EE2023/TEE2023 TUTORIAL 1 (PROBLEMS)

Q.1 Find the magnitudes and phases of the following complex numbers.

(a) $z = \dfrac{1-j1}{1+j2}$ (b) $z = (-1+j1)\times(1+j2)$

ANSWER : (a) Magnitude = $\sqrt{0.4}$, Phase = -1.8925 rads ; (b) Magnitude = $\sqrt{10}$, Phase = 3.4633 rads

Q.2 Represent each of the following complex numbers in polar form and plot the point on the complex plane.

(a) $1 + j1$ (b) $-2 + j2$ (c) $-3 - j4$

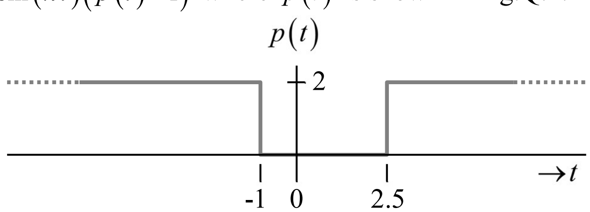
ANSWER : (a) $\sqrt{2}e^{j\pi/4}$; (b) $\sqrt{8}e^{j3\pi/4}$; (c) $5e^{-j2.2143}$

Q.3 Let $z = x + jy$ where x and y are real numbers. Provide a formula for computing the N distinct values of $\sqrt[N]{z}$. Hence, or otherwise, determine $\sqrt[6]{64}$ and $\sqrt[4]{j81}$.

ANSWER : (a) $2\exp\left(j\left(\dfrac{k\pi}{3}\right)\right)$ for $k = 0,1,\cdots,5$; (b) $3\exp\left(j\left(\dfrac{\pi}{8}+\dfrac{k\pi}{2}\right)\right)$ for $k = 0,1,\cdots,3$

Q.4 Consider the signal $x(t) = 2\sin(\pi t)(p(t)-1)$ where $p(t)$ is shown in Fig.Q.2.

Fig.Q.2



(a) Express $p(t)$ in terms of the $\text{rect}(\bullet)$ function.

(b) Sketch and label $x(t)$ and state whether or not $x(t)$ is periodic.

(c) Find an expression for $x^2(t)$. Hence, compute the average power of $x(t)$.

(d) Based on the results in (b) and (c), how would you classify $x(t)$?

ANSWER : (a) $p(t) = 2 - 2\text{rect}\left(\dfrac{t-0.75}{3.5}\right)$; (b) Not periodic ; (c) $x^2(t) = 2(1-\cos(2\pi t))$

(d) Aperiodic power signal

Q.5 In digital communications, half-cosine or raised-cosine pulses are sometimes used to pulse shape a binary waveform so as to reduce intersymbol interference. The general expressions for these pulses are

Half-cosine pulse : $x(t) = A\cos(\pi t/T)\text{rect}(t/T)$

Raised-cosine pulse : $\tilde{x}(t) = 0.5\tilde{A}\left(1+\cos\left(2\pi t/\tilde{T}\right)\right)\text{rect}\left(t/\tilde{T}\right)$.

where A, $\tilde{A}$, T and $\tilde{T}$ are positive constants. Sketch and label each pulse. Under what condition(s) will both pulses have the same energy?

ANSWER : $A^2T = \dfrac{3}{4}\tilde{A}^2\tilde{T}$

Q.6 Determine whether or not each of the following signals is periodic. If the signal is periodic, determine its fundamental frequency.

(a) $x(t) = \cos(3.2t) + \sin(1.6t) + \exp(j2.8t)$

(b) $x(t) = \cos(4t) + \sin(\pi t)$

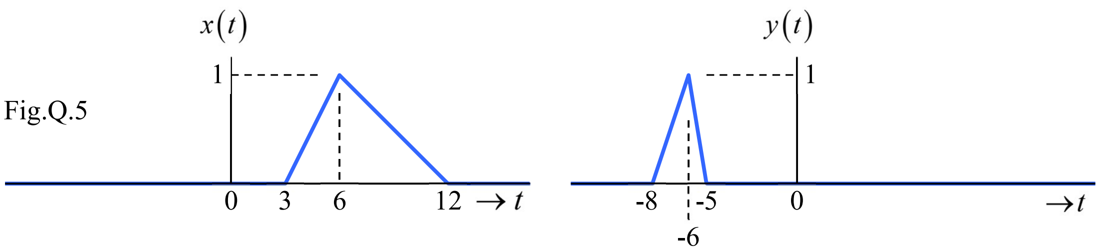
ANSWER : (a) Periodic [0.4 rad/s) ; (b) Non-periodic

Q.7 Sketches of two signals, $x(t)$ and $y(t)$, are shown in Fig.Q.5.

(a) Sketch and label the following signals: $x(t+4)$; $x(-t)$; $x(3t)$; $x(t/3)$

(b) Express $y(t)$ in terms of $x(t)$.

Fig.Q.5

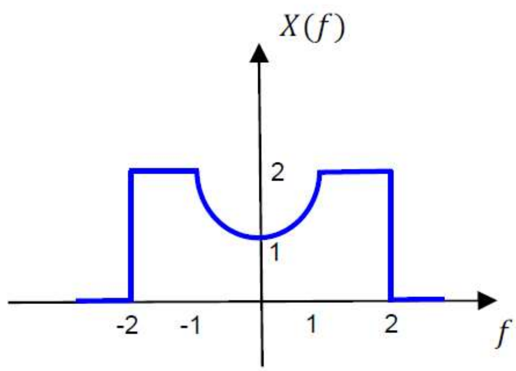


ANSWER : (b) $y(t) = x(-3(t+4))$

Q.8 Sketch the following signal:

$$x(t) = 2\delta(t+4) + \delta(t+3) + 3\delta(t+2) + 4\delta(t) + 3\delta(t-2) + \delta(t-3) + 2\delta(t-4)$$

Q.9 Consider the function $X(f)$ below. Write $X(f)$ in terms of appropriate rect(.) and cos(.) functions.

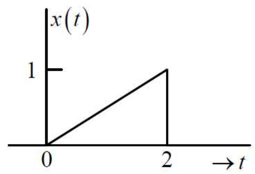


ANSWER : $X(f) = 2\,\mathrm{rect}\left(\frac{f}{4}\right) - \cos\left(\frac{\pi f}{2}\right)\mathrm{rect}\left(\frac{f}{2}\right)$

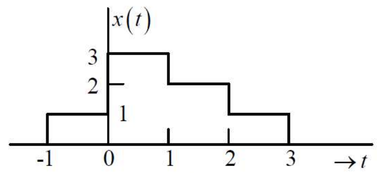
Supplementary Problems

These problems will not be discussed in class.

S.1 Express the signals shown in the figures below in terms of unit step functions.



(a)



(b)

ANSWER : (a) $x(t) = u(2-t) \bullet \int_{-\infty}^{t} 0.5u(\tau)d\tau$; (b) $x(t) = u(t+1) + 2u(t) - u(t-1) - u(t-2) - u(t-3)$

S.2 Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental period and average power.

(a) $x(t) = \cos(2t + 0.25\pi)$ *(b)* $x(t) = \cos^2(t)$

(c) $x(t) = \cos(2\pi t)u(t)$ *(d)* $x(t) = \exp(j\pi t)$

ANSWER : (a) periodic, period = π, power = ½ ; (b) periodic, period = π, power = 3/8 ; (c) non-periodic ; (d) periodic, period = 2, power = 1

S.3 Evaluate the following integrals:

(a) $\int_{-\infty}^{t} \cos(\tau)u(\tau)d\tau$ *(b)* $\int_{-\infty}^{t} \cos(\tau)\delta(\tau)d\tau$

(c) $\int_{-\infty}^{\infty} \cos(t)u(t-1)dt$ *(d)* $\int_{0}^{2\pi} t\sin\left(\frac{t}{2}\right)\delta(\pi - t)\,dt$

ANSWER : (a) $\sin(t)u(t)$; (b) $u(t)$; (c) 0 ; (d) π

S.4 Any signal $x(t)$ can be expressed as a sum of two component signals, one of which is even and one of which is odd. That is

$$x(t) = x_e(t) + x_o(t)$$

where $x_e(t) = 0.5[x(t) + x(-t)]$ is the even component and $x_o(t) = 0.5[x(t) - x(-t)]$ the odd component.

Determine the even and odd components of: *(a)* $x(t) = u(t)$ *(b)* $x(t) = \sin\left(\omega_c t + \frac{\pi}{4}\right)$.

ANSWER : (a) $\begin{cases} x_e(t) = \begin{cases} 1; & t = 0 \\ 0.5; & t \neq 0 \end{cases} \\ x_o(t) = \begin{cases} 0; & t = 0 \\ 0.5\,\mathrm{sgn}(t); & t \neq 0 \end{cases} \end{cases}$ (b) $\begin{cases} x_e(t) = \frac{1}{\sqrt{2}}\sin(\omega_c t) \\ x_e(t) = \frac{1}{\sqrt{2}}\cos(\omega_c t) \end{cases}$

Below is a list of solved problems selected from Chapter 1 of Hwei Hsu (PhD), 'The Schaum's series on Signals& Systems', 2nd Edition.

The 1st Edition can be found in the following link:
http://www.kousik.net/wp-content/uploads/2010/10/Schaums-Outline-Series-Signals_Systems.pdf

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Selected solved-problems: 1.1, 1.9, 1.10, 1.14, 1.16(a)-to-(f), 1.17, 1.18, 1.20(a)-&-(b), 1.21, 1.22, 1.27, 1.30, 1.31

These solved problems should be treated as supplementary module material catered for students who find the need for more examples or practice-problems.