NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester I : 2014/2015)

EE2023 – SIGNALS & SYSTEMS

Nov/Dec 2014 - Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES

1. This paper contains **EIGHT (8)** questions and comprises **ELEVEN (11)** printed pages.
2. Answer **ALL** questions in **Section A** and **ANY THREE (3)** questions in **Section B**.
3. This is a **CLOSED BOOK** examination.
4. Programmable calculators are not allowed.
5. Tables of Fourier Transforms, Laplace Transforms and trigonometric identities are provided in Pages 9, 10 and 11, respectively.

SECTION A : Answer ALL questions in this section

Q1. Consider the system in Figure Q1 whose transfer function is given by $G(s) = \frac{1}{s}e^{-3s}$.

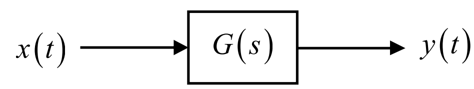

Figure Q1 : System, $G(s)$

(a) Derive the response of $G(s)$ to a unit impulse, $x(t) = \delta(t)$. Sketch the resulting impulse response.

(4 marks)

(b) <u>Derive</u> and <u>sketch</u> the response of $G(s)$ to an input, $x(t)$ given in Figure Q1. Label your sketch clearly.

(6 marks)

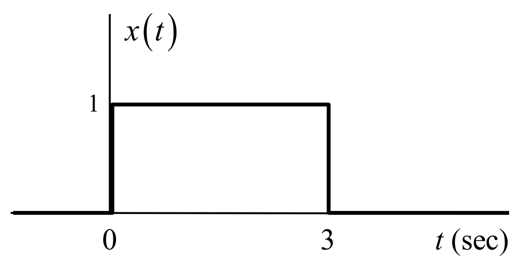

Figure Q1 : Input to $G(s)$

Q2. The signal $x(t) = \text{sinc}^2(2t)$ is sampled at 8 Hz to produce the sampled signal $x_s(t)$.

(a) Derive the expression for $x_s(t)$.

(3 marks)

(b) Derive the Fourier transform of $x_s(t)$.

(3 marks)

(c) Sketch the spectrum of $x_s(t)$.

(4 marks)

Q3. A energy signal $x(t)$ is given by

$$x(t) = \exp\left(-\pi t^2\right).$$

(a) Determine the energy spectral density, $E_x(f)$, of $x(t)$.

(4 marks)

(b) Find the 3 dB bandwidth of $x(t)$.

(3 marks)

(c) In computing the total energy of $x(t)$, would the time-domain or frequency-domain approach lead to a simpler solution, and why?

(3 marks)

Q4. The pole-zero diagrams for 2 systems (I and II) are shown in Figure Q4.

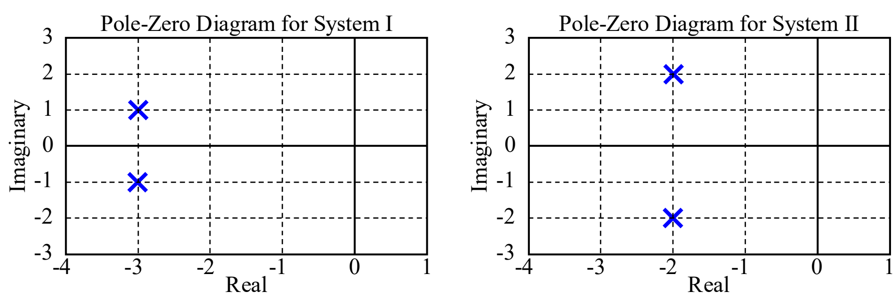

Figure Q4 : Pole-zero diagrams

(a) What is the damped natural frequency, undamped natural frequency and damping ratio of System I?

(3 marks)

(b) Following a step change of magnitude 3, the steady state output of System II is 15. Derive the transfer function of System II using the pole-zero diagram and the steady-state information.

(4 marks)

(c) Suppose Systems I and II have the same DC gain. Will System I or System II exhibit a larger overshoot following a step change in the input signal? Justify your answer.
Hint : Overshoot is governed by the damping ratio of the system.

(3 marks)

SECTION B : Answer 3 out of the 4 questions in this section

Q5. The space booster in Figure Q5 has a transfer function, $G(s)$, given by

$$\frac{\Phi(s)}{F(s)} = G(s) = \frac{1}{s^2 - 0.04}$$

where $\Phi(s)$ and $F(s)$ are Laplace transforms of $\phi(t)$ and $f(t)$, respectively.

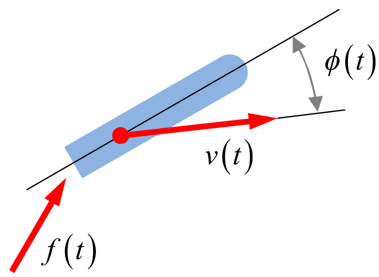

Figure Q5 : A Space Booster

(a) Describe what happens to the space booster when it is fired up with $f(t) = \delta(t)$, where $\delta(t)$ is a unit impulse function. Sketch the impulse response, $\phi(t)$.

(4 marks)

(b) A control system is then developed for the space booster such that the overall closed loop control system has a transfer function given by

$$G_{cl}(s) = \frac{K_p}{s^2 + K_D s + K_p - 0.04}.$$

i. If $K_D = 0$, what is the minimum value of K_p required for the closed loop system to have bounded outputs? Justify your answer.

(4 marks)

ii. If $K_p = 0$, why is K_D alone not able to stabilize the closed loop system?

(4 marks)

iii Design K_p and K_D so that the closed loop system has poles at $s_{1,2} = -0.2 \pm 0.3j$.

(4 marks)

iv Find the damping ratio and the undamped natural frequency of the closed loop system with poles at $s_{1,2} = -0.2 \pm 0.3j$. Sketch the impulse response of this closed loop system.

(4 marks)

Q.6 Consider the periodic signal $x(t)$ shown in Figure Q6 which comprises periodic Gaussian pulses, where

$$x(t) = \sum_{k=-\infty}^{\infty} e^{-(t-4k)^2/0.25}.$$

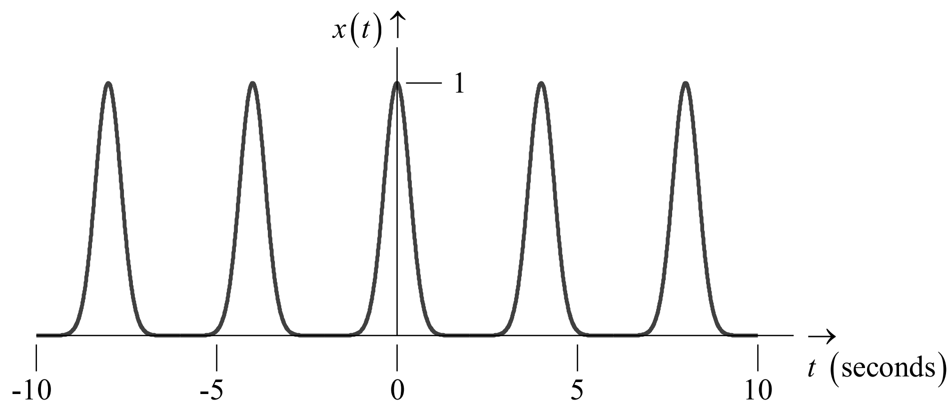

Figure Q6

(a) Derive the Fourier transform, $X(f)$, of $x(t)$.

(7 marks)

(b) Derive the Fourier series coefficient, X_k, of $x(t)$.

(3 marks)

(c) Derive an expression for the average power of $x(t)$.

(3 marks)

(d) Let the M^{th} harmonic of $x(t)$ be the harmonic that is closest to the 98% power containment bandwidth of $x(t)$. Explain how M could be found.

(7 marks)

Q7. Radio station X transmits the signal $x(t) = 10 \cdot \text{sinc}(10t) \cdot \cos(2000\pi t)$, and radio station Y transmits the signal

$$y(t) = m(t) \cdot \cos(2\pi f_c t)$$

where the spectrum of $m(t)$ is given by $M(f) = \text{tri}\left(\frac{f}{B}\right)$, and $f_c >> 2B$. Radio interference between the two stations will occur if the spectra of their transmissions overlap.

(a) Sketch of the spectrum, $X(f)$, of $x(t)$. Show all the important dimensions in your sketch.

(5 marks)

(b) Suppose $B = 8$. What is the range of f_c values that should be avoided by radio station Y so as to avoid radio interference between the two stations?

(5 marks)

(c) Suppose $f_c = 1020$. Find the maximum value of B that can be used by radio station Y without causing radio interference between the two stations.

(5 marks)

(d) Suppose $y(t)$ is sampled to form $y_s(t)$. Suggest a sampling frequency, f_s, so that $m(t)$ can be recovered without distortion by passing $y_s(t)$ through a suitably designed ideal lowpass filter. Explain your answer.

(5 marks)

Q8. Most loudspeakers are not capable of covering the entire audio spectrum with negligible distortion. Consequently, most hi-fi speaker systems use a combination of loudspeakers, each catering to a different frequency band. Filter systems are used to split the audio signal into frequency bands that can be separately routed to loudspeakers optimized for those bands i.e. a lowpass filter is used to isolate signals for the woofer loudspeaker and the output signal of a highpass filter drives the tweeter loudspeaker.

(a) Derive the transfer functions of the two filter circuits shown in Figure Q8.

(8 marks)

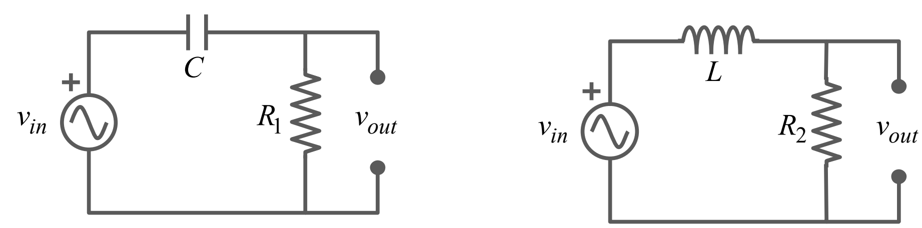

Figure Q8 : Series resistor-capacitor and resistor-inductor filtering circuits

(b) Sketch the straight line asymptotic Bode Magnitude diagrams of the resistor-capacitor (R_1C) and resistor-inductor (R_2L) filtering circuits, clearly labelling the corner frequencies and the slope of the asymptotes. Hence, or otherwise, determine if the output signal of the R_1C circuit should be used to drive the woofer or the tweeter loudspeaker?

(5 marks)

(c) A bandpass filtering system for generating audio signals in the mid-frequency range may be constructed by cascading the R_1C and R_2L filtering circuits shown in Figure Q8, and ensuring that $R_1C > \frac{L}{R_2}$.

i. Sketch the straight line asymptotic Bode Magnitude diagram of the bandpass filter, clearly labelling the corner frequencies.

(3 marks)

ii. Suppose $R_1 = R_2 = 8\Omega$ and the bandpass system should not distort signals between 800 Hz and 3000 Hz. Design suitable values for C and L.

(4 marks)

END OF QUESTIONS

This page is intentionally left blank to facilitate detachment of the formula sheet for easy reference.

Fourier Series: $\begin{cases} X_k = \dfrac{1}{T}\displaystyle\int_{\tilde{t}}^{\tilde{t}+T} x(t)\exp(-j2\pi k\,t/T)\,dt \\ x(t) = \displaystyle\sum_{k=-\infty}^{\infty} X_k \exp(j2\pi k\,t/T) \end{cases}$

Fourier Transform: $\begin{cases} X(f) = \displaystyle\int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)\,dt \\ x(t) = \displaystyle\int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)\,df \end{cases}$

FOURIER TRANSFORMS OF BASIC FUNCTIONS

	$x(t)$	$X(f)$
Constant	K	$K\delta(f)$
Unit Impulse	$\delta(t)$	**1**
Unit Step	$u(t)$	$\frac{1}{2}\left[\delta(f)+\frac{1}{j\pi f}\right]$
Sign (or Signum)	$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
Rectangle	$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}(fT)$
Triangle	$\mathrm{tri}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}^2(fT)$
Sine Cardinal	$\mathrm{sinc}\left(\frac{t}{T}\right)$	$T\,\mathrm{rect}(fT)$
Complex Exponential	$\exp(j2\pi f_o t)$	$\delta(f-f_o)$
Cosine	$\cos(2\pi f_o t)$	$\frac{1}{2}\left[\delta(f-f_o)+\delta(f+f_o)\right]$
Sine	$\sin(2\pi f_o t)$	$-\frac{j}{2}\left[\delta(f-f_o)-\delta(f+f_o)\right]$
Gaussian	$\exp\left(-\frac{t^2}{\alpha^2}\right)$	$\alpha\pi^{0.5}\exp(-\alpha^2\pi^2 f^2)$
Comb	$\sum_{m=-\infty}^{\infty}\delta(t-mT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$

FOURIER TRANSFORM PROPERTIES

	Time-domain	**Frequency-domain**
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(f)+\beta X_2(f)$
Time scaling	$x(\beta t)$	$\frac{1}{\lvert\beta\rvert}X\left(\frac{f}{\beta}\right)$
Duality	$X(t)$	$x(-f)$
Time shifting	$x(t-t_o)$	$X(f)\exp(-j2\pi f t_o)$
Frequency shifting (Modulation)	$x(t)\exp(j2\pi f_o t)$	$X(f-f_o)$
Differentiation in the time-domain	$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n X(f)$
Multiplication in the time-domain	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty}X_1(\zeta)X_2(f-\zeta)\,d\zeta$ or $X_1(f)\ast X_2(f)$
Convolution in the time-domain	$\int_{-\infty}^{\infty}x_1(\zeta)x_2(t-\zeta)\,d\zeta$ or $x_1(t)\ast x_2(t)$	$X_1(f)X_2(f)$
Integration in the time-domain	$\int_{-\infty}^{t}x(\tau)\,d\tau$	$\frac{1}{j2\pi f}X(f)+\frac{1}{2}X(0)\delta(f)$
		$\frac{1}{j2\pi f}X(f)$ **if** $X(0)=0$

Unilateral Laplace Transform: $X(s)=\int_{0^-}^{\infty} x(t)\exp(-st)\,dt$

LAPLACE TRANSFORMS OF BASIC FUNCTIONS

	$x(t)$	$X(s)$
Unit Impulse	$\delta(t)$	**1**
Unit Step	$u(t)$	$1/s$
Ramp	$t\,u(t)$	$1/s^2$
n^th order Ramp	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
Damped Ramp	$t\exp(-\alpha t)u(t)$	$1/(s+\alpha)^2$
Exponential	$\exp(-\alpha t)u(t)$	$1/(s+\alpha)$
Cosine	$\cos(\omega_o t)u(t)$	$s/(s^2+\omega_o^2)$
Sine	$\sin(\omega_o t)u(t)$	$\omega_o/(s^2+\omega_o^2)$
Damped Cosine	$\exp(-\alpha t)\cos(\omega_o t)u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_o^2}$
Damped Sine	$\exp(-\alpha t)\sin(\omega_o t)u(t)$	$\frac{\omega_o}{(s+\alpha)^2+\omega_o^2}$

LAPLACE TRANSFORM PROPERTIES

	Time-domain	*s*-domain
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(s)+\beta X_2(s)$
Time shifting	$x(t-t_o)u(t-t_o)$	$\exp(-st_o)X(s)$
Shifting in the s-domain	$\exp(s_o t)x(t)$	$X(s-s_o)$
Time scaling	$x(\alpha t)$	$\frac{1}{\lvert\alpha\rvert}X\left(\frac{s}{\alpha}\right)$
Integration in the time-domain	$\int_{0^-}^{t} x(\zeta)\,d\zeta$	$\frac{1}{s}X(s)$
Differentiation in the time-domain	$\frac{dx(t)}{dt}$	$sX(s)-x(0^-)$
	$\frac{d^n x(t)}{dt^n}$	$s^n X(s)-\sum_{k=0}^{n-1} s^{n-1-k}\left.\frac{d^k x(t)}{dt^k}\right\rvert_{t=0^-}$
Differentiation in the s-domain	$-tx(t)$	$\frac{dX(s)}{ds}$
	$(-t)^n x(t)$	$\frac{d^n X(s)}{ds^n}$
Convolution in the time-domain	$\int_{-\infty}^{\infty} x_1(\zeta)x_2(t-\zeta)\,d\zeta$	$X_1(s)X_2(s)$
Initial value theorem	$x(0^+)=\lim_{s\to\infty} sX(s)$	
Final value theorem	$\lim_{t\to\infty} x(t)=\lim_{s\to 0} sX(s)$	

	$y_{step}(t)$	$Y_{step}(s)$	**SYSTEM PARAMETERS**
Step response of 1st order system	$K\left[1-\exp\left(-\frac{t}{T}\right)\right]u(t)$	$\frac{1}{s}\cdot\frac{K}{(sT+1)}$	T: System Time-constant K: System Steady-state (or DC) Gain
Step response of 2nd order <u>underdamped</u> system: $(0<\zeta<1)$	$K\left[1-\frac{\exp(-\omega_n\zeta t)}{(1-\zeta^2)^{0.5}}\sin\left(\omega_n(1-\zeta^2)^{0.5}t+\phi\right)\right]u(t)$	$\frac{1}{s}\cdot\frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	ω_n: System Undamped Natural Frequency ζ: System Damping Factor ω_d: System Damped Natural Frequency K: System Steady-state (or DC) Gain $\sigma=\omega_n\zeta$ $\omega_d^2=\omega_n^2(1-\zeta^2)$ $\omega_n^2=\sigma^2+\omega_d^2$ $\tan(\phi)=\omega_d/\sigma$
	$K\left[1-\left(\frac{\sigma^2+\omega_d^2}{\omega_d^2}\right)^{0.5}\exp(-\sigma t)\sin(\omega_d t+\phi)\right]u(t)$	$\frac{1}{s}\cdot\frac{K(\sigma^2+\omega_d^2)}{(s+\sigma)^2+\omega_d^2}$	

2nd order system - RESONANCE - $(0\le\zeta<1/\sqrt{2})$	*RESONANCE FREQUENCY :* $\omega_r=\omega_n(1-2\zeta^2)^{0.5}$	*RESONANCE PEAK :* $M_r=\lvert H(j\omega_r)\rvert=\frac{K}{2\zeta(1-\zeta^2)^{0.5}}$

TRIGONOMETRIC IDENTITIES	
$\exp(\pm j\theta)=\cos(\theta)\pm j\sin(\theta)$	$\sin(\alpha\pm\beta)=\sin(\alpha)\cos(\beta)\pm\cos(\alpha)\sin(\beta)$
$\cos(\theta)=\frac{1}{2}\left[\exp(j\theta)+\exp(-j\theta)\right]$	$\cos(\alpha\pm\beta)=\cos(\alpha)\cos(\beta)\mp\sin(\alpha)\sin(\beta)$
$\sin(\theta)=\frac{1}{j2}\left[\exp(j\theta)-\exp(-j\theta)\right]$ $\sin^2(\theta)+\cos^2(\theta)=1$	$\tan(\alpha\pm\beta)=\frac{\tan(\alpha)\pm\tan(\beta)}{1\mp\tan(\alpha)\tan(\beta)}$
$\sin(2\theta)=2\sin(\theta)\cos(\theta)$	$\sin(\alpha)\sin(\beta)=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$
$\cos(2\theta)=\cos^2(\theta)-\sin^2(\theta)$	$\cos(\alpha)\cos(\beta)=\frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$
$\sin^2(\theta)=\frac{1}{2}\left[1-\cos(2\theta)\right]$	$\sin(\alpha)\cos(\beta)=\frac{1}{2}\left[\sin(\alpha-\beta)+\sin(\alpha+\beta)\right]$
$\cos^2(\theta)=\frac{1}{2}\left[1+\cos(2\theta)\right]$	$\mathbf{C}\cos(\theta)-\mathbf{S}\sin(\theta)=\sqrt{\mathbf{C}^2+\mathbf{S}^2}\cos\left[\theta+\tan^{-1}\left(\frac{\mathbf{S}}{\mathbf{C}}\right)\right]$