EE2023 Signals and Systems Mid-term Quiz – AY2018/2019 Semester 2

Q1(a) $X(f) = 16\,\text{sinc}^2(4f) - 4\,\text{sinc}^2(2f) - \text{sinc}^2(f)$

Q1(b) $x_p(t) = x(t) \otimes \sum_{k=-\infty}^{\infty} \delta(t - 10k)$

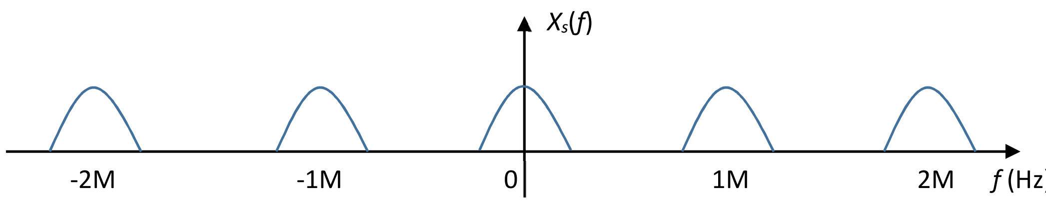
Q1(c) $X_p(f) = \sum_{k=-\infty}^{\infty} \frac{1}{10}\left[16\,\text{sinc}^2\left(\frac{2k}{5}\right) - 4\,\text{sinc}^2\left(\frac{k}{5}\right) - \text{sinc}^2\left(\frac{k}{10}\right)\right]\delta\left(f - \frac{k}{10}\right)$

Q1(d) $X_k = \frac{1}{10}\left[16\,\text{sinc}^2\left(\frac{2k}{5}\right) - 4\,\text{sinc}^2\left(\frac{k}{5}\right) - \text{sinc}^2\left(\frac{k}{10}\right)\right]$

Q2(a) The Nyquist frequency is 2,020,000 Hz.

Q2(b) $X_s(f) = \frac{f_s}{2} \sum_{k=-\infty}^{\infty} \left[X(f - kf_s - 1000000) + X(f - kf_s + 1000000)\right]$

Q2(c)



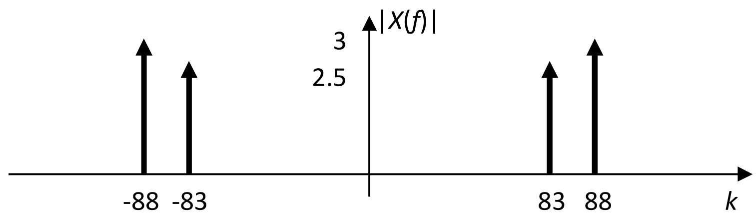
Q2(d) The original signal $x_m(t)$ can be recovered using a bandpass filter with a centre frequency of 1 MHz and a bandwidth of 20 kHz.

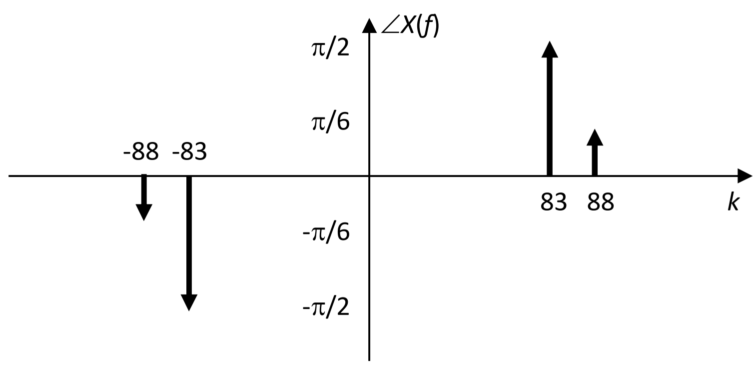
Q3(a) The fundamental frequency is 5 Hz and period is 0.2 seconds.

Q3(b)

$$x(t) = 3e^{-j\left(880\pi t+\frac{\pi}{6}\right)} - 2.5\,je^{-j(830\pi t)} + 2.5\,je^{j830\pi t} + 3e^{j\left(880\pi t+\frac{\pi}{6}\right)}$$

$$= 3e^{-j\frac{\pi}{6}}e^{-j880\pi t} + 2.5e^{-j\frac{\pi}{2}}e^{-j(830\pi t)} + 2.5\,je^{j830\pi t} + 3e^{j\frac{\pi}{6}}e^{j880\pi t}$$





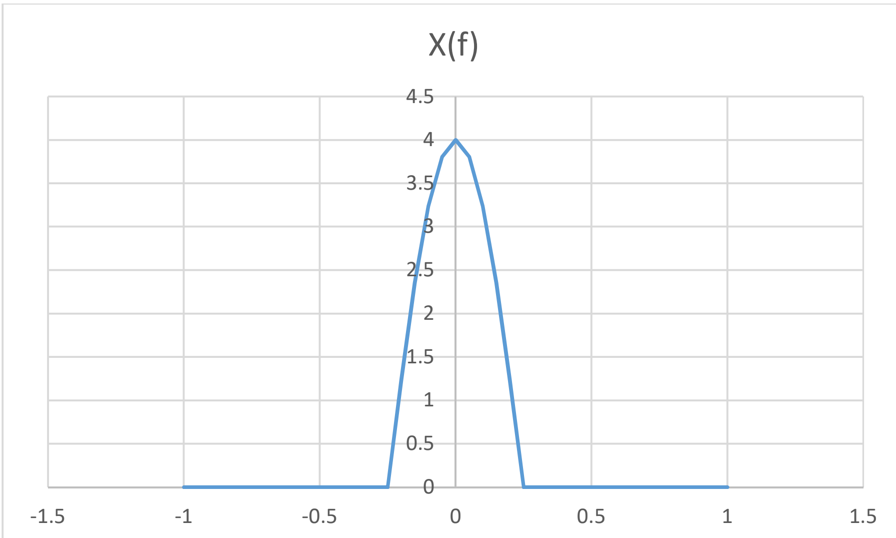
Q3(c) The average power is: $P = 3^2 + 2.5^2 + 2.5^2 + 3^2 = 30.5$

Q3(d) The frequency component that is not in tune is 415 Hz.

Q4(a)

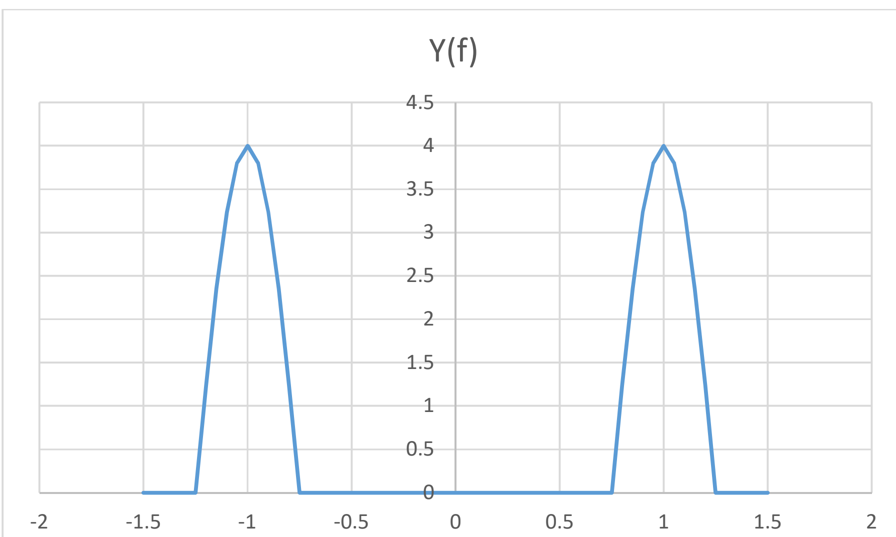
$$X(f) = 4\cos(2\pi f)\,\mathrm{rect}\left(\frac{f}{0.5}\right)$$

$$x(t) = 4.\frac{1}{2}\left[\delta(t-1) + \delta(t+1)\right] \otimes \frac{1}{2}\mathrm{sinc}\left(\frac{t}{2}\right) = \mathrm{sinc}\left(\frac{t-1}{2}\right) + \mathrm{sinc}\left(\frac{t+1}{2}\right)$$



Q4(b)i

$$Y(f) = X(f - f_0) + X(f + f_0)$$

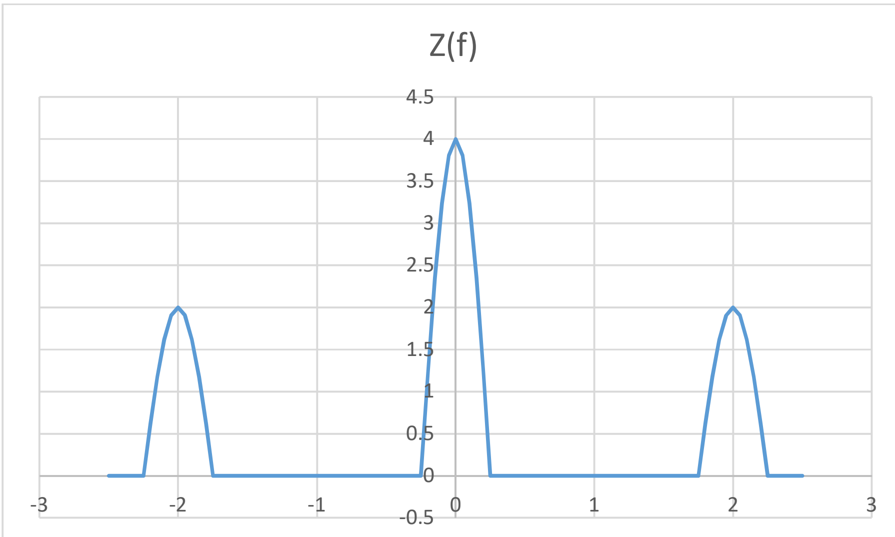


Q4(b)ii $C(f)=\frac{1}{2}\left[\delta\left(f-f_c\right)+\delta\left(f+f_c\right)\right]$

Q(b)iii

$Z(f)=Y(f)\otimes C(f)$

$z(t)=y(t).c(t)$



Hence the low pass filter will require f_1 to be 0.25 Hz.