National University of Singapore
Department of Electrical & Computer Engineering

# EE2023 Signals & Systems Tutorial 5

***Section I : Exercises that are straightforward applications of the concepts covered in class. Please attempt these problems on your own.***

1. Solve the following Laplace Transform questions :

(a) $\mathcal{L}\left\{\cos^2 \omega t\right\}$ Answer : $\dfrac{1}{2}\left[\dfrac{s}{s^2+4\omega^2}+\dfrac{1}{s}\right]$

(b) $\mathcal{L}^{-1}\left\{\dfrac{1}{(s-1)(s+2)(s+4)}\right\}$ Answer : $\dfrac{1}{15}e^{t}-\dfrac{1}{6}e^{-2t}+\dfrac{1}{10}e^{-4t}$

*Application of the shift in the s-domain function rule :* $\mathcal{L}\{e^{-\alpha t}f(t)\}=F(s+\alpha)$

(c) $\mathcal{L}^{-1}\left\{\dfrac{1}{(s+1)^2}\right\}$ Answer : $te^{-t}$

(d) $\mathcal{L}^{-1}\left\{\dfrac{s+9}{s^2+6s+13}\right\}$ Answer : $e^{-3t}\left[\cos 2t+3\sin 2t\right]$

(e) $\mathcal{L}\left\{\dfrac{3}{5}-\dfrac{\sqrt{45}}{5}e^{-2t}\sin(t+\tan^{-1}0.5)\right\}$ Answer : $\dfrac{3}{s(s^2+4s+5)}$

*Application of the shift in the time-domain function rule :* $\mathcal{L}\{f(t-t_0)U(t-t_0)\}=e^{-st_0}F(s)$

(f) $\mathcal{L}\left\{(t-1)^2U(t-1)\right\}$ Answer : $\dfrac{2}{s^3}e^{-s}$

(g) $\mathcal{L}\left\{t^2U(t-1)\right\}$ Answer : $\dfrac{2}{s^3}e^{-s}+\dfrac{2}{s^2}e^{-s}+\dfrac{1}{s}e^{-s}$

(h) $\mathcal{L}^{-1}\left\{\dfrac{se^{-2s}}{s^2+\pi^2}\right\}$ Answer : $\cos(\pi t)U(t-2)$

*Application of the derivative of transforms rule :* $F'(s)=\mathcal{L}\{-tf(t)\}$

(i) $\mathcal{L}\left\{te^{-t}\sin t\right\}$ Answer : $\dfrac{2(s+1)}{(s^2+2s+2)^2}$

(j) $\mathcal{L}^{-1}\left\{\dfrac{s}{(s^2+9)^2}\right\}$ Answer : $\dfrac{1}{6}t\sin 3t$

2. Solve the following linear second order differential equation using Laplace Transform :

$$\ddot{y}(t)+4\dot{y}(t)+3y(t)=2r(t) \text{ assuming that } r(t)=1 \text{ when } t\geq 0, y(0)=1 \text{ and } \dot{y}(0)=0$$

Answer : $y(t)=\frac{1}{2}e^{-t}-\frac{1}{6}e^{-3t}+\frac{2}{3}$

***Section II : Problems that will be discussed in class.***

1. The circuit shown in Figure 1 is operating in steady-state with the switch open prior to $t = 0$. Find expressions for $i(t)$ for $t < 0$ and for $t \geq 0$.

   ANSWER : $i(t) = 1$ for $t < 0$ and $4 - 3e^{-12.5t}$ for $t > 0$

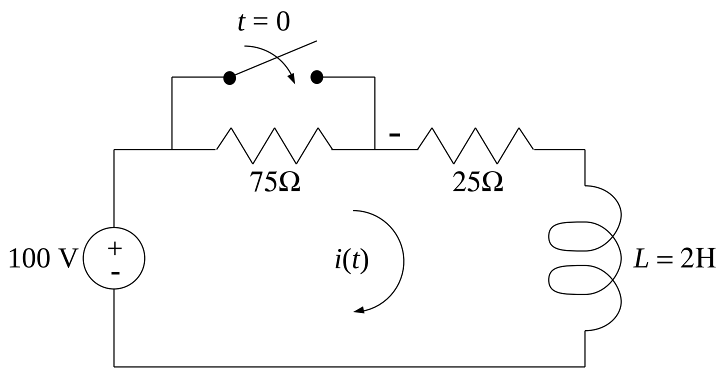


Figure 1: Series RL circuit

2. A series RLC circuit is shown in Figure 2.

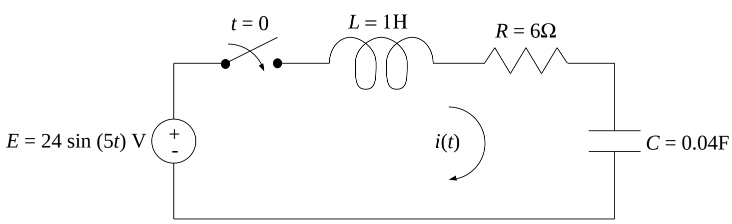


Figure 2: Series RLC circuit

   (a) Show that the differential equation relating the current $i(t)$ in the RLC circuit shown in the figure to the applied voltage $E(t)$ is

$$L\frac{d^2i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{1}{C}i(t) = \frac{dE(t)}{dt}$$

   (b) Assuming the initial current and its rate of change ($i(0)$ and $\frac{di(0)}{dt}$) are zero, find $i(t)$.

   ANSWER : $i(t) = -5e^{-3t}\sin 4t + 4\sin 5t$

3. Ah Kow is worried about an upcoming exam. His doctor advises him to take a 100mg stress relief tablet the next morning and another 50mg tablet 24 hours later. Suppose the differential equation describing the quantity of drug in Ah Kow's body is

$$\frac{\mathrm{d}^2y(t)}{\mathrm{d}t^2} + 3\frac{\mathrm{d}y(t)}{\mathrm{d}t} + 2y(t) = f(t)$$

where $y(t)$ is the quantity of drug in the body measured in mg,
$f(t)$ represents the rate at which the drug is administered into the body,
$t$ is time measured in days.

Assume that
- drugs taken in tablet form can be modelled by impulse function whose strength is equal to the quantity of drug ingested,
- there is no stress relief drug in Ah Kow's bloodstream before he takes the first tablet.

(a) Write a mathematical expression representing the input signal, $f(t)$, which models the rate at which the stress relief medicine is digested. ANSWER : $100\delta(t) + 50\delta(t-1)$

(b) What are the initial conditions ($t = 0^-$)of the system ?
ANSWER : $y(0^-) = 0, y'(0^-) = 0$

(c) Use Laplace Transform to determine the system output, $y(t)$. What is the amount of stress medicine left in Ah Kow's body by the time of the exam 4 days after he ate the first tablet ? ANSWER : 4.1634 mg

***Section III : Practice Problems. These problems will not be discussed in class.***

1. Consider the circuit shown in Figure 3. The switch opens at $t = 0$. Find an expression for $v(t)$.

ANSWER : $v(t) = 10 - 10e^{-100t}$ for $t > 0$

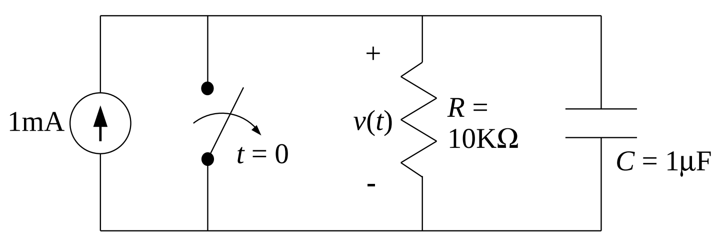


Figure 3: Parallel RC circuit