NATIONAL UNIVERSITY OF SINGAPORE

EE2023 – SIGNALS AND SYSTEMS

Online Examination

Semester I : 2020/2021 | Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES:

1. Download the e-examination paper **EE2023 Signals & Systems.pdf** from:

   LumiNUS → EE2023 → Files → Exam Paper Folder

2. This paper contains **EIGHT (8)** questions and comprises **NINE (9)** printed pages.
3. Answer **ALL** questions in **Section A** and **ANY THREE (3)** questions in **Section B**.
4. Write the answers for each question on a new page and paginate your answer script.
5. This is an **OPEN BOOK** quiz.
6. For the purpose of this examination, please take note of the digits ***a***, ***b***, ***c*** and ***d*** in your student number A0xx***abcd***X. These digits will be used in the questions in this quiz.

At the end of the examination, please do the following:

1. Print this page and use it as the cover page for your answer script. Fill in your name, student ID number and total number of pages clearly, and sign the honor code statement below.

   Student Name: ______________________ Total Pages: __________
   (including this page)

   Student ID Number: ______________________

   I have read and abided by the "Declaration of Academic Integrity in the Online Examination" when writing this quiz.

   Signature: ______________________

2. Use a scanning app (such as Microsoft Office Lens or CamScanner) to capture this Cover Page and your answer script and generate a consolidated pdf file using the filename "**EE2023Exam-A0123456J.pdf**", where you **replace A0123456J with your student ID number**.

3. Upload the consolidated pdf file to

   LumiNUS → EE2023 → Files → Exam Answer Folder

For examiner use only →

Question	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7	Q.8	TOTAL
Marks									

Before you begin, please take note of your *a, b, c* and *d* digits in your student number.

SECTION A : Answer ALL questions in this section

Q1. Given $y(t) = 4\cos(2\pi\beta t) \cdot \int_{-\infty}^{t} x(\tau)\,d\tau$ where the value of β is obtained as:

$$\beta = \frac{c+d+1}{2}$$

in which c and d are the values from your student number. Let $X(f)$ and $Y(f)$ be the Fourier transforms of $x(t)$ and $y(t)$, respectively.

Write down the value of β in your answer script and answer the following questions using this value.

(a) Express $Y(f)$ in terms of $X(f)$.

(6 marks)

(b) If $x(t) = 0.5\beta \cdot \mathrm{sinc}(0.5\beta t)$, use the result obtained in Part-(a) to find the value of $Y(0)$.

(4 marks)

Q2. Given the periodic signal $x(t) = 5 + 2\cos\left(2\lambda\pi t + \frac{\pi}{4}\right) - 3e^{j3\lambda\pi t} + 4\sin\left(5\lambda\pi t - \frac{\pi}{3}\right)$ where the value of λ is obtained as:

$$\lambda = a + 2$$

in which a is the value from your student number.

Write down the value of λ in your answer script and answer the following questions using this value.

(a) Determine the Fourier series coefficients, X_k, of the periodic signal $x(t)$.

(4 marks)

(b) Determine the Fourier transform, $X(f)$, of the periodic signal $x(t)$.

(2 marks)

(c) Sketch the magnitude and phase spectra of $x(t)$.

(4 marks)

Q3. The transfer function of a second order system is given by $G(s)=\dfrac{\beta s}{s^2+4s+\gamma}$ where the values of β and γ are obtained as:

$$\beta = c+1 \quad \text{and} \quad \gamma = d+5$$

in which c and d are the values from your student number.

Write down the values of β and γ in your answer script and answer the following questions using these values.

(a) Sketch the Bode magnitude plot for $G(s)$. Clearly indicate the slope of each straight-line segment and the frequency at which the plot changes slope each time. Other details are not required.

(5 marks)

(b) Derive the step response, $y_{step}(t)$, of the system.

(5 marks)

IMPORTANT:

To avoid penalty, all numeric expressions in the final answers must be evaluated and rounded to 4 significant figures. For example the final form of

$$\frac{1.25\times 16}{0.3}\sin\left(\frac{2.865}{1.2\times 6}\pi t+\cos\sqrt{0.4}\right)$$

should be presented as

$$66.67\sin\left(0.3979\pi t+0.8066\right).$$

No marks will be awarded if the final answer is left without substituting the parameter values.

Q4. The input-output relationship of a linear time invariant system is described as:

$$\frac{d^2}{dt^2}y(t)+\alpha\frac{d}{dt}y(t)+\beta y(t)=10x(t)$$

where $y(t)$ is the output, $x(t)$ is the input, and the values of α and β are obtained as:

$$\alpha = c+d+2 \quad \text{and} \quad \beta = c.d+c+d+1$$

in which c and d are the values from your student number.

Write down the values of α and β in your answer script and answer the following questions using these values.

(a) Derive the system transfer function $G(s)=\dfrac{Y(s)}{X(s)}$, where $Y(s)$ and $X(s)$ are the Laplace transform of $y(t)$ and $x(t)$, respectively.

(3 marks)

(b) Determine the system poles, damping ratio, ζ, undamped natural frequency, ω_n, and the system DC gain.

(4 marks)

(c) An input signal $x(t)=4\sin(3t)$ is applied to the system. Determine the steady state output of the system.

(3 marks)

SECTION B : Answer 3 out of the 4 questions in this section

Q5. Figure Q5 shows the block diagram of a signal generator where $x(t)$ is the source signal and $y(t)$ is the desired signal.

Figure Q5: Signal Generator

The spectrum of $x(t)$ is given by $X(f) = \sum_{k=-K}^{K} \frac{L}{|k|+1} \delta(f - kM)$ where f is measured in Hz and the values of K, L and M are obtained as:

$$K = a + 10, \quad L = c + 5 \quad \text{and} \quad M = d + 5$$

in which a, c and d are the values from your student number.

Write down the values of K, L and M in your answer script and answer the following questions using these values.

(a) (i) Is $x(t)$ an energy or power signal, and why? (2 marks)

(ii) Is $x(t)$ a real, imaginary or complex signal, and why? (2 marks)

(iii) Is $x(t)$ an odd or even function of t, and why? (2 marks)

(iv) Is $x(t)$ periodic, and why? (2 marks)

(v) What is the DC value of $x(t)$? (2 marks)

(b) Suppose the ideal filter in the signal generator has an impulse response given by $h(t) = 2\mu \operatorname{sinc}(t) \cos(2\pi f_o t)$ where μ and f_o are positive constants.

(i) Find the value of f_o so that $z(t)$ is a sinusoid of frequency $5M$ (Hz).

(4 marks)

(ii) With the value of f_o found in Part (a), find the value of μ so that $y(t) = \sin(10\pi M t)$.

(6 marks)

Q6. Consider the signal $x(t)$ shown in Figure Q6 where the value of ρ is obtained as:

$$\rho = b + 2$$

in which b is the value from your student number.

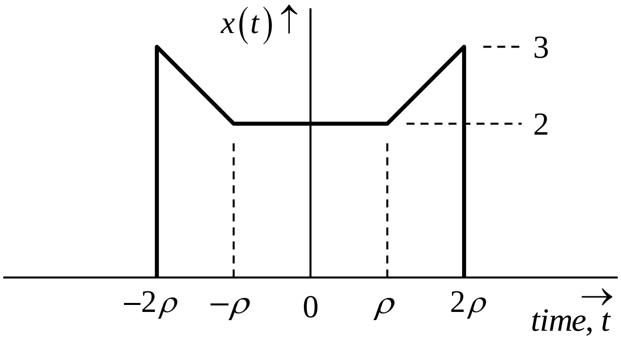

Figure Q6

Write down the value of ρ in your answer script and answer the following questions using this value.

(a) (i) Determine Fourier transform, $X(f)$, of $x(t)$.

(6 marks)

(ii) Determine the first null bandwidth of the signal $x(t)$.

(2 marks)

(b) The periodic signal $x_p(t)$ is obtained by replicating the signal $x(t)$ at periods of 10 seconds.

(i) Obtain an expression for $x_p(t)$ in terms of $x(t)$.

(2 marks)

(ii) Determine the Fourier transform, $X_p(f)$, of the periodic signal $x_p(t)$.

(5 marks)

(iii) Determine the Fourier series coefficients, $X_{p,k}$, of the periodic signal $x_p(t)$.

(2 marks)

(iv) Obtain an expression for the average power of the periodic signal $x_p(t)$.

(3 marks)

Q7. Figure Q7 shows the Bode magnitude plot for a linear time-invariant (LTI) system which has a transfer function $G(s) = \dfrac{K(s+\alpha)}{(s+\beta)(s+\gamma)(s+\lambda)}$.

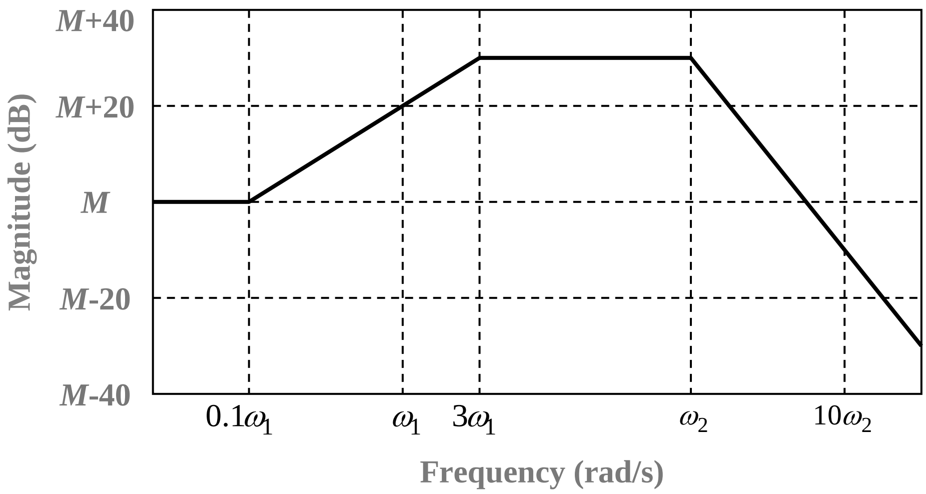

Figure Q7: Bode Magnitude for $G(s)$

In Figure Q7, the values of ω_1, ω_2 and M are obtained as:

$$\omega_1 = a+5, \quad \omega_2 = 100(b+6) \quad \text{and} \quad M = c+1$$

in which a, b and c are the values from your student number.

Write down the values of ω_1, ω_2 and M in your answer script and answer the following questions using these values.

(a) Find the values of K, α, β, γ and λ.

(9 marks)

(b) Find the low-frequency and high frequency asymptotic values of the phase response of $G(s)$.

(6 marks)

(c) Let $x(t)$ be the input and $y(t)$ the output of the LTI system. If $x(t) = 5u(t)$ where $u(t)$ is the unit step, find the steady-state value of $y(t)$, i.e. $\lim_{t\to\infty} y(t)$.

(5 marks)

Q8. Consider the circuit shown in Figure Q8, where the capacitance C and inductance L are obtained as:

$$C = b + 2 \quad \text{and} \quad L = d + 1$$

in which b and d are the values from your student number.

Write down the values of C and L in your answer script and answer the following questions using these values.

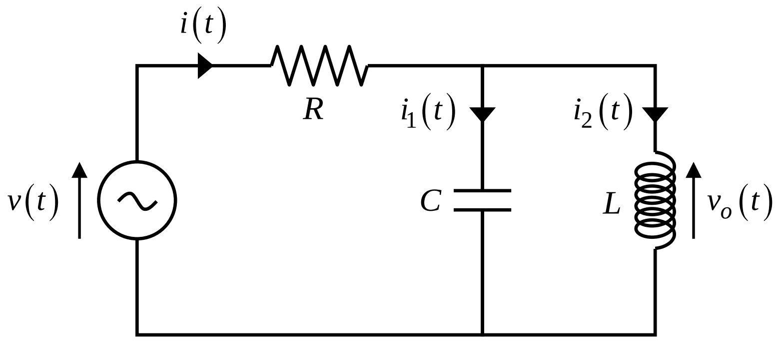

Figure Q8

(a) Derive the transfer function $G(s) = \dfrac{V_o(s)}{V(s)}$, where $V_o(s)$ and $V(s)$ are the Laplace transforms of $v_o(t)$ and $v(t)$, respectively.

(8 marks)

(b) The circuit is designed to operate as a critically damped system.

(i) Determine the value of R.

(3 marks)

(ii) Using the values for C, L and R in Part (i), what are the system poles, and the undamped natural frequency, ω_n .

(3 marks)

(iii) An input signal $v(t) = 5u(t)$ is applied to the system, where $u(t)$ is the unit step function. Derive the voltage, $v_o(t)$, across the inductor.

(6 marks)

END OF QUESTIONS