EE2023 Signals & Systems Quiz
Semester 1 AY2014/15

Date : 2 October 2014 **Time Allowed : 1.5 hours**

Instructions :

1. Answer all 4 questions. Each question carries 10 marks.
2. This is a closed book quiz.
3. Tables of Fourier transforms and trigonometric identities are given on Pages 11 and 12.
4. No programmable or graphic calculator is allowed.
5. Write your answers in the spaces indicated in this question paper. Attachment is not allowed.
6. Write your name, matric number and lecture group in the spaces indicated below.

Name : ________________________________

Matric # : ______________________________

Class Group # : _________________________

Question #	Marks
1	
2	
3	
4	
Total Marks	

For your information :

Group 1 : A/Prof Loh Ai Poh
Group 2 : A/Prof Ng Chun Sum
Group 3 : A/Prof Tan Woei Wan
Group 4 : Prof Lawrence Wong

Q.1 Consider the signal processing system shown in Figure Q1-1.

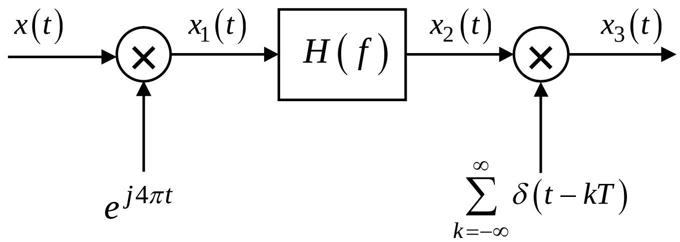

Figure Q1-1 : Signal Processing System

Assume that $x(t)$ is a real signal with its spectrum given in Figure Q1-2a. Figure Q1-2b shows the low pass filter characteristics of $H(f)$.

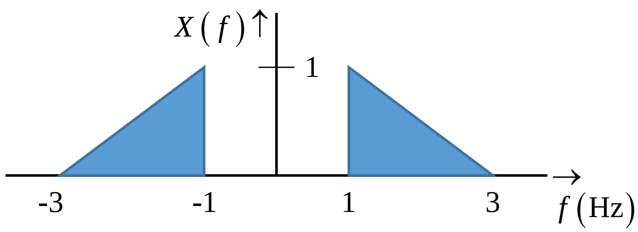

Figure Q1-2a : Spectrum of $x(t)$

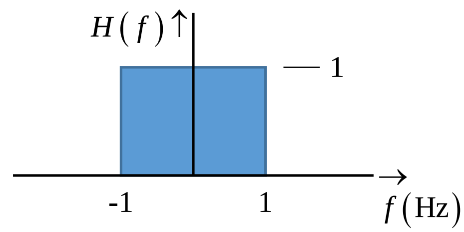

Figure Q1-2b : Filter Characteristics

(a) Sketch and label clearly the spectra of $x_1(t)$, $x_2(t)$ and $x_3(t)$. You may choose any T you wish as long as your spectrum of $x_3(t)$ is consistent.

(8 marks)

(b) Determine the maximum T such that $x(t)$ is recoverable from $x_3(t)$.

(2 marks)

Q.1 ANSWER

Q.1 ANSWER ~ continued

Q.2 A signal $x(t)$ is modeled by $x(t)=\sum_n \operatorname{sinc}(t-3n)$.

(a) Express $x(t)$ in the form of a convolution and then derive its Fourier transform $X(f)$.

(4 marks)

(b) Find the inverse Fourier transform of $X(f)$.

[*Your answer for this part* <u>*should not*</u> *contain the summation* $\sum$ *operator*.]

(3 marks)

(c) Using the result of Part (b), or otherwise, determine the Fourier series coefficients, X_k, of $x(t)$.

(3 marks)

Q.2 ANSWER

Q.2 ANSWER ~ continued

Q.3 Consider a periodic signal, $x(t)$, whose Fourier Series expansion may be expressed as

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jnt}$$

(a) What is the period of the signal $x(t)$?

(2 marks)

(b) Write down an equation for evaluating the Fourier Series coefficients, c_n.

(2 marks)

(c) Consider the generating function $x_g(t) = \begin{cases} t & -\pi < t < \pi \\ 0 & otherwise \end{cases}$ and the periodic signal

$$y(t) = \sum_{k=-\infty}^{\infty} x_g(t - 2k\pi).$$

i. Compute the Fourier Series coefficient of $y(t)$ for the frequency index $n = 0$ i.e. c_0, the DC value of $y(t)$.

(3 marks)

ii. The Fourier Series expansion for $y(t)$ is

$$y(t) = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin(nt)}{n}.$$

Using the Fourier Series expansion for $y(t)$, show that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$.

(3 marks)

Q.3 ANSWER

Q.3 ANSWER ~ continued

Q.4. (a) Determine the Fourier transform of the signal $x(t)$ that comprises two rectified cycles of a sinusoid as shown in Figure Q.4(a).

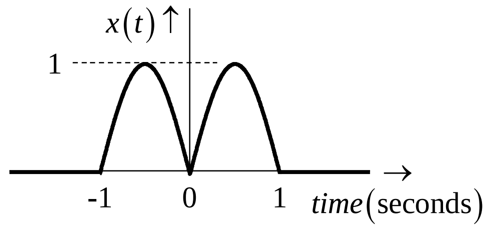

Figure Q.4(a)

(4 marks)

(b) Using the Dirac-δ replication property, derive an expression that shows the relationship between $x(t)$ and the periodic signal $x_p(t)$ shown in Figure Q.4(b).

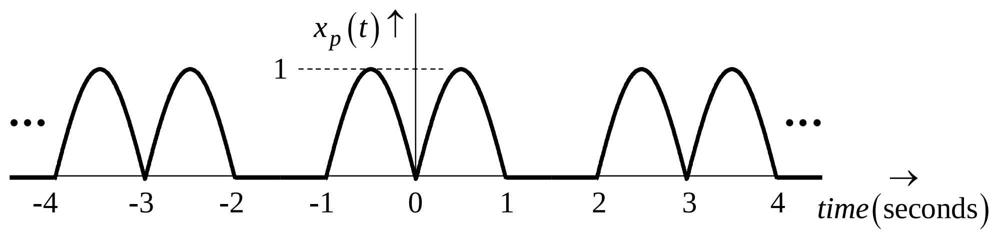

Figure Q.4(b)

(2 marks)

(c) Determine the Fourier transform and the Fourier Series coefficients of the periodic signal $x_p(t)$.

(4 marks)

Q.4 ANSWER

Q.4 ANSWER ~ continued

This page is intentionally left blank to facilitate detachment of the formula sheet for easy reference. Anything written on this page will not be graded.

Fourier Series: $$\begin{cases} X_k = \dfrac{1}{T}\displaystyle\int_{\tilde{t}}^{\tilde{t}+T} x(t)\exp(-j2\pi k\,t/T)\,dt \\ x(t) = \displaystyle\sum_{k=-\infty}^{\infty} X_k \exp(j2\pi k\,t/T) \end{cases}$$

Fourier Transform: $$\begin{cases} X(f) = \displaystyle\int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)\,dt \\ x(t) = \displaystyle\int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)\,df \end{cases}$$

FOURIER TRANSFORMS OF BASIC FUNCTIONS		
	$x(t)$	$X(f)$
Constant	K	$K\delta(f)$
Unit Impulse	$\delta(t)$	**1**
Unit Step	$u(t)$	$\frac{1}{2}\left[\delta(f)+\frac{1}{j\pi f}\right]$
Sign (or Signum)	$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
Rectangle	$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}(fT)$
Triangle	$\mathrm{tri}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}^2(fT)$
Sine Cardinal	$\mathrm{sinc}\left(\frac{t}{T}\right)$	$T\,\mathrm{rect}(fT)$
Complex Exponential	$\exp(j2\pi f_o t)$	$\delta(f-f_o)$
Cosine	$\cos(2\pi f_o t)$	$\frac{1}{2}\left[\delta(f-f_o)+\delta(f+f_o)\right]$
Sine	$\sin(2\pi f_o t)$	$-\frac{j}{2}\left[\delta(f-f_o)-\delta(f+f_o)\right]$
Gaussian	$\exp\left(-\frac{t^2}{\alpha^2}\right)$	$\alpha\pi^{0.5}\exp\left(-\alpha^2\pi^2 f^2\right)$
Comb	$\sum_{m=-\infty}^{\infty}\delta(t-mT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$

FOURIER TRANSFORM PROPERTIES		
	Time-domain	**Frequency-domain**
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(f)+\beta X_2(f)$
Time scaling	$x(\beta t)$	$\frac{1}{\lvert\beta\rvert}X\left(\frac{f}{\beta}\right)$
Duality	$X(t)$	$x(-f)$
Time shifting	$x(t-t_o)$	$X(f)\exp(-j2\pi f t_o)$
Frequency shifting (Modulation)	$x(t)\exp(j2\pi f_o t)$	$X(f-f_o)$
Differentiation in the time-domain	$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n X(f)$
Multiplication in the time-domain	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty}X_1(\zeta)X_2(f-\zeta)\,d\zeta$ or $X_1(f)\ast X_2(f)$
Convolution in the time-domain	$\int_{-\infty}^{\infty}x_1(\zeta)x_2(t-\zeta)\,d\zeta$ or $x_1(t)\ast x_2(t)$	$X_1(f)X_2(f)$
Integration in the time-domain	$\int_{-\infty}^{t}x(\tau)\,d\tau$	$\frac{1}{j2\pi f}X(f)+\frac{1}{2}X(0)\delta(f)$
		$\frac{1}{j2\pi f}X(f)$ **if** $X(0)=0$

TRIGONOMETRIC IDENTITIES	
$\exp(\pm j\theta) = \cos(\theta) \pm j\sin(\theta)$	$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$
$\cos(\theta) = \frac{1}{2}\left[\exp(j\theta) + \exp(-j\theta)\right]$	$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
$\sin(\theta) = \frac{1}{j2}\left[\exp(j\theta) - \exp(-j\theta)\right]$	$\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$
$\sin^2(\theta) + \cos^2(\theta) = 1$	
$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$	$\sin(\alpha)\sin(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$
$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$	$\cos(\alpha)\cos(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$
$\sin^2(\theta) = \frac{1}{2}\left[1 - \cos(2\theta)\right]$	$\sin(\alpha)\cos(\beta) = \frac{1}{2}\left[\sin(\alpha - \beta) + \sin(\alpha + \beta)\right]$
$\cos^2(\theta) = \frac{1}{2}\left[1 + \cos(2\theta)\right]$	$\mathbf{C}\cos(\theta) - \mathbf{S}\sin(\theta) = \sqrt{\mathbf{C}^2 + \mathbf{S}^2}\cos\left[\theta + \tan^{-1}\left(\frac{\mathbf{S}}{\mathbf{C}}\right)\right]$