Outline of Lecture

Systems & Classification of Systems

The second half of this module is about systems, specifically Linear Time Invariant (LTI) systems.

1. Systems

- Physical systems, in the broadest sense are interconnections of components, devices or subsystems. Examples are communication systems, mechanical and electronic systems, chemical systems, etc.
- In system theory, a system model is a mathematical description of a physical process that relates the input signal to the output signal.
- With an input $x(t)$ and an output $y(t)$, the system may be viewed as a transformation of $x(t)$ into $y(t)$, mathematically expressed as :

$$y(t) = \mathbf{T}[x(t)]$$

where $\mathbf{T}$ represents the operator which transforms $x(t)$ into $y(t)$.

- Systems are classified based on their basic characteristics.

2. Classification of Systems

A. Systems with memory and without memory (memoryless)

A system is said to be memoryless (or static) if its output at a given time is dependent on only the input at that time. Otherwise, the system is said to have memory (or to be dynamic).

Example 1 (Memoryless or Static System)

A resistor R is a memoryless system. Its current $i(t)$ flowing through it sets up the voltage $v(t)$ across it. The input-output relationship (Ohm's law) of the resistor is $v(t) = Ri(t)$. Clearly, the voltage $v(t)$ at time t depends only on current $i(t)$ at t.

Example 2 (Memory or Dynamic System)

A capacitor, C is a system with memory. Its current $i(t)$ flowing through it sets up the voltage $v(t)$ across it. The input-output relationship of the capacitor is

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$$

Clearly, $v(t)$ at time t depends on all values of $i(t)$ from time $-\infty$ to t.

B. Causal and Non-causal Systems A system is said to be causal (or non-anticipative) if its output, $y(t)$, at the present time t depends on only the present and/or past values of its input, $x(t)$. It is thus not possible for a causal system to produce an output before an input is applied to it.

Example 3 (Causal Systems)

$$\left.\begin{aligned} y(t) &= x(t-1) \\ y(t) &= \int_{t-3}^{t-1} x(\tau)d\tau \end{aligned}\right\} \quad \textit{Causal or Non-Anticipative Systems}$$

A system is noncausal (or anticipative) if its output, $y(t)$, at the present time depends on future values of its input, $x(t)$.

Example 4 (Non-Causal Systems)

$$\left.\begin{aligned} y(t) &= x(t+1) \\ y(t) &= \int_{t-3}^{t+1} x(\tau)d\tau \end{aligned}\right\} \textit{Non-Causal or Anticipative Systems}$$

C. Stable and Unstable Systems

- A system is bounded-input/bounded-output (BIBO) stable if for any bounded input $x(t)$ defined by $|x(t)| \le K \; \forall t, \; 0 < K < \infty$, the output $y(t)$ is also bounded ie $|y(t)| \le L \; \forall t, \; 0 < L < \infty$.

Example 5 (Stable system)

When a battery is connected to a dc motor, the motor shaft starts to rotate and accelerates to a constant angular speed. Since both the input voltage and the speed of rotation are bounded, the motor is stable.

- An unstable system is one in which not all bounded inputs lead to bounded outputs.

Example 6 (Unstable system)

In a PA system, we sometimes hear a sustained sound of rapidly increasing loudness, called howling, from the loudspeaker. This is due to the positive feedback received by the microphone from the loudspeaker, thus causing the system to electrically resonate. This can happen when the microphone is placed too near to the loudspeaker eg in a karaoke system. This is an example of an unstable system.

D. Linear and Non-Linear Systems

Let $x(t)$ and $y(t)$ be the input and output, respectively, of a system.

- A linear system is one that satisfies the superposition principle as follows :

$$\mathbf{T}[\alpha_1 x_1(t) + \alpha_2 x_2(t)] = \alpha_1 y_1(t) + \alpha_2 y_2(t)$$

where $y_1(t)$ and $y_2(t)$ are outputs corresponding to inputs $x_1(t)$ and $x_2(t)$ respectively. **T** represents the transformation when an input goes thru a system.

Example 7 (Linear Systems)

$$\left.\begin{array}{l} \textit{Resistor}: v(t) = Ri(t) \\ \textit{Capacitor}: v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau \end{array}\right\} \textit{Linear Systems}$$

- Any system that does not satisfy the superposition principle is classified as a nonlinear system.

Example 8 (Non-Linear Systems)

$$\left.\begin{array}{l} y(t) = x^2(t) \\ y(t) = \cos[x(t)] \end{array}\right\} \textit{Non-Linear Systems}$$

E. Time-Invariant and Time Varying Systems

- A system is time-invariant if a time shift (delay or advance) in the input signal, $x(t)$, causes the same time shift in the output signal, $y(t)$. Hence, a time-invariant system has the property of

$$\mathbf{T}[x(t-\tau)] = y(t-\tau), \tau \text{ is real.} \tag{1}$$

Example 9 (Time-Invariant System)

A resistor or capacitor is a time-invariant system, when measured over a short period of time. A resistor or capacitor today has the same value of resistance and capacitance when measured the next day.

- A time varying system is one which does not satisfy (1).

Example 10 (Time Varying System)

A resistor or capacitor is a time varying system, when taken over a long period of time. A resistor or capacitor today does not have the same value of resistance and capacitance when measured many years later.

3. Remarks

- We often use mathematical tools to describe or model the behaviour of many different types of signals and systems. However, not all signals and not many systems can be described precisely by mathematics.
- But it is always possible to approximate their behaviour based on physical or natural laws of Physics.
- In this module, we will only focus on linear time invariant (LTI) systems. This is a special class of systems for which their behaviours can be generalized by some mathematical tools.
- LTI systems can be described elegantly by mathematics. This leads to some nice properties that can be deduced for such systems, in many instances without having to solve their mathematical equations explicitly.

Exercise 1

State whether the following statements are TRUE / FALSE.

- *Systems which require memory are called non-dynamic systems.*
- *A non-causal system can be realized in real time.*
- *A human being is a linear time invariant system.*
- *All LTI systems are BIBO stable.*
- *A switch is a dynamic system.*
- *Behaviours of non-linear systems can be generalized using mathematical tools.*
- *When a step input is provided to a LTI system, the output responds and over time, the output reaches a steady state. This implies that the system is time varying.*