NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester II : 2016/2017)

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EE2023 / EE2023E – SIGNALS & SYSTEMS

April / May 2017 - Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES

1. This paper contains **EIGHT (8)** questions and comprises **TEN (10)** printed pages.
2. Answer **ALL** questions in **Section A** and **ANY THREE (3)** questions in **Section B**.
3. This is a **CLOSED BOOK** examination. However you are allowed to bring one self-prepared, both sides handwritten A4-size crib sheet to the examination hall.
4. Programmable calculators are not allowed.
5. Tables of Fourier Transforms, Laplace Transforms and trigonometric identities are provided in Pages 8, 9 and 10, respectively.

SECTION A : Answer ALL questions in this section

Q1. (a) When an input $x(t) = t,\ t \geq 0$ is applied to a system with a transfer function, $G(s)$, the resulting output response is given by :

$$y(t) = e^{-t} - \frac{1}{4}e^{-2t} - \frac{3}{4} + \frac{1}{2}t,\ t \geq 0.$$

Determine $G(s)$ of the system. Is the system stable? Justify your answer.

(5 marks)

(b) If a new input, $v(t) = 5\sin(10t + 0.2),\ t \geq 0$ is applied to the system in (a) above, determine the new output response, $y_1(t)$ at steady state.

(5 marks)

Q2. Consider the periodic signal $x(t) = 10\sin(3t) + 4\cos\left(4.5t + \frac{\pi}{6}\right) + e^{j\left(t + \frac{\pi}{4}\right)} + 2$.

(a) Find the fundamental frequency, f_o, and period, T, of $x(t)$.

(3 marks)

(b) Find the Fourier series coefficients, c_k, of $x(t)$ and find the Fourier transform, $X(f)$, of $x(t)$.

(5 marks)

(c) Find the average power of $x(t)$.

(2 marks)

Q3. (a) Determine the Fourier transform of the signal $x(t)$ shown in Figure Q3.

(4 marks)

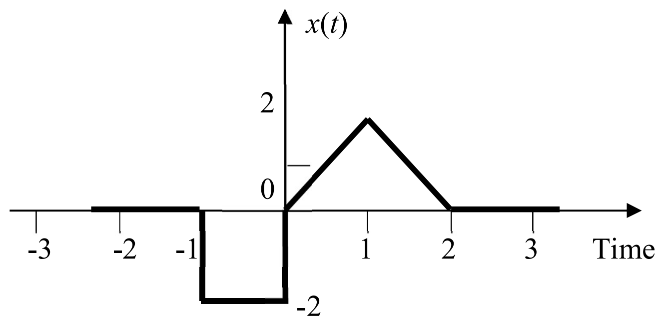

Figure Q3

(b) The periodic signal $x_p(t)$ is obtained by replicating $x(t)$ at periods of 10 seconds.

i. Determine the expression for $x_p(t)$ in terms of $x(t)$

(2 marks)

ii. Determine the Fourier transform, $X_p(f)$, of the periodic signal $x_p(t)$.

(4 marks)

Q4. The input-output relationship of a second order system is governed by the following differential equation

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 34y(t) = 40x(t).$$

(a) Derive the system transfer function.

(2 marks)

(b) Compute the unit impulse response of the system.

(4 marks)

(c) Is the system underdamped, critically damped or overdamped? What is the undamped natural frequency and DC gain of the system?

(4 marks)

SECTION B : Answer 3 out of the 4 questions in this section

Q5a) Figure Q5-1 shows an electrical circuit with L = 1mH and C = 100pF. The inductor current and capacitor voltage are both equal to zero at time, $t = 0$.

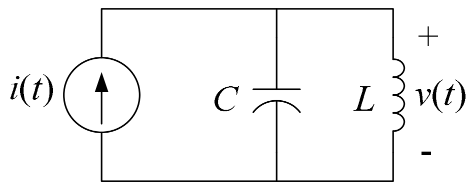

Figure Q5-1: Parallel LC circuit

(i) Derive the transfer function, $G(s) = \dfrac{V(s)}{I(s)}$, of the circuit where $I(s) = L\{i(t)\}$ and $V(s) = L\{v(t)\}$ are the Laplace transforms of the input current, $i(t)$, and output voltage, $v(t)$ respectively. Derive $G(s)$ in the form of :

$$G(s) = \frac{as}{s^2 + b}.$$

(4 marks)

(ii) Find the voltage, $v(t)$, if the current source, $i(t)$, is given by $i(t) = t$, $t \geq 0$. Sketch the voltage, $v(t)$, for $t \geq 0$. Label the quantities in your sketch clearly.

(8 marks)

(iii) Re-design the circuit such that the frequency of oscillation of the voltage across the inductor is 1000 rad/s. You may choose to change either L or C in the new design.

(3 marks)

Question 5 continues on the next page

Q5b) The unit impulse response of a system, $G_1(s)$ is shown in Figure Q5-2. Find the transfer function of $G_1(s)$.

(5 marks)

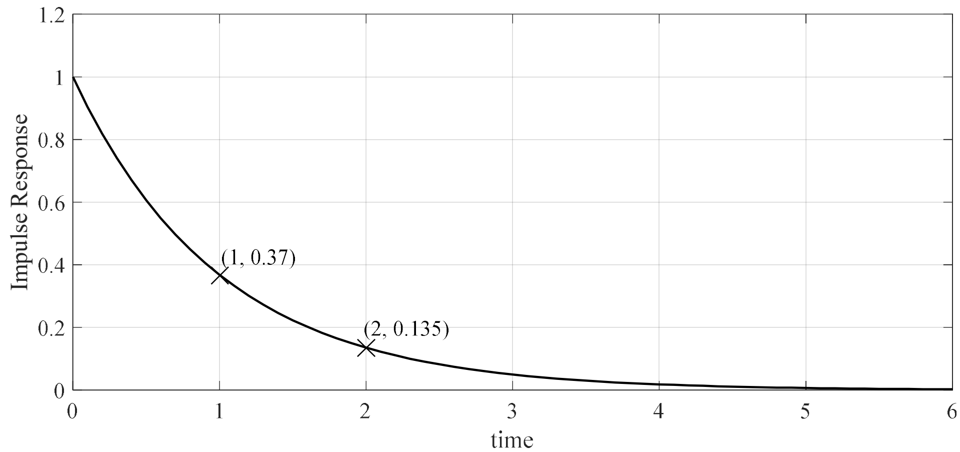

Figure Q5-2 : Unit Impulse Response

Q6. The signal $x(t)$ whose spectrum $X(f) = A \cdot \text{rect}\left(\frac{f+f_a}{\alpha}\right) + B \cdot \text{tri}\left(\frac{f+f_b}{\beta}\right)$ is shown in Figure Q6 below.

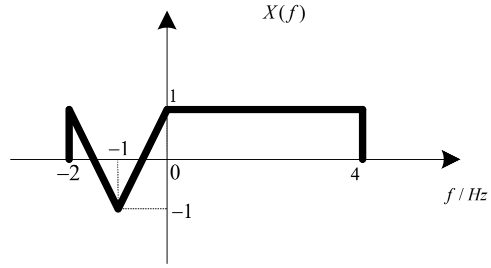

Figure Q6

(a) Find the values of the parameters A, f_a, α, B, β and f_b. (6 marks)

(b) Find the time domain signal, $x(t)$, of $X(f)$? (8 marks)

(c) Signal $y(t) = x(t)e^{j2\pi t} - 6\,\text{sinc}(6t)e^{j4\pi t}$. Sketch the magnitude and phase spectra of $y(t)$ with proper labelling.

(6 marks)

Q7. A signal $x(t)$ is given by

$$x(t) = 4\cos(2\pi f_c t)\left[\text{rect}\left(\frac{Wt}{2}\right) \otimes \text{sinc}(2Wt)\right]$$

where f_c and W are positive real constants, $f_c >> W$, and the symbol $\otimes$ denotes convolution.

(a) Determine the Fourier transform, $X(f)$, of the signal $x(t)$.

(8 marks)

(b) Find the bandwidth of $x(t)$ and determine the corresponding Nyquist sampling frequency of $x(t)$.

(5 marks)

(c) If the signal $x(t)$ is sampled at a frequency of $2f_c$ to give the sampled signal $x_s(t)$, give the expression for the Fourier transform of $x_s(t)$.

(7 marks)

Q8. Figure Q8 shows the Bode magnitude plot of a machine with transfer function $G(s)$.

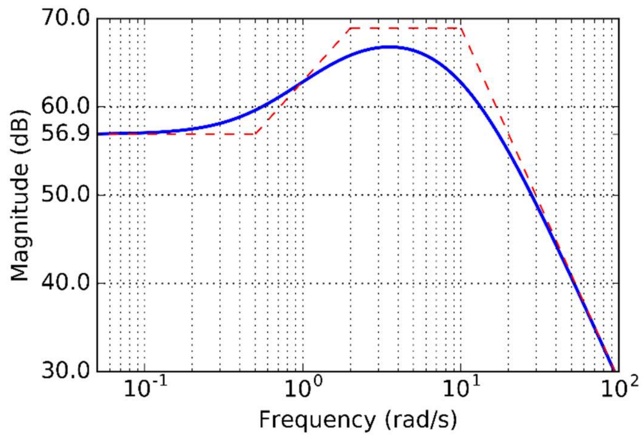

Figure Q8: Bode magnitude plot of $G(s)$.

(a) Suppose the machine is a stable system with no delay.

i. Identify the transfer function, $G(s)$.

(8 marks)

ii. Sketch the low frequency asymptote of the machine's Bode phase plot, clearly labelling the axes and important value(s).

(3 marks)

iii. Explain why $\measuredangle G(500j) = -270^o$ cannot be a valid point on the Bode phase plot for $G(s)$.

(3 marks)

(b) After extended use, the machine developed a delay. Experiments conducted revealed that $\measuredangle G(6j) = -151.4^o$. Assuming that the poles and zeros of the transfer function remain unchanged, determine the delay.

(6 marks)

END OF QUESTIONS

Fourier Series:
$$\begin{cases} X_k = \dfrac{1}{T}\int_{\tilde{t}}^{\tilde{t}+T} x(t)\exp(-j2\pi k\,t/T)\,dt \\ x(t) = \sum_{k=-\infty}^{\infty} X_k \exp(j2\pi k\,t/T) \end{cases}$$

Fourier Transform:
$$\begin{cases} X(f) = \int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)\,dt \\ x(t) = \int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)\,df \end{cases}$$

FOURIER TRANSFORMS OF BASIC FUNCTIONS

	$x(t)$	$X(f)$
Constant	K	$K\delta(f)$
Unit Impulse	$\delta(t)$	**1**
Unit Step	$u(t)$	$\frac{1}{2}\left[\delta(f)+\frac{1}{j\pi f}\right]$
Sign (or Signum)	$\operatorname{sgn}(t)$	$\frac{1}{j\pi f}$
Rectangle	$\operatorname{rect}\left(\frac{t}{T}\right)$	$T\operatorname{sinc}(fT)$
Triangle	$\operatorname{tri}\left(\frac{t}{T}\right)$	$T\operatorname{sinc}^2(fT)$
Sine Cardinal	$\operatorname{sinc}\left(\frac{t}{T}\right)$	$T\operatorname{rect}(fT)$
Complex Exponential	$\exp(j2\pi f_o t)$	$\delta(f-f_o)$
Cosine	$\cos(2\pi f_o t)$	$\frac{1}{2}\left[\delta(f-f_o)+\delta(f+f_o)\right]$
Sine	$\sin(2\pi f_o t)$	$-\frac{j}{2}\left[\delta(f-f_o)-\delta(f+f_o)\right]$
Gaussian	$\exp\left(-\frac{t^2}{\alpha^2}\right)$	$\alpha\pi^{0.5}\exp(-\alpha^2\pi^2 f^2)$
Comb	$\sum_{m=-\infty}^{\infty}\delta(t-mT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$

FOURIER TRANSFORM PROPERTIES

	Time-domain	**Frequency-domain**
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(f)+\beta X_2(f)$
Time scaling	$x(\beta t)$	$\frac{1}{\lvert\beta\rvert}X\left(\frac{f}{\beta}\right)$
Duality	$X(t)$	$x(-f)$
Time shifting	$x(t-t_o)$	$X(f)\exp(-j2\pi f t_o)$
Frequency shifting (Modulation)	$x(t)\exp(j2\pi f_o t)$	$X(f-f_o)$
Differentiation in the time-domain	$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n X(f)$
Multiplication in the time-domain	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty} X_1(\zeta)X_2(f-\zeta)\,d\zeta$ or $X_1(f)\ast X_2(f)$
Convolution in the time-domain	$\int_{-\infty}^{\infty} x_1(\zeta)x_2(t-\zeta)\,d\zeta$ or $x_1(t)\ast x_2(t)$	$X_1(f)X_2(f)$
Integration in the time-domain	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{j2\pi f}X(f)+\frac{1}{2}X(0)\delta(f)$
		$\frac{1}{j2\pi f}X(f)$ **if** $X(0)=0$

Unilateral Laplace Transform: $X(s) = \int_{0^-}^{\infty} x(t)\exp(-st)\,dt$

LAPLACE TRANSFORMS OF BASIC FUNCTIONS

	$x(t)$	$X(s)$
Unit Impulse	$\delta(t)$	1
Unit Step	$u(t)$	$1/s$
Ramp	$tu(t)$	$1/s^2$
n^th^ order Ramp	$t^n u(t)$	$\dfrac{n!}{s^{n+1}}$
Damped Ramp	$t\exp(-\alpha t)u(t)$	$1/(s+\alpha)^2$
Exponential	$\exp(-\alpha t)u(t)$	$1/(s+\alpha)$
Cosine	$\cos(\omega_o t)u(t)$	$s/(s^2+\omega_o^2)$
Sine	$\sin(\omega_o t)u(t)$	$\omega_o/(s^2+\omega_o^2)$
Damped Cosine	$\exp(-\alpha t)\cos(\omega_o t)u(t)$	$\dfrac{s+\alpha}{(s+\alpha)^2+\omega_o^2}$
Damped Sine	$\exp(-\alpha t)\sin(\omega_o t)u(t)$	$\dfrac{\omega_o}{(s+\alpha)^2+\omega_o^2}$

LAPLACE TRANSFORM PROPERTIES

	Time-domain	s-domain
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(s)+\beta X_2(s)$
Time shifting	$x(t-t_o)u(t-t_o)$	$\exp(-st_o)X(s)$
Shifting in the s-domain	$\exp(s_o t)x(t)$	$X(s-s_o)$
Time scaling	$x(\alpha t)$	$\dfrac{1}{\lvert\alpha\rvert}X\left(\dfrac{s}{\alpha}\right)$
Integration in the time-domain	$\int_{0^-}^{t} x(\zeta)\,d\zeta$	$\dfrac{1}{s}X(s)$
Differentiation in the time-domain	$\dfrac{dx(t)}{dt}$	$sX(s)-x(0^-)$
	$\dfrac{d^n x(t)}{dt^n}$	$s^n X(s)-\sum_{k=0}^{n-1} s^{n-1-k}\left.\dfrac{d^k x(t)}{dt^k}\right\rvert_{t=0^-}$
Differentiation in the s-domain	$-tx(t)$	$\dfrac{dX(s)}{ds}$
	$(-t)^n x(t)$	$\dfrac{d^n X(s)}{ds^n}$
Convolution in the time-domain	$\int_{-\infty}^{\infty} x_1(\zeta)x_2(t-\zeta)\,d\zeta$	$X_1(s)X_2(s)$
Initial value theorem	$x(0^+)=\lim_{s\to\infty} sX(s)$	
Final value theorem	$\lim_{t\to\infty} x(t)=\lim_{s\to 0} sX(s)$	

	$y_{step}(t)$	$Y_{step}(s)$	**SYSTEM PARAMETERS**
Step response of 1st order system	$K\left[1-\exp\left(-\frac{t}{T}\right)\right]u(t)$	$\frac{1}{s}\cdot\frac{K}{(sT+1)}$	T: System Time-constant K: System Steady-state (or DC) Gain
Step response of 2nd order <u>underdamped</u> system: $(0<\zeta<1)$	$K\left[1-\frac{\exp(-\omega_n\zeta t)}{(1-\zeta^2)^{0.5}}\sin\left(\omega_n(1-\zeta^2)^{0.5}t+\phi\right)\right]u(t)$	$\frac{1}{s}\cdot\frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	ω_n: System Undamped Natural Frequency ζ: System Damping Factor ω_d: System Damped Natural Frequency K: System Steady-state (or DC) Gain $\sigma=\omega_n\zeta$ $\omega_d^2=\omega_n^2(1-\zeta^2)$ $\omega_n^2=\sigma^2+\omega_d^2$ $\tan(\phi)=\omega_d/\sigma$
	$K\left[1-\left(\frac{\sigma^2+\omega_d^2}{\omega_d^2}\right)^{0.5}\exp(-\sigma t)\sin(\omega_d t+\phi)\right]u(t)$	$\frac{1}{s}\cdot\frac{K(\sigma^2+\omega_d^2)}{(s+\sigma)^2+\omega_d^2}$	

2nd order system - RESONANCE - $\left(0\le\zeta<1/\sqrt{2}\right)$	*RESONANCE FREQUENCY :* $\omega_r=\omega_n(1-2\zeta^2)^{0.5}$	*RESONANCE PEAK :* $M_r=\lvert H(j\omega_r)\rvert=\frac{K}{2\zeta(1-\zeta^2)^{0.5}}$

TRIGONOMETRIC IDENTITIES	
$\exp(\pm j\theta)=\cos(\theta)\pm j\sin(\theta)$	$\sin(\alpha\pm\beta)=\sin(\alpha)\cos(\beta)\pm\cos(\alpha)\sin(\beta)$
$\cos(\theta)=\frac{1}{2}\left[\exp(j\theta)+\exp(-j\theta)\right]$	$\cos(\alpha\pm\beta)=\cos(\alpha)\cos(\beta)\mp\sin(\alpha)\sin(\beta)$
$\sin(\theta)=\frac{1}{j2}\left[\exp(j\theta)-\exp(-j\theta)\right]$	$\tan(\alpha\pm\beta)=\frac{\tan(\alpha)\pm\tan(\beta)}{1\mp\tan(\alpha)\tan(\beta)}$
$\sin^2(\theta)+\cos^2(\theta)=1$	
$\sin(2\theta)=2\sin(\theta)\cos(\theta)$	$\sin(\alpha)\sin(\beta)=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$
$\cos(2\theta)=\cos^2(\theta)-\sin^2(\theta)$	$\cos(\alpha)\cos(\beta)=\frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$
$\sin^2(\theta)=\frac{1}{2}\left[1-\cos(2\theta)\right]$	$\sin(\alpha)\cos(\beta)=\frac{1}{2}\left[\sin(\alpha-\beta)+\sin(\alpha+\beta)\right]$
$\cos^2(\theta)=\frac{1}{2}\left[1+\cos(2\theta)\right]$	$\mathbf{C}\cos(\theta)-\mathbf{S}\sin(\theta)=\sqrt{\mathbf{C}^2+\mathbf{S}^2}\cos\left[\theta+\tan^{-1}\left(\frac{\mathbf{S}}{\mathbf{C}}\right)\right]$