EE2023 Signals & Systems Quiz
Semester 1 AY2017/18

Date : 05 October 2017 **Time Allowed : 1.5 hours**

Instructions :

1. Answer all 4 questions. Each question carries 10 marks.
2. This is a closed book quiz.
3. Tables of formulas are given on Pages 11 and 12.
4. Programmable and/or graphic calculators are not allowed.
5. Write your answers in the spaces indicated in this question paper. Attachment is not allowed.
6. Write your name, matric number and lecture group in the spaces indicated below.

Name : ________________________________

Matric # : ______________________________

Class Group # : _________________________

Question #	Marks
1	
2	
3	
4	
Total Marks	

For your information :

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Group 2 : Prof Lawrence Wong
Group 3 : A/Profs Loh Ai Poh & Tan Woei Wan
Group 4 : Dr Zhang Jianwen

Q.1 The discrete frequency spectrum of a signal $x(t)$ is given in Figure Q1 below.

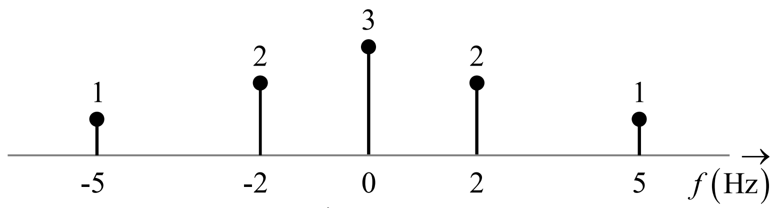

Figure Q1

(a) Is $x(t)$ a periodic signal? Justify your answer. (3 marks)

(b) Is $x(t)$ a complex signal? Justify your answer. (2 marks)

(c) Express $x(t)$ in the form of $x(t)=\sum_{k=-\infty}^{\infty} c_k e^{j2\pi k f_p t}$, showing clearly the values of c_k for $k = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5$. (3 marks)

(d) What is the Fourier transform of $x(t)$? (2 marks)

Q.1 ANSWER

Q.1 ANSWER ~ continued

Q.2 (a) Determine the Fourier transform, $X(f)$, of the signal $x(t)$ shown in Figure Q2(a) which is given by:

$$x(t)=\begin{cases}|\sin(4\pi t)|; & -0.25\le t\le 0.25\\ 0; & \text{elsewhere}\end{cases}$$

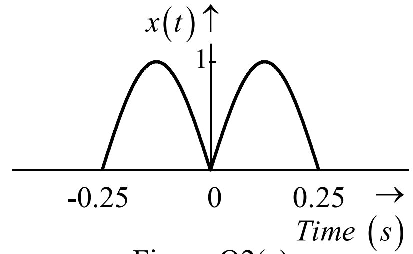

Figure Q2(a)

(4 marks)

(b) The periodic signal $x_p(t)$ is made of repeating the generating function $x(t)$ at periods of 0.75 seconds and is shown in Figure Q2(b).

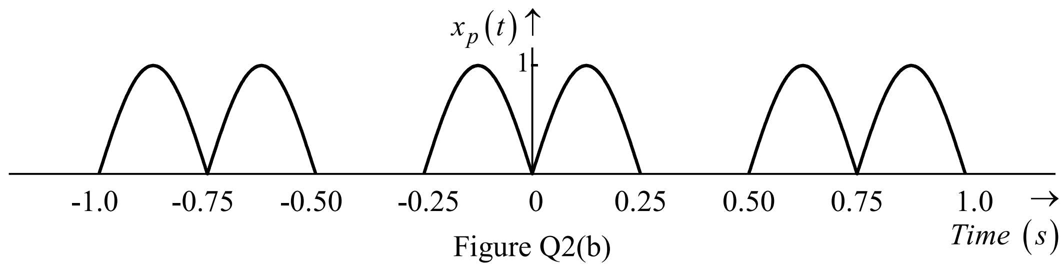

Figure Q2(b)

i. Derive an expression for $x_p(t)$ in terms of $x(t)$. (2 marks)

ii. Determine the Fourier transform, $X_p(f)$, of the periodic signal $x_p(t)$. (4 marks)

Q.2 ANSWER

Q.2 ANSWER ~ continued

Q.3 The Fourier transform, $X(f)$, of a signal $x(t)$ is given in Figure Q3.

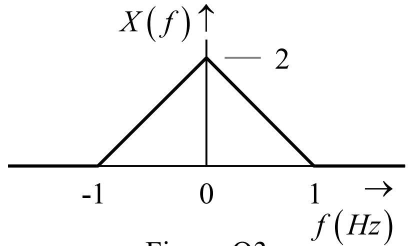

Figure Q3

(a) Find the signal $x(t)$. (3 marks)

(b) Determine the energy of $x(t)$. (3 marks)

(c) If $Y(f) = X(f) \times \sum_{k=-\infty}^{\infty} \delta(f - 0.4k)$, find the inverse Fourier transform, $y(t)$, of $Y(f)$.

(4 marks)

Q.3 ANSWER

Q.3 ANSWER ~ continued

Q.4 A rectangular pulse $x(t) = 2\ \text{rect}\left(\frac{t-2}{4}\right)$ is sent through a matched filter to produce $y(t)$ as shown in Figure Q4. The impulse response of the matched filter is given by

$$h(t) = 4\ \text{rect}\left(\frac{t-4}{4}\right).$$

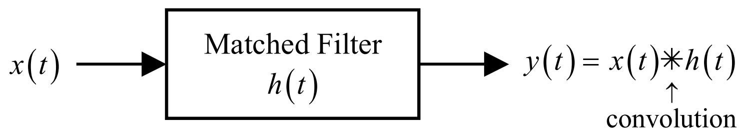

Figure Q4

(a) Find the spectrum, $Y(f)$, of $y(t)$. Hence, determine the 1st-null bandwidth of $y(t)$.

(5 marks)

(b) Find the maximum value of $y(t)$ and the time at which this maximum occurs. (5 marks)

Q.4 ANSWER

Q.4 ANSWER ~ continued

This page is intentionally left blank to facilitate detachment of the formula sheet for easy reference. Anything written on this page will not be graded.

Fourier Series: $\begin{cases} X_k = \dfrac{1}{T}\displaystyle\int_{\tilde{t}}^{\tilde{t}+T} x(t)\exp(-j2\pi k\,t/T)\,dt \\ x(t) = \displaystyle\sum_{k=-\infty}^{\infty} X_k \exp(j2\pi k\,t/T) \end{cases}$

Fourier Transform: $\begin{cases} X(f) = \displaystyle\int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)\,dt \\ x(t) = \displaystyle\int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)\,df \end{cases}$

FOURIER TRANSFORMS OF BASIC FUNCTIONS

	$x(t)$	$X(f)$
Constant	K	$K\delta(f)$
Unit Impulse	$\delta(t)$	**1**
Unit Step	$u(t)$	$\frac{1}{2}\left[\delta(f)+\frac{1}{j\pi f}\right]$
Sign (or Signum)	$\operatorname{sgn}(t)$	$\frac{1}{j\pi f}$
Rectangle	$\operatorname{rect}\left(\frac{t}{T}\right)$	$T\operatorname{sinc}(fT)$
Triangle	$\operatorname{tri}\left(\frac{t}{T}\right)$	$T\operatorname{sinc}^2(fT)$
Sine Cardinal	$\operatorname{sinc}\left(\frac{t}{T}\right)$	$T\operatorname{rect}(fT)$
Complex Exponential	$\exp(j2\pi f_o t)$	$\delta(f-f_o)$
Cosine	$\cos(2\pi f_o t)$	$\frac{1}{2}\left[\delta(f-f_o)+\delta(f+f_o)\right]$
Sine	$\sin(2\pi f_o t)$	$-\frac{j}{2}\left[\delta(f-f_o)-\delta(f+f_o)\right]$
Gaussian	$\exp\left(-\frac{t^2}{\alpha^2}\right)$	$\alpha\pi^{0.5}\exp(-\alpha^2\pi^2 f^2)$
Comb	$\sum_{m=-\infty}^{\infty}\delta(t-mT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$

FOURIER TRANSFORM PROPERTIES

	Time-domain	Frequency-domain
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(f)+\beta X_2(f)$
Time scaling	$x(\beta t)$	$\frac{1}{\lvert\beta\rvert}X\left(\frac{f}{\beta}\right)$
Duality	$X(t)$	$x(-f)$
Time shifting	$x(t-t_o)$	$X(f)\exp(-j2\pi f t_o)$
Frequency shifting (Modulation)	$x(t)\exp(j2\pi f_o t)$	$X(f-f_o)$
Differentiation in the time-domain	$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n X(f)$
Multiplication in the time-domain	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty}X_1(\zeta)X_2(f-\zeta)d\zeta$ or $X_1(f)\ast X_2(f)$
Convolution in the time-domain	$\int_{-\infty}^{\infty}x_1(\zeta)x_2(t-\zeta)d\zeta$ or $x_1(t)\ast x_2(t)$	$X_1(f)X_2(f)$
Integration in the time-domain	$\int_{-\infty}^{t}x(\tau)d\tau$	$\frac{1}{j2\pi f}X(f)+\frac{1}{2}X(0)\delta(f)$
		$\frac{1}{j2\pi f}X(f)$ **if** $X(0)=0$

Trigonometric Identities	
$\exp(\pm j\theta) = \cos(\theta) \pm j\sin(\theta)$	$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$
$\cos(\theta) = 0.5\left[\exp(j\theta) + \exp(-j\theta)\right]$	$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
$\sin(\theta) = -0.5j\left[\exp(j\theta) - \exp(-j\theta)\right]$	$\tan(\alpha \pm \beta) = \dfrac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$
$\sin^2(\theta) + \cos^2(\theta) = 1$	
$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$	$\sin(\alpha)\sin(\beta) = 0.5\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$
$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$	$\cos(\alpha)\cos(\beta) = 0.5\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$
$\sin^2(\theta) = 0.5\left[1 - \cos(2\theta)\right]$	$\sin(\alpha)\cos(\beta) = 0.5\left[\sin(\alpha - \beta) + \sin(\alpha + \beta)\right]$
$\cos^2(\theta) = 0.5\left[1 + \cos(2\theta)\right]$	$C\cos(\theta) - S\sin(\theta) = \sqrt{C^2 + S^2}\cos\left[\theta + \tan^{-1}(S/C)\right]$

Complex Unit (j) ➔ $\left(j = \sqrt{-1} = e^{j\pi/2} = e^{j90^\circ}\right)$ $\left(-j = \dfrac{1}{j} = e^{-j\pi/2} = e^{-j90^\circ}\right)$ $\left(j^2 = -1\right)$

Definitions of Basic Functions

Rectangle:

$$\operatorname{rect}\left(\frac{t}{T}\right) = \begin{cases} 1; & -T/2 \le t < T/2 \\ 0; & \text{elsewhere} \end{cases}$$

Triangle:

$$\operatorname{tri}\left(\frac{t}{T}\right) = \begin{cases} 1 - |t|/T; & |t| \le T \\ 0; & |t| > T \end{cases}$$

Sine Cardinal:

$$\operatorname{sinc}\left(\frac{t}{T}\right) = \begin{cases} \dfrac{\sin(\pi t/T)}{\pi t/T}; & t \ne 0 \\ 1; & t = 0 \end{cases}$$

Signum:

$$\operatorname{sgn}(t) = \begin{cases} 1; & t \ge 0 \\ -1; & t < 0 \end{cases}$$

Unit Impulse:

$$\delta(t) = \begin{cases} \infty; & t = 0 \\ 0; & t \ne 0 \end{cases} \qquad \int_{0^-}^{0^+} \delta(t)\,dt = 1$$

Unit Step:

$$u(t) = \begin{cases} 1; & t \ge 0 \\ 0; & t < 0 \end{cases}$$