EE2029: Introduction to Electrical Energy System

3-Phase Power

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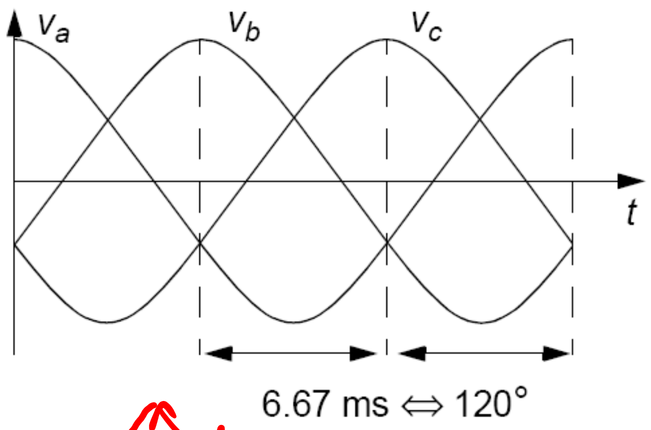
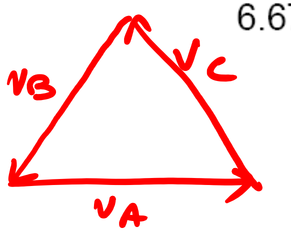
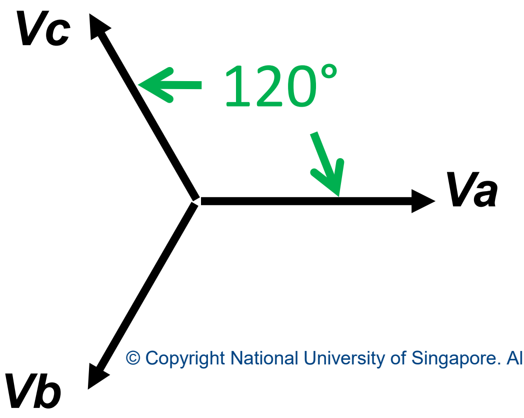
Department of Electrical and Computer Engineering

Three-Phase Voltage Sources





$V_A + V_B + V_C = 0$

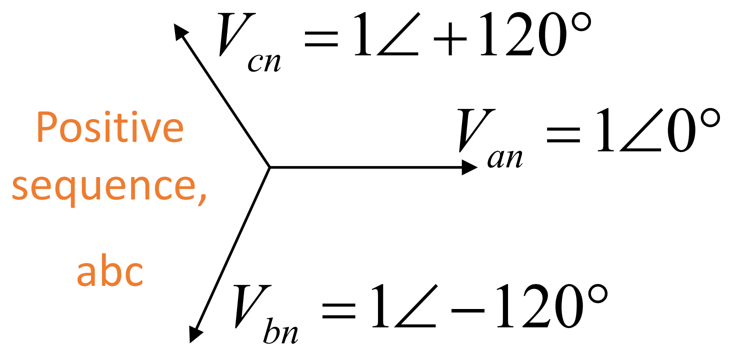


$$v_a = \boxed{\sqrt{2}|V|}\cos(\omega t)$$

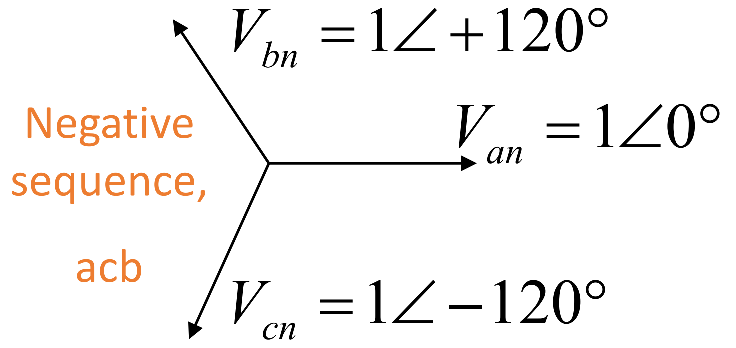
$$v_b = \boxed{\sqrt{2}|V|}\cos\left(\omega t - \frac{2\pi}{3}\right)$$

$$v_c = \boxed{\sqrt{2}|V|}\cos\left(\omega t - \frac{4\pi}{3}\right)$$

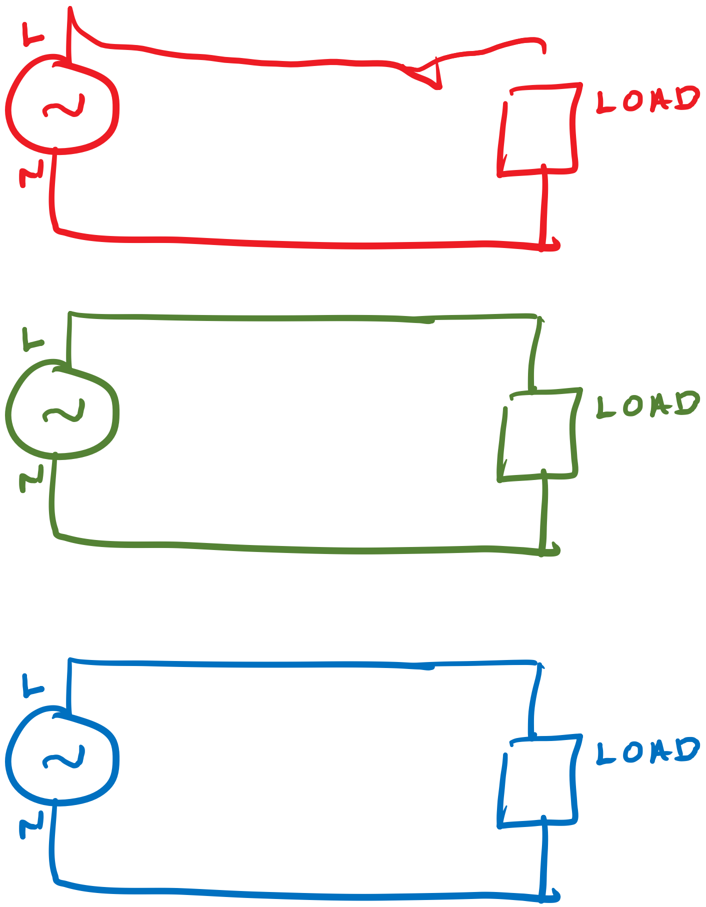
- Positive Sequence

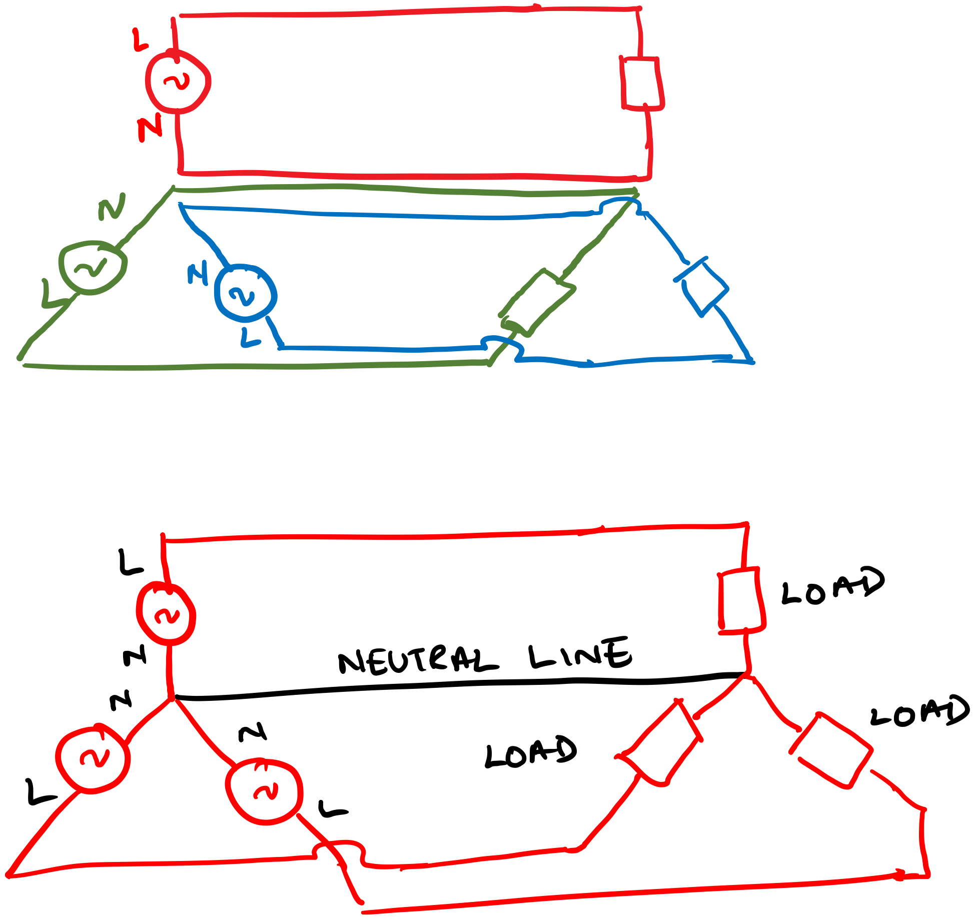


- Negative Sequence



Three Single Phase vs Three Phase Circuits

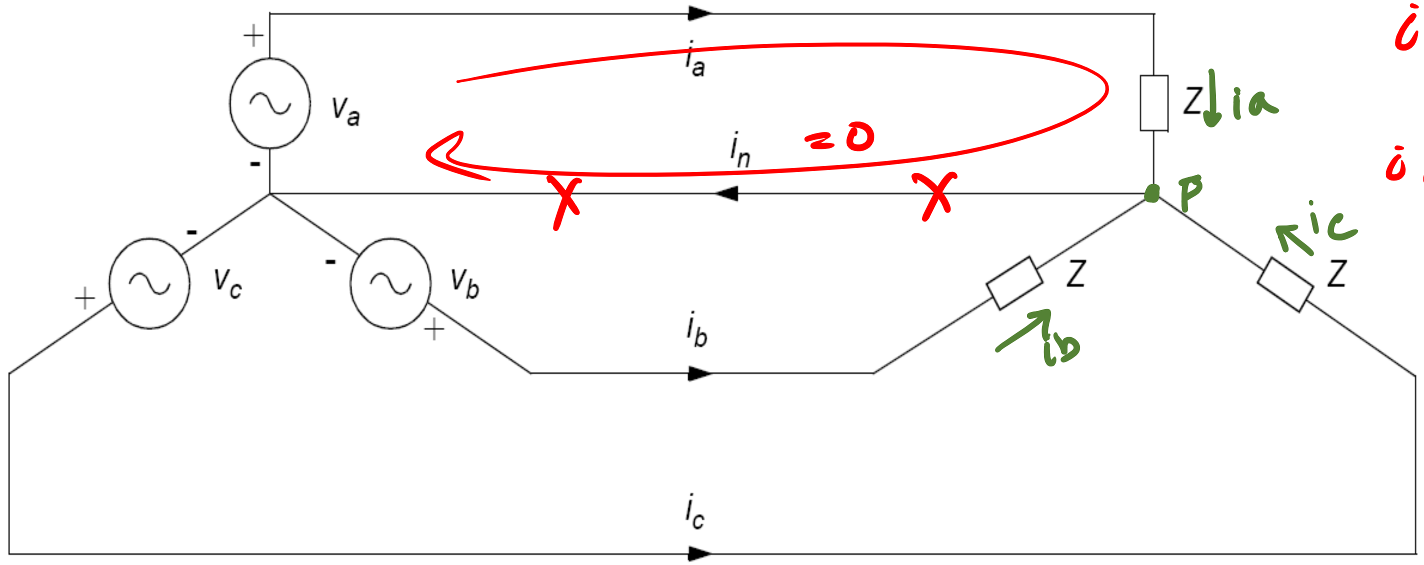


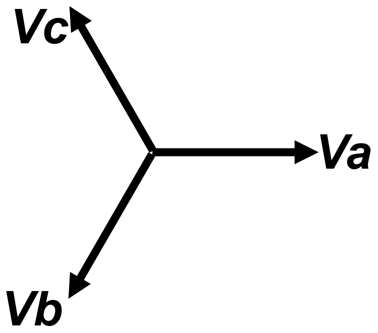


REDUCED CONDUCTORS FROM 6 TO 4 ⇒ 33.33% SAVINGS

Balanced Three-Phase Circuit

- Three-phase circuit is said to be balanced when the impedances in the 3 phases are identical





$V_a = 1\angle 0^\circ$

$V_b = 1\angle -120^\circ$

$V_c = 1\angle +120^\circ$

$i_a = \frac{V_a}{Z}$

$i_b = \frac{V_b}{Z}$

$i_c = \frac{V_c}{Z}$

KCL @ node P

$i_a + i_b + i_c = i_n$

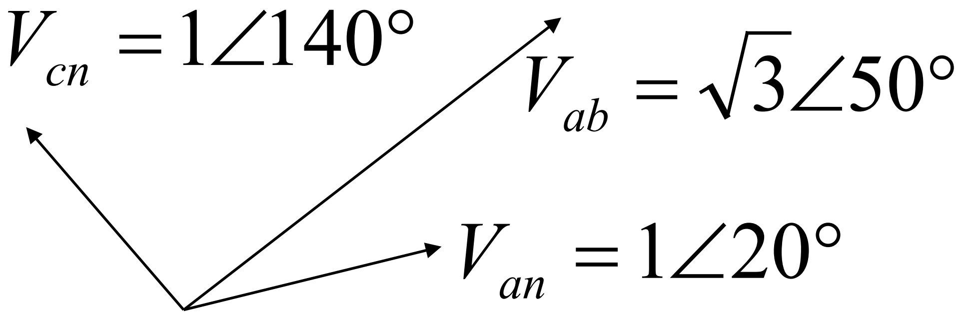
$i_a + i_b + i_c$

$= \frac{V_a}{Z} + \frac{V_b}{Z} + \frac{V_c}{Z} = \frac{1}{Z}\left(V_a + V_b + V_c\right)$

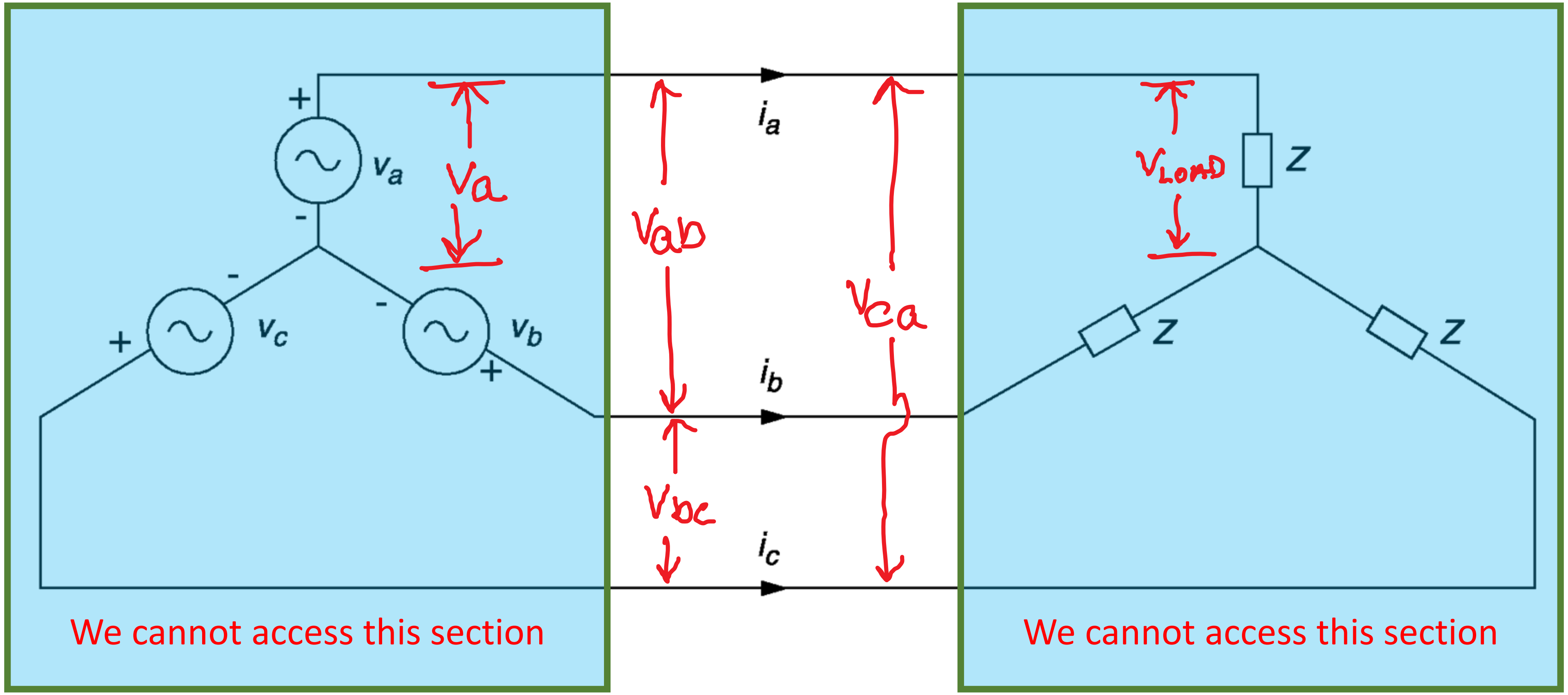
$= \frac{0}{Z} = 0 = i_n \quad \Rightarrow i_n = 0$

Example:

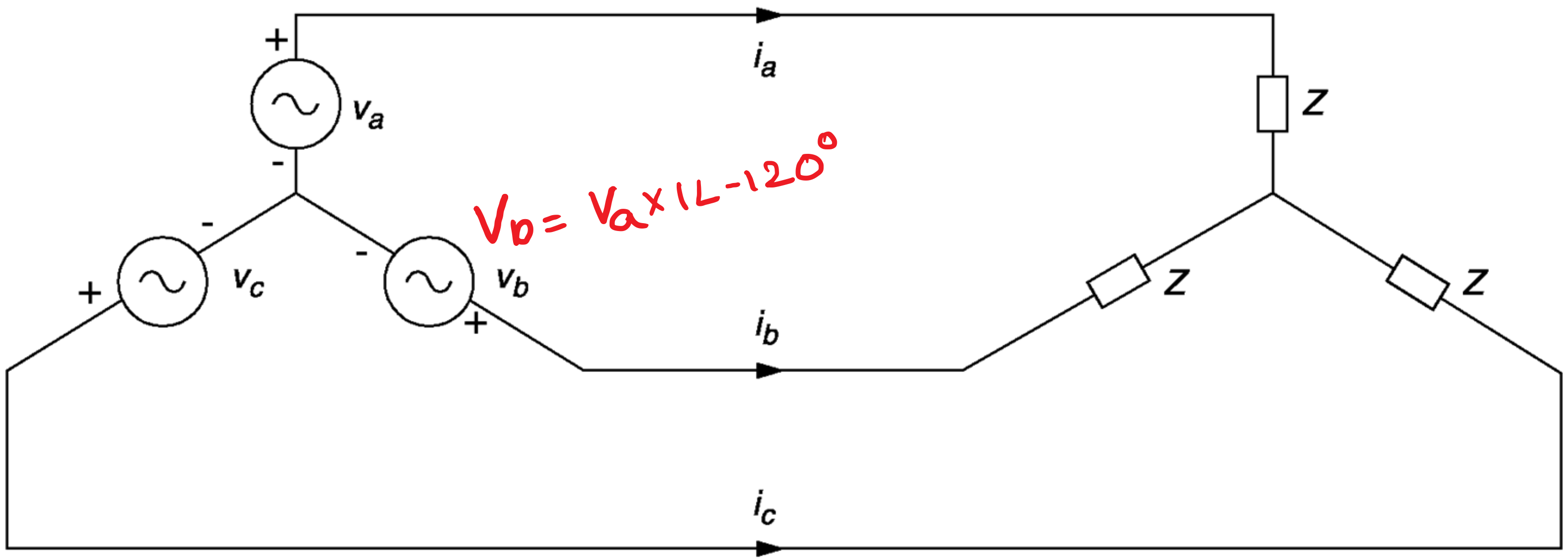
- What is the phase sequence?
- Are these balanced three-phase voltage sources?



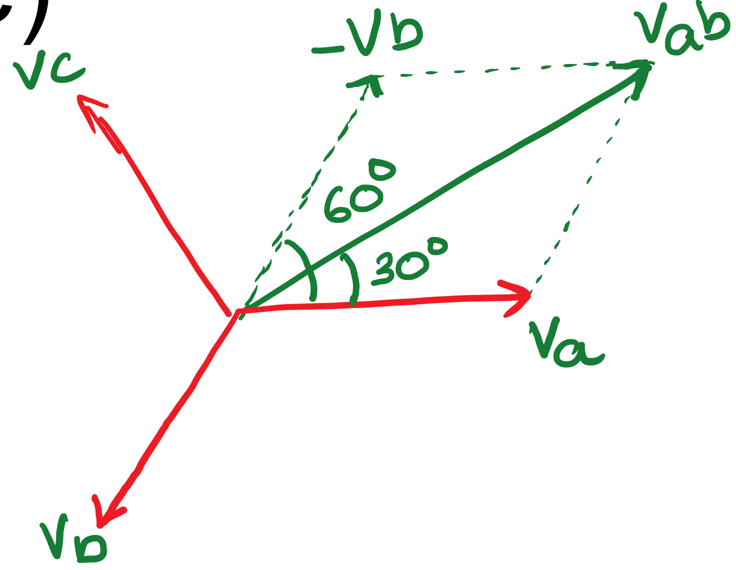
How do we Measure Voltage/Current



Line-To-Line Voltage (Line Voltage)



$V_b = V_a \times 1\angle -120^\circ$



$$V_{ab} = V_a - V_b = V_a + (-V_b)$$
$$= V_a + V_a \angle 60^\circ$$
$$= V_a \angle 0^\circ + V_a \angle 60^\circ$$
$$= \sqrt{3}\, V_a \angle 30^\circ$$

$$V_{bc} = V_b + (-V_c)$$
$$= \sqrt{3}\, V_b \angle 30^\circ$$

$$V_{ca} = V_c + (-V_a)$$
$$= \sqrt{3}\, V_c \angle 30^\circ$$

Line-To-Neutral Voltage (Phase Voltage)

$V_{ab} = \sqrt{3}\, V_a \angle 30^\circ$

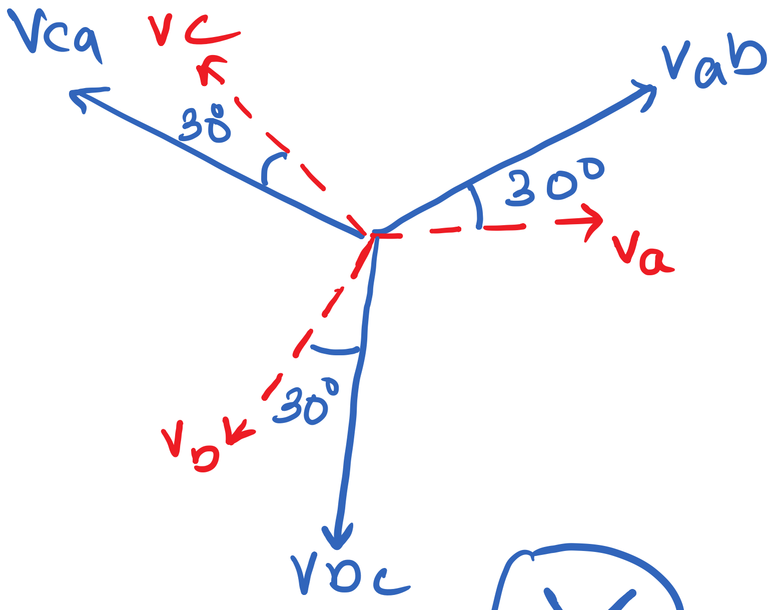
$V_{bc} = \sqrt{3}\, V_b \angle 30^\circ$

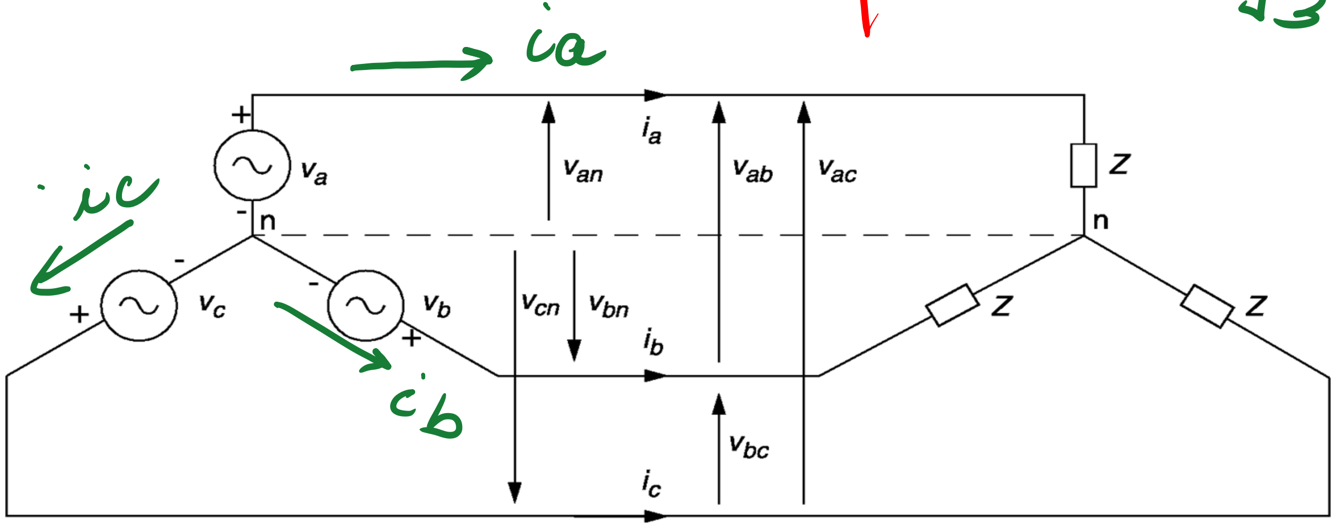
$V_{ca} = \sqrt{3}\, V_c \angle 30^\circ$

$|V_a| = \dfrac{|V_{ab}|}{\sqrt{3}}$

$|V_b| = \dfrac{|V_{bc}|}{\sqrt{3}}$

$|V_c| = \dfrac{|V_{ca}|}{\sqrt{3}}$



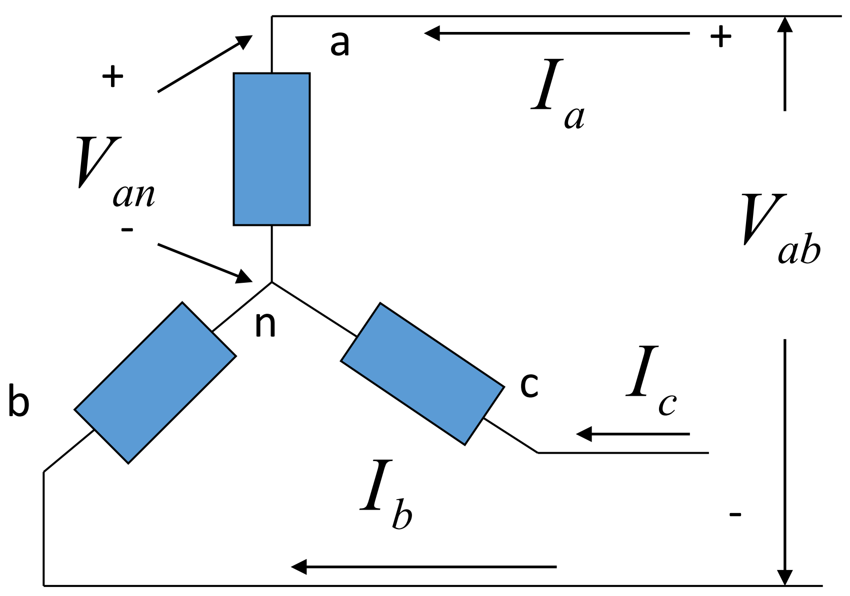


Y

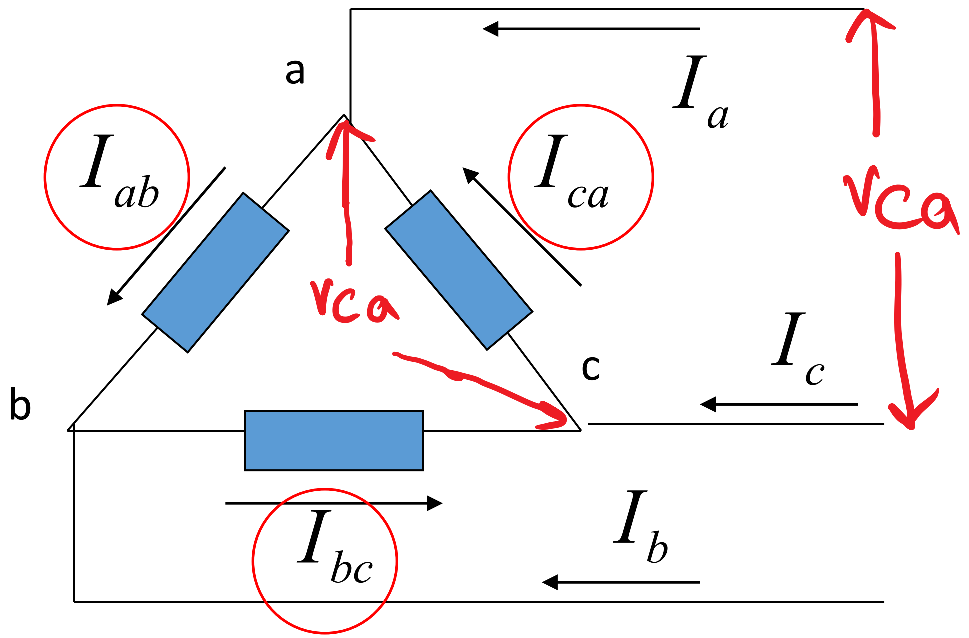
$$\boxed{V_{phase} = \frac{V_{line}}{\sqrt{3}}}$$

3-Phase Circuit Connection

Wye Connection



Delta Connection



What are these currents?

These two types of connections apply to both three-phase voltage sources and three-phase loads

Delta-Connected Load

$V_{LINE} = V_{PHASE}$

KCL @ JUNCTION A

$I_{ab} = I_{ca} + I_a$

$\Rightarrow I_a = I_{ab} - I_{ca}$

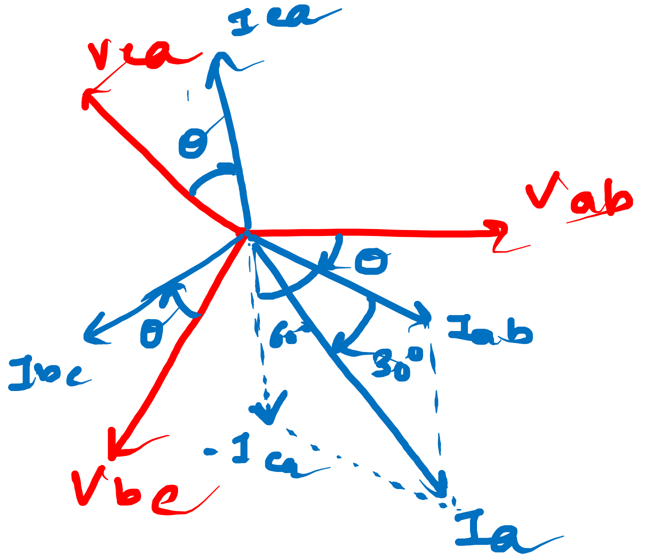
$I_a = I_{ab} + (-I_{ca})$

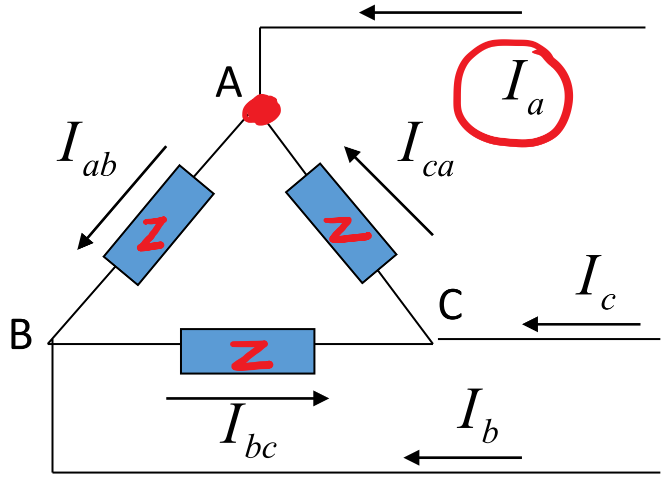
$= \sqrt{3}\, I_{ab} \angle -30^0$

$I_a = \sqrt{3}\, I_{ab} \angle -30^0$

$I_b = \sqrt{3}\, I_{bc} \angle -30^0$

$I_c = \sqrt{3}\, I_{ca} \angle -30^0$





Z - LAGGING "θ"

$\Rightarrow$ FOR Δ (DELTA)

$|I_{LINE}| = \sqrt{3}\, |I_{PHASE}|$

OR $I_{PHASE} = \dfrac{I_{LINE}}{\sqrt{3}}$

Example: For a balanced Y-connected three phase voltage source and Y-connected load system with a line voltage of 440 V and three equal resistive loads of 100 Ω per phase, assume positive sequence, what will be the magnitudes of

(a) the line-to-neutral voltage, (b) the phase current, (c) the line current?

Balanced Three-Phase Power

$S_{1\Phi} = V I^*$

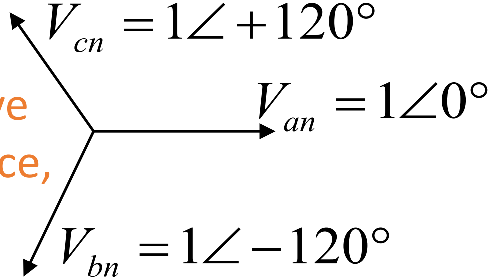
- From three phase power,

$$S_{3\Phi} = V_{an} I_a^* + V_{bn} I_b^* + V_{cn} I_c^*$$

- When the system is balanced, (assume positive sequence) we can write,

$$S_{3\Phi} = V_{an} I_a^* + V_{an}\angle -120^\circ \left(I_a \angle -120^\circ\right)^* + V_{an}\angle 120^\circ \left(I_a \angle 120^\circ\right)^*$$

Positive sequence, abc



$$\boxed{S_{3\Phi} = 3 V_{an} I_a^*} = 3(P + jQ)$$

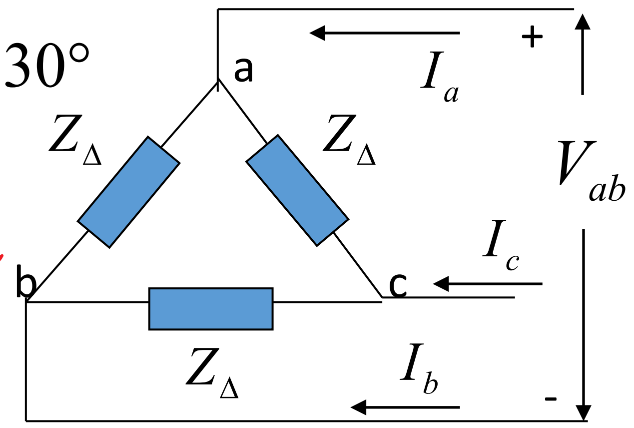
$\Rightarrow P_{3\Phi} = 3 V_{an} I_a \cos\theta$

$Q_{3\Phi} = 3 V_{an} I_a \sin\theta$

Delta/Wye Connected 3-Phase Load

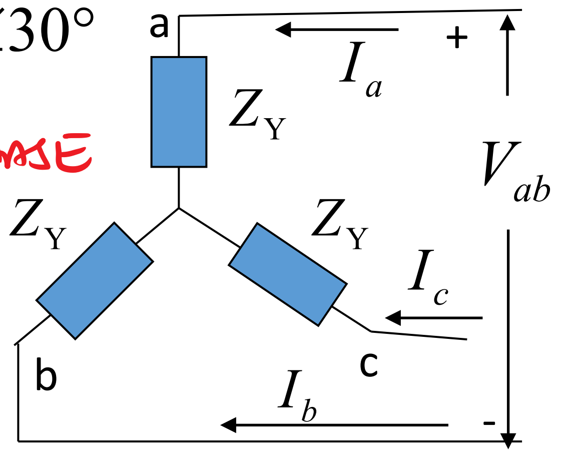
$$I_a = \sqrt{3} I_{ab} \angle -30°$$

$V_{LINE} = V_{PHASE}$



$$V_{ab} = \sqrt{3} V_{an} \angle 30°$$

$I_{LINE} = I_{PHASE}$



$$|S_{3\phi}| = 3|V_{ab}||I_{ab}|$$
$$= 3|V_{ab}|\left|\frac{I_a}{\sqrt{3}}\right|$$
$$= \sqrt{3}|V_{LINE}||I_{LINE}|$$

$$S_{3\phi} = 3|V_{an}||I_{an}|$$
$$= 3\left|\frac{V_{ab}}{\sqrt{3}}\right||I_{an}|$$
$$= \sqrt{3}|V_{LINE}||I_{LINE}|$$

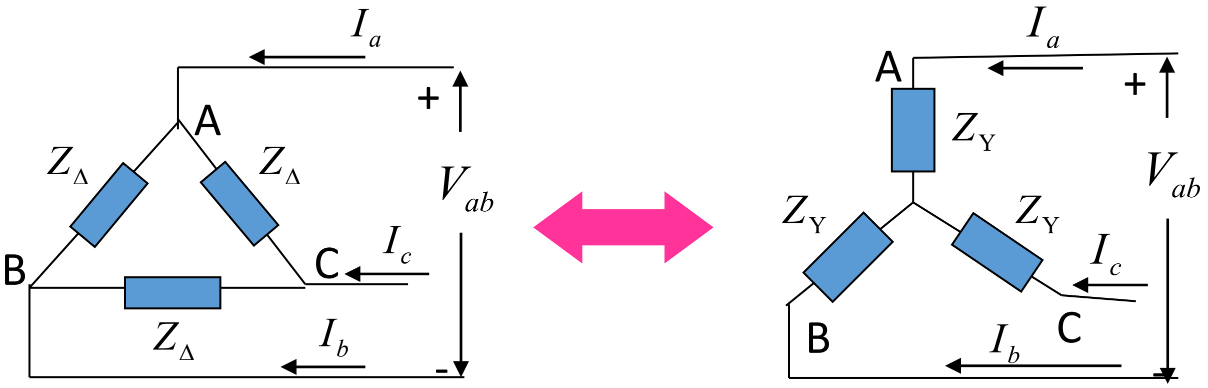
$$\Rightarrow \boxed{|S_{3\Phi}| = \sqrt{3}|V_{\text{Line-To-Line}}||I_{\text{Line}}|}$$

BOTH Δ & Y

Delta-Wye Load Transformation

$I_{ph} = \dfrac{I_{LINE}}{\sqrt{3}}$

$V_{ph} = V_{LINE}$



$V_{ph} = \dfrac{V_{LINE}}{\sqrt{3}}$

$I_{ph} = I_{LINE}$

$$Z_\Delta = \frac{V_{ph}}{I_{ph}} = \frac{V_{AB}}{I_{AB}}$$

$$Z_\Delta = \frac{V_{AB}}{I_A/\sqrt{3}} = \frac{\sqrt{3}\,V_{AB}}{I_A}$$

$$\Rightarrow \frac{V_{AB}}{I_A} = \frac{Z_\Delta}{\sqrt{3}} \quad -\ 1$$

$$Z_Y = \frac{V_{ph}}{I_{ph}} = \frac{V_{AN}}{I_{AN}}$$

$$Z_Y = \frac{V_{AB}/\sqrt{3}}{I_A} = \frac{V_{AB}}{\sqrt{3}\,I_A}$$

$$\Rightarrow \frac{V_{AB}}{I_A} = \sqrt{3}\,Z_Y \quad -\ 2$$

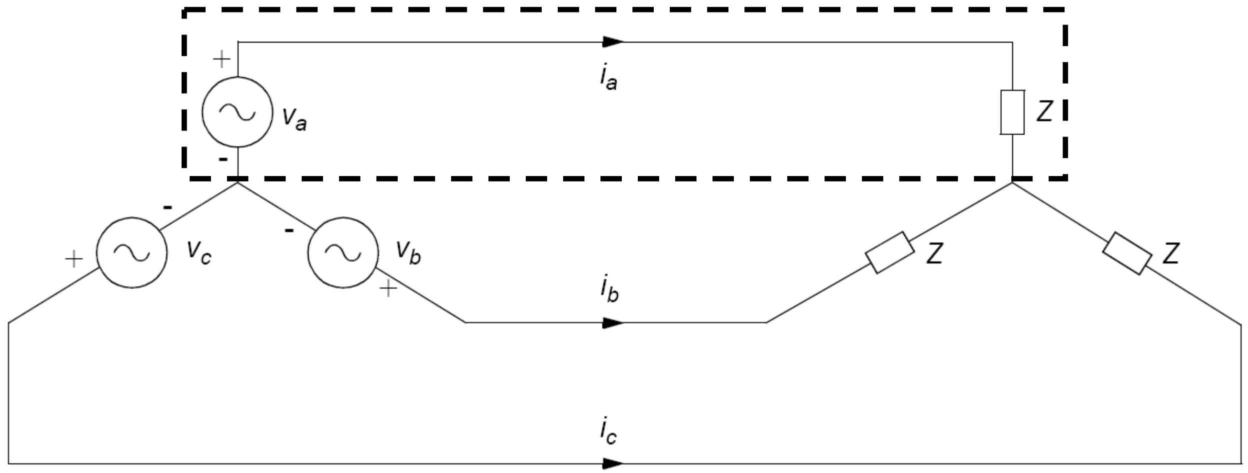
1 & 2

$$\Rightarrow \frac{Z_\Delta}{\sqrt{3}} = \sqrt{3}\,Z_Y$$

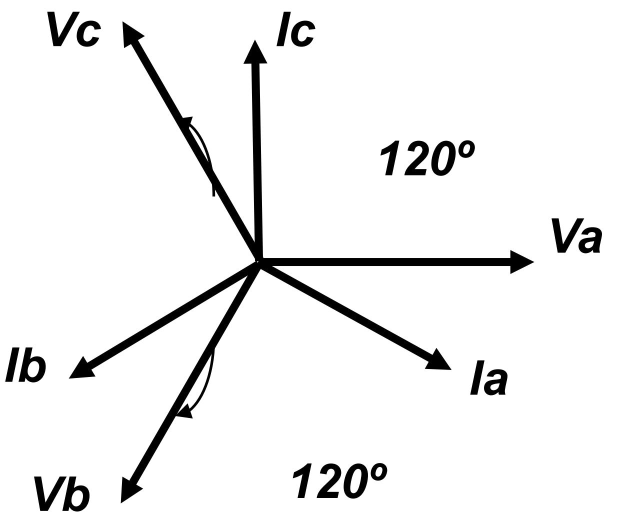
$$\Rightarrow \boxed{Z_\Delta = 3Z_Y}$$

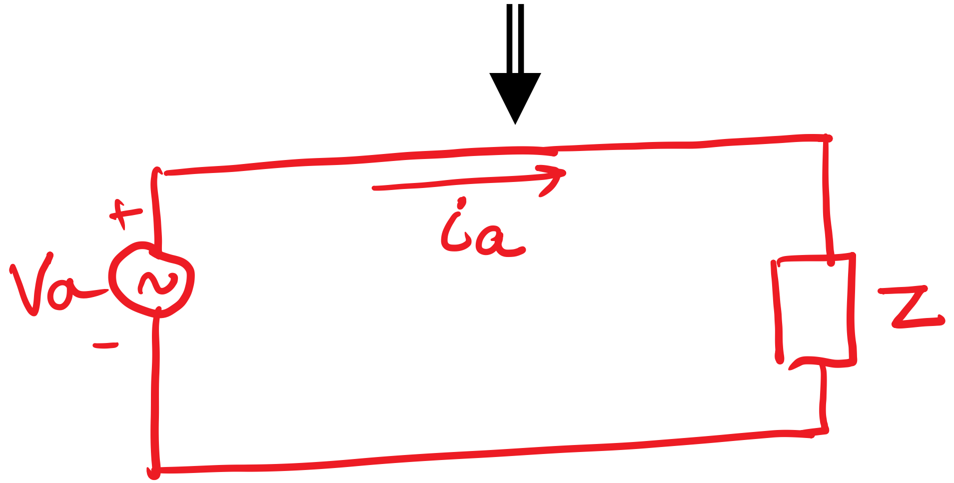
Per Phase Analysis: Assumption

It must be balanced three-phase circuit



$$I_n = I_a + I_b + I_c = 0$$

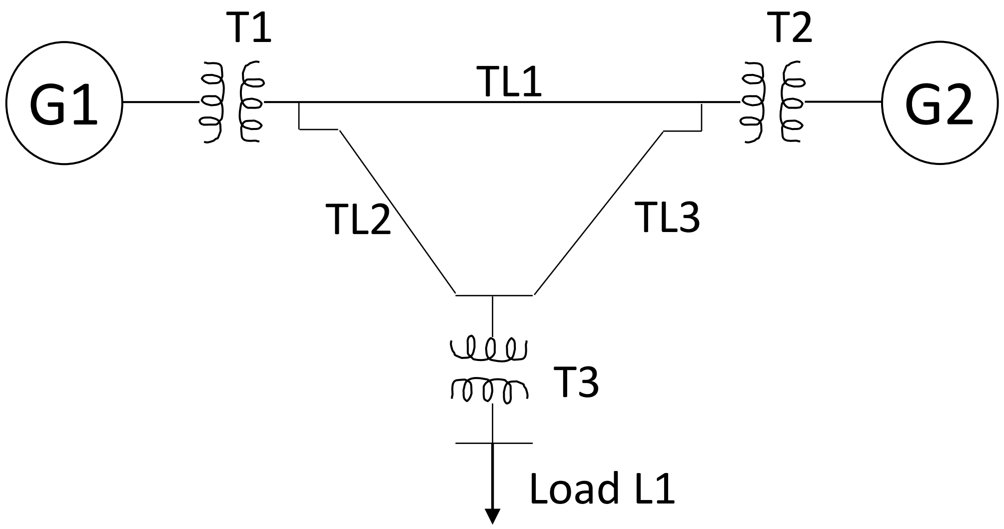




Steps of Per Phase Analysis

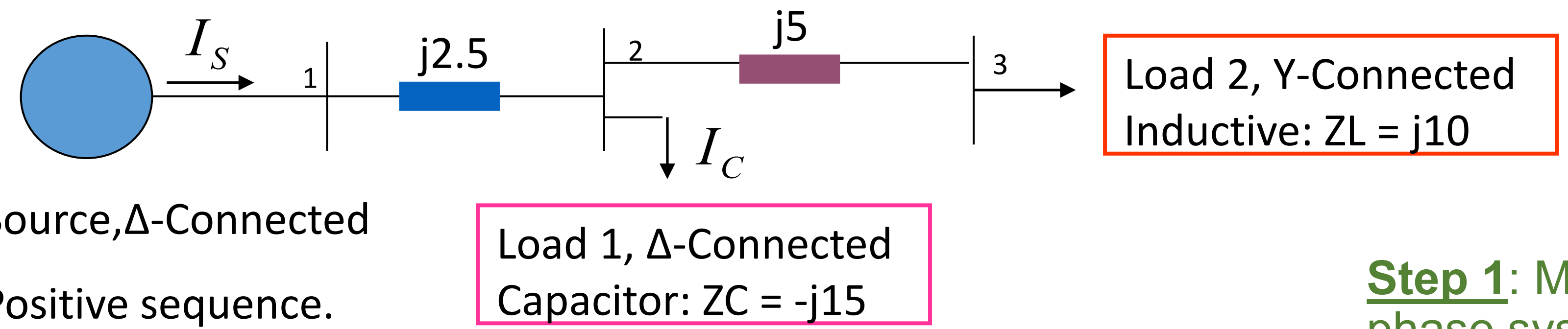
- Make sure that the three-phase system is balanced.
  - The three-phase sources need to have the same magnitude with 120 degree phase difference.
  - The three-phase impedances must be of the same value (both phase and magnitude).
- **Convert** all Delta-connected sources/loads to Wye-connected sources/loads.
- Per phase analysis reduce three-phase circuit to single-phase circuit. We can apply the same concept used in single-phase.

Single-Line Diagram



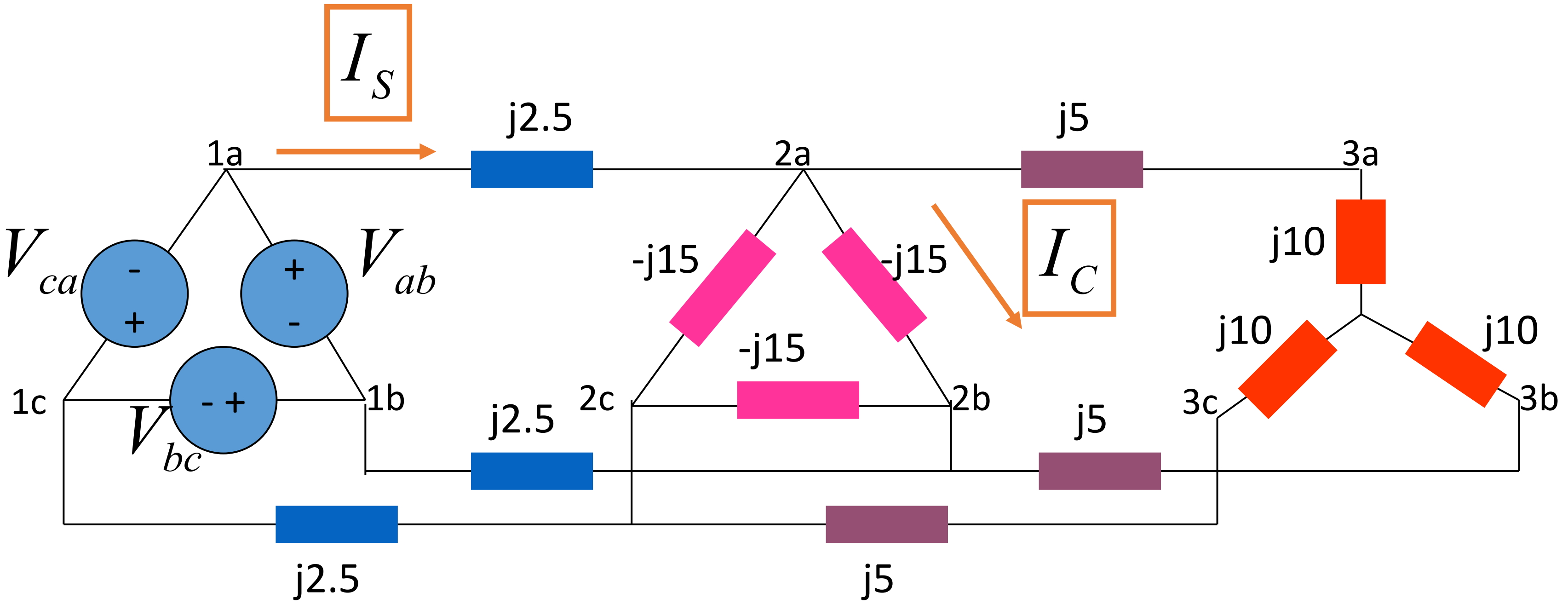
- Show the interconnections of a transmission system
  - Generator
  - Load
  - Transmission line
  - Transformer
- This is a representation of a 3Φ circuit. Each line represents three conductors in three-phase system.

Example: Given a one-line diagram, If the voltage source is $|V_{line}| = \sqrt{3}$ V. Find, the current magnitude supplied by source, $|I_S|$, and, the current magnitude through a capacitor, $|I_C|$.



Source,Δ-Connected

Positive sequence.



Step 1: Make sure that the three-phase system is balanced.

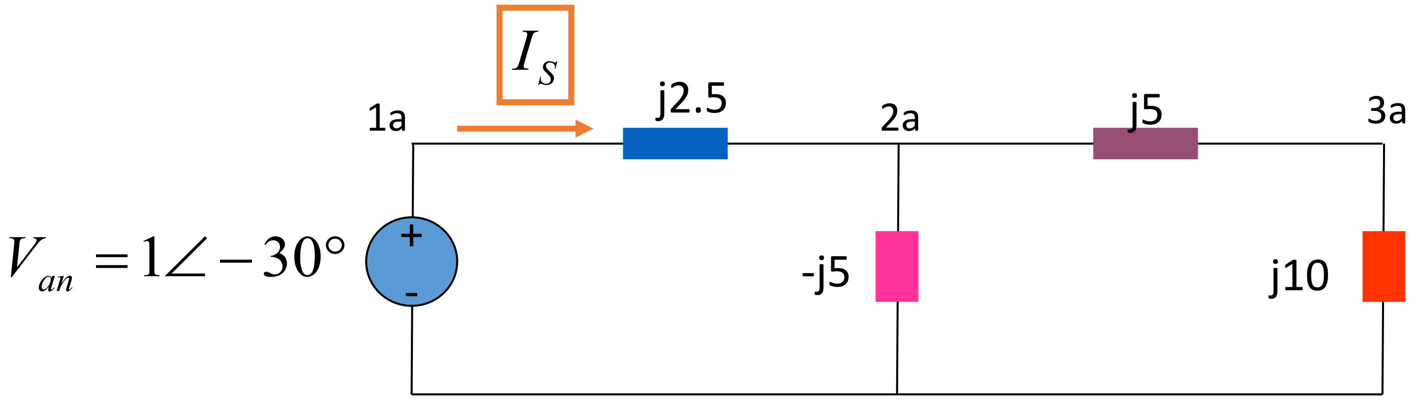
- The three-phase sources need to have the same magnitude with 120 degree phase difference.
- The three-phase impedances must be of the same value (both phase and magnitude).

Step 2: Convert from $\Delta \rightarrow Y$

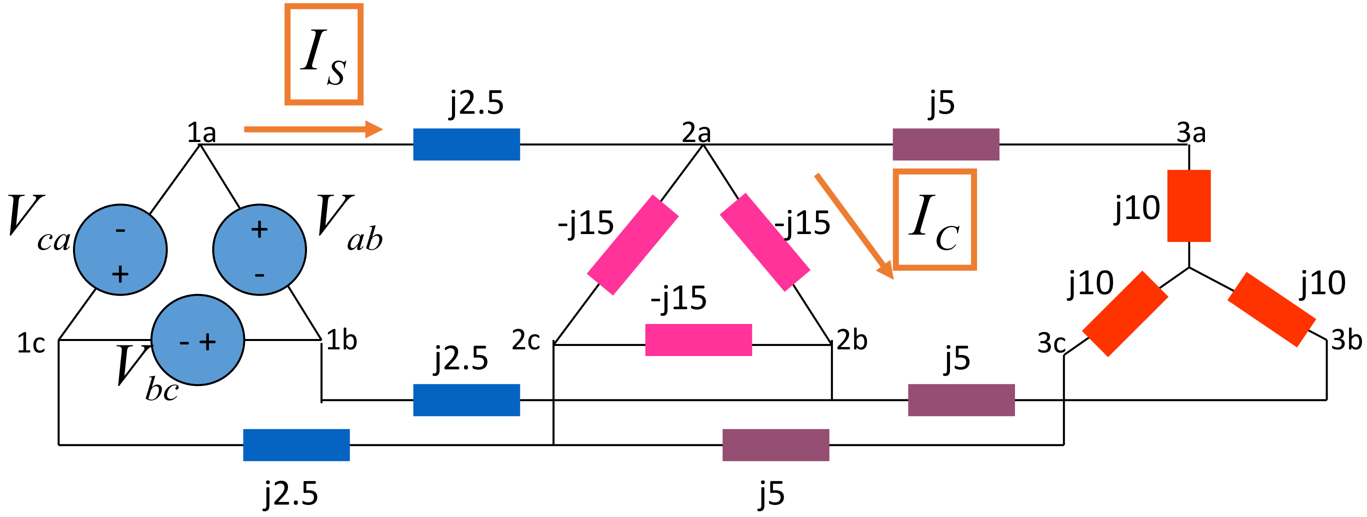
- Converting the Z_{Δ} to Z_Y : Load 1 Z_{Δ} = -j15Ω ➔ $Z_Y = \frac{Z_{\Delta}}{3} = \frac{-j15}{3} = -j5\Omega$

- Converting the Voltage Source from $\Delta \rightarrow Y$

Step 3 :Draw the 1-phase Diagram



Step 4: Find the capacitor Current I_C



Questions!!!