Q.1 Consider the signals, $x(t) = 4\text{tri}\left(\frac{t}{2}\right)$.

(a) Sketch the signal, $y(t) = \sum_{k=-\infty}^{\infty} x(t-4k)$. Label your sketch appropriately.

(b) Find $X(f)$ and $Y(f)$.

(c) Sketch $Y(f)$. Label your sketch appropriately.

Q.2 (a) Find the Fourier transform of $x(t) = 4\cos(2\pi f_c t)u(t)$, where $u(t)$ denotes the unit step function.

(b) Suppose that the Fourier transform of $x_1(t)$ is $X_1(f) = \text{rect}(\pi f)$.

i. Find the energy of $y(t)$ if $y(t) = \frac{dx_1(t)}{dt}$.

ii. What are the bandwidths of $x_1(t)$ and $y(t)$?

Q.3 Consider the trigonometric form of the Fourier series of a periodic signal, $x(t) = \sum_{k=0}^{\infty} \frac{1}{2^k}\cos(8\pi kt)$.

(a) What is the fundamental frequency of $x(t)$?

(b) Is $x(t)$ an even or odd function? Justify your answer.

(c) What is the average value of $x(t)$?

(d) Compute the average power of $x(t)$.

(e) Sketch the discrete frequency spectrum of $x(t)$.

Q.4 Consider the signal $x(t) = 2\sin(4\pi t)\sin(10\pi t) + 4\cos^2(8\pi t)$.

(a) What are the frequency components of $x(t)$?

(b) Determine the minimum sampling frequency of $x(t)$.

(c) $x(t)$ is sampled at 20 Hz. Derive and sketch the spectrum of the sampled signal. Label your sketch.

Answers to Quantitative Questions

Q.1 (b) $X(f) = 8\text{sinc}^2(2f)$

$$Y(f) = \sum_{k=-\infty}^{\infty} 2\text{sinc}^2\left(\frac{k}{2}\right)\delta\left(f - \frac{k}{4}\right)$$

Q.2 (a) $X(f) = \dfrac{2f}{j\pi\left(f^2 - f_c^2\right)} + \delta(f + f_c) + \delta(f - f_c)$.

(b)(i) Energy of $y(t)$: $\dfrac{1}{3\pi}$.

(b)(ii) Bandwidth of $x_1(t)$ = Bandwidth of $y(t) = \dfrac{1}{2\pi}$ Hz

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Q.3 (a) Fundamental frequency of $x(t)$: 4 Hz

(c) The average value (or DC value) of $x(t)$: 1

(d) Average power of $x(t)$: $\dfrac{8}{6}$

Q.4 (a) Frequency components of $x(t)$: 0 Hz, 3 Hz, 7 Hz, and 8 Hz

(b) Minimum sampling frequency of $x(t)$: 16 Hz
