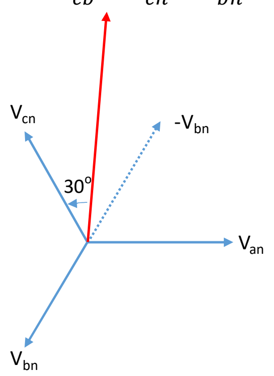
# EE2029 Introduction to Electrical Energy Systems
## (Solution for Tutorial 3 on Three-Phase Circuit Analysis)

**Solution Q.1**

Drawing the phasor diagram for abc phase sequence and showing $V_{cb}$.

$$V_{cb} = V_{cn} - V_{bn} = \sqrt{3}\,V_{cn}\angle - 30^o$$




Given $V_{cb} = 480\angle 65^o\ V$, we can get $V_{cn} = \frac{V_{cb}}{\sqrt{3}}\angle 30^o = \frac{480\angle 65^o}{\sqrt{3}}\angle 30^o = 277.13\angle 95^o\ V$.

With load impedance of $Z_Y = 60\angle - 30^o\Omega$ ,

We can get C phase current $I_c = I_{cn} = \frac{V_{cn}}{Z_Y} = \frac{277.13\angle 95^o}{60\angle -30^o} = 4.62\angle 125^o$ A

Then we can get the other line currents by adding the phase difference:

$I_a = I_c\angle - 120^o = 4.62\angle 5^o A;\ I_b = I_a\angle - 120^o = 4.62\angle - 115^o A;$

**Solution Q.2**

Line-to-neutral voltage $V_{LN} = \frac{V_{LL}}{\sqrt{3}} = \frac{500}{\sqrt{3}} = 288.675\ V$

We can take phase a voltage phasor $V_{an} = 288.675\angle 0^o\ V$

The loads are given as Wye-connected

a) $Z_Y = 30 + j0\ \Omega$

Phase a current $I_{an} = \frac{V_{an}}{Z_Y} = \frac{288.675\angle 0^o}{30+j\ 0} = 9.623\angle 0^o\ A$

$$S_{3\phi} = 3V_{an}I_{an}^* = 3 \times 288.675\angle 0^o \times 9.623\angle 0^o = 8333.76\angle 0^o VA = 8333.76 + j0\ VA$$

Average power $P_{3\phi} = 8333.76\ W$

b) $Z_Y = 30 + j72\ \Omega$

Phase a current $I_{an} = \frac{V_{an}}{Z_Y} = \frac{288.675\angle 0^o}{30+j\ 72} = 3.701\angle{-67.38^o}\ A$

$$S_{3\phi} = 3V_{an}I_{an}^* = 3 \times 288.675\angle 0^o \times 3.701\angle 67.38^o = 3205.158\angle 67.38^o = 1232.76 + j2958.6\ VA$$

Average power $P_{3\phi} = 1232.76\ W$

c) $Z_Y = 30 - j12.5\ \Omega$

Phase a current $I_{an} = \frac{V_{an}}{Z_Y} = \frac{288.675\angle 0^o}{30 - j\ 12.5} = 8.882\angle 22.62^o\ A$

$S_{3\phi} = 3V_{an}I_{an}^* = 3 \times 288.675\angle 0^o \times 8.882\angle 22.62^o = 7692.034\angle 22.62^o VA$

$= 7100.33 + j2958.49\ VA$

Average power $P_{3\phi} = 7100.33\ W$

**Solution Q.3**

Line current, $I_{line} = \frac{100}{|10 - \mathrm{j}9|} = 7.43$ A

Line-to-neutral voltage at the source,

$$|V_{Line-neutral}| = |\mathrm{I_{line}}| \times |\mathrm{Z_{total}}| = 7.43|(2 + \mathrm{j}3) + (10 - \mathrm{j}9)| = 99.7 \text{ V.}$$

Line voltage at the source,

$$V_{Line-Line} = \sqrt{3} \times |V_{Line-neutral}| = \sqrt{3} \times 99.7 = 173 \text{ V.}$$

**Solution Q.4**

We first need to combine the two loads, Δ is transformed to Y,

$$\mathrm{Z_Y} = \frac{\mathrm{Z_\Delta}}{3} = \frac{21\angle 30°}{3} = 7\angle 30°\ \Omega.$$

Now we have two balanced Y connected loads of 9∠-60⁰ Ω and 7∠30⁰ Ω in parallel.

The total load impedance per phase, $Z_{Y,total} = \dfrac{(7\ \angle 30^0)\times(9\ \angle -60^0)}{(7\ \angle 30^0 + 9\ \angle -60^0)} = 5.53\ \angle -7.87^0\ \ \Omega$

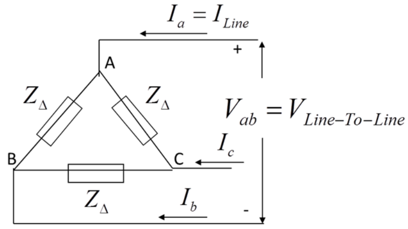
The rms line current is found from,

$$I_{line} = \frac{V_{line-to-neutral}}{|\mathrm{Z_Y}|} = \frac{V_{line-to-line}}{\sqrt{3}|\mathrm{Z_Y}|} = \frac{208}{\sqrt{3} \times 5.53} = 21.7 \text{ A}$$

The total power absorbed by the two loads is ,

$$|P_{3\Phi}| = \sqrt{3}|V_{line-to-line}||I_{line}| \times \mathrm{p.f.} = \sqrt{3} \times 208 \times 21.7 \times \cos(7.87) = 7744.15 \text{ W}$$

**Solution Q.5**




For a balanced three-phase load, $|P_{3\Phi}| = \sqrt{3}|V_{line-to-line}||I_{line}| \times \mathrm{p.f.}$ By substituting $|V_{line-to-line}| = 208$ V, $P_{3\Phi} = 2000\ W$, p.f. = 0.8 in the above equation, we have $|I_{line}| = 6.94$ A.

The phase current that pass through an impedance z can be found from

$$|I_{Phase}| = \frac{|I_{line}|}{\sqrt{3}} = 4.01 \text{ A}.$$

Let $V_{line-to-line} = 208\angle 0°$, the angle of the phase current can be found from power factor leading.

$$\angle I_{Phase} = +\cos^{-1} 0.8 = 36.87°$$

Note that the phase angle is positive because the power factor is leading.

We can find the impedance of Δ-connected load as follows.

$$Z_{\Delta} = \frac{V_{ab}}{I_{ab}} = \frac{V_{line-to-line}}{I_{phase}} = \frac{208\angle 0°}{4.01\angle 36.87°} = 51.9\angle -36.87 = 41.44 - \text{j}31.08\ \Omega$$

**Solution Q.6**

a) <u>Power triangle for the induction motor,</u>

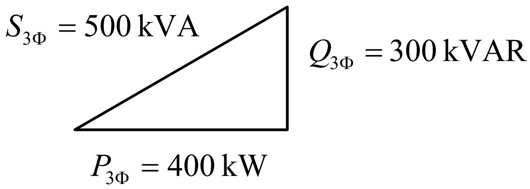
From $P_{3\Phi} = |S_{3\Phi}| \times p.f$, given that the real power, $P_{3\Phi} = 400\ kW$, we can find,

$$|S_{3\Phi}| = \frac{P_{3\Phi}}{p.f} = \frac{400}{0.8} = 500\ kVA.$$

Then, the reactive power can be found from,

$$Q_{3\Phi} = |S_{3\Phi}| \times sin(\cos^{-1}(p.f.)) = 500 \times \sin(\cos^{-1} 0.8) = 300 \text{ kVAR}.$$

Since the power factor is *lagging*, this reactive power is *absorbed* by the induction motor. The power triangle is given below.



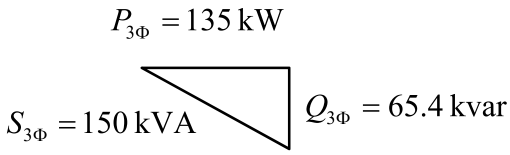

<u>Power triangle for the synchronous motor,</u>

$$S_{3\Phi} = 150\ kVA$$

$$P_{3\Phi} = |S_{3\Phi}| \times p.f = 150 \times 0.9 = 135\ kW.$$

Since the power factor is *leading*, this reactive power is *injected* by the synchronous motor.

$$Q_{3\Phi} = |S_{3\Phi}| \times sin(\cos^{-1}(p.f.)) = 150 \times \sin(-\cos^{-1} 0.9) = -65.4 \text{ kVAR}.$$




<u>Power triangle for the combined-motor load,</u>

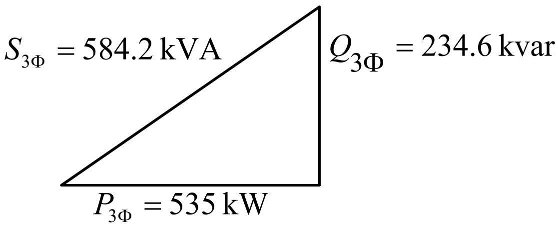
$$P_{3\Phi} = 400 + 135 = 535\ kW.$$

$$Q_{3\Phi} = 300 - 65.4 = 234.6\ kVAR$$

Since the reactive power is positive, this reactive power is *absorbed* by the combined-motor load.

The magnitude of apparent power is found below.

$$S_{3\Phi} = \sqrt{|P_{3\Phi}|^2 + |Q_{3\Phi}|^2} = 584.2\ kVA.$$




b) Power factor of the combined-motor load,

$$p.f. = \frac{P_{3\Phi}}{|S_{3\Phi}|} = 0.916$$

Since the load absorbs reactive power, the power factor is 0.916 lagging.

c) From $|S_{3\Phi}| = \sqrt{3}|V_{line-to-line}||I_{line}|$, we can find the line current below.

$$|I_{line}| = \frac{|S_{3\Phi}|}{\sqrt{3}|V_{line-to-line}|} = \frac{584.2 \times 10^3}{\sqrt{3} \times 4160} = 81.1\ A$$

d) To make the source power factor unity, the reactive power supplied by the capacitor bank, $Q_{c,3\Phi} = -234.6$ kVAR.

For a delta connected capacitor bank, the voltage applied to the capacitor at each phase is the line-to-line voltage.

$$Q_{c,1\Phi} = \frac{Q_{c,3\Phi}}{3} = -78.2\ kVAR$$

The capacitive reactance at each phase, $X_{c,1\Phi}$, can be found from $Q_{c,1\Phi} = \frac{|V_{line-to-line}|^2}{X_{c,1\Phi}}$.

We have,

$$X_{c,1\Phi} = \frac{|V_{line-to-line}|^2}{Q_{c,1\Phi}} = \frac{4160^2}{-78.2 \times 10^3} = -221.3\ \Omega.$$

The capacitive reactance is then -j221.3 Ω.

e) With the capacitor bank installed, power factor=1.

Active power delivered by the source, $P_{3\Phi} = \sqrt{3}|V_{line-to-line}||I_{line}| \times \mathrm{p.f.} = 535$ kW.

With the capacitor bank installed, power factor=1. The line current magnitude is found below.

$$|I_{line}| = \frac{|P_{3\Phi}|}{\sqrt{3}|V_{line-to-line}| \times \mathrm{p.f.}} = \frac{535 \times 10^3}{\sqrt{3} \times 4160 \times 1} = 74.3\ A$$

Note that once the power factor is adjusted to 1, the line current magnitude is reduced from 81.1 A to 74.3 A. This helps to reduce the power losses in the transmission lines.