EE2023 Mid-term Quiz: AY2019-2020 Semester 1

Q1(a). $X(f) = 10\operatorname{sinc}(10f) + 6\operatorname{sinc}^2(3f)\cos(4\pi f)$

Q1(b). $x_p(t) = x(t) \otimes \sum_k \delta(t - k15)$

Q1(c). $X_p(f) = \dfrac{2}{15}\sum_k \left[5\operatorname{sinc}\left(\dfrac{2k}{3}\right) + 3\operatorname{sinc}^2\left(\dfrac{k}{5}\right)\cos\left(\dfrac{4\pi k}{15}\right) \right]\delta\left(f - \dfrac{k}{15}\right)$

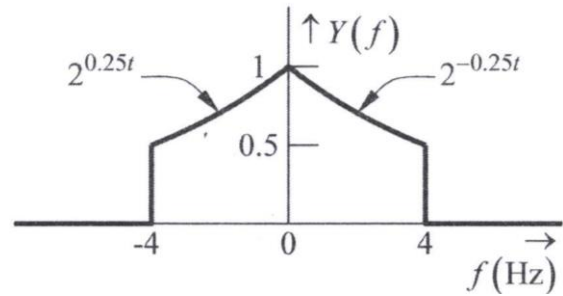Q2(a). HCF of 4 Hz and 6 Hz is 2 Hz. Hence $f_0 = 2$ and $T_0 = 0.5$.

Q2(b).

$$x(t) = \frac{3}{2}e^{j\pi/4}e^{j8\pi t} + \frac{3}{2}e^{-j\pi/4}e^{-j8\pi t} + 5e^{j12\pi t} + 10$$

$$\quad\quad\uparrow \quad\quad\quad\quad \uparrow \quad\quad\quad\quad \uparrow \quad\quad\quad \uparrow$$

$$\quad\quad c_2 \quad\quad\quad\quad c_{-2} \quad\quad\quad\quad c_3 \quad\quad\quad c_0$$
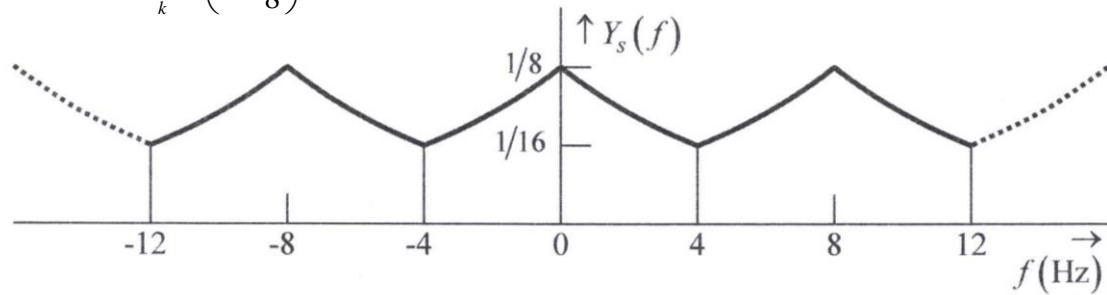
Q2(c). $X(f) = \dfrac{3}{2}e^{j\pi/4}\delta(f-4) + \dfrac{3}{2}e^{-j\pi/4}\delta(f+4) + +5\delta(f-6) + 10\delta(f)$

Q2(d). $P_x = \left(\dfrac{3}{2}\right)^2 + \left(\dfrac{3}{2}\right)^2 + 5^2 + 10^2 = 129.5$

Q3(a). $Y(f) = X(f).\text{rect}\left(\dfrac{f}{8}\right) = 2^{-0.25|f|}.\text{rect}\left(\dfrac{f}{8}\right)$

Q3(b)i. $y_s(t) = y(t).\sum\limits_k \delta\left(t - \dfrac{k}{8}\right)$

Q4(a). $A = 2 \; ; B = 2 \; ; \alpha = 2 \; ; \beta = 2$

Q4(b).
$$X(f) = 4\,\text{sinc}^2(2f) + 4\,\text{sinc}(2f)$$
$$E_x(f) = |X(f)|^2 = 16\,\text{sinc}^4(2f) + 16\,\text{sinc}^2(2f)$$

1st null bandwidth occurs at $f = 0.5$.

Q4(c). $E = 2\left[\displaystyle\int_0^1 (4-t)^2\,dt\right] + 2\left[\displaystyle\int_1^2 (2-t)^2\,dt\right] = \dfrac{76}{3}$