EE2029 Introduction to Electrical Energy Systems

Synchronous Generator Modelling

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Learning outcome

Use electrical engineering principles to explain the basic operation of the electrical generator in electrical energy system and able to identify and construct their equivalent circuits appropriately.

Outline

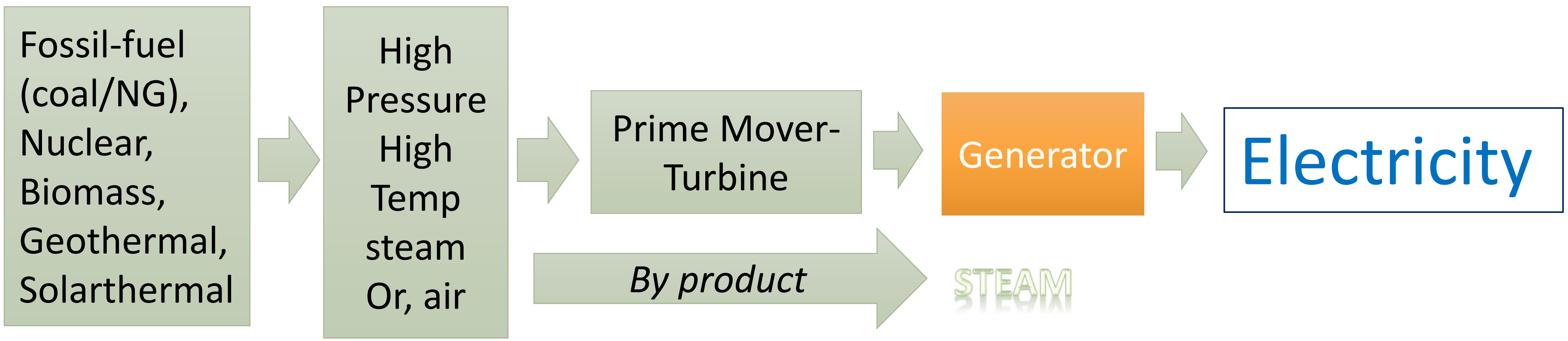
- Types of power plants
- AC generator principle
- Equivalent circuit of synchronous generators
- Calculation of complex power output of generators

How Electricity is Generated?

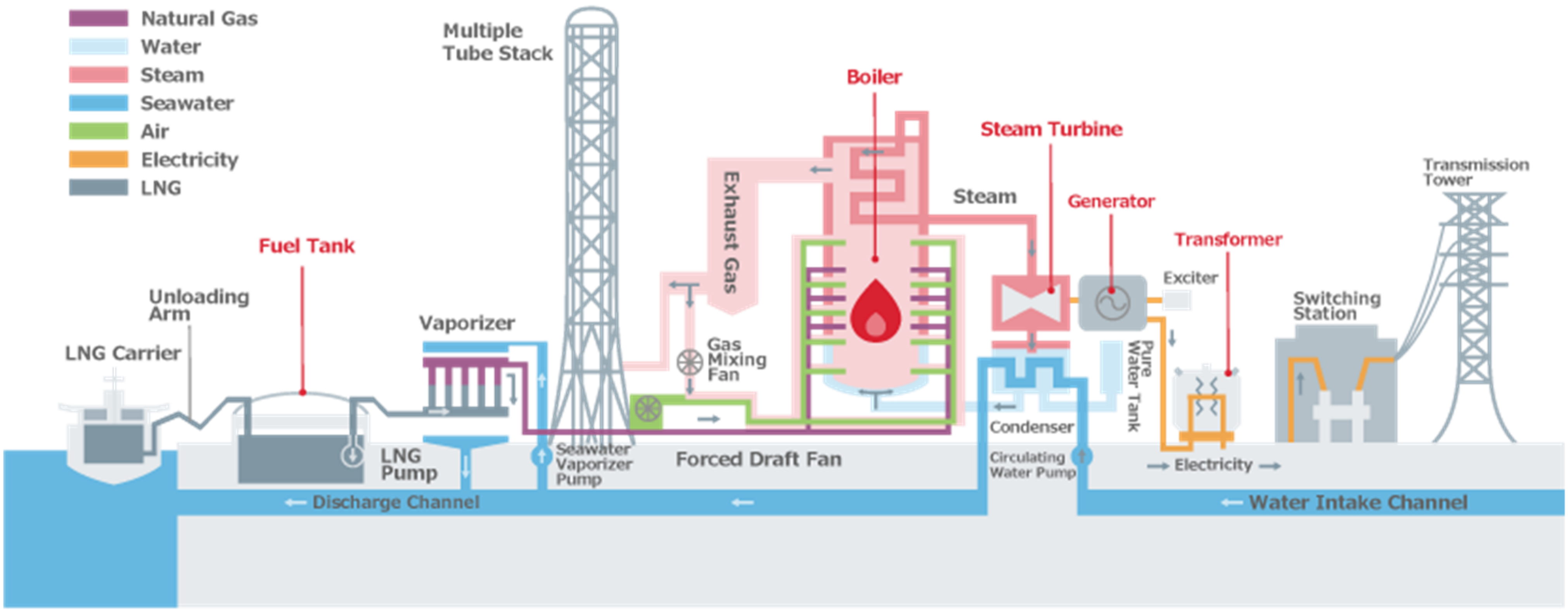
- Electricity is generated from other forms of energy to electricity through the energy conversion process.
- The most common conversion process to generate electricity is to convert mechanical energy using a generator.
- This process is called “Electromagnetic induction”.
- The mechanical energy comes from turning the turbine (prime mover).

Energy Conversion Process

- Turbine is usually moved by high pressure and high temperature steam or hot air.
- The steam is created from boiling water in a closed loop system to reduce impurities that may affect the turbine efficiency.
- The source of heat depends on fuel types.
- Heat energy is usually measured in ‘British Thermal Unit’ or ‘BTU’.

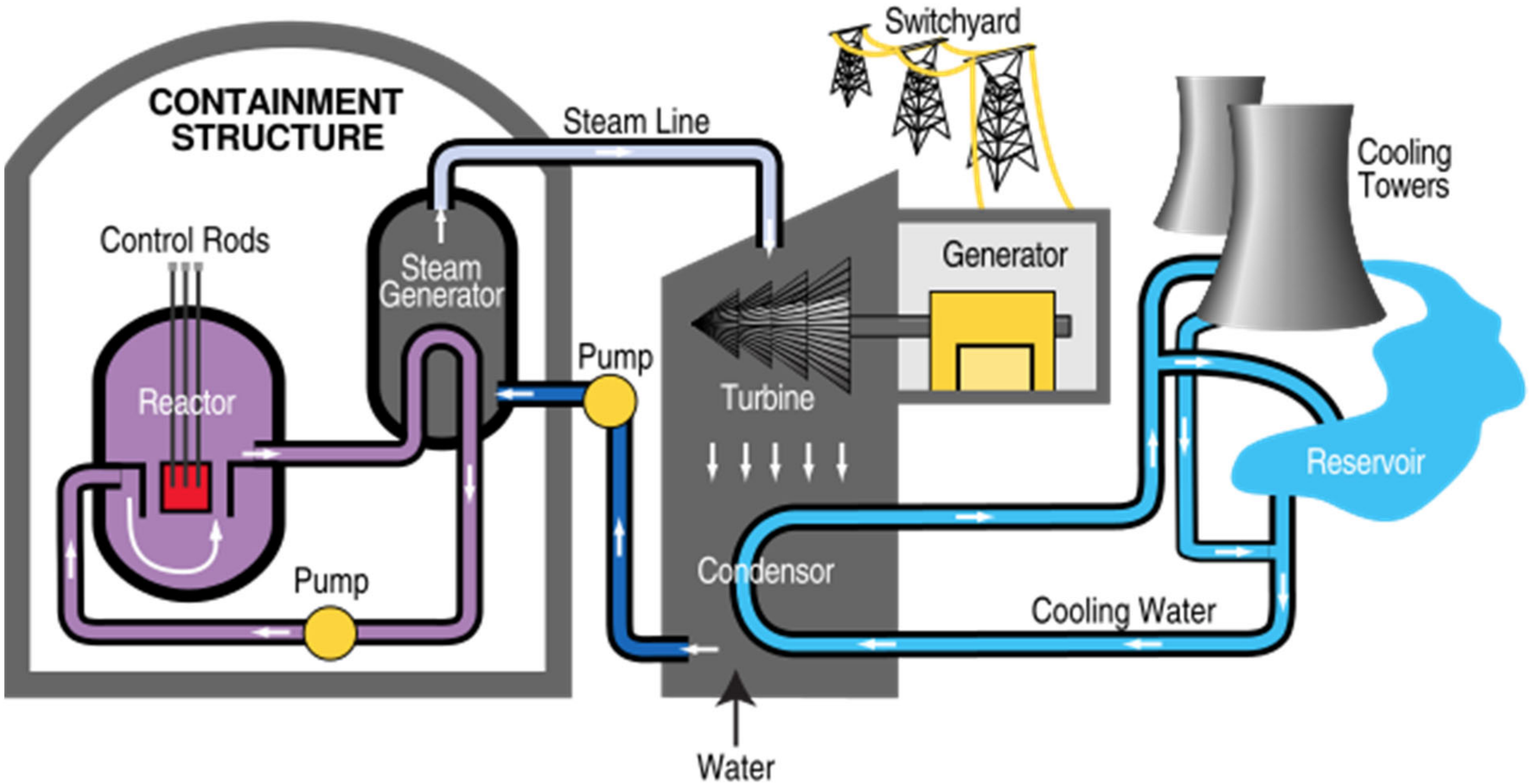


An Example of LNG fired Thermal Power Plant



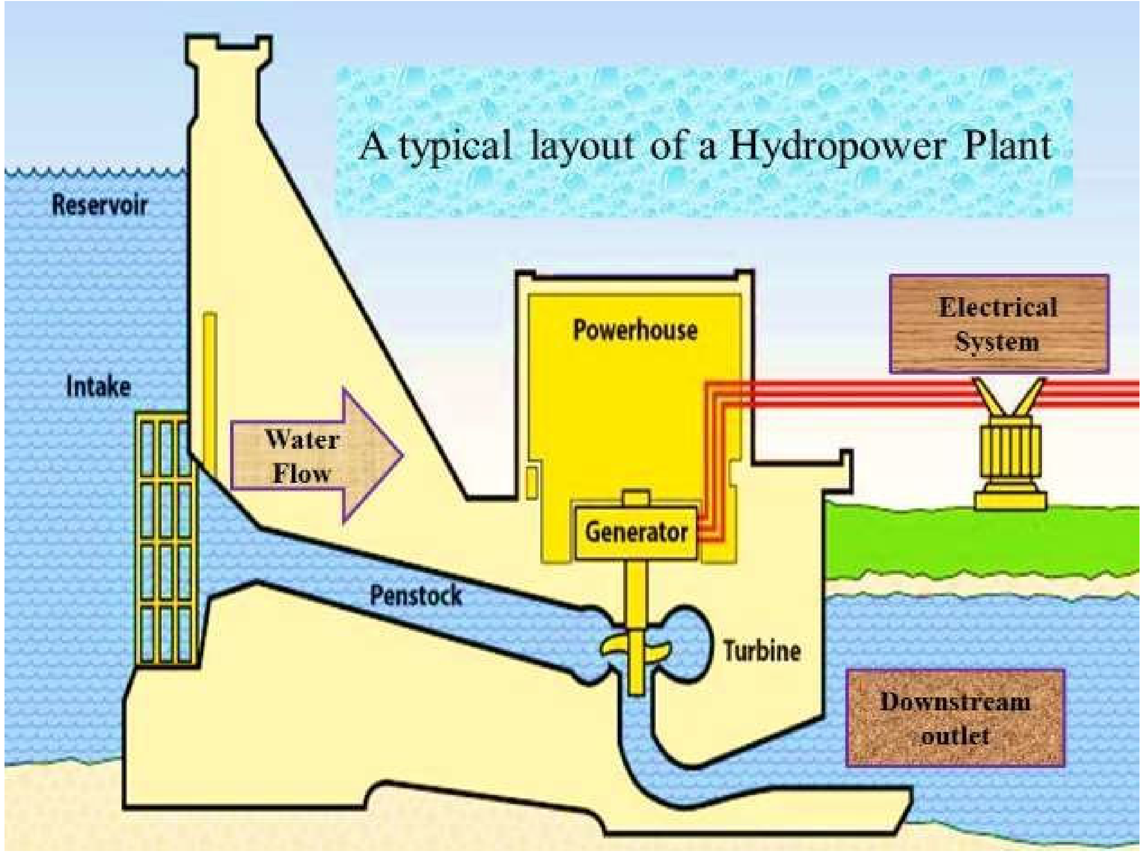
Source: https://www.jera.co.jp/english/business/thermal-power/type

Nuclear Power Plant



Source: https://en.wikipedia.org/wiki/File:PWR_nuclear_power_plant_diagram.svg

Example of hydroelectric plant



Simple generator concept

Electromagnetic induction

Main components of a generator

Types of rotor

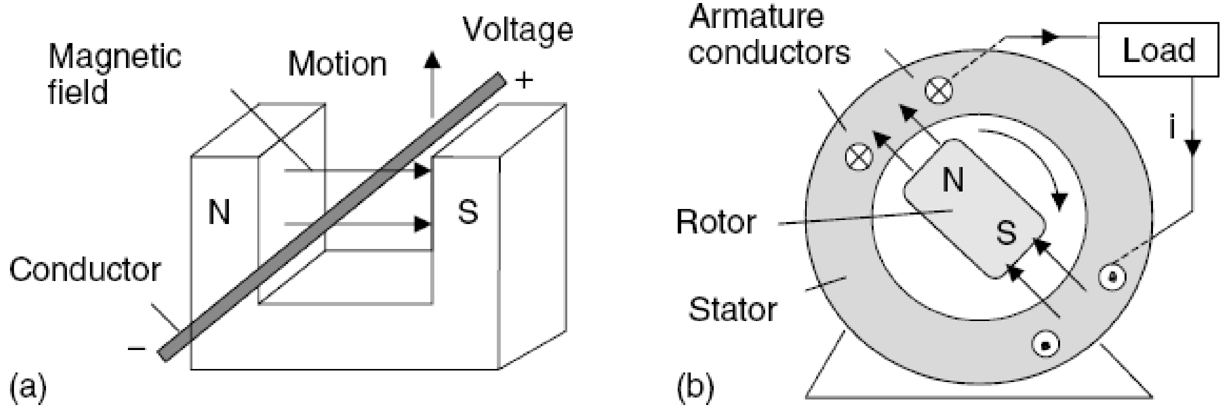
Rotor synchronous speed

Electromagnetic Induction

When a conductor sees a changing magnetic field, voltage is induced across the conductor, according to Faraday's Law.

Michael Faraday,
English chemist and physicist, 1791-1867.

$$e = N\frac{d\phi}{dt}$$



A **mechanical force** is needed to move the magnetic field to generate *"Relative motion"* between a conductor and a magnetic field.

Mechanical Input → Electrical Output

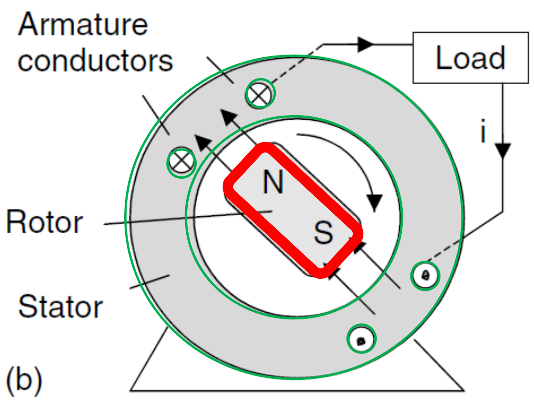
Main Components of a Generator

Rotor

- Moving part that is usually made of electromagnet materials.
- It carries the source of magnetic field.

Stator

- Stationary part that is also made of electromagnetic materials.
- It contains a set of conductors called 'armature winding' which carry the generator current.

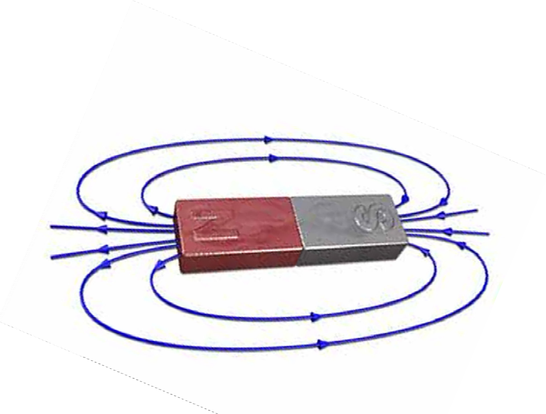


● indicates the positive current is directed out of plane of the paper.
× indicates the positive currents is directed into the plane of the paper.

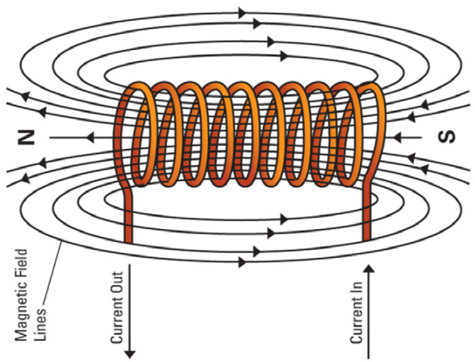
Magnetic Fields at Rotor

We can produce constant magnetic fields at the rotor by two methods.

1. Using permanent magnet. This is only suitable for small generators.
2. Create magnetic fields using DC current supplied through coil. The DC power source is called ‘exciter’. The coil is called ‘field winding’.

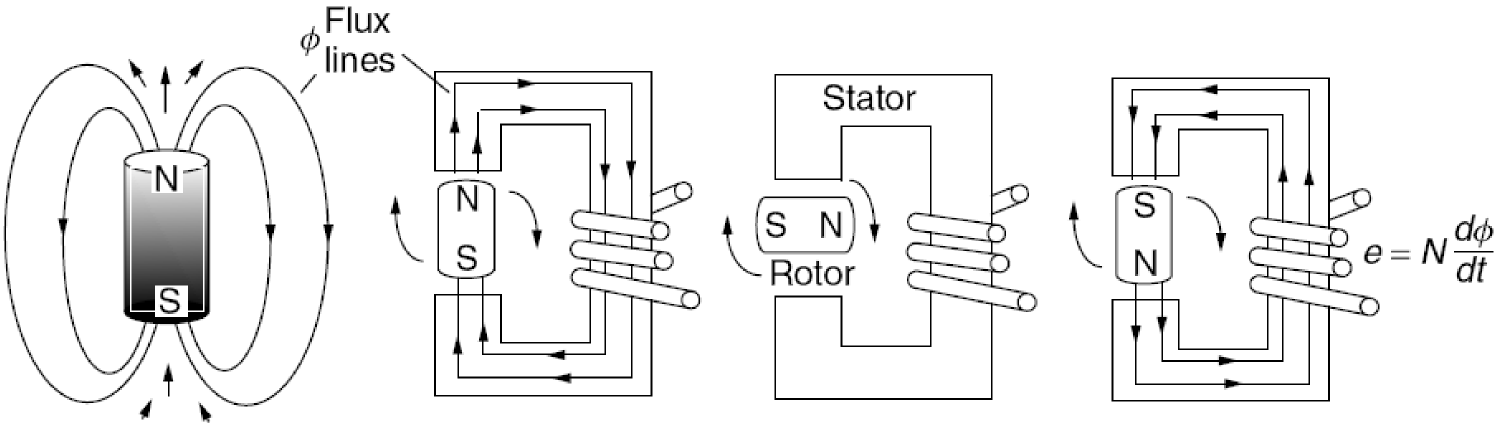


Source:
http://www.magnet.fsu.edu/education/tutorials/magnetminute/permanent.html

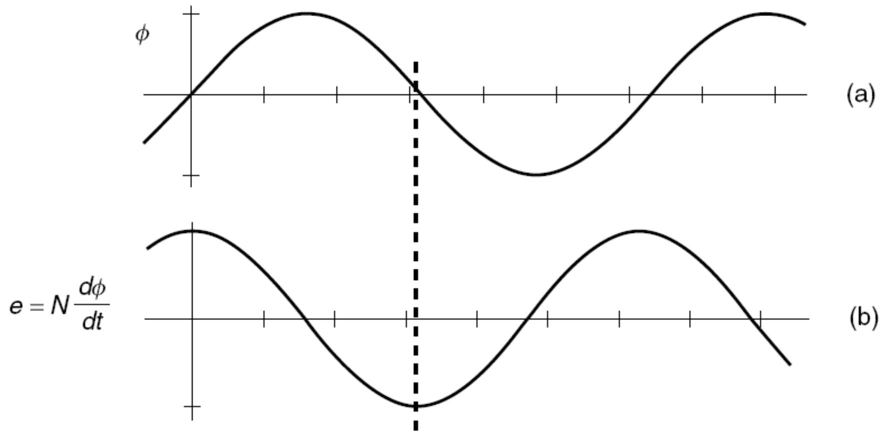


Source:
http://www.lanl.gov/news/index.php/fuseaction/1663.article/d/20085/id/13276

Electromagnetic Induction at Stator

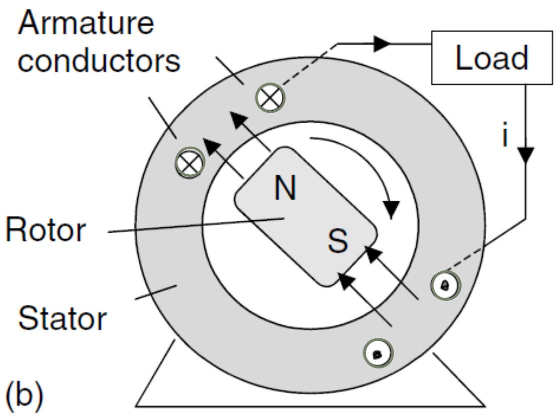


$$e = N\frac{d\phi}{dt}$$

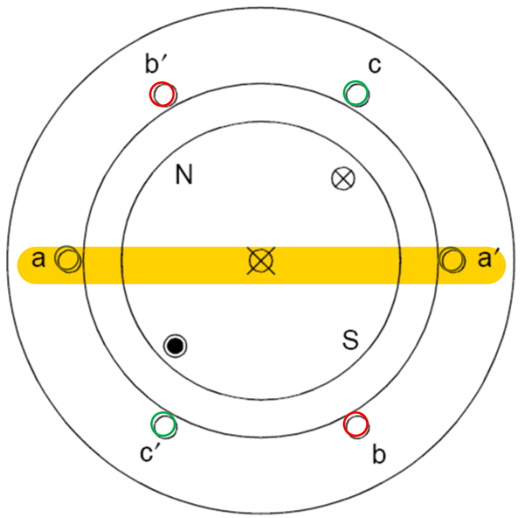


Three-Phase AC Generators

For a three-phase AC generator, the stator contains three sets of coils for phase a, b, and c.



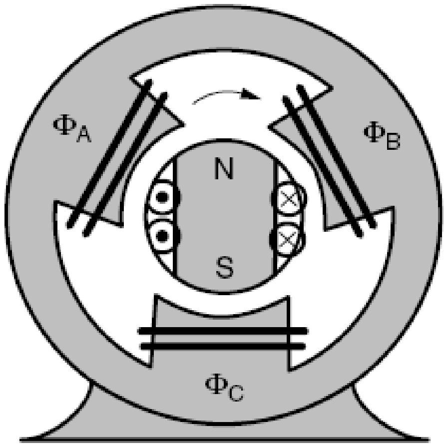
Single-Phase AC generator

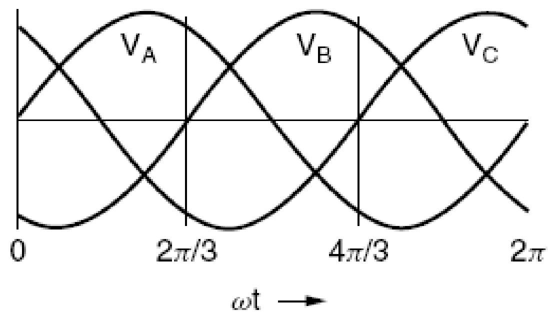


Three-Phase AC generator

Positive vs Negative Sequence

Positive and negative sequences can be achieved by **how we label the conductors** at the stator.





By swapping b to c, the voltage source will produce negative sequence.

Recall: Additional Advantages of 3-Phase Balanced Circuit

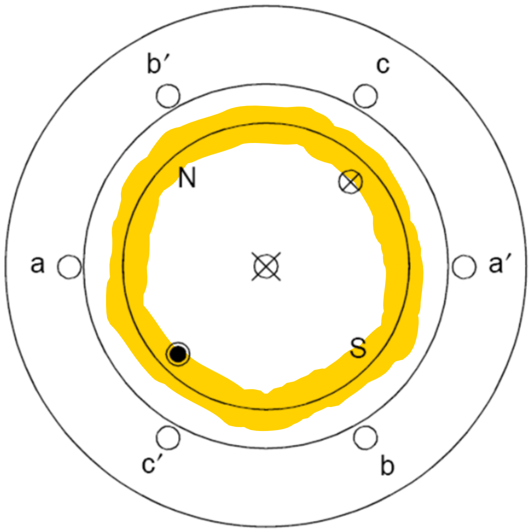
Constant power delivery to load

- This also implies constant mechanical power input.
- When mechanical power input is constant, mechanical shaft torque is also constant.
- This helps to reduce shaft vibration and noise, extending the machine's lifetime.

Types of Rotor

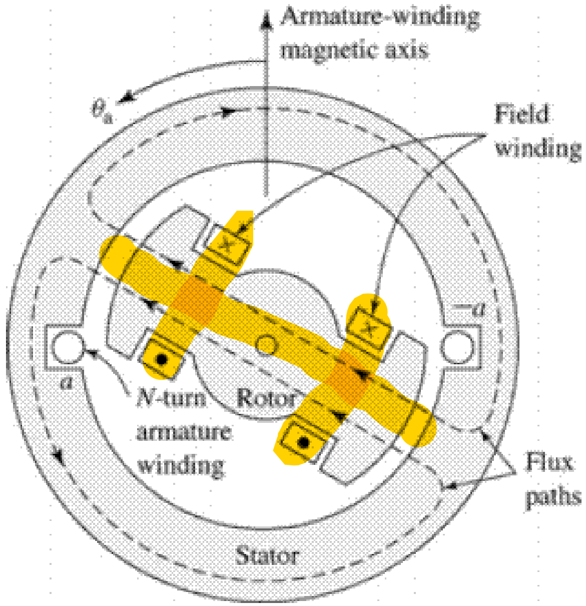
Cylindrical (Round) Rotor

- High speed application such as steam turbine at 3600 or 1800 rpm



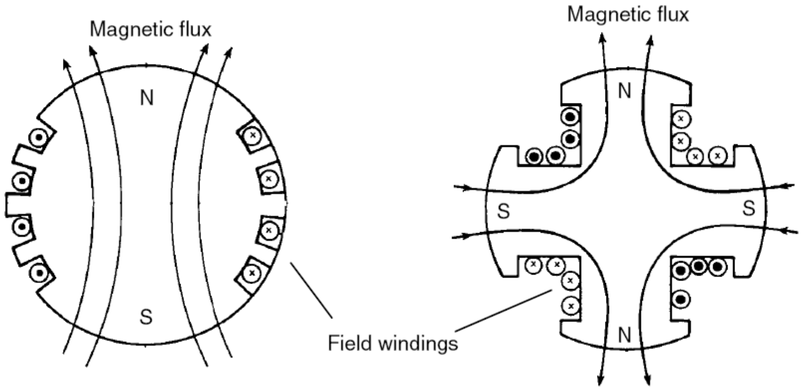
Salient Rotor

- Low speed hydro turbines at a few hundred rpm.



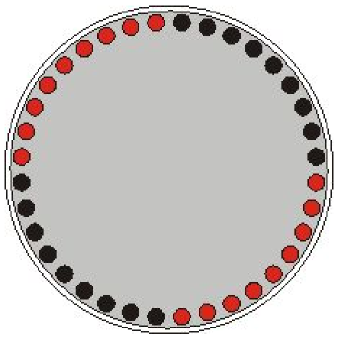
Source: http://cnx.org/content/m28323/latest/?collection=col10767/latest

Multi-Pole Rotor

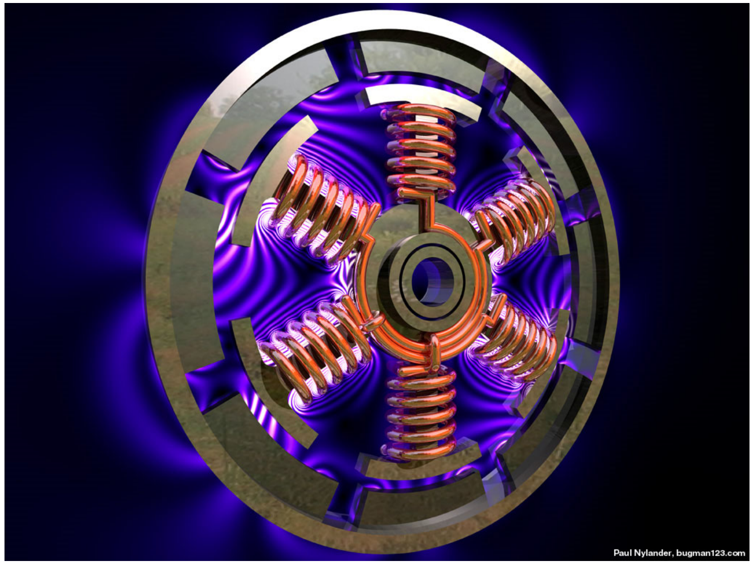


2-pole round rotor

4-pole salient-pole rotor



4-pole round rotor

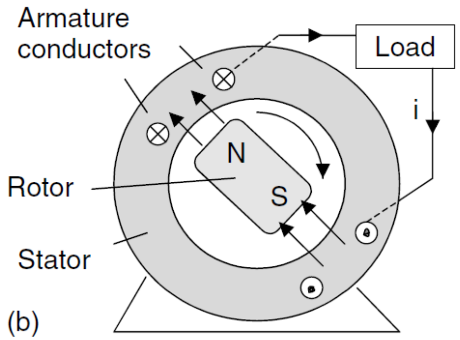


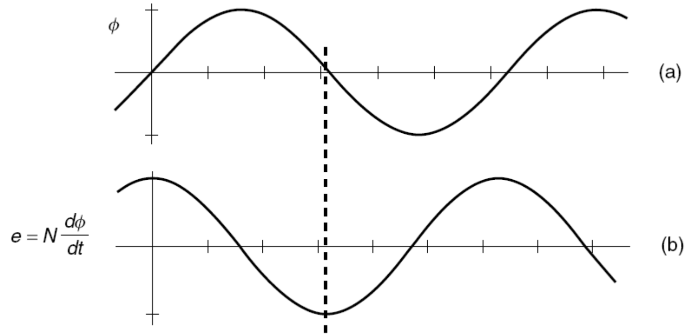
6-pole salient-pole rotor

Source: http://www.bugman123.com/Engineering/Motor-large.jpg

Rotor Speed for 2-Pole Generators

- The speed of the rotor shaft (n) is given in revolution per minute (rpm).
- For example, consider a two-pole single-phase generator on the right, the frequency of induced AC voltage is the same as the speed of rotor.
- Assume 60 Hz voltage, we can find the rotation speed by simply changing the unit from 'Hz' to 'rpm'.



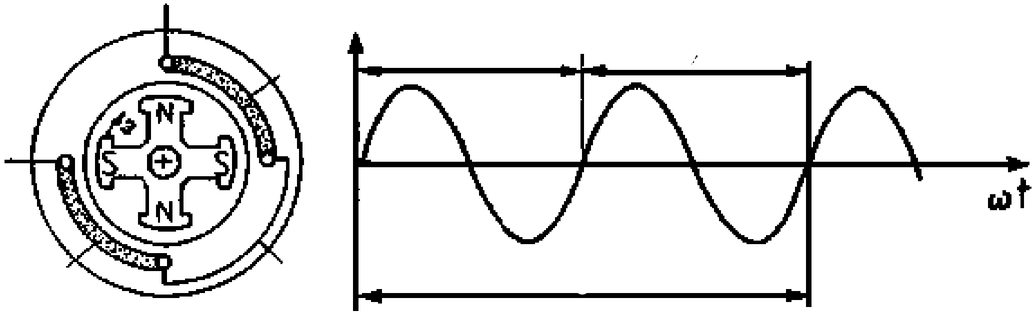


$$n\,(\text{rpm}) = \frac{1\,\text{revolution}}{\text{cycle}} \times f\,(\text{cycles/sec}) \times \frac{60\,\text{sec}}{\text{min}} = 3600\text{ rpm}$$

p = 4

Example: rotor Speed for 4-Pole Generators

Source: http://www.fastonline.org/CD3WD_40/CD3WD/ELECTRIC/GTZ021E/EN/B309_6.HTM



When the machine rotate for **1 revolution**, the induced voltage has **2 cycles**.

$$n\,(\text{rpm}) = \frac{1\ \text{revolution}}{2\ \text{cycle}} \times f\,(\text{cycles/sec}) \times \frac{60\ \text{sec}}{\text{min}}$$

p/2 = 4/2 = 2

If the frequency is 60 Hz, the rotor speed is 1800 rpm.

Rotor Speed for Multi-Pole Generators

- For p-pole generators, when the machine rotate for 1 revolution, the induced voltage has p/2 cycles.

- Then, $$n\,(\text{rpm}) = \frac{1\ \text{revolution}}{\frac{p}{2}\ \text{cycles}} \times f\,(\text{cycles/sec}) \times \frac{60\ \text{sec}}{\text{min}}$$

- We can relate the voltage frequency (Hz) to rotor speed (rpm) using,

$$f = \frac{np}{120}$$

f: voltage frequency (Hz)
n: rotor speed (rpm)
p: number of poles

Frequency (Hz) vs Rotor Speed (rpm)

$$f = \frac{np}{120}$$

f: voltage frequency (Hz)
n: rotor speed (rpm)
p: number of poles

Number of Poles	50 Hz	60 Hz
2	3000	3600
4	1500	1800
6	1000	1200
8	750	900
10	600	720
12	500	600

Synchronous Speed

- All generators connected to the system must produce AC voltage at the same frequency, f_e.
- This implies that a generator must run at a constant speed.
- We refer to this rotor speed as 'synchronous speed', n_{sync}.
- A synchronous speed is found from:

$$n_{sync} = \frac{120 f_e}{p}$$

Synchronous Generator

Connecting a generator to the grid

Excitation voltage

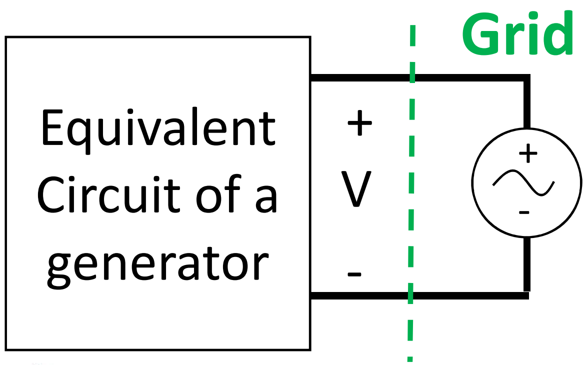
Armature reaction

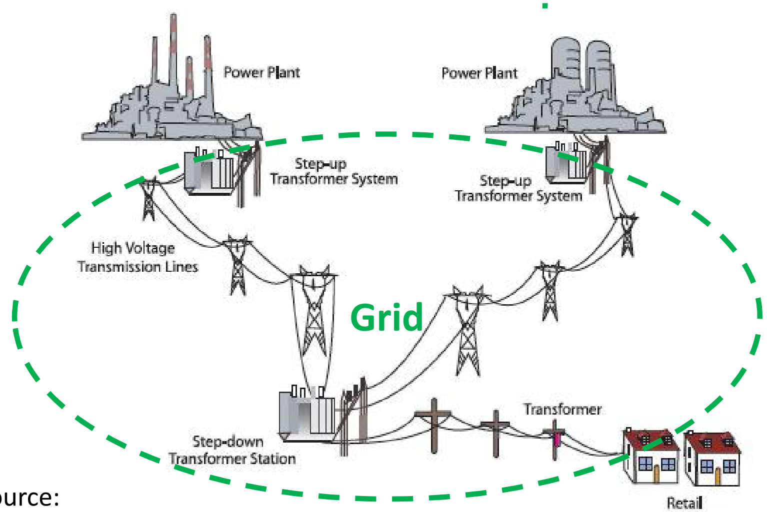
Equivalent circuit of a generator

Connecting A Generator to the Grid

Four conditions need to be met before connecting a generator to the grid:

1. The three-phase voltage must have the same **frequency** as the grid.
2. The three-phase voltage must have the same **amplitude** at its terminals as the one of the grid voltage.
3. The three-phase voltage must have the same **phase sequence** as the grid voltage.
4. The three-phase voltage must be **in phase** with the grid voltage.



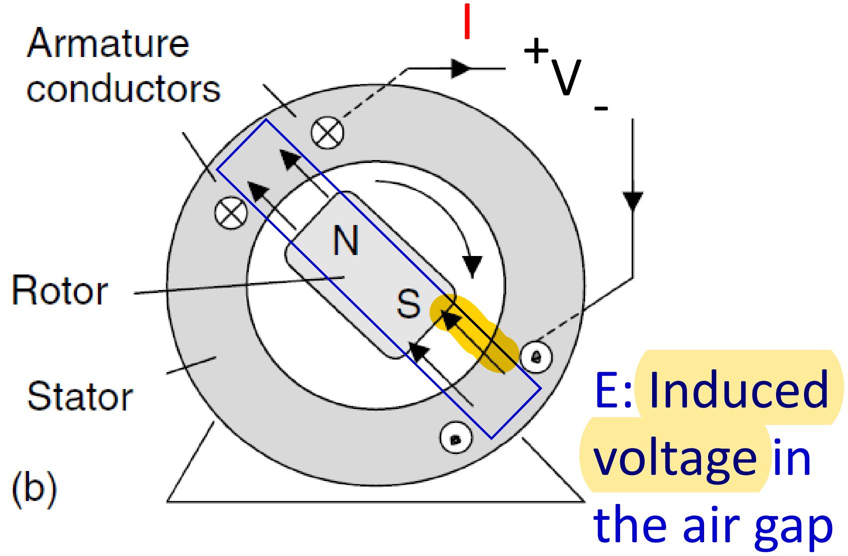


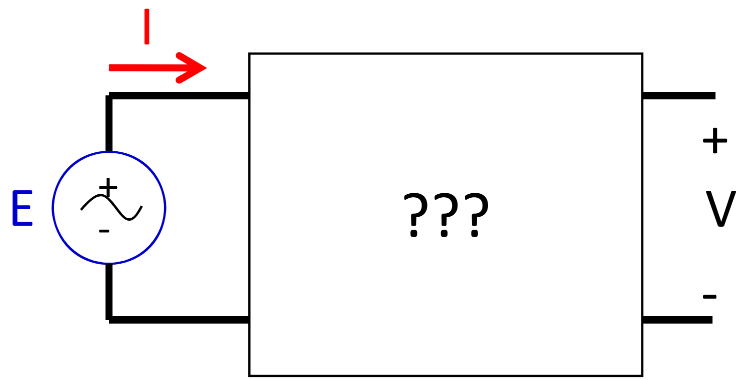
Source: http://www.thermalfluidscentral.org/encyclopedia/index.php/Generation,_Transmission,_and_Distribution_of_Electricity

Equivalent Circuit of a Generator

An equivalent circuit of a generator is given in ***per-phase*** representation.

- E = "Excitation" voltage i.e. internal Electromotive force (EMF) voltage (line-to-neutral value).
- V = "Grid" voltage i.e. terminal voltage (line-to-neutral value). We usually use this voltage angle as a reference angle.
- I = Armature current or load current.



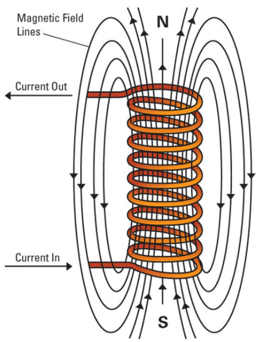


Excitation Voltage (E)

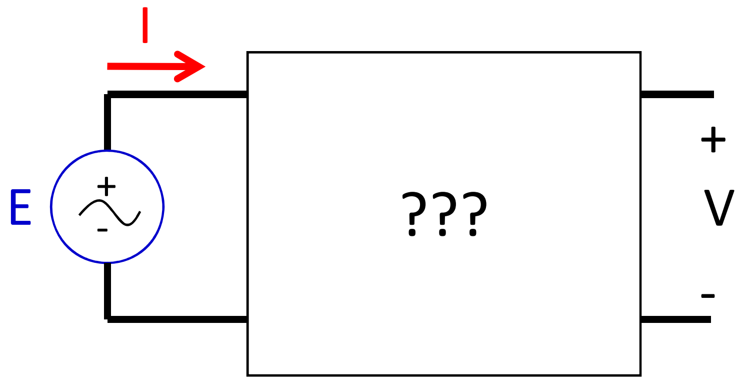
Flemings RHR

Excitation voltage (E) or induced EMF is caused by an induced magnetic flux in the air-gap.

- The magnetic flux on the rotor is created by a field winding at the rotor.



Rotating a constant magnetic field produces E.



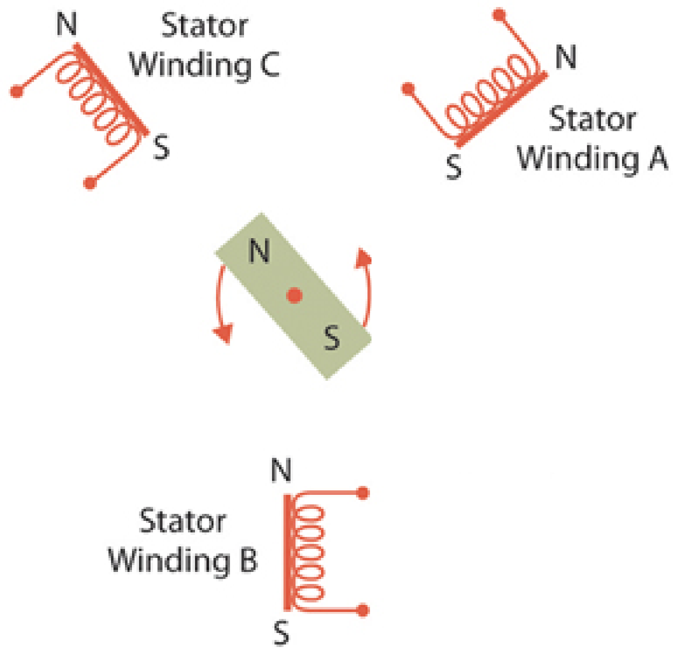
Source: http://www.lanl.gov/news/index.php/fuseaction/1663.article/d/20085/id/13276

to adjust E:
-> adjust no of coils
-> permanent magnet CANNOT

Magnetic Flux in the Air Gap

Magnetic Flux in the air gap comes from two parts.

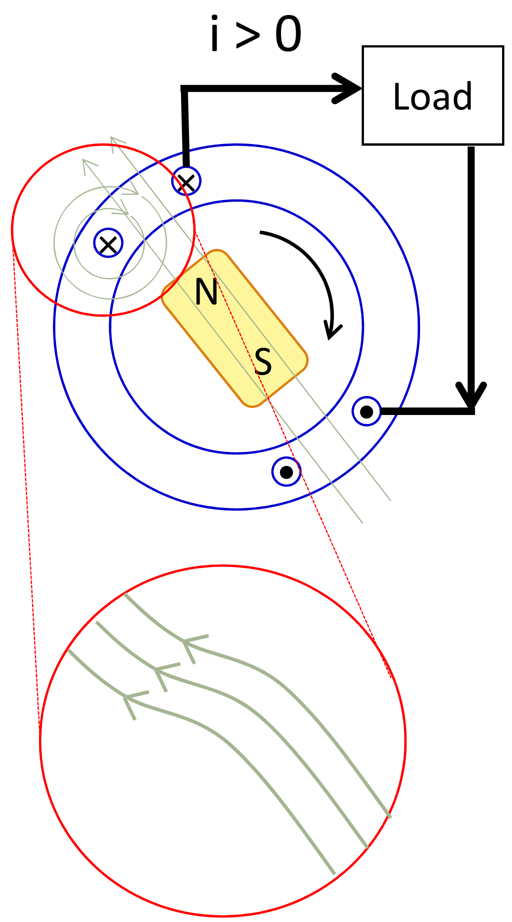
1. Field current in the rotor circuit creates a constant magnetic field around the rotor.
2. When the rotor turns, there will be induced voltage at the stator winding. After we connect stator winding to load, there will be stator (armature) currents in the stator (armature) circuit. Armature current will also create another magnetic field around it too!!



Source: http://www.ecnmag.com/article-brushless-dc-motor-control-111609.aspx

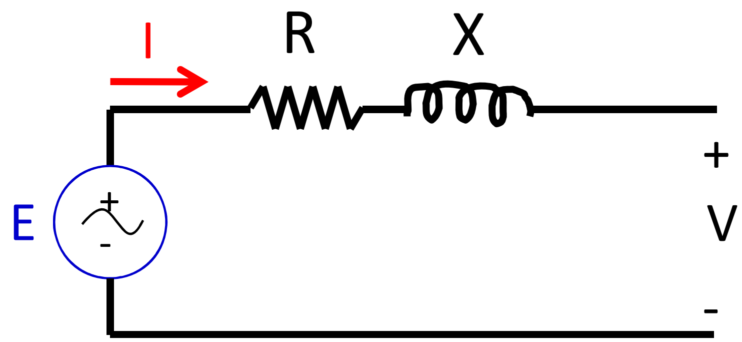
Armature Reaction

- Magnetic flux created from stator(armature) currents opposes the magnetic flux from field current.
- Flux linkage losses as a result of armature reaction are represented by an inductance called **armature reactance, Xa**.



An Equivalent Circuit

- R = resistance in the armature winding.
- X = synchronous reactance, representing flux linkage losses with a leakage reactance in the airgap, X_l and the armature reaction, X_a.



Operations of a Synchronous generator

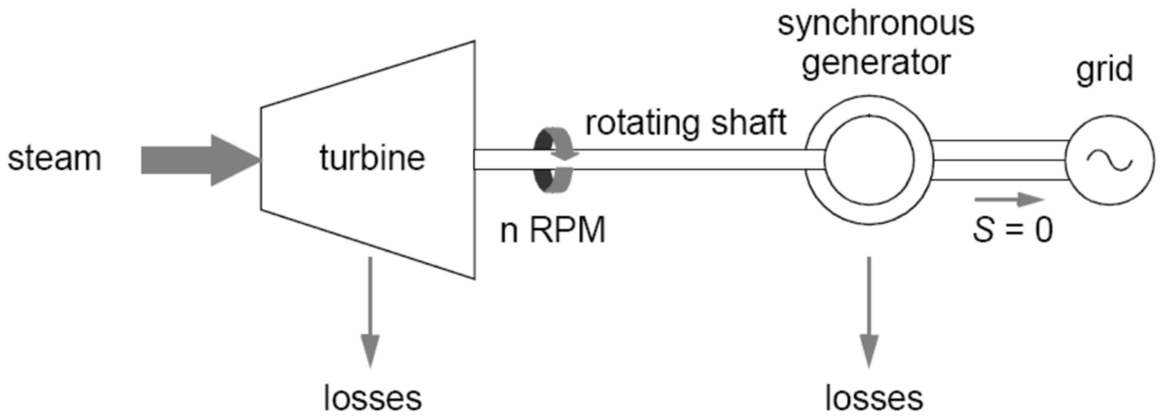
No load operation

Loaded operation

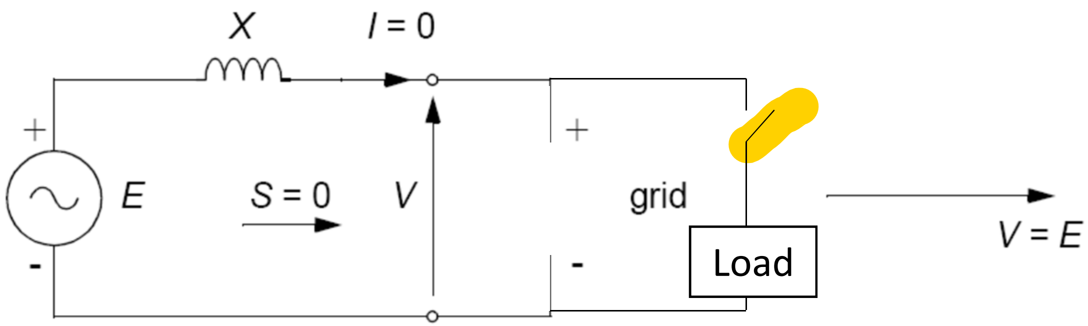
Phasor diagram

Complex power supplied by a generator

No Load Operation

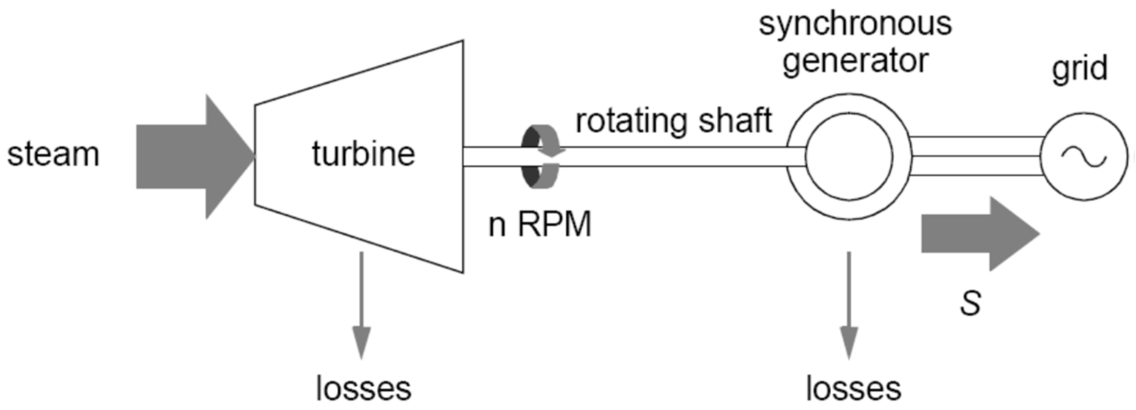


Assume that the resistance 'R' in the armature winding is negligible.



No power exchange with the grid

Loaded Operation



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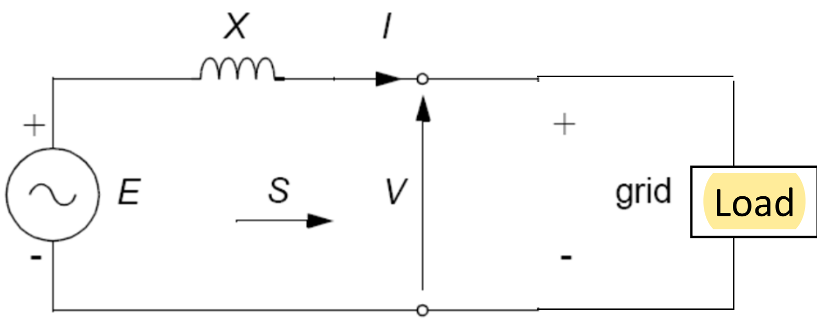
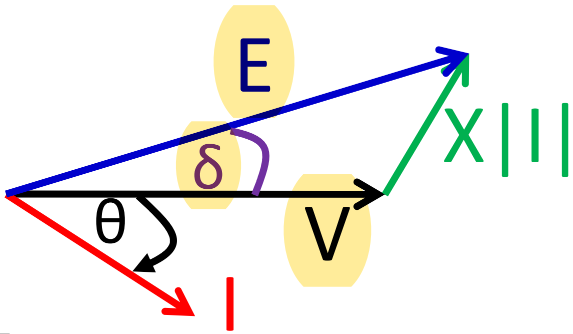


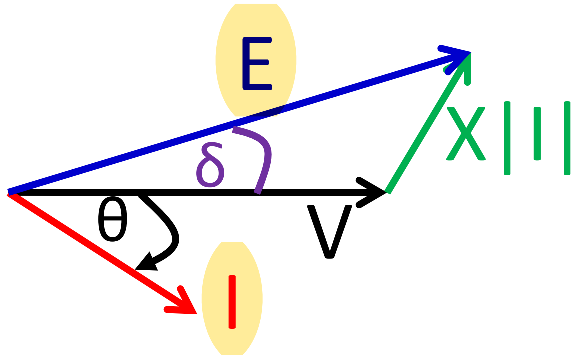


δ is called ‘power angle’, we’ll see why.

Generator injects power into the grid

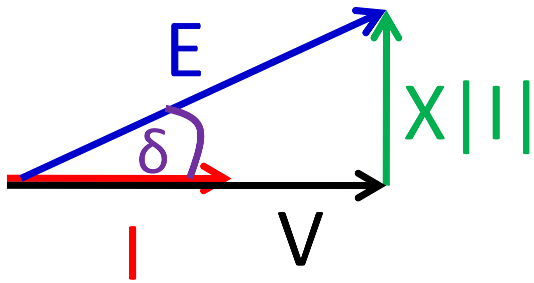
Phasor Diagram at Different Operating Conditions

Assume that the resistance 'R' in the armature winding is negligible.

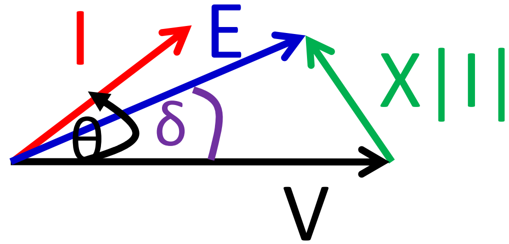


Lagging pf load

E = V + (I x jX)
-> lagging -> |E| largest



Unity pf load
θ = 0



Leading pf load

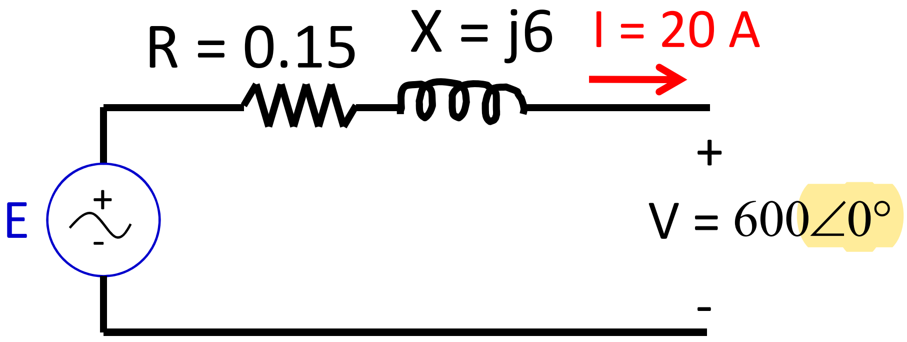
'E' Internal EMF; 'V' Terminal voltage; 'I' Armature current

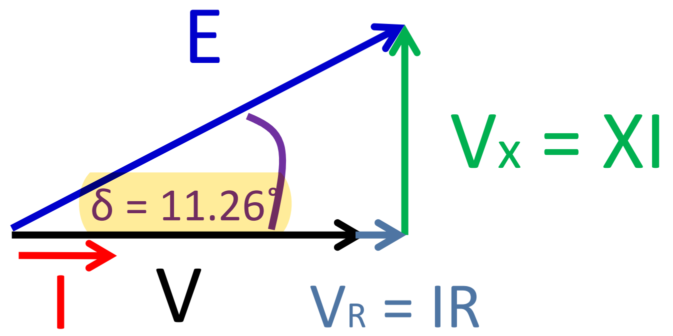
LAGGING: current is -ve!!!!!

Example

Given an equivalent circuit of three-phase wye-connected synchronous generator with a terminal voltage of 600 V per phase, stator reactance of 6 Ω per phase and an armature resistance of 0.15 Ω per phase. If the machine is connected to *a resistive load* that draws 20 A, find the internal EMF and draw a Phasor diagram.

Solution

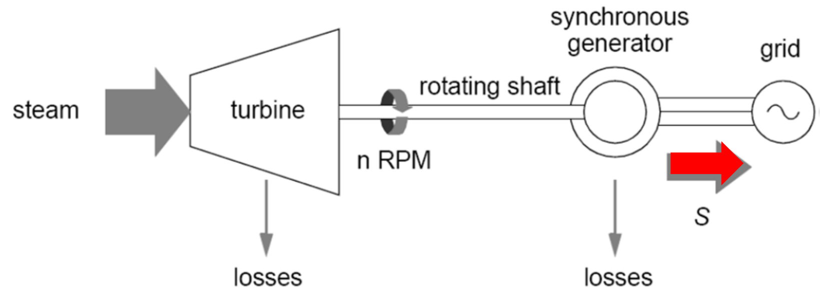


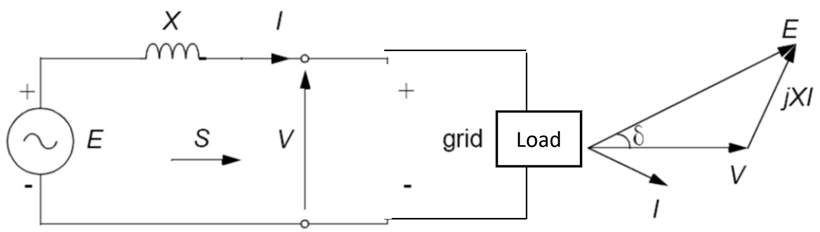


$$E = I(R + jX) + V_T = 20(0.15 + j6) + 600\angle 0^\circ = 614.82\angle 11.26^\circ$$

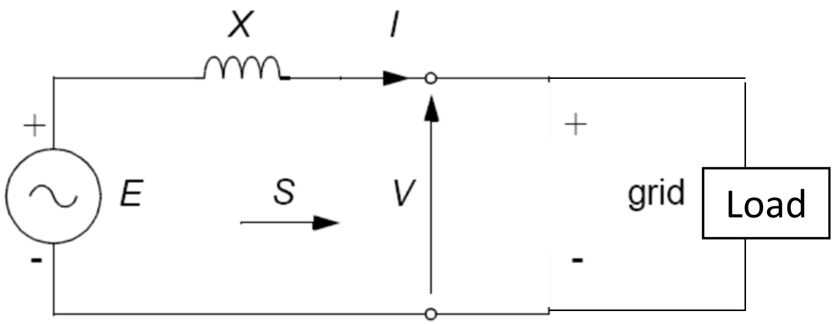
Complex Power Supplied

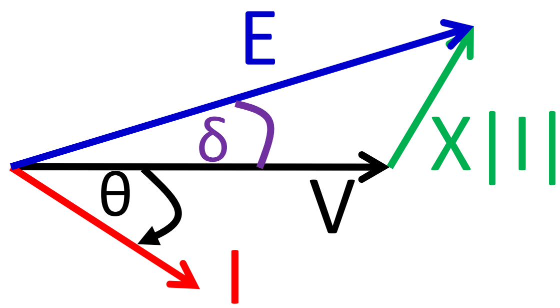
- Complex power ‘S’ supplied by a generator can be calculated.
- Using the equivalent circuit to find complex power in terms of excitation voltage ‘E’, terminal (grid) voltage ‘V’, and synchronous reactance ‘X’.
- Typically, the resistance ‘R’ in the armature winding is negligible when compared to a synchronous reactance. **In this analysis, we omit the resistance**.





Complex Power Supplied Per Phase





From $S_{1\Phi} = VI^*.$

The grid voltage magnitude V is usually known so we let $I = \dfrac{E - V}{jX}$, then we can write,

$$S_{1\Phi} = \frac{|V||E|}{X}\sin(\delta) + j\left\{\frac{|V||E|}{X}\cos(\delta) - \frac{|V|^2}{X}\right\}$$

Three-Phase Complex Power Supplied

We have,

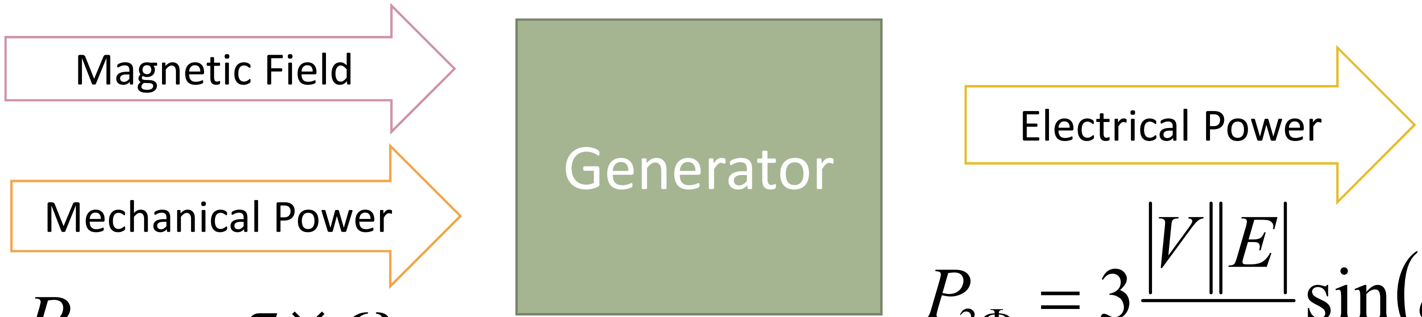
$$S_{3\Phi} = 3\frac{|V||E|}{X}\sin(\delta) + j\left\{3\frac{|V||E|}{X}\cos(\delta) - 3\frac{|V|^2}{X}\right\}$$

$$P_{3\Phi} = 3\frac{|V||E|}{X}\sin(\delta)$$

$$Q_{3\Phi} = 3\frac{|V||E|}{X}\cos(\delta) - 3\frac{|V|^2}{X}$$

Real and reactive power Output

Steady State Operation



$$P_{mech.} = \tau \times \omega$$

$$P_{3\Phi} = 3\frac{|V||E|}{X}\sin(\delta)$$

- When electrical load is increased, we need to increase mechanical power input.
- The speed of the rotor (ω) needs to be constant because rotor speed determines the voltage frequency and the frequency needs to be kept **constant.**
- We can only increase the **mechanical 'Torque'** to supply additional electrical load while maintaining the same speed.

Real Power Output

From

$$P_{3\Phi} = 3\frac{|V||E|}{X}\sin(\delta)$$

- |V| and X are constant values.
- |E| depends on the magnitude of magnetic field at the rotor.
- When the magnetic field is kept constant and mechanical power input is increased, the electrical power output will be increased.
- Since |V|, |E|, and X are kept constant, power angle will be increased.

Power Angle

From

$$P_{3\Phi} = 3\frac{|V||E|}{X}\sin(\delta)$$

Consider three cases:

Power angle	Real power output	Operation mode
δ > 0	P > 0	Supply power as generator
δ = 0	P = 0	No power exchange
δ < 0	P < 0	Absorb power as a motor

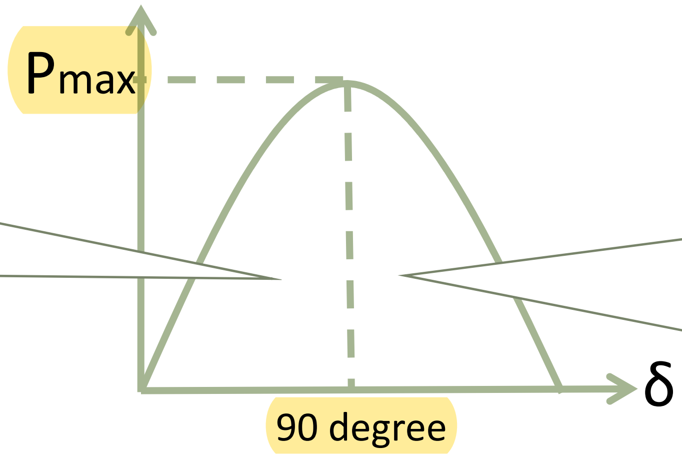
This is why δ is called ‘power angle’!

Maximum Power Transfer

- In theory, the power angle $\delta \leq 90$ degree.
- This limitation is called "Steady-state stability limit".
- Above 90 degree, generator will lose synchronism.
- The maximum power transfer is the real power output when the power angle is 90 degrees.

$$P_{3\Phi} = 3\frac{|V||E|}{X}\sin(\delta)$$

Under this region, when the mechanical power input increases, the power angle increase and as a result the electrical power output increases



Under this region, when the mechanical power input increases, the power angle increase BUT the electrical power output decreases. In order to balance mechanical power input to electrical power output, the machine will adjust its rotational speed, hence out of synchronism.

Reactive Power Output

- From $$Q_{3\Phi} = 3\frac{|V||E|}{X}\cos(\delta) - 3\frac{|V|^2}{X} = 3\frac{|V|}{X}\{|E|\cos(\delta) - |V|\}$$
- Reactive power control is done by adjusting |E|. (Although the power angle also affects the reactive power output, internal voltage magnitude dominates the final output.)
- Consider three cases,

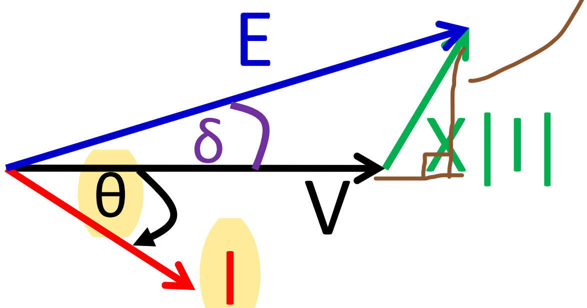
Cases	Reactive power output	Operation mode
$\|E\|\cos\delta > \|V\|$	$Q > 0$	Supply reactive power. This mode is called 'Overexcited'.
$\|E\|\cos\delta = \|V\|$	$Q = 0$	No reactive power exchange
$\|E\|\cos\delta < \|V\|$	$Q < 0$	Absorb reactive power. This mode is called 'Underexcited'.

Reactive Power Exchange

We can vary the magnitude of excitation voltage to either supply or absorb variable amount of reactive power.

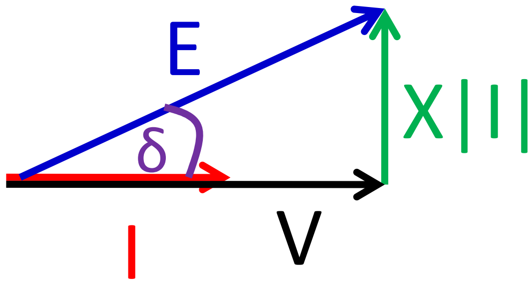
|E| cos δ

Lagging pf load



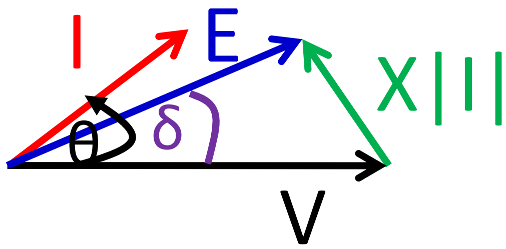
Supply reactive power ➔ Over excited

Unity pf load



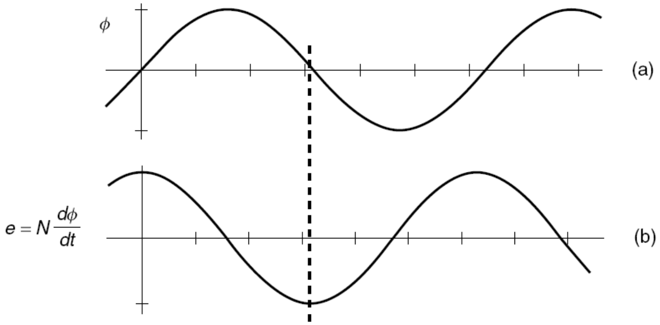
No reactive power exchange

Leading pf load

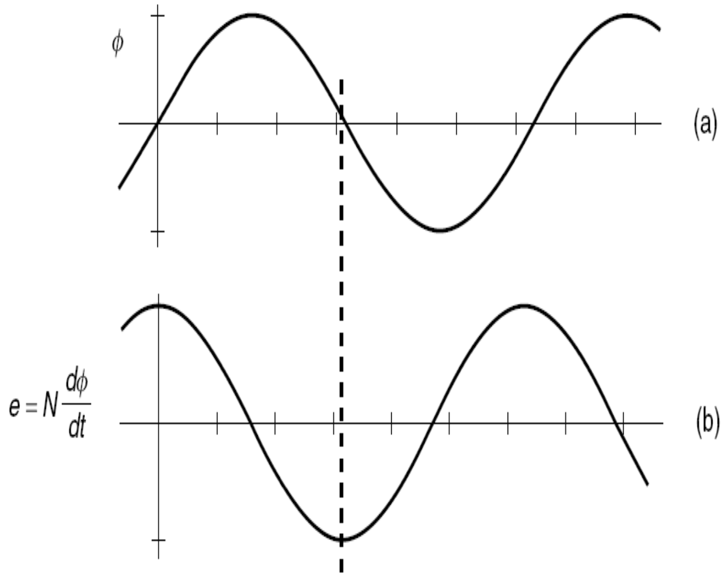


Absorb reactive power ➔ Under excited.

Adjusting Excitation Voltage



Magnetic field will still rotate at the same frequency with higher magnitude.



The magnetic field can be intensified with higher *__field current__* magnitude. As a result, excitation voltage of a generator will be increased when we increase the magnitude of magnetic field.

Example

This voltage is line-to-line voltage.

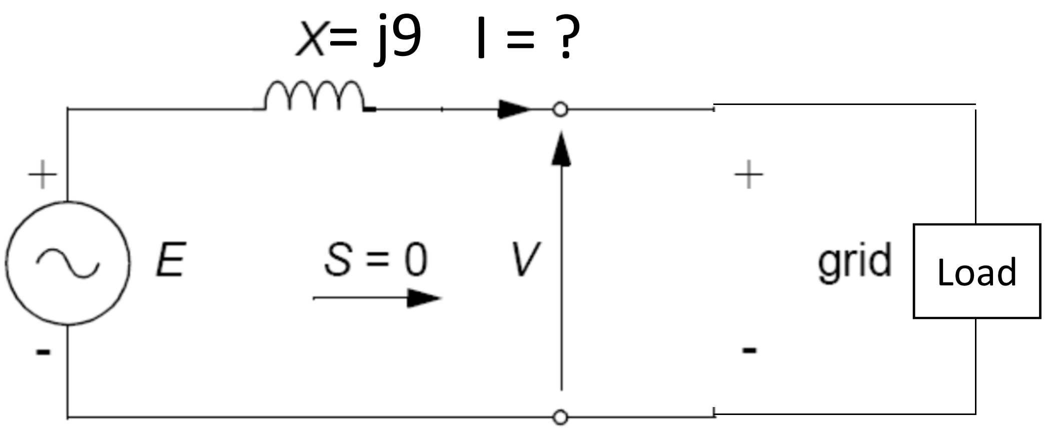
A 50-MVA, 30kV, three-phase, wye-connected 60 Hz synchronous generator has a synchronous reactance of 9 Ω per phase and a negligible resistance. The generator is supplying to the system a rated power at 0.8 p.f. lagging at the rated terminal voltage.

(a) Determine the excitation voltage per phase (E) and the power angle (δ).

(b) With the excitation held constant at the value found in (a), the driving torque is reduced until the generator is delivering 25 MW. Determine the armature current and the new power factor.

(c) what is the maximum power transfer by this generator at the current excitation?

Solution: 2(a) Excitation Voltage

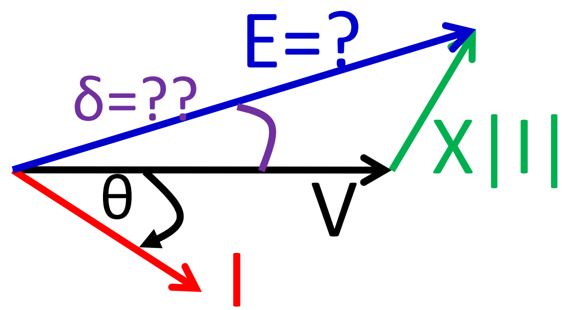


Line-to-line voltage

$$V = \frac{\boxed{30\times10^{3}}}{\sqrt{3}}\angle 0^\circ \text{ V}$$

$$E = I(jX) + V$$

➔ Need to find the current I!!!



Hint: The generator is delivering rated power to the system at a 0.8 power factor lagging.

Solution : Armature Current

From,

$$S_{3\Phi} = 3VI^* \quad \Rightarrow \quad |S_{3\Phi}| = 3|V||I|$$

- We can find the current magnitude,

$$|I| = \frac{|S_{3\Phi}|}{3|V|} = \frac{50\times10^6}{3\times\left(\frac{30\times10^3}{\sqrt{3}}\right)} = 962.25$$

Line-to-neutral voltage

- Since the angle of terminal voltage is assumed to be **zero**, current angle is found from power factor,

$$\angle I = -\cos^{-1}(p.f.) = -36.87°$$

Solution : 2(a) Excitation Voltage

We can now find the excitation voltage,

$$
\begin{aligned}
E &= I(jX) + V \\
&= 962.25\angle -36.87° \times 9\angle 90° + 17320\angle 0° \\
&= 23558.43\boxed{\angle 17.1°}
\end{aligned}
$$

Power angle = 17.1 degree

Solution: 2(b) New Power Angle

- With the **excitation held constant** at the value found in (a) (|E| = 23558 V), the driving torque is reduced until the generator is delivering 25 MW.

$$P_{3\Phi} = 3\frac{|V||E|}{X}\sin(\delta) = 25\,\text{MW}.$$

- The new power angle is found from,

$$\delta_{new} = \sin^{-1}\left(\frac{P_{3\Phi,new}X}{3|V||E|}\right) = \sin^{-1}\left(\frac{25\times 10^{6}\times 9}{3\times 17320\times 23558}\right) = 10.6^{\circ}$$

Solution 2(b) Armature Current

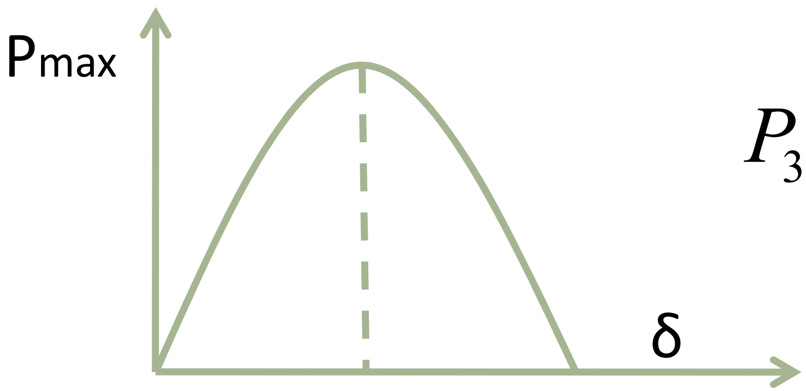
- We can now find the armature current.

$$I = \frac{E - V}{jX} = \frac{23558\angle 10.6^\circ - 17320\angle 0^\circ}{9\angle 90^\circ} = 807\angle -53.4^\circ$$

- This means that the power factor has changed too!!

$$p.f. = \cos\left(\angle V - \angle I\right) = \cos\left(0 - \left(-53.43^\circ\right)\right) = 0.596$$

Solution: 2(c) Maximum Power Transfer



$$P_{3\Phi} = 3\frac{|V||E|}{X}\sin(\delta)$$

$$P_{3\Phi,\max} = 3\frac{|V||E|}{X} = 3\frac{17320\times 23558}{9} = 136 \quad MW$$

This is a maximum possible value in theory, you can see that this value exceeds the machine's limit of 50 MVA!

Solution : Points to Note

- From (b), when we adjust *only* mechanical power input, we see that the current magnitude and power factor has changed.

$$I = 962.25\angle -36.87° \Rightarrow I = 807\angle -53.4°$$

- If we want to keep the power factor constant, we need to adjust the excitation voltage magnitude too!