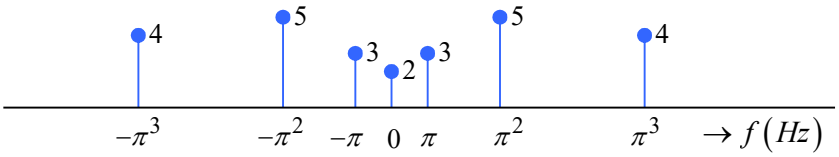
EE2023 TUTORIAL 2 (PROBLEMS)

Q.1 The discrete-frequency spectrum of a signal $x(t)$ is shown in Fig.Q.1.

Fig.Q.1



(a) What is the dc value of $x(t)$?
(b) Is $x(t)$ a power or energy signal?
(c) What is the Fourier series expansion of $x(t)$

Q.2 Determine whether or not each of the following signals is periodic. If the signal is periodic, determine its fundamental frequency and period.

(a) $x(t) = \cos(3.2t) + \sin(1.6t) + \exp(j2.8t)$

(b) $x(t) = \cos(4t) + \sin(\pi t)$

Q.3 (a) Determine the Fourier series coefficients of $x(t) = 6\sin(12\pi t) + 4\exp\left(j\left(8\pi t + \pi/4\right)\right) + 2$

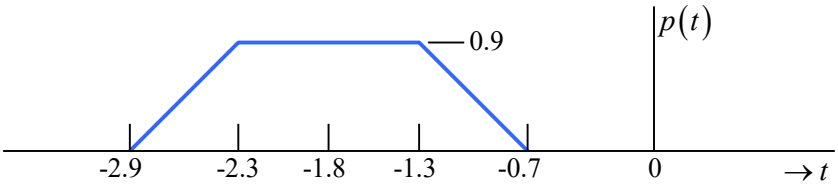
(b) Find the frequency of the 4th harmonic of $x(t) = 0.5\left(\left|\sin(\pi t)\right| + \sin(\pi t)\right)$

Q.4 Determine the Fourier series coefficients of

$$x(t) = \sum_{n=-\infty}^{\infty} 2p(t - 1.6n)$$

where $p(t)$ is given in Fig.Q.3.

Fig.Q.3



Q.5 The purpose of an analysis-synthesis system is to allow users to study the effects on a signal when certain parameters of the signal are modified. The analysis subsystem extracts the signal parameters of interest. These parameters are then altered in some desired manner and then presented to the synthesis subsystem which uses them to synthesize a modified signal.

A Fourier series analysis-synthesis system is given in Fig.Q.4.

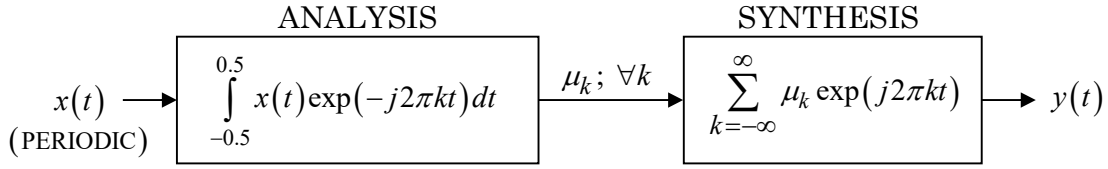


Fig.Q.4

(a) What does the analysis subsystem do?

(b) What does the synthesis subsystem do?

(c) Let $x(t)=\cos(3\pi t)$. Simply based on your understanding of the Fourier series, sketch $y(t)$ without performing any computation.

Q.6 $x(t)$, $y(t)$ and $z(t) = x(t) + y(t)$ are three periodic signals with Fourier series coefficients X_k, Y_k and Z_k, respectively. Explain whether or not $z(t) = x(t) + y(t)$ implies $Z_k = X_k + Y_k$.

Supplementary Problems
These problems will not be discussed in class.

S.1 Consider a rectified sine wave signal $x(t)$ defined by

$$x(t) = |\sin(\pi t)|.$$

(a) Sketch $x(t)$ and find its fundamental period.

(b) Find the complex exponential Fourier series of $x(t)$.

(c) Find the trigonometric Fourier series of $x(t)$.

Answer: *(a)* $period = 1$ *(b)* $x(t) = -\frac{2}{\pi}\sum_{k=-\infty}^{\infty}\frac{1}{4k^2-1}\exp(j2\pi kt)$

(c) $x(t) = \frac{2}{\pi} - \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{4k^2-1}\cos(2\pi kt)$

S.2 Find the complex exponential Fourier series of a periodic signal $x(t)$ defined by

$$x(t) = t^2; \quad -\pi < t < \pi \quad \text{and} \quad x(t+2\pi) = x(t).$$

Answer: $x(t) = \frac{\pi^2}{3} + 4\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}\cos(kt)$

S.3 The harmonic form Fourier series of a *real* periodic signal $x(t)$ with fundamental period T_0 is given by

$$x(t) = C_0 + \sum_{k=1}^{\infty} C_k \cos\left(2\pi\frac{k}{T_0}t - \theta_k\right)$$

where C_0 is known as the dc component, and the term $C_k\cos\left(2\pi\frac{k}{T_0}t - \theta_k\right)$ is referred to as the *kth-harmonic component* of $x(t)$. Express C_0, C_k and θ_k in terms of the complex exponential Fourier series coefficients X_k of $x(t)$.

Answer: $C_0 = X_0, \quad C_k = 2|X_k|, \quad \theta_k = -\tan^{-1}\left(\frac{\mathrm{Im}[X_k]}{\mathrm{Re}[X_k]}\right)$

Below is a list of solved problems selected from ***Chapter 5*** *of* ***Hwei Hsu (PhD), 'The Schaum's series on Signals & Systems,' 2nd Edition****.*

Selected solved-problems: ***5.4-to-5.13***

These solved problems should be treated as supplementary module material catered for students who find the need for more examples or practice-problems.
