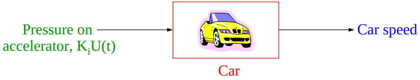
EE2023 Signals & Systems
Chapter 6 – Systems & Classification of Systems

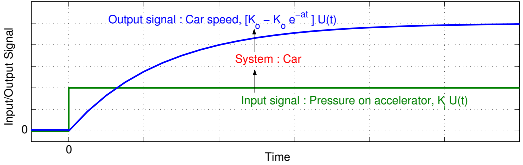
TAN Woei Wan

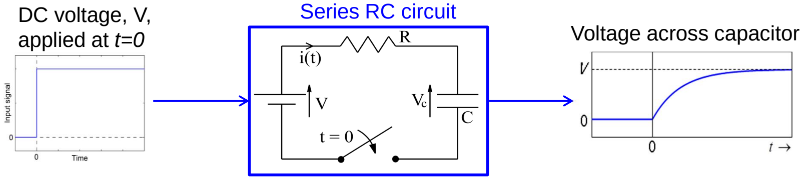
Dept. of Electrical & Computer Engineering
National University of Singapore

Definition of Systems

- Physical systems, in the broadest sense are interconnections of components, devices or subsystems. Examples are mechanical systems, communication systems, electronic systems, chemical processing systems, etc.







- A system generates a response, or output signal, for a given input.

- In system studies, a system is a mathematical model of a physical process that relates the input (or excitation) signal to the output (or response) signal.
- With an input $x(t)$ and an output $y(t)$, the system may be viewed as a transformation (or mapping) of $x(t)$ into $y(t)$, mathematically expressed as

$$y(t) = \mathcal{T}\{x(t)\}$$

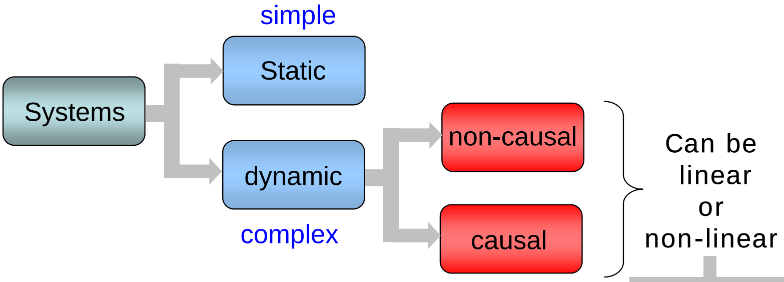
  where $\mathcal{T}$ is an operator representing some well-defined transformation rule.
- A system is commonly depicted as a "black box".

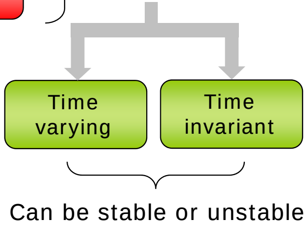
$$x(t) \longrightarrow \boxed{\begin{matrix}\text{System} \\ \mathbf{T}\end{matrix}} \longrightarrow y(t)$$

- Systems may be classified into different categories according to their basic properties and the nature of their input and output signals.

Classification of Systems – Overview

- Physical systems have many different properties.





Linear time invariant (LTI) causal dynamic systems have certain nice properties that allow easy analysis

Classification of Systems – Static and Dynamic System

- System is **static** (also called **memoryless**) if its output at a given time is dependent on the input at that time i.e.

$$y(t) = Kx(t)$$

  Examples :
  - Consider the resistor R, where the current $i(t)$ flowing through it is the input and the voltage $v(t)$ across it is the output. The resistor is a static system as the input-output relationship (Ohm's law) of the resistor is $v(t) = i(t)R$.
- Output of a **dynamic** system (also called **non-zero memory**) at time t_1 depends on the past or future values of the input x(t) in addition to the input at the present time i.e.

$$y(t_1) = \mathcal{T}\{\ldots, x(t_1+1), x(t_1), x(t_1-1), \ldots\}$$

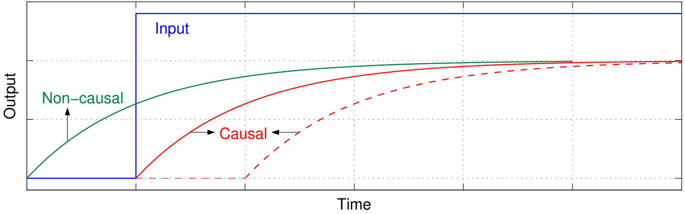
  Examples :
  - $y(t) = x(t-1)u(t-1)$ where $x(t)$ is the input and $y(t)$ is the output.
  - Capacitor C with the current $i(t)$ flowing through it taken as the input and the voltage $v(t)$ across it as output. The input-output relationship of the capacitor is $v(t) = \dfrac{1}{C}\displaystyle\int_{-\infty}^{t} i(\tau)\,d\tau$. Output voltage at time, t, depends on all values of the input current from time $-\infty$ to t.

Classification of Systems – Causal and Non-Causal System

- System is **causal** or **non-anticipatory** if its output at the time t_1 depends on only the present and/or past values of its input, $x(t)$ i.e.

$$y(t_1) = \mathcal{T}\{x(t_1), x(t_1 - 1), \ldots\}$$

Example: $y(t) = x(t-1)$ and $y(t) = \int_{t-3}^{t-1} x(\tau)\, d\tau$



It is not possible for a causal system to produce an output before an input is applied to it.

- A system called **non-causal** (or **anticipative**) if its output, $y(t)$, at the present time depends on future values of its input, $x(t)$.

Example: $y(t) = x(t+1)$ and $y(t) = \int_{t+1}^{t+3} x(\tau)\, d\tau$

Classification of Systems – Linear and non-linear System

Linear systems satisfy the properties of

- **additivity** : $y(t) = \mathcal{T}\{x_1(t) + x_2(t)\} = \mathcal{T}\{x_1(t)\} + \mathcal{T}\{x_2(t)\} = y_1(t) + y_2(t)$
- **scaling** : If $y(t) = \mathcal{T}\{x(t)\}$, then $y_1(t) = \mathcal{T}\{\alpha x(t)\} = \alpha\mathcal{T}\{x(t)\} = \alpha y(t)$

  An important property of linear systems is that a zero input yields a zero output. This is a direct result of the scaling property (set $\alpha = 0$ in the equation).

Example

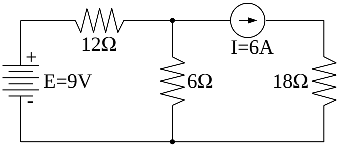
- Vending machine is a linear system : Put in twice as much money, get twice as much drinks.
- Humans are non-linear systems : Quiz mark is 60 if student studies 1 hour per day. With twice as much effort, can only score 80 marks.

Superposition Property

- The additivity and scaling properties may be combined to yield the **Superposition Property**:

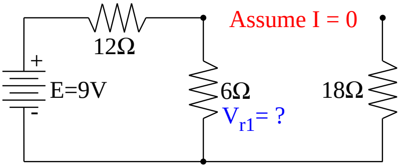
$$\mathcal{T}\{\alpha_1 x_1(t) + \alpha_2 x_2(t)\} = \alpha\mathcal{T}\{x_1(t)\} + \alpha_2\mathcal{T}\{x_2(t)\} = \alpha_1 y_1(t) + \alpha_2 y_2(t)$$

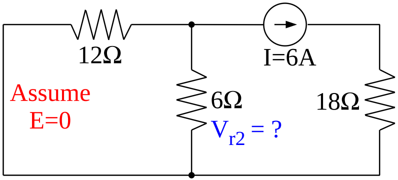
Recall how circuits are with multiple *independent* current/voltage sources are analysed in EPP



Two independent sources, E and I. What is voltage across 6Ω resistor, V_r ?

1. "Kill" current source (let $I = 0$), find V_{r1}





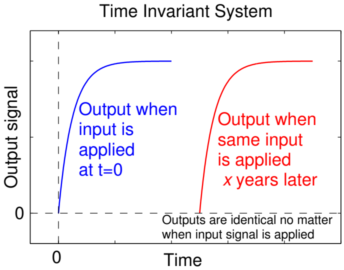
2. "Kill" voltage source (set $E = 0$), find V_{r2}
3. Sum solutions i.e. $V_r = V_{r1} + V_{r2}$.

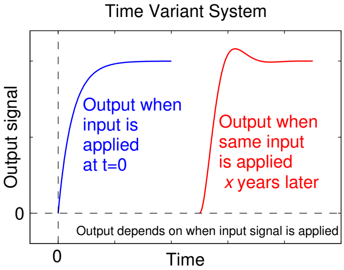
Classification of Systems – Time Invariant and Time Variant System

System is **time invariant** if a time shift (delay or advance) in the input signal leads to the same time shift in the output signal i.e.

$$\text{If } \mathcal{T}\{x(t)\} = y(t), \text{ then } \mathcal{T}\{x(t-\tau)\} = y(t-\tau)$$

for any real value of τ. A **time variant** system does not satisfy the above equation.





Time invariance property enables us to assume that an input signal is applied at $t = 0$.

Classification of Systems – Stable and Unstable System

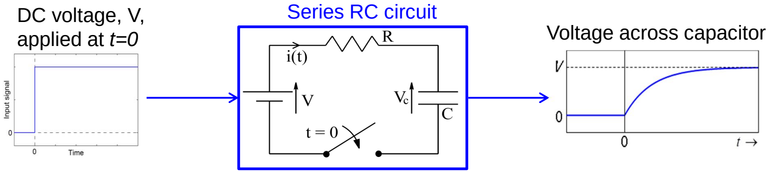
A signal, $x(t)$, is **bounded** if $|x(t)| \leq M \, \forall t$ (M is a finite value)

- System is **bounded-input bounded-output stable** if any bounded input signal, $x(t)$, results in a bounded output signal, $y(t)$.

Example of a stable system

Charging a battery

- Applied voltage (Input) is finite
- Voltage across battery (Output) is bounded

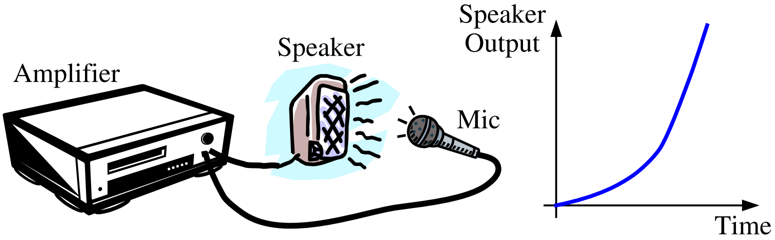


An **unstable** system is one in which not all bounded inputs lead to a bounded output.

Example of an unstable system

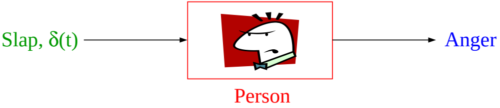
A microphone placed near a speaker

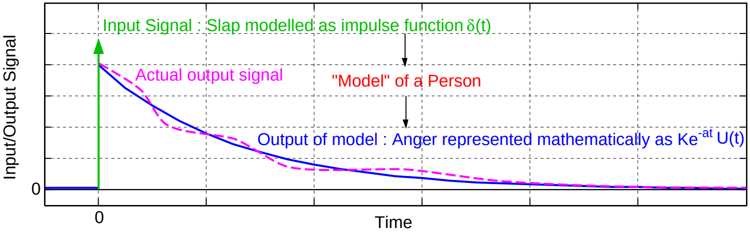
- Background noise (Input) is finite
- Speaker volume (Output) is unbounded



System Models

- Mathematics is a tool used by engineers/scientists to describe or model the behaviour of physical phenomenon. Unfortunately, not all signals and not many systems can be described precisely by mathematics.





- It is possible to approximate their behaviour using physical or natural laws of Physics.

Linear Time Invariant (LTI) Systems

- For this course, the focus of our studies is on continuous-time, linear time-invariant (LTI) systems.
  - This class of systems can be described elegantly by mathematics.
  - Their behaviours can be generalized easily. In many instances, there is no need to solve their mathematical equations explicitly.
- Relationship between the input $x(t)$ and output $y(t)$ of a LTI system is described by a linear differential equation with real constant coefficients.

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \ldots + a_o y(t) = b_m \frac{d^m x(t)}{dt^m} + b_{m-1} \frac{d^{m-1} x(t)}{dt^{m-1}} + \ldots + b_o x(t)$$

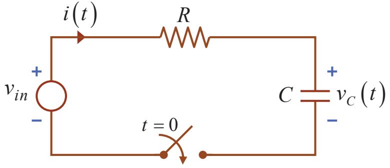
where a_k and b_k are constants and $m < n$ if the LTI system is causal.

  - Electrical Systems: $V_c(t) = \frac{1}{C} \int_0^t i(\tau) d\tau;$ $V_L(t) = L \frac{di(t)}{dt}$
  - Linear Motion (Newton's 2$^{\text{nd}}$ law): $F = m \frac{d^2 x}{dt^2}$ where $F =$ Applied force and $x(t) =$ position
  - Angular Motion (e.g. Motors): $T = J \frac{d^2 \theta}{dt^2}$ where $T =$ Applied torque, $J =$ Inertia and $\theta(t) =$ angular position.

Example

Consider a system formed by connecting a voltage source in series with a resistor and a capacitor (series RC-circuit). Circuit has

- 2 subsystems : R and C
- 4 signals : $v_{in}(t), i(t), v_R(t), v_c(t)$



$v_{in}(t)$ is the input signal
Let the voltage across capacitor i.e. $v_c(t)$ be the output signal.

- From Kirchoff's Voltage law : $v_R(t) = v_{in}(t) - v_c(t)$
- Ohm's law states that

$$i(t) = \frac{v_R(t)}{R} = \frac{v_{in}(t) - v_c(t)}{R}$$

- Substituting $i(t) = \dfrac{v_{in}(t) - v_c(t)}{R}$ into the $I - V$ relationship for a capacitor,

$$
\begin{aligned}
v_c(t) &= \frac{1}{C}\int_0^t i(\tau)\,d\tau \\
&= \frac{1}{RC}\int_0^t v_{in}(\tau) - v_c(\tau)\,d\tau \\
\frac{\mathrm{d}v_c(t)}{\mathrm{d}t} &= \frac{1}{RC}[v_{in}(t) - v_c(t)] \\
RC\frac{\mathrm{d}v_c(t)}{\mathrm{d}t} + v_c(t) &= v_{in}(t)
\end{aligned}
$$

- Input-output ($v_{in}(t)$-$v_c(t)$) relationship for a series RC is a **first order** differential equation.
- The time-domain output signal $v_c(t)$ can be derived by solving the differential equation using calculus or Laplace Transform.