NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester II : 2013/2014)

EE2023 – SIGNALS & SYSTEMS

Apr/May 2014 - Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES

1. This paper contains **EIGHT (8)** questions and comprises **ELEVEN (11)** printed pages.
2. Answer **ALL** questions in **Section A** and **ANY THREE (3)** questions in **Section B**.
3. This is a **CLOSED BOOK** examination.
4. Programmable calculators are not allowed.
5. Tables of Fourier Transforms, Laplace Transforms and trigonometric identities are provided in Pages 9, 10 and 11, respectively.

SECTION A : Answer ALL questions in this section

Q1. Ah Kow has been given the task of determining the model of a circuit which has been placed in a black box. The black box has two external connections : one at the input to connect the signal source and the other at the output to measure the output signal. It is known that the circuit is a linear time invariant system, consisting of only inductors, capacitors and resistors.

(a) Ah Kow suspects that the black box contains a circuit which can be modeled as a first order transfer function.

i. Write down the general form of the first order transfer function. State the parameters in this transfer function.

(1 mark)

ii. Suggest a simple test which he can use to determine this transfer function. Your answer should include the type of test (input) signal which he should use. For this test signal which is used, you should also sketch the output signal and <u>explain</u> how the parameters of the first order transfer function can be obtained.

(3 marks)

(b) Sometime later, Ah Kow suspects that the black box contains a circuit which can be modeled as an underdamped second order system.

i. Write down the general form of this second order transfer function and give the appropriate range of the damping ratio.

(2 mark)

ii. Suggest a simple test which he can use to determine this transfer function. Your answer should include the type of test (input) signal which he should use. For this test signal, you should also sketch the output signal and <u>explain</u> how the parameters of the underdamped second order transfer function can be obtained.

(4 marks)

Q2. Given the periodic signal $x(t) = e^{j(2t+\pi/4)} + 2\cos\left(6t + \pi / 6\right) - 3\sin(14t)$.

(a) Determine the complex Fourier series coefficients of $x(t)$.

(6 marks)

(b) Sketch the magnitude spectrum of $x(t)$.

(2 marks)

(c) Sketch the phase spectrum of $x(t)$.

(2 marks)

Q3. The energy spectral density of a signal $x(t)$ is given by

$$E_x(f) = \cos^2(\pi f)\cdot \mathrm{rect}(f).$$

(a) Draw a labeled sketch of $E_x(f)$.

(2 marks)

(b) Find the 3 dB bandwidth of $x(t)$.

(3 marks)

(c) If $x(t)$ has a phase spectrum given by $\angle X(f) = -\pi f$, find $x(t)$ and compute its value at $t = 1/6$ to 4 decimal places.

(5 marks)

Q4. The unit impulse responses of 3 systems are shown in Figure Q4. You may assume that the systems do not have zeros.

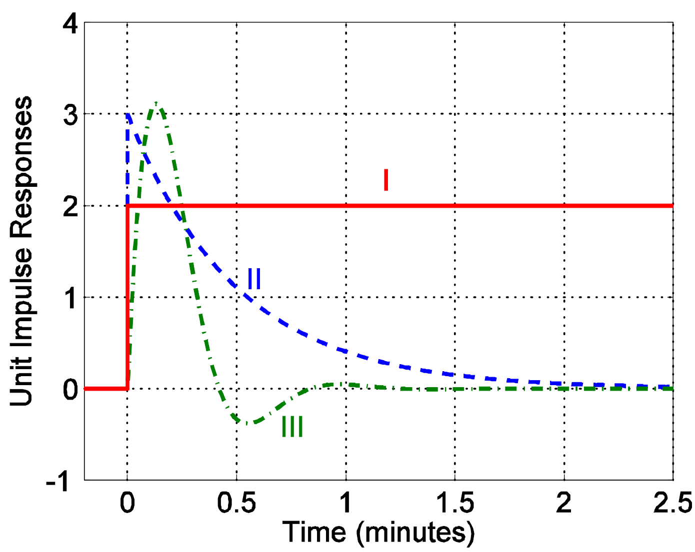

Figure Q4 : Unit Impulse Responses

(a) Determine the transfer function for System II given that its unit impulse response has a value of 0.406 at $t = 1$ minute.

(4 marks)

(b) Draw a pole-zero diagram that shows the pole(s) location(s) of the 3 systems. *Numerical values of the poles need not be given but their relative positions must be clear.*

(4 marks)

(c) Can the Final Value Theorem be used to predict the steady-state step response value for System I ? Justify your answer.

(2 marks)

SECTION B : Answer 3 out of the 4 questions in this section

Q5. A mass-damper system has a transfer function given by

$$G(s) = \frac{K}{2s^2 + 3s + C}$$

where K and C are uncertain parameters of the system.

(a) If the natural frequency of the system is 1 rad/s, find the value of C.

(2 marks)

(b) Assuming that C is uncertain, find the range of values of C for which $G(s)$ has complex poles. Sketch the corresponding region of the complex plane where the complex poles will lie for this range of C.

(5 marks)

(c) Find the range of values of C for which $G(s)$ has real poles. Sketch the corresponding region of the complex plane where the real poles will lie for this range of C.

(4 marks)

(d) For what value of C will $G(s)$ be critically damped?

(3 marks)

(e) Design the values of K and C such that the output of $G(s)$ is underdamped and the gain to a unit step input function is 2. Sketch and label the output response of $G(s)$ to the unit step function. Explain the effect of C on the frequency of the oscillations.

(6 marks)

Q.6 The periodic signal $x_p(t)$ is composed of repetitions of the pulse $x(t)$ at a period of 3 seconds, where $x(t)$ is described by

$$x(t) = \begin{cases} e^{3t}, & t \le 0 \\ 0 & \text{elsewhere} \end{cases}$$

and for t ≤ -3 seconds, we can assume that $x(t) \approx 0$.

(a) Determine the Fourier transform of $x_p(t)$.

(6 marks)

(b) Determine the Fourier series coefficients of $x_p(t)$.

(3 marks)

(c) Determine the average power of $x_p(t)$.

(4 marks)

(d) If $x_p(t)$ is sampled at a frequency 100 Hz to give the sampled signal $x_s(t)$:

i. Obtain an expression for the sampled signal $x_s(t)$ in terms of $x(t)$.

(3 marks)

ii. Determine the Fourier transform of $x_s(t)$.

(4 marks)

Q7. The radio frequency pulse used in a radar ranging system is modeled by

$$x(t) = \cos(2\pi f_o t) \cdot \text{rect}\left(\frac{t}{T}\right)$$

where $f_o > 0$ is the pulse frequency and $T > 0$ is the pulse width. The parameters f_o and T are chosen such that $f_o T >> 1$.

The pulse, $x(t)$, transmitted by the radar is reflected back to the radar by a stationary metallic object located at some distance away. The echo received by the radar has the form

$$y(t) = \beta \cdot \cos(2\pi f_o (t - \tau)) \cdot \text{rect}\left(\frac{t-\tau}{T}\right)$$

where β and τ each takes on a positive value that depends on the distance between the radar and the metallic object.

At the radar, $y(t)$ is applied to the input of a filter which produces $z(t) = x(t) \ast y(t)$ at its output. The symbol "$\ast$" denotes "convolution".

Both the transmitted and reflected pulses travel at a speed of 3×10^8 m/s.

(a) Express the spectrum, $Y(f)$, of $y(t)$ in terms of the spectrum, $X(f)$, of $x(t)$.

(4 marks)

(b) Find the spectrum, $Z(f)$, of $z(t)$. Express your answer in terms of sine cardinal $[\text{sinc}(\cdot)]$ and exponential $[\exp(\cdot)]$ functions

(6 marks)

(c) Owing to $f_o T >> 1$, we may assume that $\text{sinc}((f - f_o)T)\text{sinc}((f + f_o)T) = 0$. Applying this assumption to the $Z(f)$ found in Part (b), find $z(t)$.

(6 marks)

(d) From the result of Part (c), it is noted that $z(t)$ peaks at $t = \tau$. If $\tau = 8 \times 10^{-5}$ seconds, what is the distance between the radar and the metallic object?

(4 marks)

Q8. Figure Q8 shows the Bode magnitude plot of $G(s) = \dfrac{L\{y(t)\}}{L\{x(t)\}}$, a stable minimum phase system with no transportation lag.

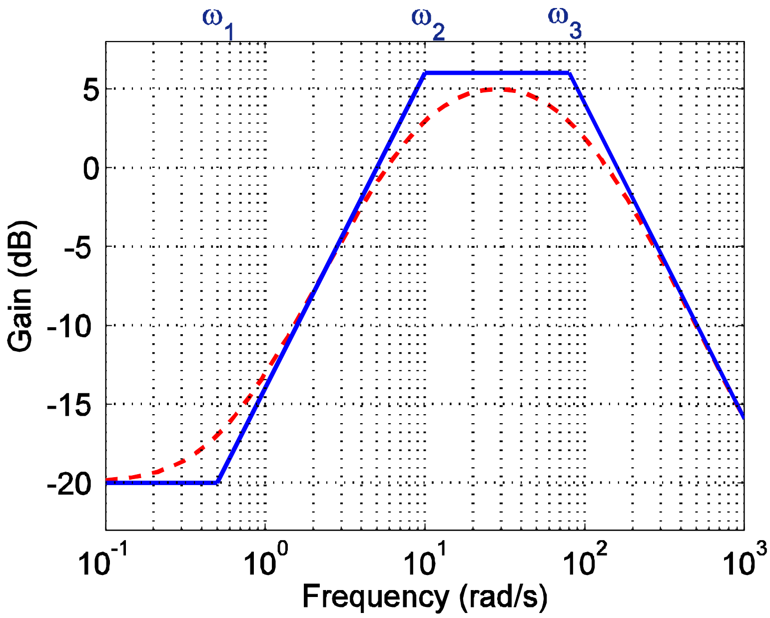

Figure Q8 : Bode magnitude plot of $G(s)$.

(a) Identify the transfer function, $G(s)$.

(8 marks)

(b) Following a breakdown, the system is refurbished. The following observations are made after analyzing data from experiments conducted on the refurbished system, $G_{new}(s)$:

- A step response test indicates that the DC gain of the new system is 0.15.
- The first two corner frequencies, ω_1 and ω_2, of the Bode magnitude plot are unchanged but the third corner frequency, ω_3, has shifted.

i. What is the low frequency asymptote of the Bode magnitude plot for $G_{new}(s)$?

(2 marks)

ii. When the input signal is $x(t) = 7\sin\left(100t + \dfrac{\pi}{6}\right)$, the steady-state output signal generated by $G_{new}(s)$ is $14\sin(100t - 0.07\pi)$. Determine the third corner frequency, ω_3, of the refurbished system, $G_{new}(s)$.

(5 marks)

iii. The steady-state output of $G(s)$ is $2.22\cos\left(t - \dfrac{5.7\pi}{18}\right)$ when $x(t) = A\cos t$ is the input signal. Estimate the steady-state output of $G_{new}(s)$ when the same input signal, $x(t) = A\cos t$, is applied.

(5 marks)

END OF QUESTIONS

This page is intentionally left blank to facilitate detachment of the formula sheet for easy reference.

Fourier Series: $$\begin{cases} X_k = \dfrac{1}{T}\displaystyle\int_{\tilde{t}}^{\tilde{t}+T} x(t)\exp(-j2\pi k\, t/T)\,dt \\ x(t) = \displaystyle\sum_{k=-\infty}^{\infty} X_k \exp(j2\pi k\, t/T) \end{cases}$$

Fourier Transform: $$\begin{cases} X(f) = \displaystyle\int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)\,dt \\ x(t) = \displaystyle\int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)\,df \end{cases}$$

FOURIER TRANSFORMS OF BASIC FUNCTIONS

	$x(t)$	$X(f)$
Constant	K	$K\delta(f)$
Unit Impulse	$\delta(t)$	**1**
Unit Step	$u(t)$	$\frac{1}{2}\left[\delta(f)+\frac{1}{j\pi f}\right]$
Sign (or Signum)	$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
Rectangle	$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}(fT)$
Triangle	$\mathrm{tri}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}^2(fT)$
Sine Cardinal	$\mathrm{sinc}\left(\frac{t}{T}\right)$	$T\,\mathrm{rect}(fT)$
Complex Exponential	$\exp(j2\pi f_o t)$	$\delta(f-f_o)$
Cosine	$\cos(2\pi f_o t)$	$\frac{1}{2}\left[\delta(f-f_o)+\delta(f+f_o)\right]$
Sine	$\sin(2\pi f_o t)$	$-\frac{j}{2}\left[\delta(f-f_o)-\delta(f+f_o)\right]$
Gaussian	$\exp\left(-\frac{t^2}{\alpha^2}\right)$	$\alpha\pi^{0.5}\exp\left(-\alpha^2\pi^2 f^2\right)$
Comb	$\sum_{m=-\infty}^{\infty}\delta(t-mT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$

FOURIER TRANSFORM PROPERTIES

	Time-domain	**Frequency-domain**
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(f)+\beta X_2(f)$
Time scaling	$x(\beta t)$	$\frac{1}{\lvert\beta\rvert}X\left(\frac{f}{\beta}\right)$
Duality	$X(t)$	$x(-f)$
Time shifting	$x(t-t_o)$	$X(f)\exp(-j2\pi f t_o)$
Frequency shifting (Modulation)	$x(t)\exp(j2\pi f_o t)$	$X(f-f_o)$
Differentiation in the time-domain	$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n X(f)$
Multiplication in the time-domain	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty}X_1(\zeta)X_2(f-\zeta)\,d\zeta$ or $X_1(f)\ast X_2(f)$
Convolution in the time-domain	$\int_{-\infty}^{\infty}x_1(\zeta)x_2(t-\zeta)\,d\zeta$ or $x_1(t)\ast x_2(t)$	$X_1(f)X_2(f)$
Integration in the time-domain	$\int_{-\infty}^{t}x(\tau)\,d\tau$	$\frac{1}{j2\pi f}X(f)+\frac{1}{2}X(0)\delta(f)$
		$\frac{1}{j2\pi f}X(f)$ **if** $X(0)=0$

Unilateral Laplace Transform: $X(s) = \int_{0^-}^{\infty} x(t)\exp(-st)\,dt$

LAPLACE TRANSFORMS OF BASIC FUNCTIONS		
	$x(t)$	$X(s)$
Unit Impulse	$\delta(t)$	1
Unit Step	$u(t)$	$1/s$
Ramp	$t\,u(t)$	$1/s^2$
n^{th} order Ramp	$t^n u(t)$	$\dfrac{n!}{s^{n+1}}$
Damped Ramp	$t\exp(-\alpha t)u(t)$	$1/(s+\alpha)^2$
Exponential	$\exp(-\alpha t)u(t)$	$1/(s+\alpha)$
Cosine	$\cos(\omega_o t)u(t)$	$s/(s^2+\omega_o^2)$
Sine	$\sin(\omega_o t)u(t)$	$\omega_o/(s^2+\omega_o^2)$
Damped Cosine	$\exp(-\alpha t)\cos(\omega_o t)u(t)$	$\dfrac{s+\alpha}{(s+\alpha)^2+\omega_o^2}$
Damped Sine	$\exp(-\alpha t)\sin(\omega_o t)u(t)$	$\dfrac{\omega_o}{(s+\alpha)^2+\omega_o^2}$

LAPLACE TRANSFORM PROPERTIES		
	Time-domain	***s*-domain**
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(s)+\beta X_2(s)$
Time shifting	$x(t-t_o)u(t-t_o)$	$\exp(-st_o)X(s)$
Shifting in the s-domain	$\exp(s_o t)x(t)$	$X(s-s_o)$
Time scaling	$x(\alpha t)$	$\dfrac{1}{\lvert\alpha\rvert}X\left(\dfrac{s}{\alpha}\right)$
Integration in the time-domain	$\int_{0^-}^{t} x(\zeta)\,d\zeta$	$\dfrac{1}{s}X(s)$
Differentiation in the time-domain	$\dfrac{dx(t)}{dt}$	$sX(s)-x(0^-)$
	$\dfrac{d^n x(t)}{dt^n}$	$s^n X(s)-\sum_{k=0}^{n-1} s^{n-1-k}\left.\dfrac{d^k x(t)}{dt^k}\right\rvert_{t=0^-}$
Differentiation in the s-domain	$-tx(t)$	$\dfrac{dX(s)}{ds}$
	$(-t)^n x(t)$	$\dfrac{d^n X(s)}{ds^n}$
Convolution in the time-domain	$\int_{-\infty}^{\infty} x_1(\zeta)x_2(t-\zeta)\,d\zeta$	$X_1(s)X_2(s)$
Initial value theorem	$x(0^+)=\lim_{s\to\infty} sX(s)$	
Final value theorem	$\lim_{t\to\infty} x(t)=\lim_{s\to 0} sX(s)$	

	$y_{step}(t)$	$Y_{step}(s)$	**SYSTEM PARAMETERS**
Step response of 1st order system	$K\left[1-\exp\left(-\frac{t}{T}\right)\right]u(t)$	$\frac{1}{s}\cdot\frac{K}{(sT+1)}$	T: System Time-constant K: System Steady-state (or DC) Gain
Step response of 2nd order underdamped system: $(0<\zeta<1)$	$K\left[1-\frac{\exp(-\omega_n\zeta t)}{(1-\zeta^2)^{0.5}}\sin\left(\omega_n(1-\zeta^2)^{0.5}t+\phi\right)\right]u(t)$	$\frac{1}{s}\cdot\frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	ω_n : System Undamped Natural Frequency ζ : System Damping Factor ω_d : System Damped Natural Frequency K : System Steady-state (or DC) Gain $\sigma=\omega_n\zeta$ $\omega_d^2=\omega_n^2(1-\zeta^2)$ $\omega_n^2=\sigma^2+\omega_d^2$ $\tan(\phi)=\omega_d/\sigma$
	$K\left[1-\left(\frac{\sigma^2+\omega_d^2}{\omega_d^2}\right)^{0.5}\exp(-\sigma t)\sin(\omega_d t+\phi)\right]u(t)$	$\frac{1}{s}\cdot\frac{K(\sigma^2+\omega_d^2)}{(s+\sigma)^2+\omega_d^2}$	
2nd order system - RESONANCE - $(0\le\zeta<1/\sqrt{2})$	*RESONANCE FREQUENCY :* $\omega_r=\omega_n(1-2\zeta^2)^{0.5}$		*RESONANCE PEAK :* $M_r=\lvert H(j\omega_r)\rvert=\frac{K}{2\zeta(1-\zeta^2)^{0.5}}$

TRIGONOMETRIC IDENTITIES

$\exp(\pm j\theta)=\cos(\theta)\pm j\sin(\theta)$	$\sin(\alpha\pm\beta)=\sin(\alpha)\cos(\beta)\pm\cos(\alpha)\sin(\beta)$
$\cos(\theta)=\frac{1}{2}[\exp(j\theta)+\exp(-j\theta)]$	$\cos(\alpha\pm\beta)=\cos(\alpha)\cos(\beta)\mp\sin(\alpha)\sin(\beta)$
$\sin(\theta)=\frac{1}{j2}[\exp(j\theta)-\exp(-j\theta)]$	$\tan(\alpha\pm\beta)=\frac{\tan(\alpha)\pm\tan(\beta)}{1\mp\tan(\alpha)\tan(\beta)}$
$\sin^2(\theta)+\cos^2(\theta)=1$	
$\sin(2\theta)=2\sin(\theta)\cos(\theta)$	$\sin(\alpha)\sin(\beta)=\frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$
$\cos(2\theta)=\cos^2(\theta)-\sin^2(\theta)$	$\cos(\alpha)\cos(\beta)=\frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$
$\sin^2(\theta)=\frac{1}{2}[1-\cos(2\theta)]$	$\sin(\alpha)\cos(\beta)=\frac{1}{2}[\sin(\alpha-\beta)+\sin(\alpha+\beta)]$
$\cos^2(\theta)=\frac{1}{2}[1+\cos(2\theta)]$	$\mathbf{C}\cos(\theta)-\mathbf{S}\sin(\theta)=\sqrt{\mathbf{C}^2+\mathbf{S}^2}\cos\left[\theta+\tan^{-1}\left(\frac{\mathbf{S}}{\mathbf{C}}\right)\right]$