating in steady-state with the switch open prior to $t = 0$. Find expressions for $i(t)$ for $t < 0$ and for $t \geq 0$.

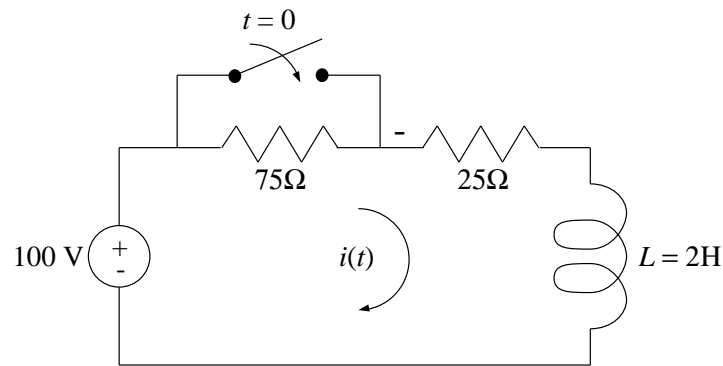ANSWER : $i(t) = 1$ for $t < 0$ and $4 - 3e^{-12.5t}$ for $t > 0$

Figure 1: Series RL circuit

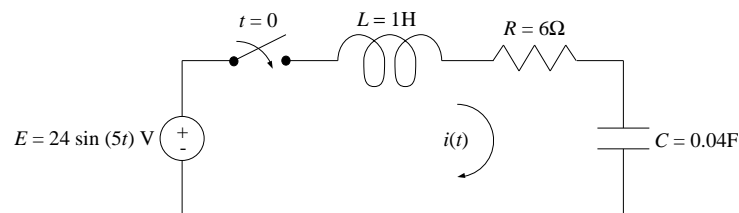2. A series RLC circuit is shown in Figure 2.

Figure 2: Series RLC circuit

(a) Show that the differential equation relating the current $i(t)$ in the RLC circuit shown in the figure to the applied voltage $E(t)$ is

$$L\frac{d^2i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{1}{C}i(t) = \frac{dE(t)}{dt}$$

(b) Assuming the initial current and its rate of change ($i(0)$ and $\frac{di(0)}{dt}$) are zero, find $i(t)$.

ANSWER : $i(t) = -5e^{-3t}\sin 4t + 4\sin 5t$

3. Ah Kow is worried about an upcoming exam. His doctor advises him to take a 100mg stress relief tablet the next morning and another 50mg tablet 24 hours later. Suppose the differential equation describing the quantity of drug in Ah Kow's body is

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = f(t)$$

where $y(t)$ is the quantity of drug in the body measured in mg,

$\quad$ $f(t)$ represents the rate at which the drug is administered into the body,

$\quad$ t is time measured in days.

Assume that $\bullet$ drugs taken in tablet form can be modelled by impulse function whose strength is equal to the quantity of drug ingested,

$\quad\quad\quad\quad$ $\bullet$ there is no stress relief drug in Ah Kow's bloodstream before he takes the first tablet.

(a) Write a mathematical expression representing the input signal, $f(t)$, which models the rate at which the stress relief medicine is digested. $\quad$ ANSWER : $100\delta(t) + 50\delta(t-1)$

(b) What are the initial conditions $(t = 0^-)$ of the system ?

$$\text{ANSWER} : y(0^-) = 0, y'(0^-) = 0$$

(c) Use Laplace Transform to determine the system output, $y(t)$. What is the amount of stress medicine left in Ah Kow's body by the time of the exam 4 days after he ate the first tablet ? $\quad\quad\quad\quad\quad\quad$ ANSWER : 4.1634 mg

Section III : Practice Problems. These problems will not be discussed in class.

1. Consider the circuit shown in Figure 3. The switch opens at $t = 0$. Find an expression for $v(t)$.
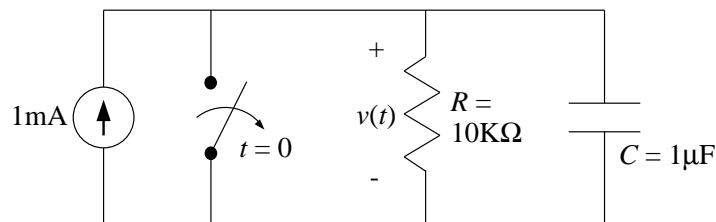
$$\text{ANSWER} : v(t) = 10 - 10e^{-100t} \text{ for } t > 0$$

Figure 3: Parallel RC circuit