Name : ______ **Group :** ______ **Student # :** ______

Due date : 12 April 2022 (Tuesday 11:59 hrs)

National University of Singapore
Department of Electrical & Computer Engineering
EE2023 Signal & Systems

In this assignment, you will model the dynamics of a vibrating system, analyze its behaviour and design a system to damp away the vibrations.

Consider a system where a motor, designed to run at a constant speed, is mounted on a non-rigid cantilever beam as shown in Figure 1. The rotating part of the motor has an eccentricity (off-centre effect) which gives rise to a sinusoidal force of frequency, ω_f. This sinusoidal force which can be written as $f(t) = A \sin \omega_f t$, will be exerted on the cantilever on which the motor is mounted. A denotes the amplitude of the force.

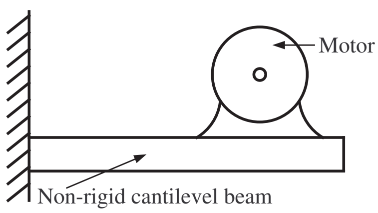

Figure 1: Motor mounted on a non-rigid cantilever beam.

The cantilever beam can be modeled as a spring-mass system as shown in Figure 2. For this model, the cantilever is of mass M, and behaves like a spring with spring constant K. The force generated by the motor causes the cantilever to vibrate in a certain manner. One way to reduce or remove the vibrations on the cantilever is to attach a vibration absorber to the cantilever so that the dynamical properties of the cantilever, together with the vibration absorber, are altered. The vibration absorber should be designed with a certain mass and material property. The model for the system with the vibration absorber is shown in Figure 3. You may assume that the mass of the motor has no effect on the bending moment of the cantilever.

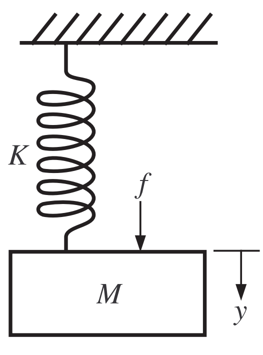

Figure 2: Model of the motor mounted on cantilever.

This assignment explores the modelling of such a system and leads up to the design of the

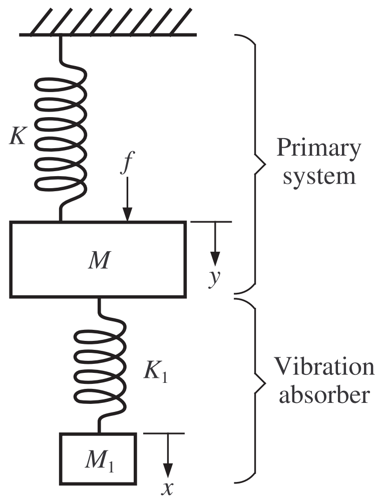

Figure 3: Model of the system with the vibration absorber.

vibration absorber to reduce or remove the vibrations.

In order for the designs to differ from one student to another, you should generate values of M and K using your student number as follows :

- Mass, M is $(C + 1)$ kg
- Spring constant, K, is $2250(D + 1)$

where C and D may be derived by reading the last 4 digits of your student number. Equate the first two digits to C and the remaining two to D. For example if your last 4 digits of your student number is 1234, then choose $C = 12$ and $D = 34$. With the values of M and K determined in this manner, you may proceed to answer the following questions.

(1) Using Newton's second law of motion, derive the differential equation governing the motion of the cantilever (Figure 2) as it vibrates due to the force generated by the motor. Essentially, obtain the relationship between the displacement of the cantilever, $y(t)$ and the force, $f(t)$, generated by the rotation of the motor. You may assume that the tension of the spring at any time t is equal to $Ky(t)$.

(2) Derive the transfer function, $G_1(s)$ (between $Y(s)$ and $F(s)$), of the cantilever. Find the poles of $G_1(s)$.

(3) Asssume that the cantilever is at equilibrium with $y(0^-) = 0$ and $\dot{y}(0^-) = 0$, before the motor was started at $t = 0$. Calculate the response of the cantilever when the vibration from the motor has a frequency, $\omega_f = 10$ rad/s. Assume the force to be sinusoidal of amplitude $A = 1$.

(4) Explain what happens to the cantilever when the vibration from the motor is of frequency, $\omega_f = \sqrt{\frac{K}{M}}$.

(5) The vibrations are expected to become violent as the forcing frequency reaches a certain value. This violent resonance condition can be controlled by adding a small mass to the system. Suppose the *vibration absorber* represented by the parameters K_1 and M_1 has been added and the system can be modelled as shown in Figure 3.

Suppose the equations of motion describing the behaviour of the system in Figure 3 are as follows :

$$\begin{aligned} M\frac{d^2y(t)}{dt^2} &= f(t) - Ky(t) - K_1\left(y(t) - x(t)\right) \\ M_1\frac{d^2x(t)}{dt^2} &= K_1\left(y(t) - x(t)\right) \end{aligned}$$

Find the transfer function, $G_2(s)$, between the cantilever displacement $Y(s)$ and the force, $F(s)$, generated by the rotation of the motor, taking into consideration the vibration absorber.

(6) What conditions can you impose on K_1 and M_1 such that $G_2(s)$ takes the form of

$$G_2(s) = \frac{C(M_1 s^2 + K_1)}{(s^2 + \omega_{n1}^2)(s^2 + \omega_{n2}^2)}?$$

where C is a constant.

(7) Design the parameters of the vibration absorber so that the vibrations can be reduced to zero. Explain how you arrive at your answer.

(8) Sketch the amplitude response ($|G_2(j\omega)|$ versus ω) of the new system.