EE2029: Introduction to Electrical Energy System

Per Unit Analysis

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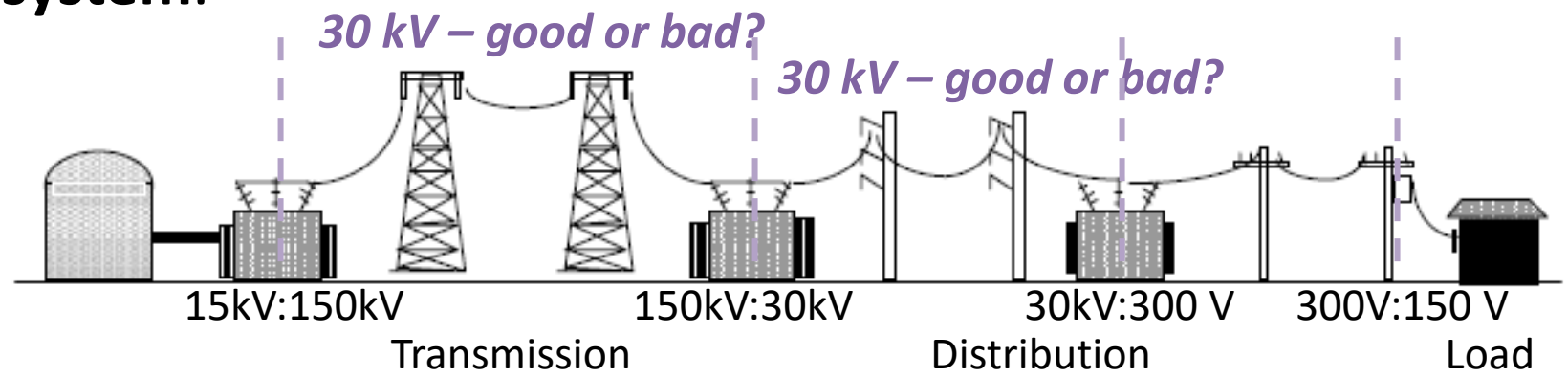
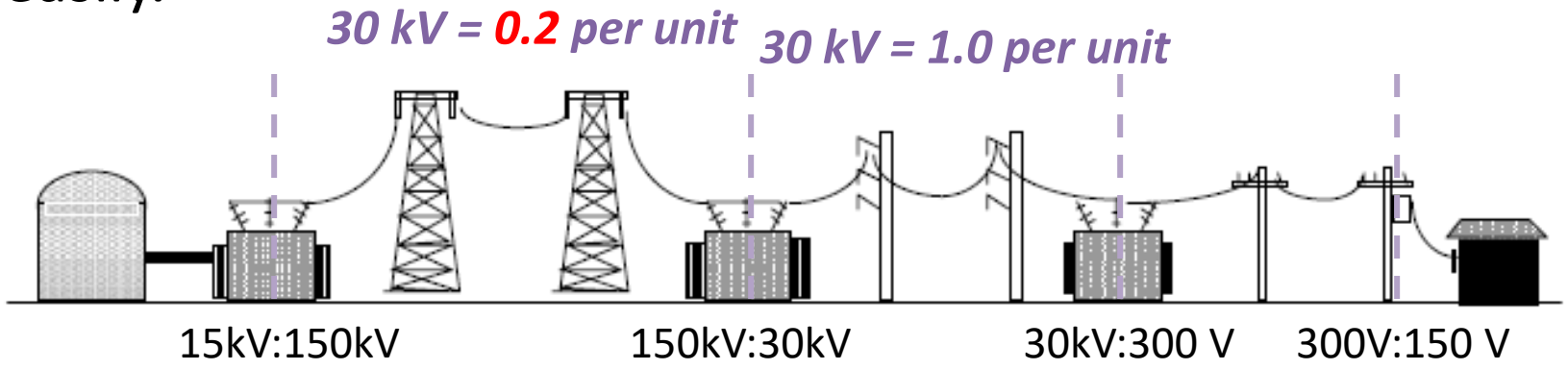
Motivations

- Transformer introduces various voltage levels.
- So far we can only reflect the load from one side of the transformer to another. Still we need to use turns ratio to find voltage and current at each side of the transformer.
- It is *difficult to calculate voltage and current* of the system at various points.
- It is **even more difficult** for system operators **to observe the current situation of the system**.



Per Unit System

- Per unit system is when we **normalize** the voltage and current at each location.
- The normalization typically follows transformer ratings.
- This usually makes the per unit value of both voltage and current to be around 1.0 per unit.
- Per unit system allows system operators to overlook abnormalities in the system easily.



Per Unit Quantity

- The per unit quantity of voltage, current, power and impedance is found from dividing the actual quantity by a base value of that quantity

$$\text{per} - \text{unit quantity} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

- Per unit value is denoted by 'p.u.'
- All base values are **real numbers**, denoted by subscript '*B*'
- The base value is *used only to normalize the quantity*

Base Values

- Complex Power Base
- Voltage Base
- Current Base
- Impedance Base

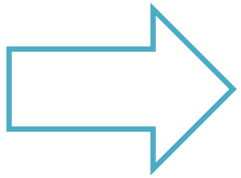
KVL, KCL, Complex Power Calculation

- We can still apply KVL, KCL, complex power calculation to the per unit value
- The actual quantity is simply found from **multiplying** **the *per unit quantity*** (normalized quantity) **with the *base value***

$$S_B = V_B I_B$$

$$V_B = Z_B I_B$$

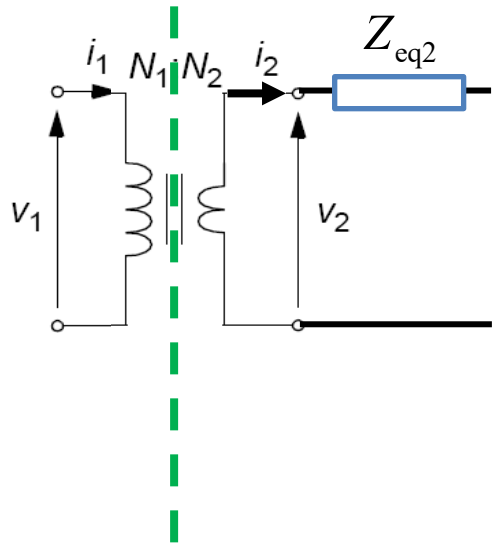
Think of Base value as 'Normalization'.



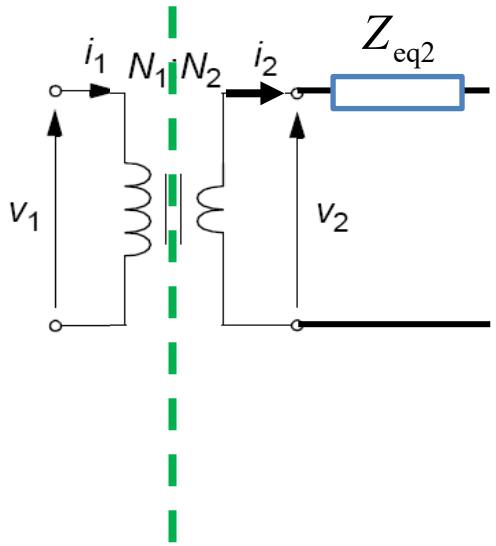
KVL, KCL, Complex Power Calculation

Example 1: Per Unit Value

- A single-phase 20kVA, 480/120 V, 60 Hz transformer has an equivalent leakage impedance referred to 120-volt winding of $Z_{eq2} = 0.0525\angle 78.13°\ \Omega$. Using the transformer rating as base values, find per-unit leakage impedance.

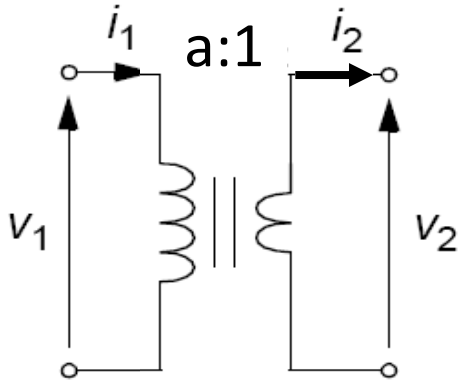


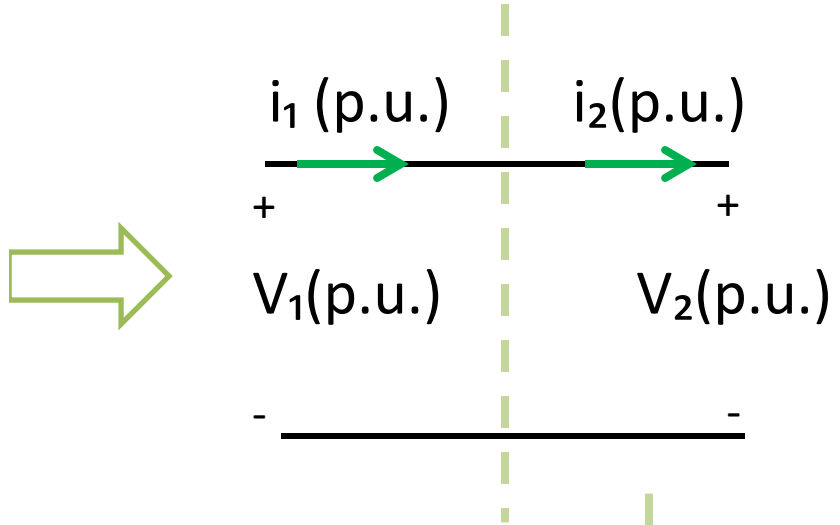
Example 1: Per Unit Value



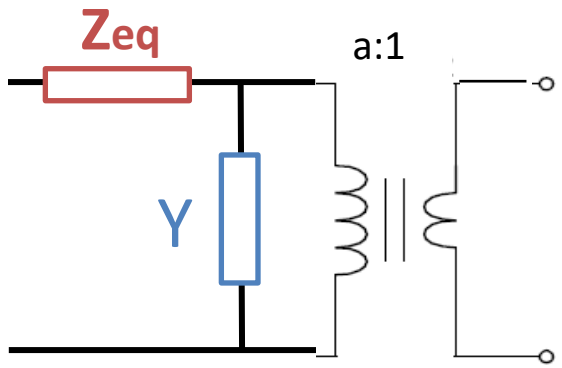
P.U. Equivalent Circuit of a Transformer

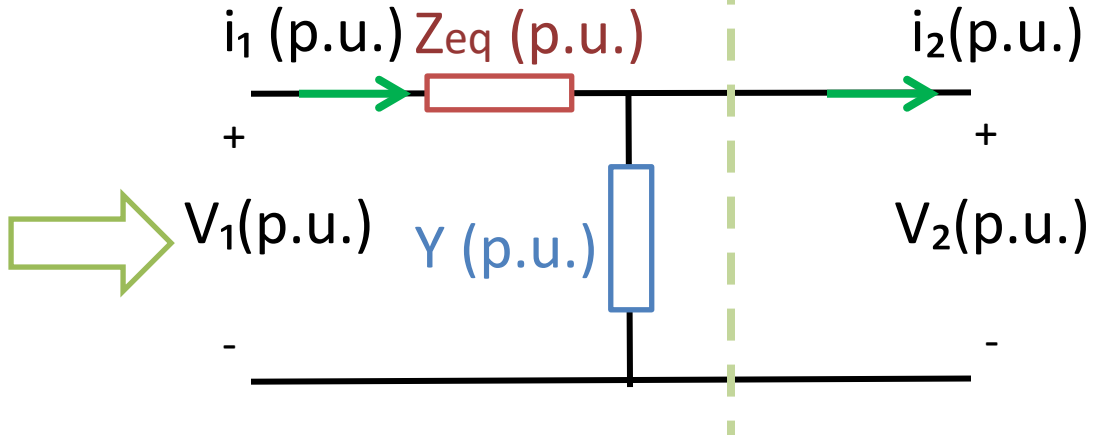
- Ideal transformer model





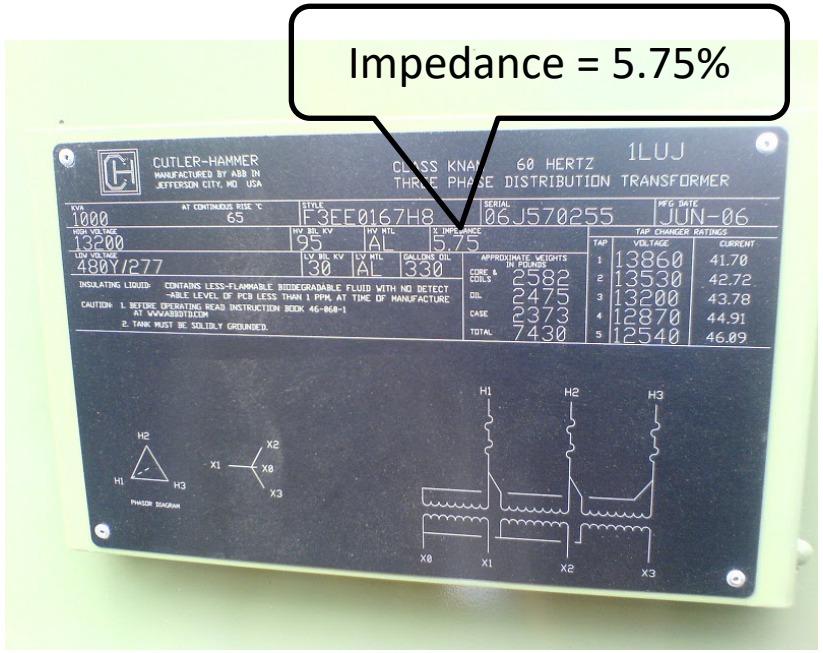
- Practical transformer model





Change of Base Value

- Manufacturers usually specify equipment impedances in per unit values together with voltage ratings (V) and apparent power rating (VA).



Advantages of Per Unit Analysis

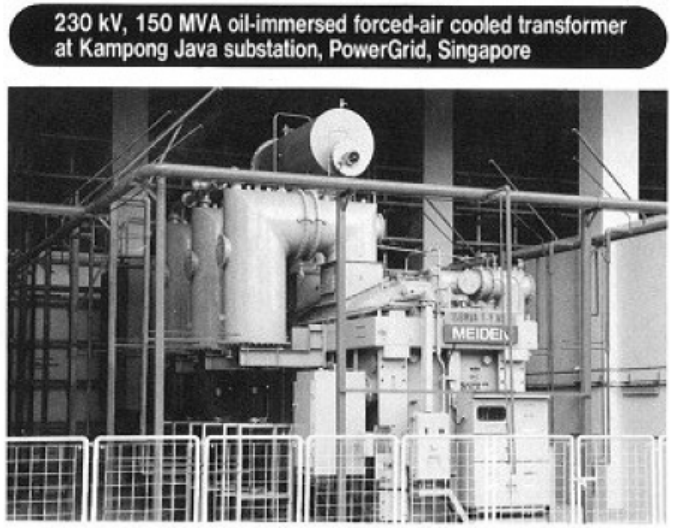
- Simplify calculation by eliminating transformers.
- Helps to spot errors in the data
  - p.u. is more uniform compare to actual impedance value of different sizes of equipment.
- Helps to detect abnormality in the system
  - Operator at control center can spot over/under voltage/current rating easily.

Steps of 1Φ Per Unit Analysis

1. Choose $S_B^{1\Phi}$ for the system.
2. Select V_B for different zones (usually follows transformer voltage ratings).
3. Calculate Z_{B} for different zones.
4. Express all quantities in p.u.
5. Draw impedance diagram and solve for p.u. quantities.
6. Convert back to actual quantities if needed.

Three-Phase Transformers

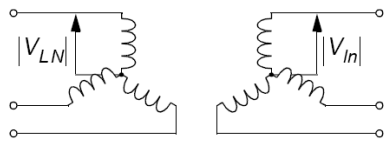
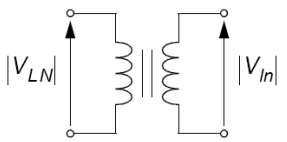
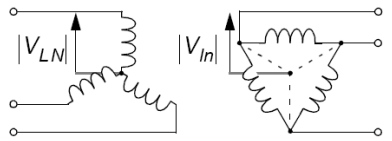
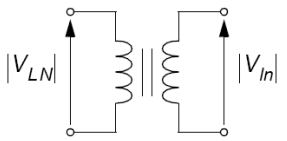
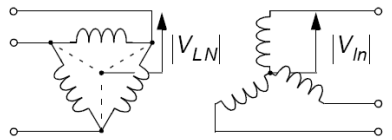
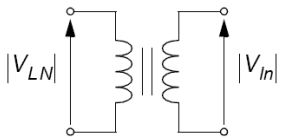
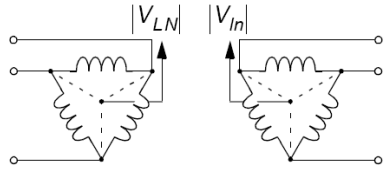
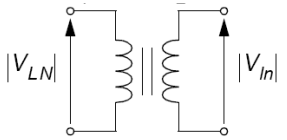
Three-phase transformers at substations



Source: http://www.meidensg.com.sg

Pole-mounted three single-phase transformers.

3Φ Transformer Connection

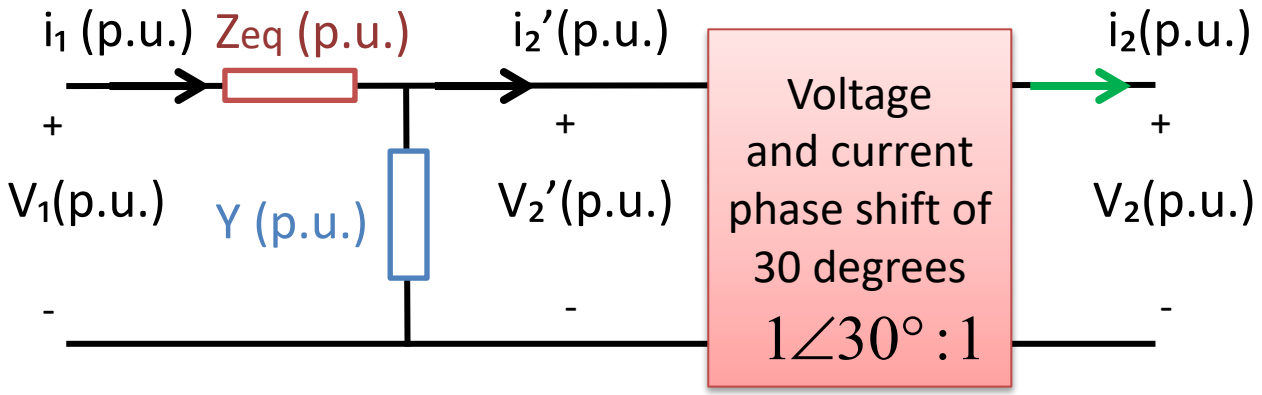
designation	winding connection	single-phase equivalent
Yy		
Yd		
Dy		
Dd		

- The voltage rating of a three phase transformer is the ratio between **line-to-line** voltage at the primary side and **line-to-line** voltage at the secondary side.
- The single-phase equivalent shows line-to-neutral voltage.
- For Y-Y and Δ-Δ transformers, voltage and current in both primary and secondary are in phase. The ratio of the voltage and current follows the turn ratio of the transformer.
- The same does not apply to Y-Δ and Δ-Y connections.

Y-Δ 3Φ Transformer: Per Phase Model

For a positive sequence voltage source,

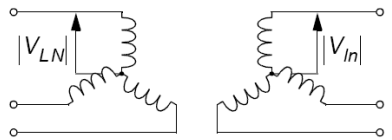
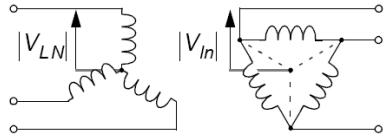
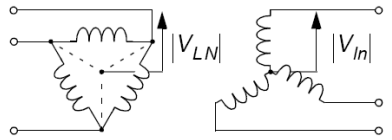
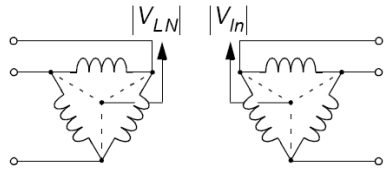
$$V_{A1-n} = \left(\frac{N_1}{N_2}\right)\sqrt{3}V_{A2-n}\angle 30° \quad \Rightarrow \quad \angle V_{A1-n} : \angle V_{A2-n} = 1\angle 30° : 1$$



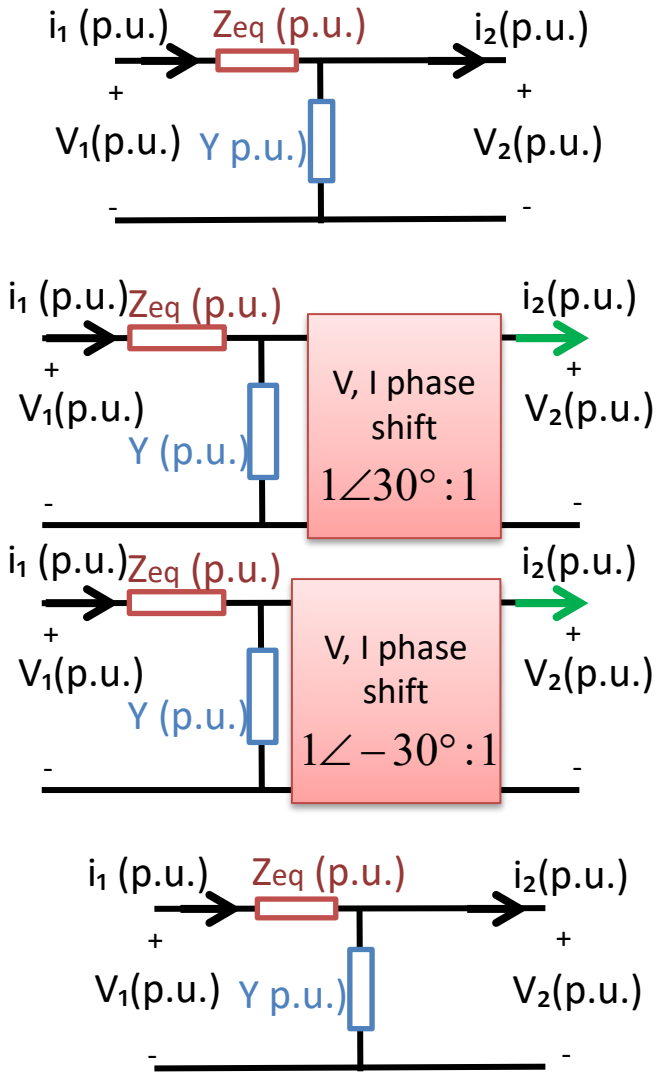
$$i_2 = i_2'\angle -30°$$

$$V_2 = V_2'\angle -30°$$

3Φ Transformer Per Unit Model

designation	winding connection
Yy	
Yd	
Dy	
Dd	

Per phase per unit circuit



3Φ Base Values

- Complex Power Base
- Voltage Base
- Current Base
- Impedance Base

Steps of Calculation: 3Φ Case

1. Choose $S_B^{3\Phi}$ for the system.
2. Select $V_B^{\text{line-to-neutral}}$ or $V_B^{\text{line-to-line}}$ for different zones.
3. Calculate Z_{B} for different zones.
4. Express all quantities in p.u.
5. Draw impedance diagram and solve for p.u. quantities.
6. Convert back to actual quantities if needed.

Note that the **per unit circuit is** the circuit in **per-phase analysis** with normalization of the voltage magnitude at different locations. This means that **the phase of voltage** in per unit analysis **refers to** the **line-to-neutral voltage**.