4.1 Three – Phase Power

Friday, February 04, 2022 1:26 PM

4.0 In-Class Lecture - 3-Phase Power - EE2029

NUS National University of Singapore

EE2029: Introduction to Electrical Energy System

3-Phase Power

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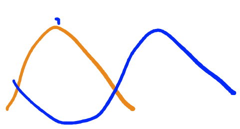
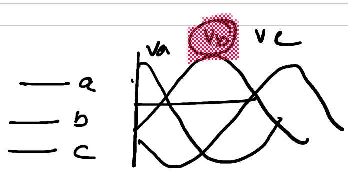
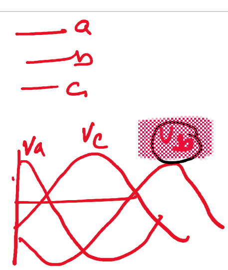
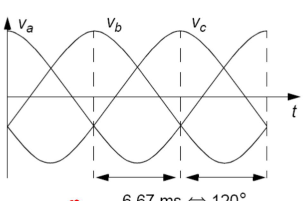
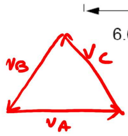






Three-Phase Voltage Sources





$V_A + V_B + V_C = 0$

$$v_a = \sqrt{2}|V|\cos(\omega t)$$

$$v_b = \sqrt{2}|V|\cos\left(\omega t - \frac{2\pi}{3}\right)$$

- Positive Sequence

Positive sequence, abc

$V_{cn} = 1\angle +120°$

$V_{an} = 1\angle 0°$

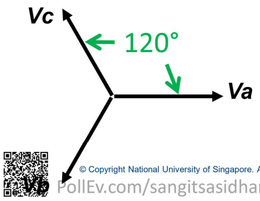
$V_{bn} = 1\angle -120°$

- Negative Sequence

$V_{\cdot} = 1\angle +120°$

V_A

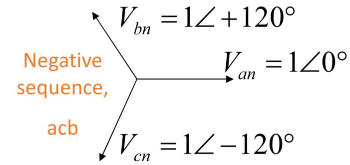
$V_A + V_B + V_C = 0$



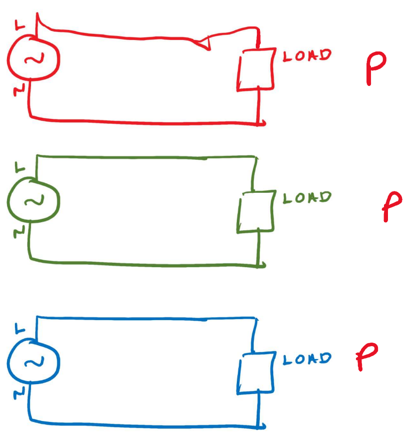
$$v_b = \boxed{\sqrt{2}|V|}\cos\left(\omega t - \frac{2\pi}{3}\right)$$

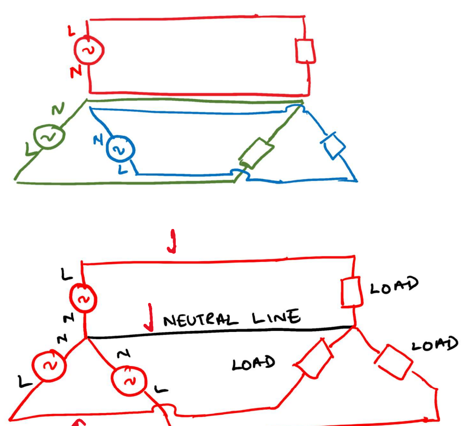
$$v_c = \boxed{\sqrt{2}|V|}\cos\left(\omega t - \frac{4\pi}{3}\right)$$

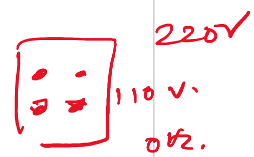
- Negative Sequence



Three Single Phase vs Three Phase Circuits







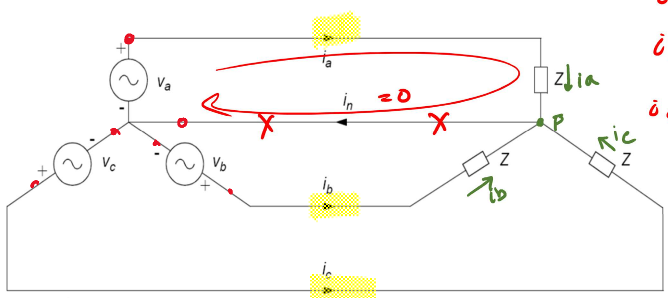
REDUCED CONDUCTORS FROM 6 TO 4 ⇒ 33.33% SAVINGS

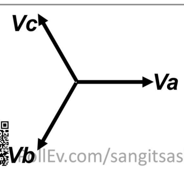
6 → 3 P.
Conn.

3P → 4

Balanced Three-Phase Circuit

- Three-phase circuit is said to be balanced when the impedances in the 3 phases are identical





$V_a = 1\angle 0^\circ$

$V_b = 1\angle -120^\circ$

$V_c = 1\angle +120^\circ$

$i_a = \frac{V_a}{Z}$

$i_b = \frac{V_b}{Z}$

$i_c = \frac{V_c}{Z}$

KCL @ node P

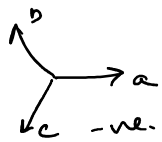
$i_a + i_b + i_c = i_n$

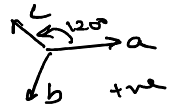
$i_a + i_b + i_c$

$= \frac{V_a}{Z} + \frac{V_b}{Z} + \frac{V_c}{Z} = \frac{1}{Z}\left(\underbrace{V_a + V_b + V_c}_{\to 0}\right)$

$= \frac{0}{Z} = 0 = i_n \quad \Big| \Rightarrow i_n = 0$

Example:

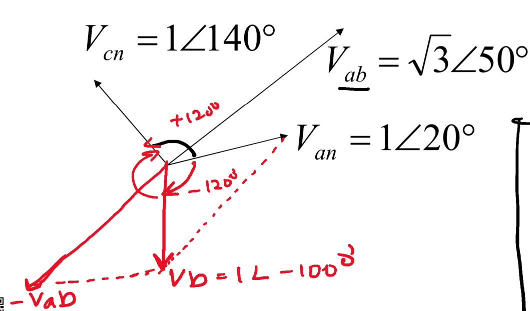




- What is the phase sequence?
- Are these balanced three-phase voltage sources?

$V_{cn} \rightarrow V_c$

$V_{ab} \rightarrow V_{an} - V_{bn}$. +ve sequence,



$V_{cn} \rightarrow V_{an}$.

$V_{cn} \rightarrow V_{an} \times 1\angle 120° = 1\angle 20° \times 1\angle 120°$

$= 1\angle 140°$

$V_{ab} = V_{an} - V_{bn}$

$V_{bn} = V_{an} - V_{ab} = V_{an} + (-V_{ab})$

$= 1\angle 20° - \sqrt{3}\angle 50°$

$= 1\angle -100°$.

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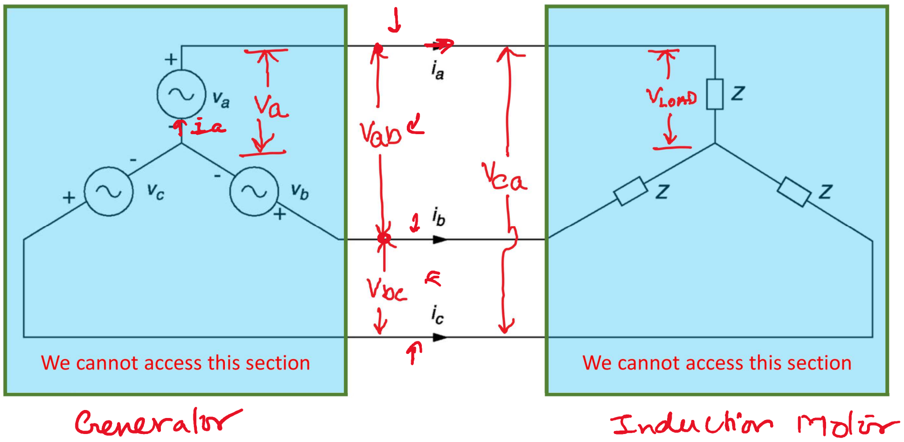
How do we Measure Voltage/Current



Measureable-Line values

Not measurable → Phase values.

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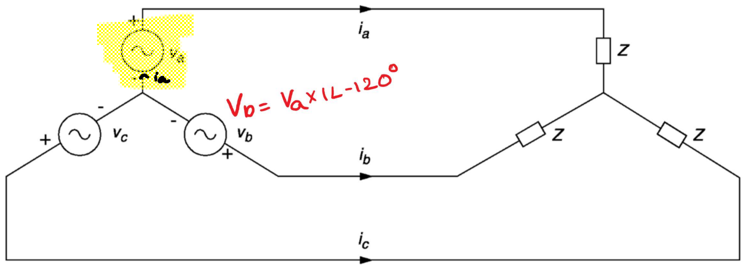
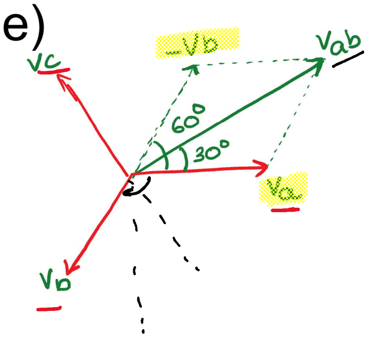
Line-To-Line Voltage (Line Voltage)





$$V_{ab} = V_a - V_b = V_a + (-V_b)$$
$$= V_a + V_a \angle 60^\circ$$
$$= V_a \angle 0^\circ + V_a \angle 60^\circ$$
$$= \sqrt{3}\, V_a \angle 30^\circ$$

$$V_{bc} = V_b + (-V_c)$$
$$= \sqrt{3}\, V_b \angle 30^\circ$$

$$V_{ca} = V_c + (-V_a)$$
$$= \sqrt{3}\, V_c \angle 30^\circ$$

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Line-To-Neutral Voltage (Phase Voltage)

$V_{ab} = \sqrt{3}\, V_a \angle 30^\circ$

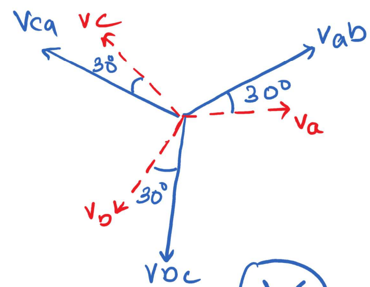
$V_{bc} = \sqrt{3}\, V_b \angle 30^\circ$

$V_{ca} = \sqrt{3}\, V_c \angle 30^\circ$

$|V_a| = \frac{|V_{ab}|}{\sqrt{3}}$

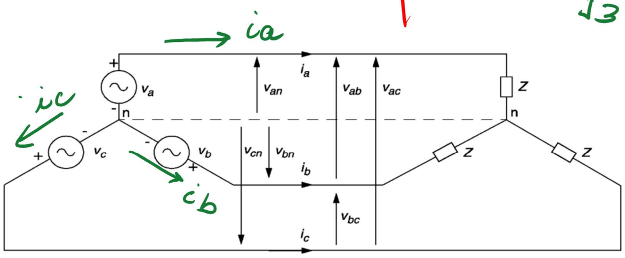
$|V_b| = \frac{|V_{bc}|}{\sqrt{3}}$

$|V_c| = \frac{|V_{ca}|}{\sqrt{3}}$

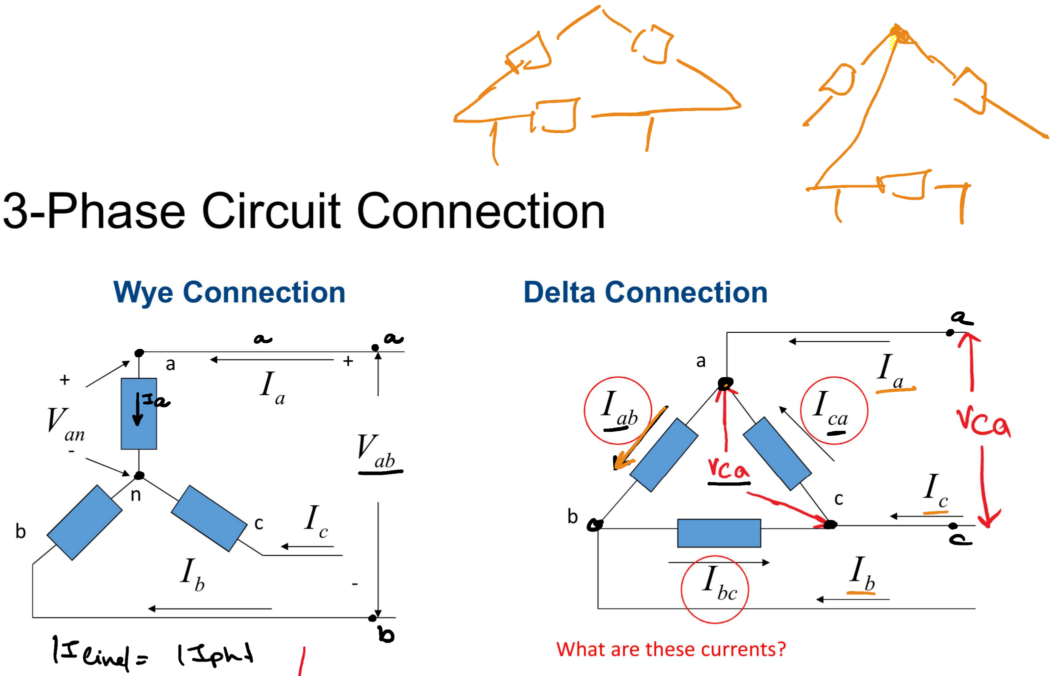


Y

$\boxed{|V_{phase}| = \frac{|V_{line}|}{\sqrt{3}}}$



3-Phase Circuit Connection



What are these currents?

These two types of connections apply to both three-phase voltage sources and three-phase loads

$|V_{line}| = \sqrt{3}|V_{ph}|$

$|V_{line}| = |V_{ph}|$

$|I_{line}| = \sqrt{3}|I_{ph}|$

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$V_{LINE} = V_{PHASE}$

Delta-Connected Load

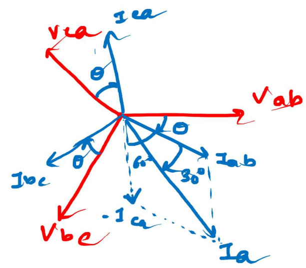
KCL @ JUNCTION A

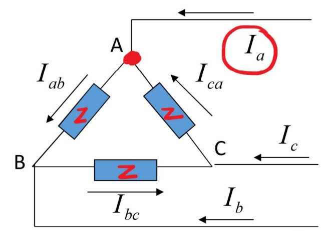
$I_{ab} = I_{ca} + I_a$

$\Rightarrow I_a = I_{ab} - I_{ca}$

$I_a = I_{ab} + (-I_{ca})$

$= \sqrt{3}\, I_{ab} \angle -30^\circ$





Z - LAGGING "θ"

$I_a = \sqrt{3}\, I_{ab} \angle -30^\circ$

$I_b = \sqrt{3}\, I_{bc} \angle -30^\circ$

$I_c = \sqrt{3}\, I_{ca} \angle -30^\circ$

$\Rightarrow$ FOR Δ (DELTA)

$|I_{LINE}| = \sqrt{3}\, |I_{PHASE}|$

OR $|I_{PHASE}| = \left|\dfrac{I_{LINE}}{\sqrt{3}}\right|$

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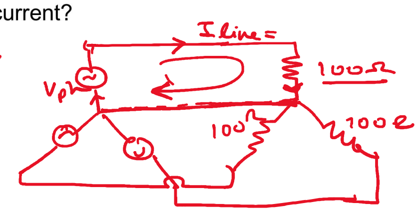
Example: For a balanced Y-connected three phase voltage source and Y-connected load system with a line voltage of 440 V and three equal resistive loads of 100 Ω per phase, assume positive sequence, what will be the magnitudes of
(a) the line-to-neutral voltage, (b) the phase current, (c) the line current?



a) $V_{line} = 440V \rightarrow V_{ph} = \frac{440}{\sqrt{3}} = 254.04V$

b) $|I_a| = \frac{|V_{ph}|}{|Z|} = \frac{254.04}{100} = 2.54A.$

c) $I_{line} = |I_a| = 2.54A.$

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Balanced Three-Phase Power

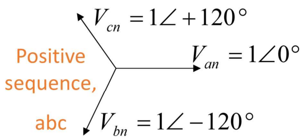
$S_{1\phi} = V I^*$

- From three phase power,

$$S_{3\Phi} = V_{an} I_a^* + V_{bn} I_b^* + V_{cn} I_c^*$$

- When the system is balanced, (assume positive sequence) we can write,

$$S_{3\Phi} = V_{an} I_a^* + V_{an}\angle -120°\left(I_a \angle -120°\right)^* + V_{an}\angle 120°\left(I_a \angle 120°\right)^*$$



$$\boxed{S_{3\Phi} = 3 \underline{V_{an}}\, \underline{I_a^*}} = 3(P + jQ)$$

$\Rightarrow P_{3\phi} = 3 V_{an} I_a \cos\theta = 3 \cdot P_{1\phi}$

$Q_{3\phi} = 3 V_{an} I_a \sin\theta = 3\, Q_{1\phi}.$

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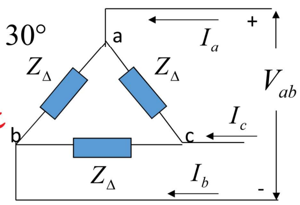
Delta/Wye Connected 3-Phase Load

$I_a = \sqrt{3} I_{ab} \angle -30°$



$V_{LINE} = V_{PHASE}$

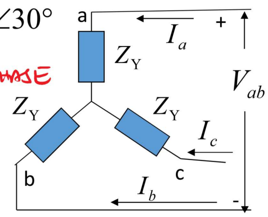
$|S_{3\phi}| = 3|V_{ab}||I_{ab}|$

$= 3|V_{ab}|\left|\frac{I_a}{\sqrt{3}}\right|$

$= \sqrt{3}|V_{LINE}||I_{LINE}|$

$V_{ab} = \sqrt{3} V_{an} \angle 30°$

$I_{LINE} = I_{PHASE}$



$|S_{3\phi}| = 3|V_{an}||I_{an}|$

$= 3\left|\frac{V_{ab}}{\sqrt{3}}\right||I_{an}|$

$= \sqrt{3}|V_{LINE}||I_{LINE}|$

$$\Longrightarrow \boxed{|S_{3\Phi}| = \sqrt{3}|V_{\text{Line-To-Line}}||I_{\text{Line}}|}$$

BOTH Δ & Y

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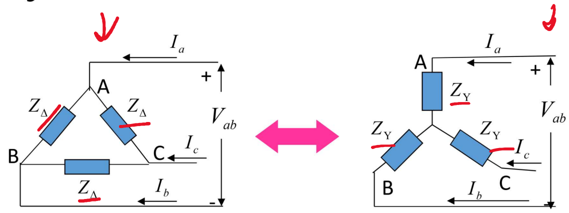
Delta-Wye Load Transformation

$I_{ph} = \frac{I_{LINE}}{\sqrt{3}}$

$V_{ph} = V_{LINE}$



$V_{ph} = \frac{V_{LINE}}{\sqrt{3}}$

$I_{ph} = I_{LINE}$

$$Z_\Delta = \frac{V_{ph}}{I_{ph}} = \frac{V_{AB}}{I_{AB}}$$

$$Z_\Delta = \frac{V_{AB}}{I_A/\sqrt{3}} = \frac{\sqrt{3}V_{AB}}{I_A}$$

$$\Rightarrow \frac{V_{AB}}{I_A} = \frac{Z_\Delta}{\sqrt{3}} \quad -1$$

$$Z_Y = \frac{V_{ph}}{I_{ph}} = \frac{V_{AN}}{I_{AN}}$$

$$Z_Y = \frac{V_{AB}/\sqrt{3}}{I_A} = \frac{V_{AB}}{\sqrt{3}\, I_A}$$

$$\Rightarrow \frac{V_{AB}}{I_A} = \sqrt{3} Z_Y \quad -2$$

1 & 2

$$\Rightarrow \frac{Z_\Delta}{\sqrt{3}} = \sqrt{3} Z_Y$$

$$\Rightarrow \boxed{Z_\Delta = 3Z_Y}$$

$$Z_Y = \frac{Z_\Delta}{3}$$

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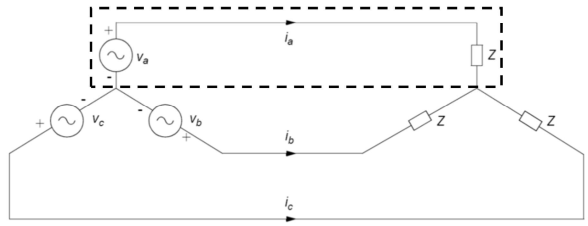
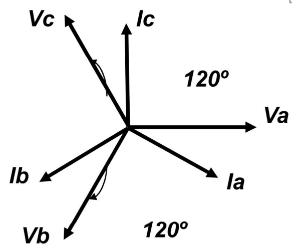
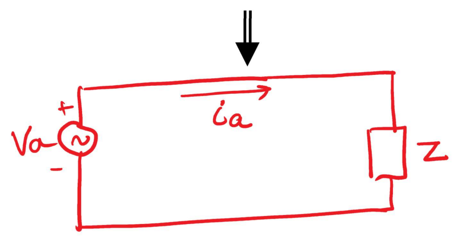
Per Phase Analysis: Assumption

It must be balanced three-phase circuit



$$I_n = I_a + I_b + I_c = 0$$





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Steps of Per Phase Analysis

- Make sure that the three-phase system is balanced.
  - The three-phase sources need to have the same magnitude with 120 degree phase difference.
  - The three-phase impedances must be of the same value (both phase and magnitude).
- **Convert** all Delta-connected sources/loads to Wye-connected sources/loads.
- Per phase analysis reduce three-phase circuit to single-phase circuit. We can apply the same concept used in single-phase.

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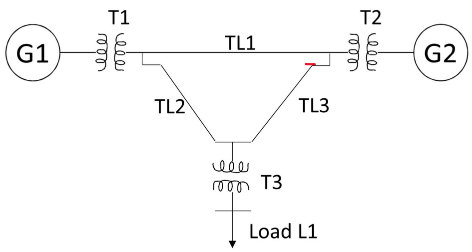
Single-Line Diagram



- Show the interconnections of a transmission system
  - Generator
  - Load
  - Transmission line
  - Transformer
- This is a representation of a 3Φ circuit. Each line represents three conductors in three-phase system.

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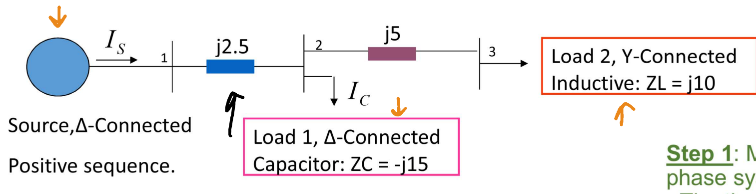
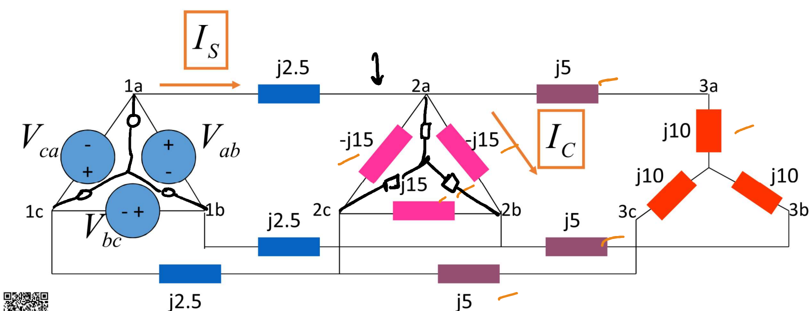
Example: Given a one-line diagram, If the voltage source is $|V_{line}| = \sqrt{3}$ V. Find, the current magnitude supplied by source, $|I_S|$, and, the current magnitude through a capacitor, $|I_C|$.





Step 1: Make sure that the three-phase system is balanced.
- The three-phase sources need to have the same magnitude with 120 degree phase difference.
- The three-phase impedances must be of the same value (both phase and magnitude).

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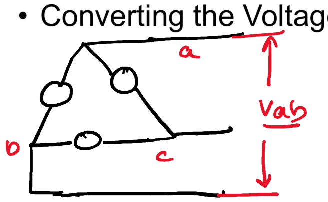
$Z'_Y = \frac{Z_\Delta}{3}$

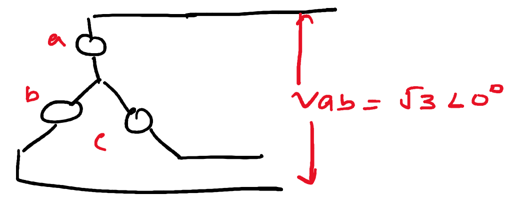
Step 2: Convert from Δ → Y

- Converting the Z_Δ to Z_Y : Load 1 Z_Δ = -j15Ω ➔ $Z_Y = \frac{Z_\Delta}{3} = \frac{-j15}{3} = -j5\Omega$
- Converting the Voltage Source from Δ → Y



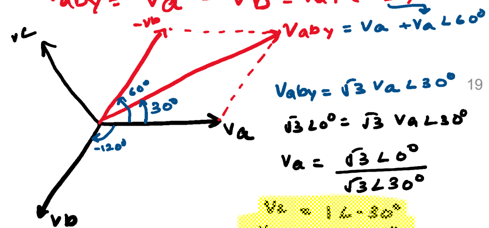
$|V_{ab_\Delta}| = \sqrt{3}\ V$

$V_{ab_\Delta} = \sqrt{3}\angle 0°$



$V_{ab_Y} = \sqrt{3}\angle 0°$

$V_{ab_Y} = V_a - V_b = V_a + (-V_b)$



$V_{ab_Y} = \sqrt{3}\,V_a\angle 30°$

$\sqrt{3}\angle 0° = \sqrt{3}\,V_a\angle 30°$

$V_a = \frac{\sqrt{3}\angle 0°}{\sqrt{3}\angle 30°}$

$V_a = 1\angle -30°$

$V_b = 1\angle -150°$

$V_c = 1\angle 90°$

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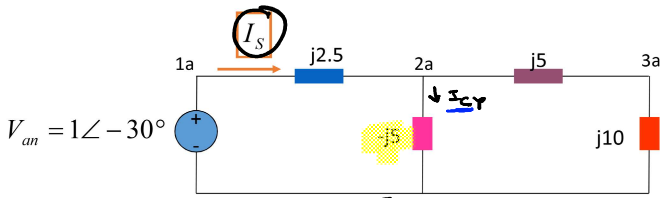
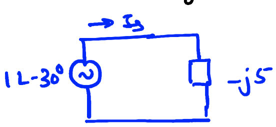
Step 3 :Draw the 1-phase Diagram



$$Z_{eq} = [(j5 + j10) \parallel (-j5)] + j2.5$$
$$= [(j15) \parallel (-j5)] + j2.5$$
$$= -j5\ \Omega$$



$$I_s = \frac{1\angle -30°}{-j5} = \frac{1\angle -30°}{5\angle -90°}$$
$$= 0.2\angle 60°\ A$$
$$|I_s| = 0.2A$$

$$I_{CY} = \frac{V_{2a}}{-j5}$$
$$= \frac{V_a - I_s(j2.5)}{-j5}$$
$$= \frac{1\angle -30 - 0.2\angle 60° (2.5\angle 90°)}{5\angle -90°}$$
$$= 0.3\angle 60°\ A$$

$$\boxed{I_{CY} = 0.3\angle 60° A}$$

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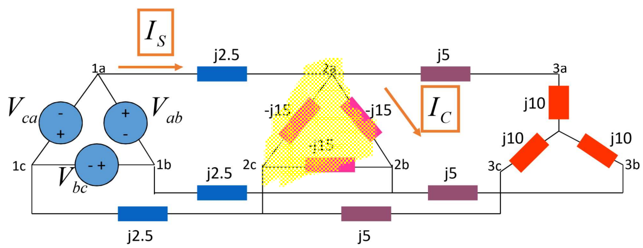
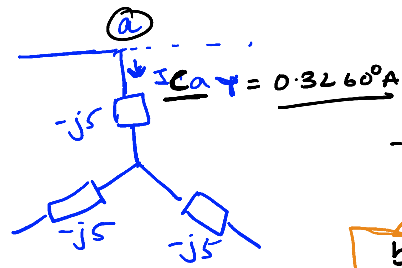
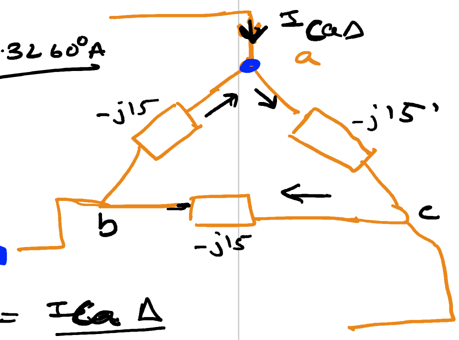
Step 4: Find the capacitor Current I_C







KCL @ junction. a

$$I_{ca\Delta} + I_{ba} = I_{ac}$$

$$I_{ca\Delta} = I_{ac} - I_{ba} = \sqrt{3}\, I_{ac} \angle -30^\circ$$

$$I_{ac} = \frac{I_{ca\Delta}}{\sqrt{3}\angle -30^\circ}$$

$$\underline{I_{caY}} = \underline{I_{ca\Delta}}$$

Line currents are same.

$$I_{ac} = \frac{0.3\angle 60^\circ}{\sqrt{3}\angle -30^\circ} = \left|\frac{\sqrt{3}}{10}\right| \angle +90^\circ A = 0.1732\angle 90^\circ A$$

$$|I_c| = 0.1732 A$$

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