EE2023 Signals & Systems Quiz
Semester 1 AY2015/16

Date : 6 October 2015 **Time Allowed : 1.5 hours**

Instructions :

1. Answer all 4 questions. Each question carries 10 marks.
2. This is a closed book quiz.
3. Tables of Fourier transforms and trigonometric identities are given on Pages 11 and 12.
4. No programmable or graphic calculator is allowed.
5. Write your answers in the spaces indicated in this question paper. Attachment is not allowed.
6. Write your name, matric number and lecture group in the spaces indicated below.

Name : ______________________________

Matric # : ____________________________

Class Group # : _______________________

Question #	Marks
1	
2	
3	
4	
Total Marks	

For your information :

Group 1 : A/Prof Loh Ai Poh
Group 2 : Prof Lawrence Wong
Group 3 : A/Prof Tan Woei Wan

Q1 Consider the signals

$$x_1(t) = \sin^2(10\pi t) + 2\cos(6\pi t) \text{ and } x_2(t) = 5x_1(t) * \text{sinc}(5t)$$

where * denotes convolution.

(a) Find the Fourier Transforms of $x_1(t)$ and $x_2(t)$.

(4 marks)

(b) Sketch both the amplitude spectra of $x_1(t)$ and $x_2(t)$. Show clearly all quantities involved.

(3 marks)

(c) Suppose $x_2(t)$ is sampled at 4Hz. Sketch its sampled spectrum.

(3 marks)

Q.1 ANSWER

Q.1 ANSWER ~ continued

Q.2 Given that the Fourier Transform of $e^{-\frac{t^2}{\alpha^2}}$ is $\alpha\sqrt{\pi}e^{-\alpha^2\pi^2 f^2}$.

(a) Find the Fourier Transform of $x_1(t) = 4e^{-4\pi^2 t^2}$.

(2 marks)

(b) Find the Fourier Transform of $x_2(t) = \int_{-\infty}^{t} e^{-\frac{\tau^2}{4}} d\tau$.

(3 marks)

(c) Find the Fourier Transform of $x_3(t) = 4\cos(16\pi t)e^{-4\pi^2 t^2}$.

(3 marks)

(d) Find the 3dB bandwidth of $x_1(t)$.

(2 marks)

Q.2 ANSWER

Q.2 ANSWER ~ continued

Q.3 Consider the periodic signal $x(t) = \sum_{n=-\infty}^{\infty} x_g(t-6n)$, where $x_g(t) = \begin{cases} t^2 & -3 < t < 3 \\ 0 & otherwise \end{cases}$.

(a) Sketch $x(t)$, clearly labelling the axes.

(2 marks)

(b) What is the fundamental cyclic frequency (Hz) of the signal $x(t)$?

(1 marks)

(c) The Fourier Series expansion of $x(t)$ is

$$x(t) = c_0 + \sum_{k=1}^{\infty} \frac{36}{k^2\pi^2}(-1)^k \cos\frac{k\pi t}{3}.$$

i. Show that the DC value of $x(t)$ is 3 i.e. $c_0 = 3$.

(4 marks)

ii. Suppose the Fourier Series expansion for $x(t)$ is truncated at the 2nd harmonic term i.e.

$$\tilde{x}(t) = 3 + \sum_{k=1}^{2} \frac{36}{k^2\pi^2}(-1)^k \cos\frac{k\pi t}{3}.$$

Determine the truncation error, $x(t) - \tilde{x}(t)$, at $t = 0$.

(3 marks)

Q.3 ANSWER

Q.3 ANSWER ~ continued

Q.4 (a) Determine the Fourier transform of the periodic signal $x(t)$ shown in Figure Q.4.

(7 marks)

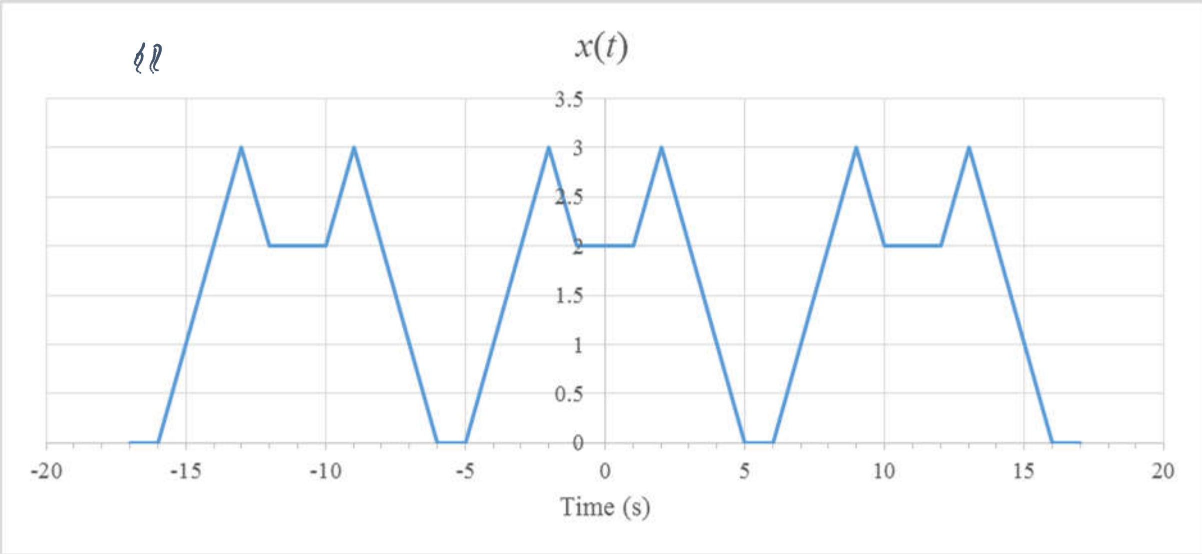


Figure Q.4

(b) Determine the average signal power of $x(t)$.

(3 marks)

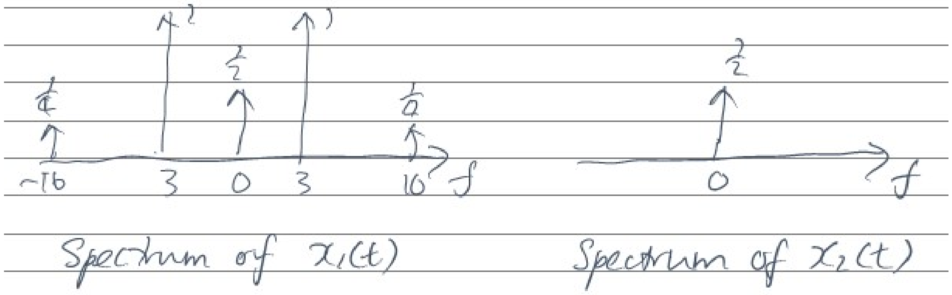
Q.4 ANSWER

Q.4 ANSWER ~ continued

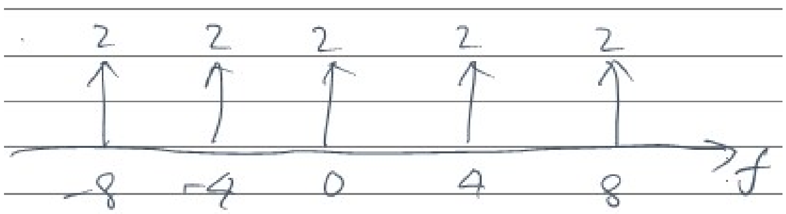
EE2023 Semester 1 2015/16 Quiz
Numeric Solution

Q1 (a)
$$X_1(f) = \frac{1}{2}\delta(f) - \frac{1}{4}[\delta(f-10) + \delta(f+10)] + [\delta(f-3) + \delta(f+3)]$$
$$X_2(f) = \frac{1}{2}\delta(f)$$

(b)



(c)



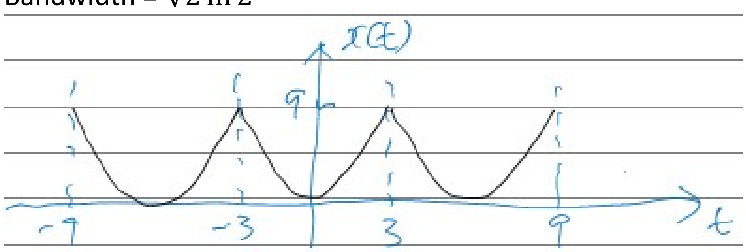
Q2 (a)
$$X_1(f) = \frac{2}{\sqrt{\pi}} e^{-0.25f^2}$$

(b)
$$X_2(f) = \frac{1}{j\sqrt{\pi}f} e^{-4\pi^2 f^2} + \sqrt{\pi}\,\delta(f)$$

(c)
$$X_3(f) = \frac{1}{\sqrt{\pi}}\left[e^{-0{,}25(f-8)^2} + e^{-0{,}25(f+8)^2}\right]$$

(d) Bandwidth = $\sqrt{2\ln 2}$

Q3 (a)



(b) $\frac{1}{6}$ Hz

(c)
$$\frac{1}{6}\int_{-3}^{3} t^2\, dt = 3$$

$x(0) = 0,\ \tilde{x}(0) = 3 + \sum_{k=1}^{2} \frac{36}{k^2\pi^2}(-1)^k$ and $x(0) - \tilde{x}(0) = -0.26$

Q4 (a)
$$X(f) = \frac{1}{11}\sum_{k=-\infty}^{\infty} 9\,\mathrm{sinc}^2\left(\frac{3k}{11}\right)\left[\exp\left(-\frac{j4\pi k}{11}\right) + \exp\left(\frac{j4\pi k}{11}\right)\right]\delta\left(f - \frac{k}{11}\right)$$
$$P = \frac{2}{11}\left[\int_0^1 2^2\, dt + \int_1^2 (t+1)^2\, dt + \int_2^5 (-t+5)^2\, dt\right] = 3.5$$