Date No.

Student Number: A01998061 Total Pages: 9
(including cover page)

I have written this quiz with no help from others

Signature: 

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$a=9$

$b=8$

$c=0$

$d=6$

1a. $\alpha = 9+1 = 10$

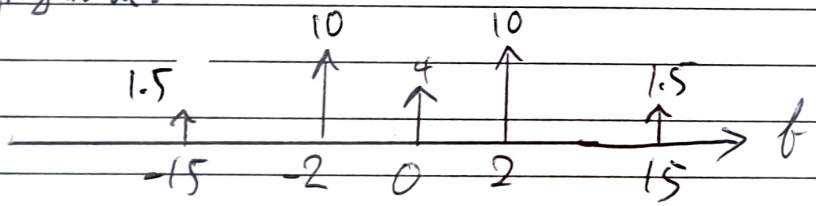
$\beta = 8+5 = 15$

$\gamma = 0+2 = 2$

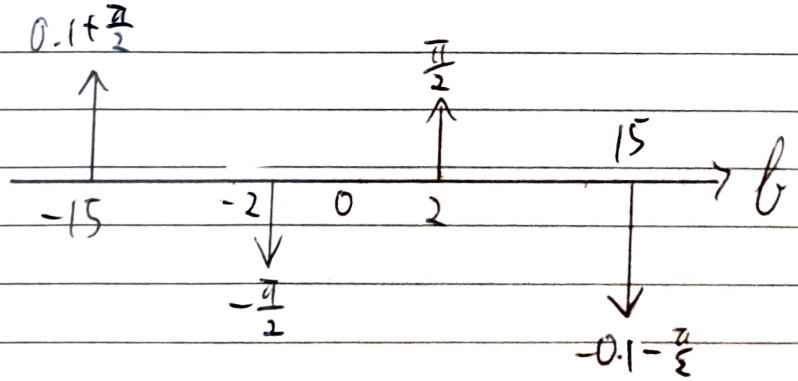
$$X(f) = 1.5j e^{j0.1}\delta(f+15) - j10\delta(f+2) + 4\delta(f) + j10\delta(f-2) - 1.5j e^{-j0.1}\delta(f-15)$$

$$= 1.5e^{j(0.1+\frac{\pi}{2})}\delta(f+15) + 10e^{-j\frac{\pi}{2}}\delta(f+2) + 4\delta(f) + 10e^{j\frac{\pi}{2}}\delta(f-2) + 1.5e^{j(-0.1-\frac{\pi}{2})}\delta(f-15)$$

magnitude:



phase



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b. $x(t)$ is not a periodic signal, there are no highest common factor among the frequency components.

c. $x(t)$ is not periodic, therefore no fourier series representation.

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2a. $\lambda = (9+10) = 19$

$X(f) = \text{rect}\left(\frac{f-19}{4}\right) + \text{rect}\left(\frac{f+19}{4}\right)$

Applying duality FT property for $X'(f) = \text{rect}\left(\frac{f}{4}\right) + \text{rect}\left(\frac{f}{4}\right)$

$x(t) = \mathcal{F}\{X(-t)\}$

$= 4\,\text{sinc}(-4t) + 4\,\text{sinc}(-4t)$

Applying time shifting property,

$x(t) = 4\,\text{sinc}(4t)\, e^{-j2\pi 19(-t)} + 4\,\text{sinc}(4t)\, e^{-j2\pi(-19)(-t)}$

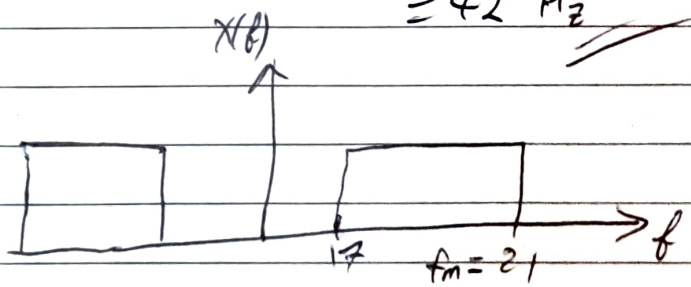
$= 4\,\text{sinc}(4t)\left[e^{+j38\pi t} + e^{-j38\pi t}\right]$

$= 4\,\text{sinc}(-4t) \cdot \left[2\cos(38\pi t)\right]$

$= 8\,\text{sinc}(-4t)\cos(38\pi t)$

b. $f_{Nyquist} = 2f_m = 2(21)$

$= 42\ \text{Hz}$

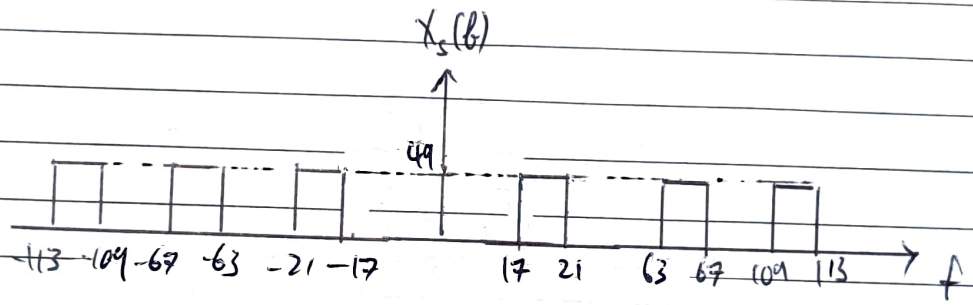


c. $f_s = 2(19+4) = 46\text{Hz}$

$$X_s(f) = f_s \sum_{-\infty}^{\infty} X(f - kf_s)$$

$$= 46 \sum_{k=-\infty}^{\infty} X(f - k46)$$

$$= 46 \sum_{k=-\infty}^{\infty} \text{rect}\left(\frac{f - k46 - 19}{4}\right) + \text{rect}\left(\frac{f - k46 + 19}{4}\right)$$



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3. $\alpha = 9 + 5 = 14$

$X(f) = 2(14)\,\mathrm{tri}\left(\frac{f}{2}\right) = 28\,\mathrm{tri}\left(\frac{f}{14}\right)$

a. Applying duality property,

$x(t) = 28\left[14\,\mathrm{sinc}^2(-14t)\right]$

$= 392\,\mathrm{sinc}^2(-14t)$

b. $\int_{-\infty}^{\infty} x(t)\,dt = \int_{-14}^{14} 28\,\mathrm{tri}\left(\frac{f}{14}\right) df$

$= 56\int_{0}^{14} 1 - \frac{f}{14}\,df$

$= 56\left[f -$

C. 3dB, $|X(B)| = \frac{|X(0)|}{\sqrt{2}}$

$28\,tri\left(\frac{B}{14}\right) = \frac{28}{\sqrt{2}}$

$tri\left(\frac{B}{14}\right) = \frac{1}{\sqrt{2}}$

$1 - \frac{|B|}{14} = \frac{1}{\sqrt{2}}$

$\frac{|B|}{14} = 1 - \frac{1}{\sqrt{2}}$

$|B| = 14 - \frac{14}{\sqrt{2}}$

$B = 4.10\,Hz\,(3\,s.f.)$

Bandwidth = 4.10 Hz

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4. $\beta = 9+2 = 11$

a. $2p = 2\beta$

$p = \beta = 11$ //

$\frac{2\pi t}{2\beta} = \frac{1}{q}\pi t$

$q = \beta = 11$ //

$r = 3$ //

b. let $x_1(t) = rect\left(\frac{t}{2p}\right) = rect\left(\frac{t}{22}\right)$

$x_2(t) = \cos\left(\frac{1}{q}\pi t\right) = \cos\left(\frac{\pi t}{11}\right)$

$X_1(f) = 22\,sinc(22f)$

$X_2(f) = \frac{1}{2}\left[\delta\left(f-\frac{1}{22}\right) + \delta\left(f+\frac{1}{22}\right)\right]$

$x(t) = 3 + x_1(t)\,x_2(t)$

$\Rightarrow X(f) = 3 + \int_{-\infty}^{\infty} X_1(\alpha) X_2(f-\alpha)\,d\alpha$

$= 3 + \int_{-\infty}^{\infty} 22\,sinc(22\alpha)\,\frac{1}{2}\left[\delta\left(f-\alpha-\frac{1}{22}\right) + \delta\left(f-\alpha+\frac{1}{22}\right)\right] d\alpha$

$= 3 + \int_{-\infty}^{\infty} 11\,sinc(22\alpha)\,\delta\left(f-\alpha-\frac{1}{22}\right) d\alpha + \int_{-\infty}^{\infty} 11\,sinc(22\alpha)\,\delta\left(f-\alpha+\frac{1}{22}\right) d\alpha$

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$$X(f) = 3 + \int_{-\infty}^{\infty} 11\,\mathrm{sinc}(22\alpha)\,\delta(f-\alpha-\tfrac{1}{22})\,d\alpha + \int_{-\infty}^{\infty} 11\,\mathrm{sinc}(22\alpha)\,\delta(f-\alpha+\tfrac{1}{22})\,d\alpha$$

$$= 3 + \int_{-\infty}^{\infty} 11\,\mathrm{sinc}(22\alpha+1)\,d\alpha + \int_{-\infty}^{\infty} 11\,\mathrm{sinc}(22\alpha-1)\,d\alpha$$

c. $T_s = 30s \Rightarrow f_s = \frac{1}{30}$ Hz

$x(t) *$ Dirac comb