11.1 Per Unit Analysis - Examples

Tuesday, 29 March 2022 11:33 am

Steps of 1Φ Per Unit Analysis

1. Choose $S_B^{1\Phi}$ for the system.
2. Select V_B for different zones (usually follows transformer voltage ratings). → Choose or given V_B for one of the zones
   → calculate V_B for other zones using transformer ratio.
3. Calculate Z_B for different zones. → $Z_B = \frac{V_B^2}{S_B}$
4. Express all quantities in p.u.
   - 4.1 → Find new p.u. impedance values if manufacturer has given impedance in p.u. instead of ohms
5. Draw impedance diagram and solve for p.u. quantities.
6. Convert back to actual quantities if needed.

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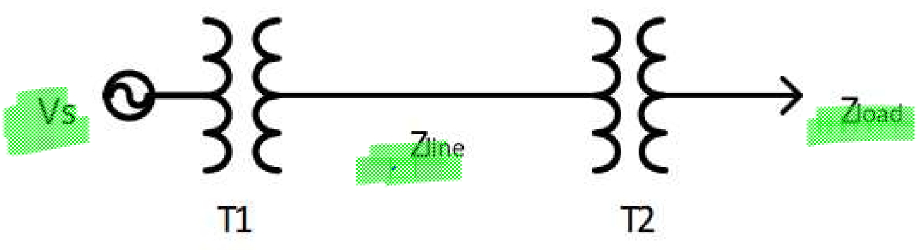
d) Draw the per unit impedance diagram for the following single-phase system and find out the load voltage and load power consumed using per unit analysis. Use a power base of 10MVA.

P

(10 marks)

T1: 12 MVA, 12kV/180kV, Leakage Reactance: 5%, → Z_{T_1} → 0.05 pu
T2: 8 MVA, 160kV/80kV, Leakage Reactance: 12%, → Z_{T_2} → 0.12 pu
Line Impedance Zline: 128+j256 Ω
Load Impedance Zload : 1024 Ω
VS = 8kV

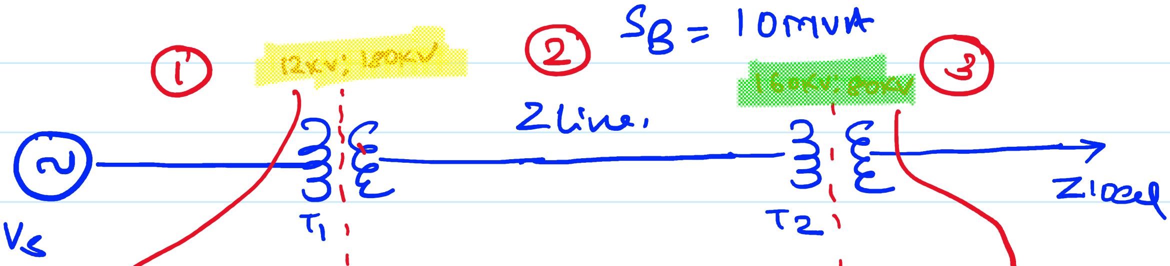


a) ✓ Per Unit Impedance Diagram

b) ✓ Load Voltage

c) ✓ Load Power

$S_B = 10\,MVA$



$V_B^1 = \frac{12}{180} \times 180\,kV$

$= 12\,kV$

$V_B^2 = 180\,kV$

$V_B^3 = \frac{80}{160} \times 180\,kV$

$V_B^3 = 90\,kV$

$Z_B^1 = \frac{(V_B^1)^2}{S_B}$

$= \frac{(12 \times 10^3)^2}{10 \times 10^6}$

$= 14.4\,\Omega$

$Z_B^2 = \frac{(180 \times 10^3)^2}{10 \times 10^6}$

$= 3240\,\Omega$

$Z_B^3 = \frac{(90 \times 10^3)^2}{10 \times 10^6}$

$= 810\,\Omega$

$V_{s,pu} = \frac{8\,kV}{12\,kV}$

$= 0.6667\ pu$

$= 0.6667\angle 0^\circ\ p.u.$

$Z_{line,pu} = \frac{128 + j256}{3240}$

$= 0.0395 + 0.079j$

$= 0.883\angle 63.43^\circ\ p.u.$

$Z_{load,pu} = \frac{1024}{810}$

$= 1.264\ p.u.$

$\Rightarrow$ Transformer change in Base Value

Transformer T_1

12 MVA, 12kV/180kV

$Z_{T_1,orig} = j0.05\ p.u.$

$Z_{T_1,old} = j0.05$ p.u.

$Z_{actual} = Z_{B,old}^{T_1} \times Z_{T_1,old}$

$\quad 12kV^2/12MVA$

$Z_{T_1,pu}^{new} = \dfrac{Z_{B,old}^{T_1} \times Z_{T_1 old}^{pu}}{Z'_B}$ → Base using transformer ratings

$= \dfrac{j.05 \times \dfrac{(12\times10^3)^2}{12\times10^6}}{14.4}$ → $\dfrac{(V'_{B,old})^2}{(S_{B,old})}$

$= j0.042$ p.u.

$Z_{pu}^{T_1,new} = \dfrac{Z_{actual}^{T_1}}{Z'_B}$

Transformer T_2

8MVA, 160kV/80kV

12% → $Z_{T_2} = j0.12$ p.u.

Primary of T_2 is in zone 2

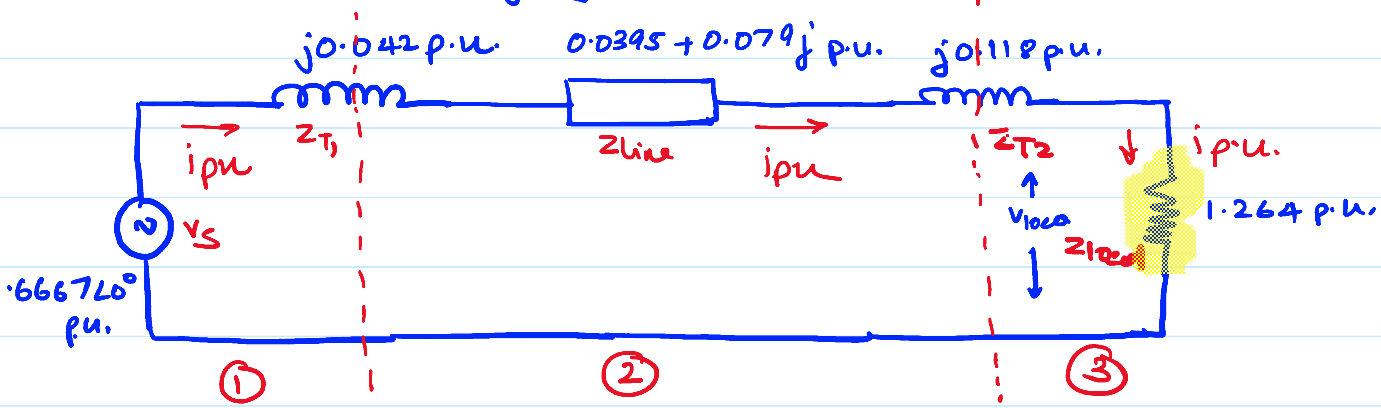
$Z_{pu,T_2}^{new} = \dfrac{Z_{B,old}^{prim} \times Z_{pu}^{old}}{Z_B^2}$

$= \dfrac{\dfrac{(160\times10^3)^2}{8\times10^6} \times j0.12}{3240}$

$= j0.118$ p.u.

Per unit impedance diagram

Per unit impedance diagram



$$i_{pu} = \frac{V_{s,pu}}{Z_{Total,\,pu}} = \frac{0.6667\angle 0^\circ}{j0.042 + j0.118 + 1.264 + 0.0395 + j0.079}$$

$$= \frac{0.6667\angle 0^\circ}{1.325\angle 10.39^\circ}$$

$$i_{p.u.} = 0.5031\angle -10.39^\circ \text{ p.u.}$$

$$V_{load\,pu} = i_{pu} \times Z_{load\,pu}$$

$$= 0.5031\angle -10.39 \times 1.264$$

$$= 0.6359\angle -10.39^\circ \text{ p.u.}$$

$$S_{load\,pu} = (V_{load\,pu})(I_{pu}^{*})$$

$$= (0.6359\angle -10.39)(0.5031\angle -10.39)^{*}$$

$$= (0.6359\angle -10.39)(0.5031\angle 10.39)$$

$$S_{load\,pu} = 0.3199\angle 0^\circ \text{ p.u.}$$

$$V_{load} = V_{load\,pu} \times V_B$$

$$= 0.6359\angle -10.39 \times 90\times 10^{3}$$

$$= 57.231\angle -10.39^\circ \text{ kV}$$

$= 0.6359\angle -10.39 \times 90\times10^{3}$

$= 57.231\angle -10.39^\circ$ kV

$S_{load} = S_{load,pu} \times S_B$

$= 0.3199\angle 0^\circ \times 10\times10^6$

$= 3.199\angle 0^\circ$ MVA

$P_{load} = 3.199$ MW

$S = P + jQ$

$= |S|\cos\theta + j|S|\sin\theta$

Casio → fx-991MS

fx-991ES

Steps of Calculation: 3Φ Case

1. Choose $S_B^{3\Phi}$ for the system.
2. Select $V_B^{\text{line-to-neutral}}$ or $V_B^{\text{line-to-line}}$ for different zones.
3. Calculate Z_B for different zones.
4. Express all quantities in p.u.
5. Draw impedance diagram and solve for p.u. quantities.
6. Convert back to actual quantities if needed.

Note that the **per unit circuit is** the circuit in **per-phase analysis** with normalization of the voltage magnitude at different locations. This means that **the phase of voltage** in per unit analysis **refers to** the **line-to-neutral voltage**.

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b) Draw the per unit impedance diagram for the following three-phase system and find out the voltage across the transmission line and the transmission line losses using per unit analysis. Use a power base of 10MVA. Find out the voltage across the transmission line.

T$_1$: 14 MVA, 11kV/220kV, Leakage Reactance: 7%, Y-Δ

T$_2$: 7 MVA, 240kV/120kV, Leakage Reactance: 14%, Δ- Y

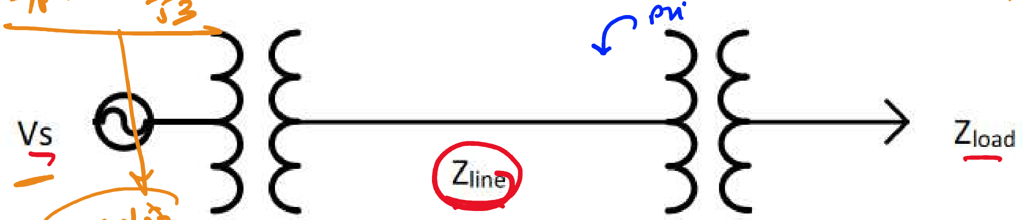
Line Impedance Z_{line}: 140+j280 Ω

Load Impedance Z_{load} : 800 Ω Y connected load

V_S = 7kV (ve)

$S_B^{3\phi} = 10MVA \rightarrow S_B^{1\phi} = \frac{10MVA}{3}$

$V_{S,ph} = \frac{7kV}{\sqrt{3}}$



$V_{S,pu} = \frac{7kV/\sqrt{3}}{12kV/\sqrt{3}}$

1 — V transmission line

Y Δ   Δ Y

① ② ③

$V_{B,1}^{line} = \frac{11}{220} \times 240$

$= 12kV$

$V_{B,2}^{line} = 240kV$

$V_{B,3}^{line} = \frac{120}{240} \times 240kV$

$= 120kV$

$V_{B,3}^{ph} = \frac{120kV}{\sqrt{3}}$

$Z_{B,1} = \frac{(V_{B,1}^{line})^2}{S_B^{3\phi}}$

$= \frac{(12\times10^3)^2}{10\times10^6}$

$= 14.4\,\Omega$

$Z_{B,2} = \frac{(240\times10^3)^2}{10\times10^6}$

$= 5760\,\Omega$

$Z_{B,3} = \frac{(120\times10^3)^2}{10\times10^6}$

$= 1440\,\Omega$

$V_{S,pu} = \frac{V_S}{}$

$Z_{line,pu} = \frac{140+j280}{}$

$Z_{load,pu} = \frac{800}{}$

$$V_{s,pu} = \frac{V_s}{V_{B,1}} = \frac{7}{12} = 0.583\angle 0^\circ \text{ p.u.}$$

$$Z_{line,pu} = \frac{140 + j280}{5760} = 0.0243 + j0.0486 \text{ p.u.} = 0.0543\angle 63.43^\circ \text{ p.u.}$$

$$Z_{load,pu} = \frac{800}{1440} = 0.556 \text{ p.u.}$$

→ Transformer change of base values.

Transformer T_1

14 MVA, 11 kV/ 220 kV, $Z^{pu}_{T_1,old} = j0.07$ p.u. Y-Δ

$$Z_{T_1,new} = \frac{Z^{T_1}_{actual}}{Z'_B} = \frac{Z^{T_1}_{pu,old} \times Z^{T_1,prim}_{B,old}}{Z'_B}$$

$$Z^{T_1,prim}_{B,old} = \frac{\left(V^{prim}_{B,old}\right)^2}{S_{B,old}} = \frac{(11 \times 10^3)^2}{14 \times 10^6}$$

$$Z_{T_1,new} = \frac{j0.07 \times \frac{(11 \times 10^3)^2}{14 \times 10^6}}{14.4} = j0.042 \text{ p.u.}$$

Transformer T_2

7 MVA, 240 kV / 120 kV, $Z_{T_2} = j0.14$ p.u. ΔY

$$Z_{T2,new} = \frac{Z_{actual}}{Z_B^2}$$

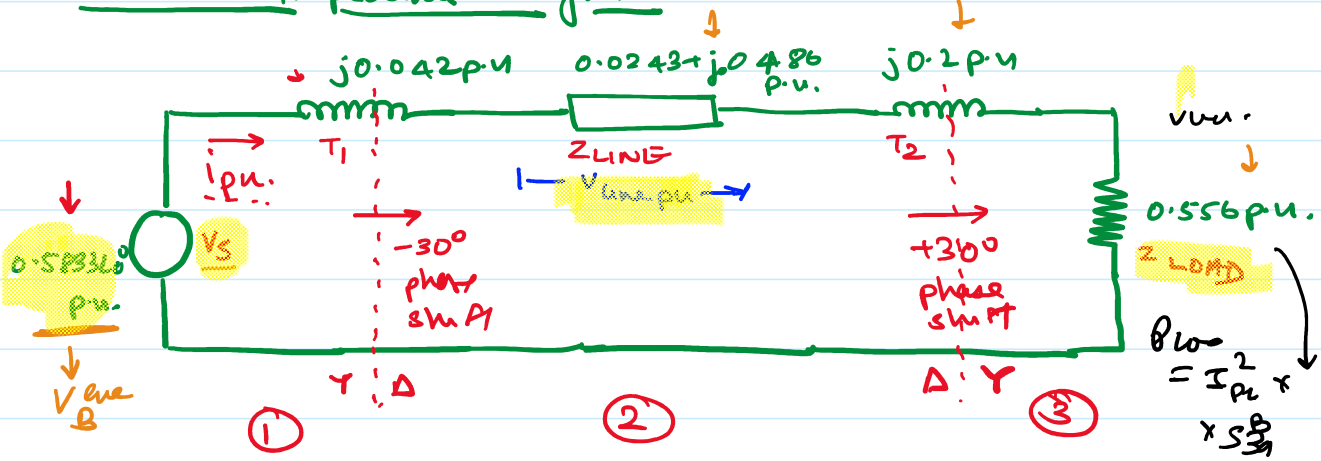
$$= \frac{Z^{T2}_{pu,old} \times Z^{T_2,primary}_{B,old}}{Z_B^2}$$

$$Z^{T_2,primary}_{B,old} = \frac{(240\times10^3)^2}{7\times10^6}$$

$$Z_{T2,new} = \frac{j0.14 \times \frac{(240\times10^3)^2}{7\times10^6}}{5760}$$

$$= j0.2 \text{ p.u.}$$

Per unit impedance diagram



$$i_{p.u} = \frac{V_{p.u.}}{Z_{Total\ p.u.}} = \frac{0.5833\angle0^\circ}{j.042 + j0.2 + 0.0243 + j.0486 + 0.556}$$

$$= \frac{0.583\angle0^\circ}{0.649\angle26.60^\circ} = 0.8988\angle-26.60^\circ \text{ p.u.}$$

$$i_{pu} = 0.8988\angle-26.60^\circ \text{ p.u.}$$

$i_{pu,\ldots} = 0.8988\angle-26.60^\circ$ pu.

$i_{pu,gen} = 0.8988\angle -26.60^\circ$ pu.

$i_{pu,line} = 0.8988\angle -26.60 - 30$
$= 0.8988\angle -56.60^\circ$ pu.

$i_{pu,load} = 0.8988\angle -26.60^\circ$ pu.

$V_{pu,line} = i_{pu,line} * Z_{line\,pu}$

$= 0.8988\angle -56.60 \times 0.0543\angle 63.43^\circ$
$= 0.0488\angle 6.83^\circ$ p.u.

$P^{pu}_{line} = (I^{pu}_{line})^2 (R^{pu}_{line})$
$= 0.8989^2 \times 0.0243$
$= 0.0196$ p.u.

$P_{line} = 0.0196 \times S^{3\phi}_{B}$
$= 196.305$ KW

$S^{pu}_{line} = (V_{pu,line})(I_{pu,line})^*$

$= 0.0488\angle 6.83 \times 0.8988\angle 56.60^\circ$
$S^{pu}_{line} = \underbrace{0.0196}_{P_{pu,line}} + \underbrace{0.039j}_{Q_{pu,line}}$ p.u.

$P^{3\phi}_{line} = 0.0196 \times 10 \times 10^6 = 196.305$ KW.

$$P^{3\phi}_{line} = 0.0196 \times 10 \times 10^{6} = 196.305\,\text{kW}.$$

$$V_{pu,line} = 0.0498 \angle 6.83^\circ$$

$$|V_{\text{transmission line}}| = 0.0488 \times \frac{\boxed{240\,\text{kV}}}{\sqrt{3}}$$

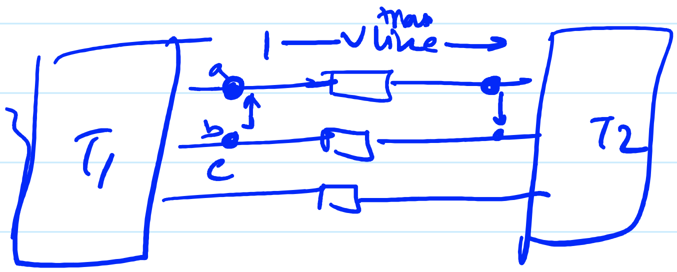
$$= 6761.93\,\text{V}$$

$$V^2_{B,line}$$

$$\downarrow$$

$$V^2_{B,phase} = \frac{V^2_{B,line}}{\sqrt{3}}$$

$$S_{3\phi} = 3 S_{1\phi}$$



$$I_B = \frac{S^{3\phi}_B}{\sqrt{3}\, V_{line\text{-}to\text{-}line}}$$

$$S = \sqrt{3}\, V_{line} \cdot I_{line}$$

$$S = 3 V^B_{ph} \cdot I^B_{ph}.$$

$$I_{ph} = \frac{S^B_{3\phi}}{3 \cdot V_{ph}}$$

$$Z_B = \frac{V_{ph}}{\frac{S_{3\phi}}{3 V_{ph}}} = \frac{V^B_{ph}}{I^B_{ph}}$$

$$Z_B = \frac{3 {V^B_{ph}}^2}{S^B_{3\phi}}$$

$>3\phi$

$$= \frac{(\sqrt{3}\, V_{ph}^{B})^2}{S_{3\phi}^{B}}$$

$$Z_B = \frac{(V_{line}^{B})^2}{S_{3\phi}^{B}}$$

$I_{load,pu}$ = ——— p.u.

Z_{load} = ——— p.u.

$S_{Load,pu} = V \cdot I^* = \overline{I Z I^*}$ p.u.

$S_{load} = I \cdot I^* \cdot Z$

$$S_{Load}^{3\phi} = \underline{S_{Load\,pu}} \times S_{3\phi}^{B} =$$

$$S_{Load}^{1\phi} = S_{Load\,pu} \times S_{1\phi}^{B}$$