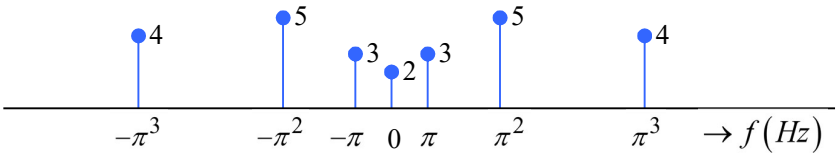
EE2023/TEE2023 TUTORIAL 2 (PROBLEMS)

Q.1 The discrete-frequency spectrum of a signal $x(t)$ is shown in Fig.Q.1.

Fig.Q.1



(a) What is the dc value of $x(t)$?
(b) Is $x(t)$ a power or energy signal?
(c) What is the Fourier series expansion of $x(t)$

Ans : (a) 2 ; (b) Power signal ; (c) No Fourier series expansion

Q.2 (a) Determine the Fourier series coefficients of $x(t)=6\sin(12\pi t)+4\exp\left(j\left(8\pi t+\pi/4\right)\right)+2$

(b) Find the frequency of the 4th harmonic of $x(t)=0.5\left(\left|\sin(\pi t)\right|+\sin(\pi t)\right)$

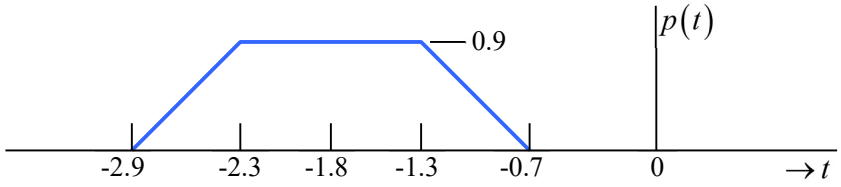
Ans : (a) $X_{-3}=j3$, $X=2$, $X_2=4e^{j\pi/4}$, $X_3=-j3$; (b) 2 Hz

Q.3 Determine the Fourier series coefficients of

$$x(t)=\sum_{n=-\infty}^{\infty}2p(t-1.6n)$$

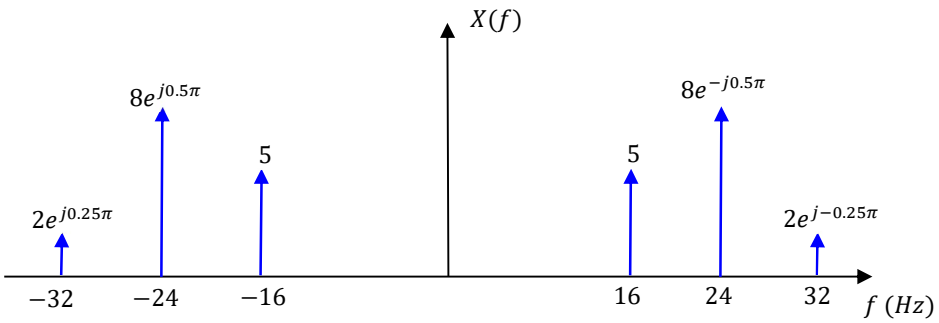
where $p(t)$ is given in Fig.Q.3.

Fig.Q.3



Ans: $X_0=1.8$

Q.4 The spectrum $X(f)$ of a periodic signal $x(t)$ is shown in the figure below. Express $x(t)$ as a function of real sinusoids.



Q.5 Consider the signal $x(t) = \cos(3\pi t)$ and define

$$y(t) = \sum_{k=-\infty}^{\infty} c_k \exp(j2\pi kt)$$

where $c_k = \int_{-0.5}^{0.5} x(t) \exp(-j2\pi kt)\, dt$. Sketch $x(t)$ and $y(t)$. Show all the important dimensions in your sketches.

Supplementary Problems
These problems will not be discussed in class.

S.1 Consider a rectified sine wave signal $x(t)$ defined by

$$x(t) = |\sin(\pi t)|.$$

(a) Sketch $x(t)$ and find its fundamental period.

(b) Find the complex exponential Fourier series of $x(t)$.

(c) Find the trigonometric Fourier series of $x(t)$.

Answer: *(a)* $period = 1$ *(b)* $x(t) = -\frac{2}{\pi}\sum_{k=-\infty}^{\infty}\frac{1}{4k^2-1}\exp(j2\pi kt)$

(c) $x(t) = \frac{2}{\pi} - \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{4k^2-1}\cos(2\pi kt)$

S.2 Find the complex exponential Fourier series of a periodic signal $x(t)$ defined by

$$x(t) = t^2; \quad -\pi < t < \pi \quad \text{and} \quad x(t+2\pi) = x(t).$$

Answer: $x(t) = \frac{\pi^2}{3} + 4\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}\cos(kt)$

S.3 The harmonic form Fourier series of a *real* periodic signal $x(t)$ with fundamental period T_0 is given by

$$x(t) = C_0 + \sum_{k=1}^{\infty} C_k \cos\left(2\pi\frac{k}{T_0}t - \theta_k\right)$$

where C_0 is known as the dc component, and the term $C_k\cos\left(2\pi\frac{k}{T_0}t - \theta_k\right)$ is referred to as the *kth-harmonic component* of $x(t)$. Express C_0, C_k and θ_k in terms of the complex exponential Fourier series coefficients X_k of $x(t)$.

Answer: $C_0 = X_0, \quad C_k = 2|X_k|, \quad \theta_k = -\tan^{-1}\left(\frac{\mathrm{Im}[X_k]}{\mathrm{Re}[X_k]}\right)$

Below is a list of solved problems selected from ***Chapter 5*** *of* ***Hwei Hsu (PhD), 'The Schaum's series on Signals & Systems,' 2nd Edition****.*

Selected solved-problems: ***5.4-to-5.13***

These solved problems should be treated as supplementary module material catered for students who find the need for more examples or practice-problems.
