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Assignment 1

$a=9,\ b=8,\ c=0,\ d=6$

1. $z=-c+jd = j6$

$|z|=6,\quad \angle z=\frac{\pi}{2}$

polar coordinates of $z=6e^{j\frac{\pi}{2}}$

2. $z=(a+10)e^{-j0.1(b+1)\pi} = 19e^{-j0.9\pi}$

cartesian coordinates of $z=19\cos(-0.9\pi)+j19\sin(-0.9\pi)$

$=-18.1-j5.87$ (3 s.f.)

3. Let $z_1=a+jb$, $z_2=c+jd$

$=9+j8$, $=j6$

$$|z|=\frac{|z_1|}{|z_2|}=\frac{\sqrt{9^2+8^2}}{6}$$

$=6.02$ (3s.f.)

$\angle z=\angle z_1-\angle z_2=\tan^{-1}\left(\frac{8}{9}\right)-\frac{\pi}{2}$

$=-0.844$ rads (3s.f.)

$z=6.02e^{-j0.844}$

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4. $z = 10(a+1) + j5(c+1) = 100 + j5$

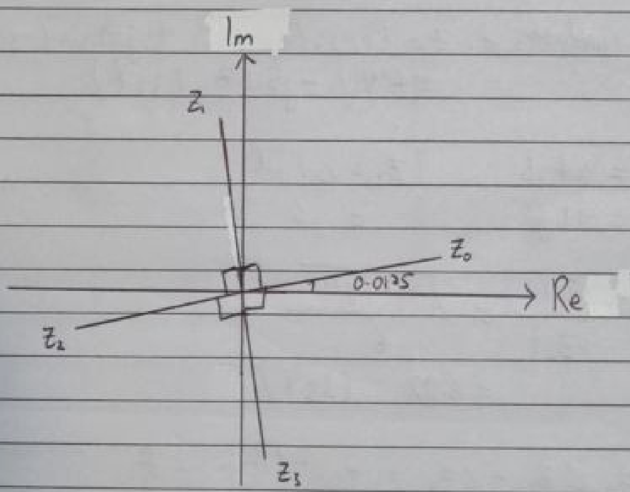
$|z| = \sqrt{100^2 + 5^2}$ , $\angle z = \tan^{-1}\left(\frac{5}{100}\right) + 2k\pi$ , $k \in \mathbb{Z}$

$\approx 100.125$ $\approx 0.049958 + 2k\pi$

$|z^{\frac{1}{4}}| = (100.125)^{\frac{1}{4}}$ , $\angle z^{\frac{1}{4}} = \frac{1}{4}(0.049958 + 2k\pi)$ , $k = 0, 1, 2, 3$

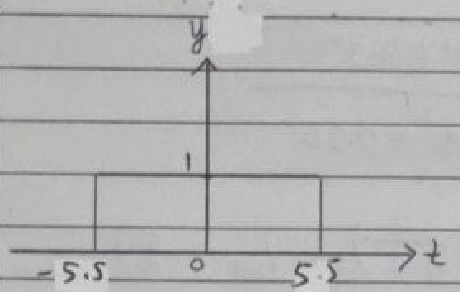
$= 3.16$ (3 s.f.) $= 0.0125 + \frac{k\pi}{2}$

$\therefore z^{\frac{1}{4}}_k = 3.16 e^{j(0.0125 + \frac{k\pi}{2})}$ , $k = 0, 1, 2, 3$

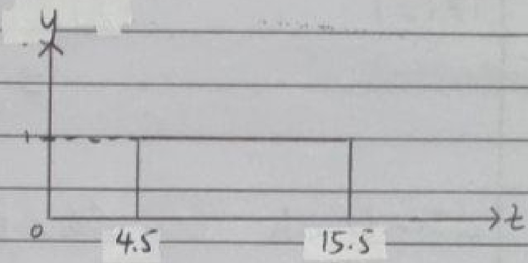


5. $y(t) = \text{rect}\left(\frac{t}{\alpha+5}\right) = \text{rect}\left(\frac{t}{11}\right)$

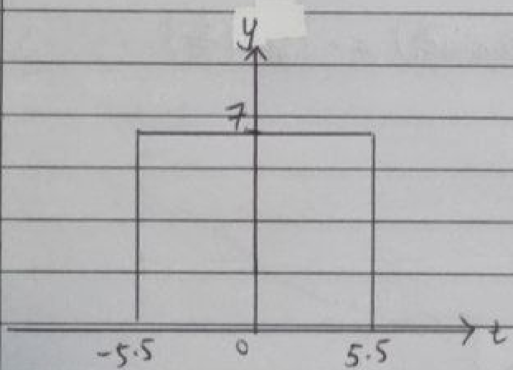
a.



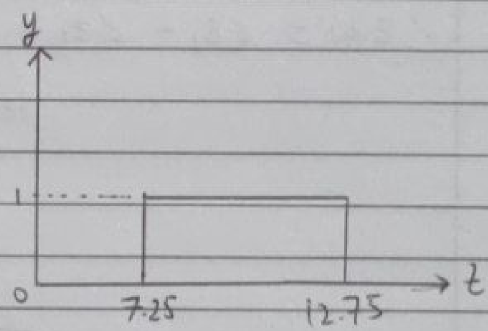

c. $y(t-(a+1)) = y(t-10)$




b. $(\alpha+1)y(t) = 7y(t)$




d. $y(2t-(a+1)) = y(2t-10)$



6. Let $z_1 = 1$, $\quad z_2 = j0.1(c+1)t + 0.5$

$$= j0.1t + 0.5$$

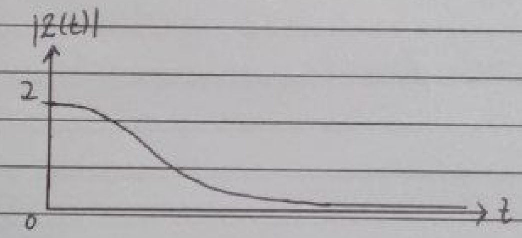
$|z_1| = 1 \qquad |z_2| = \sqrt{(0.1t)^2 + 0.5^2}$

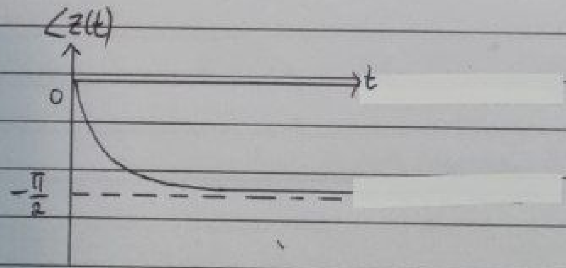
$$= \sqrt{0.01t^2 + 0.25}$$

$\angle z_1 = 0$ rads $\qquad \angle z_2 = \tan^{-1}\left(\frac{0.1t}{0.5}\right) = \tan^{-1}\left(\frac{t}{5}\right)$

$$|z(t)| = \frac{|z_1|}{|z_2|} = \frac{1}{\sqrt{0.01t^2 + 0.25}}$$

$$\angle z(t) = \angle z_1 - \angle z_2 = -\tan^{-1}\left(\frac{t}{5}\right)$$




$\angle z(t)$ tends to $-\frac{\pi}{2}$ as $t \to \infty$.

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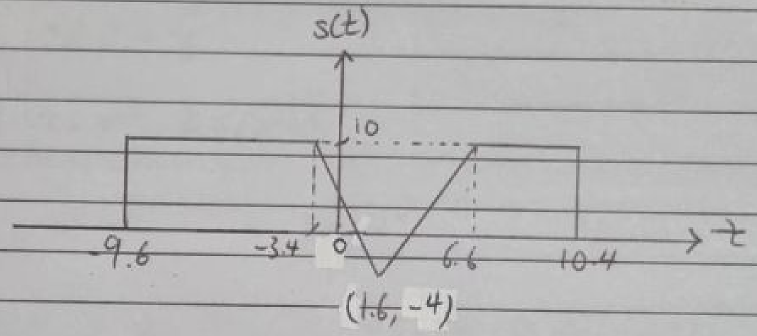
7. $s(t) = 10\,\text{rect}\left(\frac{t-b}{20}\right) - (a+5)\,\text{tri}\left(\frac{t-b}{5}\right)$

$= 10\,\text{rect}\left(\frac{t-8}{20}\right) - 14\,\text{tri}\left(\frac{t-8}{5}\right)$



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8. $V_{in}(t) = 10(d+1) = 70V$, $C = (c+5) = 5\mu F$, $L = (b+2) = 10H$

a. $$Z_{C\parallel L} = \left(\frac{1}{j\omega L} + \frac{1}{\frac{1}{j\omega C}}\right)^{-1}$$

$$= \left(\frac{1 + j\omega L \cdot j\omega C}{j\omega L}\right)^{-1}$$

$$= \frac{j\omega L}{1-\omega^2 LC}$$

$$Z = Z_{C\parallel L} + R = \frac{j10\omega}{1-\omega^2(10\times5\times10^{-6})} + 100$$

$$= 100 + \frac{j10\omega}{1-5\omega^2\times10^{-5}}\ \Omega$$

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b. $i_0 = i_1 + i_2$

$$\frac{V_{in} - V_c}{R} = C\frac{dV_c}{dt} + i_2 \quad \text{——— (1)}$$

$$V_c = L\frac{di_2}{dt}$$

$$i_2 = \int \frac{V_c}{L}\,dt \quad \text{——— (2)}$$

sub (2) into (1)

$$\frac{V_{in} - V_c}{R} = C\frac{dV_c}{dt} + \int \frac{V_c}{L}\,dt$$

differentiate w.r.t $t$

$$C\frac{d^2V_c}{dt^2} + \frac{1}{R}\frac{dV_c}{dt} - \frac{1}{R}\frac{dV_{in}}{dt} + \frac{V_c}{L} = 0$$

$$(5\times10^{-6})\frac{d^2V_c}{dt^2} + 0.01\frac{dV_c}{dt} - 0.01\frac{dV_{in}}{dt} + 0.1V_c = 0$$

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8c. From b,

$$C\frac{d^2V_c}{dt^2} + \frac{1}{R}\frac{dV_c}{dt} - \frac{1}{R}\frac{dV_{in}}{dt} + \frac{V_c}{L} = 0 \quad \text{— (1)}$$

$$I_1 = C\frac{dV_c}{dt}$$

$$\frac{dV_c}{dt} = \frac{I_1}{C} \quad \text{— (2)}$$

$$V_c = \int \frac{I_1}{C}\,dt \quad \text{— (3)}$$

$$\frac{d^2V_c}{dt} = \frac{1}{C}\frac{dI_1}{dt} \quad \text{— (4)}$$

sub (2), (3) & (4) into (1)

$$C\left(\frac{1}{C}\frac{dI_1}{dt}\right) + \frac{1}{R}\left(\frac{I_1}{C}\right) - \frac{1}{R}\frac{dV_{in}}{dt} + \frac{1}{L}\int\frac{I_1}{C}\,dt = 0$$

differentiate w.r.t t

$$\Rightarrow \frac{d^2I_1}{dt^2} + \frac{1}{RC}\frac{dI_1}{dt} - \frac{1}{R}\frac{d^2V_{in}}{dt} + \frac{I_1}{LC} = 0$$

$$\frac{d^2I_1}{dt^2} + 2000\frac{dI_1}{dt} - 0.01\frac{d^2V_{in}}{dt} + 20\,000\,I_1 = 0$$

~~d. Since steady state, $\frac{dV_c}{dt} \to 0$~~

~~From b,~~ $\cancel{C\frac{d^2V_c}{dt^2} + \frac{1}{R}\frac{dV_c}{dt} - \frac{1}{R}\frac{dV_{in}}{dt} + \frac{V_c}{L} = 0 \quad \text{— (1)}}$

~~when~~ $\cancel{\frac{dV_c}{dt} = 0,}$

~~equation (1)~~ $\cancel{\Rightarrow V_c = \frac{1}{R}\frac{dV_{in}}{dt}}$

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d. As $t \to \infty$, $\frac{dV_c}{dt} \to 0$

From b, $C\frac{d^2 v_c}{dt^2} + \frac{1}{R}\frac{dv_c}{dt} - \frac{1}{R}\frac{dV_{in}}{dt} + \frac{V_c}{L}$ — (1)

when $\frac{dv_c}{dt} = 0$,

eqn (1) $\Rightarrow \frac{V_c}{L} = \frac{1}{R}\frac{dV_{in}}{dt}$

$V_c = \frac{L}{R}\frac{dV_{in}}{dt}$

$V_{in} = 70V$

$\frac{dV_{in}}{dt} = 0$

$\therefore V_c \to \frac{L}{R}\frac{dV_{in}}{dt}$

$\Rightarrow V_c \to 0$ //