NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester I : 2016/2017)

Names of examiners: AP Loh, Lawrence Wong, WW Tan and C S Ng

EE2023 – SIGNALS & SYSTEMS

Nov/Dec 2016 - Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES

1. This paper contains **EIGHT (8)** questions and comprises **ELEVEN (11)** printed pages.
2. Answer **ALL** questions in **Section A** and **ANY THREE (3)** questions in **Section B**.
3. This is a **CLOSED BOOK** examination. However you are allowed to bring one self-prepared, both sides handwritten A4-size crib sheet to the examination hall.
4. Programmable calculators are not allowed.
5. Tables of Fourier Transforms, Laplace Transforms and trigonometric identities are provided in Pages 9, 10 and 11, respectively.

SECTION A : Answer ALL questions in this section

Q1. Consider a linear time invariant system whose impulse response is given by :

$$g(t) = te^{-3t} + e^{-9t} \qquad t \geq 0.$$

(a) Find the transfer function, $G(s)$ for this system.

(2 marks)

(b) Is the system bounded-input bounded-output stable? Justify your answer.

(2 marks)

(c) What is its DC or steady state gain?

(2 marks)

(d) Suppose the input to the system is $x(t) = 2\sin(3t)$. Find the steady state output response to $x(t)$. Sketch this response, clearly showing the important quantities.

(4 marks)

Q2. The Fourier transform $X(f)$ of the signal $x(t)$ is shown in Figure Q2. This signal is sampled at 50 Hz to obtain the sampled signal, $x_s(t)$.

(a) Obtain an expression for the Fourier transform of the sampled signal $x_s(t)$ in terms of $X(f)$.

(4 marks)

(b) Sketch and label the spectrum of the sampled signal $x_s(t)$.

(3 marks)

(c) Can the original signal $x(t)$ be recovered from the sampled signal $x_s(t)$ and if so explain how this can be done.

(3 marks)

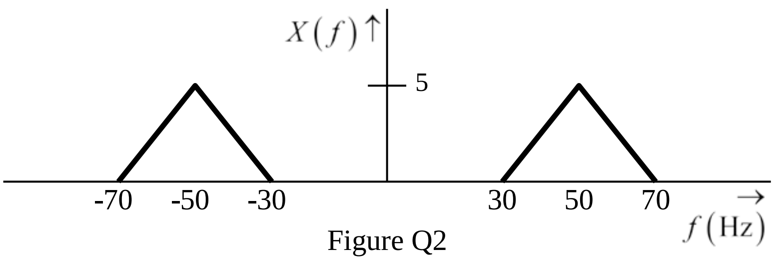


Figure Q2

Q3. Figure Q3 shows a series *RLC* circuit, where L = 2 mH and C = 5 μF.

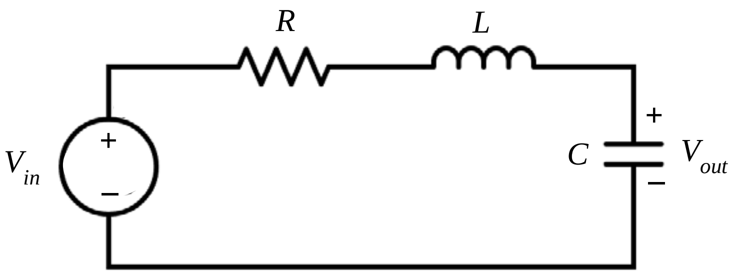


Figure Q3 : Series *RLC* circuit

(a) Derive the transfer function $\dfrac{V_{out}(s)}{V_{in}(s)}$.

(4 marks)

(b) Find R such that the series *RLC* circuit is a critically damped second order system.

(4 marks)

(c) Compute the system poles when $R = 50\Omega$.

(2 marks)

Q4. The complex envelope representation of a radio frequency pulse is given by

$$x(t) = \mathrm{rect}\left(\frac{t}{10}\right)e^{j\theta}$$

where θ is a real constant.

(a) Find the total energy, E, of $x(t)$.

(3 marks)

(b) Find the energy spectral density, $E_x(f)$, and the first-null bandwidth, B_n, of $x(t)$.

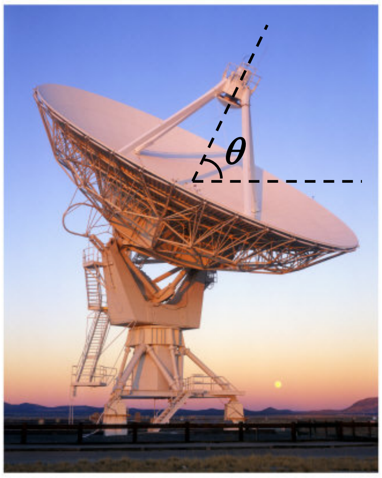
(5 marks)

(c) Suppose we delay $x(t)$ by τ seconds. How will this affect the values of E, $E_x(f)$ and $B_{n\cancel{ull}}$ found in Parts (a) and (b)? In each case, simply state whether the value will increase, remain the same, or decrease.

(2 marks)

SECTION B : Answer 3 out of the 4 questions in this section

Q5. A satellite-tracking antenna is shown in Figure Q5 below.



Picture taken from www.antenna-theory.com

Figure Q5 : Satellite-Tracking Antenna

The antenna and its drive system have a moment of inertia, J and damping parameter, B. These parameters are mainly derived from the bearing and aerodynamic friction in the system and may be assumed to be relatively constant.

The equation of motion which relates the elevation angle, $\theta(t)$ and the torque applied, $x(t)$ is as follows :

$$J\frac{d^2\theta}{dt^2}+B\frac{d\theta}{dt}=x(t).$$

(a) Derive the transfer function for the antenna system, relating the elevation angle, $\Theta(s)=L\{\theta(t)\}$ to the torque input, $X(s)=L\{x(t)\}$.

(2 marks)

(b) Assume $B=1$ and $J=2$. Suppose a gust of wind comes along and acts like the torque input on the antenna. This torque can be modelled as an impulse, $x(t)=2\delta(t)$. Calculate the response of the antenna elevation angle, $\theta(t)$ to this gust of wind. Sketch this response, clearly indicating the final position of the antenna.

(6 marks)

(c) In order to minimize the disturbance due to gusts of wind, the antenna system was re-designed and the new transfer function of the system is given by :

$$G(s)=\frac{K}{Js^2+Bs+K}$$

where $K>0$ is the parameter used to control the behaviour of the new system. Assume $J=2$ for all the cases below.

Question Q5 continues on the next page

(i) In the new design, the damping parameter, B is found to be negligible and may be assumed to be zero. Calculate the response of the elevation angle, $\theta(t)$ to the same gust of wind from part (b) above. Sketch this response. Explain how K can be chosen or designed to improve the response.

(6 marks)

(ii) The antenna engineer then decides to improve the damping of the system by changing B to 4. Design K such that the response of the elevation angle, $\theta(t)$ to any input is overdamped. For your design, sketch the response to a torque input given by $x(t) = 5u(t)$ where $u(t)$ is the unit step function.

(6 marks)

Q6. Let $G(s) = \dfrac{Y(s)}{X(s)}$ be the transfer function of a vehicle suspension system. $G(s)$ governs how road conditions, modelled by the input signal, $X(s) = L\{x(t)\}$, relate to the vertical motions of the vehicle as represented by the output signal, $Y(s) = L\{y(t)\}$. The Bode magnitude diagram of $G(s)$ is shown in Figure Q6.

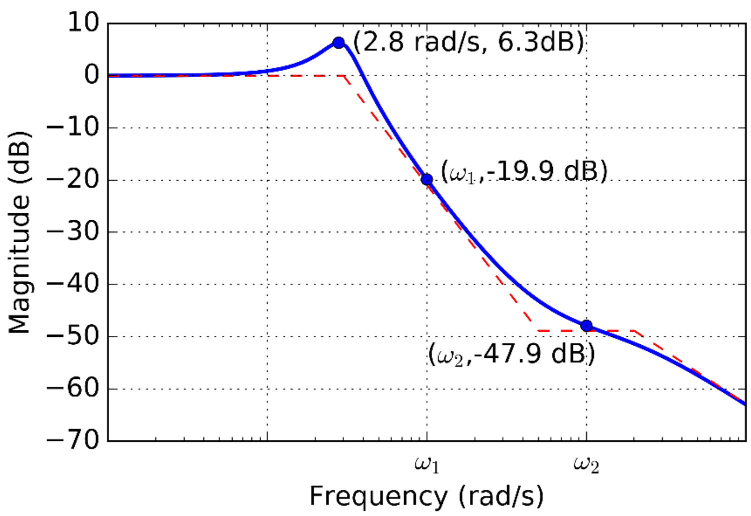


Figure Q6 : Bode magnitude diagram of $G(s)$

Question Q6 continues on the next page

(a) Suppose it is known that $G(s)$ contains an underdamped second order term, $\dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$, with a resonant frequency of 2.8 rad/s and resonant magnitude of 6.3 dB. Determine ζ and ω_n.

(6 marks)

(b) In addition to the second order term identified in part (a), the vehicle suspension system, $G(s)$, has the following characteristics:

- $G(s)$ is a third order system.
- $G(s)$ has unity DC gain.
- $G(s)$ has repeated zeros, $\left(\dfrac{s}{a}+1\right)^2$.
- $G(s)$ does not have a transportation lag.
- Corner frequencies of the asymptotic Bode magnitude diagram are at 3 rad/s, 50 rad/s and 200 rad/s

Derive the transfer function of the vehicle suspension system, $G(s)$.

(6 marks)

(c) Using the characteristics of the vehicle suspension system described in parts (a) and (b), infer the values of ω_1 and ω_2 from the x-axis labels of the Bode magnitude diagram in Figure Q6.

(2 marks)

(d) Let the road condition be modelled by $x(t) = A\sin\omega t$, where $0 < A \le 15$ and $\omega \ge 10$ rad/s. After receiving complaints from passengers about uncomfortable rides, a static gain, K, is cascaded with $G(s)$ in order to ensure that the amplitude of the steady-state motion of the vehicle body motion is less than 0.75. Determine the range of K for the re-designed system, $KG(s)$, to satisfy the design requirement.

(6 marks)

Q.7 The signal $x(t)$ shown in Figure Q7(a) has the form

$$x(t) = \alpha_1 \mathrm{rect}\left(\frac{t}{4}\right) - \alpha_2 \mathrm{rect}\left(\frac{t}{\beta_1}\right) + \alpha_3 \mathrm{tri}\left(\frac{t}{\beta_2}\right).$$

(a) Determine the values of $\alpha_1, \alpha_2, \alpha_3, \beta_1$ and β_2.

(2 marks)

(b) Determine the Fourier transform of the signal $x(t)$ shown in Figure Q7(a).

(5 marks)

(c) Obtain the expression for the periodic signal $x_p(t)$ shown in Figure Q7(b) in terms of the generating function $x(t)$.

(2 marks)

(d) Determine the Fourier transform of the periodic signal $x_p(t)$.

(6 marks)

(e) Determine the Fourier series coefficients of $x_p(t)$.

(2 marks)

(f) Determine the expression for the average power of $x_p(t)$.

(3 marks)

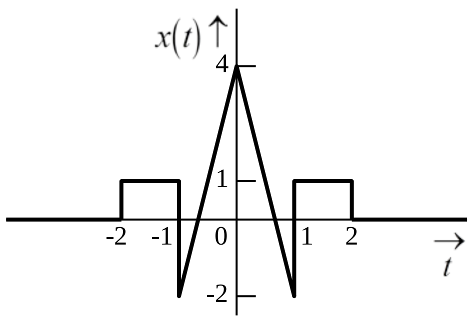


Figure Q7(a)

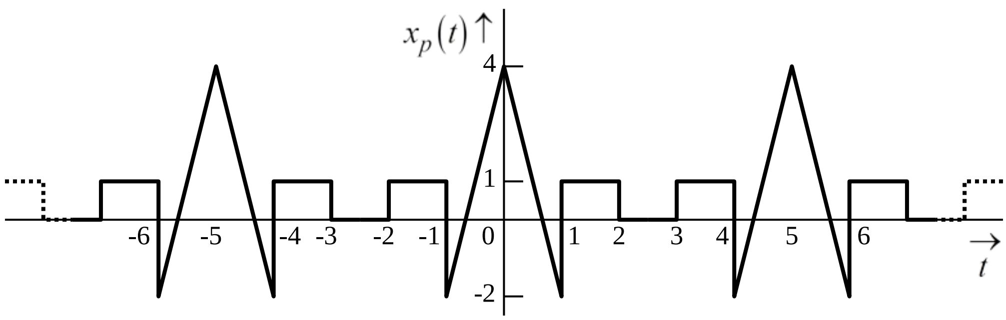


Figure Q7(b)

Q8. Figure Q8 shows an upper-single-sideband (USSB) modulator where $x(t)$ is the input message signal and $y(t)$ is the output modulated signal. In Figure Q8, $X(f)$ and $Y(f)$ are the spectra of $x(t)$ and $y(t)$, respectively, and $h(t)$ is the impulse response of the ideal bandpass filter.

[*Note that the frequency axes of the* $X(f)$ *and* $Y(f)$ *plots are labeled in* **kHz**.]

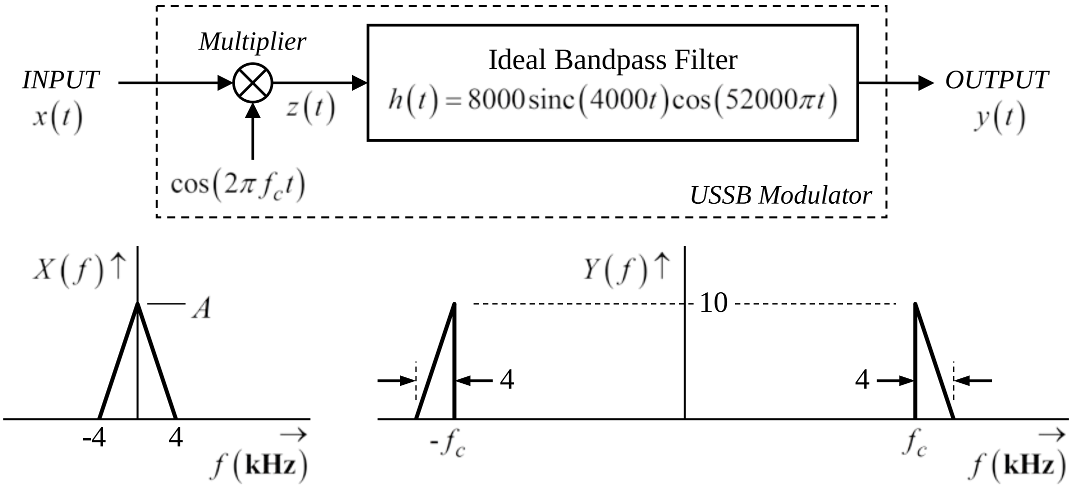


Figure Q8

(a) Find the Fourier transforms, $Z(f)$ and $H(f)$, of $z(t)$ and $h(t)$, respectively.

(6 marks)

(b) Based on the results from Part (a), determine the values of f_c and A that will produce the output spectrum, $Y(f)$, as shown in Figure Q8.

(6 marks)

(c) Suppose we sample $y(t)$ to form $y_s(t) = y(t)\cdot\sum_n \delta(t - n/f_s)$, and then send $y_s(t)$ through an ideal <u>lowpass</u> filter. Is it possible for the filter output to be a perfect reconstruction of $x(t)$? If "Yes", suggest a value for the sampling frequency f_s and sketch the corresponding spectrum of $y_s(t)$. If "No", simply say so.

(8 marks)

END OF QUESTIONS

Fourier Series: $\begin{cases} X_k = \dfrac{1}{T}\displaystyle\int_{\tilde{t}}^{\tilde{t}+T} x(t)\exp(-j2\pi k\,t/T)\,dt \\ x(t) = \sum_{k=-\infty}^{\infty} X_k \exp(j2\pi k\,t/T) \end{cases}$

Fourier Transform: $\begin{cases} X(f) = \displaystyle\int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)\,dt \\ x(t) = \displaystyle\int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)\,df \end{cases}$

FOURIER TRANSFORMS OF BASIC FUNCTIONS		
	$x(t)$	$X(f)$
Constant	K	$K\delta(f)$
Unit Impulse	$\delta(t)$	**1**
Unit Step	$u(t)$	$\frac{1}{2}\left[\delta(f)+\frac{1}{j\pi f}\right]$
Sign (or Signum)	$\operatorname{sgn}(t)$	$\frac{1}{j\pi f}$
Rectangle	$\operatorname{rect}\left(\frac{t}{T}\right)$	$T\operatorname{sinc}(fT)$
Triangle	$\operatorname{tri}\left(\frac{t}{T}\right)$	$T\operatorname{sinc}^2(fT)$
Sine Cardinal	$\operatorname{sinc}\left(\frac{t}{T}\right)$	$T\operatorname{rect}(fT)$
Complex Exponential	$\exp(j2\pi f_o t)$	$\delta(f-f_o)$
Cosine	$\cos(2\pi f_o t)$	$\frac{1}{2}\left[\delta(f-f_o)+\delta(f+f_o)\right]$
Sine	$\sin(2\pi f_o t)$	$-\frac{j}{2}\left[\delta(f-f_o)-\delta(f+f_o)\right]$
Gaussian	$\exp\left(-\frac{t^2}{\alpha^2}\right)$	$\alpha\pi^{0.5}\exp\left(-\alpha^2\pi^2 f^2\right)$
Comb	$\sum_{m=-\infty}^{\infty}\delta(t-mT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$

FOURIER TRANSFORM PROPERTIES		
	Time-domain	**Frequency-domain**
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(f)+\beta X_2(f)$
Time scaling	$x(\beta t)$	$\frac{1}{\lvert\beta\rvert}X\left(\frac{f}{\beta}\right)$
Duality	$X(t)$	$x(-f)$
Time shifting	$x(t-t_o)$	$X(f)\exp(-j2\pi f t_o)$
Frequency shifting (Modulation)	$x(t)\exp(j2\pi f_o t)$	$X(f-f_o)$
Differentiation in the time-domain	$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n X(f)$
Multiplication in the time-domain	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty}X_1(\zeta)X_2(f-\zeta)\,d\zeta$ or $X_1(f)\ast X_2(f)$
Convolution in the time-domain	$\int_{-\infty}^{\infty}x_1(\zeta)x_2(t-\zeta)\,d\zeta$ or $x_1(t)\ast x_2(t)$	$X_1(f)X_2(f)$
Integration in the time-domain	$\int_{-\infty}^{t}x(\tau)\,d\tau$	$\frac{1}{j2\pi f}X(f)+\frac{1}{2}X(0)\delta(f)$
		$\frac{1}{j2\pi f}X(f)$ **if** $X(0)=0$

Unilateral Laplace Transform: $X(s)=\int_{0^-}^{\infty} x(t)\exp(-st)\,dt$

LAPLACE TRANSFORMS OF BASIC FUNCTIONS

	$x(t)$	$X(s)$
Unit Impulse	$\delta(t)$	1
Unit Step	$u(t)$	$1/s$
Ramp	$t\,u(t)$	$1/s^2$
n[th] order Ramp	$t^n u(t)$	$\dfrac{n!}{s^{n+1}}$
Damped Ramp	$t\exp(-\alpha t)u(t)$	$1/(s+\alpha)^2$
Exponential	$\exp(-\alpha t)u(t)$	$1/(s+\alpha)$
Cosine	$\cos(\omega_o t)u(t)$	$s/(s^2+\omega_o^2)$
Sine	$\sin(\omega_o t)u(t)$	$\omega_o/(s^2+\omega_o^2)$
Damped Cosine	$\exp(-\alpha t)\cos(\omega_o t)u(t)$	$\dfrac{s+\alpha}{(s+\alpha)^2+\omega_o^2}$
Damped Sine	$\exp(-\alpha t)\sin(\omega_o t)u(t)$	$\dfrac{\omega_o}{(s+\alpha)^2+\omega_o^2}$

LAPLACE TRANSFORM PROPERTIES

	Time-domain	s-domain
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(s)+\beta X_2(s)$
Time shifting	$x(t-t_o)u(t-t_o)$	$\exp(-st_o)X(s)$
Shifting in the s-domain	$\exp(s_o t)x(t)$	$X(s-s_o)$
Time scaling	$x(\alpha t)$	$\dfrac{1}{\lvert\alpha\rvert}X\left(\dfrac{s}{\alpha}\right)$
Integration in the time-domain	$\int_{0^-}^{t} x(\zeta)\,d\zeta$	$\dfrac{1}{s}X(s)$
Differentiation in the time-domain	$\dfrac{dx(t)}{dt}$	$sX(s)-x(0^-)$
	$\dfrac{d^n x(t)}{dt^n}$	$s^n X(s)-\sum_{k=0}^{n-1} s^{n-1-k}\left.\dfrac{d^k x(t)}{dt^k}\right\rvert_{t=0^-}$
Differentiation in the s-domain	$-tx(t)$	$\dfrac{dX(s)}{ds}$
	$(-t)^n x(t)$	$\dfrac{d^n X(s)}{ds^n}$
Convolution in the time-domain	$\int_{-\infty}^{\infty} x_1(\zeta)x_2(t-\zeta)\,d\zeta$	$X_1(s)X_2(s)$

Initial value theorem	$x(0^+)=\lim_{s\to\infty} sX(s)$
Final value theorem	$\lim_{t\to\infty} x(t)=\lim_{s\to 0} sX(s)$

	$y_{step}(t)$	$Y_{step}(s)$	SYSTEM PARAMETERS
Step response of 1^{st} order system	$K\left[1-\exp\left(-\frac{t}{T}\right)\right]u(t)$	$\frac{1}{s}\cdot\frac{K}{(sT+1)}$	$\begin{pmatrix} T: \text{ System Time-constant} \\ K: \text{ System Steady-state (or DC) Gain} \end{pmatrix}$
Step response of 2^{nd} order <u>underdamped</u> system: $(\mathbf{0<\zeta<1})$	$K\left[1-\frac{\exp(-\omega_n\zeta t)}{(1-\zeta^2)^{0.5}}\sin\left(\omega_n(1-\zeta^2)^{0.5}t+\phi\right)\right]u(t)$	$\frac{1}{s}\cdot\frac{K\omega_n{}^2}{s^2+2\zeta\omega_n s+\omega_n{}^2}$	$\begin{pmatrix} \omega_n: \text{ System Undamped Natural Frequency} \\ \zeta: \text{ System Damping Factor} \\ \omega_d: \text{ System Damped Natural Frequency} \\ K: \text{ System Steady-state (or DC) Gain} \end{pmatrix}$ $\begin{pmatrix} \sigma=\omega_n\zeta \\ \omega_d^2=\omega_n^2(1-\zeta^2) \\ \omega_n^2=\sigma^2+\omega_d^2 \\ \tan(\phi)=\omega_d/\sigma \end{pmatrix}$
	$K\left[1-\left(\frac{\sigma^2+\omega_d^2}{\omega_d^2}\right)^{0.5}\exp(-\sigma t)\sin(\omega_d t+\phi)\right]u(t)$	$\frac{1}{s}\cdot\frac{K(\sigma^2+\omega_d{}^2)}{(s+\sigma)^2+\omega_d{}^2}$	

2^{nd} order system - RESONANCE - $(\mathbf{0\le\zeta<1/\sqrt{2}})$	*RESONANCE FREQUENCY :* $\omega_r=\omega_n(1-2\zeta^2)^{0.5}$	*RESONANCE PEAK :* $M_r=\lvert H(j\omega_r)\rvert=\frac{K}{2\zeta(1-\zeta^2)^{0.5}}$

TRIGONOMETRIC IDENTITIES	
$\exp(\pm j\theta)=\cos(\theta)\pm j\sin(\theta)$	$\sin(\alpha\pm\beta)=\sin(\alpha)\cos(\beta)\pm\cos(\alpha)\sin(\beta)$
$\cos(\theta)=\frac{1}{2}[\exp(j\theta)+\exp(-j\theta)]$	$\cos(\alpha\pm\beta)=\cos(\alpha)\cos(\beta)\mp\sin(\alpha)\sin(\beta)$
$\sin(\theta)=\frac{1}{j2}[\exp(j\theta)-\exp(-j\theta)]$	$\tan(\alpha\pm\beta)=\frac{\tan(\alpha)\pm\tan(\beta)}{1\mp\tan(\alpha)\tan(\beta)}$
$\sin^2(\theta)+\cos^2(\theta)=1$	
$\sin(2\theta)=2\sin(\theta)\cos(\theta)$	$\sin(\alpha)\sin(\beta)=\frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$
$\cos(2\theta)=\cos^2(\theta)-\sin^2(\theta)$	$\cos(\alpha)\cos(\beta)=\frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$
$\sin^2(\theta)=\frac{1}{2}[1-\cos(2\theta)]$	$\sin(\alpha)\cos(\beta)=\frac{1}{2}[\sin(\alpha-\beta)+\sin(\alpha+\beta)]$
$\cos^2(\theta)=\frac{1}{2}[1+\cos(2\theta)]$	$\mathbf{C}\cos(\theta)-\mathbf{S}\sin(\theta)=\sqrt{\mathbf{C}^2+\mathbf{S}^2}\cos\left[\theta+\tan^{-1}\left(\frac{\mathbf{S}}{\mathbf{C}}\right)\right]$