EE2023/TEE2023/EE2023E TUTORIAL 8 (PROBLEMS)

Section I : Exercises that are straightforward applications of the concepts covered in class. Please attempt these problems on your own.

1. Polar coordinates will help us understand complex numbers geometrically. On the one hand, the usual rectangular coordinates x and y specify a complex number $z = x + jy$ by giving the distance x right and the distance y up from the origin 0. On the other hand, polar coordinates specify the same point z by saying how far r away from the origin, and the angle θ for the line from the origin to the point. Represent each of the following complex numbers in polar form and plot the point on the complex plane :

   (a) $1 + j$ (b) $-2 + 2j$ (c) $-3 -4j$

   ANSWER : a) $\sqrt{2}\angle 45^{\circ} = \sqrt{2}e^{j\pi/4}$
   b) $\sqrt{8}\angle 135^{\circ} = \sqrt{8}e^{j3\pi/4}$
   c) $5\angle(-126.9^{\circ} = 5e^{-j2.21}$

2. Consider the first order system $G(s) = \dfrac{2}{0.2s+1}$.

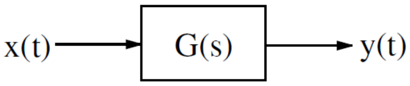

Figure 1: Open loop system, $G(s)$

   Suppose that the input is a sinusoidal signal $x(t) = \sin(3t)$ (See Figure 1).
   (a) Find the output of the system
   (b) Identify the steady-state response.
   (c) Show that the amplitude ratio and phase shift of the steady-state response are equal to values given by $|G(j\omega)|$ and $\angle G(j\omega)$ where ω is the frequency of the sinusoidal input.

   ANSWER : $y_{ss}(t) = 1.71\sin(3t - 0.54)$

3. The steady-state output of a first order system, $G(s)$, is $4.5\sin\left(5t - 38^{\circ}\right)$. Assuming that $|G(5j)| = 0.75$ and $\angle G(5j) = -68^{\circ}$, identify the function(s) that may be the input signal.

   ANSWER : $6\sin\left(5t + \dfrac{\pi}{6} \pm 2n\pi\right) = 6\cos\left(5t - \dfrac{\pi}{3} \pm 2n\pi\right)$
   Since $\cos(\omega t - \pi/2) = \sin(\omega t)$

4. The magnitude response for the system $G(s)$ is shown in Figure 2.

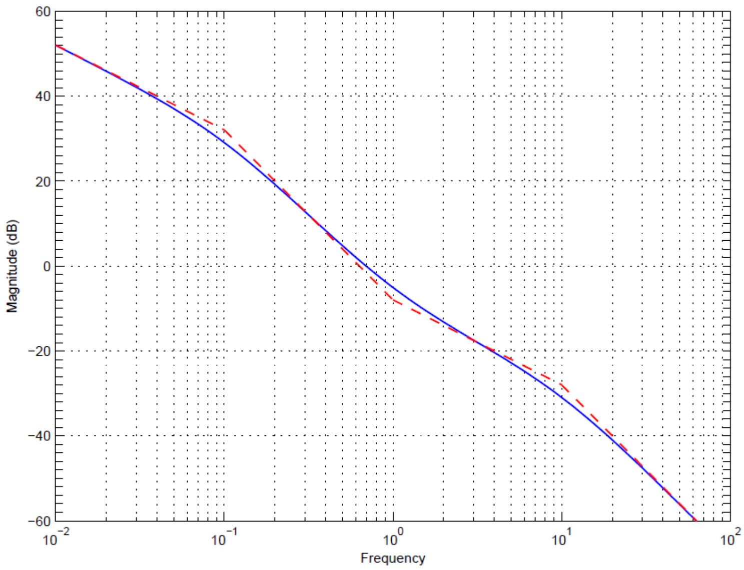

Figure 2: Magnitude plot for $G(s)$

(a) What is the slope of the high frequency asymptote?

ANSWER : -40 dB/decade

(b) $G(s)$ has how many pole(s) , zeros and integrators?

ANSWER : 3 poles, 1 zero and 1 integrator

(c) The low frequency asymptote of the magnitude response is $\dfrac{K}{s^N}$. Find the value of K.

ANSWER : $K = 4$

Section II : Problems that will be discussed in class.

1. A car suspension system and a very simplified version of the system are shown in Figure 3(a) and 3(b) respectively.

   The transfer function of the simplified car suspension system is

$$\frac{X_o(s)}{X_i(s)} = \frac{bs+k}{ms^2+bs+k}$$

   Suppose a car (m = 1 kg, k = 1 N/m and $b=\sqrt{2}$ N/ms^{-1}) is travelling on a road that has speed reducing stripes and the input to the simplified car suspension system, x_i, may be modelled by the periodic square wave of frequency ω = 1 rad/s, shown in Figure 4.

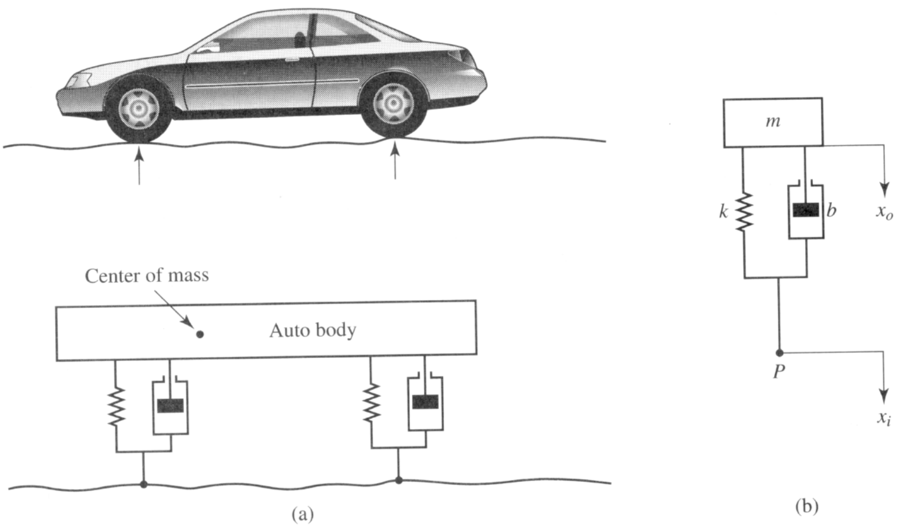

Figure 3: (a) Automobile suspension system, (b) Simplified suspension system

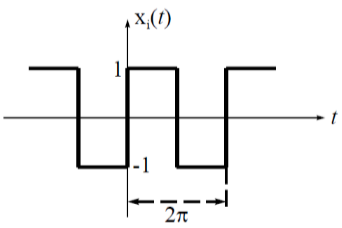

Figure 4: Input waveform, $x_i(t)$

   Determine the steady-state displacement of the car body, $x_{o,ss}(t)$.

   Hint : The Fourier Series representation of the periodic square wave shown in Figure 4 is

$$x_i(t) = \frac{4}{\pi}\left[\sin(t) + \frac{1}{3}\sin(3t) + \frac{1}{5}\sin(5t) + \ldots\right]$$

   ANSWER :

$$x_i(t) = \frac{4}{\pi}\left[1.2247\sin(t-0.6155) + 0.1605\sin(3t-1.3147) + 0.05708\sin(5t-1.4248) + \ldots\right]$$

2. A high speed recorder monitors the temperature of an air stream as sensed by a thermocouple. The following observations were made:
   - The recorded temperature shows an essentially sinusoidal variation after about 1 second.
   - The maximum recorded temperature is about 52°C and the minimum is 48°C at 2 cycles per minute.

   The information indicates that the recorded steady-state temperature may be expressed as $50+2\sin(4\pi t)$. If the system (thermocouple and high speed recorder) has unity steady-state gain and first order dynamics with a time constant of approximately 1 minute under these conditions, estimate the actual maximum and minimum air temperatures.

   ANSWER : Maximum = 75.2°C and Minimum = 24.8°C

3. Figure 5 shows the magnitude plot of $G(s)=\dfrac{A(s+\alpha)}{(s+\beta)(S+\gamma)(s+\lambda)}$.

   (a) Using the approximate (straight line asymptotes) magnitude response, determine A, α, β, γ and λ.

   ANSWER : A = 5000, α = 4, β = 10, γ = λ = 20

(b) Write down the transfer function of another system that may have the magnitude response shown in Figure 5.

ANSWER : $\dfrac{5000(s\pm 4)}{(s\pm 10)(s\pm 20)^2}$; $\dfrac{5000(s\pm 4)e^{-sL}}{(s\pm 10)(s\pm 20)^2}$

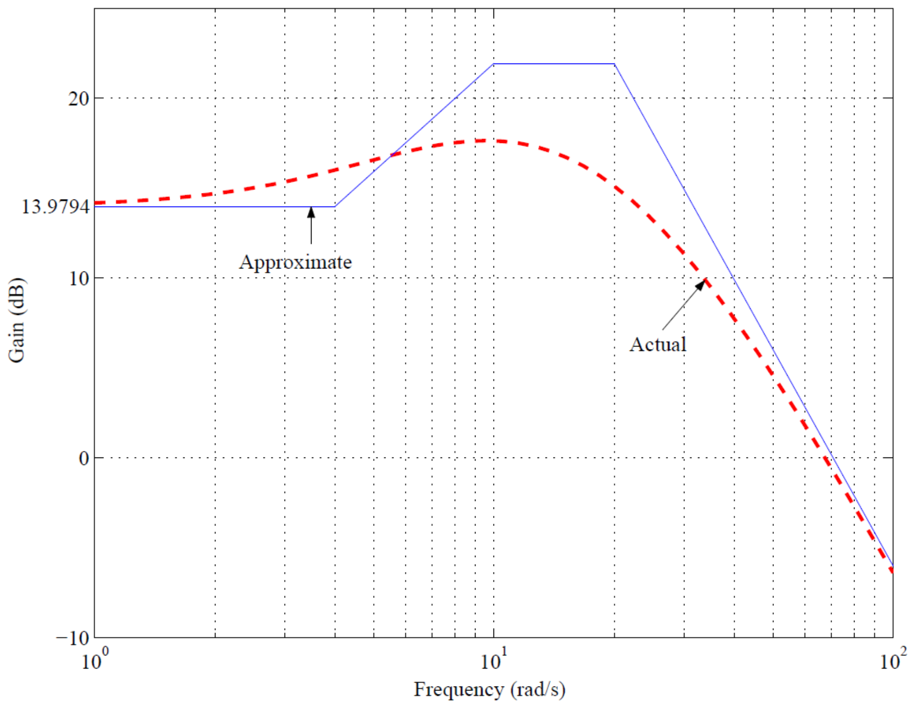

Figure 5: Magnitude response of $G(s)=\dfrac{A(s+\alpha)}{(s+\beta)(s+\gamma)(s+\lambda)}$

Section III : Practice Problems. These problems will not be discussed in class.

1. Find the steady-state current owing through the capacitor ($\lim_{t\to\infty} i_C(t)$), inductor ($\lim_{t\to\infty} i_L(t)$) and resistor ($\lim_{t\to\infty} i_R(t)$) in the circuit shown in Figure 6.

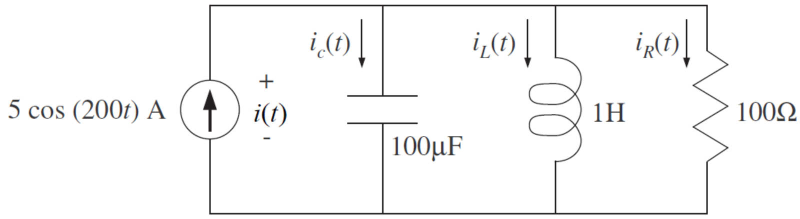

Figure 6: Parallel RLC circuit

ANSWER : $\lim_{t\to\infty} x_C(t) = \frac{20}{\sqrt{13}}\cos\left(200t + 33.7^\circ\right)$

$$\lim_{t\to\infty} x_L(t) = \frac{5}{\sqrt{13}}\cos\left(200t - 146.3^\circ\right)$$

$$\lim_{t\to\infty} x_R(t) = \frac{10}{\sqrt{13}}\cos\left(200t - 56.3^\circ\right)$$

2. Figure 7 shows the Bode diagram of a system whose transfer function is

$$G(s) = \frac{A(s+a)}{(s+b)\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}$$

What are the values of A, a, b, ς and ω_n ?

ANSWER : $A = 12$, $a = 30$, $b = 9$, $\varsigma = 0.25$, $\omega_n = 2$

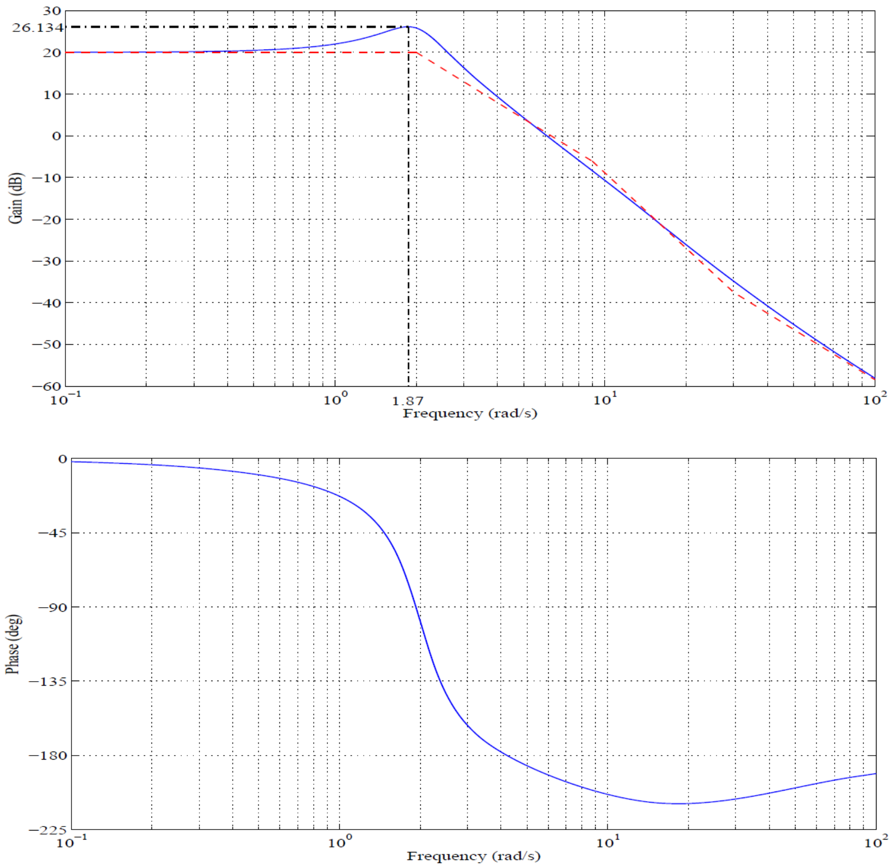

Figure 7: Bode diagram