SECTION B : Answer 3 out of the 4 questions in this section

Q5. Consider the circuit in Figure Q5 below. Assume zero initial conditions in all cases.

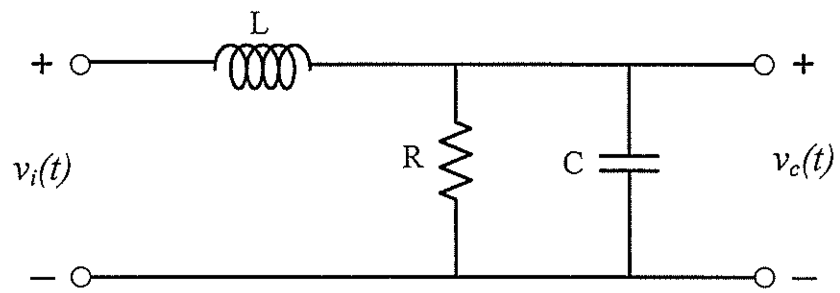

Figure Q5 : R-L-C Circuit

(a) Derive the transfer function, $G(s)=\dfrac{V_c(s)}{V_i(s)}$, where $V_i(s)=L\{v_i(t)\}$ and $V_c(s)=L\{v_c(t)\}$ are the Laplace transforms of $v_i(t)$ and $v_c(t)$ respectively.

(4 marks)

(b) Find the unit impulse response of the circuit if $LC = RC = 0.25$.

(4 marks)

(c) Find the total response of the voltage across the capacitor if $v_i(t)=u(t)$ where $u(t)$ is the unit step function. Assume $LC = RC = 0.25$.

(4 marks)

(d) Based on the transfer function from part (a), what type of system do you get if $R=\infty$? Justify your answer.

(4 marks)

(e) Sketch the unit step response of the circuit if $LC = 0.25$ and $R=\infty$. Label your sketch appropriately.

(4 marks)

Q5a) Figure Q5-1 shows an electrical circuit with $L = 1\text{mH}$ and $C = 100\text{pF}$. The inductor current and capacitor voltage are both equal to zero at time, $t = 0$.

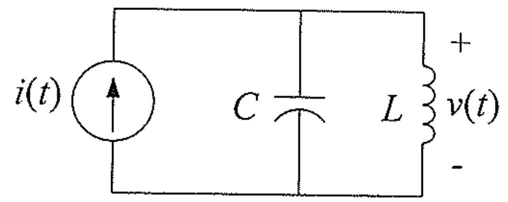

Figure Q5-1: Parallel LC circuit

(i) Derive the transfer function, $G(s) = \dfrac{V(s)}{I(s)}$, of the circuit where $I(s) = L\{i(t)\}$ and $V(s) = L\{v(t)\}$ are the Laplace transforms of the input current, $i(t)$, and output voltage, $v(t)$ respectively. Derive $G(s)$ in the form of :

$$G(s) = \frac{as}{s^2 + b}.$$

(4 marks)

(ii) Find the voltage, $v(t)$, if the current source, $i(t)$, is given by $i(t) = t$, $t \geq 0$. Sketch the voltage, $v(t)$, for $t \geq 0$. Label the quantities in your sketch clearly.

(8 marks)

(iii) Re-design the circuit such that the frequency of oscillation of the voltage across the inductor is 1000 rad/s. You may choose to change either L or C in the new design.

(3 marks)

Q5b) The unit impulse response of a system, $G_1(s)$ is shown in Figure Q5-2. Find the transfer function of $G_1(s)$.

(5 marks)

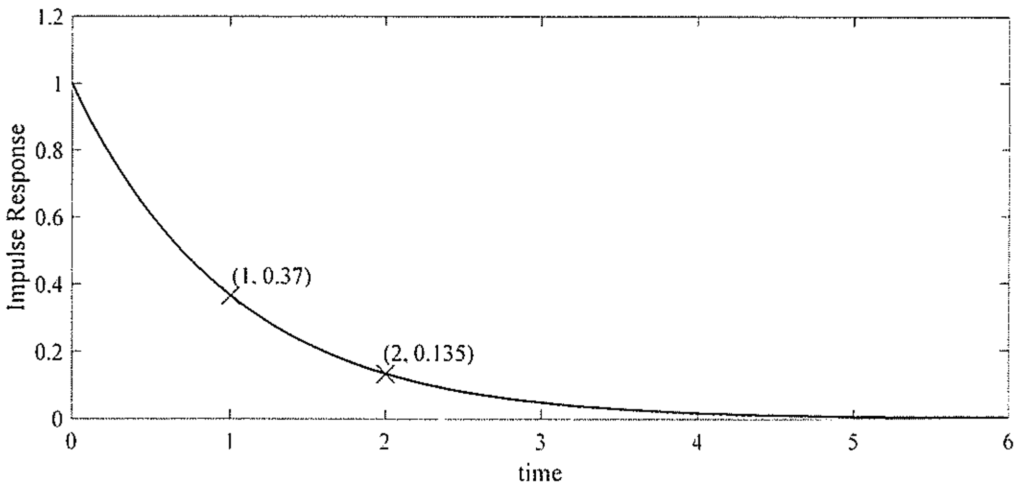

Figure Q5-2 : Unit Impulse Response

Q8. A single time-constant (STC) circuit is shown in Figure Q8-1.

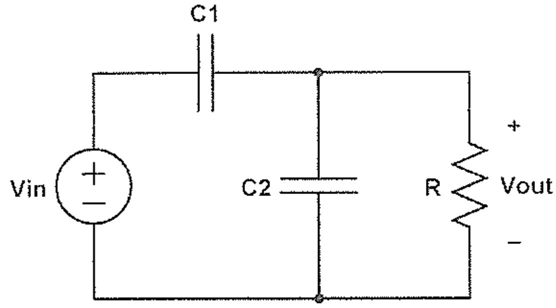

Figure Q8-1 : STC circuit

(a) Derive the transfer function of the STC circuit, $G(s)=\dfrac{V_{out}(s)}{V_{in}(s)}$.

(6 marks)

(b) The Bode Magnitude diagram of the STC circuit when $C_1 = 0.5\mu F$ is shown in Figure Q8-2.

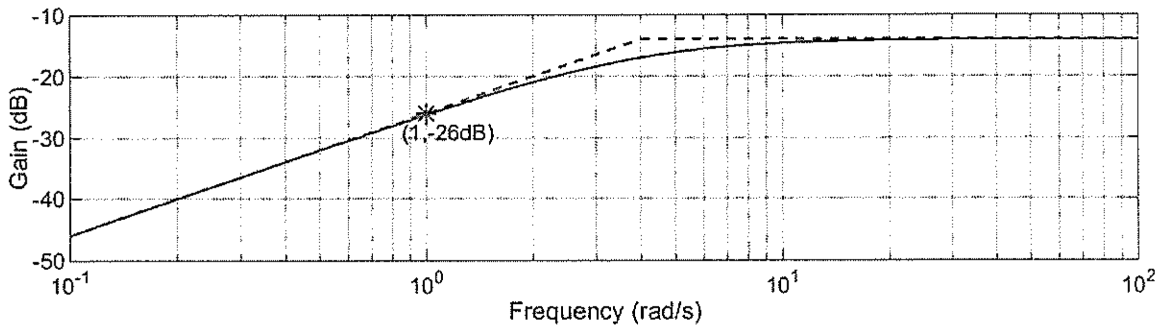

Figure Q8-2 : Bode Magnitude diagram of STC circuit when $C_1 = 0.5\mu F$

i. Find R and C_2.

Hint : Equation of the low frequency asymptote is $20\log_{10}\omega RC_1$.

(6 marks)

ii. What is the gain of the STC circuit at frequencies higher than 10 rad/s ?

(2 marks)

(c) Derive an expression for the phase response of the STC circuit, $\angle G(j\omega)$. Hence, or otherwise, sketch the Bode phase diagram, clearly labelling the phase at low and high frequencies.

(6 marks)

END OF QUESTIONS