Image source: www.elex.com.np

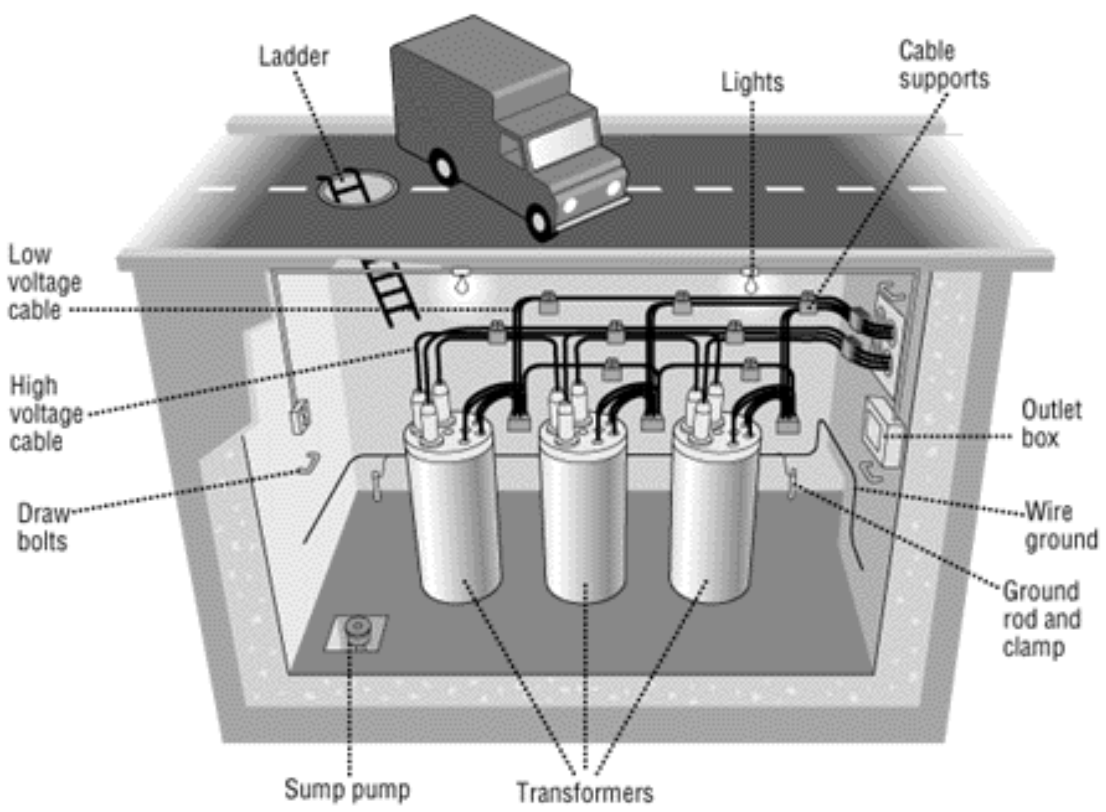
EE2029 - Introduction to Electrical Energy Systems

Transformers

Current-Voltage Relationship
Transformer Core Modeling
Equivalent Circuit
Open and Short Circuit Tests
Per Unit Analysis
Three-Phase Transformer

Learning Outcomes

- Able to construct an equivalent circuit of a transformer.
- Able to estimate the transformer parameters through open-circuit and short-circuit tests.
- Apply per unit analysis to normalise different voltage ratings to a common reference.
- Apply three-phase circuit theory to model three-phase transformer of different connection types, e.g. delta-wye or wye-wye.



underground substation

Image source: https://slideplayer.com/slide/12883106/

Overview

- Power grid operates at many voltage levels, ranging from 120/240 V to 765 kV.
- Transformers *transfer* energy between different voltage levels.
- Ability to *inexpensively* change voltage levels is a key advantage of AC systems over DC systems.
- Ideal transformer has:
  - no real power losses
  - no leakage flux
  - magnetic core has infinite permeability

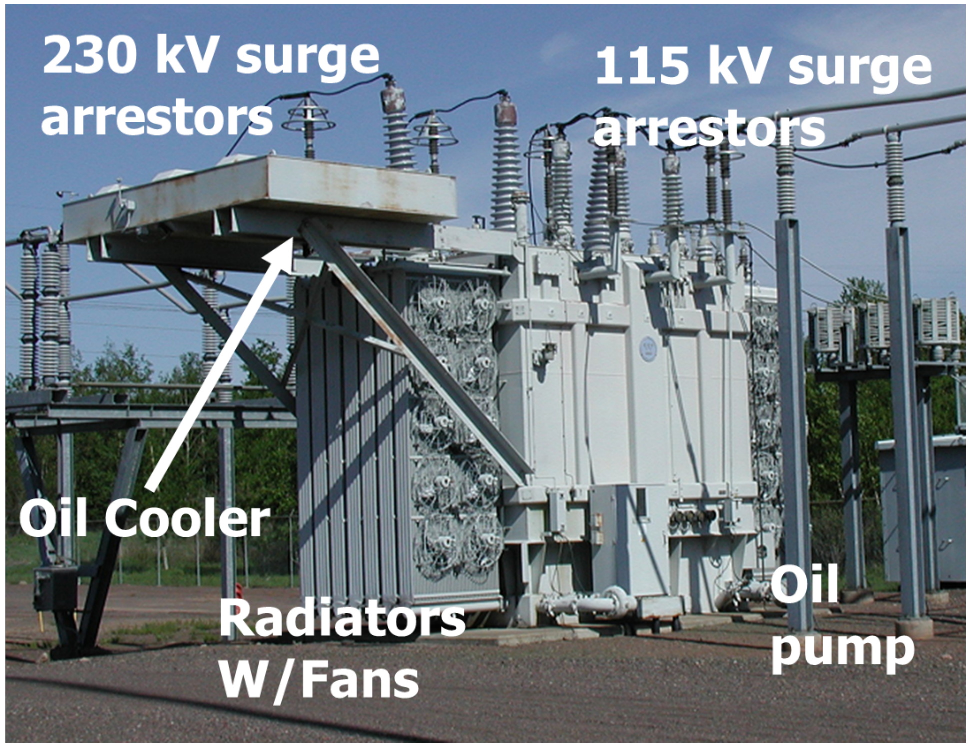
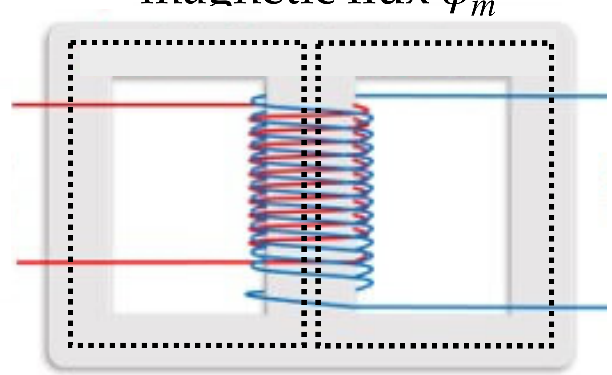


Image source: www.elex.com.np

Ideal Transformer Relationships

- For a transformer, **primary** side takes power, and **secondary** transfer power.
  - Primary is usually the side with the higher voltage.
- Voltage relationship:



shell type transformer

primary

$$\lambda_1 = N_1\phi_m$$

$$V_1 = \frac{d\lambda_1}{dt} = N_1\frac{d\phi_m}{dt}$$

secondary

$$\lambda_2 = N_2\phi_m$$

$$V_2 = \frac{d\lambda_2}{dt} = N_2\frac{d\phi_m}{dt}$$

$$\frac{d\phi_m}{dt} = \frac{V_1}{N_1} = \frac{V_2}{N_2} \rightarrow \frac{V_1}{V_2} = \frac{N_1}{N_2} = a = \text{turns ratio}$$

- Current relationship:

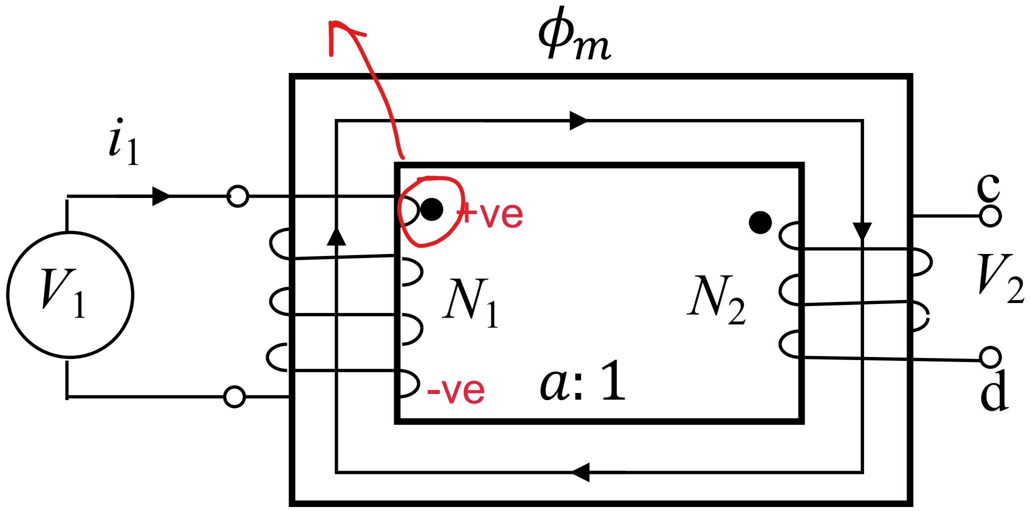
$$mmf = \oint \mathbf{H}\,d\mathbf{l} = N_1 i_1 + N_2 i_2 \Rightarrow H \times length = N_1 i_1 + N_2 i_2' \quad \text{or} \quad \frac{B \times length}{\mu} = N_1 i_1 + N_2 i_2'$$

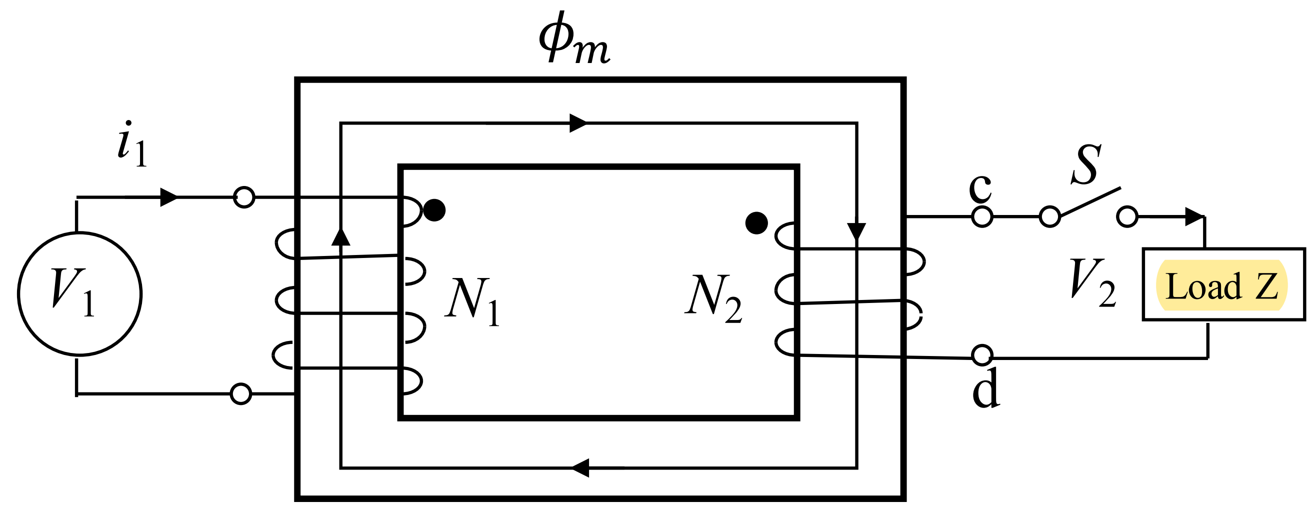
Assume uniform flux density in the core: $\dfrac{\phi \times length}{\mu \times area} = N_1 i_1 + N_2 i_2'$ = 0 when miu = 0)

permittivity (miu) = infinite --> field intensity around the closed path = 0

Current-Voltage Relationship

the dot shows +ve terminal





core type transformer

$i_2 = i_2' + i_{leakage}$ Aassume $i_{leakage} = 0$

If μ is infinite $\rightarrow N_1 i_1 + N_2 i_2 = 0$

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} \quad \text{or} \quad \frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a}$$

$$\begin{array}{ccccc} \text{primary (source)} & & \text{turns ratio} & & \text{secondary (sink)} \\ \begin{bmatrix} V_1 \\ i_1 \end{bmatrix} & = & \begin{bmatrix} a & 0 \\ 0 & 1/a \end{bmatrix} & & \begin{bmatrix} V_2 \\ i_2 \end{bmatrix} \end{array}$$

Making $i_2 = V_2 / Z$:

$$V_1 = aV_2$$

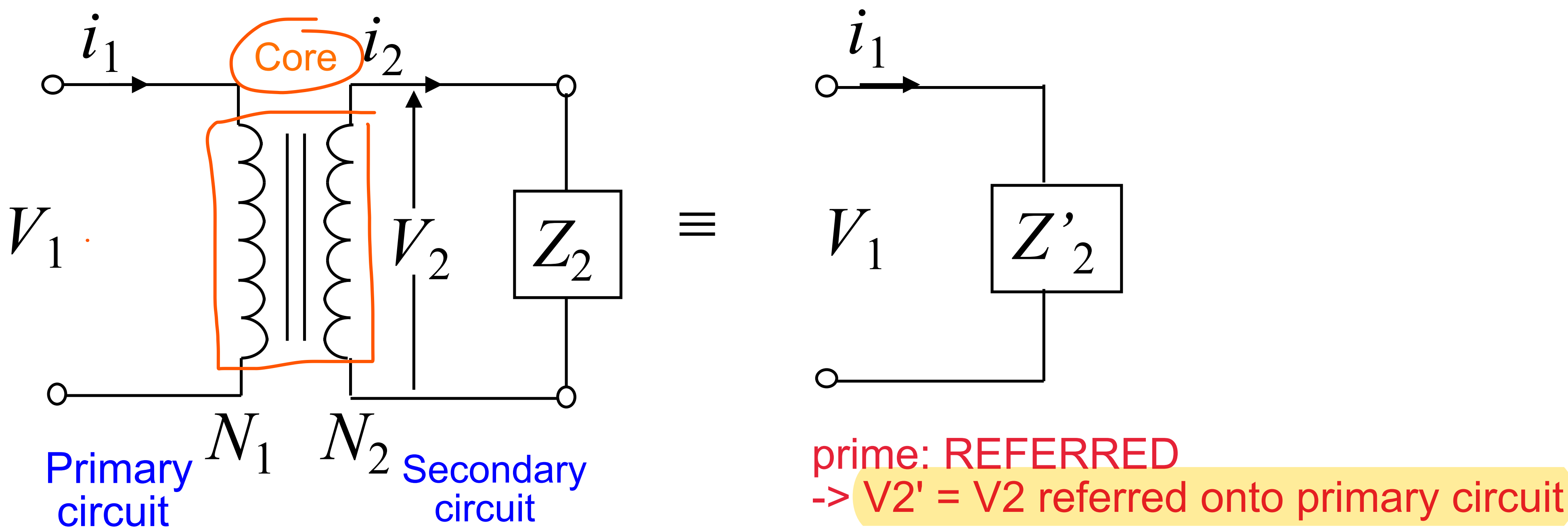
$$\frac{V_1}{i_1} = a^2 Z$$

$i_1 = \frac{aV_2}{Z_2'}$?

$$i_1 = \frac{1}{a}\frac{V_2}{Z}$$

where a = N1/N2

Impedance Transfer



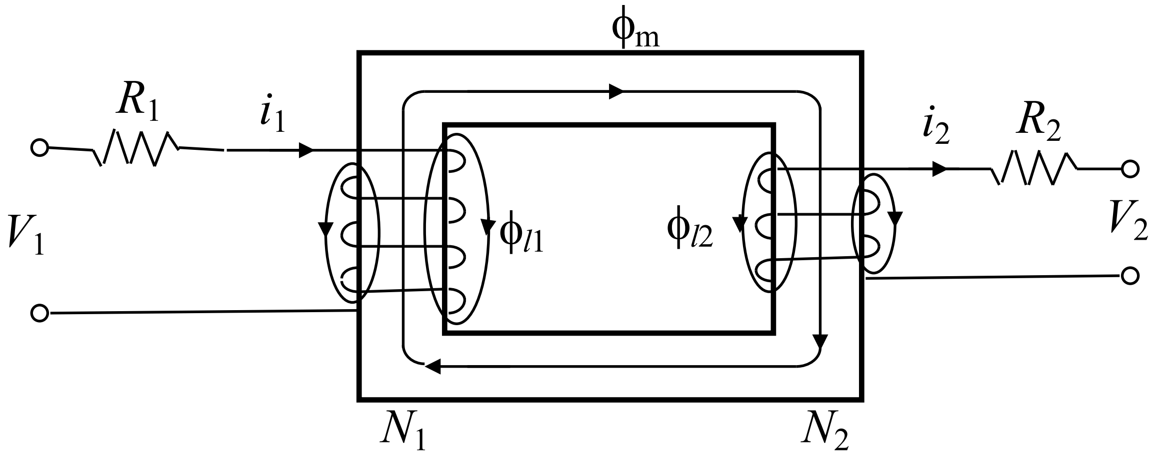
Secondary impedance transferred to primary side: $Z_2' = Z_1 = a^2 Z_2$

Primary impedance transferred to secondary side: $Z_1' = Z_2 = \dfrac{1}{a^2} Z_1$

Impedance transfer is very useful because it eliminates the *coupled magnetic element* in the electrical circuit.

Transformer Core Modeling

Actual Transformer Characteristics



- Real transformers have:
  - Losses
  - Leakage flux
  - Finite permeability of magnetic core (i.e. saturation)
- Real power losses:
  - Resistance in windings (i^2 R)
  - Core losses due to eddy currents and hysteresis

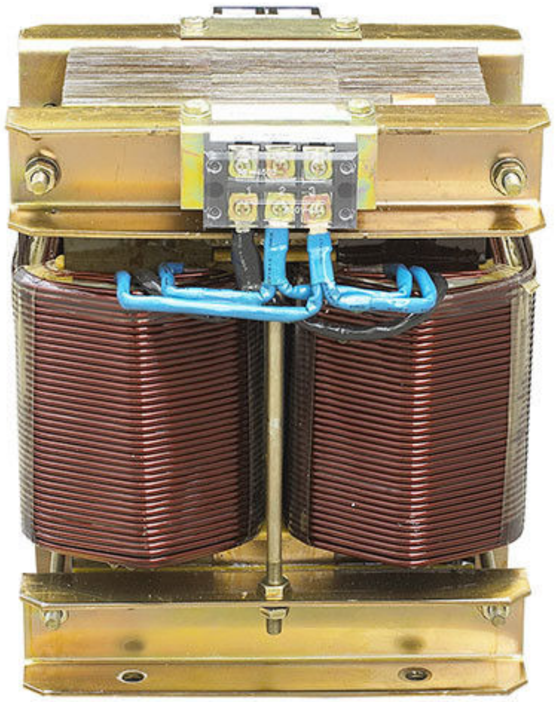
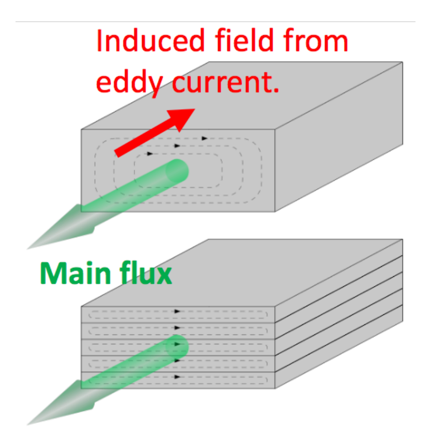
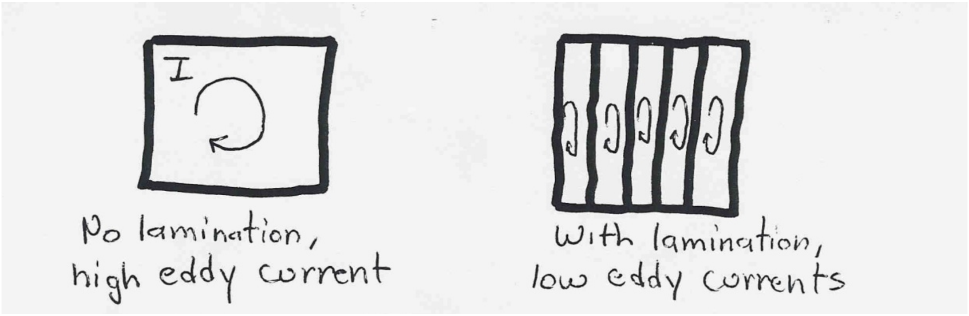


Image source: Ravi Electricals

Transformer Core losses

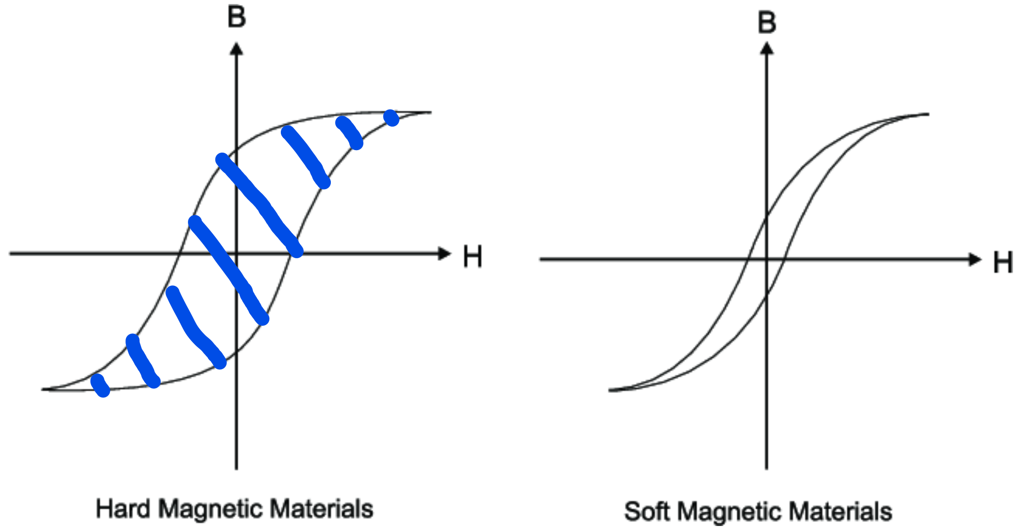
- Eddy currents arise because of changing flux in the core.
  - Refers to the small *circulating current* in the magnetic core caused by the flux that passes through the core.
  - Magnitude of eddy current losses depends on the strength of the main flux, thus the voltage supplied.
- Eddy current loss produces heat, and is represented as a **resistance parallel** to the ideal transformer.
- Eddy current loss can be reduced by making the cores from thin sheets of steel i.e. the core is laminated. The thinner the sheets, the smaller are the eddy current losses.





Transformer Hysteresis Loss

- Hysteresis losses are proportional to the area of BH curve and frequency.
  - These losses can be reduced by using material with a thin BH curve.



- Hysteresis loop *characterises* how ferromagnetic material is magnetized.
  - This loss is proportional to the frequency of electricity.
- Hysteresis loss generates heat, and is represented as a **resistance** *parallel* to the ideal transformer.

Effect of Leakage Flux

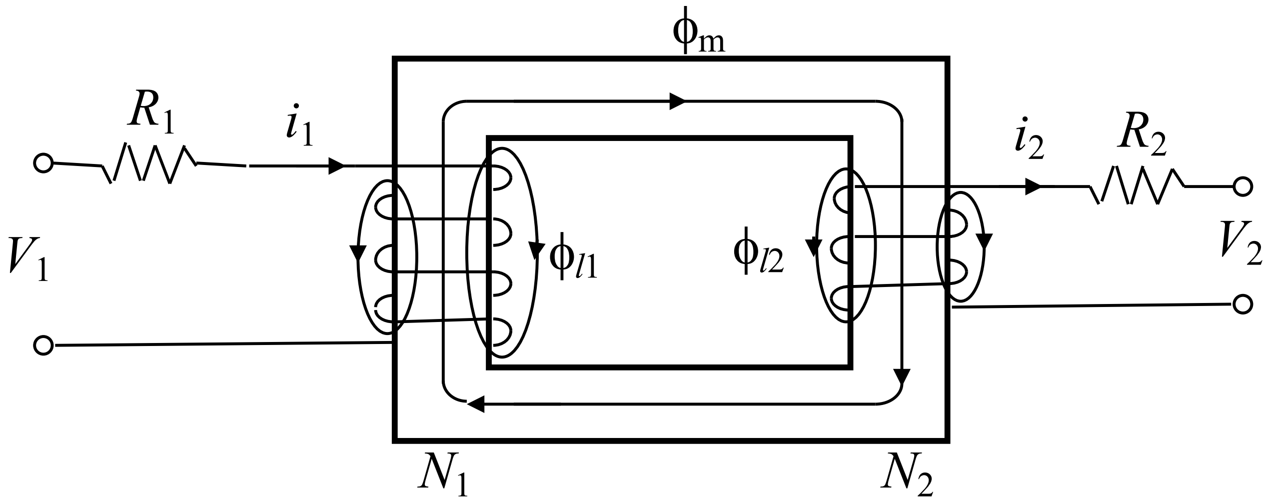
- Not all fluxes are within the transformer core:

$$\begin{cases} \lambda_1 = \lambda_{l1} + N_1\phi_m \\ \lambda_2 = \lambda_{l2} + N_2\phi_m \end{cases}$$

- Suppose the magnetic medium is linear:

$$\begin{cases} \lambda_{l1} = L_{l1} i_1 \\ \lambda_{l2} = L_{l2} i_2^{'} \end{cases}$$

$$\begin{cases} V_1 = R_1 i_1 + L_{l1}\dfrac{di_1}{dt} + N_1\dfrac{d\phi_m}{dt} \\ V_2 = R_2 i_2 + L_{l2}\dfrac{di_2}{dt} + N_2\dfrac{d\phi_m}{dt} \end{cases}$$



Effect of Finite Core Permeability

- Finite core permeability means a non-zero mmf is needed to maintain ϕ_m in the core:

$$N_1 i_1 - N_2 i_2 = \Gamma \phi_m$$

r = reluctance = opposition to magnetic flux

- This value is usually modelled as a magnetising current:

$$i_1 = \frac{\Gamma \phi_m}{N_1} + \frac{N_2}{N_1} i_2$$

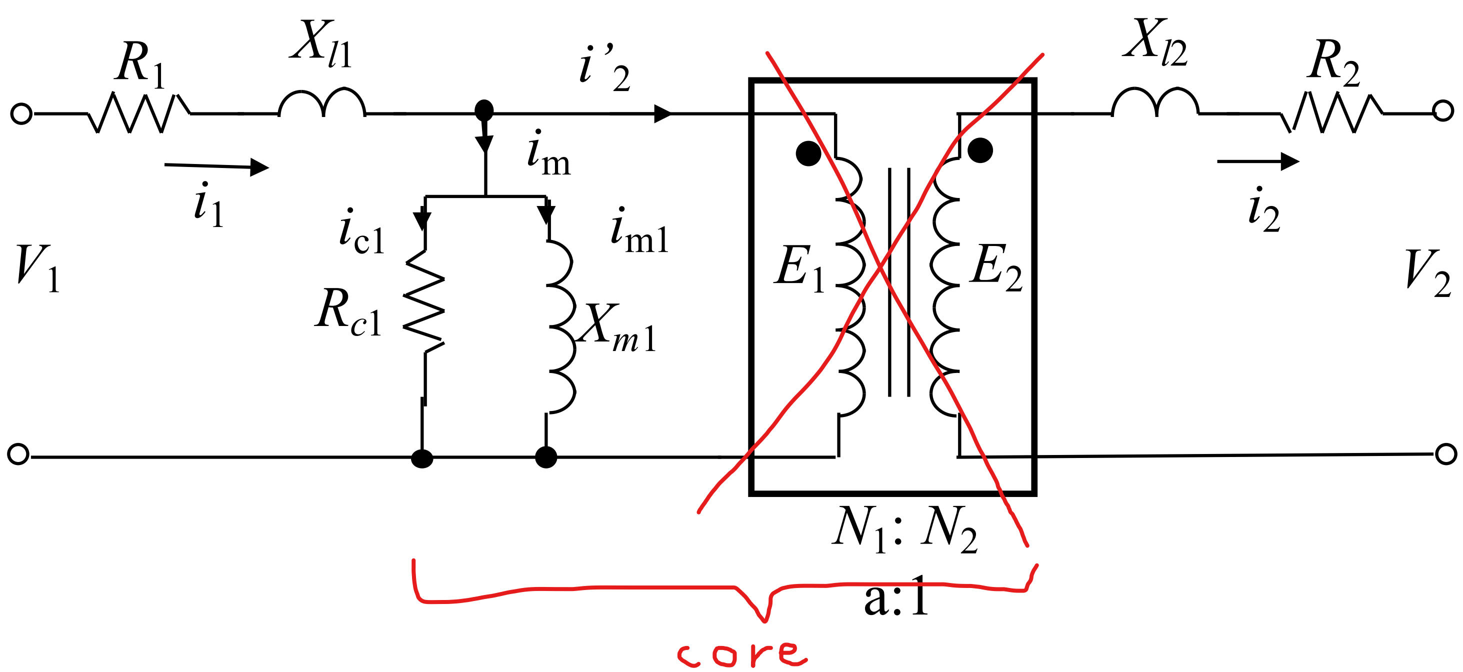
$$i_1 = i_m + \frac{N_2}{N_1} i_2$$

$$i_m = \frac{\Gamma \phi_m}{N_1}$$

Equivalent Circuit

Transformer Equivalent Circuit

- Using the previous relationships, we can derive an equivalent circuit model for the real transformer:



Core loss can be represented by R_{c1}

In practice, I_m is required to establish flux ϕ_m in core. This effect can be represented by a magnetising inductance L_{m1}

*Goal: Merge primary & secondary circuits!!

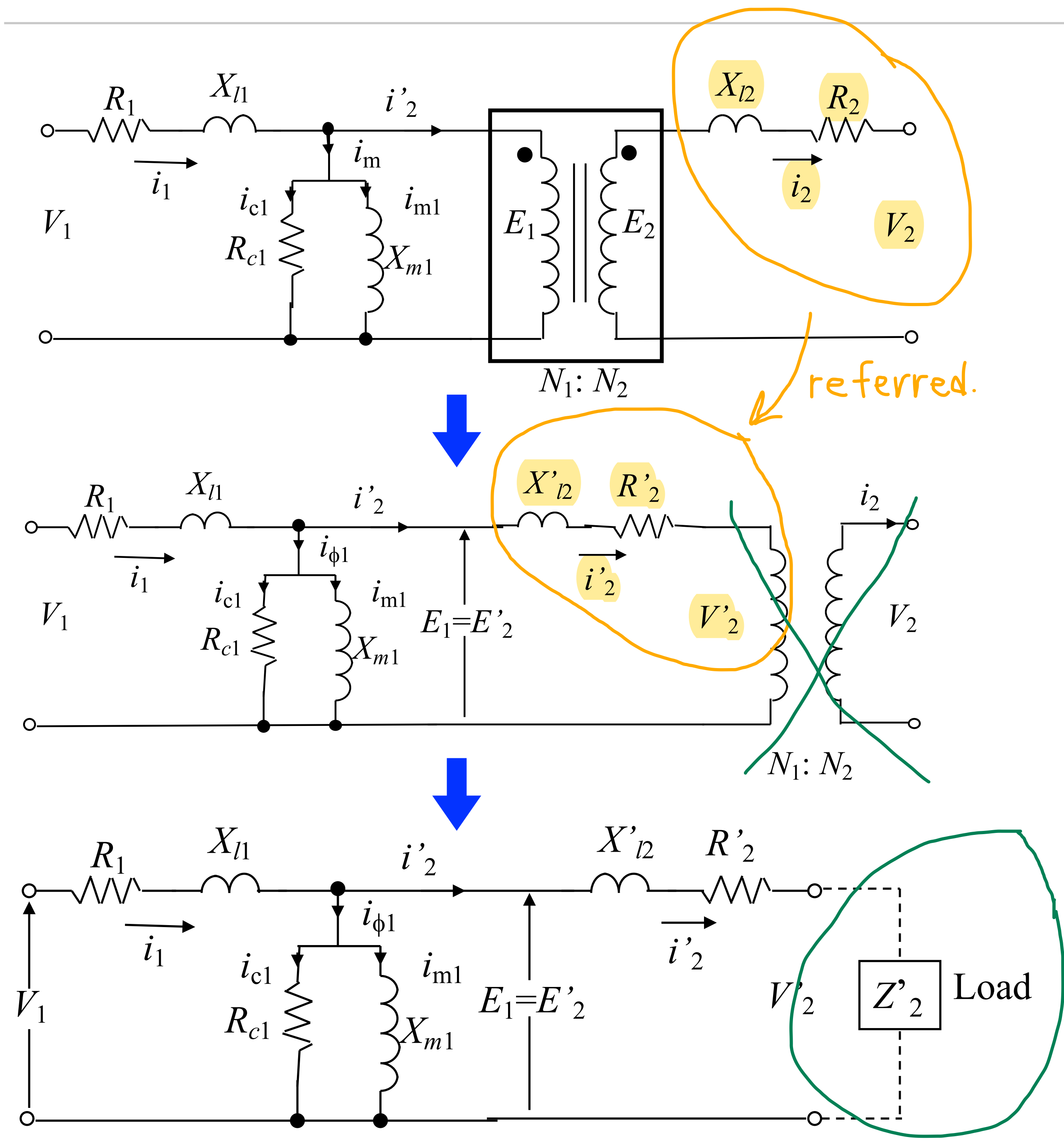
- We can then simplify the circuit by stating:

$$R_2' = a^2 R_2 \qquad R_e = R_1 + R_2'$$

$$X_2' = a^2 X_2 \qquad X_e = X_1 + X_2'$$

Note $X = 2\pi f L$

Transformer Equivalent Circuit



$$V_1 = \underbrace{E_2\frac{N_1}{N_2}}_{E_1} + i_1(R_1 + jX_{l1})$$

KVL.

$$V_1 = [V_2 + i_2(R_2 + jX_{l2})]\frac{N_1}{N_2} + i_1(R_1 + jX_{l1})$$

$$V_1 = \left[V_2\frac{N_1}{N_2} + i_2'\left(\frac{N_1}{N_2}\right)^2(R_2 + jX_{l2})\right] + i_1(R_1 + jX_{l1})$$

$$V_1 = [V_2' + i_2'(R_2' + jX_2')] + i_1(R_1 + jX_{l1})$$

where $E_1 = E_2' = aE_2$ $\quad X_{l2}' = a^2X_{l2}$

$V_2' = aV_2$ $\quad R_2' = a^2R_2$

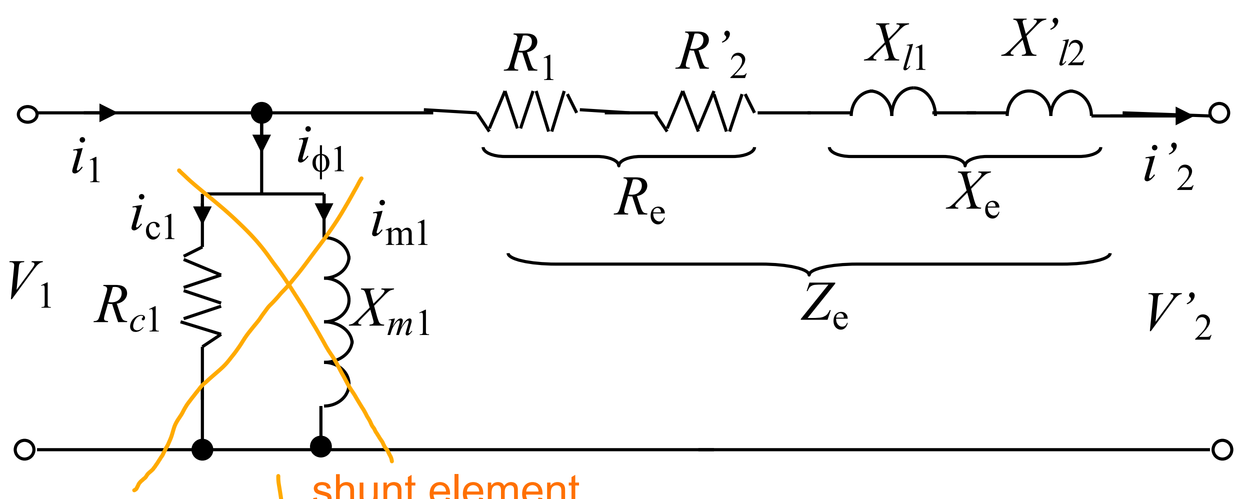
$i_2' = i_2/a$

$$i_2 = i_2'\left(\frac{N_1}{N_2}\right),\quad R_2' = R_2\left(\frac{N_1}{N_2}\right)^2,\quad X_2' = X_{l2}\left(\frac{N_1}{N_2}\right)^2$$

Approximated Equivalent Circuit

I_1R_1 and I_1X_{l1} are very small compared to $V_1 \Rightarrow |E_1| \approx |V_1|$

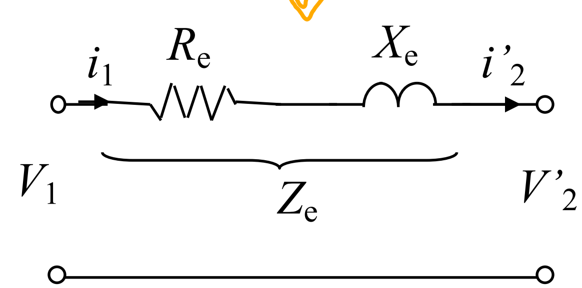
Ze = R1 + R2' +j(Xl1 + Xl2')
= Re + jXe



shunt element

Shunt branch can be *moved* to the supply terminal. The magnetising current is very small and can be neglected.

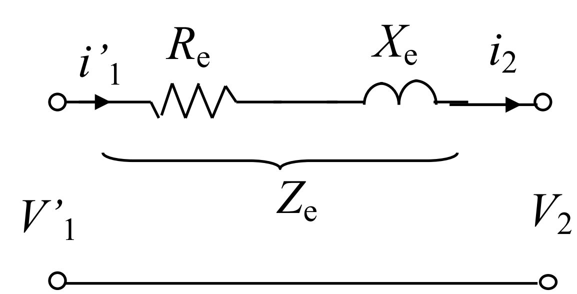
$$R_e = R_1 + a^2 R_2, \quad X_e = X_1 + a^2 X_2$$
$$\Rightarrow \quad Z_e = Z_1 + a^2 Z_2$$



Referred to Primary

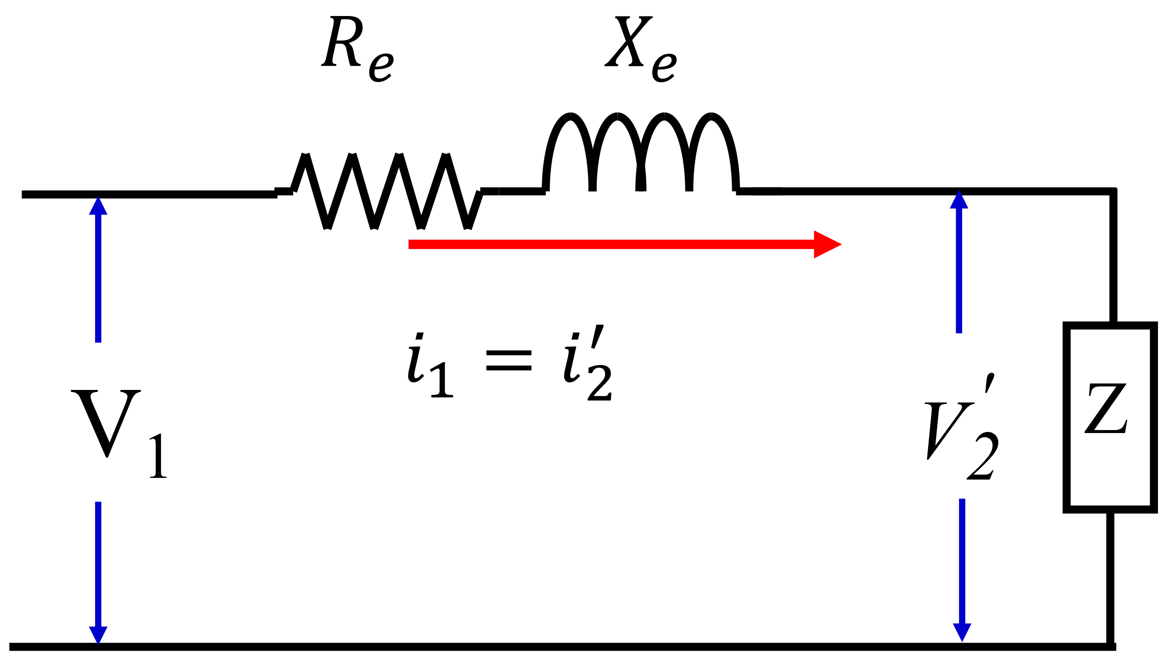
$$V_1' = \frac{V_1}{a} \qquad R_e = \frac{R_e}{a^2} = R_2 + R_1'$$

$$i_1' = ai_1 = i_2 \qquad X_e = \frac{X_e}{a^2} = X_{l2} + X_{l1}'$$



Referred to Secondary

Approximated Equivalent Circuit



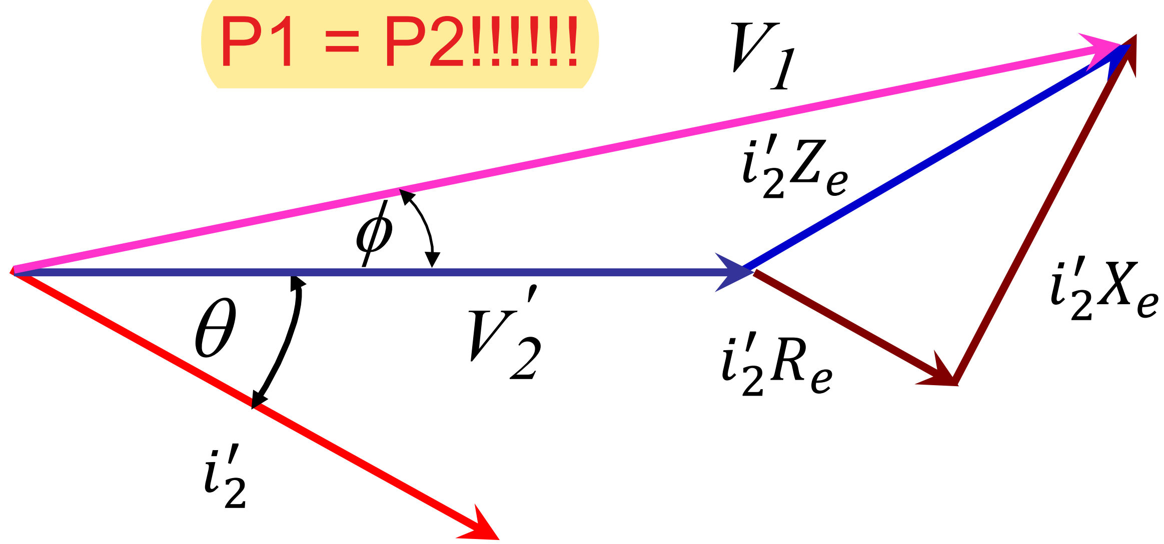
no shunt element!!

$$\bar{V}_1 = \bar{V}_2' + \bar{i}_2'\left(R_e + jX_e\right)$$

$$i_1 = i_2'$$

i2 = i2' + ileakage,2

P1 = P2!!!!!!



In this case, V2 is LAGGING V1, since an INDUCTOR is involved!!

Example 1 - Voltage Regulation

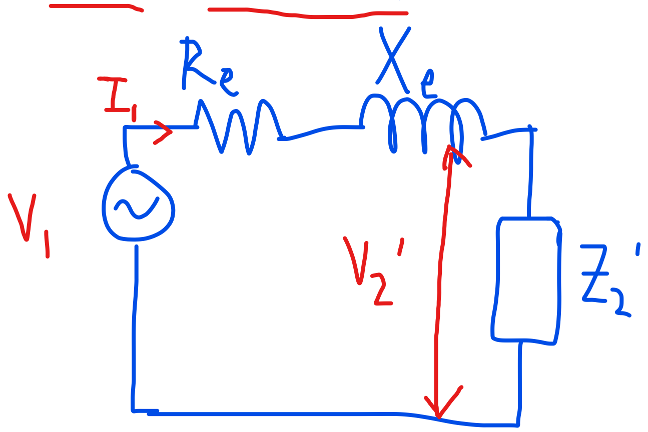
A single-phase transformer has 2000 turns on the primary winding and 500 turns on the secondary. Winding resistances are $R_1 = 2.0\Omega$, $R_2 = 0.125\Omega$. Leakage reactances are $X_1 = 8.0\Omega$ and $X_2 = 0.50\Omega$. The resistance load $Z_2 = 12\Omega$. If applied voltage at the terminals of the primary wiring is 1200V, find V_2 and the voltage regulation. Neglect the magnetising current.

$$\boxed{a = \frac{N_1}{N_2} = \frac{2000}{500} = 4}$$

$$Re = Re_1 + Re_2' = Re_1 + Re_2(a^2) = 4.0\Omega$$

$$X_e = X_1 + X_2' = X_1 + X_2(a^2) = 16\Omega$$

$$Z_2' = Z_2(a^2) = 192\Omega$$



$$I_1 = \frac{V_1}{R_e + jX_e + Z_2'} = \frac{1200\angle 0^\circ}{4 + j16 + 192} = 6.10\angle -4.67^\circ A$$

$$\boxed{I_1 = a\frac{V_2}{Z_2'}}$$

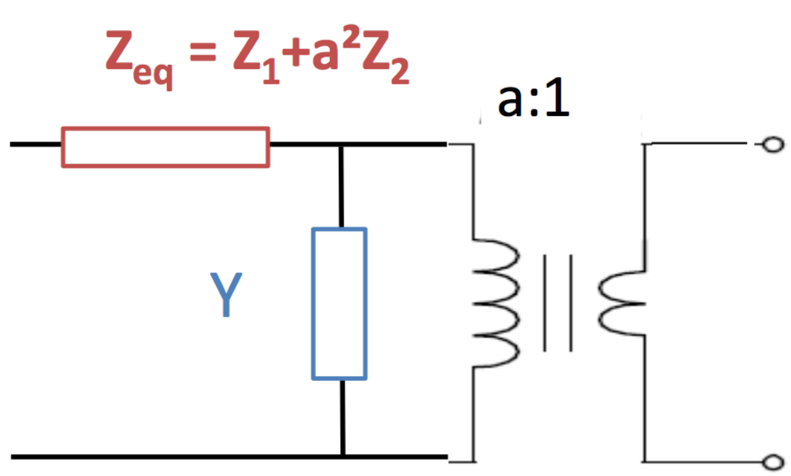
$$\Rightarrow V_2 = \frac{192(6.10\angle -4.67^\circ)}{4} = 292.2\angle -4.67^\circ V$$

$$\boxed{\text{Regulation} = \frac{V_1' - V_2}{V_2}} = \frac{\left(\frac{V_1}{a}\right) - V_2}{V_2} = \frac{\left(\frac{1200}{4}\right) - 292.2}{292.2}$$

$$= 2.42\%$$

Equivalent Circuit Parameters

- **Series impedance:**
  - Z_1 and Z_2 are series impedances representing the resistive loss and flux linkage loss in the two windings.
  - Find using **Short-Circuit Test**
- **Shunt admittance:**
  - Y is a shut admittance representing iron core loss and magnetising susceptance.
  - Find using **Open-Circuit Test**

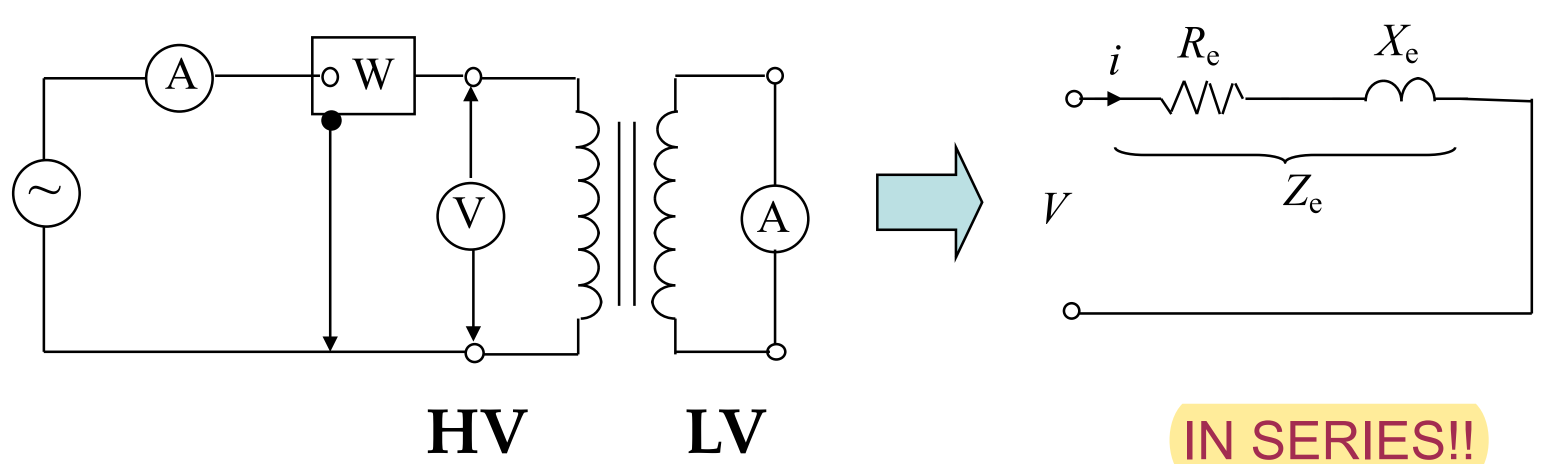


Note that by convention, the primary side of a transformer is the side with a higher number of turns. This means that a > 1.

Short-Circuit Test

(secondary) (primary)

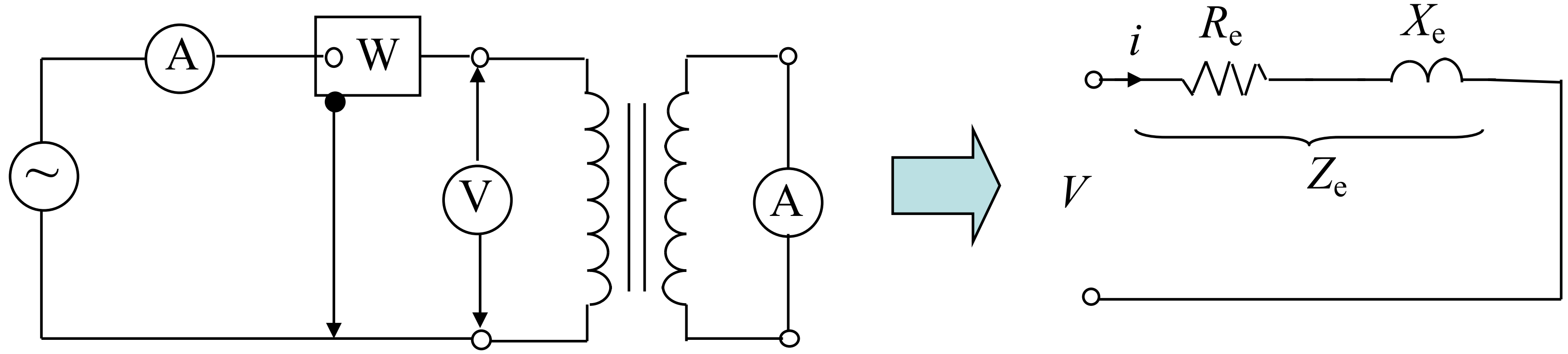
- Short-circuited the **LV side**, and rated current is made to flow in the **HV side** by applying an appropriate voltage.
  - This is because the current on HV side is smaller —> safer to do.
  - The applied voltage should be around 5-10% of the rated voltage due to the short circuit.
  - Large amount of current passing through the impedance. This will allow more accurate calculation of the series impedance
- Measure real power, and voltage at the primary side.



IN SERIES!!

Equivalent Circuit Parameters (Short-Circuit)

- Short-circuit Test:



$$R_e = \frac{P_{SC}}{i_{SC}^2} \qquad X_e = \sqrt{Z_e^2 - R_e^2} \qquad R_e = Z_{SC}\cos(\theta_{SC}) \qquad Z_{SC} = \frac{V_{SC}}{i_{SC}}$$

$$Z_e = \frac{V_{SC}}{i_{SC}} \qquad X_e = Z_{SC}\sin(\theta_{SC})$$

$$\boxed{R_1 + a^2 R_2 = R_e, \quad X_1 + a^2 X_2 = X_e}$$

Primary resistance R_1 can be measured, and *a* is known —> R_2 can be found.
Leakage reactance is assumed to be divided equally, $X_1 = a^2X_2 = 0.5\ X_e$.

Example 2 - Short-Circuit Test

rated $|S|$. primary

A single-phase transformers rated at 15MVA, 11.5/69kV. If the 11.5kV winding (designated as secondary winding) is short-circuited, the rated current flows when the voltage applied to the primary winding is 5.5kV. The input power is 105.8kW. Find R_1 and X_1 referred to the high-voltage (HV) winding.

losses.

For 69kV (rated I_1):

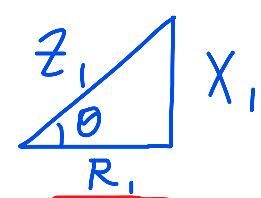
$\boxed{|S| = |I_1||V_1|}$

$\Rightarrow |I_1| = \dfrac{15\times10^6}{69\times10^3} = 217.4\,A$

$\boxed{P_1 = |I_1|^2 R_1} = 105.8\times10^3\,W$

$\Rightarrow R_1 = 2.24\,\Omega$ //

$\boxed{|Z_1| = \dfrac{V_1}{|I_1|}} = \dfrac{5.5\times10^3}{217.4} = 25.30\,\Omega$



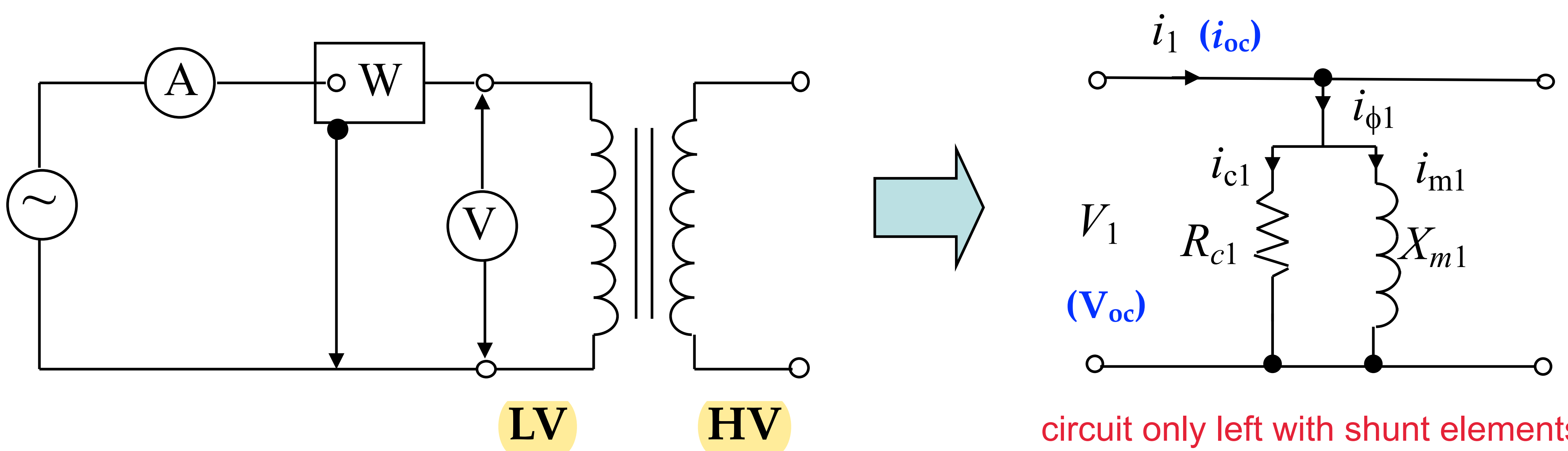
$\Rightarrow \boxed{X_1 = \sqrt{|Z_1|^2 - |R_1|^2}} = \sqrt{25.30^2 - 2.24^2}$

$= 25.20\,\Omega$ //

Open-Circuit Test

can ignore X2 & R2
-> too insigificantly small

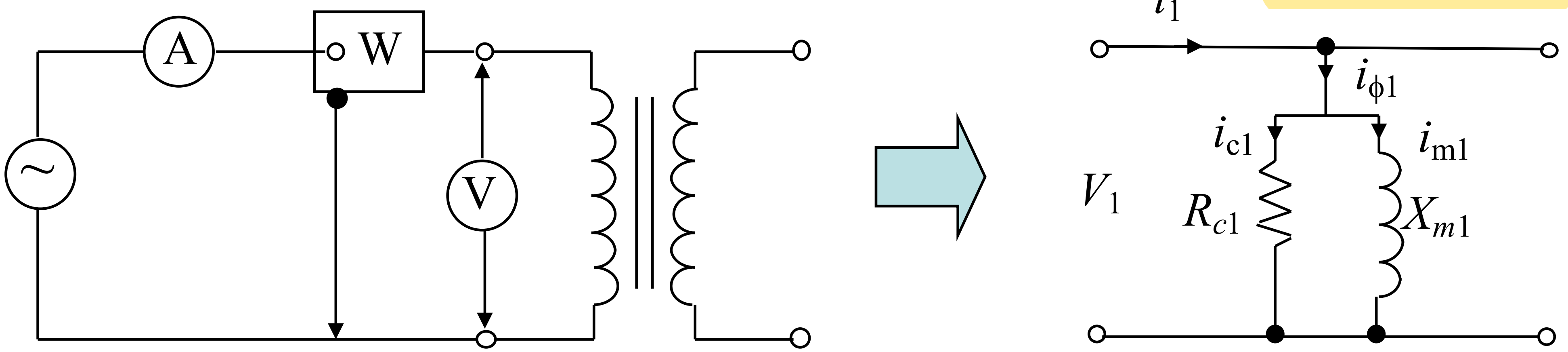
- Open-circuit test measures the magnetising reactance X_{m1} and R_{c1}.
  - This test is performed with all instrumentation on the **LV side** with the HV side being open-circuited
  - Rated voltage is applied to the LV side —> measure real power and current at LV.
  - From the approximate equivalent circuit, the equivalent circuit for the open-circuit test reduces to the circuit on the right.



circuit only left with shunt elements!!

Equivalent Circuit Parameters (Open-Circuit)

- Open-circuit test is used to find R_{c1} // X_{m1} and thus R_{c1} then X_{m1}.



KCL: i1 = ic1 + im1 = ioc

$$R_{c1} = \frac{V_{OC}^2}{P_{OC}} \qquad i_{c1} = \frac{V_{OC}}{R_{c1}} \qquad i_{c1} = i_{OC}\cos(\theta_{OC}) \qquad R_{c1} = \frac{V_1}{i_{c1}}$$

$$X_{m1} = \frac{V_{OC}}{i_{m1}} \qquad i_{m1} = \sqrt{i_{OC}^2 - i_{c1}^2} \qquad i_{m1} = i_{OC}\sin(\theta_{OC}) \qquad X_{m1} = \frac{V_1}{i_{m1}}$$

Example 3 - Open-Circuit Test

Using the same transformer from the example on short-circuit test, the open-circuit test with 11.5kV voltage led to an input power of 66.7kW and a current of 30.4A. Find the values of R_{c1} and X_{m1} referred to the primary winding (HV) side. Also, compute the efficiency of the transformer for a load of 12MW at 0.8 lagging power factor at the rated voltage.

$$a = \frac{N_1}{N_2} = 6$$

$$P = \frac{|V_2|^2}{R_e} = \frac{|V_2|^2}{\left(\frac{R_{c1}}{a^2}\right)}$$

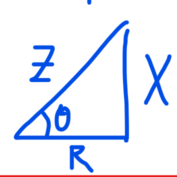
R_1'

$$\Rightarrow R_{c1} = \frac{(11.5\times10^3)^2(6^2)}{66.7\times10^3} = 71.4k\Omega$$

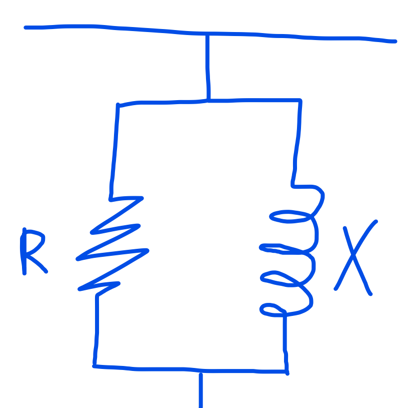
$$|V_2| = I_2 \cdot |Z_e| = I_2 \left|\frac{Z_{m1}}{a^2}\right|$$

Z_1'

$$\Rightarrow |Z_{m1}| = 6^2\left[\frac{11.5\times10^3}{30.4}\right] = 13.6k\Omega$$



$$\boxed{X_{m1} = \frac{1}{\sqrt{\left(\frac{1}{|Z_{m1}|}\right)^2 - \left(\frac{1}{R_{c1}}\right)^2}}} = 13.9k\Omega$$



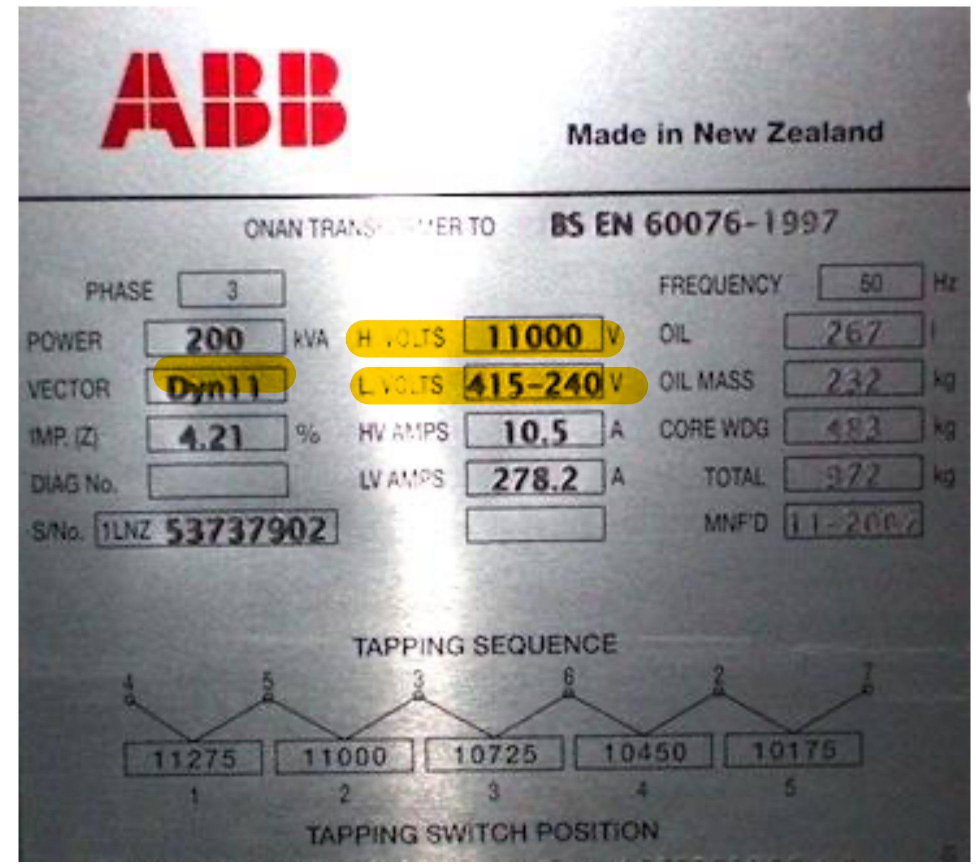
R // X.

$$\boxed{\eta = \frac{\text{output}}{\text{input}}\times100\%} = \frac{12\times10^6}{(12\times10^6)+\underbrace{(105.8\times10^3)}_{\text{SC losses}}+\underbrace{(66.7\times10^3)}_{\text{OC losses}}}$$

$$= 98.6\%$$

Transformer Parameters

- Parameters of the model are calculated based upon:
  - **Nameplate data:** gives the rated voltages and power
  - **Open-circuit test:** rated voltage is applied to primary with secondary open; measure the primary current and losses (the test may also be done applying the voltage to the secondary, calculating the values, then referring the values back to the primary side).
  - **Short-circuit test:** with secondary shorted, apply voltage to primary to get rated current to flow; measure voltage and losses.



Dyn11

Delta connected

Wye connected & with neutral

30 degrees phase lead

Example 4 - Transformer Parameters

A single-phase, 100 MVA, 200/80 kV transformer has the following test data:

$P \quad S$

- Open-circuit test at LV side: 20 amps, with 10 kW losses
- Short-circuit test at HV side: 30 kV, with 500 kW losses

Find the model parameters of the transformer.

Short Circuit Test:

$$\boxed{I_{sc} = \frac{P}{V_1}} = \frac{100\times10^6}{200} = 500A$$

$$\boxed{Z_e = \frac{V_{loss}}{I_{sc}}} = \frac{30\times10^3}{500} = 60\Omega$$

$$\Rightarrow |R_e + jX_e| = 60\Omega$$

$$\boxed{P_{sc} = I_{sc}^2 R_e}$$

$$\Rightarrow R_e = \frac{500\times10^3}{500^2} = 2\Omega$$

$$X_e = \sqrt{|Z_e|^2 - R_e^2} = \sqrt{60^2 - 2^2} = 60\Omega$$

Open Circuit Test:

$$\boxed{a = \frac{N_1}{N_2} = \frac{V_1}{V_2}} = \frac{200}{8} = 2.5$$

$$\boxed{R_{c1} = a^2 \frac{V_2^2}{P_{oc}}} = 2.5^2\left[\frac{(80\times10^3)^2}{10\times10^3}\right] = 4M\Omega$$

Ignore R_e & jX_e : (too insignificantly small)

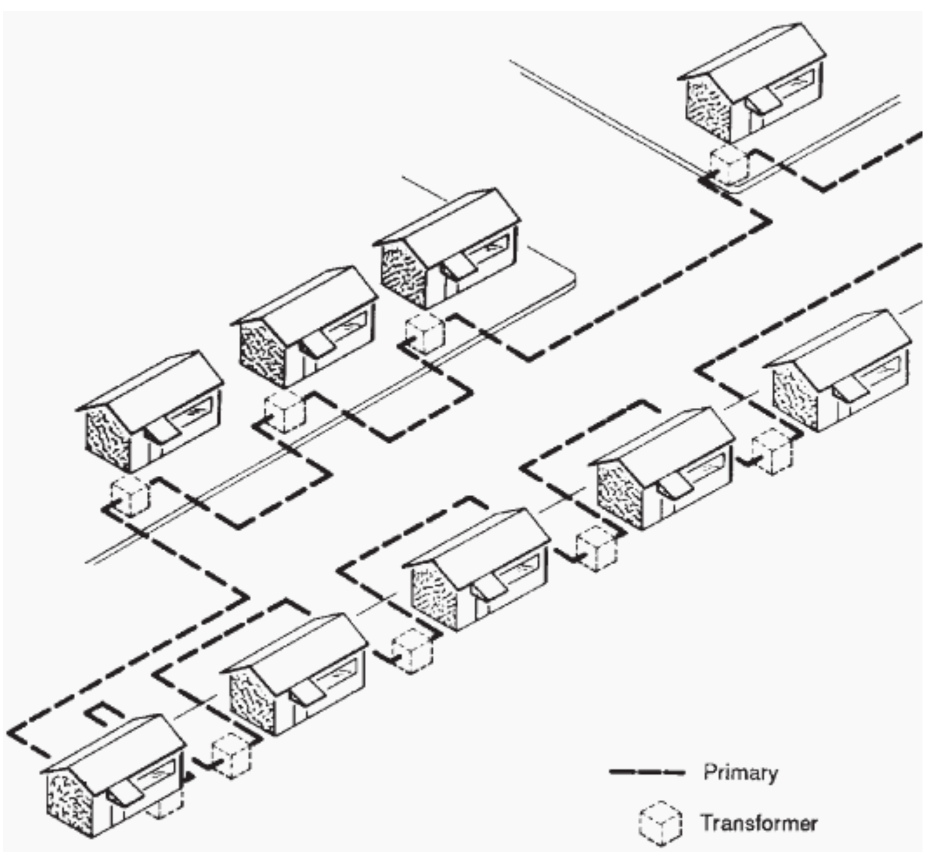
$$\boxed{|Z_{m1}| = a^2 \frac{V_2}{I_{oc}}} = 2.5^2\left[\frac{80\times10^3}{200}\right] = 25k\Omega$$

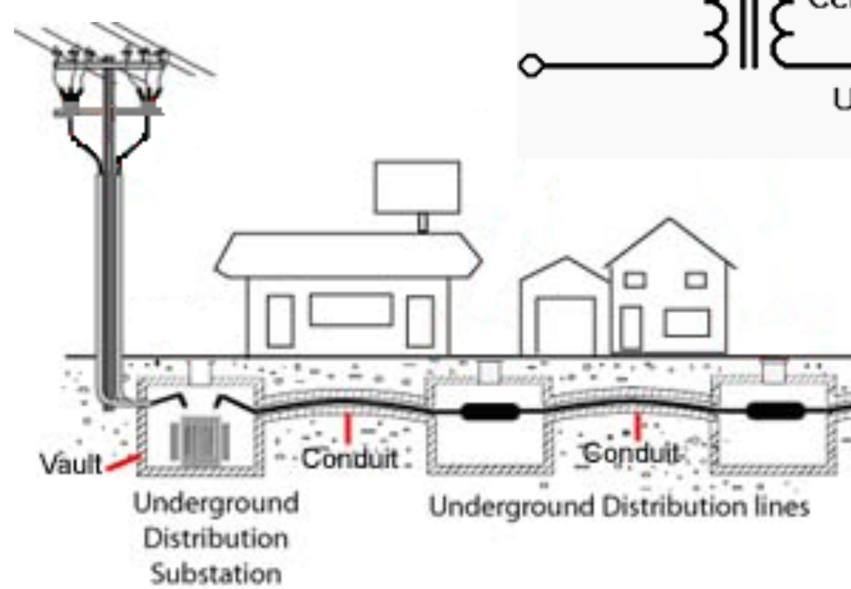
$$\Rightarrow |R_{c1} // jX_{m1}| = 25k\Omega$$

$$\therefore \boxed{X_{m1} = \frac{1}{\sqrt{\left(\frac{1}{|Z_{m1}|}\right)^2 - \left(\frac{1}{R_{c1}}\right)^2}}} = 25k\Omega$$

Residential Distribution Transformer

- Single-phase transformers are commonly used in residential distribution systems.
- Most distribution systems are 4-wire, with a multi-grounded, common neutral.





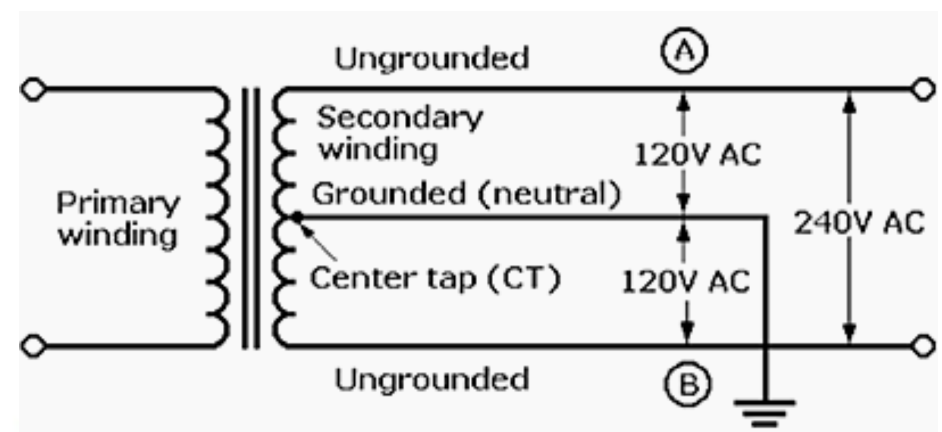


Image sources: http://resources.arcfmsolution.com