EE2029 - Introduction to Electrical Energy Systems

Line Conductors

Electromagnetic Background
Conductor Modeling
Bundled Conductor

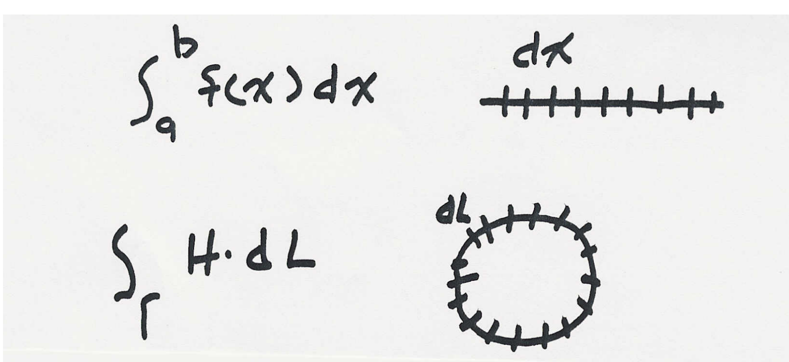
Electromagnetic Background

Ampere's Circuital Law

- The law states the relationship between a **current** and the **magnetic field** created by it.
  - Line integral of magnetic field intensity (H) along an imaginary closed path is equal to the product of current (I) enclosed by the path and permeability of the medium.

$$mmf = \oint H\, dl = I$$

  - dl = vector differential path length
  - Line integral is a *generalisation* of traditional integration.



Integration along the x-axis

Integration along a general path, which may be closed.

Magnetic Flux Density

- Magnetic fields are usually measured in terms of flux density (B).

$$B = \mu H$$

  - μ is the permeability, i.e., $\mu = \mu_0 \mu_r$
  - μ_0 is the permeability of free space (constant) $= 4\pi \times 10^{-7}$ H/m
  - μ_r is the relative permeability of the material to free space, e.g. 1 for air.

- Magnetic flux is a measurement of total magnetic field passing through a given area.
  - A useful tool for helping to describe effects of magnetic force on something occupying a given area.
  - Total flux (ϕ) passing through a surface A is $\phi = \int_A \mathbf{B} d\mathbf{a}$
    - where d**a** = vector with direction *normal* to surface.

Example: Magnetic Field of Single Wire

Assume we have an infinitely long wire with current of 1000A. How much magnetic flux passes through a 1 meter square, located between 4 and 5 meters from the wire?

$$\boxed{mmf = \oint H\, dl = I}$$

curved SA of cylinder $= 2\pi r \cancel{L}$

$\therefore \oint H\, dl = 2\pi r$

In this case,

$mmf = 2\pi r H = I$

$\Rightarrow H = \dfrac{I}{2\pi r}$ —— (1)

(air)

Sub. (1) in.

$$\boxed{\phi = \int_A B\, da} = \int_A \mu H\, da = \int_A \mu_0 \cancel{\mu_r} H\, da = \int_4^5 \mu_0 \frac{I}{2\pi r}(1)\, dr$$

$\boxed{B = \mu H}$ $\boxed{\mu = \mu_0 \mu_r}$

$$= \left[\mu_0 \frac{I}{2\pi} \ln r\right]_4^5$$

$$= \mu_0 \frac{I}{2\pi}(\ln 5 - \ln 4)$$

$$= 4\pi \times 10^{-7}\left(\frac{1000}{2\pi}\right)[\ln 5 - \ln 4]$$

$$= 4.46 \times 10^{-5}\ \mathrm{Wb}$$

Faraday's Law of Induction

- Induced electromotive force (V) in any closed circuit is equal to the negative of time rate of change of magnetic flux enclosed by the circuit.

$N\phi = \lambda$

$$V = \frac{d\lambda}{dt} = \frac{d(N\phi)}{dt}$$

- Flux linkages (λ) is the amount of flux *linking* an N turn coiled.
    - For simplicity, we assume the flux links to all turns and there are no leakages.
- Flux linkages can be related to **inductance** for RLC circuit analysis.
    - For a linear magnetic system, i.e. $B = \mu H$, inductance (L) can be defined as the constant relating the current to the flux linkage.

$$L = \lambda / I$$

Flux Linkages of Single Conductor

- To development models of transmission lines, we first need to determine the inductance of a single, infinitely long wire.
- To do this we need to determine the wire's total flux linkage, including
  - Flux linkages **outside** of the wire
  - Flux linkages **within** the wire (skin effect)
- Assume that the current density within the wire is uniform and that the wire has a radius of r.

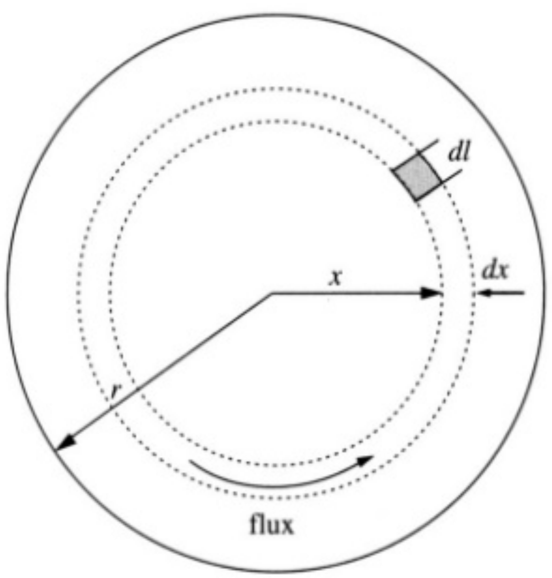
$$\oint H_x dl = I_x \Rightarrow 2\pi x H_x = I_x$$

$$\boxed{I_x = \frac{\pi x^2}{\pi r^2} I}$$

only possible since current density is uniform

$$H_x = \frac{x}{2\pi r^2} I \quad \Rightarrow B_x = \mu H_x = \frac{\mu x I}{2\pi r^2}$$

, where x = any distance into the wire.



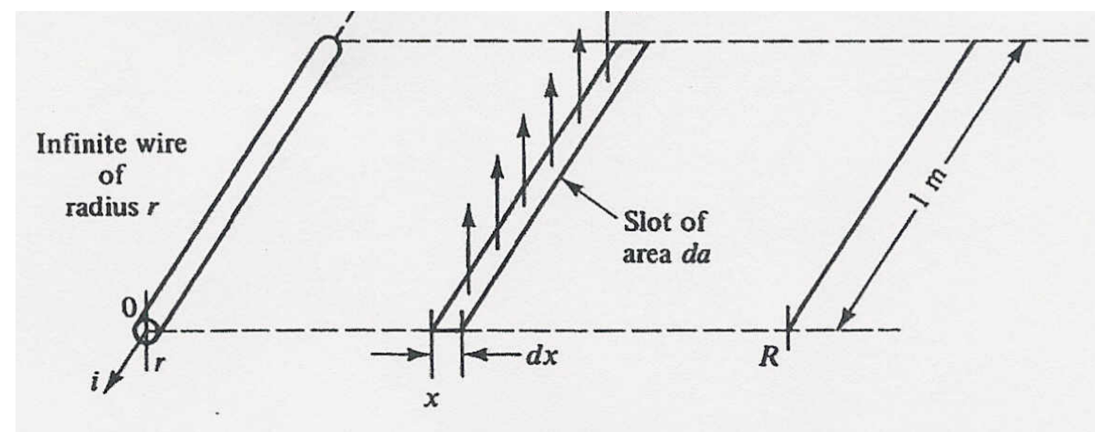
Flux Linkages Outside of Conductor

- Suppose a wire is a single loop that is closed at infinity —> $\lambda=\phi$ as $N=1$.
- Flux linking the wire out to a distance of R from the centre of the wire is:

$$\phi = \int_A B\,da = \text{length} \int_r^R \mu_0 \frac{I}{2\pi x}\,dx$$

$$\lambda = \phi = l \int_r^R \mu_0 \frac{I}{2\pi x}\,dx$$

N=1



- Since length is infinity, we can represent the expression as per unit length.

divide by L.

$$\frac{\lambda}{l} = \phi = \int_r^R \mu_0 \frac{I}{2\pi x}\,dx = \frac{\mu_0}{2\pi} I \ln\frac{R}{r}$$

distance from centre

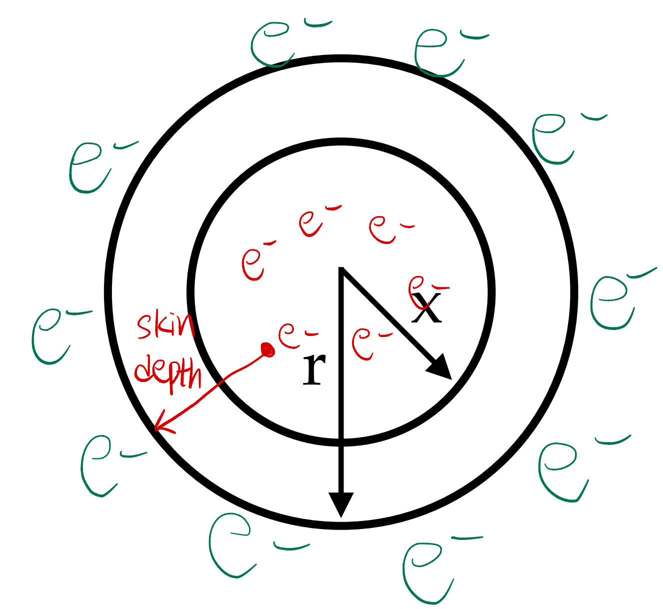
radius of wire

flux linkages per unit L.

Flux Linkages Inside of Conductor

tendency of current to 'crowd' towards surface of wire under AC current.

- Current inside conductor tends to travel on the surface of conductor due to **skin effect.** The penetration of current into the conductor is approximated using the skin depth.



$$s = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

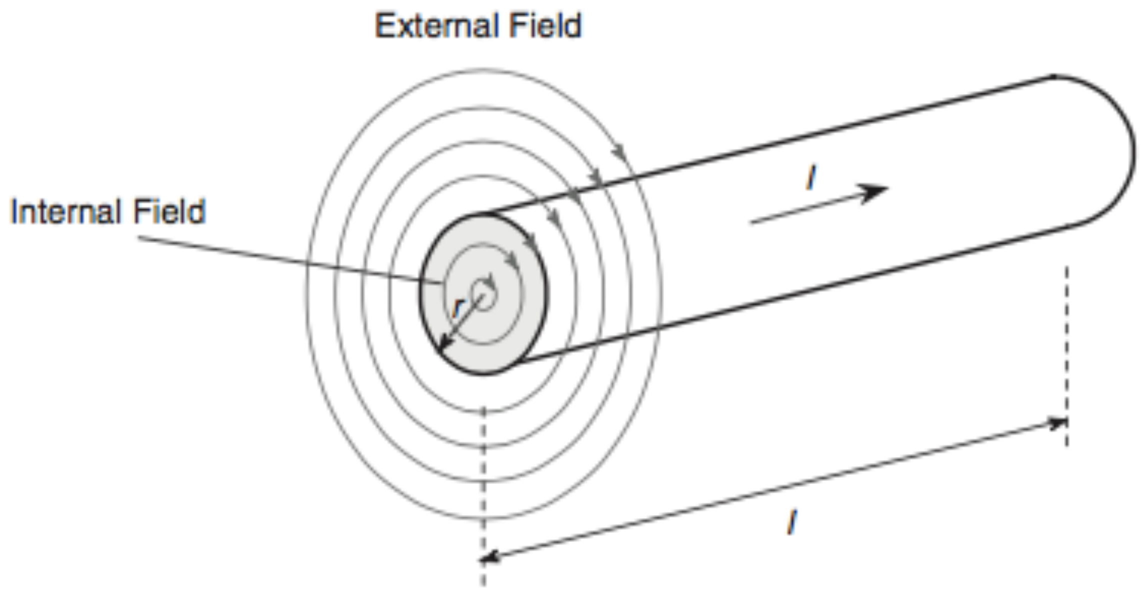
  - f is the frequency, and σ is the conductivity.
  - For copper wire, skin depth is about $0.066\ \mathrm{m}/\sqrt{f} \approx 0.0093\ \mathrm{m}$
  - For simplicity, assume a uniform current density.

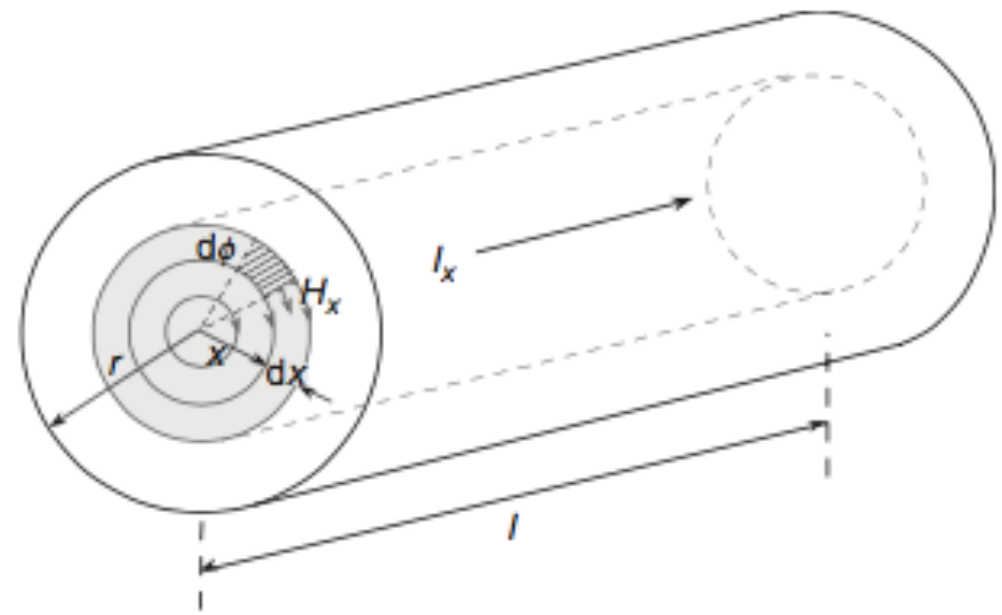
(ratio) assuming current density is uniformly distributed.

- Current enclosed within a distance x of centre: $I_e = \frac{\pi x^2}{\pi r^2} I$, $H_x = \frac{I_e}{2\pi x} = \frac{Ix}{2\pi r^2}$

- Flux only links part of the current: $\lambda_{inside} = \int_0^r \mu \frac{Ix}{2\pi r^2} \frac{x^2}{r^2} dx = \frac{\mu}{2\pi} \int_0^r \frac{Ix^3}{r^4} dx = \frac{\mu_0 \mu_r}{8\pi} I$

Flux Linkages Inside of Wire





$$I_e = \frac{\pi x^2}{\pi r^2} I, \quad H_x = \frac{I_e}{2\pi x} = \frac{Ix}{2\pi r^2}$$

$$\lambda_{inside} = \int_0^r \mu \frac{Ix}{2\pi r^2} \frac{x^2}{r^2} dx = \frac{\mu}{2\pi} \int_0^r \frac{Ix^3}{r^4} dx = \frac{\mu_0 \mu_r}{8\pi} I$$

$$\lambda_{inside} = \frac{I}{2} \times 10^{-7} \quad \text{given} \quad \mu_r = 1 \;\&\; \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \quad \Rightarrow L_{inside} = \frac{1}{2} \times 10^{-7} \text{ H/m}$$

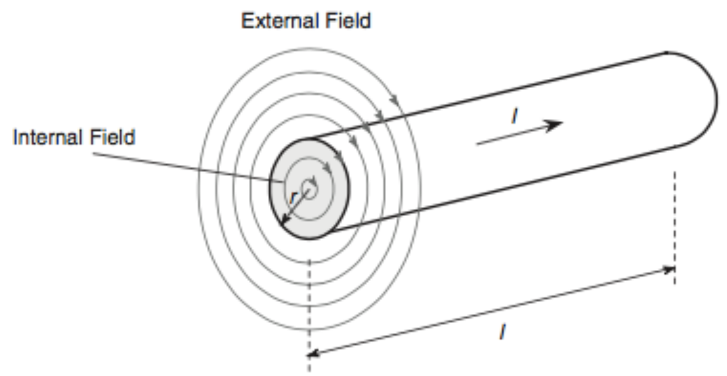
$L = \frac{\lambda}{I}$

Line Total Flux and Inductance

- In summary:

$$\lambda_{total} = \frac{\mu_0}{2\pi} I \ln\frac{R}{r} + \frac{\mu_0\mu_r}{8\pi} I$$

External + Internal



$$= \frac{\mu_0}{2\pi} I \left(\ln\frac{R}{r} + \frac{\mu_r}{4} \right)$$

$$L = \frac{\lambda}{I}$$

$$L = \frac{\mu_0}{2\pi} \left(\ln\frac{R}{r} + \frac{\mu_r}{4} \right)$$

- Inductance can be simplified using the following identities:

Useful identities

$$\begin{cases} \ln(ab) = \ln a + \ln b, \\ \ln\frac{a}{b} = \ln a - \ln b, \\ a = \ln\left(e^a\right) \end{cases}$$

$$L = \frac{\mu_0}{2\pi} \left(\ln\frac{R}{r} + \frac{\mu_r}{4} \right) = \frac{\mu_0}{2\pi} \left(\ln R - \left(\ln r + \ln e^{-\frac{\mu_r}{4}} \right) \right)$$

$$= \frac{\mu_0}{2\pi} \left(\ln R - \ln\left(r e^{-\frac{\mu_r}{4}} \right) \right)$$

$$= \frac{\mu_0}{2\pi} \ln\frac{R}{r'}$$

$$= 2\times 10^{-7} \ln\frac{R}{r'}$$

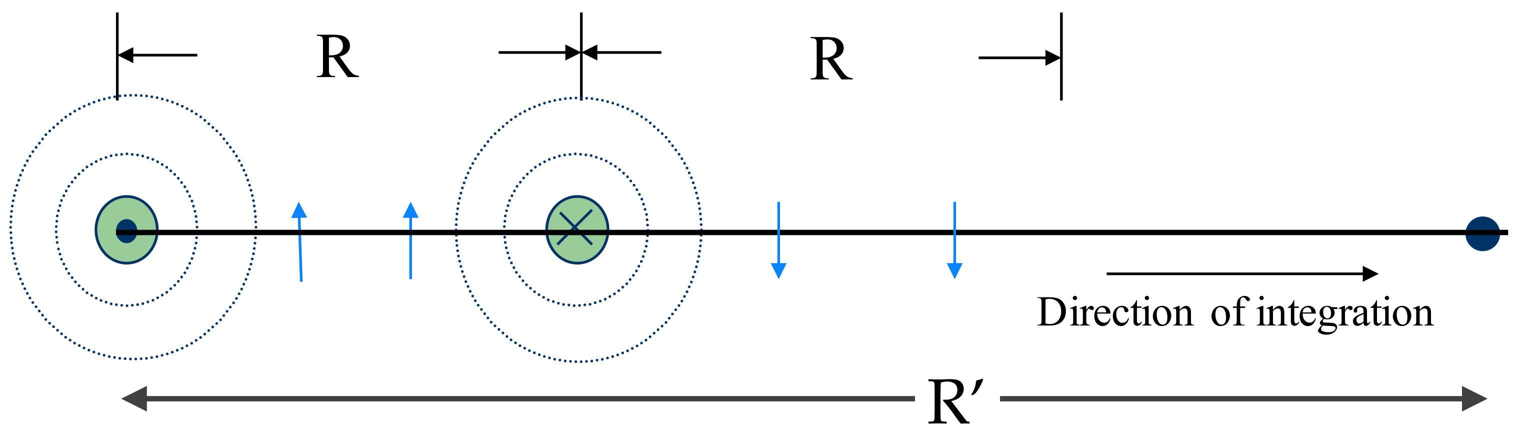
$$r' = r e^{-\frac{\mu_r}{4}} \approx 0.78r \text{ for } \mu_r = 1$$

Conductor Modeling

Two Conductor Line Inductance

$\lambda_{left} = \lambda_{\text{due to left}} + \lambda_{\text{due to right}}$

- Consider the case of two wires, each carrying the same current (I), but in opposite directions. The wires are separated by distance (R).
  - To determine the inductance of each conductor, apply integration as before. In this case, there will be some field cancellation due to the opposite flow of current.
  - Integrate for the left to an arbitrary distance of R', the total flux linkages are:



$$\lambda_{left} = \frac{\mu_0}{2\pi} I \ln \frac{R'}{r'} - \frac{\mu_0}{2\pi} I \ln\left(\frac{R'-R}{R}\right)$$

Left Current **Right Current** where $r' = re^{-\frac{\mu_r}{4}}$

$$\lambda_{left} = \frac{\mu_0}{2\pi} I \ln \frac{R'}{r'} - \frac{\mu_0}{2\pi} I \ln\left(\frac{R'-R}{R}\right)$$

$$= \frac{\mu_0}{2\pi} I \left(\ln \frac{R'}{r'} - \ln\left(\frac{R'-R}{R}\right)\right)$$

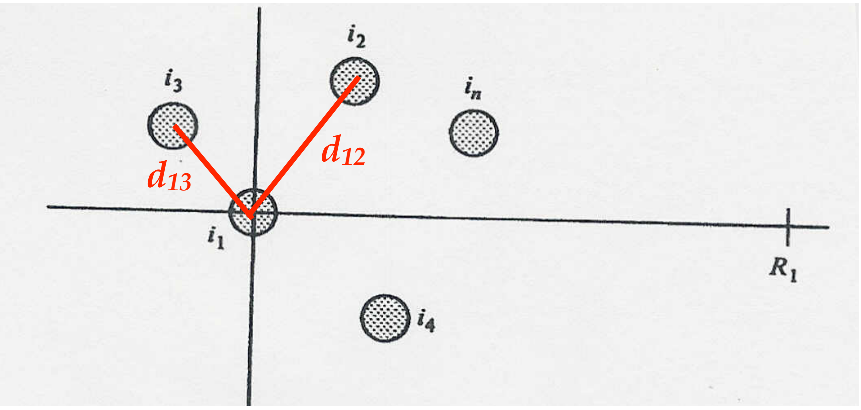
$$= \frac{\mu_0}{2\pi} I \left(\ln R' - \ln r' - \ln(R'-R) + \ln R \right)$$

$$= \frac{\mu_0}{2\pi} I \left(\ln \frac{R}{r'} + \ln \frac{R'}{R'-R} \right)$$

$$= \frac{\mu_0}{2\pi} I \left(\ln \frac{R}{r'} \right) \text{ as } R' \rightarrow \infty$$

Multiple Conductors

- Assume there are k conductors, each with current i_k, arranged in an arbitrary geometry. How to find flux linkages of each wire?
  - Each conductor's flux linkage, λ_k, depends upon its own current and the current in all the other conductors.
  - To derive λ_1, the same approach is taken by integrating from conductor 1 along the right along the axis.



$$\lambda_1 = \frac{\mu_0}{2\pi}\left[i_1 \ln \frac{R_1}{r'_1} + i_2 \ln \frac{R_2}{d_{12}} + \ldots + i_n \ln \frac{R_n}{d_{1n}}\right]$$

$$\lambda_1 = \frac{\mu_0}{2\pi}\left[i_1 \ln \frac{1}{r'_1} + i_2 \ln \frac{1}{d_{12}} + \ldots + i_n \ln \frac{1}{d_{1n}}\right] + \frac{\mu_0}{2\pi}\left[i_1 \ln R_1 + i_2 \ln R_2 + \ldots + i_n \ln R_n\right]$$

Multiple Conductors

- As R_1 goes to infinity, $R_1 = R_2 = \ldots = R_n$ yielding the following relation:

$$\lambda_1 = \frac{\mu_0}{2\pi}\left[i_1 \ln \frac{1}{r'_1} + i_2 \ln \frac{1}{d_{12}} + \ldots + i_n \ln \frac{1}{d_{1n}}\right] + \cancel{\frac{\mu_0}{2\pi}\left(\sum_{j=1}^{n} i_j\right)} \ln R_1 \quad 0$$

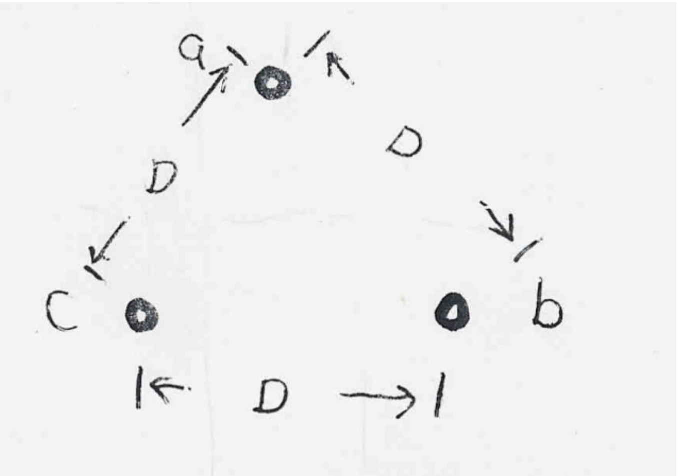
$$\lambda_1 = \frac{\mu_0}{2\pi}\left[i_1 \ln \frac{1}{r'_1} + i_2 \ln \frac{1}{d_{12}} + \ldots + i_n \ln \frac{1}{d_{1n}}\right]$$

$$\lambda_1 = L_{11}i_1 + L_{12}i_2 + \ldots + L_{1n}i_n$$

- Flux linkages of each conductor will consist of *self* and *mutual* inductances.
- Mutual elements only exist in *unbalanced system*, where the influence from the three-phase currents is not zero.

Example - Line Inductance

Calculate the per-phase reactance for a balanced three-phase, 50 Hz transmission line with a conductor geometry of an equilateral triangle with D = 5m, r = 1.24 cm (Rook conductor) and a length of 5 km. Assume the system is balanced, i.e. $i_1 + i_2 + i_3 = 0$. Note that D = R for this problem.



Since balanced system : $i_a + i_b + i_c = 0$ balanced system.

$\Rightarrow \boxed{i_a = -(i_b + i_c)}$

$$\boxed{\lambda_a = \frac{\mu_0}{2\pi}\left[i_a \ln\frac{1}{r'} + i_b \ln\frac{1}{D} + i_c \ln\frac{1}{D}\right]}$$

$$= \frac{\mu_0}{2\pi}\left[i_a \ln\frac{1}{r'} - i_a \ln\frac{1}{D}\right]$$

$$= \frac{\mu_0}{2\pi}\left[i_a\left(\ln\frac{1}{r'} - \ln\frac{1}{D}\right)\right]$$

$$= \frac{\mu_0}{2\pi} i_a \ln\left(\frac{D}{r'}\right)$$

$$\Rightarrow \boxed{L_a = \frac{\lambda_a}{i_a}} = \frac{\mu_0}{2\pi}\ln\left(\frac{D}{r'}\right) = \frac{\mu_0}{2\pi}\ln\left(\frac{D}{re^{-\mu r/4}}\right)$$

$r' = re^{-\mu r/4}$

$$= \frac{4\pi\times10^{-7}}{2\pi}\ln\left[\frac{5}{0.0124e^{-1/4}}\right]$$

$$= 1.25\times10^{-6}\ H/m$$

For per unit length:

$$\boxed{X_a = \omega L_a} = 2\pi f L_a = 2\pi(50)(1.25\times10^{-6})$$

per unit length!

$$= 3.93\times10^{-4}\ \Omega/m$$

$\therefore$ For $l = 5km$:

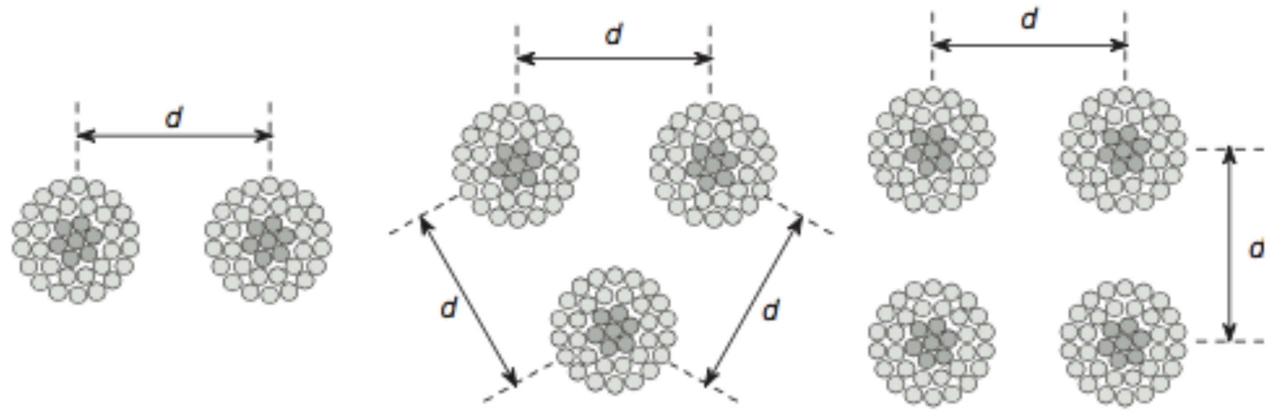
$$X_{a,total} = 3.93\times10^{-4}(5\times10^{3})$$

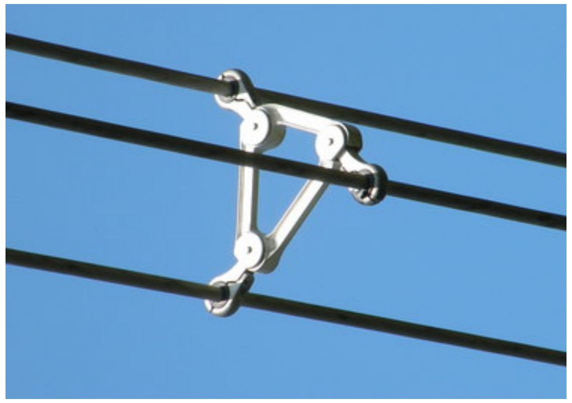
$$= 1.96\Omega$$

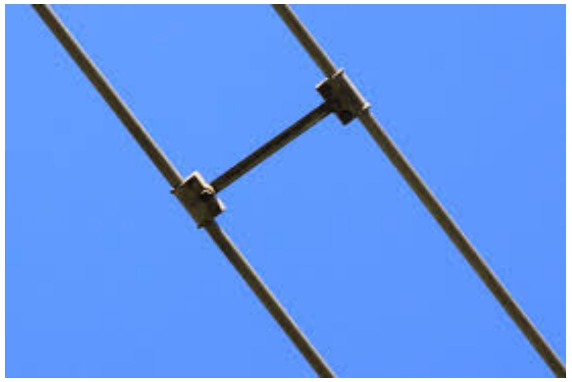
Bundled Conductors

Conductor Bundling (>110kV)

- To increase the capacity of high voltage transmission lines it is very common to use a number of conductors per phase.
  - This is known as conductor bundling.
  - Typical values are two conductors for 345 kV lines, three for 500 kV, and four for 765 kV.







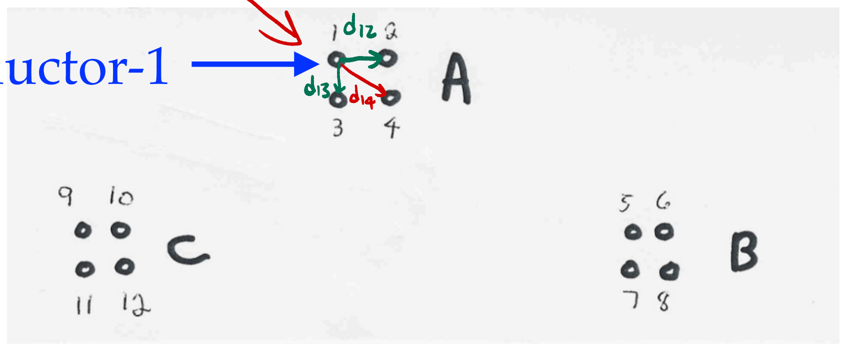
Bundled Conductor Flux Linkages

- Define d_{ij} as the distance between conductors i and j. λ for each conductor can be defined as:

diagonal!

$$\lambda_1 = \frac{\mu_0}{2\pi}\left[\begin{array}{l} \frac{i_a}{4}\left(\ln\frac{1}{r'}+\ln\frac{1}{d_{12}}+\ln\frac{1}{d_{13}}+\ln\frac{1}{d_{14}}\right)+ \\ \frac{i_b}{4}\left(\ln\frac{1}{d_{15}}+\ln\frac{1}{d_{16}}+\ln\frac{1}{d_{17}}+\ln\frac{1}{d_{18}}\right)+ \\ \frac{i_c}{4}\left(\ln\frac{1}{d_{19}}+\ln\frac{1}{d_{110}}+\ln\frac{1}{d_{111}}+\ln\frac{1}{d_{112}}\right) \end{array}\right]$$

conductor-1



- Simplified:

$$\lambda_1 = \frac{\mu_0}{2\pi}\left[\begin{array}{l} i_a \ln\left(\frac{1}{\boxed{\left(r' d_{12} d_{13} d_{14}\right)^{1/4}}}\right)+ \\ i_b \ln\left(\frac{1}{\boxed{\left(d_{15} d_{16} d_{17} d_{18}\right)^{1/4}}}\right)+ \\ i_c \ln\left(\frac{1}{\left(d_{19} d_{110} d_{111} d_{112}\right)^{1/4}}\right) \end{array}\right]$$

GMR ("radius" per phase)

GMD (distance of one conductor to another phase)

Bundled Inductance

- For a balanced system: $D_{ab} = D_{ac} = D_{bc} = D$ & $i_a = -i_b - i_c$
- We can derive the inductance from the flux linkage in previous slide:

$$\lambda_1 = \frac{\mu_0}{2\pi}\left[i_a \ln\left(\frac{1}{R_n}\right) - i_a \ln\left(\frac{1}{D}\right)\right]$$

$$= \frac{\mu_0}{2\pi} i_a \ln\left(\frac{D}{R_n}\right)$$

$$= \frac{\mu_0}{2\pi} 4 I_1 \ln\left(\frac{D}{R_n}\right)$$

GMD

$$L_1 = \frac{\mu_0}{2\pi} 4 \ln\left(\frac{D}{R_n}\right)$$

GMR

Remember **D = GMD & $\mathbf{R_n}$ = GMR**

- Each bundle has n conductors —> $L_a = L_1/n$ —> $L_a = \frac{\mu_0}{2\pi}\ln\left(\frac{D}{R_n}\right)$

Inductance per phase per unit length in a transmission line

Example - Bundled Inductance

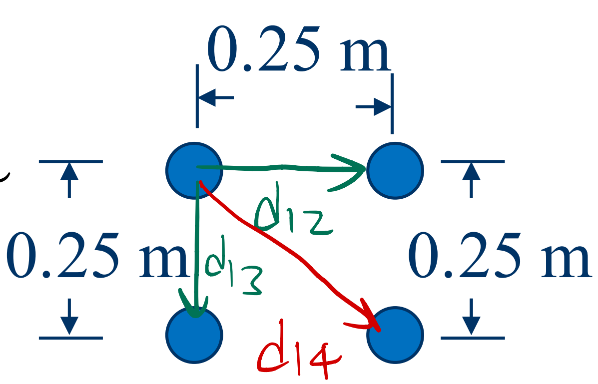
D=GMD=5m

Consider the previous example of the three-phase lines that are symmetrically spaced 5 meters apart using conductors with a radius of r = 1.24 cm. However, now assume each phase has 4 conductors arranged in a square bundle, and spaced by 0.25 meters apart. What is the new inductance per meter?

$$\boxed{r' = re^{-\mu_r/4}} = 0.0124e^{-1/4} = 9.67\times10^{-3}\,\text{m}$$

$$\boxed{R_n = GMR = (r' d_{12} d_{13} d_{14})^{1/4}} = \left[9.67\times10^{-3}(0.25)(0.25)\left(\sqrt{0.25^2+0.25^2}\right)\right]^{1/4}$$

$$= 0.12\,\text{m}$$



$$D = GMD = 5\,\text{m}$$

$$\boxed{L_a = \frac{\mu_0}{2\pi}\ln\left(\frac{D}{R_n}\right)} = \frac{4\pi\times10^{-7}}{2\pi}\ln\left[\frac{5}{0.12}\right]$$

$$= 7.46\times10^{-7}\,\text{H/m}$$

ACSR Data Table

TABLE A8.1. BARE ALUMINUM CONDUCTORS, STEEL REINFORCED (ACSR) ELECTRICAL PROPERTIES OF MULTILAYER SIZES *(Cont'd)*

Code Word	Size (kcmil)	Stranding Al./St.	Number of Aluminum Layers	Resistance: dc 20°C (Ohms/Mile)	Resistance: ac-60 Hz 25°C (Ohms/Mile)	Resistance: ac-60 Hz 50°C (Ohms/Mile)	Resistance: ac-60 Hz 75°C (Ohms/Mile)	GMR (ft)	Phase-to-Neutral, 60 Hz Reactance at One ft Spacing: Inductive Ohms/Mile X_a	Phase-to-Neutral, 60 Hz Reactance at One ft Spacing: Capacitive Megohm-Miles X'_a
Flicker	477	24/7	2	0.1889	0.194	0.213	0.232	0.0283	0.432	0.0992
Hawk	477	26/7	2	0.1883	0.193	0.212	0.231	0.0290	0.430	0.0988
Hen	477	30/7	2	0.1869	0.191	0.210	0.229	0.0304	0.424	0.0980
Osprey	556.5	18/1	2	0.1629	0.168	0.184	0.200	0.0284	0.432	0.0981
Parakeet	556.5	24/7	2	0.1620	0.166	0.183	0.199	0.0306	0.423	0.0969
Dove	556.5	26/7	2	0.1613	0.166	0.182	0.198	0.0313	0.420	0.0965
Eagle	556.5	30/7	2	0.1602	0.164	0.180	0.196	0.0328	0.415	0.0957
Peacock	605	24/7	2	0.1490	0.153	0.168	0.183	0.0319	0.418	0.0957
Squab	605	26/7	2	0.1485	0.153	0.167	0.182	0.0327	0.415	0.0953

GMR is equivalent to r'

Inductance and Capacitance assume a D_m of 1 ft

1 ft ≈ 0.3 m

1 inch ≈ 0.0254 m

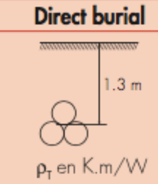
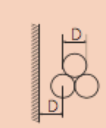
Singaporean Environment

	Single Phase	Three Phase
Phase Conductor (Line)	Brown	Line 1 Brown Line 2 Black Line 3 Grey
Neutral Conductor	Blue	
Protective Conductor (Earth)	Green-and-Yellow	

Nominal section area	Conductor diameter	Thickness of insulation	DC conductor resistance at 20°C	Electrostatic capacitance	Aluminium screen: Sectional area*	Aluminium screen: Outside diameter of cable*	Aluminium screen: Weight of cable*	Copper wire/lead sheath: Sectional area* copper screen	Copper wire/lead sheath: Outside diameter of cable*	Copper wire/lead sheath: Weight of cable*	Copper wire/alu sheath: Sectional area* copper screen	Copper wire/alu sheath: Outside diameter of cable*	Copper wire/alu sheath: Weight of cable*	Corrugated Alu sheath: Sectional area*	Corrugated Alu sheath: Outside diameter of cable*	Corrugated Alu sheath: Weight of cable*	Lead sheath: Sectional area*	Lead sheath: Outside diameter of cable*	Lead sheath: Weight of cable*
mm^2	mm	mm	Ω/km	μF/km	mm^2	mm	kg/m	mm^2	mm	kg/m	mm^2	mm	kg/m	mm^2	mm	kg/m	mm^2	mm	kg/m
400 R	23.2	21.6	0.0470	0.14	310	85	9	145	91	16	165	87	10	480	97	10	1290	93	23
500 R	26.7	22.1	0.0366	0.15	300	90	10	135	96	18	160	92	11	510	102	11	1280	97	24
630 R	30.3	20.4	0.0283	0.17	300	90	11	135	96	19	160	92	12	510	102	12	1290	97	26
800 R	34.7	18.4	0.0221	0.20	300	90	13	135	97	21	160	93	14	510	102	14	1290	98	27
1000 R	38.8	18.1	0.0176	0.21	290	94	15	130	100	24	155	96	16	560	107	16	1290	101	29
1000 S	40.0	18.5	0.0176	0.22	300	97	16	120	104	25	155	100	17	640	111	17	1280	104	30
1200 S	42.5	19.5	0.0151	0.22	290	102	18	115	109	27	150	104	19	740	116	19	1280	109	32
1600 S	48.9	18.5	0.0113	0.25	300	107	23	110	114	33	150	109	24	770	121	24	1270	113	37
1600 S En	48.9	18.5	0.0113	0.25	300	107	23	110	114	33	150	109	24	770	121	24	1270	113	37
2000 S	57.2	18.5	0.0090	0.28	290	115	26	95	123	38	145	118	27	940	130	28	1280	121	40
2000 S En	57,2	18.5	0.0090	0.28	290	115	26	95	123	38	145	118	27	940	130	28	1280	121	40
2500 S En	63.5	19.2	0.0072	0.30	280	123	32	80	131	45	140	126	33	1080	139	35	1260	128	46

*Indicative value

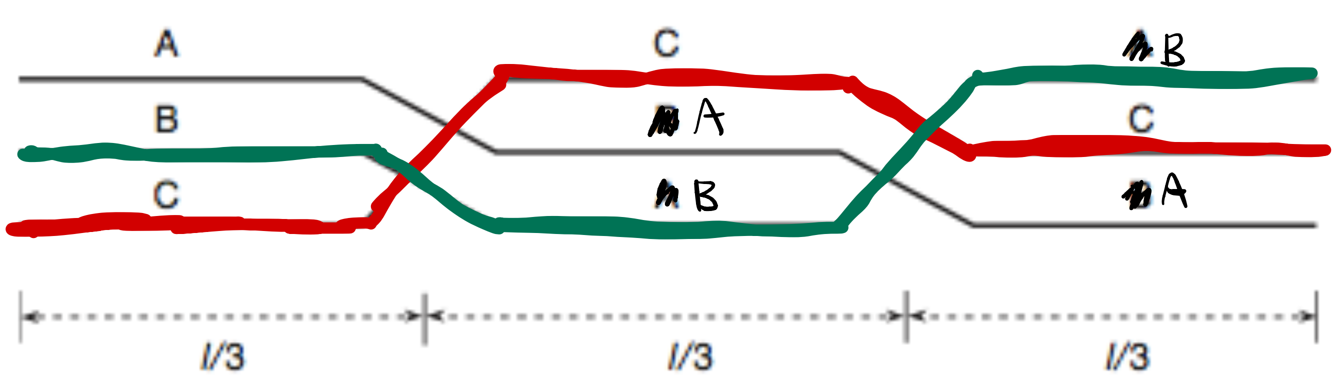
R : round stranded
S : segmental stranded
S En : segmental stranded enamelled

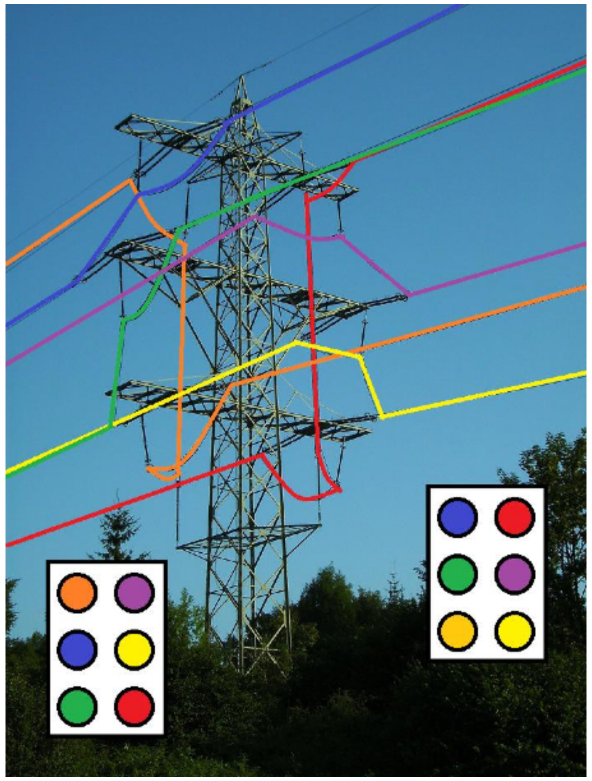
Laying conditions : Trefoil formation	
Direct burial	In air, in gallery
	

Lines Transposition

- To keep system **balanced**, the conductors are rotated over the length of a transmission line. Each phase occupies each position on tower for an equal distance. This is known as transposition.

- An aerial view of conductor positions over the length of transmission line:





Circuit Breaker and Conductor Sizing

- Overcurrent devices (breaker or fuse) to be sized no less than 100% of the noncontinuous load, and 125% of the continuous load.

- Conductors to be sized no less than 100% of the noncontinuous load, and 125% of the continuous load.

Example: What size branch-circuit overcurrent protection device and conductor is required for a 23A continuous load (75°C terminals)?

- Breaker current: 23A $\boxed{\text{x } 125\%}$ = 28.75A or 30A
- Select conductor size (gauge) based on this current.

∴ We use the 30-Amp breaker.