National University of Singapore Department of Electrical & Computer Engineering

EE2023 Signals and Systems Assignment 2

- 1. Show that the signal $x(t) = 3\cos(15t + \pi/6) + \sin(20t)$ is periodic. Find its fundamental frequency.

 Ans: 5 rad/s
- 2. Find and sketch the Fourier transform of the following time domain signals:

(i)
$$x(t) = Ae^{-\beta t}\cos(\omega_0 t)u(t) \operatorname{Re}(\beta) > 0$$

Ans:
$$X(f) = \frac{A}{2} \left[\frac{1}{\beta + i2\pi(f + f_0)} + \frac{1}{\beta + i2\pi(f - f_0)} \right]$$

(ii)
$$x(t) = A\sin(\omega_1 t) + B\cos(\omega_2 t)$$

$$Ans: X(f) = \frac{A}{2j} \left[\delta \left(f - \frac{\omega_1}{2\pi} \right) - \delta \left(f + \frac{\omega_1}{2\pi} \right) \right] + \frac{B}{2} \left[\delta \left(f - \frac{\omega_2}{2\pi} \right) + \delta \left(f + \frac{\omega_2}{2\pi} \right) \right]$$

(iii)
$$x(t) = 6 \operatorname{sinc} (0.5t)$$

$$Ans: X(f) = 12rect(2f)$$

(iv)
$$x(t) = 6 \operatorname{rect}\left(\frac{t-4}{3}\right)$$

Ans:
$$X(f) = 18sinc(3f)e^{-j8\pi f}$$

3. Given $e^{-|t|} \leftrightarrow \frac{1}{\pi(f^2+1)}$, find the Fourier transform of the following:

(i)
$$x(t) = \frac{d}{dt}e^{-|t|}$$

Ans:
$$X(f) = \frac{j2f}{(f^2+1)}$$

(ii)
$$x(t) = \frac{1}{2\pi(t^2+1)}$$

Ans:
$$X(f) = \frac{1}{2\pi(t^2+1)}$$

(iii)
$$x(t) = \frac{4\cos(2t)}{t^2 + 1}$$

Ans:
$$X(f) = 2\pi [e^{-|f-1/\pi|} + e^{-|f+1/\pi|}]$$

4. Show that systems which transmit pulses at very high frequencies have larger bandwidth.