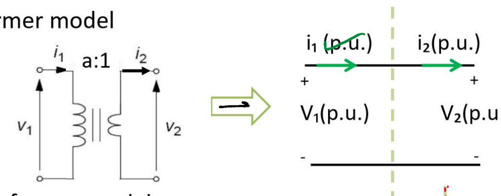
10.2 Per Unit Analysis

Friday, 25 March 2022 2:59 pm

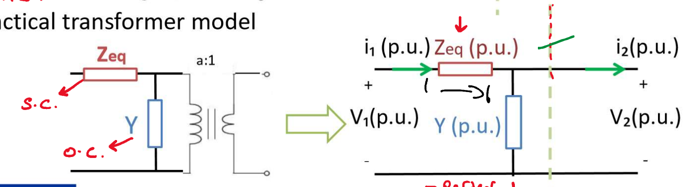
P.U. Equivalent Circuit of a Transformer

- Ideal transformer model



Simplified.

- Practical transformer model



$Z_{eq\,pu} = \dfrac{Z_{eq}}{Z_{B.primary}}$ → Reflected to the primary

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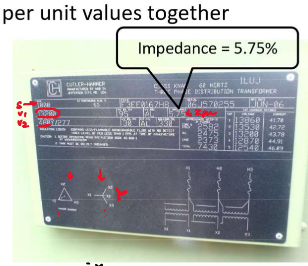
$Y_{p.u} = \dfrac{Y}{Y_{B\text{-}primary}}$ $Y_{B,primary} = \dfrac{1}{Z_{B,primary}}$.

Change of Base Value

- Manufacturers usually specify equipment impedances in per unit values together with voltage ratings (V) and apparent power rating (VA).



$Z_{pu} = 0.0575$ → By manufacturer.

$Z_{pu\,old} = \dfrac{Z_{actual}}{Z_{base}}$. $Z_{base} = \dfrac{V_B^2}{S_B}$ $V_1 : V_2$ S

0.5pu → Base is 100 → Actual value = 50

→ Base is 10 → p.u. value → $\frac{50}{10} = 5$ pu

$Z^1_{Bold} = \dfrac{(V^1_{Bold})^2}{S_{Bold}}$; $Z^2_{Bold} = \dfrac{(V^2_{Bold})^2}{S_{Bold}}$.

$V^1_{Bold} = V_{1\,rat}$ $S_{Bold} = S_{rated}$

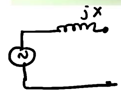
$V^2_{Bold} = V_{2\,rat}$.

$Z^1_{actual} = Z_{pu.old} \times Z^1_{Bold}$

Zone 1 → $Z^1_B = \dfrac{(V^1_B)^2}{S_B}$ → Chosen by me to solve the system

$Z^{new}_{pu} = \dfrac{Z^1_{actual}}{Z^1_B}$

$Z^{new}_{pu} = \dfrac{Z_{pu\,old} \times Z^1_{Bold}}{Z^1_B}$



S_{rated}, V_{rated}.

$X = 0.03$ p.u

$X = \dfrac{X_{actual}}{Z_B}$ → $\dfrac{V_{rat}^2}{S_{rat}}$

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$Z_B \to 100\ \Omega$

$jX \to j10 \to 10\angle 90$

$Y_{pu} = \dfrac{10\angle 90}{100} = 0.1\angle 90°$ pu. RIGHT ✓

WRONG BASE

$Z_B \to 100\angle 90°\ \Omega$

$jX \to j10 \to 10\angle 90°$

$X_{pu} = \dfrac{10\angle 90}{100\angle 90°} = 0.1$ pu. X WRONG

Advantages of Per Unit Analysis

0.1pu

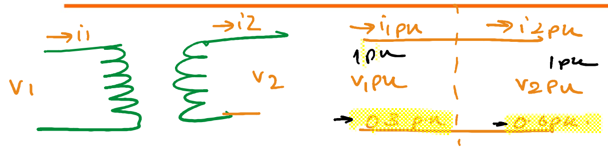
$G_1 \leftarrow 4V \xrightarrow{V_B} 1.5pu$

$G_2 \leftarrow 40V \xrightarrow{V_B} 0.05$

$G_1 \rightarrow$ 1000KVA (V) $\rightarrow jx \rightarrow j5$

$G_2 \rightarrow$ 10000KVA (V) $\rightarrow j5$

- Simplify calculation by eliminating transformers.
- Helps to spot errors in the data
  - p.u. is more uniform compare to actual impedance value of different sizes of equipment.
- Helps to detect abnormality in the system
  - Operator at control center can spot over/under voltage/current rating easily.



$V_1 pu = V_2 pu$

$i_1 pu = i_2 pu$.

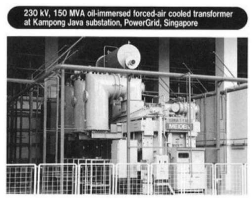
Steps of 1Φ Per Unit Analysis

1. Choose $S_B^{1\Phi}$ for the system.
2. Select V_B for different zones (usually follows transformer voltage ratings). → Choose or given V_B for one of the zone
   → Calculate V_B for other zones using transformer ratio.
3. Calculate Z_B for different zones. $\rightarrow Z_B = \frac{V_B^2}{S_B}$
4. Express all quantities in p.u.
   4.1
5. Draw impedance diagram and solve for p.u. quantities.
6. Convert back to actual quantities if needed.

→ 4.1 → Find new p.u. impedance values if manufacturer has given impedance in p.u. instead of ohms

Three-Phase Transformers

Three-phase transformers at substations



Source: http://www.meidensg.com.sg

Pole-mounted three single-phase transformers.

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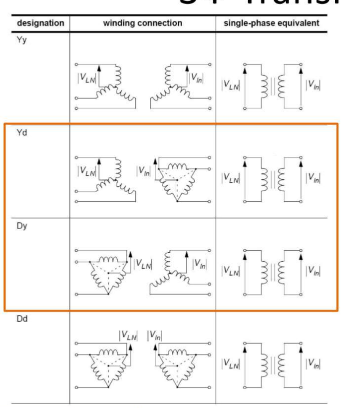
3Φ Transformer Connection



- The voltage rating of a three phase transformer is the ratio between **line-to-line** voltage at the primary side and **line-to-line** voltage at the secondary side.
- The single-phase equivalent shows line-to-neutral voltage.
- For Y-Y and Δ-Δ transformers, voltage and current in both primary and secondary are in phase. The ratio of the voltage and current follows the turn ratio of the transformer.
- The same does not apply to Y-Δ and Δ-Y connections.

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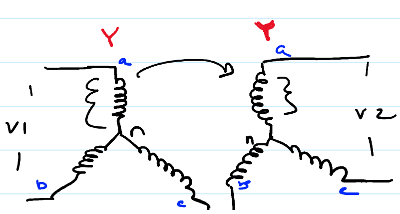
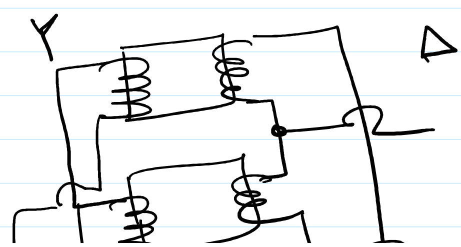
⇒ 3Φ Transformer connections,

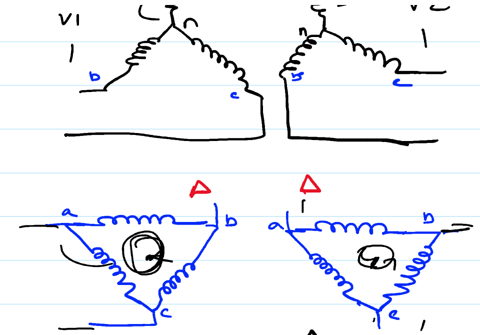


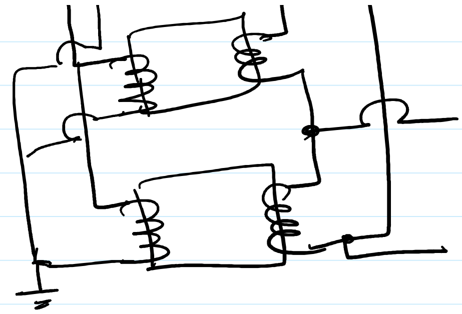
Voltage rating of a 3φ transformer.
→ Ratio of the line voltage of primary to the line voltage of secondary.

→ Voltage & current in both primary...



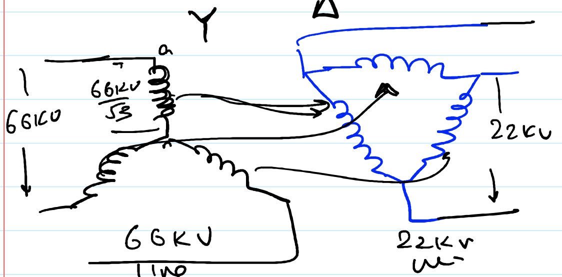
to the unit voltage of secondary.



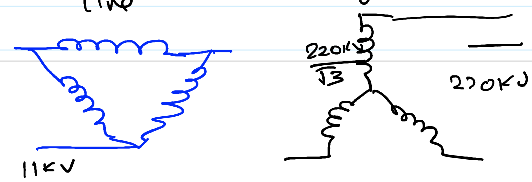


→ Voltage & current in both primary and secondary are in phase with each other.
→ Y-Y, Δ-Δ

→ Y-Δ or Δ-Y configuration
→ Phase shift of ±30° in both voltage & current of secondary with respect to primary,



→ Y-Δ → Step down transformer

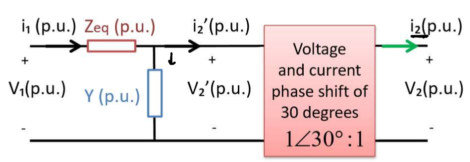


Δ-Y → Step up transformer

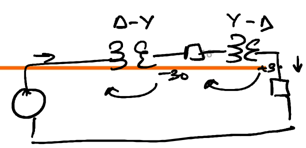
Y-Δ 3Φ Transformer: Per Phase Model

For a positive sequence voltage source,

$$V_{A1-n} = \boxed{\left(\frac{N_1}{N_2}\right)\sqrt{3}}V_{A2-n}\angle 30^\circ \quad \Rightarrow \quad \angle V_{A1-n} : \angle V_{A2-n} = 1\angle 30^\circ : 1$$



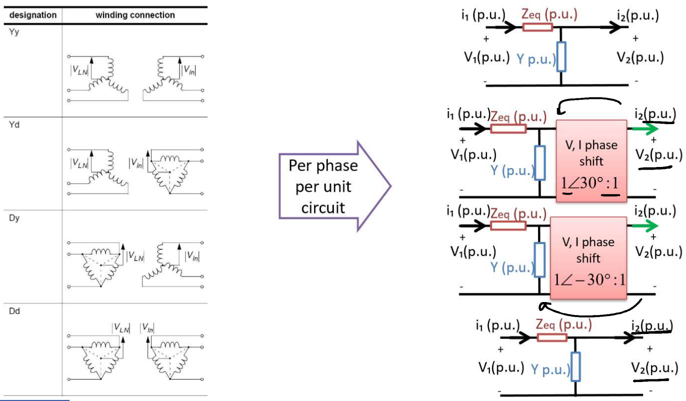
$$i_2 = i_2'\angle -30^\circ$$
$$V_2 = V_2'\angle -30^\circ$$



ΔY → $i_2 \rightarrow i_2' \angle 30^\circ$

$V_2 \rightarrow V_2' \angle 30^\circ$

3Φ Transformer Per Unit Model



3φ system

→ Voltage is -line-to line voltage.

→ Current is line current.

→ Apparent power → 3φ power.

$$|S_{3\varphi}| = \sqrt{3}\,|V_{line-line}|\,|I_{line}|$$

3Φ Base Values

- Complex Power Base → Single $S_{3\varphi}$ for the whole system
- Voltage Base → Each zone different Base value.
  → $V^B_{line-line}$
  $V^B_{line}/\sqrt{3} = V^B_{ph}$
- Current Base → Different for each zone
  $I^B_{line} = \dfrac{S_B}{\sqrt{3}\,V_{line}}$
- Impedance Base → Different for each zone
  $$Z_B = \frac{V_{line}^2}{S_B^{3\varphi}}$$

Steps of Calculation: 3Φ Case

1. Choose $S_B^{3\Phi}$ for the system.
2. Select $V_B^{\text{line-to-neutral}}$ or $V_B^{\text{line-to-line}}$ for different zones.
3. Calculate $\underline{Z}_{\mathrm{B}}$ for different zones.
4. Express all quantities in p.u.
5. Draw impedance diagram and solve for p.u. quantities.
6. Convert back to actual quantities if needed.

Note that the **per unit circuit is** the circuit in **per-phase analysis** with normalization of the voltage magnitude at different locations. This means that **the phase of voltage** in per unit analysis **refers to** the **line-to-neutral voltage**.