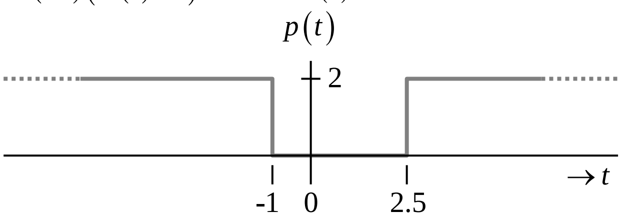
EE2023 TUTORIAL 1 (PROBLEMS)

Q.1 Let $z = x + jy$ where x and y are real numbers. Provide a formula for computing the N distinct values of $\sqrt[N]{z}$. Hence, or otherwise, determine $\sqrt[6]{64}$ and $\sqrt[4]{j81}$.

Q.2 Consider the signal $x(t) = 2\sin(\pi t)(p(t) - 1)$ where $p(t)$ is shown in Fig.Q.2.

Fig.Q.2



(a) Express $p(t)$ in terms of the $\text{rect}(\bullet)$ function.

(b) Sketch and label $x(t)$ and state whether or not $x(t)$ is periodic.

(c) Find an expression for $x^2(t)$. Hence, compute the average power of $x(t)$.

(d) Based on the results in (b) and (c), how would you classify $x(t)$?

Q.3 In digital communications, half-cosine or raised-cosine pulses are sometimes used to pulse shape a binary waveform so as to reduce intersymbol interference. The general expressions for these pulses are

Half-cosine pulse : $x(t) = A\cos(\pi t/T)\text{rect}(t/T)$

Raised-cosine pulse : $\tilde{x}(t) = 0.5\tilde{A}\left(1 + \cos\left(2\pi t/\tilde{T}\right)\right)\text{rect}\left(t/\tilde{T}\right)$.

where A, $\tilde{A}$, T and $\tilde{T}$ are positive constants. Sketch and label each pulse. Under what condition(s) will both pulses have the same energy?

Q.4 Determine whether or not each of the following signals is periodic. If the signal is periodic, determine its fundamental frequency.

(a) $x(t) = \cos(3.2t) + \sin(1.6t) + \exp(j2.8t)$

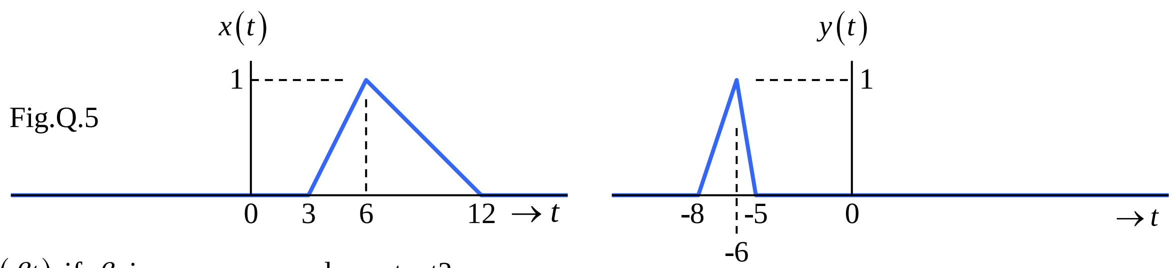
(b) $x(t) = \cos(4t) + \sin(\pi t)$

Q.5 Sketches of two signals, $x(t)$ and $y(t)$, are shown in Fig.Q.5.

(a) Sketch and label the following signals: $x(t+4)$; $x(-t)$; $x(3t)$; $x(t/3)$

(b) Express $y(t)$ in terms of $x(t)$.

Fig.Q.5

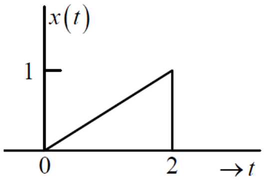


Q.6 What is $\delta(\beta t)$ if β is a non-zero real constant?

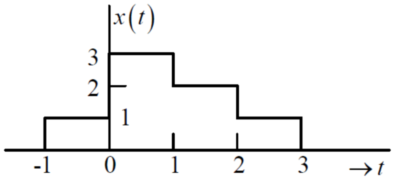
Supplementary Problems

These problems will not be discussed in class.

S.1 Express the signals shown in the figures below in terms of unit step functions.



(a)



(b)

Answer: (a) $x(t) = u(2-t) \bullet \int_{-\infty}^{t} 0.5u(\tau)d\tau$

(b) $x(t) = u(t+1) + 2u(t) - u(t-1) - u(t-2) - u(t-3)$

S.2 Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental period and average power.

(a) $x(t) = \cos\left(2t + 0.25\pi\right)$ *(b)* $x(t) = \cos^2\left(t\right)$

(c) $x(t) = \cos\left(2\pi t\right)u\left(t\right)$ *(d)* $x(t) = \exp\left(j\pi t\right)$

Answer: *(a)* *periodic, period = π, power = ½* *(b)* *periodic, period = π, power = 3/8*

(c) *non-periodic* *(d)* *periodic, period = 2, power = 1*

S.3 Evaluate the following integrals:

(a) $\int_{-\infty}^{t} \cos(\tau)u(\tau)d\tau$ *(b)* $\int_{-\infty}^{t} \cos(\tau)\delta(\tau)d\tau$

(c) $\int_{-\infty}^{\infty} \cos(t)u(t-1)dt$ *(d)* $\int_{0}^{2\pi} t\sin\left(\frac{t}{2}\right)\delta\left(\pi - t\right)dt$

Answer: *(a)* $\sin(t)u(t)$ *(b)* $u(t)$

(c) *0* *(d)* π

S.4 Any signal $x(t)$ can be expressed as a sum of two component signals, one of which is even and one of which is odd. That is

$$x(t) = x_e(t) + x_o(t)$$

where $x_e(t) = 0.5\left[x(t) + x(-t)\right]$ is the even component and $x_o(t) = 0.5\left[x(t) - x(-t)\right]$ the odd component.

Determine the even and odd components of: *(a)* $x(t) = u(t)$ *(b)* $x(t) = \sin\left(\omega_c t + \frac{\pi}{4}\right)$.

Answer: (a)
$$\begin{cases} x_e(t) = \begin{cases} 1; & t = 0 \\ 0.5; & t \neq 0 \end{cases} \\ x_o(t) = \begin{cases} 0; & t = 0 \\ 0.5\,\mathrm{sgn}(t); & t \neq 0 \end{cases} \end{cases}$$

(b)
$$\begin{cases} x_e(t) = \dfrac{1}{\sqrt{2}}\sin\left(\omega_c t\right) \\ x_e(t) = \dfrac{1}{\sqrt{2}}\cos\left(\omega_c t\right) \end{cases}$$

Below is a list of solved problems selected from Chapter 1 of Hwei Hsu (PhD), 'The Schaum's series on Signals& Systems', 2nd Edition.

The 1st Edition can be found in the following link:
http://www.kousik.net/wp-content/uploads/2010/10/Schaums-Outline-Series-Signals_Systems.pdf

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Selected solved-problems: 1.1, 1.9, 1.10, 1.14, 1.16(a)-to-(f), 1.17, 1.18, 1.20(a)-&-(b), 1.21, 1.22, 1.27, 1.30, 1.31

These solved problems should be treated as supplementary module material catered for students who find the need for more examples or practice-problems.