Q.2 A system is modelled by a steady state or DC gain of one and has zeros and poles given by:

$$z = -2, \; p_1 = +j2, \; p_2 = -j2$$

where z denotes the zero while p_1 and p_2 are the poles.

(a) What is the order of the transfer function model of the system?

(2 marks)

(b) Derive the transfer function of the system.

(5 marks)

(c) What is the damping ratio and natural frequency of the system?

(3 marks)

Q.3 The response of a second order system to the input signal, $2\delta(t)$, is

$$8\, e^{-2t} \sin\left(2\sqrt{3}t\right).$$

(a) Derive the system transfer function.

(5 marks)

(b) Find the system poles.

(2 marks)

(c) What is the damping ratio, undamped natural frequency and DC gain of the system?

(3 marks)

Q4. The input-output relationship of a mass-spring-damper system is governed by the following differential equation:

$$\frac{d^2 y(t)}{dt^2} + 50\frac{dy(t)}{dt} + 400y(t) = 600x(t).$$

(a) Derive the system transfer function, $G(s) = \dfrac{Y(s)}{X(s)}$, where $X(s) = L\{x(t)\}$ and $Y(s) = L\{y(t)\}$ are the Laplace transforms of the input, $x(t)$, and output, $y(t)$, respectively.

(2 marks)

(b) What is the DC gain of the system?

(1 marks)

(c) Is the system underdamped, critically damped or overdamped?

(3 marks)

(d) The input signal, $x(t) = 10\cos(30t)$, is applied to the mass-spring-damper system.

Determine the steady-state output signal, $\lim_{t\to\infty} y(t)$.

Q2. Consider the underdamped 2nd order system with a transfer function, $G(s)$, given by

$$G(s)=\frac{80}{s^2+4s+16}.$$

(a) Determine the damping ratio of the system.

(2 marks)

(b) Suppose a step function of magnitude 7 is applied to the system, what is the steady state value of the output signal? Sketch the response of the system, assuming zero initial conditions. You <u>do not need to derive</u> the response due to the DC signal of 7.

(5 marks)

(b) What is the frequency at which the magnitude of the frequency response of the system is at a maximum? Find this maximum frequency response value.

(3 marks)

Q1. Consider the following differential equation which describes the motion of a vehicle :

$$\frac{d^2y(t)}{dt^2}+5\frac{dy}{dt}+4y(t)=x(t),$$

where $y(t)$ is the vehicle's position from a reference point, and $x(t)$ represents thrust.

(a) Derive the transfer function of this vehicle, relating the thrust to the vehicle's position. Is the system stable? Justify your answer.

(4 marks)

(b) Find the vehicle's steady state (final) position if a thrust of $x(t)=2u(t)$ is applied while the initial conditions of the vehicle are $y(0)=6$ and $\left.\frac{dy(t)}{dt}\right|_{t=0}=0$. You may assume $u(t)$ to be the unit step function.

(3 marks)

(c) Give an alternative method to find the vehicle's final position for the same input in part (b) above.

(3 marks)

Q4. The input-output relationship of a second order system is governed by the following differential equation

$$\frac{d^2y(t)}{dt^2}+6\frac{dy(t)}{dt}+34\,y(t)=40x(t).$$

(a) Derive the system transfer function.

(2 marks)

(b) Compute the unit impulse response of the system.

(4 marks)

(c) Is the system underdamped, critically damped or overdamped? What is the undamped natural frequency and DC gain of the system?

(4 marks)

Q1. (a) When an input $x(t)=t,\ t\geq 0$ is applied to a system with a transfer function, $G(s)$, the resulting output response is given by :

$$y(t)=e^{-t}-\frac{1}{4}e^{-2t}-\frac{3}{4}+\frac{1}{2}t,\ t\geq 0.$$

Determine $G(s)$ of the system. Is the system stable? Justify your answer.

(5 marks)

(b) If a new input, $v(t)=5\sin(10t+0.2),\ t\geq 0$ is applied to the system in (a) above, determine the new output response, $y_1(t)$ at steady state.

(5 marks)

Q3. The transfer function of a linear time-invariant system is given by

$$\tilde{H}(s)=\frac{200}{(s+10)^2}.$$

(a) Find the magnitude response, $\left|\tilde{H}(j\omega)\right|$, and phase response, $\angle\tilde{H}(j\omega)$, of the system.

(5 marks)

(b) Let $x(t)$ and $y(t)$ be the input and output signals of the system, respectively. Find $y(t)$ if $x(t)=20\sin(10t+15°)$.

(5 marks)

Q.6 A thermal cycling chamber is often used to test satellite components to ensure the reliability of the components when they are operating in space. The function of the thermal cycling chamber is to provide heating and cooling cycles to the satellite components. Hence this chamber must be able to respond well to input signals to achieve the desired heating and cooling cycles. The input and output of the chamber are illustrated in Figure Q6 below.

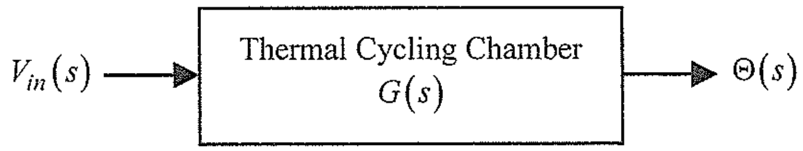

Figure Q6

$V_{in}(s)=\mathcal{L}\{v_{in}(t)\}$ is the input signal to the chamber, in Volts and $\Theta(s)=\mathcal{L}\{\theta(t)\}$ is the measured chamber temperature in degree Celsius.

Suppose the chamber can be modeled by a first order transfer function given by

$$G(s)=\frac{\Theta(s)}{V_{in}(s)}=\frac{4}{3s+2}.$$

(a) What is the steady state gain and time constant of the chamber?

(2 marks)

(b) If the input signal to the chamber is $v_{in}(t)=100u(t)$ Volts, what will be the steady state temperature in the chamber? Sketch the response to this input. You may assume $u(t)$ to be the unit step function and all initial conditions are zero.

(3 marks)

(c) If the input signal is $v_{in}(t)=50\sin(0.1t)\,u(t)$ Volts, what will be the steady state temperature in the chamber? Assume zero initial conditions. Sketch the response.

(4 marks)

(d) The satellite engineer wishes to set the desired chamber temperature between 50 degree Celsius to 150 degree Celsius with a sinusoidal waveform of frequency 2 rad/s.

(i) If the input signal is of the form $v_{in}(t)=\left[x_0+x_1\sin(2t)\right]u(t)$ Volts, what values of x_0 and x_1 should the engineer choose in order to achieve the desired chamber temperature profile?

(3 marks)

(ii) With the values of x_0 and x_1 chosen above, find the steady state temperature of the chamber. What are the maximum and minimum temperatures that the engineer is able to achieve in the chamber? Assume all initial conditions to be zero.

(*Hint : If you are unable to give the right values of x_0 and x_1, you may just assume any value and show how you can solve this part.*)

(5 marks)

(iii) Is the engineer able to achieve his desired temperature profile in the chamber? Why?

(3 marks)

Q1. A rotating load is connected to a DC motor which has a model given by:

$$\frac{\Omega(s)}{V(s)} = \frac{K}{s\tau + 1},$$

where $\Omega(s) = L\{\omega(t)\}$ and $V(s) = L\{v(t)\}$ denote the Laplace transforms of the motor speed, $\omega(t)$ in rad/s and the input voltage, $v(t)$ in volts, respectively.

A test results in the output load reaching a speed of 1 rad/s within 0.5 seconds when a constant input of 100 V is applied to the motor terminals. The steady state output speed from the same test is found to be 2 rad/s.

(a) Determine the parameters, K and τ of the motor model.

(6 marks)

(b) Sketch the output response derived from the test, clearly indicating the quantities obtained from the test result.

(4 marks)

SECTION B : Answer 3 out of the 4 questions in this section

Q5. Consider a control system with a transfer function given by:

$$G(s) = \frac{K_1 s + K_2}{s^2 + 2s(1+K_1) + 2K_2}.$$

The goal is to design the parameters, K_1 and K_2, such that the system achieves certain dynamical characteristics. The strategy to achieve this is to design such that the poles of $G(s)$ lie in the shaded regions in Figure Q5 below.

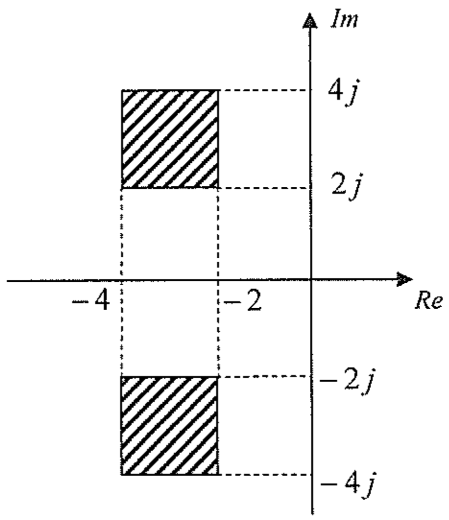

Figure Q5 : Regions where Desired Poles Lie

(a) Given the desired pole locations in Figure Q5, determine the ranges of values of damping ratio, ς, and natural frequency, ω_n, that are achievable by the control system. (4 marks)

(b) Give two possible sets of poles which will achieve a damping ratio of $\varsigma = 0.707$. (4 marks)

(c) Determine K_1 and K_2 such that the lowest achievable damping ratio is attained by the control system. (6 marks)

(d) Determine K_1 and K_2 such that the system output has the fastest decay in its transient response and has a damping ratio of $\varsigma = 0.707$. (6 marks)

Q1. A pendulum system has a transfer function given by

$$G(s)=\frac{K}{2s^2+3s+C}$$

where K and C are uncertain parameters of the system.

(a) If the undamped natural frequency of the pendulum is 1 rad/s, find C.

(2 marks)

(b) Find the range of values of C for which $G(s)$ has complex poles.

(3 marks)

(c) Design the values of K and C such that the output of $G(s)$ is underdamped and the gain to a unit step input is 2. Sketch and label this output response of $G(s)$.

(5 marks)

Q4. Figure Q4 shows the input signal, $x(t)$, and output signal, $y(t)$, of a system with a transfer function, $G(s)$.

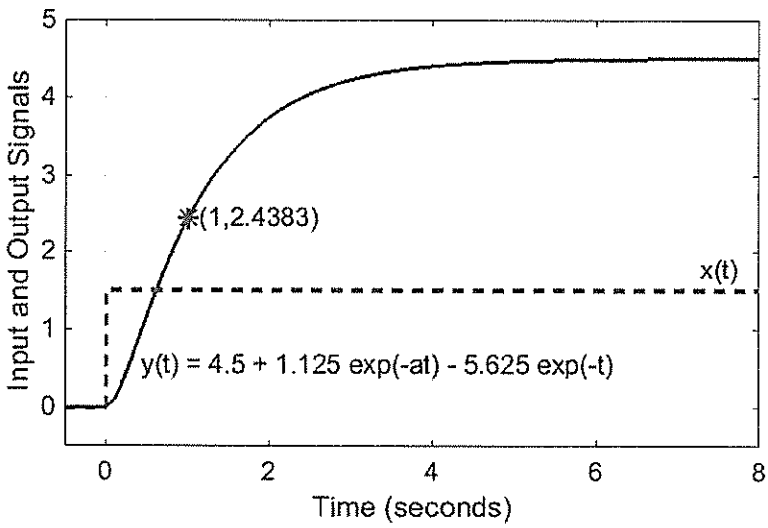

Figure Q4 : Input signal (dashed) and Output signal (solid)

(a) Write down an expression for the input signal, $x(t)$.

(2 marks)

(b) What is the DC gain of the system?

(2 marks)

(c) Find the poles of the system, $G(s)$. Round the numerical values to the nearest integer.

(3 marks)

(d) Suppose $G(s)$ is cascaded with an unity gain differentiator to form the new system $H(s) = sG(s)$. Determine the output signal of the new system, $H(s)$, corresponding to the input $x(t)$ shown in Figure Q4.

SECTION B : Answer 3 out of the 4 questions in this section

Q5. Answer **ALL** the short questions (a) to (d).

(a) Find the Laplace transform of $x(t) = te^{-2t} \cos 3t \, u(t)$ where $u(t)$ represents the unit step function.

(4 marks)

(b) The unit step response of an electrical circuit is given by

$$y(t) = 6t - 9e^{-t} + 5e^{-3t} - 12 \qquad t \geq 0.$$

Determine the unit impulse response of the circuit.

(4 marks)

(c) The poles and zeros of a system are given as follows :

Poles : $s = -1, \ -1 \pm j4$
Zeros : $s = \pm j1$

If the steady state response of the system to a unit step input is 1, determine the transfer function of the system.

(6 marks)

(d) If the input and total response of a linear time invariant system are given by $x(t) = 2\cos 10t$ and $y(t) = 5e^{-10t} + 5\sqrt{2}\cos\left(10t + \dfrac{\pi}{4}\right)$ respectively, show that the transfer function of the system is given by

$$G(s) = \frac{5s}{s+10}.$$

(6 marks)