NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester II : 2015/2016)

EE2023 – SIGNALS & SYSTEMS

April / May 2016 - Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES

1. This paper contains **EIGHT (8)** questions and comprises **ELEVEN (11)** printed pages.
2. Answer **ALL** questions in **Section A** and **ANY THREE (3)** questions in **Section B**.
3. This is a **CLOSED BOOK** examination.
4. Programmable calculators are not allowed.
5. Tables of Fourier Transforms, Laplace Transforms and trigonometric identities are provided in Pages 9, 10 and 11, respectively.

SECTION A : Answer ALL questions in this section

Q1. A pendulum system has a transfer function given by

$$G(s) = \frac{K}{2s^2 + 3s + C}$$

where K and C are uncertain parameters of the system.

(a) If the undamped natural frequency of the pendulum is 1 rad/s, find C.

(2 marks)

(b) Find the range of values of C for which $G(s)$ has complex poles.

(3 marks)

(c) Design the values of K and C such that the output of $G(s)$ is underdamped and the gain to a unit step input is 2. Sketch and label this output response of $G(s)$.

(5 marks)

Q2. Figure Q2 shows the half-cosine amplitude spectrum, $X(f)$, of the signal $x(t)$.

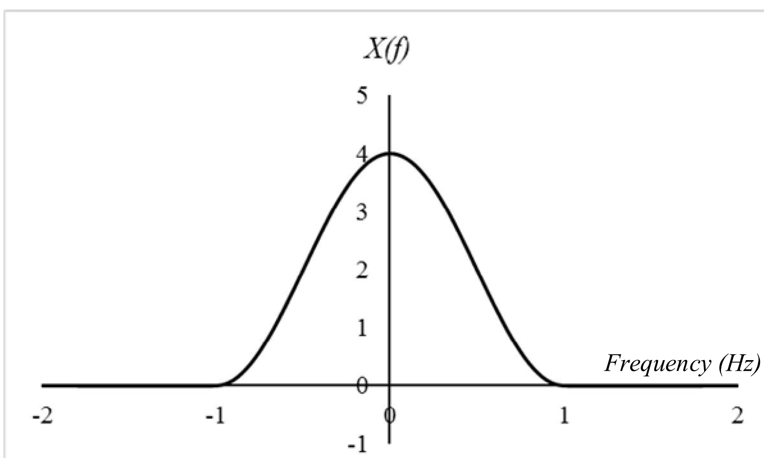

Figure Q2 : Amplitude Spectrum, $X(f)$

(a) What is the energy of signal $x(t)$?

(3 marks)

(b) What is the 3dB bandwidth of signal $x(t)$?

(3 marks)

(c) Determine the expression for signal $x(t)$.

(4 marks)

Q3. Consider the time-domain periodic signal, $x(t) = 2 + \cos(12t + \frac{\pi}{3}) + \sin(16t)$.

(a) The complex exponential Fourier series expansion of $x(t)$ is given by :

$$x(t) = \sum_{k=-\infty}^{\infty} c_k \exp\left(j2\pi \frac{k}{T_p} t \right).$$

Find T_p and c_k.

(5 marks)

(b) Determine the Fourier transform $X(f)$ of $x(t)$.

(2 marks)

(c) Sketch the magnitude spectrum and phase spectrum of $x(t)$ with proper labelling.

(3 marks)

Q4. The input-output relationship of a mass-spring-damper system is governed by the following differential equation:

$$\frac{d^2 y(t)}{dt^2} + 50\frac{dy(t)}{dt} + 400y(t) = 600x(t).$$

(a) Derive the system transfer function, $G(s) = \dfrac{Y(s)}{X(s)}$, where $X(s) = L\{x(t)\}$ and $Y(s) = L\{y(t)\}$ are the Laplace transforms of the input, $x(t)$, and output, $y(t)$, respectively.

(2 marks)

(b) What is the DC gain of the system?

(1 marks)

(c) Is the system underdamped, critically damped or overdamped?

(3 marks)

(d) The input signal, $x(t) = 10\cos(30t)$, is applied to the mass-spring-damper system. Determine the steady-state output signal, $\lim_{t\to\infty} y(t)$.

(4 marks)

SECTION B : Answer 3 out of the 4 questions in this section

Q5. Consider the circuit in Figure Q5 below. Assume zero initial conditions in all cases.

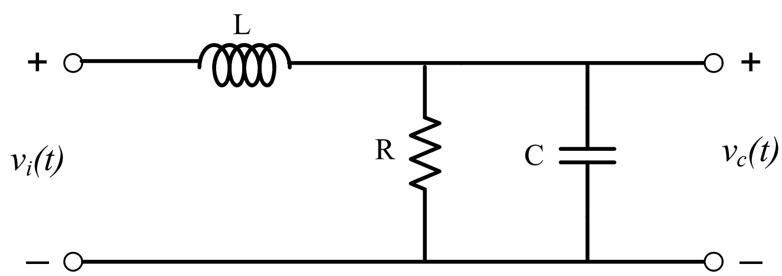

Figure Q5 : R-L-C Circuit

(a) Derive the transfer function, $G(s)=\dfrac{V_c(s)}{V_i(s)}$, where $V_i(s)=L\{v_i(t)\}$ and $V_c(s)=L\{v_c(t)\}$ are the Laplace transforms of $v_i(t)$ and $v_c(t)$ respectively.

(4 marks)

(b) Find the unit impulse response of the circuit if $LC = RC = 0.25$.

(4 marks)

(c) Find the total response of the voltage across the capacitor if $v_i(t)=u(t)$ where $u(t)$ is the unit step function. Assume $LC = RC = 0.25$.

(4 marks)

(d) Based on the transfer function from part (a), what type of system do you get if $R=\infty$? Justify your answer.

(4 marks)

(e) Sketch the unit step response of the circuit if $LC = 0.25$ and $R=\infty$. Label your sketch appropriately.

(4 marks)

Q.6 The signal $x(t)$ is shown in Figure Q6.

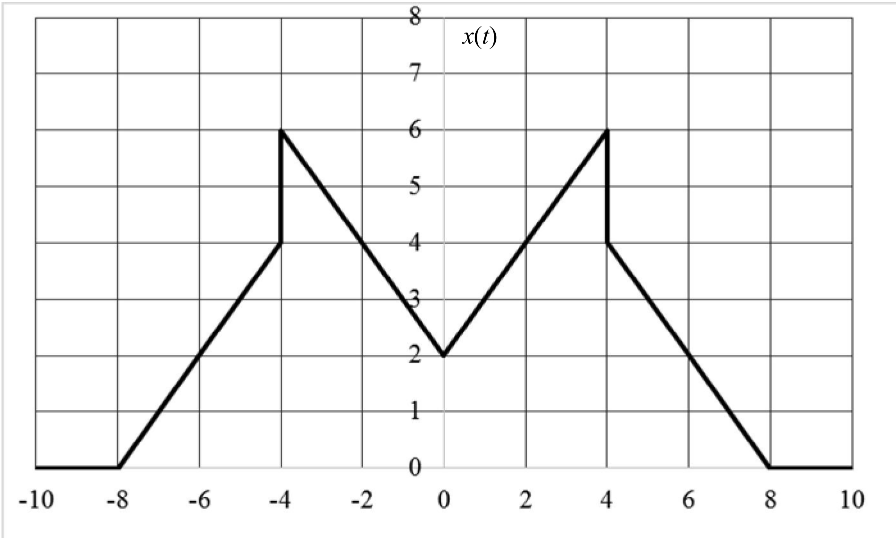

Figure Q.6 – Signal $x(t)$

(a) Determine the Fourier transform, $X(f)$, of the signal $x(t)$.

(6 marks)

(b) The signal $x(t)$ is sampled at 0.5 Hz to give the sampled signal $x_s(t)$. Obtain the Fourier transform, $X_s(f)$, of the signal $x_s(t)$.

(6 marks)

(c) The periodic signal $x_p(t)$ comprises repetitions of the pulse $x(t)$ at periodic intervals of 20 seconds.

i. Sketch the signal $x_p(t)$.

(2 marks)

ii. Determine the Fourier transform of the periodic signal $x_p(t)$.

(6 marks)

Q7. Two time-domain periodic signals are given by $x(t) = 2\text{sinc}(2.5t - 0.5) * \sum_{n=-\infty}^{\infty} \delta(t - 2n)$ and $y(t) = x(t)\cos(20\pi t)$.

(a) Find fundamental frequency, f_p of $x(t)$ and its Fourier transform, $X(f)$.

(8 marks)

(b) Determine the complex exponential Fourier series coefficients, c_k, of $x(t)$ and sketch the magnitude spectrum of $x(t)$ with proper labelling. Find the power, P_1 of $x(t)$.

(6 marks)

(c) Derive the Fourier transform, $Y(f)$, of $y(t)$ in terms of $X(f)$ and find the power, P_2 of $y(t)$?

(6 marks)

8. The Bode diagram of a system, $G(s)$, is shown in Figure Q8-1.

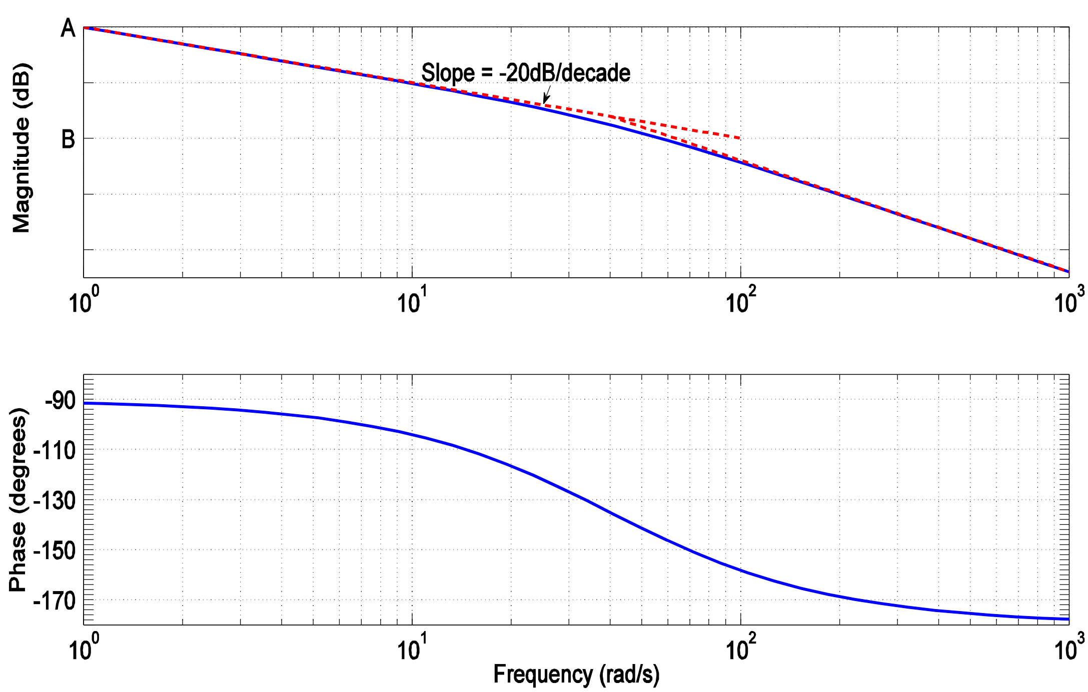

Figure Q8-1 : The Bode diagram of a system, $G(s)$.

Q8 continues on Page 7

The values on the y-axis of the Bode Magnitude diagram in Figure Q8-1 have not been marked, and needs to be deduced using the following information:

- The steady-state output signal for an input sinusoidal waveform, with an angular frequency **ω = 1 rad/s**, is also a sinusoidal waveform having the same frequency, but with an amplitude **10 times** that of the input, and phase difference $\boldsymbol{\phi}$.

(a) What is the value of ϕ?

(2 marks)

(b) Determine the values of A and B on the y-axis of the Bode Magnitude diagram.

(4 marks)

(c) Identify the system transfer function, $G(s)$.

(6 marks)

(d) Suppose the input signal, $x(t)$, shown in Figure Q8-2 is applied to the system, $G(s)$.

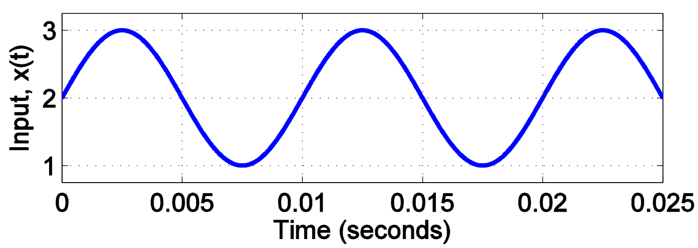

Figure Q8-2 : Input signal, $x(t)$

i. Derive the equation of the input signal, $x(t)$.

(3 marks)

ii. Explain why the output signal is essentially the step response of $G(s)$. Hence, or otherwise, sketch the output signal.

(5 marks)

END OF QUESTIONS

This page is intentionally left blank to facilitate detachment of the formula sheet for easy reference.

Fourier Series: $$\begin{cases} X_k = \dfrac{1}{T}\displaystyle\int_{\tilde{t}}^{\tilde{t}+T} x(t)\exp(-j2\pi k\,t/T)\,dt \\ x(t) = \displaystyle\sum_{k=-\infty}^{\infty} X_k \exp(j2\pi k\,t/T) \end{cases}$$

Fourier Transform: $$\begin{cases} X(f) = \displaystyle\int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)\,dt \\ x(t) = \displaystyle\int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)\,df \end{cases}$$

FOURIER TRANSFORMS OF BASIC FUNCTIONS

	$x(t)$	$X(f)$
Constant	K	$K\delta(f)$
Unit Impulse	$\delta(t)$	**1**
Unit Step	$u(t)$	$\frac{1}{2}\left[\delta(f)+\frac{1}{j\pi f}\right]$
Sign (or Signum)	$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
Rectangle	$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}(fT)$
Triangle	$\mathrm{tri}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}^2(fT)$
Sine Cardinal	$\mathrm{sinc}\left(\frac{t}{T}\right)$	$T\,\mathrm{rect}(fT)$
Complex Exponential	$\exp(j2\pi f_o t)$	$\delta(f-f_o)$
Cosine	$\cos(2\pi f_o t)$	$\frac{1}{2}\left[\delta(f-f_o)+\delta(f+f_o)\right]$
Sine	$\sin(2\pi f_o t)$	$-\frac{j}{2}\left[\delta(f-f_o)-\delta(f+f_o)\right]$
Gaussian	$\exp\left(-\frac{t^2}{\alpha^2}\right)$	$\alpha\pi^{0.5}\exp(-\alpha^2\pi^2 f^2)$
Comb	$\sum_{m=-\infty}^{\infty}\delta(t-mT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$

FOURIER TRANSFORM PROPERTIES

	Time-domain	**Frequency-domain**
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(f)+\beta X_2(f)$
Time scaling	$x(\beta t)$	$\frac{1}{\lvert\beta\rvert}X\left(\frac{f}{\beta}\right)$
Duality	$X(t)$	$x(-f)$
Time shifting	$x(t-t_o)$	$X(f)\exp(-j2\pi f t_o)$
Frequency shifting (Modulation)	$x(t)\exp(j2\pi f_o t)$	$X(f-f_o)$
Differentiation in the time-domain	$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n X(f)$
Multiplication in the time-domain	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty}X_1(\zeta)X_2(f-\zeta)\,d\zeta$ or $X_1(f)\ast X_2(f)$
Convolution in the time-domain	$\int_{-\infty}^{\infty}x_1(\zeta)x_2(t-\zeta)\,d\zeta$ or $x_1(t)\ast x_2(t)$	$X_1(f)X_2(f)$
Integration in the time-domain	$\int_{-\infty}^{t}x(\tau)\,d\tau$	$\frac{1}{j2\pi f}X(f)+\frac{1}{2}X(0)\delta(f)$ $\frac{1}{j2\pi f}X(f)$ **if** $X(0)=0$

Unilateral Laplace Transform: $X(s) = \int_{0^-}^{\infty} x(t)\exp(-st)\,dt$

LAPLACE TRANSFORMS OF BASIC FUNCTIONS		
	$x(t)$	$X(s)$
Unit Impulse	$\delta(t)$	1
Unit Step	$u(t)$	$1/s$
Ramp	$t\,u(t)$	$1/s^2$
n^{th} order Ramp	$t^n u(t)$	$\dfrac{n!}{s^{n+1}}$
Damped Ramp	$t\exp(-\alpha t)u(t)$	$1/(s+\alpha)^2$
Exponential	$\exp(-\alpha t)u(t)$	$1/(s+\alpha)$
Cosine	$\cos(\omega_o t)u(t)$	$s/(s^2+\omega_o^2)$
Sine	$\sin(\omega_o t)u(t)$	$\omega_o/(s^2+\omega_o^2)$
Damped Cosine	$\exp(-\alpha t)\cos(\omega_o t)u(t)$	$\dfrac{s+\alpha}{(s+\alpha)^2+\omega_o^2}$
Damped Sine	$\exp(-\alpha t)\sin(\omega_o t)u(t)$	$\dfrac{\omega_o}{(s+\alpha)^2+\omega_o^2}$

LAPLACE TRANSFORM PROPERTIES		
	Time-domain	***s*-domain**
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(s)+\beta X_2(s)$
Time shifting	$x(t-t_o)u(t-t_o)$	$\exp(-st_o)X(s)$
Shifting in the s-domain	$\exp(s_o t)x(t)$	$X(s-s_o)$
Time scaling	$x(\alpha t)$	$\dfrac{1}{\lvert\alpha\rvert}X\left(\dfrac{s}{\alpha}\right)$
Integration in the time-domain	$\int_{0^-}^{t} x(\zeta)\,d\zeta$	$\dfrac{1}{s}X(s)$
Differentiation in the time-domain	$\dfrac{dx(t)}{dt}$	$sX(s)-x(0^-)$
	$\dfrac{d^n x(t)}{dt^n}$	$s^n X(s)-\sum_{k=0}^{n-1} s^{n-1-k}\left.\dfrac{d^k x(t)}{dt^k}\right\rvert_{t=0^-}$
Differentiation in the s-domain	$-tx(t)$	$\dfrac{dX(s)}{ds}$
	$(-t)^n x(t)$	$\dfrac{d^n X(s)}{ds^n}$
Convolution in the time-domain	$\int_{-\infty}^{\infty} x_1(\zeta)x_2(t-\zeta)\,d\zeta$	$X_1(s)X_2(s)$
Initial value theorem	$x(0^+)=\lim_{s\to\infty} sX(s)$	
Final value theorem	$\lim_{t\to\infty} x(t)=\lim_{s\to 0} sX(s)$	

	$y_{step}(t)$	$Y_{step}(s)$	**SYSTEM PARAMETERS**
Step response of 1^{st} order system	$K\left[1-\exp\left(-\frac{t}{T}\right)\right]u(t)$	$\frac{1}{s}\cdot\frac{K}{(sT+1)}$	T: System Time-constant K: System Steady-state (or DC) Gain
Step response of 2^{nd} order <u>underdamped</u> system: $(0<\zeta<1)$	$K\left[1-\frac{\exp(-\omega_n\zeta t)}{(1-\zeta^2)^{0.5}}\sin\left(\omega_n(1-\zeta^2)^{0.5}t+\phi\right)\right]u(t)$	$\frac{1}{s}\cdot\frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	ω_n: System Undamped Natural Frequency ζ: System Damping Factor ω_d: System Damped Natural Frequency K: System Steady-state (or DC) Gain $\sigma=\omega_n\zeta$ $\omega_d^2=\omega_n^2(1-\zeta^2)$ $\omega_n^2=\sigma^2+\omega_d^2$ $\tan(\phi)=\omega_d/\sigma$
	$K\left[1-\left(\frac{\sigma^2+\omega_d^2}{\omega_d^2}\right)^{0.5}\exp(-\sigma t)\sin(\omega_d t+\phi)\right]u(t)$	$\frac{1}{s}\cdot\frac{K(\sigma^2+\omega_d^2)}{(s+\sigma)^2+\omega_d^2}$	

2^{nd} order system - RESONANCE - $(0\le\zeta<1/\sqrt{2})$	*RESONANCE FREQUENCY :* $\omega_r=\omega_n(1-2\zeta^2)^{0.5}$	*RESONANCE PEAK :* $M_r=\lvert H(j\omega_r)\rvert=\frac{K}{2\zeta(1-\zeta^2)^{0.5}}$

TRIGONOMETRIC IDENTITIES	
$\exp(\pm j\theta)=\cos(\theta)\pm j\sin(\theta)$	$\sin(\alpha\pm\beta)=\sin(\alpha)\cos(\beta)\pm\cos(\alpha)\sin(\beta)$
$\cos(\theta)=\frac{1}{2}[\exp(j\theta)+\exp(-j\theta)]$	$\cos(\alpha\pm\beta)=\cos(\alpha)\cos(\beta)\mp\sin(\alpha)\sin(\beta)$
$\sin(\theta)=\frac{1}{j2}[\exp(j\theta)-\exp(-j\theta)]$	$\tan(\alpha\pm\beta)=\frac{\tan(\alpha)\pm\tan(\beta)}{1\mp\tan(\alpha)\tan(\beta)}$
$\sin^2(\theta)+\cos^2(\theta)=1$	
$\sin(2\theta)=2\sin(\theta)\cos(\theta)$	$\sin(\alpha)\sin(\beta)=\frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$
$\cos(2\theta)=\cos^2(\theta)-\sin^2(\theta)$	$\cos(\alpha)\cos(\beta)=\frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$
$\sin^2(\theta)=\frac{1}{2}[1-\cos(2\theta)]$	$\sin(\alpha)\cos(\beta)=\frac{1}{2}[\sin(\alpha-\beta)+\sin(\alpha+\beta)]$
$\cos^2(\theta)=\frac{1}{2}[1+\cos(2\theta)]$	$\mathbf{C}\cos(\theta)-\mathbf{S}\sin(\theta)=\sqrt{\mathbf{C}^2+\mathbf{S}^2}\cos\left[\theta+\tan^{-1}\left(\frac{\mathbf{S}}{\mathbf{C}}\right)\right]$