EE2023 Signals & Systems Quiz
Semester 2 AY2018/19

Date : 7 March 2019 **Time Allowed : 1.5 hours**

Instructions :

1. Answer all 4 questions. Each question carries 10 marks.
2. This is a closed book quiz. However, you are allowed to bring a help sheet comprising one single sheet of paper of A4 size.
3. Tables of formulas are given on Pages 15 and 16.
4. Programmable and/or graphic calculators are not allowed.
5. Write your answers in the spaces indicated in this question paper. Attachment is not allowed.
6. Write your name, matric number and lecture group in the spaces indicated below.

Name : ________________________________

Matric # : ______________________________

Group : ________________________________

Question #	**Marks**
1	
2	
3	
4	
Total Marks	

Q1. The signal $x(t)$ is shown in Figure Q1(a).

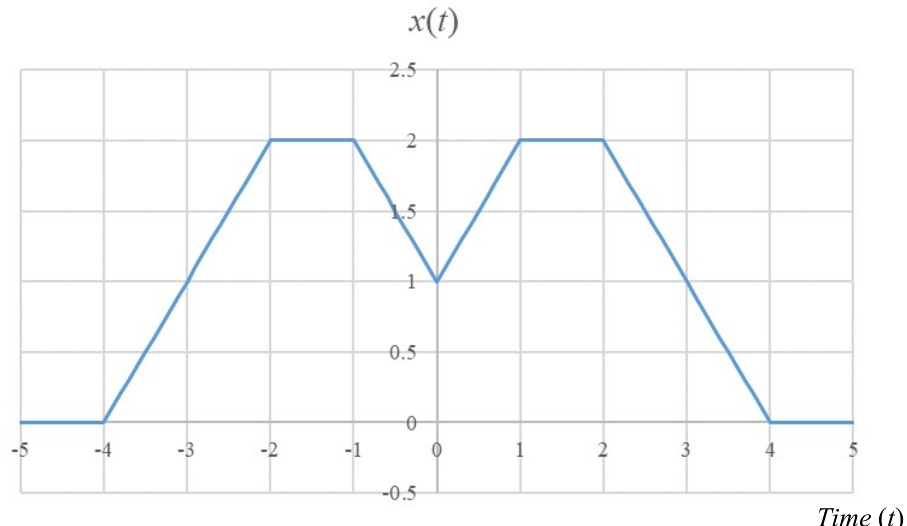

Figure Q1(a)

(a) Determine the Fourier transform, $X(f)$, of $x(t)$.

(4 marks)

(b) The periodic signal, $x_p(t)$, can be obtained by replicating $x(t)$ at a period of 10 seconds as shown in Figure Q1(b). Obtain an expression for $x_p(t)$ in terms of $x(t)$ and the Dirac δ-function.

(1 marks)

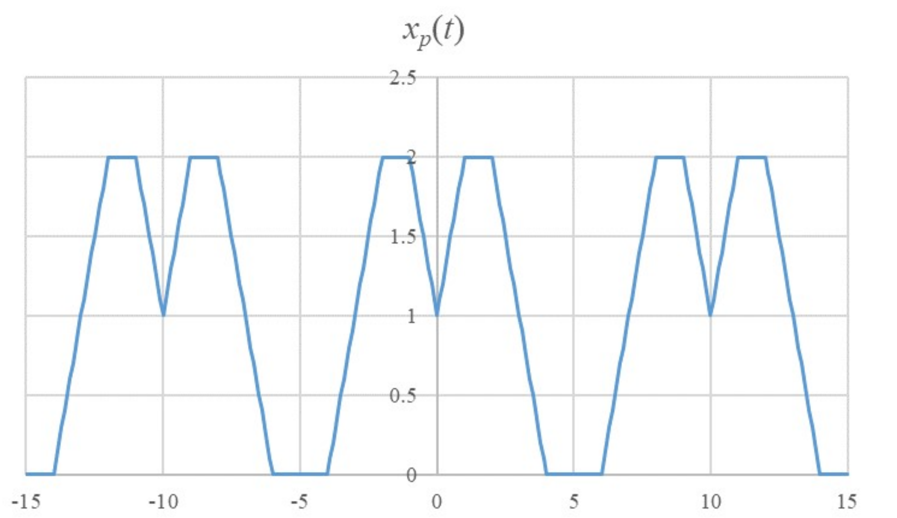

Figure Q1(b)

(c) Determine the Fourier transform, $X_p(f)$, of the periodic signal $x_p(t)$.

(4 marks)

(d) Determine the Fourier series coefficients, $X_{p,k}$, of the periodic signal $x_p(t)$.

(1 marks)

Q1. ANSWER

Q1. ANSWER ~ continued

Q2. A lowpass audio signal, $x(t)$, with a bandwidth of 10 kHz, is modulated to obtain the signal $x_m(t) = x(t)\cos\left(2000000\pi t\right)$. This modulated signal, $x_m(t)$, is then sampled at a frequency of f_s Hz to give the sampled signal $x_s(t)$.

(a) What is the Nyquist frequency?

(1 mark)

(b) Determine the Fourier transform of the sampled signal, $X_s(f)$.

(4 marks)

(c) If the sampling frequency is 1 MHz

i. Sketch the magnitude spectrum of the sampled signal $\left|X_s(f)\right|$.

(3 marks)

ii. Can the signal $x_m(t)$ be recovered from the sampled signal $x_s(t)$, and if so explain how.

(2 marks)

Q2. ANSWER

Q2. ANSWER ~ continued

Q3. The Fourier Series expansion of a signal, $x(t)$, that is produced when two keys on a piano are pressed simultaneously is

$$x(t) = 3e^{-j\left(880\pi t+\frac{\pi}{6}\right)} - 2.5je^{-j(830\pi t)} + 2.5je^{j830\pi t} + 3e^{j\left(880\pi t+\frac{\pi}{6}\right)}.$$

(a) What is the fundamental period of $x(t)$?

(2 marks)

(b) Draw the magnitude and phase spectrum of $x(t)$. Use the Fourier Series index to label the x-axis of the graphs.

(5 marks)

(c) Determine the average power of $x(t)$.

(2 marks)

(d) The fundamental frequencies in hertz of notes generated by a theoretically ideal piano obey the following equation

$$440 \times 2^{\frac{n}{12}}$$

where n is an integer. Which frequency component in $x(t)$ is not in tune?

(1 mark)

Q3. ANSWER

Q3. ANSWER ~ continued

Q4. Consider the signal, $x(t)$, with its Fourier Transform given by :

$$X(f)=\begin{cases} 4\cos 2\pi f & -0.25 \le f \le 0.25 \\ 0 & \text{elsewhere} \end{cases} \qquad (4.1)$$

(a) Sketch $X(f)$ and find $x(t)$. (*Hint: Write* $X(f)$ *in terms of an appropriate* rect(.) *function.*)

(5 marks)

(b) Suppose $Y(f)$ is another signal given by :

$$Y(f)=X(f-f_0)+X(f+f_0).$$

$X(f)$ is given in eqn. (4.1). $Y(f)$ is then convolved with a carrier signal $c(t)=\cos 2\pi f_c t$ and subsequently low pass filtered (LPF) to output $\tilde{z}(t)$, as shown in Figure Q4-1 below.

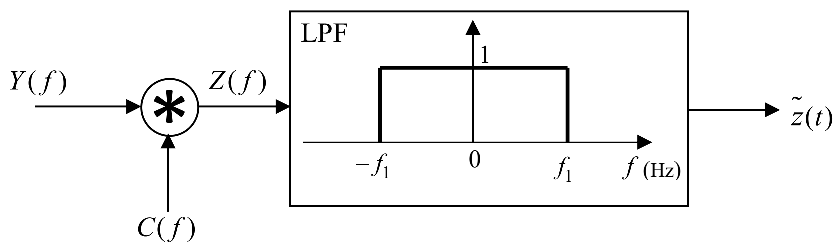

Figure Q4-1 : Signal Processing with an ideal LPF.

$Z(f)=Y(f)*C(f)$, where * represents convolution.

i. Sketch $Y(f)$ for $f_0=1$ Hz.

(1 mark)

ii. Find $C(f)$.

(1 mark)

iii. Sketch $Z(f)$ for $f_c=f_0=1$ Hz. Hence determine a suitable f_1 for which $\tilde{z}(t)=x(t)$.

(3 marks)

Q4. ANSWER

Q4. ANSWER ~ continued

This page is intentionally left blank to facilitate detachment of the formula sheet for easy reference. Anything written on this page will not be graded.

Fourier Series:
$$\begin{cases} X_k = \dfrac{1}{T}\displaystyle\int_{\tilde{t}}^{\tilde{t}+T} x(t)\exp(-j2\pi k\,t/T)\,dt \\ x(t) = \displaystyle\sum_{k=-\infty}^{\infty} X_k \exp(j2\pi k\,t/T) \end{cases}$$

Fourier Transform:
$$\begin{cases} X(f) = \displaystyle\int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)\,dt \\ x(t) = \displaystyle\int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)\,df \end{cases}$$

FOURIER TRANSFORMS OF BASIC FUNCTIONS

	$x(t)$	$X(f)$
Constant	K	$K\delta(f)$
Unit Impulse	$\delta(t)$	**1**
Unit Step	$u(t)$	$\frac{1}{2}\left[\delta(f)+\frac{1}{j\pi f}\right]$
Sign (or Signum)	$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
Rectangle	$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}(fT)$
Triangle	$\mathrm{tri}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}^2(fT)$
Sine Cardinal	$\mathrm{sinc}\left(\frac{t}{T}\right)$	$T\,\mathrm{rect}(fT)$
Complex Exponential	$\exp(j2\pi f_o t)$	$\delta(f-f_o)$
Cosine	$\cos(2\pi f_o t)$	$\frac{1}{2}\left[\delta(f-f_o)+\delta(f+f_o)\right]$
Sine	$\sin(2\pi f_o t)$	$-\frac{j}{2}\left[\delta(f-f_o)-\delta(f+f_o)\right]$
Gaussian	$\exp\left(-\frac{t^2}{\alpha^2}\right)$	$\alpha\pi^{0.5}\exp(-\alpha^2\pi^2 f^2)$
Comb	$\sum_{m=-\infty}^{\infty}\delta(t-mT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$

FOURIER TRANSFORM PROPERTIES

	Time-domain	**Frequency-domain**
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(f)+\beta X_2(f)$
Time scaling	$x(\beta t)$	$\frac{1}{\lvert\beta\rvert}X\left(\frac{f}{\beta}\right)$
Duality	$X(t)$	$x(-f)$
Time shifting	$x(t-t_o)$	$X(f)\exp(-j2\pi f t_o)$
Frequency shifting (Modulation)	$x(t)\exp(j2\pi f_o t)$	$X(f-f_o)$
Differentiation in the time-domain	$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n X(f)$
Multiplication in the time-domain	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty}X_1(\zeta)X_2(f-\zeta)\,d\zeta$ or $X_1(f)\ast X_2(f)$
Convolution in the time-domain	$\int_{-\infty}^{\infty}x_1(\zeta)x_2(t-\zeta)\,d\zeta$ or $x_1(t)\ast x_2(t)$	$X_1(f)X_2(f)$
Integration in the time-domain	$\int_{-\infty}^{t}x(\tau)\,d\tau$	$\frac{1}{j2\pi f}X(f)+\frac{1}{2}X(0)\delta(f)$
		$\frac{1}{j2\pi f}X(f)$ **if** $X(0)=0$

Trigonometric Identities

$\exp(\pm j\theta) = \cos(\theta) \pm j\sin(\theta)$	$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$
$\cos(\theta) = 0.5\left[\exp(j\theta) + \exp(-j\theta)\right]$	$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
$\sin(\theta) = -0.5j\left[\exp(j\theta) - \exp(-j\theta)\right]$	$\tan(\alpha \pm \beta) = \dfrac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$
$\sin^2(\theta) + \cos^2(\theta) = 1$	
$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$	$\sin(\alpha)\sin(\beta) = 0.5\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$
$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$	$\cos(\alpha)\cos(\beta) = 0.5\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$
$\sin^2(\theta) = 0.5\left[1 - \cos(2\theta)\right]$	$\sin(\alpha)\cos(\beta) = 0.5\left[\sin(\alpha - \beta) + \sin(\alpha + \beta)\right]$
$\cos^2(\theta) = 0.5\left[1 + \cos(2\theta)\right]$	$C\cos(\theta) - S\sin(\theta) = \sqrt{C^2 + S^2}\cos\left[\theta + \tan^{-1}(S/C)\right]$

Complex Unit (j) ➔ $\left(j = \sqrt{-1} = e^{j\pi/2} = e^{j90^\circ}\right)$ $\left(-j = \dfrac{1}{j} = e^{-j\pi/2} = e^{-j90^\circ}\right)$ $\left(j^2 = -1\right)$

Definitions of Basic Functions

Rectangle:

$$\mathrm{rect}\left(\frac{t}{T}\right) = \begin{cases} 1; & -T/2 \le t < T/2 \\ 0; & \text{elsewhere} \end{cases}$$

Triangle:

$$\mathrm{tri}\left(\frac{t}{T}\right) = \begin{cases} 1 - |t|/T; & |t| \le T \\ 0; & |t| > T \end{cases}$$

Sine Cardinal:

$$\mathrm{sinc}\left(\frac{t}{T}\right) = \begin{cases} \dfrac{\sin(\pi t/T)}{\pi t/T}; & t \ne 0 \\ 1; & t = 0 \end{cases}$$

Signum:

$$\mathrm{sgn}(t) = \begin{cases} 1; & t \ge 0 \\ -1; & t < 0 \end{cases}$$

Unit Impulse:

$$\delta(t) = \begin{cases} \infty; & t = 0 \\ 0; & t \ne 0 \end{cases} \qquad \int_{0^-}^{0^+} \delta(t)\,dt = 1$$

Unit Step:

$$u(t) = \begin{cases} 1; & t \ge 0 \\ 0; & t < 0 \end{cases}$$