Name :   **Group :**   **Matric # :**

EE2023 Signal & Systems (AY2020/21)

Due Date : 6 pm, 15 April 2021

In this assignment, you will analyze the behaviour of a booster rocket, and design a control system to automatically regulate the heading angle of the booster.

1. Dynamics of a booster rocket

A booster rocket (or engine) is either the first stage of a multi-stage launch vehicle, or a shorter-burning rocket used in parallel with longer-burning sustainer rockets to augment the space vehicle's takeoff thrust and payload capability. Figure 1 is an illustration of the booster rocket and the signals in the system.

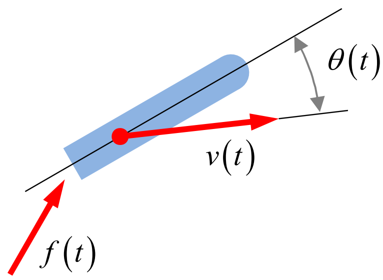

Figure 1: Booster rocket

The heading of the booster rocket, $\theta(t)$, following the application of a force, $f(t)$, may be represented by the following transfer function :

$$G(s) = \frac{\theta(s)}{F(s)} = \frac{1}{s^2 - \alpha} \tag{1}$$

where $\theta(s) = \mathcal{L}\{\theta(t)\}$ and $F(s) = \mathcal{L}\{f(t)\}$. For the purpose of this assignment, set the parameter α to $\frac{50+C}{1000}$, where C may be derived by reading the last 2 digits of your student matriculation number. For example, if your matriculation number is A0**1234*, then let $C = 34$.

i. What are the values of the system poles? Using the information that system poles provide about system behaviour, describe what happens to the space booster when an input signal is applied.

ii. Sketch the impulse response, $\theta(t)$.

2. Analysis of the booster rocket control system

A feedback control system needs to be developed to automatically manipulate the heading angle. Figure 2 on the next page shows the block diagram of the *automatic heading control system*. The control system uses a navigational sensor to measure the heading angle of the booster rocket, and the output signal of the controller, $f(t)$, is determined by the following expression

$$f(t) = K_p e(t) + K_d \frac{\mathrm{d}e(t)}{\mathrm{d}t} + K_i \int_0^t e(\tau)d\tau \tag{2}$$

where the heading angle error, $e(t) = \theta_r(t) - \theta(t)$, $\theta_r(t)$ is the desired heading of the rocket booster at time t. K_p, K_d and K_i are constants to be designed.

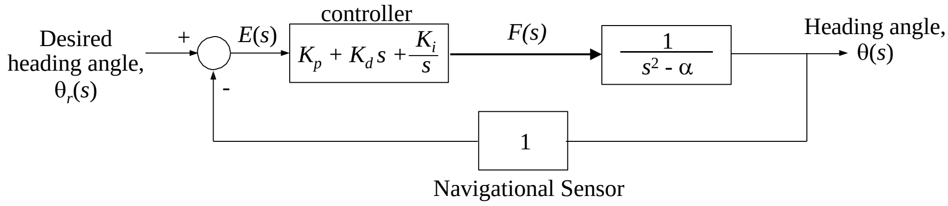

Figure 2: Automatic booster rocket heading control system

i. Derive the transfer function $G_r(s) = \dfrac{\theta(s)}{\theta_r(s)}$. What is the order of the booster rocket control system?

ii. Which of the 3 design parameters (K_p, K_d and K_i) should be set to zero for $G_r(s) = \dfrac{\theta(s)}{\theta_r(s)}$ to be a second order system? With this second order system, set the remaining two parameters to zero in turn. For each of the two scenarios, derive the range of values that will yield a bounded output signal.

iii. Design K_p, K_d and K_i such that the automatic heading control system is a second order system with system poles $s_{1,2} = -1 \pm 3j$.

iv. Sketch the response of the system designed in part iii when the desired heading angle is 30 degrees i.e. $\theta_r(t) = 30u(t)$ where $u(t)$ is the unit step function. The steady-state heading angle should be clearly labeled.