2.2 AC Fundamentals - Tutorial

Friday, January 21, 2022 3:49 PM

1) a) $v_1(t) = 40\sin 628t + 60\cos(628t - 45) + 30\cos 628t$

$$V_1 = \frac{40}{\sqrt{2}}\angle -90 + \frac{60}{\sqrt{2}}\angle -45^\circ + \frac{30}{\sqrt{2}}\angle 0^\circ$$

$$= \frac{1}{\sqrt{2}}\left(40\angle -90 + 60\angle -45 + 30\angle 0\right)$$

$$= \frac{109.72}{\sqrt{2}}\angle -48.69^\circ$$

$$\Rightarrow v_1(t) = 109.72\cos(628t - 48.69)$$

b) $v_2(t) = 20\sin 314t + 10\cos(314t + 60^\circ) - 5\sin(314t - 20^\circ)$

$$V_2 = \frac{20}{\sqrt{2}}\angle -90 + \frac{10}{\sqrt{2}}\angle 60^\circ + \frac{5}{\sqrt{2}}\angle 70^\circ$$

$$= \frac{9.44}{\sqrt{2}}\angle -44.71^\circ$$

$$v_2(t) = 9.44\cos(314t - 44.71^\circ)$$

$-5\sin(314t - 20^\circ)$

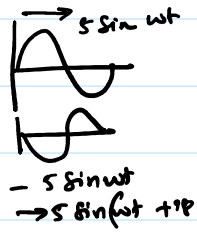
$\rightarrow -5\cos(314t - 20 - 90)$

$-5\cos(314t - 110^\circ)$

$5\cos(314t - 110 + 180^\circ)$

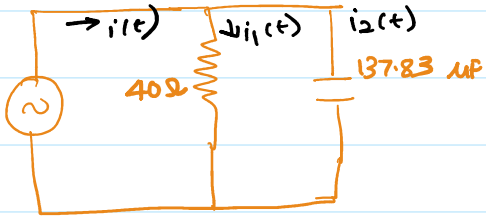
$5\cos(314t + 70^\circ)$

$-1 \rightarrow 1\angle 180^\circ$



2) $i(t) = A\cos(314.16t + \beta)$ A

$v(t) = 282.84\cos(314.16t + 10^\circ)$ V



$i(t) = i_1(t) + i_2(t)$

$$i_1(t) = \frac{v(t)}{R} = \frac{282.84\cos(314.16t + 10^\circ)}{40}$$

$$= 7.071\cos(314.16t + 10^\circ)\ A$$

$i_2(t) = ?$

$$i_C(t) = C\frac{dv(t)}{dt}$$

$$i_2(t) = 137.83\times10^{-6} \times \frac{d}{dt}(282.84\cos(314.16t + 10^\circ)$$

$$= \underbrace{137.83\times10^{-6}}_{C} \times 282.84 \times \underbrace{314.16}_{\omega}(-\sin(314.16t + 10^\circ)$$

$$= -12.24\sin(314.16t + 10^\circ)$$

$$= 12.24\cos(314.16t + 100^\circ)\ A$$

$$i(t) = i_1(t) + i_2(t)$$

$$= 7.071\cos(314.16t + 10^\circ) + 12.24\cos(314.16t + 100^\circ)$$

$$= 14.14\cos(314.16t + 70^\circ)\ A$$

Using Phasors

$$V = \frac{282.84}{\sqrt{2}}\angle 10^\circ \qquad I = \frac{I_m}{\sqrt{2}}\angle\beta$$

$$R = 40\Omega \qquad Z_C = \frac{-j}{\omega C} = \frac{-j}{314.16\times137.83\times10^{-6}}$$

$$Z_C = -j23.09\Omega$$

$$I_1 = \frac{V}{R} = \frac{282.84\angle10^\circ}{\sqrt{2}\times40} = 5\angle10^\circ\ A$$

$$I_2 = \frac{V}{Z_C} = \frac{282.84\angle10^\circ}{\sqrt{2}\times23.09\angle-90^\circ} = 8.66\angle100^\circ$$

$$I = I_1 + I_2 = 5\angle10^\circ + 8.66\angle100^\circ$$

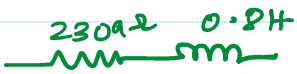
$$= 10\angle70^\circ$$

$i(t) = 10\sqrt{2}\cos(314.16t + 70^\circ)\ A$

$A = 10\sqrt{2}$ $\quad \beta = 70^\circ$

3) $i(t) = 2.828\cos(5000t + 30^\circ)$ ←

$R = 2309\Omega \quad L = 0.8H$



a) Find $Z_{RL} = R + j\omega L$

$= 2309 + j5000 \times 0.8 = 2309 + j4000$

$= 4618.6\angle 60^\circ\ \Omega$

$I = \dfrac{2.828\angle 30^\circ}{\sqrt{2}} = 2\angle 30^\circ A$

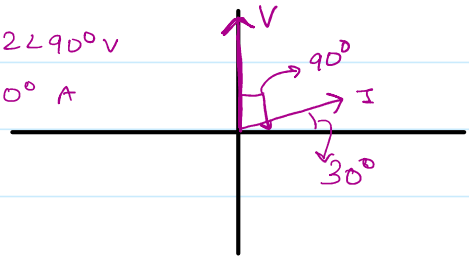
b) $V = IZ = 2\angle 30^\circ \times 4618.6\angle 60^\circ$

$= 9237.2\angle 90^\circ\ V$

$v(t) = 9237.2\sqrt{2}\cos(5000t + 90^\circ)\ V$

$V \rightarrow 9237.2\angle 90^\circ V$

$I \rightarrow 2\angle 30^\circ\ A$

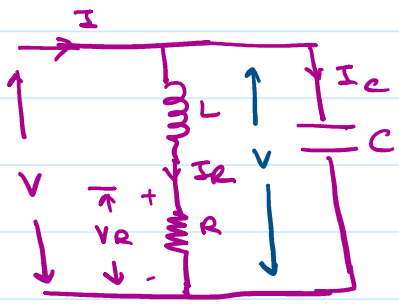


Q4 $L = 2H, \quad R = 3\Omega, \quad C = 0.2F$

$v_R = 8.48\cos 2t\ V$

$V_R = \dfrac{8.48\angle 0^\circ V}{\sqrt{2}}$

$V_R = 6\angle 0^\circ V$



$$I_R = \frac{V_R}{R} = \frac{6\angle 0^\circ}{3} = 2\angle 0^\circ \text{ A}$$

$V_L = I_R \cdot Z_L$

$Z_L = j\omega L = j\,2.2 = j4\,\Omega.$

$V_L = 2\angle 0^\circ \cdot 4\angle 90^\circ = 8\angle 90^\circ \text{ V}$

$I = I_R + I_C$

$I_C = \frac{V}{Z_C}$

$Z_C = \frac{-j}{\omega C} = \frac{-j}{2\pi \times 0.2}$

$Z_C = -j2.5\,\Omega.$

$V = V_R + V_L = 6\angle 0^\circ + 8\angle 90$

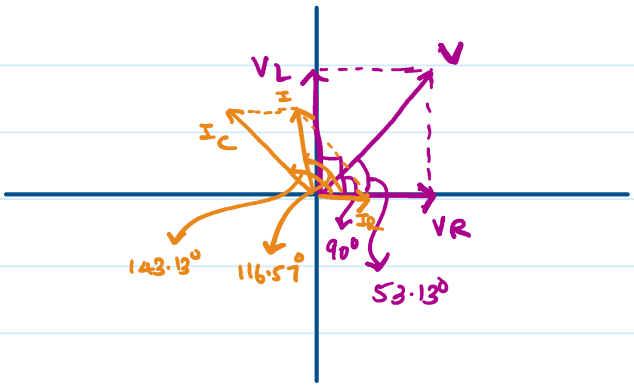
$= 10\angle 53.13^\circ \text{ V}$

$I_R = \frac{V}{R + j\omega L} = \frac{10\angle 53.13}{2 + j4} = 2\angle 0^\circ$

$I_C = \frac{10\angle 53.13^\circ}{2.5\angle -90^\circ} = 4\angle 143.13^\circ \text{ A}$

$I = I_R + I_C = 2\angle 0 + 4\angle 143.13^\circ$

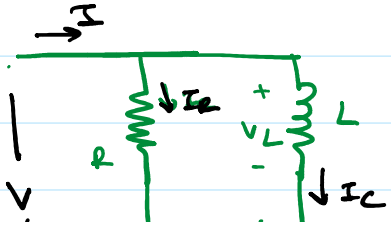
$I = 2.68\angle 116.57^\circ \text{ A}$

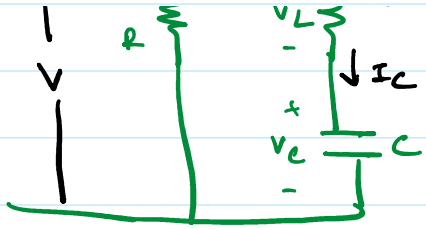


I , I_C , I_R

V , V_R , V_L

5)

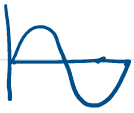




$R = 2\Omega,\ L = 3.25\,mH,\ C = 100\,\mu F$

$v_c = 100\sqrt{2}\cos(2000t - 90^\circ)\,V$

$\omega = 2000\ rad/s$



a) $\underline{V_c} = 100\angle -90^\circ$

$Z_L = j\omega L = j\,3.25\times10^{-3}\times 2000$

$Z_c = \frac{-j}{\omega C} = \frac{-j}{2000\times 100\times10^{-6}} =$

$\underline{I_C} = \frac{V_C}{Z_C} = 100\angle -90 \times 2000\times100\times10^{-6}\angle +90^\circ$

$= 20\angle 0^\circ\ A$

$\underline{V_L} = I_C \times Z_L$

$= 20\angle 0 \times 6.5\angle 90^\circ = 130\angle 90^\circ\ V$

b) I_R ? I ?

$\underline{V} = V_L + v_c = 100\angle -90 + 130\angle 90$

$= 30\angle 90^\circ\ V$

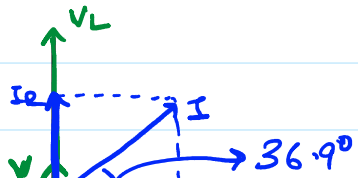
$\underline{I_R} = \frac{V}{R} = \frac{30\angle 90}{2} = 15\angle 90\ A$

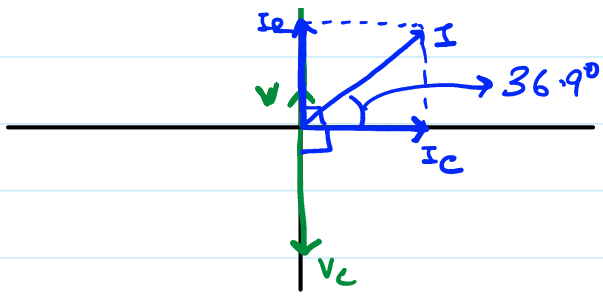
$\underline{I} = I_R + I_C = 15\angle 90 + 20\angle 0^\circ$

$= 25\angle 36.9^\circ\ \underline{A}$

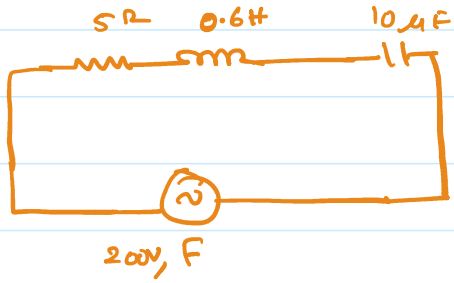
d) $i(t) = 25\sqrt{2}\cos(2000t + 36.9^\circ)\,A$

c) Phasor diagram.





6)



$$Z = 5 + j\omega(0.6) - \frac{j}{\omega(10\times10^{-6})}$$

I is max when Z is minimum. $\Rightarrow (Z = R)$

$$\omega(0.6) = \frac{1}{\omega(10\times10^{-6})} \quad (|Z_L| = |Z_C|$$

$$\omega^2 = \frac{1}{6\times10^{-6}} \Rightarrow$$

$$\omega = 408.25 \text{ rad/s}$$

$$f = \frac{\omega}{2\pi} = 64.97 \text{ Hz.}$$

$$V = 200$$

$$I = \frac{V}{R} = \frac{200}{5} = 40\text{A.} \quad \Rightarrow V_R = I.R = 200\text{V}$$

$$|V_L| = |I| \times |j\omega L| = |I| \times |Z_L|$$

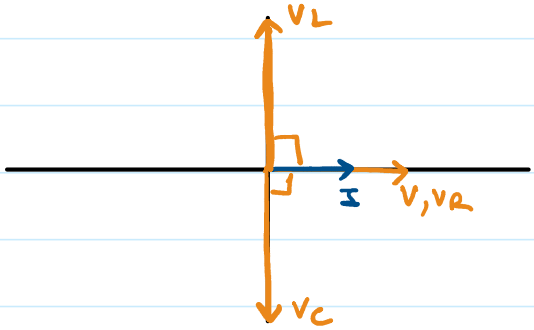
$$= 40 \times 408.25 \times 0.6$$

$$= 9798\text{V}$$

$$|V_C| = |I| \times \left|\frac{1}{j\omega C}\right| = 9798\text{V}$$

$|V| = 200\,V.$

Ratio $\frac{|V_L|}{|V|} = \frac{9798}{200} = 48.99$

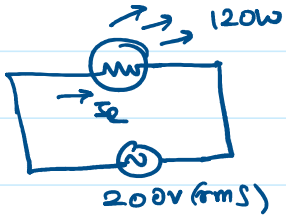


$V \rightarrow 200\angle 0^\circ \qquad V_0 \rightarrow 200\angle 0^\circ$

$V_L = I \cdot \omega L \angle 90^\circ$

$V_C = I \times \frac{1}{j\omega C} = \frac{I}{\omega C} \angle -90$

7) $V_R = 200\,V \qquad P_R = 120\,W$

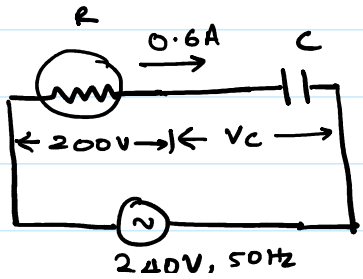


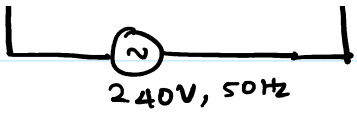
a) $I_R = \frac{P_R}{V_R} = \frac{120}{200} = 0.6A.$

b) $R = \frac{V_R}{I_R} = \frac{200}{0.6} = 333.33\,\Omega$

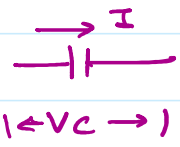
$= \frac{P}{I_R^2} = \frac{120}{0.6 \times 0.6} = 333.33\,\Omega.$

c)

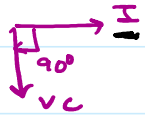




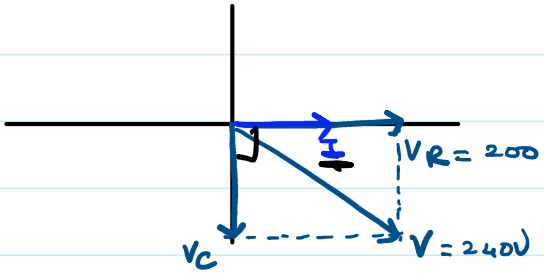
$\omega = 2\pi F = 314.16$ rad/s



I leads V_c by 90°



$V_R = 200\angle 0^\circ$



$V_R + V_C = V$

$|V_C| = \sqrt{240^2 - 200^2} = 132.66\,V$

$V_C = 132.66\angle -90^\circ\,V$

$|V_C| = |I||Z_C| = |I|\left|\frac{1}{\omega C}\right|$

$C = \frac{0.6}{314.16 \times 132.66} = 14.4\,\mu F$

b) Method (1)

$Z = R - \frac{j}{\omega C}$

$|Z| = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

$|I| = \frac{|V|}{|Z|} \rightarrow I = \frac{V}{Z}$ (Ohm's Law)

$|I|\angle\theta = \frac{|V|}{|Z|}\angle(\angle V - \angle Z)$

$|Z|$

$$|Z| = \frac{|V|}{|I|} = \frac{240}{0.6} = 400\,\Omega.$$

$$R = 333.33\,\Omega.$$

$$\sqrt{Z^2 - R^2} = \frac{1}{\omega C_1}$$

$$\sqrt{400^2 - 333.33^2} = \frac{1}{\omega C} = 221.11\,\Omega.$$

$$C = \frac{1}{314.16 \times 221.11} = 14.4\,\mu F$$

$$I = |I| \angle \theta_1$$

$$V = |V| \angle \theta_2$$

$$Z = |Z| \angle \theta_3$$

$$I = \frac{V}{Z} = \frac{|V| \angle \theta_2}{|Z| \angle \theta_3}$$

$$|I| \angle \theta_1 = \frac{|V|}{|Z|} \angle [\theta_2 - \theta_3]$$

$$\theta_1 = \theta_2 - \theta_3$$

$$|I| = \frac{|V|}{|Z|}$$

I