# EE2023 Signals & Systems

AY2019/20-1

Midterm Quiz (Close Book)

Date: 3 October 2019 | Time Allowed: 1.5 Hours

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<u>INSTRUCTIONS TO CANDIDATES</u>:

1. Answer all 4 questions. Each question carries 10 marks.
2. This is a closed book quiz. However, you are allowed to bring a help sheet comprising one single sheet of paper of A4 size.
3. Tables of formulas are given on Pages 15 & 16, which you may detach for easy reference. You need not hand in these two pages.
4. Programmable and/or graphic calculators are not allowed.
5. Write your **answers** in the spaces indicated in this question paper. Attachment is not allowed.
6. Write your **name, student number** and **seat number** in the spaces indicated below.

Name :______________________________

Student № :______________________________

Seat № :______________________________

| **Question №** | **Marks** |
|---|---|
| **Q.1** | |
| **Q.2** | |
| **Q.3** | |
| **Q.4** | |
| **Total Marks** | |

Q.1 The signal $x(t)$ is shown in Figure Q1.

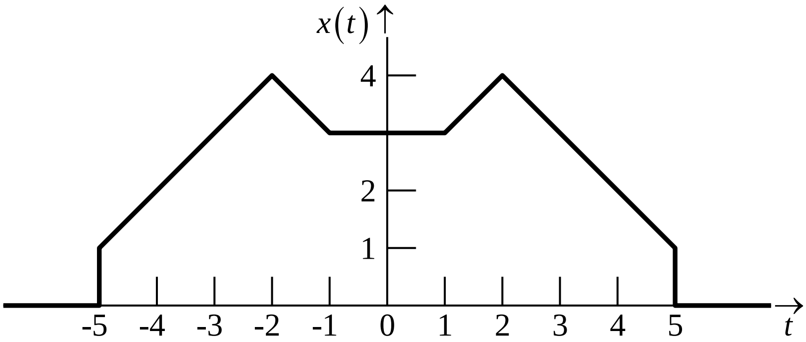



Figure Q1

(a) Determine the Fourier transform, $X(f)$, of $x(t)$. (5 marks)

(b) The periodic signal, $x_p(t)$, can be obtained by replicating $x(t)$ at a period of 15 seconds. Obtain an expression for $x_p(t)$ in terms of $x(t)$ and the Dirac $\delta$-function.

(1 mark)

(c) Determine the Fourier transform, $X_p(f)$, of the periodic signal $x_p(t)$. (4 marks)

Q.1 ANSWER

Q.1 ANSWER ~ continued

Q.1 ANSWER ~ continued

Q.2 The periodic signal $x(t)$ is given by

$$x(t) = 3\cos\left(8\pi t + \frac{\pi}{4}\right) + 5e^{j12\pi t} + 10\,.$$

(a) What is the fundamental frequency and period of $x(t)$? (2 marks)

(b) Determine the Fourier Series coefficients of $x(t)$. (4 marks)

(c) Determine the Fourier transform, $X(f)$, of $x(t)$. (2 marks)

(d) What is the average power of $x(t)$? (2 marks)

Q.2 ANSWER

Q.2 ANSWER ~ continued

Q.2 ANSWER ~ continued

Q.3 The spectrum, $X(f)$, of a signal $x(t)$ is shown in Figure Q3.

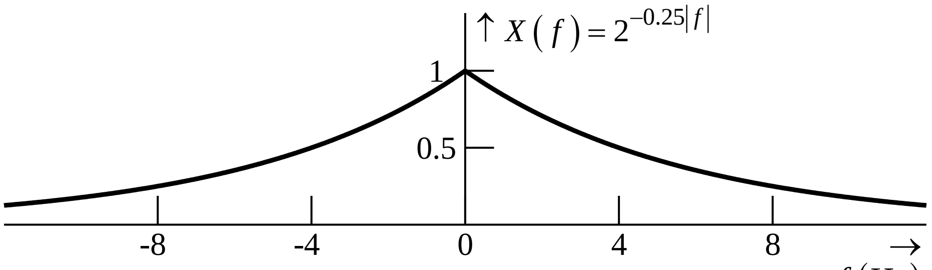



Figure Q3

The signal $x(t)$ is filtered to produce $y(t)=x(t)\ast\left[8\mathrm{sinc}(8t)\right]$, where '$\ast$' denotes convolution. Let $Y(f)$ be the spectrum of $y(t)$.

(a) Find an expression for $Y(f)$ and sketch it. Label your sketch adequately. (4 marks)

(b) The signal $y(t)$ is sampled at Nyquist sampling frequency to form $y_s(t)$.

i. Sketch the spectrum, $Y_s(f)$, of $y_s(t)$. Label your sketch adequately. (3 marks)

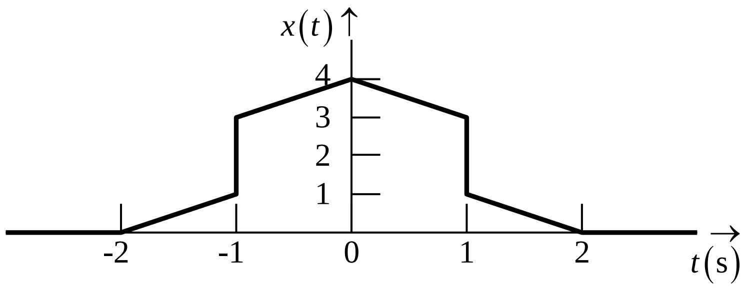
ii. Specify the reconstruction filter for exact recovery of $y(t)$ from $y_s(t)$. (3 marks)

Q.3 ANSWER

Q.3 ANSWER ~ continued

Q.3 ANSWER ~ continued

Q.4 An energy pulse $x(t) = A\,\text{tri}\left(\frac{t}{\alpha}\right) + B\,\text{rect}\left(\frac{t}{\beta}\right)$ is shown in Figure Q4.




$$x(t) = \begin{cases} t+2; & -2 \le t < -1 \\ t+4; & -1 \le t < 0 \\ -t+4; & 0 \le t < 1 \\ -t+2; & 1 \le t < 2 \\ 0; & \text{elsewhere} \end{cases}$$

Figure Q4

(a) Find the values of $A$, $B$, $\alpha$ and $\beta$. (2 mark)

(b) Find the energy spectral density and the 1st-null bandwidth of $x(t)$. (5 marks)

(c) Derive the total energy of $x(t)$. (3 marks)

Q.4 ANSWER

Q.4 ANSWER ~ continued

Q.4 ANSWER ~ continued

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**Fourier Series:** $$\begin{cases} c_k = \frac{1}{T}\int_{\tilde{t}}^{\tilde{t}+T} x(t)\exp(-j2\pi k\, t/T)\,dt \\ x(t) = \sum_{k=-\infty}^{\infty} c_k \exp(j2\pi k\, t/T) \end{cases}$$

**Fourier Transform:** $$\begin{cases} X(f) = \int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)\,dt \\ x(t) = \int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)\,df \end{cases}$$

| FOURIER TRANSFORMS OF BASIC FUNCTIONS | | |
|---|---|---|
| | $x(t)$ | $X(f)$ |
| **Constant** | $K$ | $K\delta(f)$ |
| **Unit Impulse** | $\delta(t)$ | **1** |
| **Unit Step** | $u(t)$ | $\frac{1}{2}\left[\delta(f)+\frac{1}{j\pi f}\right]$ |
| **Sign (or Signum)** | $\operatorname{sgn}(t)$ | $\frac{1}{j\pi f}$ |
| **Rectangle** | $\operatorname{rect}\left(\frac{t}{T}\right)$ | $T\operatorname{sinc}(fT)$ |
| **Triangle** | $\operatorname{tri}\left(\frac{t}{T}\right)$ | $T\operatorname{sinc}^2(fT)$ |
| **Sine Cardinal** | $\operatorname{sinc}\left(\frac{t}{T}\right)$ | $T\operatorname{rect}(fT)$ |
| **Complex Exponential** | $\exp(j2\pi f_o t)$ | $\delta(f-f_o)$ |
| **Cosine** | $\cos(2\pi f_o t)$ | $\frac{1}{2}\left[\delta(f-f_o)+\delta(f+f_o)\right]$ |
| **Sine** | $\sin(2\pi f_o t)$ | $-\frac{j}{2}\left[\delta(f-f_o)-\delta(f+f_o)\right]$ |
| **Gaussian** | $\exp\left(-\frac{t^2}{\alpha^2}\right)$ | $\alpha\pi^{0.5}\exp\left(-\alpha^2\pi^2 f^2\right)$ |
| **Comb** | $\sum_{m=-\infty}^{\infty}\delta(t-mT)$ | $\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$ |

| FOURIER TRANSFORM PROPERTIES | | |
|---|---|---|
| | **Time-domain** | **Frequency-domain** |
| **Linearity** | $\alpha x_1(t)+\beta x_2(t)$ | $\alpha X_1(f)+\beta X_2(f)$ |
| **Time scaling** | $x(\beta t)$ | $\frac{1}{\lvert\beta\rvert}X\left(\frac{f}{\beta}\right)$ |
| **Duality** | $X(t)$ | $x(-f)$ |
| **Time shifting** | $x(t-t_o)$ | $X(f)\exp(-j2\pi ft_o)$ |
| **Frequency shifting (Modulation)** | $x(t)\exp(j2\pi f_o t)$ | $X(f-f_o)$ |
| **Differentiation in the time-domain** | $\frac{d^n}{dt^n}x(t)$ | $(j2\pi f)^n X(f)$ |
| **Multiplication in the time-domain** | $x_1(t)x_2(t)$ | $\int_{-\infty}^{\infty}X_1(\zeta)X_2(f-\zeta)\,d\zeta$ or $X_1(f)\ast X_2(f)$ |
| **Convolution in the time-domain** | $\int_{-\infty}^{\infty}x_1(\zeta)x_2(t-\zeta)\,d\zeta$ or $x_1(t)\ast x_2(t)$ | $X_1(f)X_2(f)$ |
| **Integration in the time-domain** | $\int_{-\infty}^{t}x(\tau)\,d\tau$ | $\frac{1}{j2\pi f}X(f)+\frac{1}{2}X(0)\delta(f)$ |
| | | $\frac{1}{j2\pi f}X(f)$ **if** $X(0)=0$ |

## Trigonometric Identities

| | |
|---|---|
| $\exp(\pm j\theta) = \cos(\theta) \pm j\sin(\theta)$ | $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$ |
| $\cos(\theta) = 0.5\left[\exp(j\theta) + \exp(-j\theta)\right]$ | $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$ |
| $\sin(\theta) = -0.5j\left[\exp(j\theta) - \exp(-j\theta)\right]$ | $\tan(\alpha \pm \beta) = \dfrac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$ |
| $\sin^2(\theta) + \cos^2(\theta) = 1$ | |
| $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ | $\sin(\alpha)\sin(\beta) = 0.5\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$ |
| $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$ | $\cos(\alpha)\cos(\beta) = 0.5\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$ |
| $\sin^2(\theta) = 0.5\left[1 - \cos(2\theta)\right]$ | $\sin(\alpha)\cos(\beta) = 0.5\left[\sin(\alpha - \beta) + \sin(\alpha + \beta)\right]$ |
| $\cos^2(\theta) = 0.5\left[1 + \cos(2\theta)\right]$ | $C\cos(\theta) - S\sin(\theta) = \sqrt{C^2 + S^2}\cos\left[\theta + \tan^{-1}(S/C)\right]$ |

## Definitions of Basic Functions

Rectangle:

$$\mathrm{rect}\left(\frac{t}{T}\right) = \begin{cases} 1; & -T/2 \le t < T/2 \\ 0; & \text{elsewhere} \end{cases}$$

Triangle:

$$\mathrm{tri}\left(\frac{t}{T}\right) = \begin{cases} 1 - |t|/T; & |t| \le T \\ 0; & |t| > T \end{cases}$$

Sine Cardinal:

$$\mathrm{sinc}\left(\frac{t}{T}\right) = \begin{cases} \dfrac{\sin(\pi t/T)}{\pi t/T}; & t \ne 0 \\ 1; & t = 0 \end{cases}$$

Signum:

$$\mathrm{sgn}(t) = \begin{cases} 1; & t \ge 0 \\ -1; & t < 0 \end{cases}$$

Unit Impulse:

$$\delta(t) = \begin{cases} \infty; & t = 0 \\ 0; & t \ne 0 \end{cases} \qquad \int_{0^-}^{0^+} \delta(t)\,dt = 1$$

Unit Step:

$$u(t) = \begin{cases} 1; & t \ge 0 \\ 0; & t < 0 \end{cases}$$