Tutorial - Line Conductors

$$I_e = \frac{\pi x^2}{\pi r^2} I \quad , \quad H_x = \frac{I_e}{2\pi x} = \frac{Ix}{2\pi r^2}$$

$$\phi = \frac{\lambda}{l} = \int_A B \, da \qquad \Rightarrow \lambda_{int} = \int_0^r \mu \frac{Ix}{2\pi r^2} \cdot \frac{x^2}{r^2} dx$$

$$= \frac{\mu}{2\pi} \int_0^r \frac{Ix^3}{r^4} dx$$

$$\lambda_{int} = \frac{\mu_0 \mu_r}{8\pi} I.$$

Assume $\mu_0 = 4\pi \times 10^{-7} H/m$, $\mu_r = 1$

$$\Rightarrow \lambda_{int} = \frac{4\pi \times 10^{-7} \times 1}{8\pi} I = \frac{1}{2} \times 10^{-7} I$$

$$\lambda_{int} = L_{int} \cdot I$$

$$\Rightarrow L_{int} = \frac{1}{2} \times 10^{-7} \ H/m$$

$$\lambda_{tot} = \frac{\mu_0}{2\pi} I \ln \frac{R}{r} + \frac{\mu_0 \mu_r}{8\pi} I$$

$$= \frac{\mu_0}{2\pi} I \left(\ln \frac{R}{r} + \frac{\mu_r}{4} \right)$$

$$L_{tt} = \frac{\mu_0}{2\pi} \left(\ln \frac{R}{r} + \frac{\mu_r}{4} \right)$$

$$= \frac{\mu_0}{2\pi} \left(\ln R - \left(\ln r + \ln e^{-\frac{\mu_r}{4}} \right) \right)$$

$$= \frac{\mu_0}{2\pi} \left(\ln R - \ln r e^{-\frac{\mu_r}{4}} \right)$$

$$= \frac{\mu_0}{2\pi} \ln \frac{R}{r'}$$

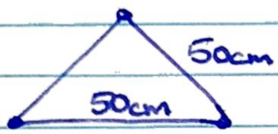
$$L_{tot} = 2 \times 10^{-7} \ln \frac{R}{r'} \qquad \text{where } r' = r e^{-\frac{\mu_r}{4}}$$

$$\Rightarrow L_x = L_y = 2\times10^{-7} \ln \frac{R}{r'}$$

$$= 2\times10^{-7} \ln\left(\frac{0.5\,m}{\frac{0.012\,m}{2} e^{-1/4}}\right)$$

$$\underline{L_{tot} = 9.346\times10^{-7}\,H/m}$$

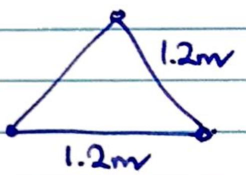
2.



$$GMR = \sqrt[3]{1.56\times50\times50} = \underline{15.7\,cm}$$

note. $r' = 1.56$ cm.

3.



$$r' = r \cdot e^{-1/4}$$

$$= \frac{0.01}{2} \times e^{-1/4}$$

$$r' = 3.894\times10^{-3}\,m$$

$$L = 2\times10^{-7} \ln \frac{1.2}{3.894\times10^{-3}} = \underline{1.146\times10^{-6}\,H/m}$$

$$X_L = 1.146\times10^{-6} \times 2\pi\times60 = 0.432\times10^{-4}\,\Omega/m$$

$$\Rightarrow X_L = 0.432\times10^{-4}\times1000 = \underline{0.432\,\Omega/km}$$

4. $r = \frac{1.345}{2 \times 12} = 0.056$ ft. Note. 1 ft = 12 inches.

$GMR = \frac{0.5328}{12} = 0.0444$ ft or 0.0135 m

$GMD = \sqrt[3]{D_{ab} D_{ac} D_{bc}} = \sqrt[3]{35 \times 70 \times 35}$

$GMD = 44.097$ ft or 13.44 m

$L = 2 \times 10^{-7} \ln\left(\frac{13.44}{0.0135}\right) = 1.38 \times 10^{-6}$ H/m

$\Rightarrow \underline{L = 1.38 \text{ mH/km}}$

Assume mutual inductance due to non-equal GMD to be small and negligible. Note that transposition of lines usually can also remove/reduce the mutual inductance.