NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester I : 2015/2016)

EE2023 – SIGNALS & SYSTEMS

Nov/Dec 2015 - Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES

1. This paper contains **EIGHT (8)** questions and comprises **ELEVEN (11)** printed pages.
2. Answer **ALL** questions in **Section A** and **ANY THREE (3)** questions in **Section B**.
3. This is a **CLOSED BOOK** examination.
4. Programmable calculators are not allowed.
5. Tables of Fourier Transforms, Laplace Transforms and trigonometric identities are provided in Pages 9, 10 and 11, respectively.

SECTION A : Answer ALL questions in this section

Q1. A rotating load is connected to a DC motor which has a model given by:

$$\frac{\Omega(s)}{V(s)} = \frac{K}{s\tau + 1},$$

where $\Omega(s) = L\{\omega(t)\}$ and $V(s) = L\{v(t)\}$ denote the Laplace transforms of the motor speed, $\omega(t)$ in rad/s and the input voltage, $v(t)$ in volts, respectively.

A test results in the output load reaching a speed of 1 rad/s within 0.5 seconds when a constant input of 100 V is applied to the motor terminals. The steady state output speed from the same test is found to be 2 rad/s.

(a) Determine the parameters, K and τ of the motor model.

(6 marks)

(b) Sketch the output response derived from the test, clearly indicating the quantities obtained from the test result.

(4 marks)

Q2. The Fourier transform, $X(f)$ of the signal $x(t)$ is a half-cosine shaped amplitude spectrum as shown in Figure Q2.

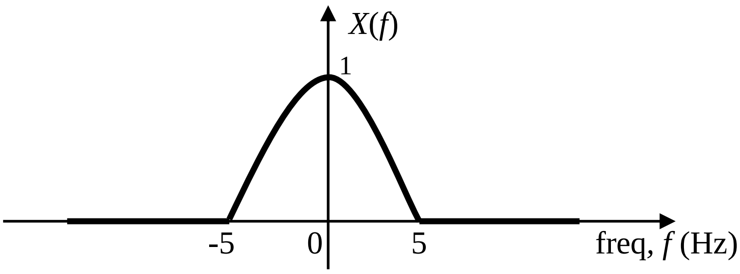

Figure Q2

(a) Provide the expression for $X(f)$ in terms of the frequency f.

(2 marks)

(b) Derive the energy of the signal $x(t)$.

(6 marks)

(c) The signal $x(t)$ is sampled at 15 Hz. Sketch the amplitude spectrum of the sampled signal.

(2 marks)

Q3. Consider the signal $x(t)$ given by:

$$x(t) = 3 + je^{-j14t} + \cos\left(8t + \frac{\pi}{4}\right) + (2+3j)e^{j6t} - je^{j14t}$$

(a) What is the fundamental frequency of $x(t)$?

(2 marks)

(b) Obtain the Fourier series coefficients of $x(t)$.

(4 marks)

(c) Sketch the magnitude and phase spectra of $x(t)$.

(4 marks)

Q4. Figure Q4 shows the input signal, $x(t)$, and output signal, $y(t)$, of a system with a transfer function, $G(s)$.

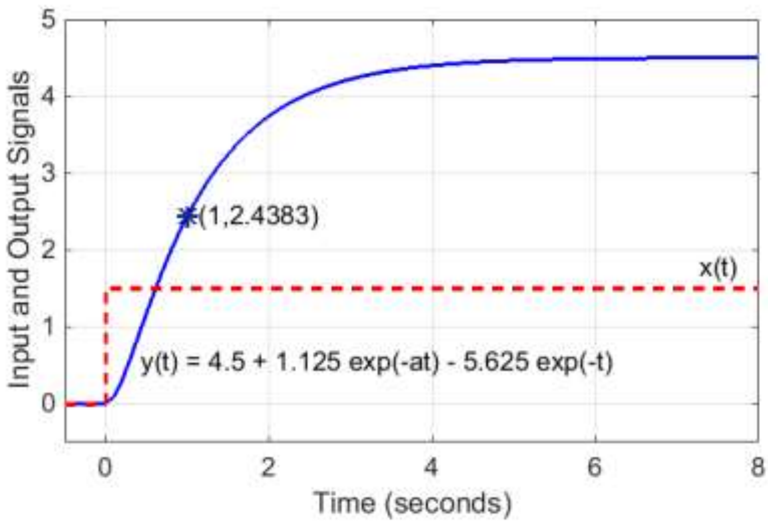

Figure Q4 : Input signal (dashed) and Output signal (solid)

(a) Write down an expression for the input signal, $x(t)$.

(2 marks)

(b) What is the DC gain of the system?

(2 marks)

(c) Find the poles of the system, $G(s)$. Round the numerical values to the nearest integer.

(3 marks)

(d) Suppose $G(s)$ is cascaded with an unity gain differentiator to form the new system $H(s) = sG(s)$. Determine the output signal of the new system, $H(s)$, corresponding to the input $x(t)$ shown in Figure Q4.

(3 marks)

SECTION B : Answer 3 out of the 4 questions in this section

Q5. Consider a control system with a transfer function given by:

$$G(s) = \frac{K_1 s + K_2}{s^2 + 2s(1+K_1) + 2K_2}.$$

The goal is to design the parameters, K_1 and K_2, such that the system achieves certain dynamical characteristics. The strategy to achieve this is to design such that the poles of $G(s)$ lie in the shaded regions in Figure Q5 below.

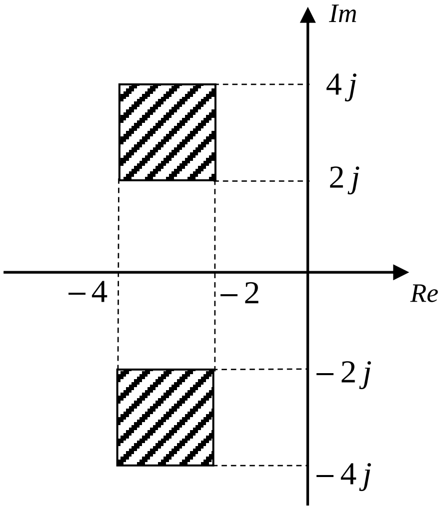

Figure Q5 : Regions where Desired Poles Lie

(a) Given the desired pole locations in Figure Q5, determine the ranges of values of damping ratio, ς, and natural frequency, ω_n, that are achievable by the control system. (4 marks)

(b) Give two possible sets of poles which will achieve a damping ratio of $\varsigma = 0.707$.

(4 marks)

(c) Determine K_1 and K_2 such that the lowest achievable damping ratio is attained by the control system.

(6 marks)

(d) Determine K_1 and K_2 such that the system output has the fastest decay in its transient response and has a damping ratio of $\varsigma = 0.707$.

(6 marks)

Q.6 Suppose $x(t) = 2\,\mathrm{sinc}(2t)$, $y(t) = \left[\sum_{k=-\infty}^{\infty} 2\,\mathrm{rect}\left(\frac{t-4k}{2}\right)\right] - 1$ and $z(t) = x(t) \otimes y(t)$, where the symbol $\otimes$ denotes the convolution operator.

(a) Determine the Fourier transforms $X(f)$, $Y(f)$ and $Z(f)$ of $x(t)$, $y(t)$ and $z(t)$, respectively, and sketch their corresponding amplitude spectra.

(14 marks)

(b) Determine the average power of $z(t)$.

(3 marks)

(c) If $z(t)$ is sampled at a frequency of 2.5 Hz, sketch the amplitude spectrum of the sampled signal.

(3 marks)

Q7. (a) The amplitude spectrum of the signal $x(t)$ is shown in Figure Q7-1, and the signal $y(t) = x(t)\cos(40\pi t)$ is sampled at a frequency of 20 Hz to obtain the sampled signal $y_s(t)$.

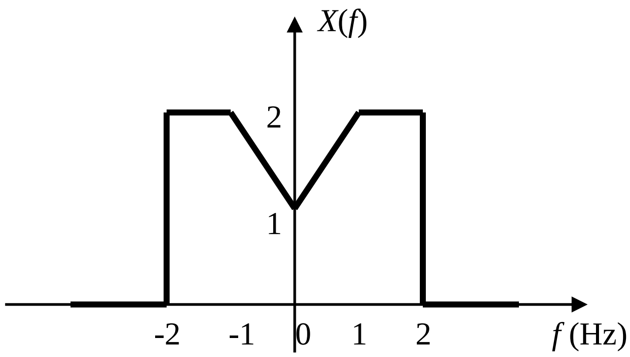

Figure Q7-1

i. What is the Nyquist frequency for signal $y(t)$?

(2 marks)

ii. The signal $y(t)$ is sampled at 20 Hz to give the signal $y_s(t)$. Determine the Fourier transform, $Y_s(f)$, of the sampled signal $y_s(t)$ and sketch its magnitude spectrum.

(6 marks)

iii. Can the signal $y(t)$ be recovered from the sampled signal $y_s(t)$? Explain your answer.

(2 marks)

Question 7 continues on Page 6.

(b) The amplitude spectrum of the signal $a(t)$ is shown in Figure Q7-2. Assume that the phase spectrum is zero.

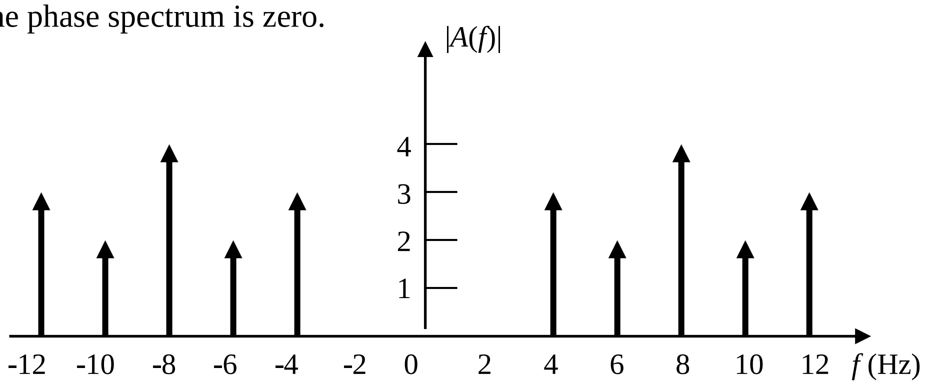

Figure Q7-2

i. Derive the signal $a(t)$.

(4 marks)

ii. Is $a(t)$ a power or an energy signal? Find the corresponding power or energy?

(4 marks)

iii. What is the bandwidth of the signal $a(t)$?

(2 marks)

Q8. A single time-constant (STC) circuit is shown in Figure Q8-1.

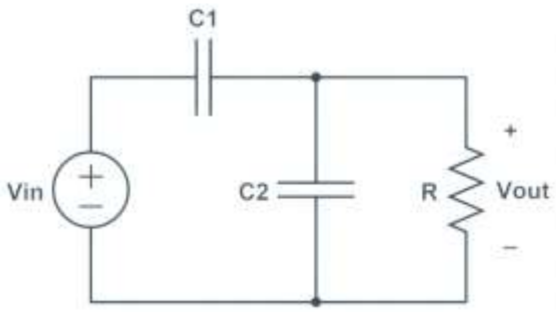

Figure Q8-1 : STC circuit

(a) Derive the transfer function of the STC circuit, $G(s) = \dfrac{V_{out}(s)}{V_{in}(s)}$.

(6 marks)

(b) The Bode Magnitude diagram of the STC circuit when $C_1 = 0.5\mu F$ is shown in Figure Q8-2.

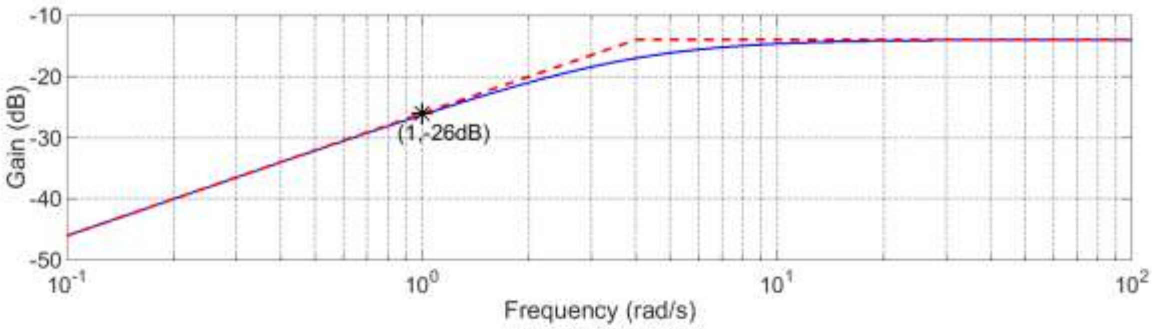

Figure Q8-2 : Bode Magnitude diagram of STC circuit when $C_1 = 0.5\mu F$

i. Find R and C_2.

Hint : Equation of the low frequency asymptote is $20\log_{10} \omega RC_1$.

(6 marks)

ii. What is the gain of the STC circuit at frequencies higher than 10 rad/s ?

(2 marks)

(c) Derive an expression for the phase response of the STC circuit, $\angle G(j\omega)$. Hence, or otherwise, sketch the Bode phase diagram, clearly labelling the phase at low and high frequencies.

(6 marks)

END OF QUESTIONS

This page is intentionally left blank to facilitate detachment of the formula sheet for easy reference.

Fourier Series: $$\begin{cases} X_k = \dfrac{1}{T}\displaystyle\int_{\tilde{t}}^{\tilde{t}+T} x(t)\exp(-j2\pi k\,t/T)\,dt \\ x(t) = \sum_{k=-\infty}^{\infty} X_k \exp(j2\pi k\,t/T) \end{cases}$$

Fourier Transform: $$\begin{cases} X(f) = \displaystyle\int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)\,dt \\ x(t) = \displaystyle\int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)\,df \end{cases}$$

FOURIER TRANSFORMS OF BASIC FUNCTIONS

	$x(t)$	$X(f)$
Constant	K	$K\delta(f)$
Unit Impulse	$\delta(t)$	**1**
Unit Step	$u(t)$	$\frac{1}{2}\left[\delta(f)+\frac{1}{j\pi f}\right]$
Sign (or Signum)	$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
Rectangle	$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}(fT)$
Triangle	$\mathrm{tri}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}^2(fT)$
Sine Cardinal	$\mathrm{sinc}\left(\frac{t}{T}\right)$	$T\,\mathrm{rect}(fT)$
Complex Exponential	$\exp(j2\pi f_o t)$	$\delta(f-f_o)$
Cosine	$\cos(2\pi f_o t)$	$\frac{1}{2}\left[\delta(f-f_o)+\delta(f+f_o)\right]$
Sine	$\sin(2\pi f_o t)$	$-\frac{j}{2}\left[\delta(f-f_o)-\delta(f+f_o)\right]$
Gaussian	$\exp\left(-\frac{t^2}{\alpha^2}\right)$	$\alpha\pi^{0.5}\exp\left(-\alpha^2\pi^2 f^2\right)$
Comb	$\sum_{m=-\infty}^{\infty}\delta(t-mT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$

FOURIER TRANSFORM PROPERTIES

	Time-domain	**Frequency-domain**
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(f)+\beta X_2(f)$
Time scaling	$x(\beta t)$	$\frac{1}{\lvert\beta\rvert}X\left(\frac{f}{\beta}\right)$
Duality	$X(t)$	$x(-f)$
Time shifting	$x(t-t_o)$	$X(f)\exp(-j2\pi ft_o)$
Frequency shifting (Modulation)	$x(t)\exp(j2\pi f_o t)$	$X(f-f_o)$
Differentiation in the time-domain	$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n X(f)$
Multiplication in the time-domain	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty}X_1(\zeta)X_2(f-\zeta)\,d\zeta$ or $X_1(f)\ast X_2(f)$
Convolution in the time-domain	$\int_{-\infty}^{\infty}x_1(\zeta)x_2(t-\zeta)\,d\zeta$ or $x_1(t)\ast x_2(t)$	$X_1(f)X_2(f)$
Integration in the time-domain	$\int_{-\infty}^{t}x(\tau)\,d\tau$	$\frac{1}{j2\pi f}X(f)+\frac{1}{2}X(0)\delta(f)$
		$\frac{1}{j2\pi f}X(f)$ **if** $X(0)=0$

Unilateral Laplace Transform: $X(s) = \int_{0^-}^{\infty} x(t)\exp(-st)\,dt$

LAPLACE TRANSFORMS OF BASIC FUNCTIONS		
	$x(t)$	$X(s)$
Unit Impulse	$\delta(t)$	1
Unit Step	$u(t)$	$1/s$
Ramp	$t\,u(t)$	$1/s^2$
n^{th} order Ramp	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
Damped Ramp	$t\exp(-\alpha t)u(t)$	$1/(s+\alpha)^2$
Exponential	$\exp(-\alpha t)u(t)$	$1/(s+\alpha)$
Cosine	$\cos(\omega_o t)u(t)$	$s/(s^2+\omega_o^2)$
Sine	$\sin(\omega_o t)u(t)$	$\omega_o/(s^2+\omega_o^2)$
Damped Cosine	$\exp(-\alpha t)\cos(\omega_o t)u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_o^2}$
Damped Sine	$\exp(-\alpha t)\sin(\omega_o t)u(t)$	$\frac{\omega_o}{(s+\alpha)^2+\omega_o^2}$

LAPLACE TRANSFORM PROPERTIES		
	Time-domain	**s-domain**
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(s)+\beta X_2(s)$
Time shifting	$x(t-t_o)u(t-t_o)$	$\exp(-st_o)X(s)$
Shifting in the s-domain	$\exp(s_o t)x(t)$	$X(s-s_o)$
Time scaling	$x(\alpha t)$	$\frac{1}{\lvert\alpha\rvert}X\left(\frac{s}{\alpha}\right)$
Integration in the time-domain	$\int_{0^-}^{t} x(\zeta)\,d\zeta$	$\frac{1}{s}X(s)$
Differentiation in the time-domain	$\frac{dx(t)}{dt}$	$sX(s)-x(0^-)$
	$\frac{d^n x(t)}{dt^n}$	$s^n X(s)-\sum_{k=0}^{n-1} s^{n-1-k}\left.\frac{d^k x(t)}{dt^k}\right\rvert_{t=0^-}$
Differentiation in the s-domain	$-tx(t)$	$\frac{dX(s)}{ds}$
	$(-t)^n x(t)$	$\frac{d^n X(s)}{ds^n}$
Convolution in the time-domain	$\int_{-\infty}^{\infty} x_1(\zeta)x_2(t-\zeta)\,d\zeta$	$X_1(s)X_2(s)$
Initial value theorem	$x(0^+)=\lim_{s\to\infty} sX(s)$	
Final value theorem	$\lim_{t\to\infty} x(t)=\lim_{s\to 0} sX(s)$	

	$y_{step}(t)$	$Y_{step}(s)$	**SYSTEM PARAMETERS**
Step response of 1st order system	$K\left[1-\exp\left(-\frac{t}{T}\right)\right]u(t)$	$\frac{1}{s}\cdot\frac{K}{(sT+1)}$	$\begin{pmatrix} T: \text{ System Time-constant} \\ K: \text{ System Steady-state (or DC) Gain} \end{pmatrix}$
Step response of 2nd order <u>underdamped</u> system: $(\mathbf{0}<\boldsymbol{\zeta}<\mathbf{1})$	$K\left[1-\frac{\exp(-\omega_n\zeta t)}{(1-\zeta^2)^{0.5}}\sin\left(\omega_n(1-\zeta^2)^{0.5}t+\phi\right)\right]u(t)$	$\frac{1}{s}\cdot\frac{K\omega_n{}^2}{s^2+2\zeta\omega_n s+\omega_n{}^2}$	$\begin{pmatrix} \omega_n: \text{ System Undamped Natural Frequency} \\ \zeta: \text{ System Damping Factor} \\ \omega_d: \text{ System Damped Natural Frequency} \\ K: \text{ System Steady-state (or DC) Gain} \end{pmatrix}$ $\begin{pmatrix} \sigma=\omega_n\zeta \\ \omega_d^2=\omega_n^2(1-\zeta^2) \\ \omega_n^2=\sigma^2+\omega_d^2 \\ \tan(\phi)=\omega_d/\sigma \end{pmatrix}$
	$K\left[1-\left(\frac{\sigma^2+\omega_d^2}{\omega_d^2}\right)^{0.5}\exp(-\sigma t)\sin(\omega_d t+\phi)\right]u(t)$	$\frac{1}{s}\cdot\frac{K(\sigma^2+\omega_d{}^2)}{(s+\sigma)^2+\omega_d{}^2}$	
2nd order system - RESONANCE - $(\mathbf{0}\le\boldsymbol{\zeta}<\mathbf{1}/\sqrt{\mathbf{2}})$	*RESONANCE FREQUENCY :* $\omega_r=\omega_n(1-2\zeta^2)^{0.5}$		*RESONANCE PEAK :* $M_r=\lvert H(j\omega_r)\rvert=\frac{K}{2\zeta(1-\zeta^2)^{0.5}}$

TRIGONOMETRIC IDENTITIES	
$\exp(\pm j\theta)=\cos(\theta)\pm j\sin(\theta)$	$\sin(\alpha\pm\beta)=\sin(\alpha)\cos(\beta)\pm\cos(\alpha)\sin(\beta)$
$\cos(\theta)=\frac{1}{2}\left[\exp(j\theta)+\exp(-j\theta)\right]$	$\cos(\alpha\pm\beta)=\cos(\alpha)\cos(\beta)\mp\sin(\alpha)\sin(\beta)$
$\sin(\theta)=\frac{1}{j2}\left[\exp(j\theta)-\exp(-j\theta)\right]$	$\tan(\alpha\pm\beta)=\frac{\tan(\alpha)\pm\tan(\beta)}{1\mp\tan(\alpha)\tan(\beta)}$
$\sin^2(\theta)+\cos^2(\theta)=1$	
$\sin(2\theta)=2\sin(\theta)\cos(\theta)$	$\sin(\alpha)\sin(\beta)=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$
$\cos(2\theta)=\cos^2(\theta)-\sin^2(\theta)$	$\cos(\alpha)\cos(\beta)=\frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$
$\sin^2(\theta)=\frac{1}{2}\left[1-\cos(2\theta)\right]$	$\sin(\alpha)\cos(\beta)=\frac{1}{2}\left[\sin(\alpha-\beta)+\sin(\alpha+\beta)\right]$
$\cos^2(\theta)=\frac{1}{2}\left[1+\cos(2\theta)\right]$	$\mathbf{C}\cos(\theta)-\mathbf{S}\sin(\theta)=\sqrt{\mathbf{C}^2+\mathbf{S}^2}\cos\left[\theta+\tan^{-1}\left(\frac{\mathbf{S}}{\mathbf{C}}\right)\right]$