EE2029 – Introduction to Electrical Energy Systems
(Tutorial #1 AC Fundamentals)

(Tutorial #1 solutions)

Solution Q1:

(a) $V_1 = \frac{40}{\sqrt{2}}\angle{-90^\circ} + \frac{60}{\sqrt{2}}\angle{-45^\circ} + \frac{30}{\sqrt{2}}\angle{0^\circ} = \frac{109.72}{\sqrt{2}}\angle{-48.69^\circ}$

Hence, $v_1(t) = 109.72\, cos\,(628t - 48.69^\circ)$

(b) $V_2 = \frac{20}{\sqrt{2}}\angle{-90^\circ} + \frac{10}{\sqrt{2}}\angle{60^\circ} + \frac{5}{\sqrt{2}}\angle{70^\circ} = \frac{9.44}{\sqrt{2}}\angle{-44.71^\circ}$

Hence, $v_2(t) = 9.44\, cos\,(314t - 44.71^\circ)$

Solution Q2:

The current through the resistor and capacitor are respectively,

$$\begin{aligned} i_i(t) &= \frac{v(t)}{R} = 7.071\, cos\,(314.16t + 10^\circ) \\ i_2(t) &= C\frac{dv}{dt} = -12.247\, sin\,(314.16t + 10^\circ) \end{aligned}$$

So, the current supplied by the source is

$$\begin{aligned} i(t) &= i_1(t) + i_2(t) \\ &= 7.071\, cos\,(314.16t + 10^\circ) - 12.247\, sin\,(314.16t + 10^\circ) \end{aligned}$$

In phasor form, this current can be written as:

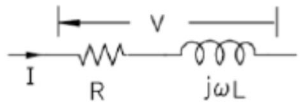
$$I \;=\; 5\angle 10^\circ + 8.66\angle 100^\circ \;=\; 10\angle 70^\circ$$

The time domain expression for the current is

$$i(t) = 14.14\, cos\,(314.16t + 70^\circ)$$

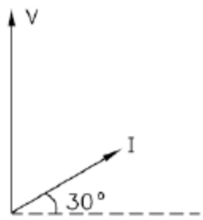
Solution Q3:

Current $i(t) = 2\sqrt{2}\cos\,(5000t + 30^0)mA$



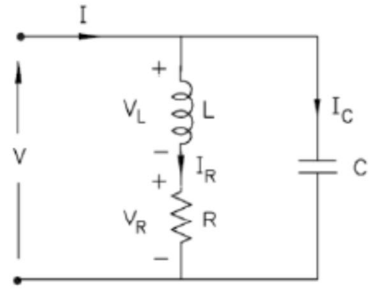
The impedance and voltage phasors can be determined as follows:

$$
\begin{aligned}
I &= 2\times 10^{-3}\angle 30^\circ \ \ \mathrm{A} \\
Z &= R + j\omega L = 2309 + j5000\times 0.8 = 2309 + j4000\Omega \\
V &= Z\,I = (2309 + j4000)\times 2\times 10^{-3}\angle 30^\circ = 9.24\angle 90^\circ \ \ \mathrm{V} \\
v(t) &= 9.24\,\sqrt{2}\,cos(5000t + 90^\circ) \ \ \mathrm{V}
\end{aligned}
$$



Solution Q4:

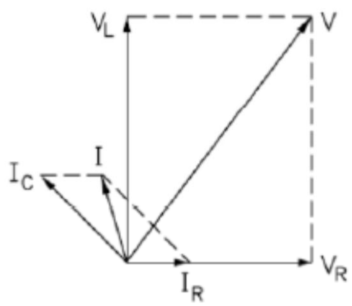
L = 2H, R = 3Ω, C = 0.2μF and $v_R = 6\sqrt{2}cos2t$ volts.



As $v_R = 6\sqrt{2}\,cos\,2t$, we have $V_R = 6\angle 0°$. Hence,

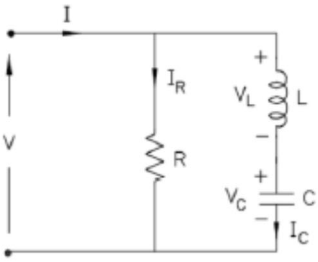
$$\begin{aligned}
I_R &= \frac{V_R}{R} = \frac{6\angle 0°}{3} = 2\angle 0° \text{ A} \\
V_L &= j\omega L I_R = j2 \times 2 \times 2\angle 0° = 8\angle 90° \text{ V} \\
V &= V_R + V_L = 6\angle 0° + 8\angle 90° = 6 + j8 = 10\angle 53.13° \text{ V} \\
I_C &= j\omega C V = j2 \times 0.2 \times (6 + j8) = -3.2 + j2.4 = 4\angle 143.13° \text{ A} \\
I &= I_L + I_C = 2 - 3.2 + j2.4 = -1.2 + j2.4 = 2.68\angle 116.57° \text{ A}
\end{aligned}$$

The phasor diagram can be drawn as follows:



Solution Q5:

R = 2Ω, L = 3.25mH and C = 100μF, $v_C = 100\sqrt{2}\cos\,(2000t - 90^0)$ volts



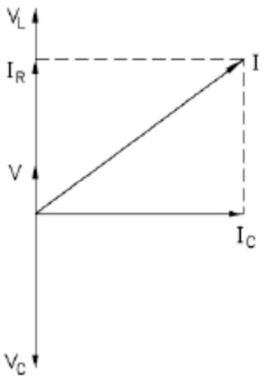
(a) As $v_C = 100\sqrt{2}\,cos(2000t - 90°)$ volts, we get

$$\begin{aligned} V_C &= 100\angle{-90°} \\ I_C &= j\omega C V_C = j2000 \times 10^{-4} \times 100\angle{-90°} = 20\angle 0° \ \ \text{A} \\ V_L &= j\omega L I_C = j2000 \times 3.25 \times 10^{-3} \times 20\angle 0° = 130\angle 90° \ \ \text{A} \end{aligned}$$

(b) So, the voltage V across the circuit and hence currents I_R and I are

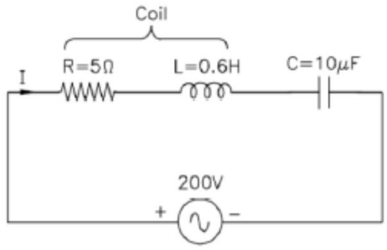
$$\begin{aligned} V &= V_C + V_L = 100\angle{-90°} + 130\angle 90° = -j100 + j130 = j30 \ \ \text{V} \\ I_R &= \frac{V}{R} = \frac{j30}{2} = 15\angle 90° \ \ \text{A} \\ I &= I_C + I_R = 20\angle 0° + 15\angle 90° = 20 + j15 = 25\angle 36.9°\,\text{A} \end{aligned}$$

(c) Phasor diagram:



(d) $i(t) = 25\,\sqrt{2}\,cos(2000t + 36.9°)$ A

Solution Q6:



$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$
$$I = \frac{V}{Z}$$

Current flow in the circuit will be maximum when the impedance is minimum. Hence,

$$\omega L = \frac{1}{\omega C}$$
$$\omega = \frac{1}{\sqrt{LC}} = 408.248 \text{ rad/s}$$
$$f = \frac{\omega}{2\pi} = 64.97 \text{ Hz}$$
$$I = \frac{V}{R} = \frac{200}{5} = 40 \text{ A}$$

$$|V_L| = \omega L I = 9798 \text{ V}$$
$$|V_C| = \frac{I}{\omega C} = 9798 \text{ V}$$
$$\frac{|V_L|}{|V|} = \frac{\omega L I}{R I} = \frac{\omega_o L}{R} = \frac{244.95}{5} = 48.99$$

Solution Q7.

(a)

$$P = V_{rms} I_{rms} \quad \Rightarrow \quad I_{rms} = \frac{120\ W}{200\ V} = 0.6\ \text{Amp}$$

(b)

$$R_{lamp} = \frac{200\ V}{0.6\ A} = 333.33\ \Omega$$

(c)

With the capacitor connected in series, the total impedance is Z = R_{lamp} – jX_C.

When connected to 240 V source, we still want the voltage across the lamp to the rated value of 200 V, and hence 0.6A current through the lamp.

$$|Z| = \frac{240\ V}{0.6\ A} = 400\ \Omega$$

$$Z = \sqrt{R_{lamp}^2 + X_C^2} \quad \Rightarrow \quad X_C^2 = Z^2 - R_{lamp}^2$$

$$X_C = \sqrt{400^2 - (333.33)^2} = 221.11\ \Omega$$

$$X_C = \frac{1}{\omega C} \quad \Rightarrow \quad C = \frac{1}{\omega X_C} = \frac{1}{(2\pi \times 50)(221.11)} = 14.4\ \mu\text{F}$$