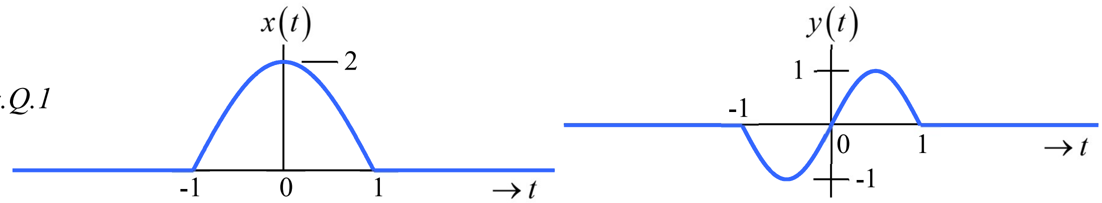
EE2023/TEE2023 TUTORIAL 3 (PROBLEMS)

Q.1 A half-cosine pulse $x(t)$ and a sine pulse $y(t)$ are shown in Fig.Q.1.

Fig.Q.1



(a) Derive the spectrum of $x(t)$ using the forward Fourier transform equation and show how the derivation can be simplified by applying relevant Fourier transform properties.

Ans: $X(f)=\dfrac{2\cos(2\pi f)}{\pi\left(0.25-4f^2\right)}$, Hint: $x(t)=2\cos(0.5\pi t)\cdot \text{rect}(0.5t)$

(b) Using the results of Part-(a), determine the spectrum of $y(t)$.

Ans: $Y(f)=\dfrac{1}{j2}\left[\dfrac{\sin(2\pi f)}{\pi\left(0.25-f^2\right)}\right]$

Q.2 The energy spectral density of a signal $x(t)$ is given by :

$$E_x(f) = 16\, e^{-2|f|} \text{ Joules/Hz.}$$

(a) Find the 3dB bandwidth of $x(t)$.

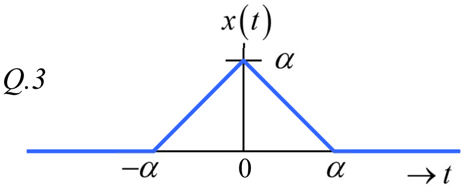
Ans : 3 dB bandwidth $= \frac{1}{2}\ln 2$ *Hz*

(b) Find $X(f)$ if the phase spectrum of $x(t)$ is given by $\angle X(f) = -0.5f$.

Ans: $X(f) = 4e^{-|f|}\, e^{-j0.5f}$

Q.3 Fig.Q.3 shows the plot of a triangular pulse $x(t)$.

Fig.Q.3



Determine the magnitude and phase spectra of $x(t)$. Hence, or otherwise, find the energy spectral density and total energy of $\frac{dx(t)}{dt}$.

$$\text{Ans: } \begin{cases} \text{Magnitude spectrum: } |X(f)| = \alpha^2 \text{sinc}^2(\alpha f), \ \text{Phase spectrum: } \angle X(f) = 0 \\ E_y(f) = 4\pi^2 f^2 \alpha^4 \text{sinc}^4(\alpha f), \ E = 2\alpha \end{cases}$$

Q.4 The spectrum of a lowpass energy signal $x(t)$ is given by $X(f) = \exp(-\alpha|f|)$ where α is a positive constant.

(a) The 99% energy containment bandwidth of a signal is defined as the smallest bandwidth that contains at least 99% of the total signal energy. Find the 99% energy containment bandwidth of $x(t)$?

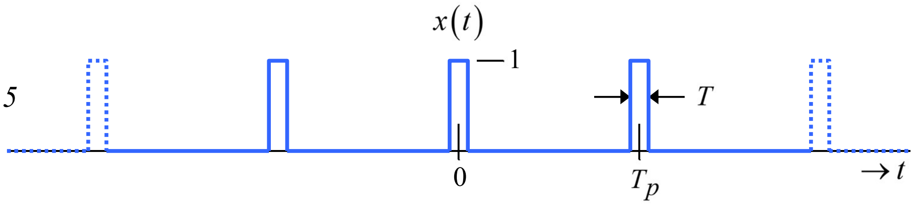
Ans: $\frac{1}{\alpha}\ln(10)$ Hz

(b) Find the 3dB bandwidth of $x(t)$. How many percent of the total energy of $x(t)$ does its 3dB bandwidth contain?

Ans: $\frac{1}{2\alpha}\ln(2)$ Hz, 50%

Q.5 A periodic pulse train $x(t)$ is shown in Fig.Q.5.

Fig.Q.5



(a) Derive the power spectral density, $P_x(f)$, of $x(t)$.

Ans: $X(f) = \sum_{k=-\infty}^{\infty} \left(\frac{T}{T_p}\right)^2 \text{sinc}^2\left(k\frac{T}{T_p}\right)\delta\left(f - \frac{k}{T_p}\right)$

(b) What is the average power of $x(t)$?

Ans: Average Power of $x(t) = \frac{T}{T_p}$

(c) The 99% power containment bandwidth of a power signal is defined as the smallest bandwidth that contains at least 99% of the average signal power. Provide a formula for computing the 99% power containment bandwidth of $x(t)$?

Supplementary Problems

These problems will not be discussed in class.

S.1 Find the Fourier transform of each of the following signals:

(a) $x(t)=\cos(2\pi f_c t)u(t)$ *(b)* $x(t)=\sin(2\pi f_c t)u(t)$

(c) $x(t)=\exp(-\alpha t)\cos(\omega_c t)u(t);\ \ \alpha>0$ *(d)* $x(t)=\exp(-\alpha t)\sin(\omega_c t)u(t);\ \ \alpha>0$

Answer: *(a)* $X(f)=\frac{1}{4}\left[\delta(f-f_c)+\delta(f+f_c)\right]+\dfrac{jf}{2\pi\left(f_c^2-f^2\right)}$

(b) $X(f)=\frac{j}{4}\left[\delta(f+f_c)-\delta(f-f_c)\right]+\dfrac{f_c}{2\pi\left(f_c^2-f^2\right)}$

(c) $X(\omega)=\dfrac{\alpha+j\omega}{(\alpha+j\omega)^2+\omega_c^2}$

(d) $X(\omega)=\dfrac{\omega_c}{(\alpha+j\omega)^2+\omega_c^2}$

S.2 Given: $\Im\{\exp(-\alpha t)u(t)\}=\dfrac{1}{\alpha+j2\pi f}$. Find the inverse Fourier transform of $\dfrac{1}{(\alpha+j2\pi f)^n}$.

Answer: $\dfrac{t^{n-1}}{(n-1)!}\exp(-\alpha t)u(t)$

S.3 Given: $\Im\{\exp(-\alpha t)u(t)\}=\dfrac{1}{j\omega+\alpha}$. Find the inverse Fourier transform of $\dfrac{1}{2-\omega^2+j3\omega}$.

Answer: $\left[\exp(-t)-\exp(-2t)\right]u(t)$

S.4 Given: $\Im\{x(t)\}=\mathrm{rect}(\pi f)$. Find the value of $\int_{-\infty}^{\infty}|y(t)|^2\,dt$ if $y(t)=\dfrac{dx(t)}{dt}$.

Answer: $\dfrac{1}{3\pi}$

S.5 Given: $\Im\left\{\dfrac{\pi}{\alpha}\exp(-2\pi\alpha|t|)\right\}=\dfrac{1}{\alpha^2+f^2}$. Determine the 99% energy containment bandwidth for the signal $x(t)=\dfrac{1}{\alpha^2+t^2}$.

Answer: $\dfrac{0.366}{\alpha}$

S.6 Let $\omega=2\pi f$. Using the fact that the Fourier transform is a <u>one-to-one</u> linear transformation, show that $\delta(f)=2\pi\delta(\omega)$.

Hint : Show that $\Im^{-1}\{\delta(f)\}=\Im^{-1}\{2\pi\delta(\omega)\}=1$

Below is a list of solved problems selected from ***Chapter 5*** *of* ***Hwei Hsu (PhD), 'The Schaum's series on Signals & Systems,' 2nd Edition****.*

Selected solved-problems: 5.19-to-5.27, 5.32, 5.34, 5.40, 5.42, 5.42, 5.57

These solved problems should be treated as supplementary module material catered for students who find the need for more examples or practice-problems.