EE2023 TUTORIAL 1 (PROBLEMS)

Q.1 Let z be a complex number. Provide a formula for computing the distinct values of $z^{1/N}$ where N is a positive integer. Hence, or otherwise, determine $64^{1/6}$ and $(j81)^{1/4}$

Q.2 Consider the signal $x(t) = 2\sin(\pi t)(p(t)-1)$ where $p(t)$ is shown in Fig.Q.2.

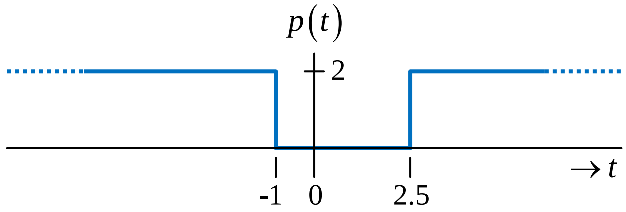


Fig.Q.2

(a) Express $p(t)$ in terms of the $\text{rect}(\bullet)$ function.

(b) Sketch and label $x(t)$ and state whether or not $x(t)$ is periodic.

(c) Find an expression for $x^2(t)$. Hence, compute the average power of $x(t)$.

(d) Based on the results in (b) and (c), How would you classify $x(t)$?

Q.3 In digital communications, half-cosine or raised-cosine pulses are sometimes used to pulse shape a binary waveform so as to reduce intersymbol interference. The general expressions for these pulses are

$$\text{Half-cosine pulse} \quad : \; x(t) = A\cos(\pi t/T)\,\text{rect}(t/T)$$
$$\text{Raised-cosine pulse} \quad : \; \tilde{x}(t) = 0.5\tilde{A}\left(1+\cos\left(2\pi t/\tilde{T}\right)\right)\text{rect}\left(t/\tilde{T}\right).$$

where A, $\tilde{A}$, T and $\tilde{T}$ are positive constants. Sketch and label each pulse. Under what condition(s) will both pulses have the same energy?

Q.4 Sketches of two signals, $x(t)$ and $y(t)$, are shown in Fig.Q.4.

(a) Sketch and label the following signals: $x(t+4)$; $x(-t)$; $x(3t)$; $x(t/3)$

(b) Express $y(t)$ in terms of $x(t)$.

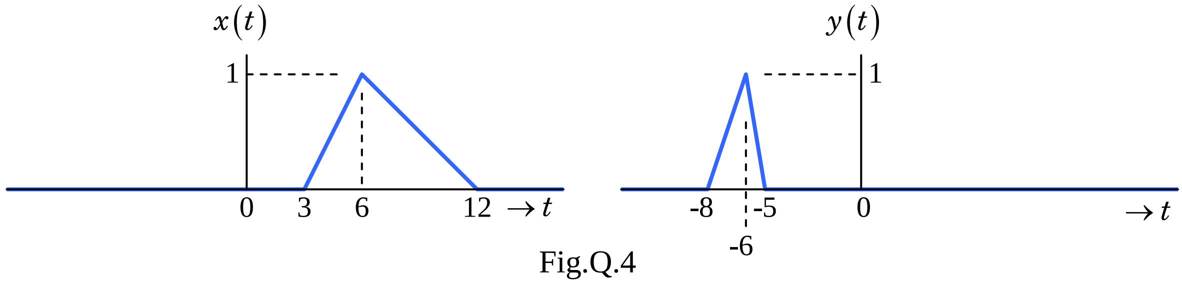
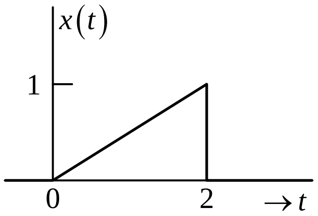


Fig.Q.4

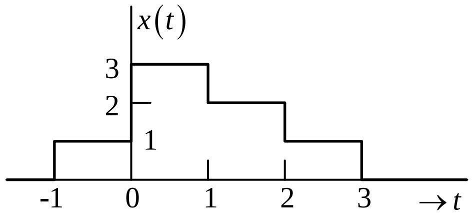
Supplementary Problems

These problems will not be discussed in class.

S.1 Express the signals shown in the figures below in terms of unit step functions.



(a)



(b)

Answer: (a) $x(t) = u(2-t) \cdot \int_{-\infty}^{t} 0.5u(\tau)d\tau$

(b) $x(t) = u(t+1) + 2u(t) - u(t-1) - u(t-2) - u(t-3)$

S.2 Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental period and average power.

(a) $x(t) = \cos(2t + 0.25\pi)$ (b) $x(t) = \cos^2(t)$

(c) $x(t) = \cos(2\pi t)u(t)$ (d) $x(t) = \exp(j\pi t)$

Answer: (a) *periodic, period* $= \pi$, *power* $= 1/2$ (b) *periodic, period* $= \pi$, *power* $= 3/8$

(c) *non-periodic* (d) *periodic, period* $= 2$, *power* $= 1$

S.3 Evaluate the following integrals:

(a) $\int_{-\infty}^{t} \cos(\tau)u(\tau)d\tau$ (b) $\int_{-\infty}^{t} \cos(\tau)\delta(\tau)d\tau$

(c) $\int_{-\infty}^{\infty} \cos(t)u(t-1)\delta(t)dt$ (d) $\int_{0}^{2\pi} t\sin\left(\frac{t}{2}\right)\delta(\pi - t)dt$

Answer: (a) $\sin(t)u(t)$ (b) $u(t)$

(c) 0 (d) π

S.4 Any signal $x(t)$ can be expressed as a sum of two component signals, one of which is even and one of which is odd. That is

$$x(t) = x_e(t) + x_o(t)$$

where $x_e(t) = 0.5[x(t) + x(-t)]$ is the even component and $x_o(t) = 0.5[x(t) - x(-t)]$ the odd component.

Determine the even and odd components of : (a) $x(t) = u(t)$ (b) $x(t) = \sin\left(\omega_c t + \frac{\pi}{4}\right)$

Answer: (a) $\begin{cases} x_e(t) = \begin{cases} 1; & t = 0 \\ 0.5; & t \neq 0 \end{cases} \\ x_o(t) = \begin{cases} 0; & t = 0 \\ 0.5\,\mathrm{sgn}(t); & t \neq 0 \end{cases} \end{cases}$ (b) $\begin{cases} x_e(t) = \frac{1}{\sqrt{2}}\cos(\omega_c t) \\ x_o(t) = \frac{1}{\sqrt{2}}\sin(\omega_c t) \end{cases}$

Below is a list of solved problems selected from ***Chapter 1*** *of* ***Hwei Hsu (PhD), 'The Schaum's series on Signals & Systems,' 2nd Edition****.*

Selected solved-problems: ***1.1, 1.9, 1.10, 1.14, 1.16(a)-to-(f), 1.17, 1.18, 1.20(a)-&-(b), 1.21, 1.22, 1.27, 1.30, 1.31***

These solved problems should be treated as supplementary module material catered for students who find the need for more examples or practice-problems.