EE2023 Signals & Systems Quiz
Semester 2 AY2019/20

Date : 5 March 2020 **Time Allowed : 1.5 hours**

Instructions :

1. Answer all 4 questions. Each question carries 10 marks.
2. This is a closed book quiz. However you are allowed to bring in an A4 sized cheat sheet.
3. No programmable or graphic calculators allowed.
4. Please enter your name and matric number in the spaces below.
5. Please staple this page to your written answer scripts.

Name : ______________________________

Matric # : ____________________________

Lecture Group # : ______________________

For your information :
Group 1 : A/Prof Loh Ai Poh (LT3)
Group 2 : A/Prof Tan Woei Wan (LT4)
Group 3 : Prof Lawrence Wong (LT7)

Question #	**Marks**
1	
2	
3	
4	
Total Marks	

Q.1 The signal $x(t)$ is shown in Figure Q1 has the form of $x(t) = a.\text{rect}\left(\frac{t}{b}\right) - c.\text{tri}\left(\frac{t}{d}\right) + e.\text{tri}\left(\frac{t}{f}\right)$, where *a*, *b*, *c*, *d*, *e* and *f* are positive constants.

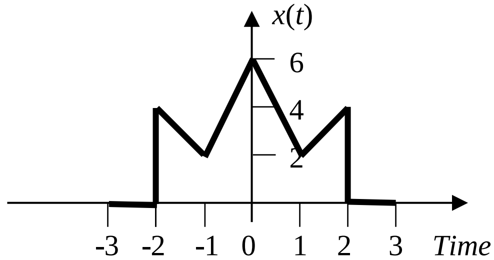

Figure Q1

(a) Determine the values of the constants *a*, *b*, *c*, *d*, *e*, and *f*.

(3 marks)

(b) Determine the Fourier transform, $X(f)$, of $x(t)$. (3 marks)

(c) The periodic signal, $x_p(t)$, can be obtained by replicating $x(t)$ at a period of 5 seconds. Obtain an expression for $x_p(t)$ in terms of $x(t)$ and the Dirac δ-function.

(1 mark)

(d) Determine the Fourier transform, $X_p(f)$, of the periodic signal $x_p(t)$. (3 marks)

Q1 ANSWER

Q1 ANSWER CONTINUES

Q.2 The continuous-frequency spectra plots of a periodic signal, $x(t)$, is shown in Figure Q2.

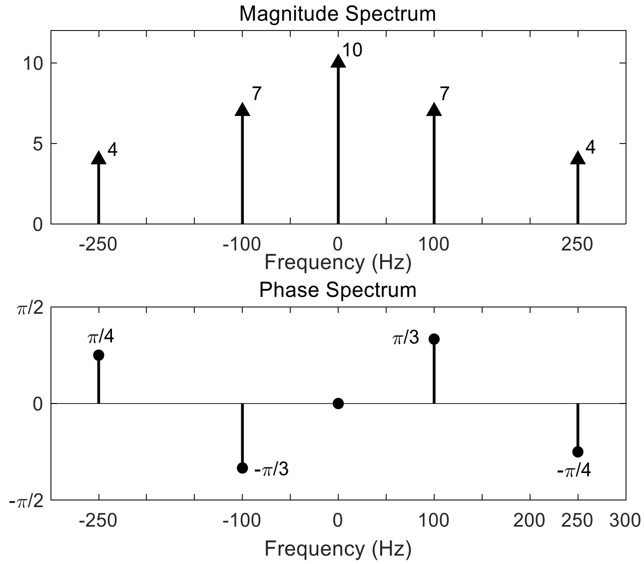

Figure 2: Spectra plots of $x(t)$

(a) Find the fundamental period, T_p, of $x(t)$.

(2 marks)

(b) What are the values of the coefficients, c_k $(k = -\infty, \dots, 0, \dots, \infty)$, of the complex exponential Fourier Series expansion of $x(t)$?

(6 marks)

(c) Compute the average power of $x(t)$.

(2 marks)

Q2 ANSWER

Q2 ANSWER CONTINUES

Q3. Consider the system shown in Figure Q3a below.

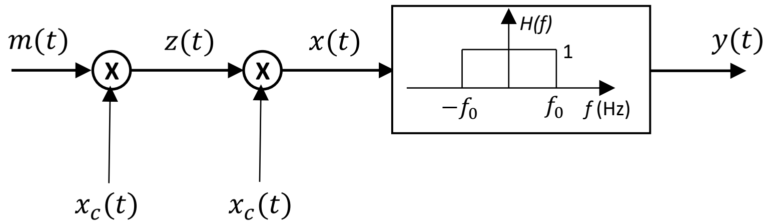

Fig. Q3a

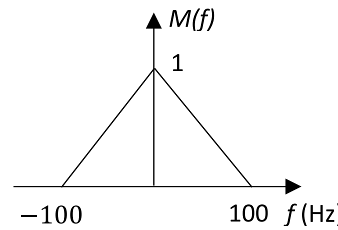

Fig. Q3b : Spectrum of $m(t)$

The spectrum of $m(t)$ is given in Figure Q3b. Assume that $x_c(t) = 2cos(1000\pi t)$.

(a) Sketch the spectra of $z(t)$ and $x(t)$.

(6 marks)

(b) Sketch the spectrum of $y(t)$ when $f_0 = 300$ Hz. Can $m(t)$ be recovered from $y(t)$?

(4 marks)

In your sketches, show all amplitudes clearly.

Q3 ANSWER

Q3 ANSWER CONTINUES

Q4. Consider the signal $x(t)$ in Figure Q4 below.

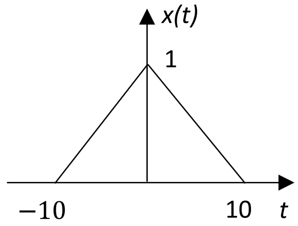

Fig Q4

(a) Find the Fourier transform of $x(t)$ and find its 1st null bandwidth.

(4 marks)

(b) If $y(t) = x(t).\sum_{k=-\infty}^{\infty}\delta(t-k)$, sketch $y(t)$.

(3 marks)

(c) If $z(t) = x(t) * \sum_{k=-\infty}^{\infty}\delta(t-20k)$, where * denotes convolution, sketch $z(t)$.

(3 marks)

In your sketches, show all amplitudes clearly.

Q4 ANSWER

Q4 ANSWER CONTINUES

Additional pages for your answers

Fourier Series: $\begin{cases} X_k = \dfrac{1}{T}\int_{\tilde{t}}^{\tilde{t}+T} x(t)\exp(-j2\pi k\, t/T)\,dt \\ x(t) = \sum_{k=-\infty}^{\infty} X_k \exp(j2\pi k\, t/T) \end{cases}$

Fourier Transform: $\begin{cases} X(f) = \int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)\,dt \\ x(t) = \int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)\,df \end{cases}$

FOURIER TRANSFORMS OF BASIC FUNCTIONS

	$x(t)$	$X(f)$
Constant	K	$K\delta(f)$
Unit Impulse	$\delta(t)$	**1**
Unit Step	$u(t)$	$\frac{1}{2}\left[\delta(f)+\frac{1}{j\pi f}\right]$
Sign (or Signum)	$\operatorname{sgn}(t)$	$\frac{1}{j\pi f}$
Rectangle	$\operatorname{rect}\left(\frac{t}{T}\right)$	$T\operatorname{sinc}(fT)$
Triangle	$\operatorname{tri}\left(\frac{t}{T}\right)$	$T\operatorname{sinc}^2(fT)$
Sine Cardinal	$\operatorname{sinc}\left(\frac{t}{T}\right)$	$T\operatorname{rect}(fT)$
Complex Exponential	$\exp(j2\pi f_o t)$	$\delta(f-f_o)$
Cosine	$\cos(2\pi f_o t)$	$\frac{1}{2}\left[\delta(f-f_o)+\delta(f+f_o)\right]$
Sine	$\sin(2\pi f_o t)$	$-\frac{j}{2}\left[\delta(f-f_o)-\delta(f+f_o)\right]$
Gaussian	$\exp\left(-\frac{t^2}{\alpha^2}\right)$	$\alpha\pi^{0.5}\exp\left(-\alpha^2\pi^2 f^2\right)$

FOURIER TRANSFORM PROPERTIES

	Time-domain	Frequency-domain
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(f)+\beta X_2(f)$
Time scaling	$x(\beta t)$	$\frac{1}{\lvert\beta\rvert}X\left(\frac{f}{\beta}\right)$
Duality	$X(t)$	$x(-f)$
Time shifting	$x(t-t_o)$	$X(f)\exp(-j2\pi f t_o)$
Frequency shifting (Modulation)	$x(t)\exp(j2\pi f_o t)$	$X(f-f_o)$
Differentiation in the time-domain	$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n X(f)$
Multiplication in the time-domain	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty} X_1(\zeta)X_2(f-\zeta)\,d\zeta$ or $X_1(f) \ast X_2(f)$
Convolution in the time-domain	$\int_{-\infty}^{\infty} x_1(\zeta)x_2(t-\zeta)\,d\zeta$ or $x_1(t) \ast x_2(t)$	$X_1(f)X_2(f)$
Integration in the time-domain	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{j2\pi f}X(f)+\frac{1}{2}X(0)\delta(f)$

FOURIER TRANSFORMS OF BASIC FUNCTIONS		
	$\boldsymbol{x(t)}$	$\boldsymbol{X(f)}$
Comb	$\sum_{m=-\infty}^{\infty} \delta(t - mT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right)$

FOURIER TRANSFORM PROPERTIES		
	Time-domain	**Frequency-domain**
		$\frac{1}{j2\pi f} X(f)$ **if** $X(0) = 0$

TRIGONOMETRIC IDENTITIES	
$\exp(\pm j\theta) = \cos(\theta) \pm j\sin(\theta)$	$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$
$\cos(\theta) = \frac{1}{2}\left[\exp(j\theta) + \exp(-j\theta)\right]$	$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
$\sin(\theta) = \frac{1}{j2}\left[\exp(j\theta) - \exp(-j\theta)\right]$	$\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$
$\sin^2(\theta) + \cos^2(\theta) = 1$	
$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$	$\sin(\alpha)\sin(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$
$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$	$\cos(\alpha)\cos(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$
$\sin^2(\theta) = \frac{1}{2}\left[1 - \cos(2\theta)\right]$	$\sin(\alpha)\cos(\beta) = \frac{1}{2}\left[\sin(\alpha - \beta) + \sin(\alpha + \beta)\right]$
$\cos^2(\theta) = \frac{1}{2}\left[1 + \cos(2\theta)\right]$	$\mathbf{C}\cos(\theta) - \mathbf{S}\sin(\theta) = \sqrt{\mathbf{C}^2 + \mathbf{S}^2}\cos\left[\theta + \tan^{-1}\left(\frac{\mathbf{S}}{\mathbf{C}}\right)\right]$