

Fourier Series:
$$\begin{cases} X_k = \frac{1}{T} \int_{\tilde{t}}^{\tilde{t}+T} x(t) \exp(-j2\pi k t/T) dt \\ x(t) = \sum_{k=-\infty}^{\infty} X_k \exp(j2\pi k t/T) \end{cases}$$

Fourier Transform:
$$\begin{cases} X(f) = \int_{-\infty}^{\infty} x(t) \exp(-j2\pi f t) dt \\ x(t) = \int_{-\infty}^{\infty} X(f) \exp(j2\pi f t) df \end{cases}$$

FOURIER TRANSFORMS OF BASIC FUNCTIONS		
	$x(t)$	$X(f)$
Constant	K	$K\delta(f)$
Unit Impulse	$\delta(t)$	1
Unit Step	$u(t)$	$\frac{1}{2} \left[\delta(f) + \frac{1}{j\pi f} \right]$
Sign (or Signum)	$\text{sgn}(t)$	$\frac{1}{j\pi f}$
Rectangle	$\text{rect}\left(\frac{t}{T}\right)$	$T \text{sinc}(fT)$
Triangle	$\text{tri}\left(\frac{t}{T}\right)$	$T \text{sinc}^2(fT)$
Sine Cardinal	$\text{sinc}\left(\frac{t}{T}\right)$	$T \text{rect}(fT)$
Complex Exponential	$\exp(j2\pi f_o t)$	$\delta(f - f_o)$
Cosine	$\cos(2\pi f_o t)$	$\frac{1}{2} [\delta(f - f_o) + \delta(f + f_o)]$
Sine	$\sin(2\pi f_o t)$	$-\frac{j}{2} [\delta(f - f_o) - \delta(f + f_o)]$
Gaussian	$\exp\left(-\frac{t^2}{\alpha^2}\right)$	$\alpha\pi^{0.5} \exp(-\alpha^2\pi^2 f^2)$
Comb	$\sum_{m=-\infty}^{\infty} \delta(t - mT)$	$\frac{1}{T} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right)$

FOURIER TRANSFORM PROPERTIES		
	Time-domain	Frequency-domain
Linearity	$\alpha x_1(t) + \beta x_2(t)$	$\alpha X_1(f) + \beta X_2(f)$
Time scaling	$x(\beta t)$	$\frac{1}{ \beta } X\left(\frac{f}{\beta}\right)$
Duality	$X(t)$	$x(-f)$
Time shifting	$x(t - t_o)$	$X(f) \exp(-j2\pi f t_o)$
Frequency shifting (Modulation)	$x(t) \exp(j2\pi f_o t)$	$X(f - f_o)$
Differentiation in the time-domain	$\frac{d^n}{dt^n} x(t)$	$(j2\pi f)^n X(f)$
Multiplication in the time-domain	$x_1(t) x_2(t)$	$\int_{-\infty}^{\infty} X_1(\zeta) X_2(f - \zeta) d\zeta$ or $X_1(f) * X_2(f)$
Convolution in the time-domain	$\int_{-\infty}^{\infty} x_1(\zeta) x_2(t - \zeta) d\zeta$ or $x_1(t) * x_2(t)$	$X_1(f) X_2(f)$
Integration in the time-domain	$\int_{-\infty}^t x(\tau) d\tau$	$\frac{1}{j2\pi f} X(f) + \frac{1}{2} X(0) \delta(f)$
		$\frac{1}{j2\pi f} X(f) \quad \text{if } X(0) = 0$

Trigonometric Identities	
$\exp(\pm j\theta) = \cos(\theta) \pm j \sin(\theta)$	$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$
$\cos(\theta) = 0.5[\exp(j\theta) + \exp(-j\theta)]$	$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
$\sin(\theta) = -0.5j[\exp(j\theta) - \exp(-j\theta)]$	$\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$
$\sin^2(\theta) + \cos^2(\theta) = 1$	
$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$	$\sin(\alpha)\sin(\beta) = 0.5[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$	$\cos(\alpha)\cos(\beta) = 0.5[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$
$\sin^2(\theta) = 0.5[1 - \cos(2\theta)]$	$\sin(\alpha)\cos(\beta) = 0.5[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$
$\cos^2(\theta) = 0.5[1 + \cos(2\theta)]$	$C \cos(\theta) - S \sin(\theta) = \sqrt{C^2 + S^2} \cos[\theta + \tan^{-1}(S/C)]$
Complex Unit (j) \rightarrow $(j = \sqrt{-1} = e^{j\pi/2} = e^{j90^\circ}) \quad \left(-j = \frac{1}{j} = e^{-j\pi/2} = e^{-j90^\circ}\right) \quad (j^2 = -1)$ 	

Definitions of Basic Functions
Rectangle: $\text{rect}\left(\frac{t}{T}\right) = \begin{cases} 1; & -T/2 \leq t < T/2 \\ 0; & \text{elsewhere} \end{cases}$
Triangle: $\text{tri}\left(\frac{t}{T}\right) = \begin{cases} 1 - t /T; & t \leq T \\ 0; & t > T \end{cases}$
Sine Cardinal: $\text{sinc}\left(\frac{t}{T}\right) = \begin{cases} \frac{\sin(\pi t/T)}{\pi t/T}; & t \neq 0 \\ 1; & t = 0 \end{cases}$
Signum: $\text{sgn}(t) = \begin{cases} 1; & t \geq 0 \\ -1; & t < 0 \end{cases}$
Unit Impulse: $\delta(t) = \begin{cases} \infty; & t = 0 \\ 0; & t \neq 0 \end{cases} \quad \int_{0^-}^{0^+} \delta(t) dt = 1$
Unit Step: $u(t) = \begin{cases} 1; & t \geq 0 \\ 0; & t < 0 \end{cases}$