EE2029 – Introduction to Electrical Energy Systems
Tutorial # 3 Line Conductors

$D = 0.012m$

1. A 60-Hz single-phase, two-wire overhead line has solid cylindrical copper conductors with 1.2 cm diameter. The conductors are arranged in a horizontal configuration with 0.5 m spacing. Find the inductance of each conductor due to internal flux linkages only, and the inductance of each conductor due to both internal and external flux linkages.

$L_a = \frac{\mu_0}{2\pi} \ln\left(\frac{GMD}{GMR}\right)$

(Answer: $L_{int}=0.5\times10^{-7}$ H/m, $L=9.346\times10^{-7}$ H/m)

bundled conductors qn.

2. Find the GMR of three symmetrically spaced conductors configured as an equilateral triangle. The spacing between conductors is 50 cm. Assume that $r = 2$ cm, $r' = 2e^{-0.25} = 1.56$ cm.

(Answer: 15.7 cm)

3. A 60 Hz three-phase, three-wire overhead line has solid cylindrical conductors arranged in the form of an equilateral triangle with 1.2 m conductor spacing. Conductor diameter is 1 cm. Calculate the line inductance in H/m, and the inductive reactance in Ω/km.

↳ normal multiple conductors qn.

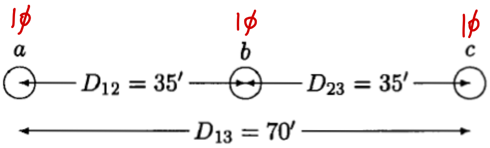
$L_a = \frac{\mu_0}{2\pi} \ln\left(\frac{R}{r'}\right)$

(Answer: 1.146×10^{-6} H/m, 0.432 Ω/km)

4. A 500 kV three-phase line is composed of one ACSR conductor per-phase with horizontal conductor configuration as shown below. The conductors have a diameter of 1.345 inches and a GMR of 0.5328 inch. Find the line inductance per km.

bundled conductors qn.

$L_a = \frac{\mu_0}{2\pi} \ln\left(\frac{GMD}{GMR}\right)$



(Answer: 1.38 mH/km)

1. For internal flux linkages:

$$I_e = \frac{\pi x^2}{\pi r^2} I \quad , \quad H_x = \frac{I_e}{2\pi x} = \frac{Ix}{2\pi r^2}$$

(assuming uniform current density)

$$\boxed{\lambda_{inside} = \int_0^r \mu \frac{Ix}{2\pi r^2} \frac{x^2}{r^2} dx = \frac{\mu}{2\pi}\int_0^r \frac{Ix^3}{r^4} dx = \frac{\mu_0 \mu_r}{8\pi} I} \text{ — (1)}$$

$$\boxed{L = \frac{\lambda}{I}}$$

$$\Rightarrow L_{inside} = \frac{\mu_0 \mu_r}{8\pi} \text{ — (2)}$$

Assume: $\mu_0 = 4\pi \times 10^{-7}$ H/m, $\mu_r \overset{(air)}{=} 1$ H/m,

$$\therefore L_{inside} = \frac{4\pi \times 10^{-7}(1)}{8\pi} = \frac{1}{2} \times 10^{-7} \text{ H/m}$$

For total flux linkage:

$$\boxed{\begin{aligned}\lambda_{total} &= \lambda_{outside} + \lambda_{inside} \\ &= \frac{\mu_0}{2\pi} I \ln\frac{R}{r} + \frac{\mu_0 \mu_r}{8\pi} I\end{aligned}}$$

$$\lambda_{out} = \frac{\mu_0}{2\pi} I \ln\frac{R}{r}$$

$$= \frac{\mu_0}{2\pi} I \left(\ln\frac{R}{r} + \frac{\mu_r}{4}\right) \text{ — (3)}$$

$$L = \frac{\lambda}{I} \text{ — (4)}$$

$$\Rightarrow \boxed{L_{total} = \frac{\mu_0}{2\pi}\left(\ln\frac{R}{r} + \frac{\mu_r}{4}\right)}$$

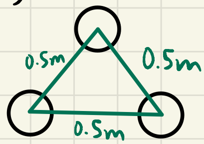
$$= \frac{\mu_0}{2\pi}\left[\ln R - \left(\ln r + \ln e^{-\mu_r/4}\right)\right]$$

$$= \frac{\mu_0}{2\pi}\left[\ln R - \ln\left(re^{-\mu_r/4}\right)\right]$$

$$\boxed{= \frac{\mu_0}{2\pi} \ln\frac{R}{r'}}$$

$$= \frac{4\pi \times 10^{-7}}{2\pi} \ln\left[\frac{0.5}{\left(\frac{0.012}{2}\right)e^{-1/4}}\right] = 9.346 \times 10^{-7} \text{ H/m}$$

2. Type of Qn: Bundled Conductors



$r = 0.02m$

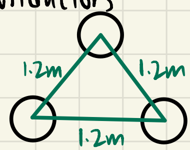
$r' = re^{-\mu_r/4} = 0.02e^{-1/4} = 0.0156m$

$$\boxed{R_n = GMR = [r' d_{12} d_{13}]^{1/n}}$$

n = no. of conductors

$$= [0.0156(0.5)(0.5)]^{1/3} = 0.157m$$

3. Type of Qn: Normal Multiple Conductors



$d = 0.01m \Rightarrow r = 0.005m$

$R = 1.2m$

Assume: $\mu_0 = 4\pi \times 10^{-7}$ H/m, $\mu_r = 1$ H/m, (air)

$\boxed{r' = re^{-\mu_r/4}} = 0.005e^{-1/4} = 0.003894m$

$$\Rightarrow \boxed{L = \frac{\mu_0}{2\pi} \ln\left(\frac{R}{r'}\right)}$$

$$= \frac{4\pi \times 10^{-7}}{2\pi} \ln\left[\frac{1.2}{0.003894}\right] = 1.146 \times 10^{-6} \text{ H/m}$$

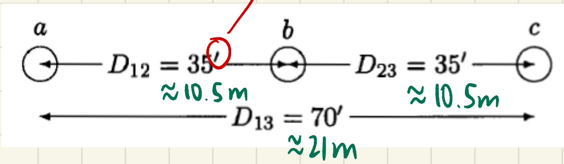
$$\boxed{X = \omega L = 2\pi f \cdot L}$$

$$= 2\pi(60)(1.146 \times 10^{-6}) = 4.320 \times 10^{-4}\ \Omega/m$$

$$= 0.432\ \Omega/km$$

4.

1ft ≈ 0.3m



≈ 10.5 m   ≈ 10.5 m

≈ 21 m

1 inch ≈ 0.0254 m

$d = 1.345\,\text{inch} \approx 0.03416\,\text{m}$

$GMR = 0.5328\,\text{inch}$

$\approx 0.01353\,\text{m}$

Assume: $\mu_0 = 4\pi \times 10^{-7}\ \text{H/m}$, $\mu_r = 1\ \text{H/m}$ (air),

$GMR = 0.01353\,\text{m}$ (given)

$$\boxed{GMD = [D_{12} D_{13} D_{23}]^{1/n}} = [D_{12} D_{13} D_{23}]^{1/3}$$

why??

$$= [10.5(21)(10.5)]^{1/3} = 13.23\,\text{m}$$

$$L_{1\phi,\text{total}} = \frac{\mu_0}{2\pi} n \ln\left(\frac{GMD}{GMR}\right) = \frac{\mu_0}{2\pi} 3 \ln\left(\frac{GMD}{GMR}\right) \quad \text{—— (1)}$$

per phase, per unit length.

$$L_a = \frac{L_{1\phi,\text{total}}}{n} \quad \text{—— (2)}$$

$$\Rightarrow \boxed{L_a = \frac{\mu_0}{2\pi} \ln\left(\frac{GMD}{GMR}\right)}$$

$$= \frac{4\pi \times 10^{-7}}{2\pi} \ln\left(\frac{13.23}{0.01353}\right) = 1.377 \times 10^{-6}\ \text{H/m}$$

$$= 0.00138\ \text{H/km} \,/\!\!/$$