

## 12.2 Induction Motors

Friday, 8 April 2022 3:42 PM

Q1  $n_r = (1-s)n_s$   $s = \frac{n_s - n_r}{n_s}$   
 At rest:  $n_r = 0$   $s = 1$  if  $n_r = 0$   
 $\Rightarrow 0 = (1-s)n_s$   
 $s = 1$  or 100%

Q2 3 $\phi$ , 2 pole, 60 Hz,  $n_r = 3502$  rpm

$P_{in} = 15.7$  kW,  $I_1 = 22.6$  A

$r_1 = 0.2 \Omega$  / phase,

$P_{acL} = ?$

$P_{core} = 0$  W

$P_{scL} = 3 I_1^2 r_1$   
 $= 3 \times 22.6^2 \times 0.2$   
 $= 306.46$  W

$P_{me} = P_{in} - P_{core} - P_{scL}$   
 $= 15.7 \text{ kW} - 0 - 306.46$   
 $= 15393.54$  W

$P_{me} = 3 I_2^2 \frac{R_2'}{s}$

$s \cdot P_{me} = 3 I_2^2 R_2'$

$n_s = \frac{60}{2} \times 60 = 3600$  rpm

$s = \frac{n_s - n_r}{n_s} = \frac{3600 - 3502}{3600}$

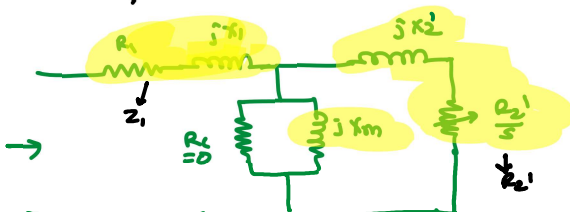
$= 0.027$  or 2.7%

$s \cdot P_{me} = 3 I_2^2 R_2' = P_{acL}$

$P_{acL} = 0.027 \times 15393.54$   
 $= 419$  W

$P_{in} = 3 V_1 I_1 \cos \theta$   $P_{scL} \rightarrow$  Stator copper loss  
 $P_{scL} = 3 I_1^2 R_1$   
 $P_{core} = \frac{3 B_m^2}{\rho_c}$   
 $P_{me} = P_{mech} = P_{in} - P_{scL} - P_{core}$   
 $P_{me} = 3 I_2^2 \frac{R_2'}{s}$   $P_{acL} \rightarrow$  Rotor copper loss  
 $P_{acL} = 3 I_2^2 R_2'$   
 $P_{mech} = P_{me} - P_{acL} = 3 I_2^2 \frac{R_2'}{s} - 3 I_2^2 R_2'$   
 $= 3 I_2^2 R_2' \left( \frac{1}{s} - 1 \right)$   
 $= 3 I_2^2 R_2' \left( \frac{1-s}{s} \right)$   
 $P_{mech\ loss} = P_{friction} + P_{windage}$   
 $P_{out} = P_{mech} - P_{mech\ loss}$   
 $P_{out} = T_m \cdot \omega_m = P_{mech}$   
 $\eta = \frac{P_{out}}{P_{in}}$

3) 3 $\phi$ , Y, 220V, 7.5 kW, 60 Hz, 6 pole  
 $R_1 = 0.1 \Omega$ ,  $R_2' = 0.1 \Omega$ ,  $r_1 = 0.2 \Omega$ ,  $X_2' = 0.2 \Omega$   
 $X_m = 10 \Omega$ ,  $s = 0.01$



$n_s = \frac{120 f}{P} = \frac{120 \times 60}{6} = 1200$  rpm

$n_r = (1-s)n_s = (1-0.01)1200$   
 $= 1188$  rpm

$\omega_r = 1188 \times \frac{2\pi}{60} = 124.4$  rad/s

$$z_2' = \frac{R_2' + jX_2'}{0.01} = 10 + j0.2 \Omega$$

$$z_2' || jx_m = (10 + j0.2) || (j10) = 4.9 + j5 \Omega$$

$$Z_{eq} = Z_1 + (z_2' || jx_m) = (0.1 + j0.2) + (4.9 + j5) \Omega = 5 + j5.2 \Omega$$

$$Z_{eq} = 1Z_{eq} \angle Z_{eq}$$

$\downarrow$   
 $\cos(\angle Z_{eq})$   
 P.F. of the I.N.

If load change  $\rightarrow$  S change  $\rightarrow$   $z_2'$  change  
 Power factor  $\leftarrow$   $\angle Z_{eq}$  change  $\leftarrow$   $Z_{eq}$  change

$$S = \frac{n_s - n_r}{n_s}$$

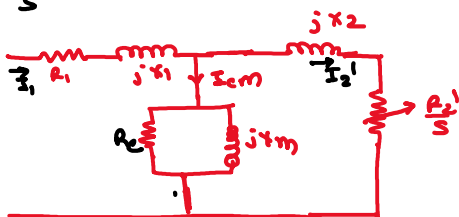
$$1 \angle \omega = n_s - n_r$$

$$n_r = 0$$

4)  $z_2' = 5.41 + j3.1 \Omega$ , 7.5 kW  
 $I_1 = 18.8 A$ ,  $s = 2\%$ ,  $\omega = 120 \text{ rad/s}$   
 $P_{mech} = 2408 W$   
 $P_{shaft} = ?$   
 $T_{output} = ?$

$$z_2' = 5.41 + j3.1$$

$$\frac{P_2'}{S} = 5.41$$



$$I_1 = I_2' + I_{em}$$

$$I_{em} = \text{very small}$$

$$I_2' \approx I_1 = 18.8 A$$

$$P_{in} = 3 I_2'^2 \cdot \frac{R_2'}{s}$$

$$= 3 \times 18.8^2 \times 5.41$$

$$= 5736.3 W$$

$$P_{elec} = P_{in} - P_{acL}$$

$$= P_{in} - s \cdot P_{in}$$

$$= 5736.3 (1 - 0.02)$$

$$= 5621.6 W$$

$$P_{in} = 3 V_1 I_1 \cos \theta$$

$P_{acL} \rightarrow$  Stator copper loss

$$P_{acL} = 3 I_1^2 R_1$$

$$P_{core} = \frac{3 E_1^2}{R_c}$$

$$P_{mech} = P_{in} - P_{acL} - P_{core}$$

$$P_{mech} = 3 I_2'^2 \cdot \frac{R_2'}{s}$$

$P_{acL} \rightarrow$  rotor copper loss

$$P_{acL} = 3 I_2'^2 \cdot R_2'$$

$$= s P_{mech}$$

$$P_{elec} = P_{mech} - P_{acL} = 3 I_2'^2 \cdot \frac{R_2'}{s} - 3 I_2'^2 \cdot R_2'$$

$$= 3 I_2'^2 R_2' \left( \frac{1}{s} - 1 \right)$$

$$= 3 I_2'^2 R_2' \left( \frac{1-s}{s} \right)$$

$$\rightarrow P_{mech-loss} = P_{mech} + P_{elec}$$

Shaft Power  
 $= P_{elec} - P_{mech-loss}$   
 $= 5621.6 - 403$   
 $= 5218.6 \text{ W}$

$P_{out} = P_{elec} - P_{mech-loss}$   
 $P_{out} = T_m \cdot \omega_m = P_{shaft}$   
 $\eta = \frac{P_{out}}{P_{in}}$

$T_m = \frac{P_{shaft}}{\omega} = \frac{5218.6}{123.2} = 42.4 \text{ N}\cdot\text{m}$

5)  $\rightarrow$  480V, 60Hz, 50 H.P, 3 $\phi$ ,  $\tilde{I}_1 = 60 \text{ A}$

$\cos \theta_1 = 0.85 \text{ lag}$

$P_{scl} = 2000 \text{ W}$ ,  $P_{rcl} = 700 \text{ W}$

$P_{mech-loss} = 600 \text{ W}$ ,  $P_{conv} = 1800 \text{ W}$

$P_{in} = 3 V_L \tilde{I}_L \cos \theta$

$P_{in} = 3 V_L \tilde{I}_L \cos \theta$   
 $= 3 \times 480 \times 60 \times 0.85$   
 $= 42400.6 \text{ W}$

a)  $P_{me} = P_{in} - P_{scl} - P_{rcl}$   
 $= 42400.6 - 2000 - 1800$   
 $= 38600.6 \text{ W}$

b)  $P_{elec} = P_{conv} = P_{converted}$   
 $= P_{me} - P_{rcl}$   
 $= 38600.6 - 700$   
 $= 37900.6 \text{ W}$

c)  $P_{out} = P_{elec} - P_{mech-loss}$   
 $= 37900.6 - 600$   
 $= 37300.6 \text{ W}$

d)  $\eta = \frac{P_{out}}{P_{in}} \times 100\%$   
 $= \frac{37300.6}{42400.6} \times 100\%$   
 $= 87.97\%$

$\rightarrow P_{in} = 3 V_L \tilde{I}_L \cos \theta$   $P_{scl} \rightarrow$  Stator copper loss  
 $\checkmark \rightarrow P_{scl} = 3 \tilde{I}_1^2 R_1$   
 $\checkmark \rightarrow P_{rcl} = 3 \tilde{I}_2^2 R_2$   
 $P_{me} \rightarrow P_{mech} = P_{in} - P_{scl} - P_{rcl}$   
 $P_{me} = 3 \tilde{I}_2^2 \frac{R_2}{s}$   $P_{rcl} \rightarrow$  Rotor copper loss  
 $\checkmark \rightarrow P_{rcl} = 3 \tilde{I}_2^2 R_2$   
 $P_{elec} = P_{me} - P_{rcl} = 3 \tilde{I}_2^2 \frac{R_2}{s} - 3 \tilde{I}_2^2 R_2$   
 $= 3 \tilde{I}_2^2 R_2 \left( \frac{1}{s} - 1 \right)$   
 $= 3 \tilde{I}_2^2 R_2 \left( \frac{1-s}{s} \right)$   
 $\rightarrow P_{mech-loss} = P_{friction} + P_{windage}$   
 $P_{out} = P_{elec} - P_{mech-loss}$   
 $P_{out} = T_m \cdot \omega_m = P_{shaft}$   
 $\eta = \frac{P_{out}}{P_{in}}$

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