# *EE2023/TEE2023/EE2023E TUTORIAL 8 (PROBLEMS)*

## ***Section I : Exercises that are straightforward applications of the concepts covered in class. Please attempt these problems on your own.***

1. Consider the first order system $G(s) = \dfrac{2}{0.2s+1}$.

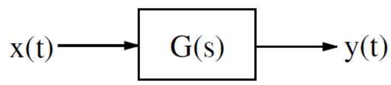


Figure 1: Open loop system, $G(s)$

Suppose that the input is a sinusoidal signal $x(t) = \sin(3t)$ (See Figure 1).

(a) Find the output of the system

(b) Identify the steady-state response.

(c) Show that the amplitude ratio and phase shift of the steady-state response are equal to values given by $|G(j\omega)|$ and $\angle G(j\omega)$ where $\omega$ is the frequency of the sinusoidal input.

ANSWER : $y_{ss}(t) = 1.71\sin\left(3t - 0.54\right)$

2. The steady-state output of a first order system, $G(s)$, is $4.5\sin\left(5t - 38^{\mathrm{o}}\right)$. Assuming that $|G(5j)| = 0.75$ and $\angle G\left(5j\right) = -68^{\mathrm{o}}$, identify the function(s) that may be the input signal.

ANSWER : $6\sin\left(5t + \dfrac{\pi}{6} \pm 2n\pi\right) = 6\cos\left(5t - \dfrac{\pi}{3} \pm 2n\pi\right)$

Since $\cos\left(\omega t - \pi/2\right) = \sin\left(\omega t\right)$

3. The magnitude response for the system $G(s)$ is shown in Figure 2.

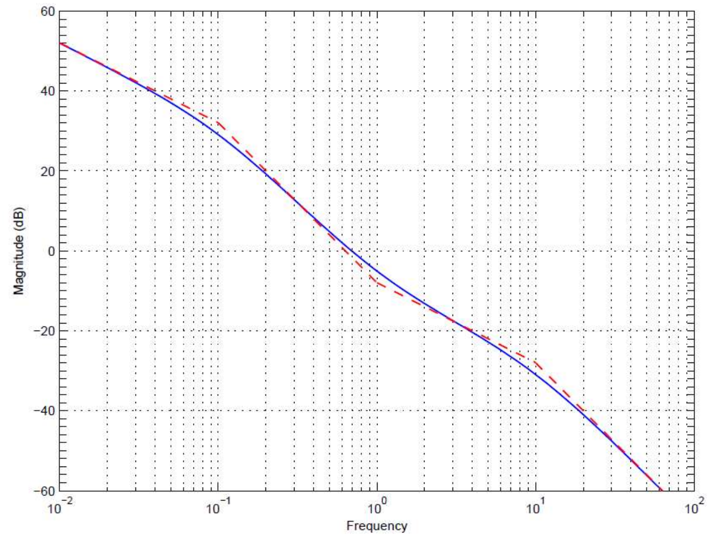


Figure 2: Magnitude plot for $G(s)$

(a) What is the slope of the high frequency asymptote?

ANSWER : -40 dB/decade

(b) How many pole(s), zeros and integrators does $G(s)$ have?

ANSWER : 3 poles, 1 zero and 1 integrator

(c) The low frequency asymptote of the magnitude response is $\frac{K}{s^N}$. Find the value of $K$.

ANSWER : $K = 4$

## *Section II : Problems that will be discussed in class.*

1. A car suspension system and a very simplified version of the system are shown in Figure 3(a) and 3(b) respectively.

The transfer function of the simplified car suspension system is

$$\frac{X_o(s)}{X_i(s)} = \frac{bs+k}{ms^2+bs+k}$$

Suppose a car ($m$ = 1 kg, $k$ = 1 N/m and $b = \sqrt{2}$ N/ms$^{-1}$) is travelling on a road that has speed reducing stripes and the input to the simplified car suspension system, $x_i$, may be modelled by the periodic square wave of frequency $\omega$ = 1 rad/s, shown in Figure 4.

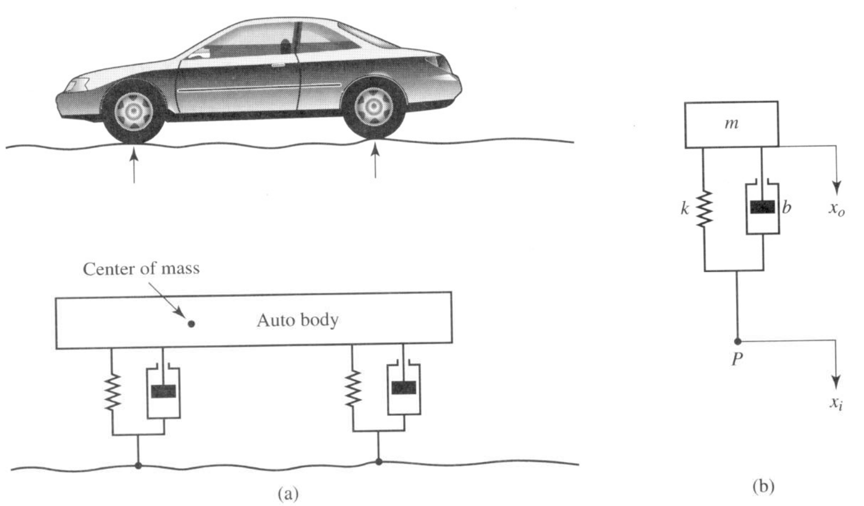


Figure 3: (a) Automobile suspension system, (b) Simplified suspension system

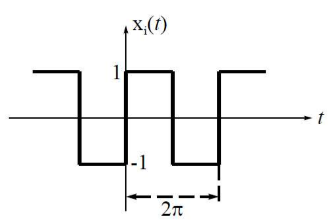


Figure 4: Input waveform, $x_i(t)$

Determine the steady-state displacement of the car body, $x_{o,ss}(t)$.

*Hint* : The Fourier Series representation of the periodic square wave shown in Figure 4 is

$$x_i(t) = \frac{4}{\pi}\left[\sin(t) + \frac{1}{3}\sin(3t) + \frac{1}{5}\sin(5t) + \ldots\right]$$

ANSWER :

$$x_i(t) = \frac{4}{\pi}\left[1.2247\sin(t-0.6155) + 0.1605\sin(3t-1.3147) + 0.05708\sin(5t-1.4248) + \ldots\right]$$

2. A high speed recorder monitors the temperature of an air stream as sensed by a thermocouple. The following observations were made:
   - The recorded temperature shows an essentially sinusoidal variation after about 1 second.
   - The maximum recorded temperature is about 52ºC and the minimum is 48ºC at 2 cycles per minute.

   The information indicates that the recorded steady-state temperature may be expressed as $50+2\sin(4\pi t)$. If the system (thermocouple and high speed recorder) has unity steady-state gain and first order dynamics with a time constant of approximately 1 minute under these conditions, estimate the actual maximum and minimum air temperatures.

   ANSWER : Maximum = 75.2ºC and Minimum = 24.8ºC

3. Figure 5 shows the magnitude plot of $G(s)=\dfrac{A(s+\alpha)}{(s+\beta)(S+\gamma)(s+\lambda)}$.

   (a) Using the approximate (straight line asymptotes) magnitude response, determine $A$, $\alpha$, $\beta$, $\gamma$ and $\lambda$.

   ANSWER : $A$ = 5000, $\alpha$ = 4, $\beta$ = 10, $\gamma$ = $\lambda$ = 20

   (b) Write down the transfer function of another system that may have the magnitude response shown in Figure 5.

   ANSWER : $\dfrac{5000(s\pm4)}{(s\pm10)(s+20)^2}$ ; $\dfrac{5000(s\pm4)e^{-sL}}{(s\pm10)(s+20)^2}$

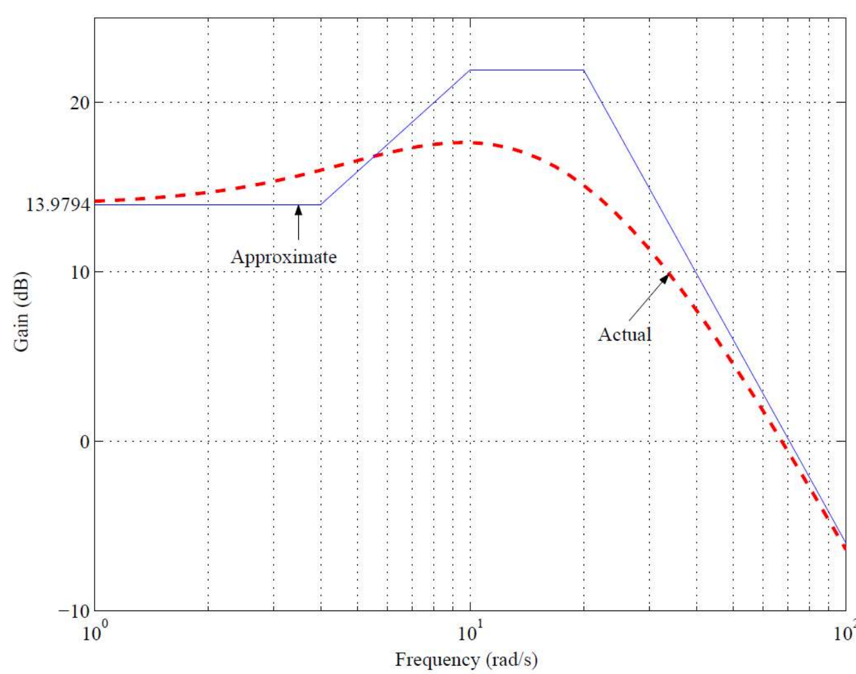


Figure 5: Magnitude response of $G(s)=\dfrac{A(s+\alpha)}{(s+\beta)(s+\gamma)(s+\lambda)}$

4. Consider a system modelled by the transfer function,

$$G(s) = \frac{K\left(-\dfrac{s}{\alpha}+1\right)}{\left(\dfrac{s}{\beta}+1\right)\left(\dfrac{s}{\gamma}+1\right)^2}$$

Using the pole-zero map and Bode magnitude plot of $G(s)$ shown in Figure 6, answer the following questions.

(a) Identify the corner frequencies ($\omega_1$, $\omega_2$ and $\omega_3$) of the Bode magnitude plot of $G(s)$

(b) What is the value of the repeated pole?

(c) Determine the DC gain, $K$.

(d) Is the system stable?

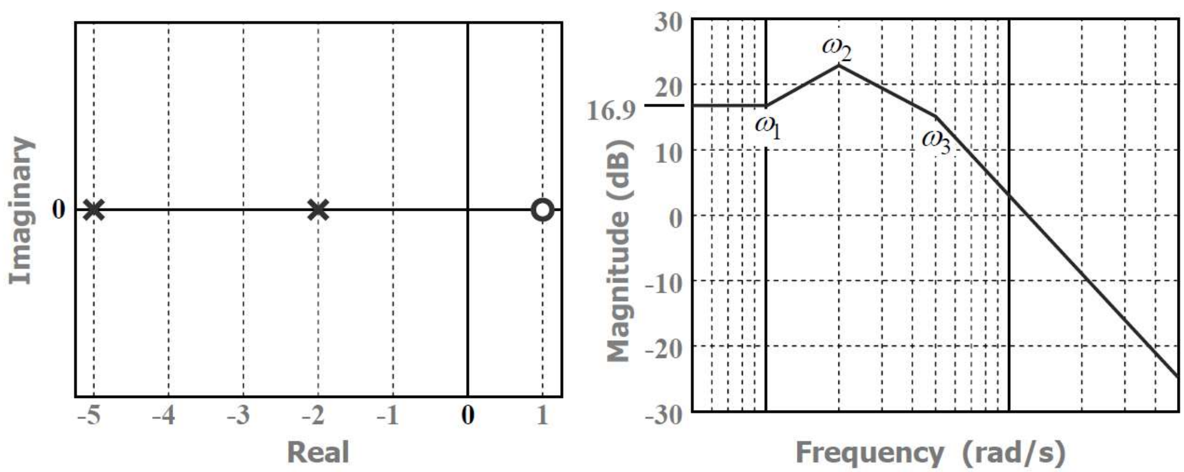


Figure 6

***Section III : Practice Problems. These problems will not be discussed in class.***

1. Find the steady-state current owing through the capacitor ( $\lim_{t\to\infty} i_C(t)$ ), inductor ( $\lim_{t\to\infty} i_L(t)$ ) and resistor ( $\lim_{t\to\infty} i_R(t)$ ) in the circuit shown in Figure 7.

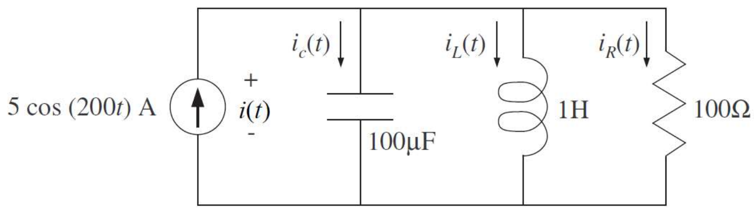


Figure 7: Parallel RLC Circuit

ANSWER : $\lim_{t\to\infty} x_C(t) = \frac{20}{\sqrt{13}}\cos\left(200t + 33.7^{\circ}\right)$

$\lim_{t\to\infty} x_L(t) = \frac{5}{\sqrt{13}}\cos\left(200t - 146.3^{\circ}\right)$

$\lim_{t\to\infty} x_R(t) = \frac{10}{\sqrt{13}}\cos\left(200t - 56.3^{\circ}\right)$

2. Figure 8 shows the Bode diagram of a system whose transfer function is

$$G(s) = \frac{A(s+a)}{(s+b)\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}$$

What are the values of $A$, $a$, $b$, $\varsigma$ and $\omega_n$?

ANSWER : $A = 12$, $a = 30$, $b = 9$, $\varsigma = 0.25$, $\omega_n = 2$

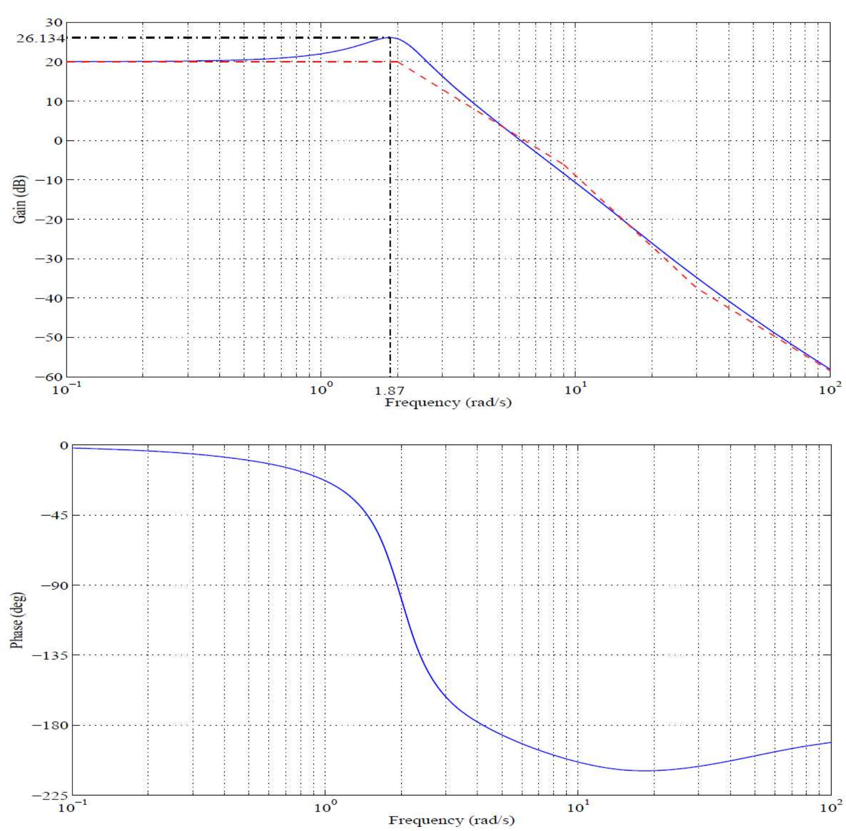


Figure 8: Bode Diagram