EE2023 Signals & Systems

AY2018/19-1

Midterm Quiz (Close Book)

Date: 4 Oct 2018 Time Allowed: 1.5 Hours

INSTRUCTIONS TO CANDIDATES:

1. Answer all 4 questions. Each question carries 10 marks.
2. This is a closed book quiz. However, you are allowed to bring a help sheet comprising one single sheet of paper of A4 size.
3. Tables of formulas are given on Pages 11 & 12, which you may detach for easy reference. You need not hand in these two pages.
4. Programmable and/or graphic calculators are not allowed.
5. Write your **answers** in the spaces indicated in this question paper. Attachment is not allowed.
6. Write your **name, matric number** and **seat number** in the spaces indicated below.

Name :________________________________

Matric # :________________________________

Seat # :________________________________

Question #	**Marks**
1	
2	
3	
4	
Total Marks	

Q.1 The signal $x(t) = e^{-3|t|}$ is shown in Figure Q1(a).

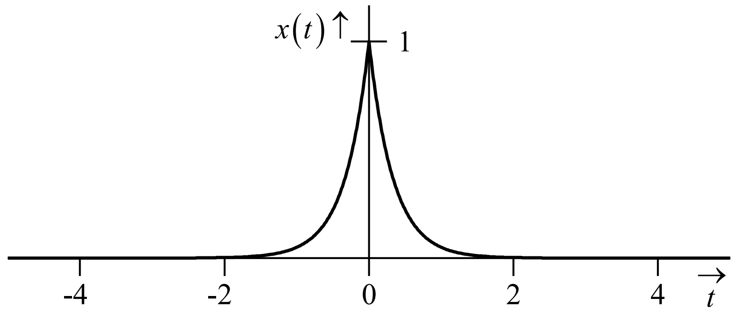

Figure Q1(a)

(a) Determine the Fourier transform, $X(f)$, of $x(t)$. (3 marks)

(b) The periodic signal $x_p(t)$ can be obtained by replicating $x(t)$ at a period of 3 seconds as shown in Figure Q1(b). Obtain an expression for $x_p(t)$ in terms of $x(t)$ and the Dirac δ-function. (2 marks)

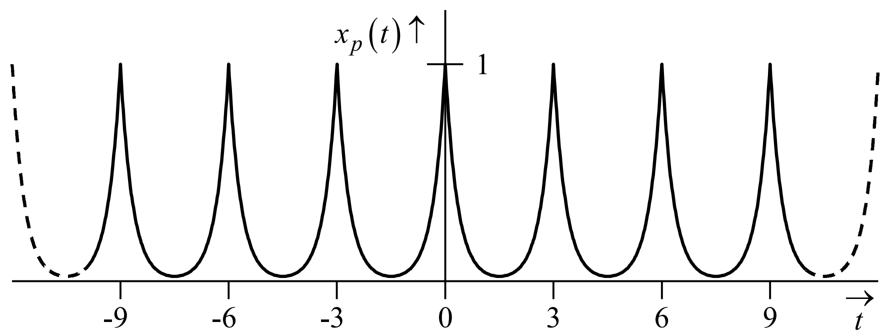

Figure Q1(b)

(c) Determine the Fourier transform, $X_p(f)$, of the periodic signal $x_p(t)$. (3 marks)

(d) Determine the Fourier series coefficients, c_k, of the periodic signal $x_p(t)$.

(2 marks)

Q.1 ANSWER

Q.1 ANSWER ~ continued

Q.2 The signal $x(t)$ has a half-cosine shaped spectrum, $X(f)$, as shown in Figure Q2.

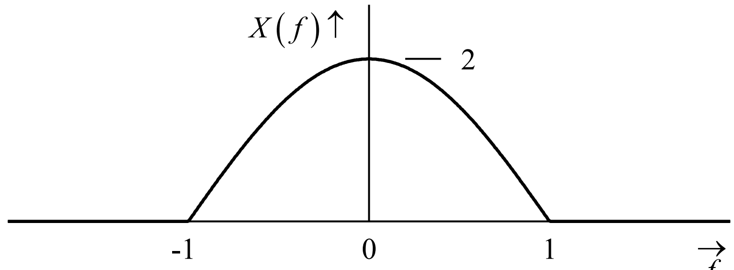

Figure Q2

(a) i. Express the spectrum $X(f)$ as a function of frequency f. (3 marks)

ii. What is the Nyquist sampling frequency for $x(t)$? (1 mark)

(b) Instead of sampling at the Nyquist frequency, the signal $x(t)$ is sampled at 4 Hz to obtain the signal $x_s(t)$.

i. Derive an expression for the sampled signal $x_s(t)$ in terms of $x(t)$. (2 marks)

ii. Let $X_s(f)$ denote the spectrum of $x_s(t)$. Derive an expression for $X_s(f)$ and sketch it. (4 marks)

Q.2 ANSWER

Q.2 ANSWER ~ continued

Q.3 Figure Q3 shows the discrete-frequency magnitude and phase spectra of a periodic signal $x(t)$.

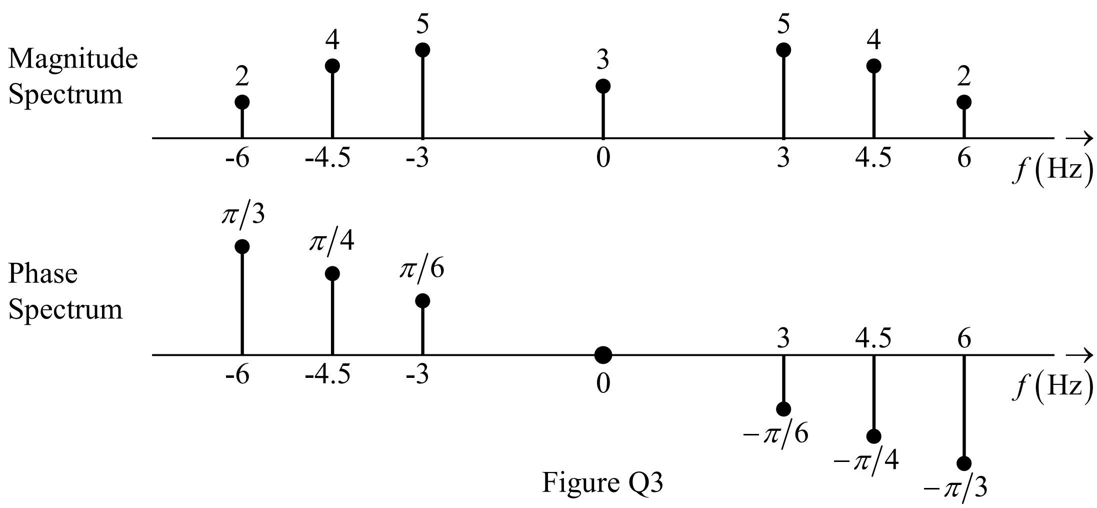

Figure Q3

(a) The Fourier series expansion of $x(t)$ is given by $x(t) = \sum_{k=-\infty}^{\infty} c_k \exp\left(j2\pi k f_p t\right)$. Find the values of f_p and c_k $(\forall \text{ integer } k)$. (5 marks)

(b) Draw a labeled sketch of the power spectral density, $P_x(f)$, of $x(t)$. (2 marks)

(c) What are the DC value and average power of $x(t)$? (3 marks)

Q.3 ANSWER

Q.3 ANSWER ~ continued

Q.4 Consider the signal $x(t) = 4\operatorname{sinc}^2(2t) - \operatorname{sinc}^2(t)$. Let $X(f)$ denote the Fourier transform of $x(t)$.

(a) By applying appropriate formulas given in the tables of Fourier transform pairs and Fourier transform properties, find $X(f)$ and draw a labeled sketch of it. (5 marks)

(b) Derive the 3dB bandwidth of $x(t)$. (3 marks)

(c) Determine the value of $\int_{-\infty}^{\infty} x(t)\,dt$. (2 marks)

Q.4 ANSWER

Q.4 ANSWER ~ continued

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Fourier Series: $\begin{cases} c_k = \dfrac{1}{T}\int_{\tilde{t}}^{\tilde{t}+T} x(t)\exp(-j2\pi k\,t/T)\,dt \\ x(t) = \sum_{k=-\infty}^{\infty} c_k \exp(j2\pi k\,t/T) \end{cases}$

Fourier Transform: $\begin{cases} X(f) = \int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)\,dt \\ x(t) = \int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)\,df \end{cases}$

FOURIER TRANSFORMS OF BASIC FUNCTIONS		
	$x(t)$	$X(f)$
Constant	K	$K\delta(f)$
Unit Impulse	$\delta(t)$	**1**
Unit Step	$u(t)$	$\frac{1}{2}\left[\delta(f)+\frac{1}{j\pi f}\right]$
Sign (or Signum)	$\operatorname{sgn}(t)$	$\frac{1}{j\pi f}$
Rectangle	$\operatorname{rect}\left(\frac{t}{T}\right)$	$T\operatorname{sinc}(fT)$
Triangle	$\operatorname{tri}\left(\frac{t}{T}\right)$	$T\operatorname{sinc}^2(fT)$
Sine Cardinal	$\operatorname{sinc}\left(\frac{t}{T}\right)$	$T\operatorname{rect}(fT)$
Complex Exponential	$\exp(j2\pi f_o t)$	$\delta(f-f_o)$
Cosine	$\cos(2\pi f_o t)$	$\frac{1}{2}\left[\delta(f-f_o)+\delta(f+f_o)\right]$
Sine	$\sin(2\pi f_o t)$	$-\frac{j}{2}\left[\delta(f-f_o)-\delta(f+f_o)\right]$
Gaussian	$\exp\left(-\frac{t^2}{\alpha^2}\right)$	$\alpha\pi^{0.5}\exp\left(-\alpha^2\pi^2 f^2\right)$
Comb	$\sum_{m=-\infty}^{\infty}\delta(t-mT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$

FOURIER TRANSFORM PROPERTIES		
	Time-domain	**Frequency-domain**
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(f)+\beta X_2(f)$
Time scaling	$x(\beta t)$	$\frac{1}{\lvert\beta\rvert}X\left(\frac{f}{\beta}\right)$
Duality	$X(t)$	$x(-f)$
Time shifting	$x(t-t_o)$	$X(f)\exp(-j2\pi f t_o)$
Frequency shifting (Modulation)	$x(t)\exp(j2\pi f_o t)$	$X(f-f_o)$
Differentiation in the time-domain	$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n X(f)$
Multiplication in the time-domain	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty}X_1(\zeta)X_2(f-\zeta)\,d\zeta$ or $X_1(f)\ast X_2(f)$
Convolution in the time-domain	$\int_{-\infty}^{\infty}x_1(\zeta)x_2(t-\zeta)\,d\zeta$ or $x_1(t)\ast x_2(t)$	$X_1(f)X_2(f)$
Integration in the time-domain	$\int_{-\infty}^{t}x(\tau)\,d\tau$	$\frac{1}{j2\pi f}X(f)+\frac{1}{2}X(0)\delta(f)$
		$\frac{1}{j2\pi f}X(f)$ **if** $X(0)=0$

Trigonometric Identities	
$\exp(\pm j\theta) = \cos(\theta) \pm j\sin(\theta)$	$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$
$\cos(\theta) = 0.5\left[\exp(j\theta) + \exp(-j\theta)\right]$	$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
$\sin(\theta) = -0.5j\left[\exp(j\theta) - \exp(-j\theta)\right]$	$\tan(\alpha \pm \beta) = \dfrac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$
$\sin^2(\theta) + \cos^2(\theta) = 1$	
$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$	$\sin(\alpha)\sin(\beta) = 0.5\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$
$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$	$\cos(\alpha)\cos(\beta) = 0.5\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$
$\sin^2(\theta) = 0.5\left[1 - \cos(2\theta)\right]$	$\sin(\alpha)\cos(\beta) = 0.5\left[\sin(\alpha - \beta) + \sin(\alpha + \beta)\right]$
$\cos^2(\theta) = 0.5\left[1 + \cos(2\theta)\right]$	$C\cos(\theta) - S\sin(\theta) = \sqrt{C^2 + S^2}\cos\left[\theta + \tan^{-1}(S/C)\right]$

Definitions of Basic Functions
Rectangle: $\operatorname{rect}\left(\frac{t}{T}\right) = \begin{cases} 1; & -T/2 \le t < T/2 \\ 0; & \text{elsewhere} \end{cases}$
Triangle: $\operatorname{tri}\left(\frac{t}{T}\right) = \begin{cases} 1 - \lvert t\rvert/T; & \lvert t\rvert \le T \\ 0; & \lvert t\rvert > T \end{cases}$
Sine Cardinal: $\operatorname{sinc}\left(\frac{t}{T}\right) = \begin{cases} \dfrac{\sin(\pi t/T)}{\pi t/T}; & t \ne 0 \\ 1; & t = 0 \end{cases}$
Signum: $\operatorname{sgn}(t) = \begin{cases} 1; & t \ge 0 \\ -1; & t < 0 \end{cases}$
Unit Impulse: $\delta(t) = \begin{cases} \infty; & t = 0 \\ 0; & t \ne 0 \end{cases} \qquad \int_{0^-}^{0^+} \delta(t)\,dt = 1$
Unit Step: $u(t) = \begin{cases} 1; & t \ge 0 \\ 0; & t < 0 \end{cases}$