EE2023/TEE2023 TUTORIAL 4 (PROBLEMS)

Q.1 The signal $x(t)$ shown in Fig.Q.1 is sampled at 5 Hz to form a continuous-time signal $\tilde{x}(t)$. Let $\tilde{X}(f)$ be the spectrum of $\tilde{x}(t)$. Sketch and label $\tilde{x}(t)$, $\left|\tilde{X}(f)\right|$ and $\angle\tilde{X}(f)$.

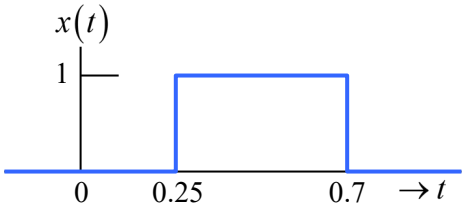


Fig.Q.1

Ans : $\tilde{x}(t)=\delta(t-0.4)+\delta(t-0.6)$

$\left|\tilde{X}(f)\right|=2\left|\cos(0.2\pi f)\right|$

$\angle\tilde{X}(f)=-\pi f$

Q.2 The signal $x(t)$ shown in Fig.Q.2 is sampled at 0.25 Hz to form a continuous-time signal $\tilde{x}(t)$. Let $\tilde{X}(f)$ be the spectrum of $\tilde{x}(t)$. Determine $\tilde{X}(f)$ and show that it is periodic.

[Hint: $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$, where -1 < r < 1]

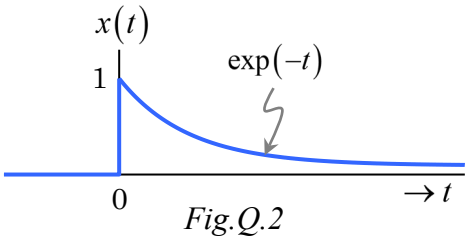


Fig.Q.2

Ans : $\tilde{X}(f+0.25)=\tilde{X}(f)=\sum_{n=0}^{\infty}\exp(-4n)\exp(-j8\pi nf)=\frac{1}{1-\exp(-4)\exp(-j8\pi f)}$

Q.3 Fig.Q.3 shows a periodic signal

$$x(t)=\sum_{n=-\infty}^{\infty} g(t-nT_p)$$

and its generating function $g(t)$ which has a Fourier transform of $G(f)$.

Let $X(f)$ and X_k be the Fourier transform and Fourier series coefficients of $x(t)$, respectively. Show how $X(f)$ can be obtained directly from $G(f)$. What is the relationship between X_k and $G(f)$? Is the generation function of a periodic signal unique, and why?

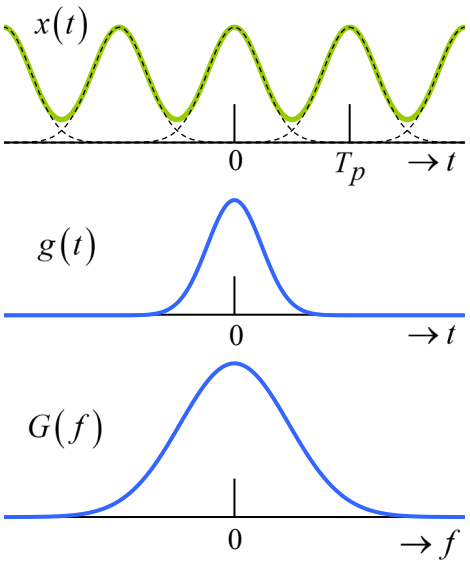
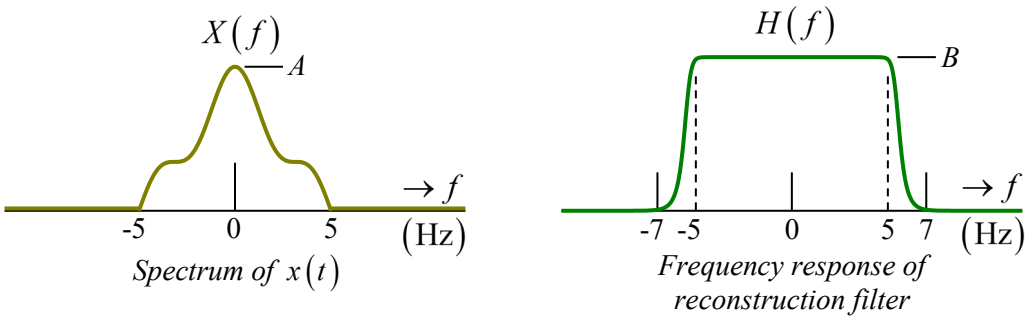


Fig.Q.3

Ans : $X_k=\frac{1}{T_p}G\left(\frac{k}{T_p}\right)$

Q.4 A signal $x(t)$ is sampled, stored and later reconstructed from the stored samples using a reconstruction filter. The spectrum of $x(t)$ and the frequency response of the reconstruction filter are shown in Fig.Q.4. What is the Nyquist sampling frequency for $x(t)$? What sampling frequency would you recommend so that $x(t)$ can be reconstructed from its samples without distortion, and why? Illustrate your answer.

Fig.Q.4

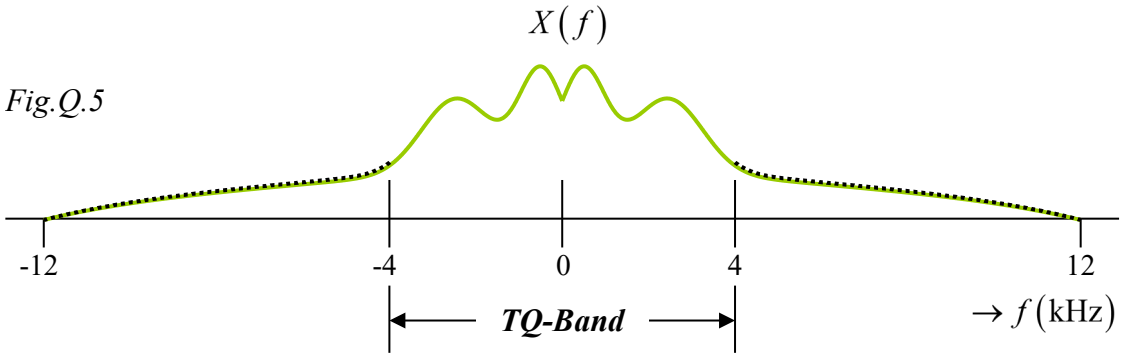


Ans : Nyquist rate = 10 Hz ; Recommended sampling frequency = 12 Hz

Q.5 Speech studies have shown that perceptual cues pertaining to speech intelligibility and speaker identity are mainly found within the first 4 kHz of a speech spectrum. We shall refer to this frequency band as the *TQ-Band* , where *TQ* stands for '*Telephone Quality*'.

The spectrum of a speech signal $x(t)$ is shown in Fig.Q.5 where the *TQ-Band* is indicated.

Fig.Q.5



In order to send $x(t)$ over a digital telephone network, the first step is to sample $x(t)$ without corrupting its *TQ-Band* . Suggest a method for sampling $x(t)$ in each of the following situations.

Situation A: Anti-aliasing lowpass filter is not available and frequency aliasing is prohibited.

Situation B: Lowest sampling frequency must be used at all cost.

Situation C: Anti-aliasing lowpass filter is not available and frequency aliasing is permitted.

Comment on the advantage and disadvantage of each method.

Ans : Situation A = 24 Hz ; Situation B = 8 Hz ; Situation C = 16 Hz

Supplementary Problems

These problems will not be discussed in class.

S.1 A bandlimited lowpass signal $x(t)$ of bandwidth $20(kHz)$ is sampled at a rate of f_S samples/sec to form $x_S(t)$. $x_S(t)$ is then sent through a reconstruction filter having frequency response $H(f)$ so that $x(t)$ is exactly reproduced at the filter's output. Find the smallest applicable value of f_S and specify the corresponding $H(f)$.

Answer : $f_S = 40\ kHz, \quad H(f) = \frac{1}{40000} \cdot \text{rect}\left(\frac{f}{40000}\right)$

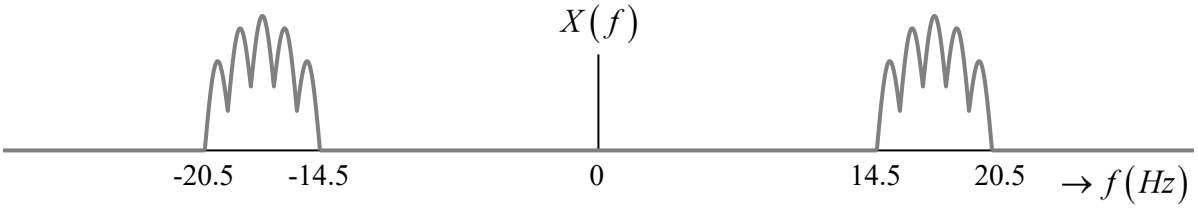
S.2 The sampled version of a signal $x(t)$ has the form $x_S(t) = \sum_{n=-5}^{5} x(5n)\delta(t-5n)$.

(a) Determine the sampling frequency used.

(b) If $x(t) = \text{tri}(t)$, can it be perfectly reconstructed from $x_S(t)$, and why?

Answer : *(a)* 0.2 *Hz* *(b)* *No*

S.3 The spectrum of a bandpass signal $x(t)$ is shown in the figure below.



(a) What is the Nyquist sampling frequency for $x(t)$.

(b) Determine the lowest sampling frequency that can be used so that $x(t)$ may be reconstructed from its sampled version without distortion. Specify the reconstruction filter.

Answer : *(a)* 41 *Hz* *(b)* 7 *Hz*

Below is a list of solved problems selected from ***Chapter 5*** *of* ***Hwei Hsu (PhD), 'The Schaum's series on Signals & Systems,' 2nd Edition****.*

Selected solved-problems: ***5.58, 5.59, 5.60***

These solved problems should be treated as supplementary module material catered for students who find the need for more examples or practice-problems

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