EE2023/TEE2023/EE2023E TUTORIAL 5 (PROBLEMS)

Section I : Exercises that are straightforward applications of the concepts covered in class. Please attempt these problems on your own.

1. Solve the following Laplace Transform questions :

(a) $\mathcal{L}\left\{\cos^2(wt)\right\}$ Answer: $\frac{1}{2}\left[\frac{s}{s^2+4\omega^2}+\frac{1}{s}\right]$

(b) $\mathcal{L}^{-1}\left\{\frac{1}{(s-1)(s+2)(s+4)}\right\}$ Answer: $\frac{1}{15}e^{t}-\frac{1}{6}e^{-2t}+\frac{1}{10}e^{-4t}$

Application of the shift in the s-domain function rule : $\mathcal{L}\left\{e^{-\alpha t}f(t)\right\}=F(s+\alpha)$

(c) $\mathcal{L}\left\{\frac{1}{(s+1)^2}\right\}$ Answer: te^{-t}

(d) $\mathcal{L}^{-1}\left\{\frac{s+9}{s^2+6s+13}\right\}$ Answer: $e^{-3t}\left[\cos(2t)+3\sin(2t)\right]$

(e) $\mathcal{L}\left\{\frac{3}{5}-\frac{\sqrt{45}}{5}e^{-2t}\sin\left(t+\tan^{-1}0.5\right)\right\}$ Answer: $\frac{3}{s\left(s^2+4s+5\right)}$

Application of the shift in the time-domain function rule : $\mathcal{L}\left\{f(t-t_0)u(t-t_0)\right\}=e^{-st_0}F(s)$

(f) $\mathcal{L}\left\{(t-1)^2u(t-1)\right\}$ Answer: $\frac{2}{s^3}e^{-s}$

(g) $\mathcal{L}\left\{t^2u(t-1)\right\}$ Answer: $\frac{2}{s^3}e^{-s}+\frac{2}{s^2}e^{-s}+\frac{1}{s}e^{-s}$

(h) $\mathcal{L}^{-1}\left\{\frac{se^{-2s}}{s^2+\pi^2}\right\}$ Answer: $\cos(\pi t)u(t-2)$

Application of the derivative of transforms rule : $F'(s)=\mathcal{L}\left\{-tf(t)\right\}$

(i) $\mathcal{L}\left\{te^{-t}\sin(t)\right\}$ Answer: $\frac{2(s+1)}{\left(s^2+2s+2\right)^2}$

(j) $\mathcal{L}^{-1}\left\{\frac{s}{\left(s^2+9\right)^2}\right\}$ Answer: $\frac{1}{6}t\sin(3t)$

2. Solve the following linear second order differential equation using Laplace Transform :

$\ddot{y}(t)+4\dot{y}(t)+3y(t)=2r(t)$ assuming that $r(t)=1$ when $t\geq 0$, $y(0)=1$ and $\dot{y}(0)=0$

Answer: $y(t)=\frac{1}{2}e^{-t}-\frac{1}{6}e^{-3t}+\frac{2}{3}$

Section II – Problems that will be discussed in class.

1. The circuit shown in Figure 1 is operating in steady-state with the switch open prior to t = 0. Find expressions for $i(t)$ for $t < 0$ and for $t \geq 0$.

Answer : $i(t) = 1$ for $t < 0$ and $4\text{-}3e^{-12:5t}$ for $t > 0$

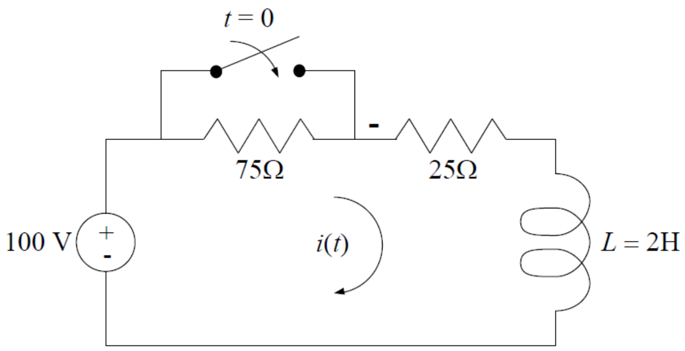

Figure 1: Series RL circuit

2. A series RLC circuit is shown in Figure 2.

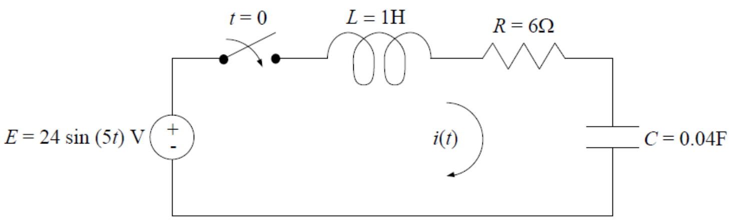

Figure 2: Series RLC circuit

(a) Show that the differential equation relating the current $i(t)$ in the RLC circuit shown in the figure to the applied voltage $E(t)$ is

$$L\frac{d^2}{dt^2}i(t) + R\frac{d}{dt}i(t) + \frac{1}{C}i(t) = \frac{d}{dt}E(t)$$

(b) Assuming the initial current and its rate of change ($i(0)$ and $\frac{d}{dt}i(0)$) are zero, fnd $i(t)$.

Answer : $i(t) = -5e^{-3t}\sin(4t) + 4\sin(5t)$

3. Ah Kow is worried about an upcoming exam. His doctor advises him to take a 100mg stress relief tablet the next morning and another 50mg tablet 24 hours later. Suppose the differential equation describing the quantity of drug in Ah Kow's body is

$$\frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) + 2y(t) = f(t)$$

where $y(t)$ is the quantity of drug in the body measured in mg,
$f(t)$ represents the rate at which the drug is administered into the body,
t is time measured in days.

Assume that:

- drugs taken in tablet form can be modelled by impulse function whose strength is equal to the quantity of drug ingested,
- there is no stress relief drug in Ah Kow's bloodstream before he takes the first tablet.

(a) Write a mathematical expression representing the input signal, $f(t)$, which models the rate at which the stress relief medicine is digested.

Answer : $100\delta(t) + 50\delta(t - 1)$

(b) What are the initial conditions ($t = 0^-$)of the system ?

Answer : $y(0^-) = 0$; $y'(0^-) = 0$

(c) Use Laplace Transform to determine the system output, $y(t)$. What is the amount of stress medicine left in Ah Kow's body by the time of the exam 4 days after he ate the first tablet?

Answer : 4.1634 mg

Section III : Practice Problems. These problems will not be discussed in class.

1 Consider the circuit shown in Figure 3. The switch opens at $t = 0$. Find an expression for $v(t)$.

Answer : $v(t) = 10 - 10e^{-100t}$ for $t > 0$

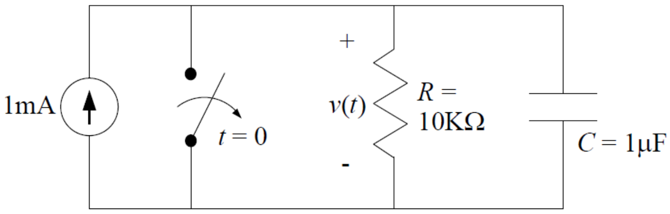

Figure 3: Parallel RC circuit