EE2023 TUTORIAL 2 (PROBLEMS)

Q.1 The discrete-frequency spectrum of a signal $x(t)$ is shown in Fig.Q.1.

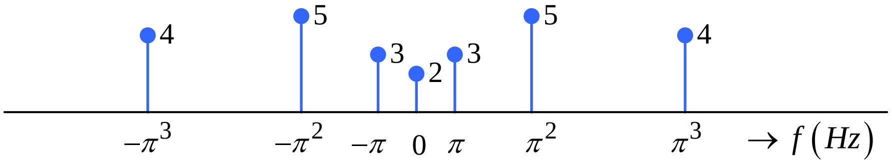

Fig.Q.1

(a) What is the dc value of $x(t)$?

(b) Is $x(t)$ a power or energy signal?

(c) What is the Fourier series expansion of $x(t)$?

Q.2 Determine whether or not each of the following signals is periodic. If the signal is periodic, determine its fundamental frequency and period.

(a) $x(t) = \cos(3.2t) + \sin(1.6t) + \exp(j2.8t)$

(b) $x(t) = \cos(4t) + \sin(\pi t)$

Q.3 (a) Determine the Fourier series coefficients of $x(t) = 6\sin(12\pi t) + 4\exp\left(j(8\pi t + \pi/4)\right) + 2$

(b) Find the frequency of the 4$^{\text{th}}$ harmonic of $x(t) = 0.5\left(\left|\sin(\pi t)\right| + \sin(\pi t)\right)$

Q.4 Determine the Fourier series coefficients of

$$x(t) = \sum_{n=-\infty}^{\infty} 2p(t - 1.6n)$$

where $p(t)$ is given in Fig.Q.4.

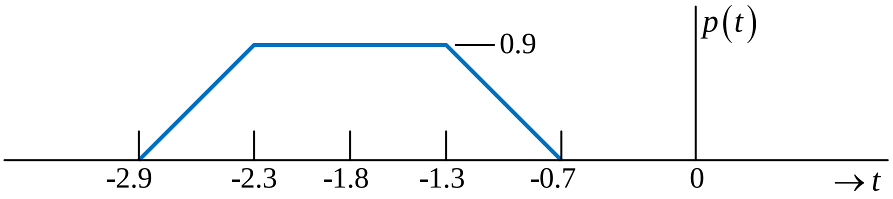

Fig.Q.4

Q.5 Consider the signal $x(t) = \cos(3\pi t)$ and define

$$y(t) = \sum_{k=-\infty}^{\infty} c_k \exp(j2\pi kt)$$

where $c_k = \int_{-0.5}^{0.5} x(t)\exp(-j2\pi kt)\,dt$. Sketch $x(t)$ and $y(t)$. Show all the important dimensions in your sketches

Supplementary Problems

These problems will not be discussed in class.

S.1 Consider a rectified sine wave signal $x(t)$ defined by

$$x(t) = |\sin(\pi t)|.$$

(a) Sketch $x(t)$ and find its fundamental period.

(b) Find the complex exponential Fourier series of $x(t)$.

(c) Find the trigonometric Fourier series of $x(t)$.

Answer: (a) $period = 1$ (b) $x(t) = -\frac{2}{\pi}\sum_{k=-\infty}^{\infty}\frac{1}{4k^2-1}\exp(j2\pi kt)$

(c) $x(t) = \frac{2}{\pi} - \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{4k^2-1}\cos(2\pi kt)$

S.2 Find the complex exponential Fourier series of a periodic signal $x(t)$ defined by

$$x(t) = t^2; \quad -\pi < t < \pi \qquad \text{and} \qquad x(t+2\pi) = x(t).$$

Answer: $x(t) = \frac{\pi^2}{3} + 4\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}\cos(kt)$

S.3 The harmonic form Fourier series of a real periodic signal $x(t)$ with fundamental period T_0 is given by

$$x(t) = h_0 + \sum_{k=1}^{\infty} h_k \cos\left(2\pi\frac{k}{T_0}t - \theta_k\right)$$

where h_0 is known as the dc component, and the term $h_k \cos\left(2\pi\frac{k}{T_0}t - \theta_k\right)$ is referred to as the kth-harmonic component of $x(t)$. Express h_0, h_k and θ_k in terms of the complex exponential Fourier series coefficients c_k of $x(t)$.

Answer: $h_0 = c_0$, $h_k = 2|c_k|$, $\theta_k = -\tan^{-1}\left(\frac{\operatorname{Im}[c_k]}{\operatorname{Re}[c_k]}\right)$

Below is a list of solved problems selected from ***Chapter 5*** *of* ***Hwei Hsu (PhD), 'The Schaum's series on Signals & Systems,' 2nd Edition****.*

Selected solved-problems: 5.4-to-5.13

These solved problems should be treated as supplementary module material catered for students who find the need for more examples or practice-problems.
