EE2023 Signals & Systems Quiz
Semester 1 AY2016/17

Date : 4 Oct 2016 **Time Allowed : 1.5 hours**

Instructions :

1. Answer all 4 questions. Each question carries 10 marks.
2. This is a closed book quiz.
3. Tables of Fourier transforms and trigonometric identities are given on Pages 11 and 12.
4. Programmable and/or graphic calculators are not allowed.
5. Write your answers in the spaces indicated in this question paper. Attachment is not allowed.
6. Write your name, matric number and lecture group in the spaces indicated below.

Name : ______________________________

Matric # : ____________________________

Class Group # : _______________________

Question #	Marks
1	
2	
3	
4	
Total Marks	

For your information :

Group 1 : A/Prof Loh Ai Poh
Group 2 : Prof Lawrence Wong
Group 3 : A/Prof Tan Woei Wan
Group 4 : A/Prof Ng Chun Sum

Q.1 Consider the signal $x(t) = 6\,\mathrm{tri}\left(\frac{t}{6}\right)$.

(a) Sketch $x(t)$, clearly labelling the axes. (1 marks)

(b) $x(t)$ is sampled at a sampling frequency of 0.5 Hz. Write down the sampled signal, $x_s(t)$. (2 marks)

(c) Sketch $x_s(t)$. (2 marks)

(d) Find the spectrum of the sampled signal. (3 marks)

(e) Can the original signal $x(t)$ be reconstructed from $x_s(t)$? Justify your answer. (2 marks)

Q.1 ANSWER

Q.1 ANSWER ~ continued

Q.2 The signal $x(t)$ is shown in Figure Q.2(a) and the periodic signal $x_p(t)$ is shown in Figure Q.2(b).

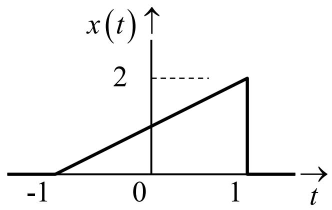


Figure Q.2(a)

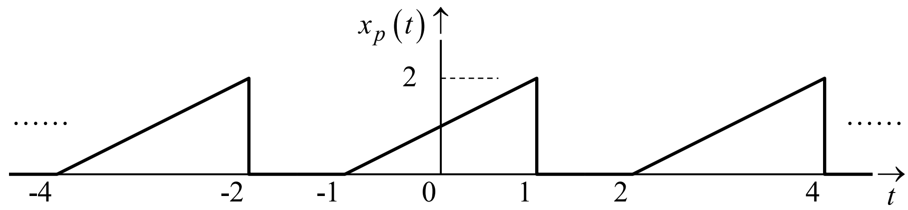


Figure Q.2(b)

(a) Using the replication property of the Dirac-δ function, express $x_p(t)$ in terms of $x(t)$.

(2 marks)

(b) Derive the Fourier transform, $X(f)$, of the signal $x(t)$. (4 marks)

(c) Derive the Fourier transform, $X_p(f)$, of the periodic signal $x_p(t)$. (4 marks)

Q.2 ANSWER

Q.2 ANSWER ~ continued

Q.3 Let the time-domain representation of a recorded voice signal be

$$x(t) = \cos(2t + \pi) + (10 + 2j)\sin(6t).$$

(a) What is the fundamental frequency of $x(t)$?

(1 marks)

(b) Determine the magnitude, $|c_k|$, and phase, $\angle c_k$, of the Fourier series coefficients, c_k, of $x(t)$, where $k = -\infty, \cdots, -1, 0, 1, \cdots \infty$.

(7 marks)

(c) The voice signal, $x(t)$, may be identified by comparing its magnitude spectrum against a database comprising the magnitude spectrum of known words. Does the magnitude spectrum of $x(t)$ match the database record shown in Figure Q.3? Explain how you arrived at your conclusion.

(2 marks)

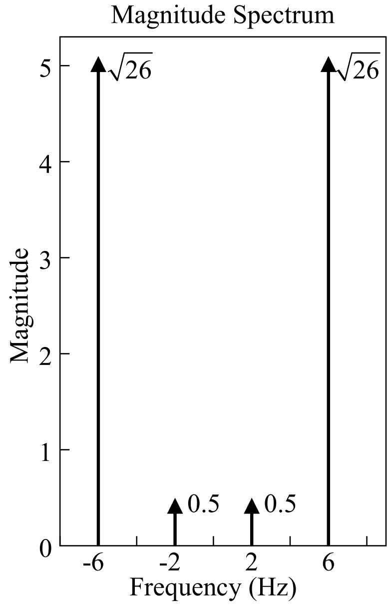


Figure Q.3

Q.3 ANSWER

Q.3 ANSWER ~ continued

Q.4 The spectrum of a signal $x(t)$ is given by

$$X(f) = jf \exp\left(-\frac{|f|^3}{250}\right).$$

(a) Find the phase spectrum, $\angle X(f)$, and the energy spectral density, $E_x(f)$, of $x(t)$.

(5 marks)

(b) $x(t)$ is propagated through an ideal lowpass filter of bandwidth $B(\text{Hz})$. What is the minimum value of B such that at least 96.5% of the energy of $x(t)$ is retained after filtering. Round your answer to 1 decimal place. $\left[\text{Hint: } \int x^{n-1} e^{kx^n}\, dx = \frac{e^{kx^n}}{kn}\right]$

(5 marks)

Q.4 ANSWER

Q.4 ANSWER ~ continued

This page is intentionally left blank to facilitate detachment of the formula sheet for easy reference. Anything written on this page will not be graded.

Fourier Series: $$\begin{cases} X_k = \dfrac{1}{T}\int_{\tilde{t}}^{\tilde{t}+T} x(t)\exp(-j2\pi k\,t/T)\,dt \\ x(t) = \sum_{k=-\infty}^{\infty} X_k \exp(j2\pi k\,t/T) \end{cases}$$

Fourier Transform: $$\begin{cases} X(f) = \int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)\,dt \\ x(t) = \int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)\,df \end{cases}$$

FOURIER TRANSFORMS OF BASIC FUNCTIONS

	$x(t)$	$X(f)$
Constant	K	$K\delta(f)$
Unit Impulse	$\delta(t)$	**1**
Unit Step	$u(t)$	$\frac{1}{2}\left[\delta(f)+\frac{1}{j\pi f}\right]$
Sign (or Signum)	$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
Rectangle	$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}(fT)$
Triangle	$\mathrm{tri}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}^2(fT)$
Sine Cardinal	$\mathrm{sinc}\left(\frac{t}{T}\right)$	$T\,\mathrm{rect}(fT)$
Complex Exponential	$\exp(j2\pi f_o t)$	$\delta(f-f_o)$
Cosine	$\cos(2\pi f_o t)$	$\frac{1}{2}\left[\delta(f-f_o)+\delta(f+f_o)\right]$
Sine	$\sin(2\pi f_o t)$	$-\frac{j}{2}\left[\delta(f-f_o)-\delta(f+f_o)\right]$
Gaussian	$\exp\left(-\frac{t^2}{\alpha^2}\right)$	$\alpha\pi^{0.5}\exp(-\alpha^2\pi^2 f^2)$
Comb	$\sum_{m=-\infty}^{\infty}\delta(t-mT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$

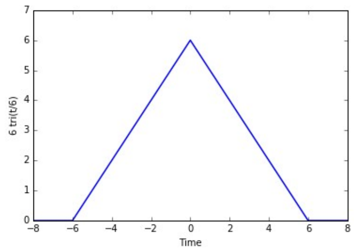
FOURIER TRANSFORM PROPERTIES

	Time-domain	**Frequency-domain**
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(f)+\beta X_2(f)$
Time scaling	$x(\beta t)$	$\frac{1}{\lvert\beta\rvert}X\left(\frac{f}{\beta}\right)$
Duality	$X(t)$	$x(-f)$
Time shifting	$x(t-t_o)$	$X(f)\exp(-j2\pi f t_o)$
Frequency shifting (Modulation)	$x(t)\exp(j2\pi f_o t)$	$X(f-f_o)$
Differentiation in the time-domain	$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n X(f)$
Multiplication in the time-domain	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty}X_1(\zeta)X_2(f-\zeta)\,d\zeta$ or $X_1(f)\ast X_2(f)$
Convolution in the time-domain	$\int_{-\infty}^{\infty}x_1(\zeta)x_2(t-\zeta)\,d\zeta$ or $x_1(t)\ast x_2(t)$	$X_1(f)X_2(f)$
Integration in the time-domain	$\int_{-\infty}^{t}x(\tau)\,d\tau$	$\frac{1}{j2\pi f}X(f)+\frac{1}{2}X(0)\delta(f)$
		$\frac{1}{j2\pi f}X(f)$ **if** $X(0)=0$

TRIGONOMETRIC IDENTITIES	
$\exp(\pm j\theta) = \cos(\theta) \pm j\sin(\theta)$	$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$
$\cos(\theta) = \frac{1}{2}\left[\exp(j\theta) + \exp(-j\theta)\right]$	$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
$\sin(\theta) = \frac{1}{j2}\left[\exp(j\theta) - \exp(-j\theta)\right]$	$\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$
$\sin^2(\theta) + \cos^2(\theta) = 1$	
$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$	$\sin(\alpha)\sin(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$
$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$	$\cos(\alpha)\cos(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$
$\sin^2(\theta) = \frac{1}{2}\left[1 - \cos(2\theta)\right]$	$\sin(\alpha)\cos(\beta) = \frac{1}{2}\left[\sin(\alpha - \beta) + \sin(\alpha + \beta)\right]$
$\cos^2(\theta) = \frac{1}{2}\left[1 + \cos(2\theta)\right]$	$\mathbf{C}\cos(\theta) - \mathbf{S}\sin(\theta) = \sqrt{\mathbf{C}^2 + \mathbf{S}^2}\cos\left[\theta + \tan^{-1}\left(\frac{\mathbf{S}}{\mathbf{C}}\right)\right]$

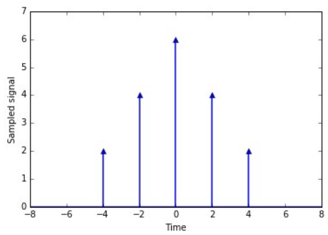
EE2023 Mid-Term Quiz
Numerical Solution

1.a)



b) $x_s(t) = \sum_{k=-\infty}^{\infty} 6\, tri\left(\frac{2k}{6}\right) \delta(t-2k)$

c)



d) $X_s(f) = 18 \sum_{k=-\infty}^{\infty} sinc^2\big(6(f-0.5k)\big)$

e) The original signal cannot be reconstructed from the sampled signal because the cardinal sine function is band unlimited.

2.a) $x_p(t) = \sum_{k=-\infty}^{\infty} x(t-3k) = x(t) * \sum_{k=-\infty}^{\infty} \delta(t-3k)$

b) $x(t) = \int_{-\infty}^{t} rect\left(\frac{\tau}{2}\right) d\tau - 2u(t-1)$

$X(f) = \frac{1}{j\pi f}\left[sinc(2f) - e^{-j2\pi f}\right] + \left[1 - e^{-j2\pi f}\right]\delta(f)$

c) $X_p(f) = \frac{1}{3}\sum_{k=-\infty}^{\infty} X\left(\frac{k}{3}\right)\delta\left(f-\frac{k}{3}\right) = \sum_{k=-\infty}^{\infty}\left\{\frac{1}{j\pi k}\left[sinc\left(\frac{2k}{3}\right) - e^{-j\frac{2\pi k}{3}}\right] + \frac{1}{3}\left[1 - e^{-j\frac{2\pi k}{3}}\right]\right\}\delta\left(f-\frac{k}{3}\right)$

3.a) 2 rad/s

b) $|c_k| = \begin{cases} \frac{1}{2} & k=1 \\ \frac{1}{2} & k=-1 \\ \sqrt{26} & k=3 \\ \sqrt{26} & k=-3 \\ 0 & otherwise \end{cases}$ $\qquad \measuredangle c_k = \begin{cases} \pi & k=1 \\ -\pi & k=-1 \\ -1.373\ rad & k=3 \\ 1.768\ rad & k=-3 \\ 0 & otherwise \end{cases}$

c) The magnitude spectrum of x(t) does not match Figure Q.3 because the frequency components should be at ± 2 rad/s and ± 4 rad/s.

4.a) $\measuredangle X(f) = \frac{\pi}{2} sgn(f)$ $\qquad E_x(f) = f^2 e^{\frac{-|f^3|}{125}}$

b) $B = \sqrt[3]{-125 \ln 0.035}$