Q.5 Let signals $x(t) = 40\,\text{sinc}(20t - 1)$ and $y(t) = x(t)\cos(2\pi \times 10^3 t)$.

(a) Find the Fourier transform of $x(t)$.

(7 marks)

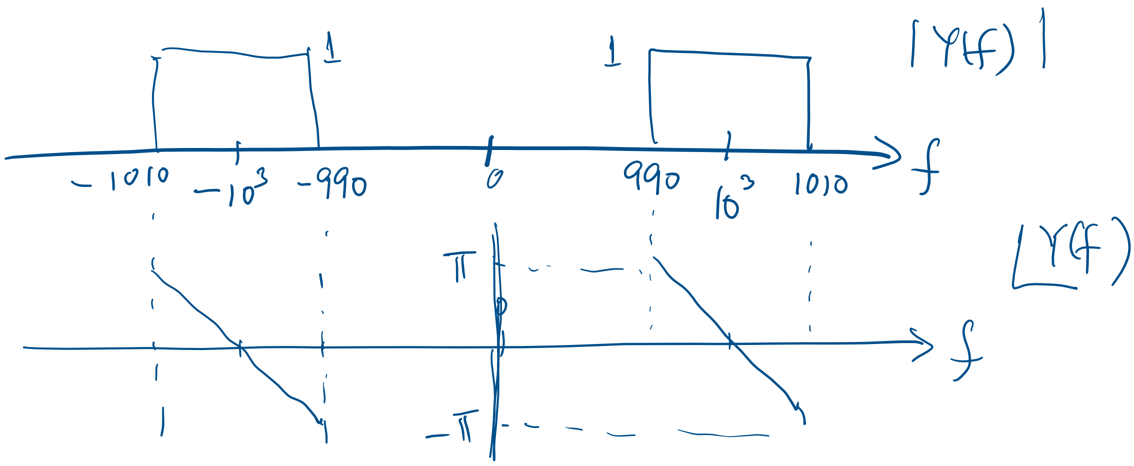
Ans : $X(f) = 2rect\left(\frac{f}{20}\right)e^{-j0.1\pi f}$

(b) Find the Fourier transform of $y(t)$ and plot its spectrum with proper labeling.

(13 marks)

Ans : $Y(f) = rect\left(\frac{f+10^3}{20}\right)e^{-j0.1\pi(f+10^3)} + rect\left(\frac{f-10^3}{20}\right)e^{-j0.1\pi(f-10^3)}$

Sketch :



Q3. Consider the time-domain periodic signal, $x(t) = 2 + \cos\left(12t + \frac{\pi}{3}\right) + \sin(16t)$.

(a) The complex exponential Fourier series expansion of $x(t)$ is given by :

$$x(t) = \sum_{k=-\infty}^{\infty} c_k \exp\left(j2\pi \frac{k}{T_p} t\right).$$

Find T_p and c_k.

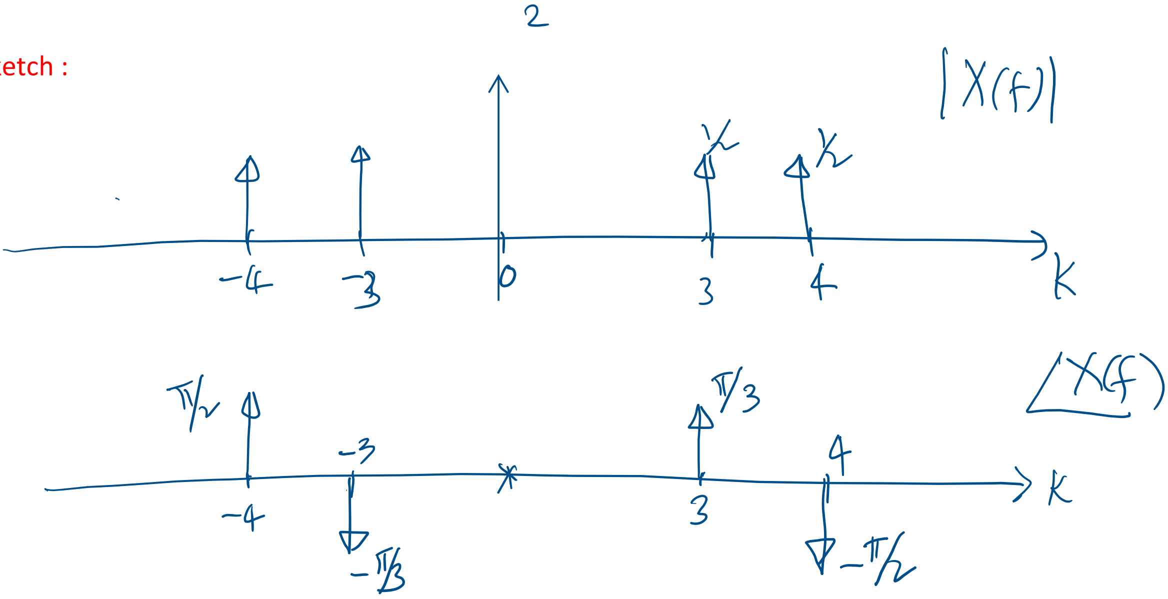
Ans : $T_p = 0.5\pi,\ \ c_0 = 2,\ c_{\pm 3} = \frac{1}{2}e^{\pm j\frac{\pi}{3}},\ c_{\pm 4} = \frac{1}{2}e^{\mp j\frac{\pi}{2}},\ c_k = 0\ k\ elsewhere$

(b) Determine the Fourier transform $X(f)$ of $x(t)$.

Ans : $X(f) = 2\delta(f) + \frac{1}{2}e^{j\frac{\pi}{3}}\delta\left(f - 3\frac{2}{\pi}\right) + \frac{1}{2}e^{-j\frac{\pi}{3}}\delta\left(f + 3\frac{2}{\pi}\right) + \frac{1}{2}e^{-j\frac{\pi}{2}}\delta\left(f - 4\frac{2}{\pi}\right) + \frac{1}{2}e^{j\frac{\pi}{2}}\delta\left(f + 4\frac{2}{\pi}\right)$

(c) Sketch the magnitude spectrum and phase spectrum of $x(t)$ with proper labelling.

Sketch :



Q6. The signal $x(t)$ whose spectrum $X(f) = A \cdot \text{rect}\left(\dfrac{f+f_a}{\alpha}\right) + B \cdot \text{tri}\left(\dfrac{f+f_b}{\beta}\right)$ is shown in Figure Q6 below.

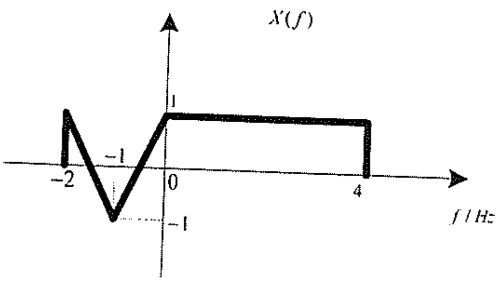


Figure Q6

(a) Find the values of the parameters A, f_a, α, B, β and f_b. (6 marks)

Ans : $A = 1, f_\alpha = -1, \alpha = 6, B = -2, f_b = 1, \beta = 1$

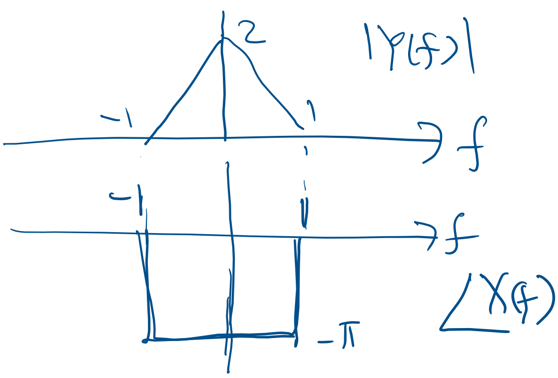
(b) Find the time domain signal, $x(t)$, of $X(f)$?

Ans : $x(t) = 6sinc(6t)e^{j2\pi t} - 2sinc^2(t)e^{-j2\pi t}$

(c) Signal $y(t) = x(t)e^{j2\pi t} - 6\text{sinc}(6\overset{t}{\cancel{t}})e^{j4\pi t}$. Sketch the magnitude and phase spectra of $y(t)$ with proper labelling.

Ans : $Y(f) = -2tri(f)$

Sketch :



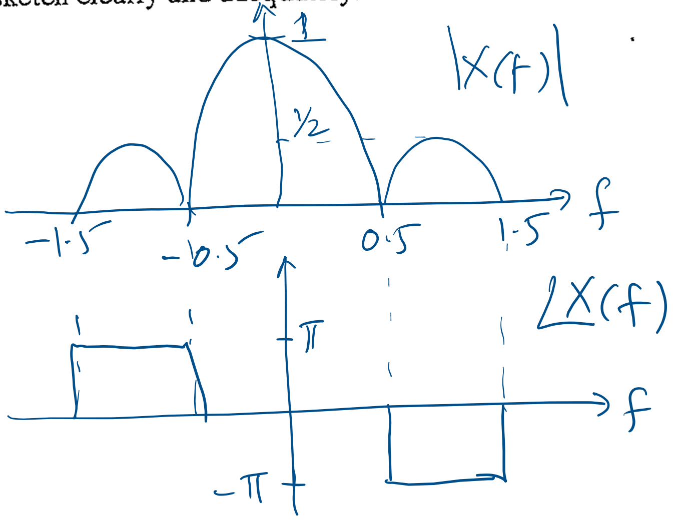
Q3. The spectrum, $X(f)$, of an energy signal, $x(t)$, is given by

$$X(f)=\cos(\pi f)\,\mathrm{rect}(f)*\left[-\frac{1}{2}\delta(f+1)+\delta(f)-\frac{1}{2}\delta(f-1)\right].$$

(a) Sketch the magnitude spectrum, $|X(f)|$, and phase spectrum, $\angle X(f)$, of $x(t)$. Label your sketch clearly and adequately.

(6 marks)

Sketch :



(b) Compute the energy of $x(t)$ contained within its 1st-null bandwidth.

(4 marks)

Ans : E=0.5 Joules

Q3. Consider the signal $x(t)$ given by:

$$x(t)=3+je^{-j14t}+\cos\left(8t+\frac{\pi}{4}\right)+(2+3j)e^{j6t}-je^{j14t}$$

(a) What is the fundamental frequency of $x(t)$?

(2 marks)

Ans : $f_p = 2\ rad\ s^{-1}\ or \frac{1}{\pi}\ Hz$

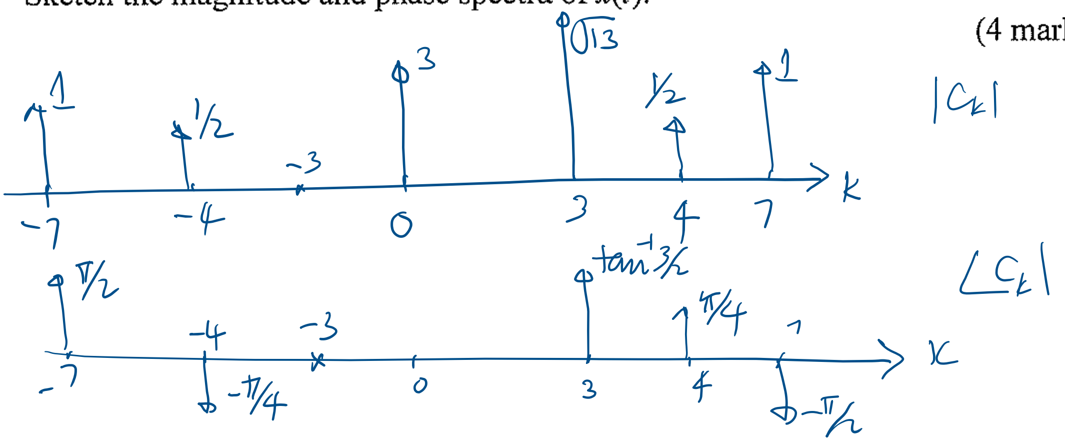
(b) Obtain the Fourier series coefficients of $x(t)$.

(4 marks)

Ans : $c_{0=3,}\ c_3 = (2+3j),\ c_{\pm 4} = \frac{1}{2}e^{\pm j\frac{\pi}{4}},\ c_{\pm 7} = \mp j\ or\ e^{\mp j\frac{\pi}{2}},$

(c) Sketch the magnitude and phase spectra of $x(t)$.

(4 marks)

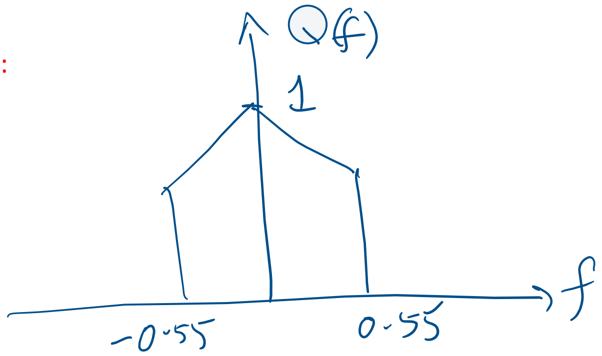


Q6. An energy pulse is modeled by $q(t) = \text{sinc}^2(t) * [1.1\,\text{sinc}(1.1t)]$, where '*' denotes convolution. The spectrum, $Q(f)$, of $q(t)$ is sampled in the frequency domain to form $X(f) = Q(f)\sum_k \delta(f - 0.1k)$. Let $x(t)$ denote the inverse Fourier transform of $X(f)$.

(a) Draw a labeled sketch of the spectrum, $Q(f)$, of $q(t)$.

(4 marks)

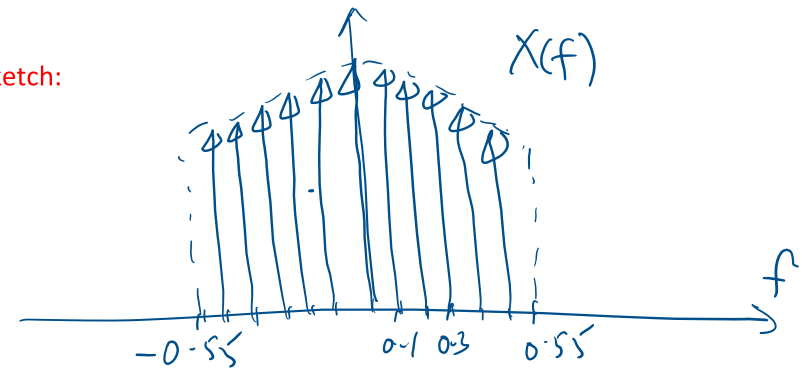
Sketch :



(b) Draw a labeled sketch of the spectrum, $X(f)$, of $x(t)$.

(4 marks)

Sketch:



(c) By inspection of the sketch in Part (b), or otherwise, determine whether or not $x(t)$ is periodic. If $x(t)$ is periodic, find its fundamental frequency, its Fourier series coefficients, c_k, and its average power.

(12 marks)

Ans : periodic, $f_p = 0.1\ Hz$

$c_0 = 1, c_{\pm 1} = 0.9, c_{\pm 2} = 0.8, c_{\pm 3} = 0.7, c_{\pm 4} = 0.6, c_{\pm 5} = 0.5$

$P_{av} = 6.1\ Watts$

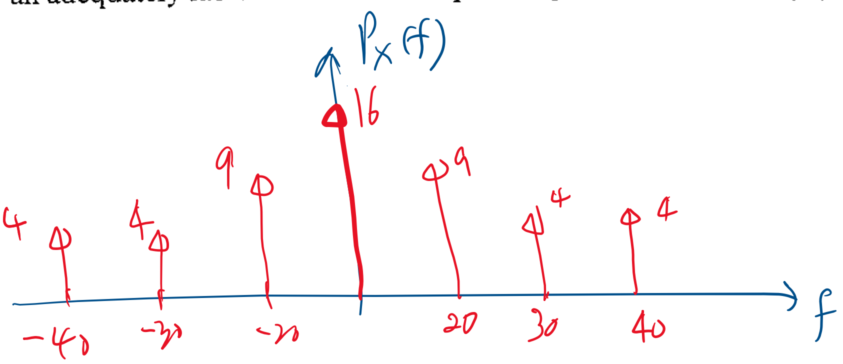
Q.4 The continuous-frequency spectrum of a signal $x(t)$ is given by

$$X(f) = 2\delta(f+40) + 2\delta(f+30) + 3\delta(f+20) + 4$$
$$+3\delta(f-20) + 2\delta(f-30) + 2\delta(f-40)$$

(a) Draw an adequately labeled sketch of the power spectral density, $P_x(f)$, of $x(t)$.

(4 marks)

Sketch :



(b) What is the average power of $x(t)$?

(2 marks)

Ans : Average Power = 50 Watts

(c) The 80% power containment bandwidth of a power signal is defined as the smallest bandwidth that contains at least 80% of the average signal power. What is the 80% power containment bandwidth of $x(t)$?

(4 marks)

Ans : 80% power containment bandwidth = 20 Hz

Q2. The Fourier transform, $X(f)$ of the signal $x(t)$ is a half-cosine shaped amplitude spectrum as shown in Figure Q2.

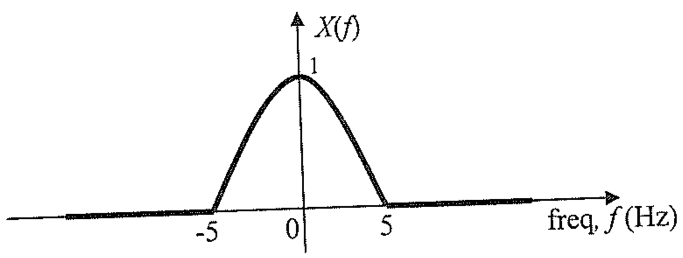


Figure Q2

(a) Provide the expression for $X(f)$ in terms of the frequency f.

(2 marks)

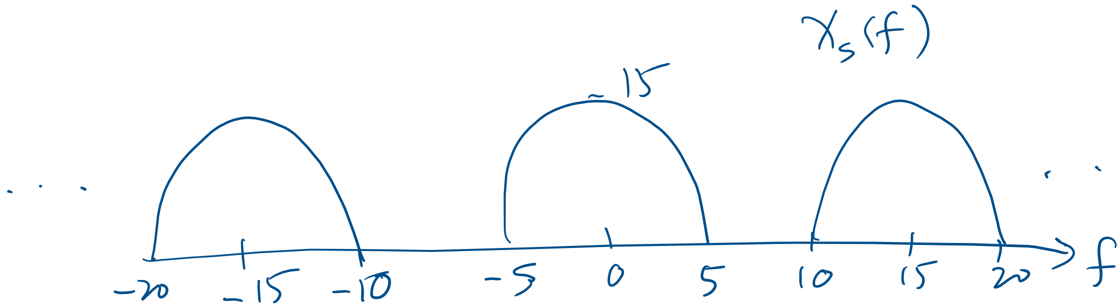
Ans : $X(f) = \cos(0.1\pi f)\, rect\left(\frac{f}{10}\right)$

(b) Derive the energy of the signal $x(t)$. (6 marks)

Ans : E=5 Joules

(c) The signal $x(t)$ is sampled at 15 Hz. Sketch the amplitude spectrum of the sampled signal. (2 marks)

Ans : $X_s(f) = 15\sum X(f-15k)$



Q2. Consider the periodic signal $x(t) = 10\sin(3t) + 4\cos\left(4.5t + \frac{\pi}{6}\right) + e^{j\left(t+\frac{\pi}{4}\right)} + 2$.

(a) Find the fundamental frequency, f_o, and period, T, of $x(t)$. (3 marks)

Ans : $f_0 = 0.5\ rad\ s^{-1}\ or\ \frac{1}{4\pi}\ Hz,\ T = 4\pi$

(b) Find the Fourier series coefficients, c_k, of $x(t)$ and find the Fourier transform, $X(f)$, of $x(t)$. (5 marks)

Ans : $c_0 = 2, c_2 = e^{j\frac{\pi}{4}},\ c_{\pm 6} = \mp 5j,\ c_{\pm 9} = 2e^{\pm j\frac{\pi}{6}}$

$X(f) = \sum c_k \delta\left(f - \frac{k}{4\pi}\right)$

(c) Find the average power of $x(t)$.

Ans $P_{av} = 63\ Watts$

Q7. (a) The amplitude spectrum of the signal $x(t)$ is shown in Figure Q7-1, and the signal $y(t) = x(t)\cos(40\pi t)$ is sampled at a frequency of 20 Hz to obtain the sampled signal $y_s(t)$.

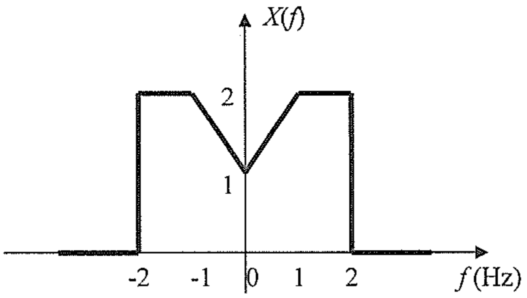


Figure Q7-1

i. What is the Nyquist frequency for signal $y(t)$?

(2 marks)

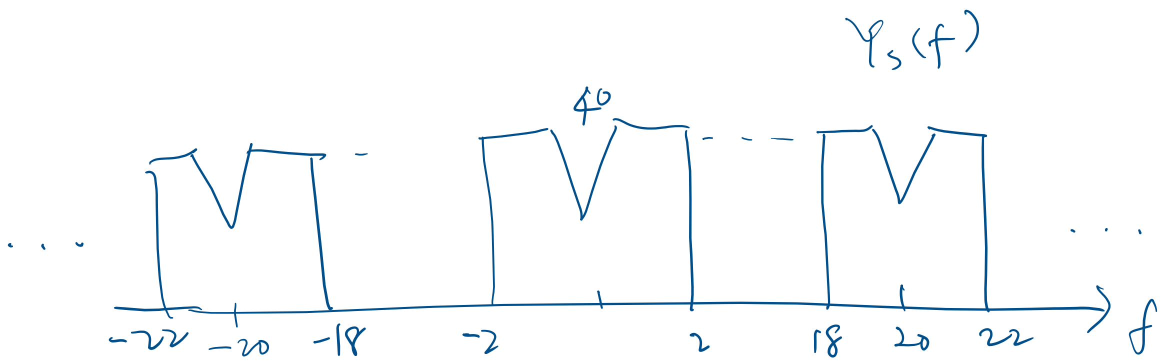
Ans : $Nyquist\ frequency = 44\ Hz$

ii. The signal $y(t)$ is sampled at 20 Hz to give the signal $y_s(t)$. Determine the Fourier transform, $Y_s(f)$, of the sampled signal $y_s(t)$ and sketch its magnitude spectrum.

(6 marks)

Ans : $Y_s(f) = 20 \sum Y(f - 20k)$

Sketch :



iii. Can the signal $y(t)$ be recovered from the sampled signal $y_s(t)$? Explain your answer.

(2 marks)

Ans : Yes, $y(t)$ can be recovered with a bandpass filter of bandwidth of 4 Hz, centered at 20Hz.

(b) The amplitude spectrum of the signal $a(t)$ is shown in Figure Q7-2. Assume that the phase spectrum is zero.

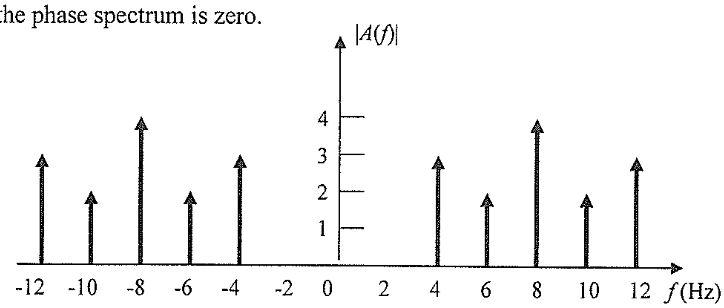


Figure Q7-2

i. Derive the signal $a(t)$.

(4 marks)

Ans : $a(t) = 6\cos(8\pi t) + 4\cos(12\pi t) + 8\cos(16\pi t) + 4\cos(20\pi t) + 6\cos(24\pi t)$

ii. Is $a(t)$ a power or an energy signal? Find the corresponding power or energy?

Ans : $a(t)$ is a power signal, $P_{av} = 84\ Watts$

iii. What is the bandwidth of the signal $a(t)$?

Ans : Bandwidth of $a(t)$ is 12 Hz

Q1. The spectrum, $X(f)$, of a periodic signal $x(t)$ is shown in Figure Q1.

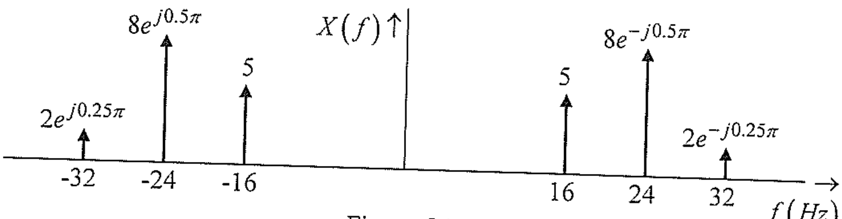


Figure Q1

(a) Find the dc value and average power of $x(t)$. (5 marks)

Ans : DC value = 0, Power = 186 Watts

(b) Express $x(t)$ as a function of real sinusoids. (5 marks)

Ans : $x(t) = 10\cos(32\pi t) + 16\sin(48\pi t) + 4\cos(64\pi t - 0.25\pi)$

Q7. Two time-domain periodic signals are given by $x(t) = 2\mathrm{sinc}(2.5t - 0.5) * \sum_{n=-\infty}^{\infty} \delta(t - 2n)$ and $y(t) = x(t)\cos(20\pi t)$.

(a) Find fundamental frequency, f_p, of $x(t)$ and its Fourier transform, $X(f)$.

(8 marks)

Ans : fundamental frequency = 0.5 Hz

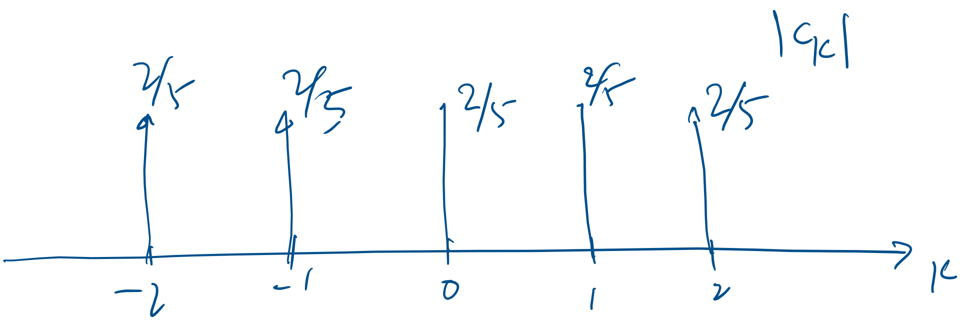
$$X(f) = \frac{2}{5}\sum rect\left(\frac{n}{5}\right)e^{-j0.2\pi n}\delta\left(f - \frac{n}{2}\right) = \frac{2}{5} + \frac{2}{5}e^{j0.5\pi t}e^{-j0.2\pi} + \frac{2}{5}e^{j2\pi t}e^{-j0.4\pi}$$
$$+\frac{2}{5}e^{-j0.5\pi t}e^{j0.2\pi} + \frac{2}{5}e^{-j2\pi t}e^{j0.4\pi}$$

(b) Determine the complex exponential Fourier series coefficients, c_k, of $x(t)$ and sketch the magnitude spectrum of $x(t)$ with proper labelling. Find the power, P_1 of $x(t)$.

Ans : $c_0 = \frac{2}{5},\ c_{\pm1} = \frac{2}{5}e^{\mp j0.2\pi}, c_{\pm2} = \frac{2}{5}e^{\mp 0.4\pi}, c_k = 0\ elsewhere$

$P_1 = \frac{4}{5}Watts$

Sketch :



(c) Derive the Fourier transform, $Y(f)$, of $y(t)$ in terms of $X(f)$ and find the power, P_2 of $y(t)$?

Ans : $Y(f) = X(f) * \frac{1}{2}[\delta(f+10) + \delta(f-10)],\ P_2 = \frac{2}{5}Watts$

Q.6 Suppose $x(t) = 2\,\mathrm{sinc}(2t)$, $y(t) = \left[\sum_{k=-\infty}^{\infty} 2\,\mathrm{rect}\left(\frac{t-4k}{2}\right)\right] - 1$ and $z(t) = x(t) \otimes y(t)$, where the symbol $\otimes$ denotes the convolution operator.

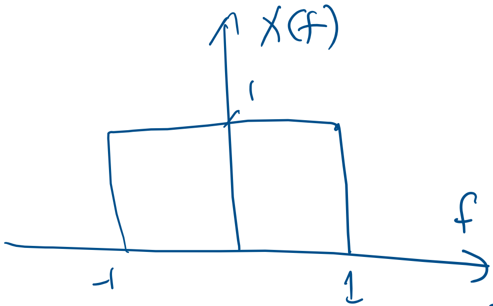
(a) Determine the Fourier transforms $X(f)$, $Y(f)$ and $Z(f)$ of $x(t)$, $y(t)$ and $z(t)$, respectively, and sketch their corresponding amplitude spectra.

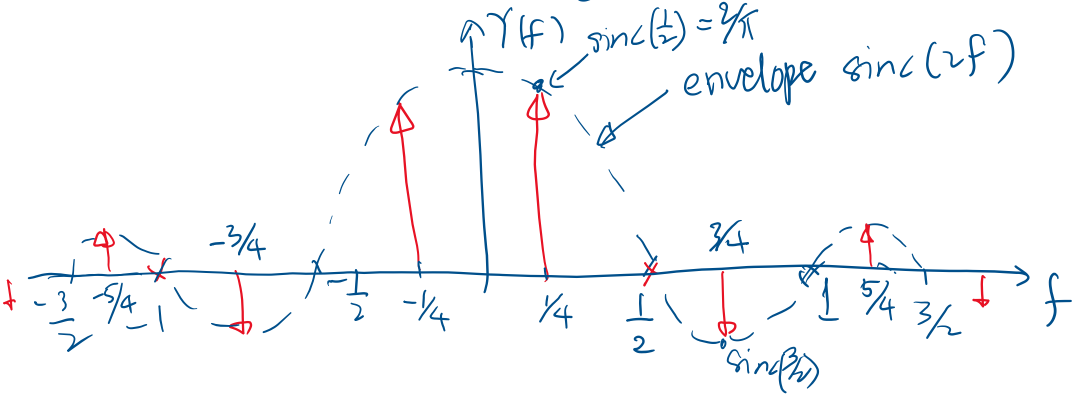
(14 marks)

Ans : $X(f) = rect\left(\frac{f}{2}\right)$, $Y(f) = \left[\sum sinc\left(2\frac{k}{4}\right)\delta\left(f - \frac{k}{4}\right)\right] - \delta(f)$

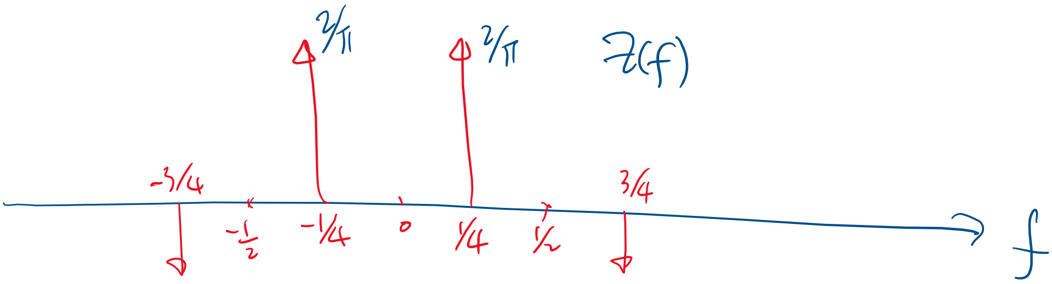
$$Z(f) = sinc\left(-\frac{3}{2}\right)\delta\left(f + \frac{3}{4}\right) + sinc\left(-\frac{1}{2}\right)\delta\left(f + \frac{1}{4}\right) + sinc\left(\frac{3}{2}\right)\delta\left(f - \frac{3}{4}\right) + sinc\left(\frac{1}{2}\right)\delta\left(f - \frac{1}{4}\right)$$

Sketch :





I am sketching the actual spectrum, not amplitude spectrum!

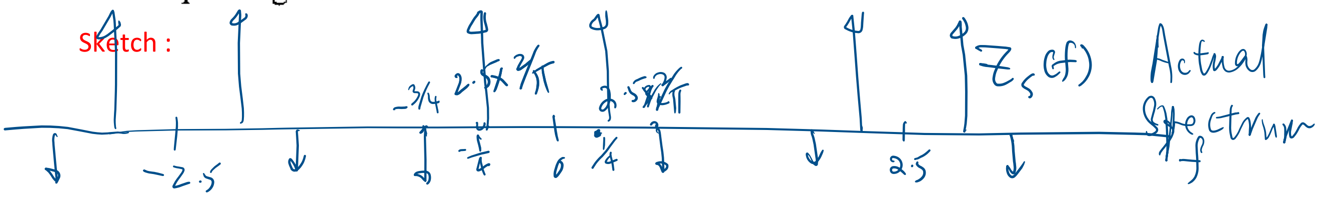


(b) Determine the average power of $z(t)$.

Ans : $P_z = \frac{80}{9\pi^2}$

(c) If $z(t)$ is sampled at a frequency of 2.5 Hz, sketch the amplitude spectrum of the sampled signal.

Sketch :



Q2. The energy spectral density of a signal $x(t)$ is given by

$$E_x(f) = 16\exp(-2|f|) \quad \text{Joules/Hz}.$$

(a) Find the 3dB bandwidth of $x(t)$. (5 marks)

Ans : 3 $dB\ bandwidth = \frac{\ln 2}{2}$ Hz

(b) Find $X(f)$ if the phase spectrum of $x(t)$ is given by $\angle X(f) = -0.5f$.

(5 marks)

Ans : $X(f) = 4e^{-|f|}e^{-j0.5f}$

Q2. Figure Q2 shows the half-cosine amplitude spectrum, $X(f)$, of the signal $x(t)$.

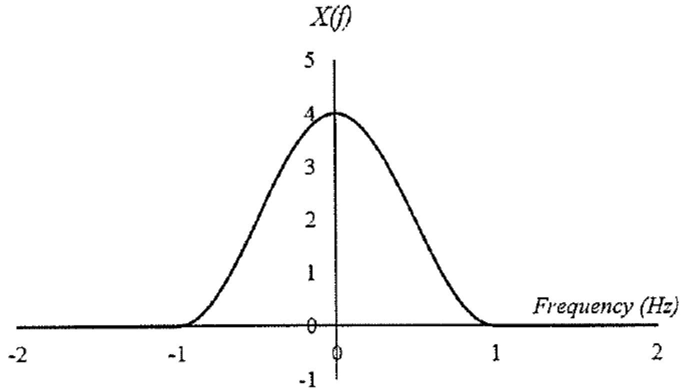


Figure Q2 : Amplitude Spectrum, $X(f)$

(a) What is the energy of signal $x(t)$?

(3 marks)

Ans : E=16 Joules

(b) What is the 3dB bandwidth of signal $x(t)$?

(3 marks)

Ans : 3 dB bandwidth = 0.5 Hz

(c) Determine the expression for signal $x(t)$.

(4 marks)

Ans : $x(t) = 4\left[sinc\left(2\left(t+\frac{1}{4}\right)\right) + sinc\left(2\left(t-\frac{1}{4}\right)\right)\right]$

Q7. A signal $x(t)$ is given by

$$x(t) = 4\cos(2\pi f_c t)\left[\text{rect}\left(\frac{Wt}{2}\right)\otimes \text{sinc}(2Wt)\right]$$

where f_c and W are positive real constants, $f_c >> W$, and the symbol $\otimes$ denotes convolution.

(a) Determine the Fourier transform, $X(f)$, of the signal $x(t)$.

(8 marks)

Ans : $Y(f) = \frac{1}{W^2} sinc\left(\frac{f}{0.5W}\right) rect(\left(\frac{f}{2W}\right)$

(b) Find the bandwidth of $x(t)$ and determine the corresponding Nyquist sampling frequency of $x(t)$.

(5 marks)

Ans : Bandwidth = 2W, Nyquist sampling frequency = $2(f_c + W)$ Hz

(c) If the signal $x(t)$ is sampled at a frequency of $2f_c$ to give the sampled signal $x_s(t)$, give the expression for the Fourier transform of $x_s(t)$.

(7 marks)

Ans : $X_s(f) = \frac{4f_c}{W^2}\sum\left[sinc\left(\frac{2}{W}(f + f_c - 2kf_c)\right) rect\left(\frac{f+f_c-2kf_c}{2W}\right) + sinc\left(\frac{2}{W}(f - f_c - 2kf_c)\right) rect\left(\frac{f-f_c-2kf_c}{2W}\right)\right]$

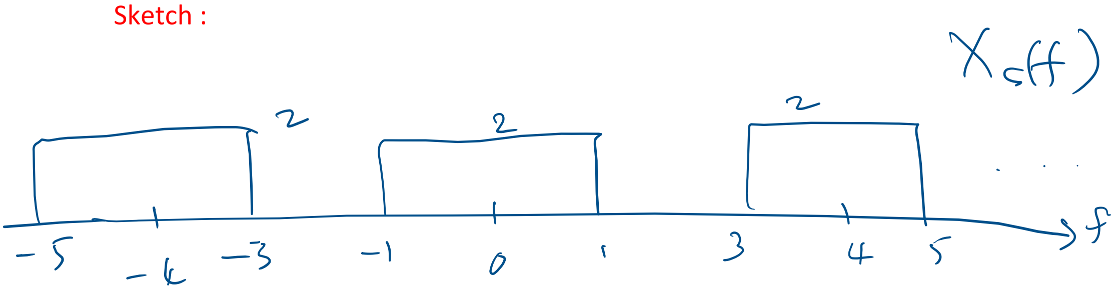
Q.1 The signal $x(t) = \text{sinc}(2t)$ is sampled at 4 Hz to obtain the sampled signal, $x_s(t)$.

(a) Derive the Fourier transform, $X_s(f)$, of the sampled signal $x_s(t)$ and sketch its spectrum.

(6 marks)

Ans : $X_s(f) = 2\sum rect\left(\frac{f-4k}{2}\right)$

Sketch :



(b) What is the Nyquist sampling frequency?

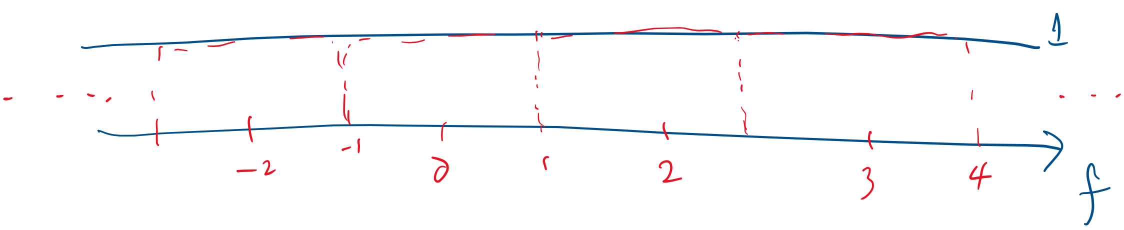
(2 marks)

Ans : Nyquist sampling frequency is 2 Hz.

(c) If $x(t)$ is sampled at a frequency of 2 Hz, sketch and label the sampled signal.

(2 marks)

Sketch :



Q4. The signal $x(t) = \mathrm{sinc}^2(5t)$ is sampled at 15Hz to produce the signal $x_s(t)$.

(a) Derive the Fourier transform of the sampled signal $x_s(t)$.

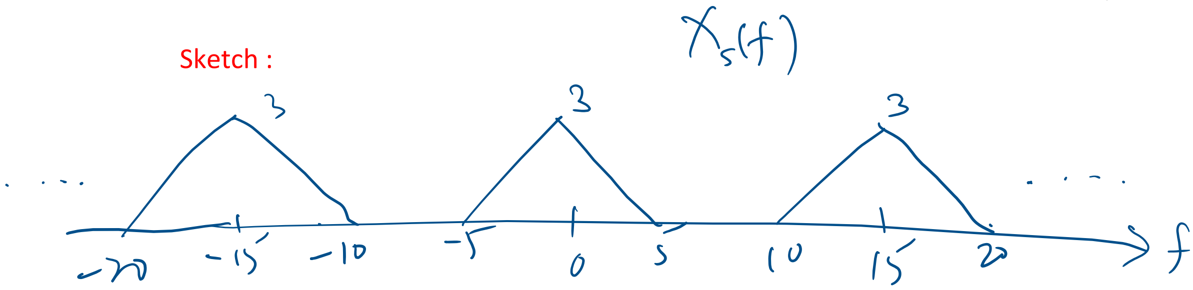
(6 marks)

Ans : $X_s(f) = 3\sum tri\left(\frac{f-15k}{5}\right)$

(b) Sketch the spectrum of the sampled signal $x_s(t)$.

(4 marks)

Sketch :



Q.8 Figure Q8(a) shows a vowel synthesizer which consists of a linear time-invariant (LTI) filter driven by an impulse train

$$e(t) = 2\sum_{n=-\infty}^{\infty} \delta(t-10n).$$

The impulse response, $h(t)$, of the LTI filter is plotted in Figure Q8(b) where $h(t) = 0$ for $t < 0$, and

$$H(f) = \Im\{h(t)\} = \frac{40}{(13-20f^2)+j4f}$$

is the Fourier transform of $h(t)$.

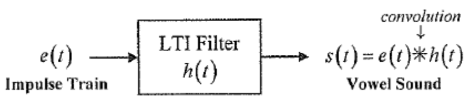


Figure Q8(a)

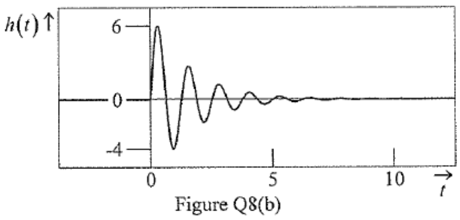
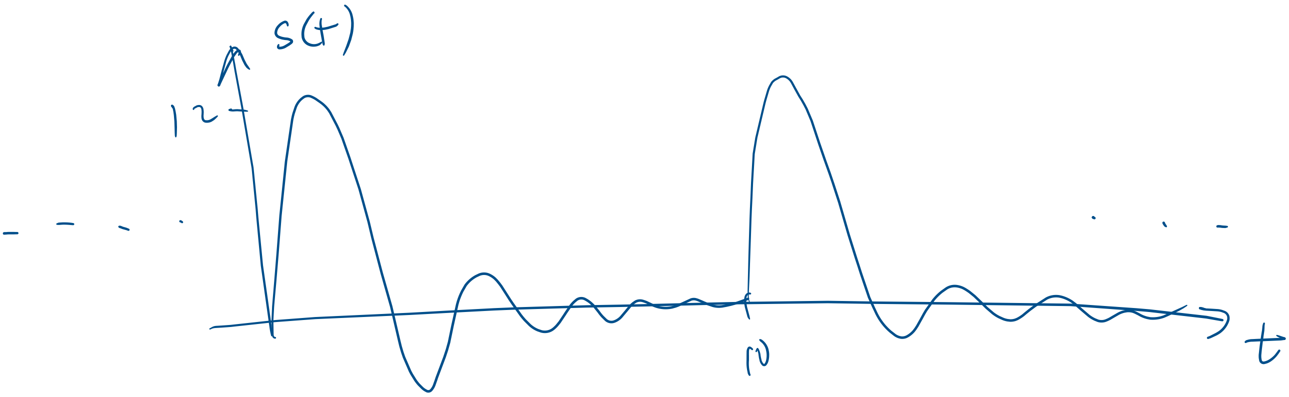


Figure Q8(b)

(a) Draw an adequately labeled sketch of $s(t)$.

(5 marks)

Sketch :



(b) Derive the spectrum, $S(f)$, of $s(t)$.

(6 marks)

Ans : $S(f) = 8\sum \frac{1}{\left(13-20\left(\frac{n}{10}\right)^2\right)+j4\left(\frac{n}{10}\right)}\delta(f - \frac{n}{10})$

(c) i. Find the complex exponential Fourier series coefficients, c_k, of $s(t)$.

(5 marks)

Ans : $c_k = \frac{8}{\left(13-20\left(\frac{k}{10}\right)^2\right)+j4\left(\frac{k}{10}\right)}$

ii Based on the value of c_0 alone, can we claim with absolute certainty that the average power of $s(t)$ is greater than the average value of $s(t)$? Explain your answer.

(4 marks)

Ans : No, because if the average value $c_0 < 1$, then $c_0^2 < c_0$, assuming that all other values of $c_k = 0$.

Q6. Figure Q6 shows a lower-single-sideband (LSSB) modulator where $x(t)$ is the input message signal and $y(t)$ is the output modulated signal. In Figure Q6, $X(f)$ and $Y(f)$ are the Fourier transforms of $x(t)$ and $y(t)$, respectively, and $h(t)$ is the impulse response of the ideal lowpass filter.

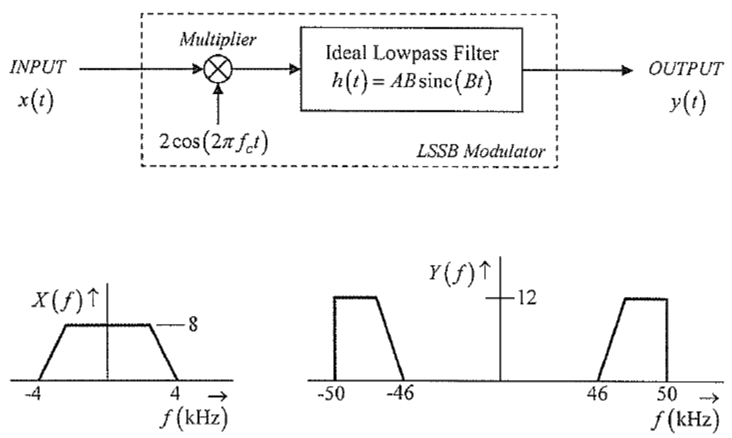


Figure Q6

(a) Find the values of f_c, A and B. (12 marks)

Ans : $B = 100, A = 1.5, f_c = 50\ kHz$

(b) Suppose we apply $y(t)$ to the input of another LSSB modulator to produce $z(t)$ at its output. Find the relationship between $z(t)$ and $x(t)$ if the LSSB modulator is identical to the one used in Part (a).

(8 marks)

Ans : $Z(f) = \frac{9}{4}X(f)$