# Tutorials for EE2029

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# Tutorial 4 Generator Modeling

Learning objectives

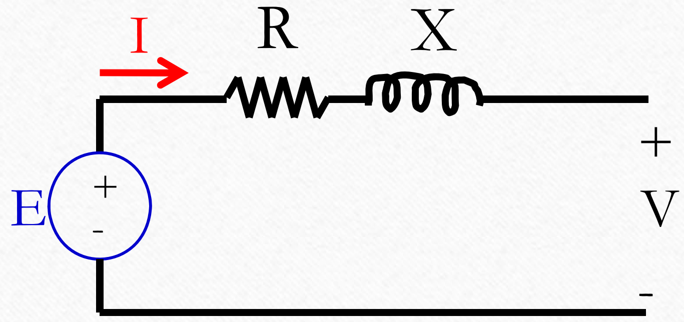
- To be able to draw an equivalent circuit of synchronous generators
- To be able to calculate complex power output of generators

# Approach to the Tutorials

- Recall prior knowledge
- Revise the key mathematical tools used in the topic
- Revise key concepts and formula
  - Express the concepts in your own words
- Explain your thinking as you solve the mathematical problems
- Solve the tutorials and consult lecturer if answers do not match

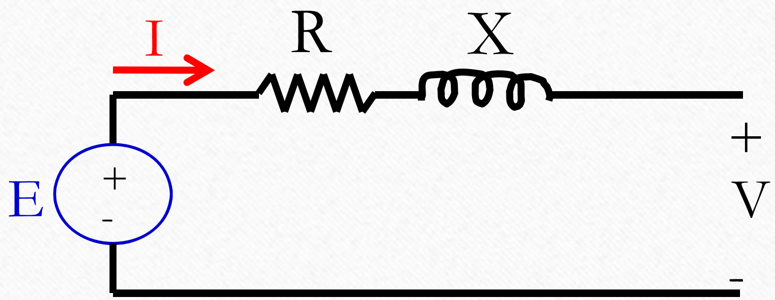
# Key concepts and formula

- In synchronous generator, $f = \frac{np}{120}$
  - $f$ voltage frequency
  - $n$ synchronous speed in RPM
  - $p$ number of poles in the rotor
- Alternately $n_{sync} = \frac{120 f_e}{p}$



# Key concepts and formula

- Single phase equivalent circuit of a synchronous generator




  - R : Resistance of armature winding
  - X : Synchronous reactance
  - V : Grid voltage
  - E : Excitation voltage

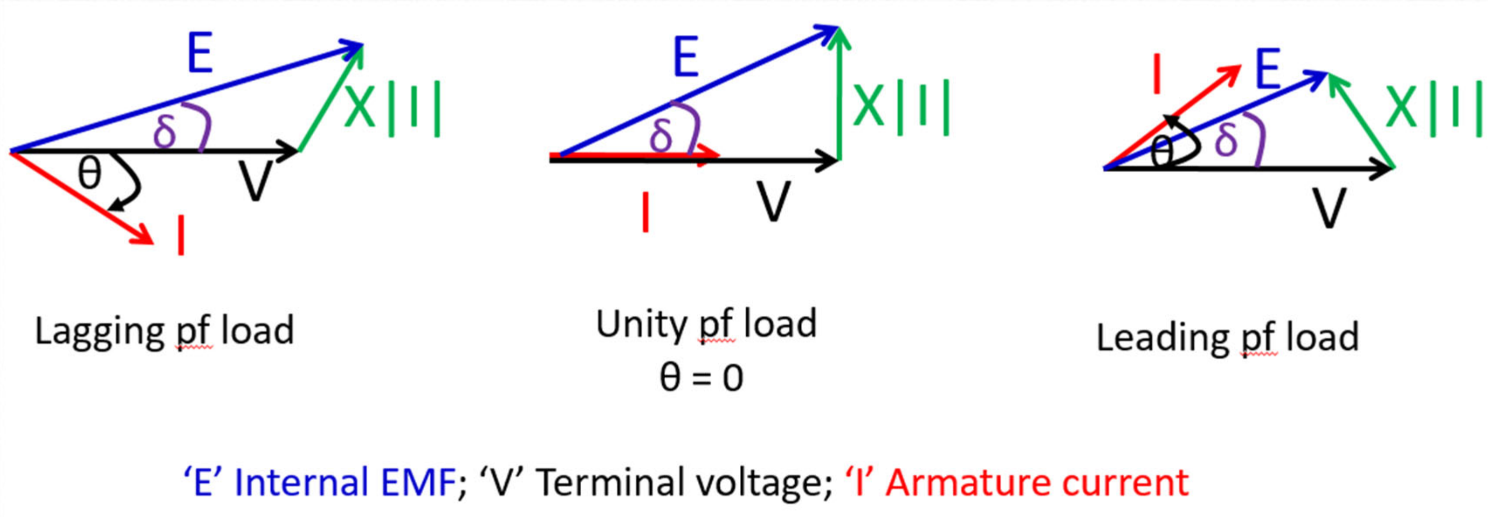
- $\boldsymbol{E} = \boldsymbol{V} + \boldsymbol{I}(\boldsymbol{R} + \boldsymbol{jX})$

# Key concepts and formula

- Complex Power Supplied Per Phase $S_{1\Phi} = \frac{|V||E|}{X}\sin(\delta) + j\left\{\frac{|V||E|}{X}\cos(\delta) - \frac{|V|^2}{X}\right\}$
- Complex three phase real power $P_{3\Phi} = 3\frac{|V||E|}{X}\sin(\delta)$
- Complex three phase reactive power $Q_{3\Phi} = 3\frac{|V||E|}{X}\cos(\delta) - 3\frac{|V|^2}{X}$

# Key concepts and formula

- Phasor Diagram at Different Operating Conditions



1. A 60kVA three-phase wye-connected 440V, 60 Hz synchronous generator has a resistance of 0.15Ω and a synchronous reactance of 3.5Ω per phase. At rated load and unity power factor, find the internal excitation voltage magnitude and the power angle.

(Answer: 382.89 V, 46.03°)

- 60 kVA is the maximum apparent power of the generator – size of the generator
- Given 440V is line-to-line RMS value , we find the phase voltage $V = \frac{440}{\sqrt{3}} \angle 0^o$ V
- Rated load : 60kVA and unity power factor; $I = \frac{60{,}000}{\sqrt{3} \times 440} \angle 0^o A$
- Using per phase equivalent circuit of the synchronous generator $E = V + I \times (0.15 + j3.5)$
- Internal excitation voltage $E = |E| \angle \delta$ where $|E|$ is the excitation voltage, $\delta$ is power angle

2. A 1MVA 11kV three-phase wye-connected synchronous generator supplies a three-phase load of 600kW, 0.8 leading power factor. The synchronous reactance is 24Ω per phase and the armature resistance is negligible. Find the power angle.

(Answer: 7.44°)

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- 1 MVA, 11kV three-phase wye-connected synchronous generator
  - 11 kV is line-to-line voltage at the terminal
  - Per-phase voltage $V = \frac{11{,}000}{\sqrt{3}} \angle 0^o$
- 600kW, 0.8 leading power factor. Here $P_{3\emptyset} = 600kW$
  - $|I| = \frac{P_{3\emptyset}}{\sqrt{3} \times V_{LL} \times (Power\ factor)}$;
  - $\theta = cos^{-1}(Power\ factor)$
  - $I = |I| \angle -\theta$ if lagging power factor; $I = |I| \angle \theta$ if leading power factor
- Synchronous reactance $X$, resistance is negligible : $E = V + I \times (jX)$
  - The argument of $E$ phasor is the power angle

3. Calculate the excitation voltage for a three-phase wye-connected 2500 kVA, 6.6 kV synchronous generator operating at full load and 0.9 p.f. lagging. The per phase synchronous reactance is 4Ω and the per phase armature resistance is negligible. What will be the internal excitation voltage when the generator is operating at full load with 0.9 p.f. leading? Explain whether the machine is overexcited or underexcited in each case.

(Answer: 4265.11∠10.64° V overexcited, 3518.42 ∠12.93° V underexcited)

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- 2500kVA, 6.6kV three-phase synchronous generator: $|s_{3\phi}| = 2500kVA, |V_{LL}| = 6.6kV$
- Per-phase terminal voltage $V = \frac{6600}{\sqrt{3}}\angle 0^o$
- Full load = 2500kVA= $|S_{3\emptyset}|$, $|I| = \frac{|S_{3\emptyset}|}{\sqrt{3} \times |V_{LL}|}$
- Power factor is 0.9 leading → $\theta = cos^{-1}(Power\ factor)$
- $I = |I|\angle -\theta$ if power factor is lagging; $I = |I|\angle\theta$ if power factor is leading
- Neglecting armature resistance, internal excitation voltage $E = V + I \times jX$
- Over excited if $|E|cos\delta > |V|$; Under excited if $|E|cos\delta < |V|$

4. A 75kVA, 2.2kV, 60 Hz three-phase wye-connected synchronous generator has a resistance of 0.2 Ω, a synchronous reactance of 6Ω per phase and is operated at full load. Draw a phasor diagram of the excitation voltage, load current, and terminal voltage when the load is (i) 0.85 lagging power factor, (ii) unity power factor, and (iii) 0.85 leading power factor. Use the terminal voltage as the reference with the angle of zero degree.

(Answer: (i) Load current = 19.68∠-31.8$^0$ A, Excitation voltage = 1339.36∠4.2$^0$ V,

(ii) Load current = 19.68∠0$^0$ A, Excitation voltage = 1279.57∠5.3$^0$ V,

(iii) Load current = 19.68∠31.8$^0$ A, Excitation voltage = 1215.61∠4.8$^0$ V)

- Find the phasor for per-phase voltage $V = \frac{2200}{\sqrt{3}}\angle 0^o$
- Determine the current for each case $|I| = \frac{|s_{3\emptyset}|}{\sqrt{3}\times|V_{LL}|} = \frac{75000}{\sqrt{3}\times 2200} A$
- Find $\theta = cos^{-1}(Power\ factor)$
- $I = |I|\angle -\theta$ for lagging power factor; $I = |I|\angle 0^o$ for unity power factor,
  $I = |I|\angle\theta$ for leading power factor
- Draw $IR$ part along the current, $jIX$ as perpendicular to $IR$, draw $E$.

5. A 1MVA 11kV three-phase wye-connected synchronous generator has a synchronous reactance of 5 Ω and a negligible armature resistance. At a certain field current the generator delivers rated load at 0.9 lagging power factor at 11kV. For the same excitation, what is the armature current and power factor when the input torque is reduced such that the real power output is half of the previous case?

(Answer: $33.34\angle -44.88^{0}$ A, 0.71 lagging)

- The generator is rated 1MVA of $|S_{rated}| = 1\ MVA$
- Its line-to-line terminal voltage $V_{LL} = 11kV$
- Synchronous reactance $X = 5\ \Omega$
- When the generator delivers rated load at 0.9 lagging power factor at 11kV
  - $|S_{Load}| = 1\ MVA$
  - We can find the line current magnitude from the formula, $|S_{3\phi}| = \sqrt{3}|V_{LL}||I_L|$
  - For the per-phase equivalent circuit $V = \frac{V_{LL}}{\sqrt{3}}\angle 0^o$
  - We can find the angle of the line current from the power factor; lagging power means negative phase angle for current
  - Then we can find the excitation voltage phasor from $E = V + (R + jX)\ I_L$

# Qn 5 continued

- If field excitation is maintained, then magnitude of E will remain same
- Power output is reduced to half means the power angle ($\delta$) will change to a new value:
  - $\frac{P_{new}}{P_{old}} = \frac{\sin \delta_{new}}{\sin \delta_{old}}$ can be used to find $\delta_{new}$
- This give new excitation voltage : $E_{new} = |E_{old}|\angle\delta_{new}$
- Then we can find the new armature current $I_{new} = \frac{E_{new}-V}{jX}$
- New power factor can be obtained from the phase angle of the new current $I_{new}$.

# Qn 5 continued

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- New power factor can be obtained from the phase angle of the new current $I_{new}$.

6. An 11kV three-phase wye-connected generator has a synchronous reactance of 6 Ω per phase and a negligible armature resistance. For a given field current, the open-circuit line-to-line excitation voltage is 12kV. Calculate the maximum power developed by the generator. Determine the armature current and power factor at the maximum power condition.

(Answer: 22 MW, 1566.43 $\angle 42.51^0$ A, 0.74 leading )

- Maximum power from generator happens when power angle $(\delta)$ is $90^o$.
  - $P_{max} = 3\frac{|E||V|}{X}\sin 90^o = 3\frac{|E||V|}{X}$
- The terminal voltage and excitation voltage are given as line voltage and need to be divided by $\sqrt{3}$ .
  - $V = \frac{11000}{\sqrt{3}}, E = \frac{12000}{\sqrt{3}}$
- At maximum power transfer phasor for per phase excitation voltage is $E = \frac{12000}{\sqrt{3}}\angle 90^o$
- Per phase terminal voltage $V = \frac{11000}{\sqrt{3}}\angle 0^o$
- Armature current during maximum power transfer can be calculated as $I = \frac{E-V}{jX}$
- Power factor can be obtained from the phase angle of the armature current

7. Two three-phase generators (G1 and G2) supply a three-phase load through separate three-phase lines as shown in Fig. 1 below.

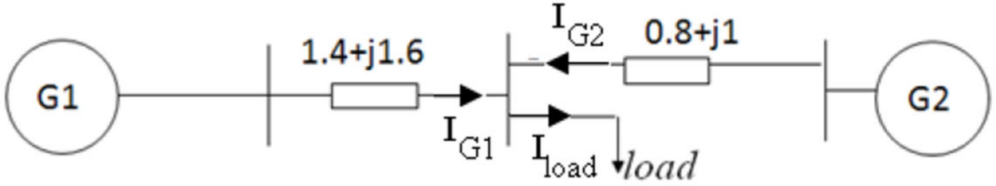
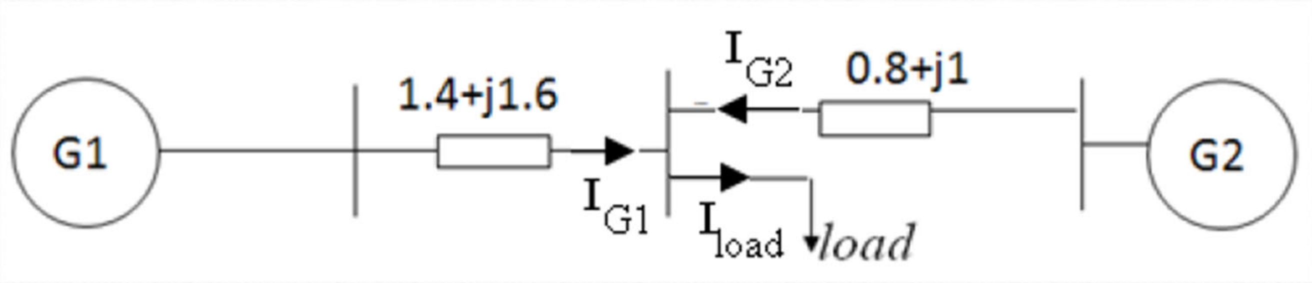



Fig. 1 Single-line diagram of a three-phase circuit

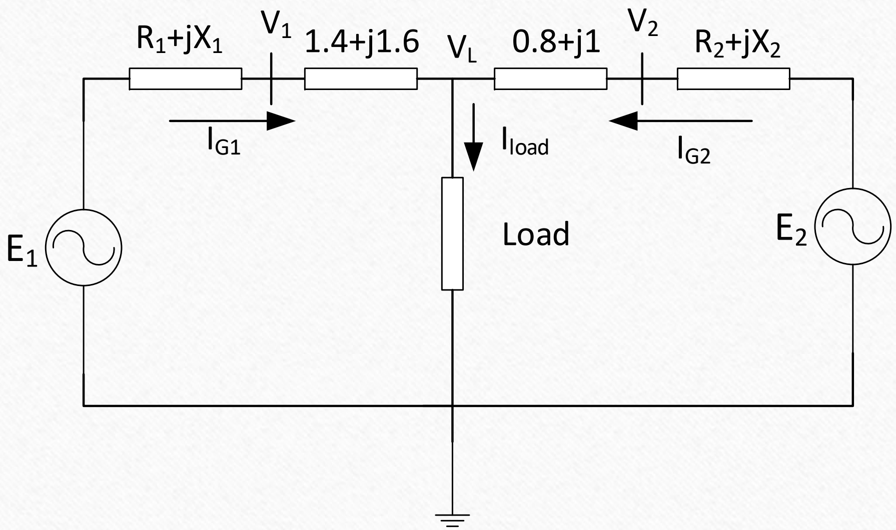
Each generator has a synchronous reactance of 3 Ω per phase and a negligible armature resistance. The three-phase Y-connected load absorbs 30 kW at 0.8 power factor lagging. The line impedance is 1.4+j1.6 Ω per phase between generator G1 and the load, and 0.8+j1 Ω per phase between generator G2 and the load. If generator G1 supplies 15 kW at 0.8 power factor lagging, with terminal voltage of 460 V line-to-line, assume balanced operation, determine internal excitation voltage magnitude (per phase) and power angle of both generators. Use terminal voltage of generator 1 as a reference angle.

(Answer: For G1, 313.07 V, 10.39$^0$, For G2, 335.73 V, 12.74$^0$)

# Qn. 7




- Draw the per phase equivalent circuit of the two generators, the lines and the loads



# Qn.7 continued

- Given the power output of generator 1 at the terminal $V_{1,LL}$
- We can find the current $I_{G1.}$
  - Taking generator 1terminal voltage as reference $V_1 = \frac{|V_{1,LL}|}{\sqrt{3}} \angle 0^o$
  - $|I_{G1}| = \frac{P_{3\emptyset}}{\sqrt{3} \times power\ factor \times |V_{1,LL}|}$; Phase angle$= -cos^{-1}(power\ factor)$ for lagging
  - Then the generator excitation voltage $E_1 = V_1 + I_{G1} \times (R_1 + j\, X_1)$
- Then we can find the load voltage phasor $V_L = V_1 - I_{G1} \times (1.4 + j1.6)$
- Then find load current phasor
  - $|I_{load}| = \frac{P_{load}}{3 \times |V_L| \times (Power\ factor)}$
  - Phase angle of the load current can be obtained as $\angle I_L = \angle V_L - \cos^{-1}(load\ power\ factor)$

# Qn.7 continued

- Find $I_{G2}$ by applying KCL $I_{G2} = I_{load} - I_{G1}$
- Excitation voltage of the generator 2:
  - $V_2 = V_{load} + I_{G2} \times (0.8 + j1)$
  - $E_2 = V_2 + I_{G2} \times (R_2 + jX_2)$