EE2023 Signals & Systems Quiz
Semester 2 AY2017/18

Date : 8 March 2018 **Time Allowed : 1.5 hours**

Instructions :

1. Answer all 4 questions. Each question carries 10 marks.
2. This is a closed book quiz. However, you are allowed to bring one self-prepared, both sides handwritten A4-size crib sheet.
3. Tables of formulas are given on Pages 11 and 12.
4. Programmable and/or graphic calculators are not allowed.
5. Write your answers in the spaces indicated in this question paper. Attachment is not allowed.
6. Write your name, matric number and lecture group in the spaces indicated below.

Name : ______________________________

Matric # : ____________________________

____

Question #	**Marks**
1	
2	
3	
4	
Total Marks	

Q.1 The periodic signal $x(t)$ is given by

$$x(t) = 4\cos(6\pi t) + 3\sin\left(15\pi t + \pi / 3\right) + 7e^{j9\pi t}$$

(a) What is the fundamental frequency and period of $x(t)$?

(2 marks)

(b) Determine the Fourier Series coefficients of $x(t)$.

(4 marks)

(c) Determine the Fourier transform, $X(f)$, of $x(t)$.

(2 marks)

(d) What is the average power of $x(t)$?

(2 marks)

Q.1 ANSWER

Q.1 ANSWER ~ continued

Q.2 (a) Determine the Fourier transform, $X(f)$, of the signal $x(t)$ shown in Figure Q2(a) which is given by:

(4 marks)

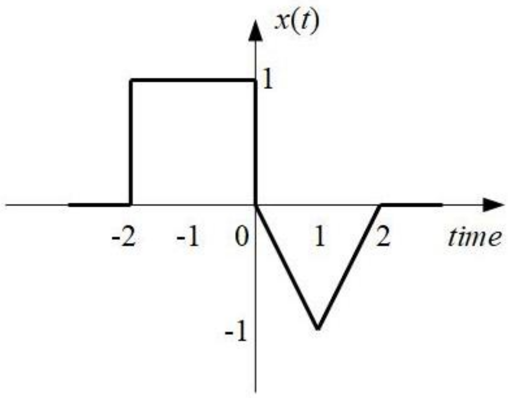

Figure Q.2(a)

(b) The periodic signal $x_p(t)$ is obtained by repeating the generating function $x(t)$ at a period of 5 seconds and is shown in Figure Q.2(b).

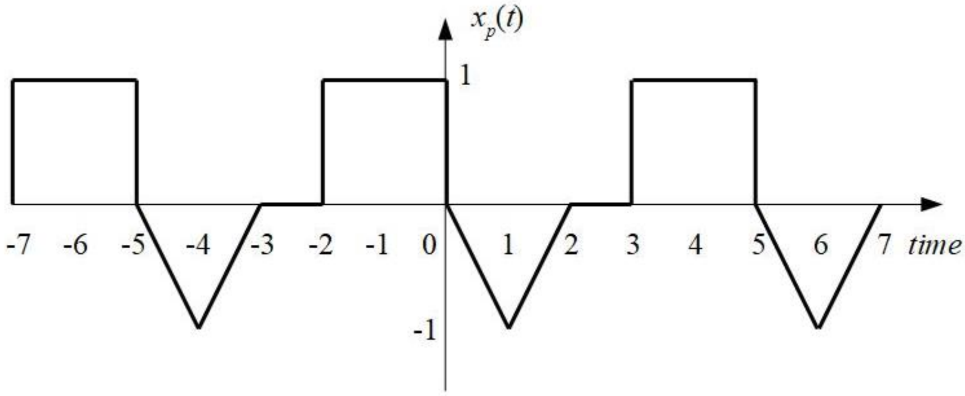

Figure Q.2(b)

i. Derive an expression for $x_p(t)$ in terms of $x(t)$. (2 marks)

ii. Determine the Fourier transform, $X_p(f)$, of the periodic signal $x_p(t)$. (4 marks)

Q.2 ANSWER

Q.2 ANSWER ~ continued

Q.3 The Fourier transform, $X(f)$, of a signal $x(t)$ is shown in Figure Q3.

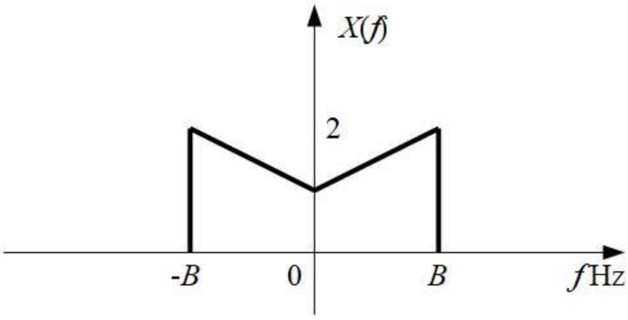

Figure Q.3

Consider also the signal $y(t) = x(t)\cos(20\pi t)$.

(a) If the bandwidth of $x(t)$ is $B = 4$ Hz, sketch and label the magnitude spectrum of $y(t)$.

(3 marks)

(b) If $y(t)$ is sampled at a sampling frequency of 30 Hz to give the sampled signal $y_s(t)$, obtain the expression for $y_s(t)$ in terms of the comb function.

(3 marks)

(c) Sketch the magnitude spectrum of the sampled signal $y_s(t)$.

(4 marks)

Q.3 ANSWER

Q.3 ANSWER ~ continued

Q.4 A signal $x(t) = 2\ \text{tri}\left(\frac{t-1}{2}\right)$ is sent through a filter to produce $y(t)$ as shown in Figure Q.4. The impulse response of the filter is given by $h(t) = 4\ \text{tri}\left(\frac{t-2}{2}\right)$.

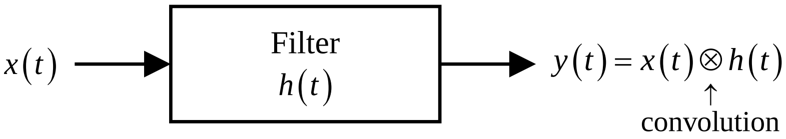

Figure Q.4

(a) Find the spectrum, $Y(f)$, of $y(t)$. Hence, determine the 1st-null bandwidth of $y(t)$.

(5 marks)

(b) Find the maximum value of $y(t)$ and the time at which this maximum occurs.

(5 marks)

Q.4 ANSWER

Q.4 ANSWER ~ continued

This page is intentionally left blank to facilitate detachment of the formula sheet for easy reference. Anything written on this page will not be graded.

Fourier Series: $\begin{cases} \boldsymbol{X_k = \frac{1}{T}\int_{\tilde{t}}^{\tilde{t}+T} x(t)\exp(-j2\pi k\, t/T)\,dt} \\ \boldsymbol{x(t) = \sum_{k=-\infty}^{\infty} X_k \exp(j2\pi k\, t/T)} \end{cases}$

Fourier Transform: $\begin{cases} \boldsymbol{X(f) = \int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)\,dt} \\ \boldsymbol{x(t) = \int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)\,df} \end{cases}$

FOURIER TRANSFORMS OF BASIC FUNCTIONS

	$\boldsymbol{x(t)}$	$\boldsymbol{X(f)}$
Constant	K	$K\delta(f)$
Unit Impulse	$\delta(t)$	**1**
Unit Step	$u(t)$	$\frac{1}{2}\left[\delta(f)+\frac{1}{j\pi f}\right]$
Sign (or Signum)	$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
Rectangle	$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}(fT)$
Triangle	$\mathrm{tri}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}^2(fT)$
Sine Cardinal	$\mathrm{sinc}\left(\frac{t}{T}\right)$	$T\,\mathrm{rect}(fT)$
Complex Exponential	$\exp(j2\pi f_o t)$	$\delta(f-f_o)$
Cosine	$\cos(2\pi f_o t)$	$\frac{1}{2}[\delta(f-f_o)+\delta(f+f_o)]$
Sine	$\sin(2\pi f_o t)$	$-\frac{j}{2}[\delta(f-f_o)-\delta(f+f_o)]$
Gaussian	$\exp\left(-\frac{t^2}{\alpha^2}\right)$	$\alpha\pi^{0.5}\exp(-\alpha^2\pi^2 f^2)$
Comb	$\sum_{m=-\infty}^{\infty}\delta(t-mT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$

FOURIER TRANSFORM PROPERTIES

	Time-domain	**Frequency-domain**
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(f)+\beta X_2(f)$
Time scaling	$x(\beta t)$	$\frac{1}{\lvert\beta\rvert}X\left(\frac{f}{\beta}\right)$
Duality	$X(t)$	$x(-f)$
Time shifting	$x(t-t_o)$	$X(f)\exp(-j2\pi f t_o)$
Frequency shifting (Modulation)	$x(t)\exp(j2\pi f_o t)$	$X(f-f_o)$
Differentiation in the time-domain	$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n X(f)$
Multiplication in the time-domain	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty}X_1(\zeta)X_2(f-\zeta)\,d\zeta$ or $X_1(f)\ast X_2(f)$
Convolution in the time-domain	$\int_{-\infty}^{\infty}x_1(\zeta)x_2(t-\zeta)\,d\zeta$ or $x_1(t)\ast x_2(t)$	$X_1(f)X_2(f)$
Integration in the time-domain	$\int_{-\infty}^{t}x(\tau)\,d\tau$	$\frac{1}{j2\pi f}X(f)+\frac{1}{2}X(0)\delta(f)$
		$\frac{1}{j2\pi f}X(f)$ **if** $X(0)=0$

Trigonometric Identities	
$\exp(\pm j\theta) = \cos(\theta) \pm j\sin(\theta)$	$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$
$\cos(\theta) = 0.5\left[\exp(j\theta) + \exp(-j\theta)\right]$	$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
$\sin(\theta) = -0.5j\left[\exp(j\theta) - \exp(-j\theta)\right]$	$\tan(\alpha \pm \beta) = \dfrac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$
$\sin^2(\theta) + \cos^2(\theta) = 1$	
$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$	$\sin(\alpha)\sin(\beta) = 0.5\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$
$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$	$\cos(\alpha)\cos(\beta) = 0.5\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$
$\sin^2(\theta) = 0.5\left[1 - \cos(2\theta)\right]$	$\sin(\alpha)\cos(\beta) = 0.5\left[\sin(\alpha-\beta) + \sin(\alpha+\beta)\right]$
$\cos^2(\theta) = 0.5\left[1 + \cos(2\theta)\right]$	$C\cos(\theta) - S\sin(\theta) = \sqrt{C^2 + S^2}\cos\left[\theta + \tan^{-1}(S/C)\right]$

Complex Unit (j) ➔ $\left(j = \sqrt{-1} = e^{j\pi/2} = e^{j90^\circ}\right)$ $\left(-j = \dfrac{1}{j} = e^{-j\pi/2} = e^{-j90^\circ}\right)$ $\left(j^2 = -1\right)$

Definitions of Basic Functions

Rectangle:

$$\mathrm{rect}\left(\frac{t}{T}\right) = \begin{cases} 1; & -T/2 \le t < T/2 \\ 0; & \text{elsewhere} \end{cases}$$

Triangle:

$$\mathrm{tri}\left(\frac{t}{T}\right) = \begin{cases} 1 - |t|/T; & |t| \le T \\ 0; & |t| > T \end{cases}$$

Sine Cardinal:

$$\mathrm{sinc}\left(\frac{t}{T}\right) = \begin{cases} \dfrac{\sin(\pi t/T)}{\pi t/T}; & t \ne 0 \\ 1; & t = 0 \end{cases}$$

Signum:

$$\mathrm{sgn}(t) = \begin{cases} 1; & t \ge 0 \\ -1; & t < 0 \end{cases}$$

Unit Impulse:

$$\delta(t) = \begin{cases} \infty\ ; & t = 0 \\ 0\ ; & t \ne 0 \end{cases} \qquad \int_{0^-}^{0^+} \delta(t)\,dt = 1$$

Unit Step:

$$u(t) = \begin{cases} 1\ ; & t \ge 0 \\ 0\ ; & t < 0 \end{cases}$$