EE2023 TUTORIAL 3 (PROBLEMS)

Q.1 A half-cosine pulse $x(t)$ and a sine pulse $y(t)$ are shown in Fig.Q.1.

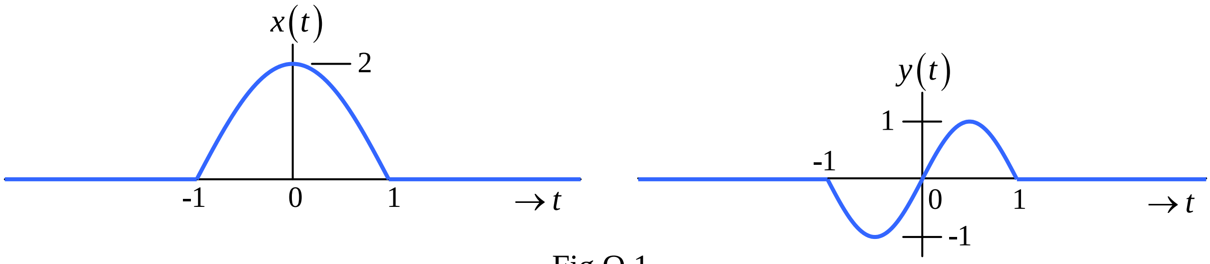

Fig.Q.1

(a) Derive the spectrum of $x(t)$ using the forward Fourier transform equation and show how the derivation can be simplified by applying relevant Fourier transform properties.

(b) Using the results of Part-(a), determine the spectrum of $y(t)$.

Q.2 (a) Show that Fig.Q.2(a)(I) and Fig.Q.2(a)(II) are plots of the same function $u(t-\gamma)$, where $u(\cdot)$ denotes the unit step function. Hence, express $\int_{-\infty}^{t} x(\gamma)d\gamma$ as a convolution integral.

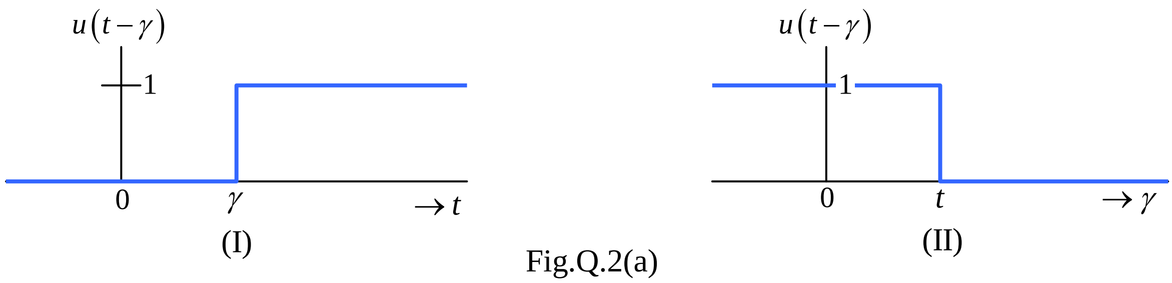

Fig.Q.2(a)

(b) Evaluate $\left[\cos(t)u(t)\right] \ast u(t)$ where $\ast$ denotes convolution.

Q.3 Fig.Q.3 shows the plot of a triangular pulse $x(t)$.

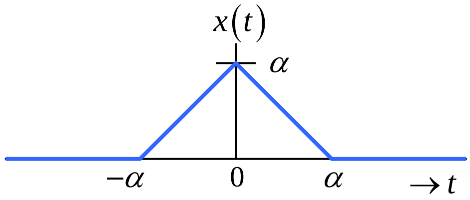

Fig.Q.3

Determine the magnitude and phase spectra of $x(t)$. Hence, or otherwise, find the energy spectral density and total energy of $\dfrac{dx(t)}{dt}$.

Q.4 The spectrum of a lowpass energy signal $x(t)$ is given by $X(f) = \exp(-\alpha|f|)$ where α is a positive constant.

(a) The 99% energy containment bandwidth of a signal is defined as the smallest bandwidth that contains at least 99% of the total signal energy. Find the 99% energy containment bandwidth of $x(t)$?

(b) Find the 3dB bandwidth of $x(t)$. How many percent of the total energy of $x(t)$ does its 3dB bandwidth contain?

Q.5 A periodic pulse train, $x(t)$ is shown in Fig.Q.5.

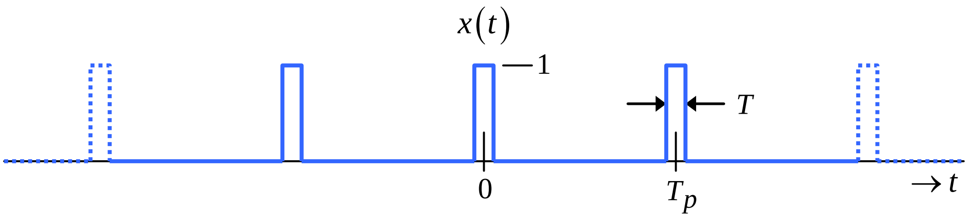

Fig.Q.5

(a) Derive the power spectral density, $P_x(f)$, of $x(t)$.

(b) What is the average power of $x(t)$?

(c) The 99% power containment bandwidth of a power signal is defined as the smallest bandwidth that contains at least 99% of the average signal power. Provide a method for computing the 99% power containment bandwidth of $x(t)$.

Supplementary Problems

These problems will not be discussed in class.

S.1 Find the Fourier transform of each of the following signals:

(a) $x(t)=\cos(2\pi f_c t)u(t)$ (b) $x(t)=\sin(2\pi f_c t)u(t)$

(c) $x(t)=\exp(-\alpha t)\cos(\omega_c t)u(t);\ \ \alpha>0$ (d) $x(t)=\exp(-\alpha t)\sin(\omega_c t)u(t);\ \ \alpha>0$

Answer: (a) $X(f)=\frac{1}{4}\left[\delta(f-f_c)+\delta(f+f_c)\right]+\dfrac{jf}{2\pi\left(f_c^2-f^2\right)}$

(b) $X(f)=\frac{j}{4}\left[\delta(f+f_c)-\delta(f-f_c)\right]+\dfrac{f_c}{2\pi\left(f_c^2-f^2\right)}$

(c) $X(\omega)=\dfrac{\alpha+j\omega}{(\alpha+j\omega)^2+\omega_c^2}$

(d) $X(\omega)=\dfrac{\omega_c}{(\alpha+j\omega)^2+\omega_c^2}$

S.2 Given: $\Im\{\exp(-\alpha t)u(t)\}=\dfrac{1}{\alpha+j2\pi f}$. Find the inverse Fourier transform of $\dfrac{1}{(\alpha+j2\pi f)^n}$.

Answer: $\dfrac{t^{n-1}}{(n-1)!}\exp(-\alpha t)u(t)$

S.3 Given: $\Im\{\exp(-\alpha t)u(t)\}=\dfrac{1}{j\omega+\alpha}$. Find the inverse Fourier transform of $\dfrac{1}{2-\omega^2+j3\omega}$.

Answer: $\left[\exp(-t)-\exp(-2t)\right]u(t)$

S.4 Given: $\Im\{x(t)\}=\mathrm{rect}(\pi f)$. Find the value of $\int_{-\infty}^{\infty}|y(t)|^2\,dt$ if $y(t)=\dfrac{dx(t)}{dt}$.

Answer: $\dfrac{1}{3\pi}$

S.5 Given: $\Im\left\{\dfrac{\pi}{\alpha}\exp(-2\pi\alpha|t|)\right\}=\dfrac{1}{\alpha^2+f^2}$. Determine the 99% energy containment bandwidth for the signal $x(t)=\dfrac{1}{\alpha^2+t^2}$.

Answer: $\dfrac{0.366}{\alpha}$

S.6 Let $\omega=2\pi f$. Using the fact that the Fourier transform is a <u>one-to-one</u> linear transformation, show that $\delta(f)=2\pi\delta(\omega)$.

Hint : Show that $\Im^{-1}\{\delta(f)\}=\Im^{-1}\{2\pi\delta(\omega)\}=1$

*Below is a list of solved problems selected from **Chapter 5** of **Hwei Hsu (PhD), 'The Schaum's series on Signals & Systems,' 2nd Edition**.*

Selected solved-problems: 5.19-to-5.27, 5.32, 5.34, 5.40, 5.42, 5.42, 5.57

These solved problems should be treated as supplementary module material catered for students who find the need for more examples or practice-problems.