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EE2029 - Introduction to Electrical Energy Systems

Transformers

Current-Voltage Relationship
Transformer Core Modeling
Equivalent Circuit
Open and Short Circuit Tests
Per Unit Analysis
Three-Phase Transformer

Per Unit Analysis

Per Unit Calculation

- A key problem in analysing power systems is the large number of transformers.
  - It would be very difficult to continually have to refer impedances to the different sides of the transformers.
- This problem is avoided by a normalisation of all variables.
  - This normalisation is known as per unit analysis.

$$\text{quantity in per unit} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

Per Unit Conversion (Single-Phase)

1. Pick a 1-ϕ VA base for the entire system, S_{base}
2. Pick a voltage base for each different voltage level, V_{base}.
    - Bases are related by transformer turns ratios.
    - Voltages are line-to-neutral.
3. Calculate the impedance base, $Z_{base} = (V_{base})^2 / S_{base}$
4. Calculate the current base, $I_{base} = V_{base} / Z_{base}$
5. Convert actual values to per unit

Note, per unit conversion only affects magnitudes, not angles. Per unit quantities have no units (i.e., a voltage is 1 p.u. NOT 1 p.u. volt)

Example - Per Unit

A single-phase transformer is rated at 110/440V, 2.5kVA. Leakage reactance measured from the low-voltage side is 0.06Ω. Determine the leakage reactance in per unit.

$$\boxed{Z_{base,LV} = \frac{(V_{base,LV})^2}{S_{base}}} = \frac{110^2}{2500} = 4.84\Omega$$

In per unit:

$$\boxed{X_{2,pu} = \frac{X_2}{Z_{base,LV}}} = \frac{0.06}{4.84} = 0.0124\,p.u. //$$

At HV side: $X_2' = a^2 X_2 = \left(\frac{V_1}{V_2}\right)^2 X_2 = 0.06\left(\frac{440}{110}\right)^2 = 0.96\Omega$

$$\boxed{Z_{base,HV} = \frac{(V_{base,HV})^2}{S_{base}}} = \frac{440^2}{2500} = 77.5\Omega$$

In per unit:

$$\boxed{X_{2,pu}' = \frac{X_2'}{Z_{base,HV}}} = \frac{0.96}{77.5} = 0.0124\,p.u. //$$

Per Unit Solution

1. Convert to per unit (p.u.)
    - Note that most grid data are given in per unit.
2. Solve
3. Convert back to actual as necessary

Note: Neglect the presence of the small magnetising current. The voltage bases for the transformer terminals must have the same ratio as the turns ratio of the transformer windings.

the selected voltage bases must have same turns ratio!

Example - Multiple Terminals

Solve for the load voltage V_L, load power P_L, and input complex power S_{in} in the circuit shown below using per unit analysis with an S_{base} of 100 MVA, and voltage bases of 8 kV, 80 kV, and 16 kV. The load is 10Ω from the diagram.

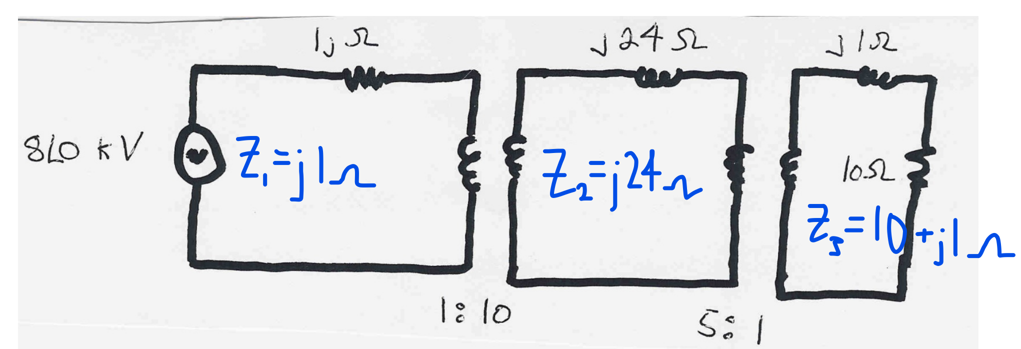
$$\boxed{Z_{base,left} = \frac{V_{base,left}^2}{S_{base}}} = \frac{8kV^2}{100MVA} = 0.64\Omega$$

$$Z_{base,mid} = \frac{80kV^2}{100MVA} = 64\Omega \;,\; Z_{base,right} = \frac{16kV^2}{100MVA} = 2.56\Omega$$

$$\Rightarrow \boxed{Z_{left,pu} = \frac{actual}{base}} = \frac{j1}{0.64} = j1.56 p.u. \qquad \Rightarrow Z_{mid,pu} = \frac{24j}{64} = j0.375\Omega$$

$$\Rightarrow Z_{right,pu} = \frac{10+j1}{2.56} = 3.91 + j0.39 p.u.$$

$$\therefore Z_{total} = j1.56 + j0.375 + 3.91 + j0.39$$
$$= 3.91 + j2.325\Omega$$



left mid right

(on paper)

Per Unit Conversion (Three-Phase)

Procedure is very similar to 1-ϕ except we use a 3-ϕ VA base.

1. Pick a 3-ϕ VA base for the entire system.
2. Pick a voltage base for each different voltage level, i.e. V_{base}.
   - Given voltage values are **line-to-line** values.
3. Calculate the impedance base.

$$Z_{base} = \frac{V_{base,LL}^2}{S_{base}^{3\phi}} = \frac{\left(\sqrt{3}V_{base,LN}\right)^2}{3S_{base}^{1\phi}} = \frac{V_{base,LN}^2}{S_{base}^{1\phi}}$$

Exactly the same impedance bases as with single-phase!

Per Unit Conversion (Three-Phase)

4. Calculate the current base, i.e. I_{base}.

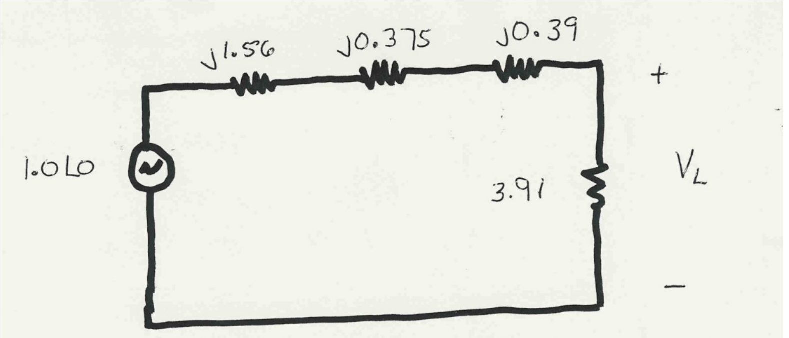
$$I_{base}^{3\phi} = \frac{S_{base}^{3\phi}}{\sqrt{3}V_{base,LL}} = \frac{3S_{base}^{1\phi}}{\sqrt{3}\sqrt{3}V_{base,LN}} = \frac{S_{base}^{1\phi}}{V_{base,LN}} = I_{base}^{1\phi}$$

Exactly the <u>same</u> current bases as with single-phase!

5. Convert actual values to per unit.

Example - Three-phase Per Unit

Solve for the current, load voltage and load power in the previous circuit, but assuming a three-phase power base of 300 MVA, and line-to-line voltage bases of 13.8 kV, 138 kV, and 27.6 kV. Assume the generator is Y-connected, and its line-to-line voltage is 13.8 kV.



Per Unit Change of MVA Base

- Parameters for equipment are often given using power rating of equipment as the MVA base.

- To analyse an *entire system*, all per unit data must be on a **common power base** (i.e. subscript base').

$$Z_{base} \rightarrow Z_{actual} \rightarrow Z_{base'}$$

$$Z_{base} \times \frac{\cancel{V^2_{base}}}{S_{base}} / \frac{\cancel{V^2_{base}}}{S_{base'}}$$

$$Z_{base} \times \frac{S_{base'}}{S_{base}} = Z_{base'}$$

Example: A 54 MVA transformer has a leakage reactance of 3.69%. What is the reactance on a 100 MVA base?

$$X_e = 0.0369 \times \frac{100}{54} = 0.0683 \text{ p.u.}$$

Transformer Reactance

- Transformer reactance is often specified as a percentage, say 10%.
  - This is a per unit value (divide by 100) on the power base of the transformer.

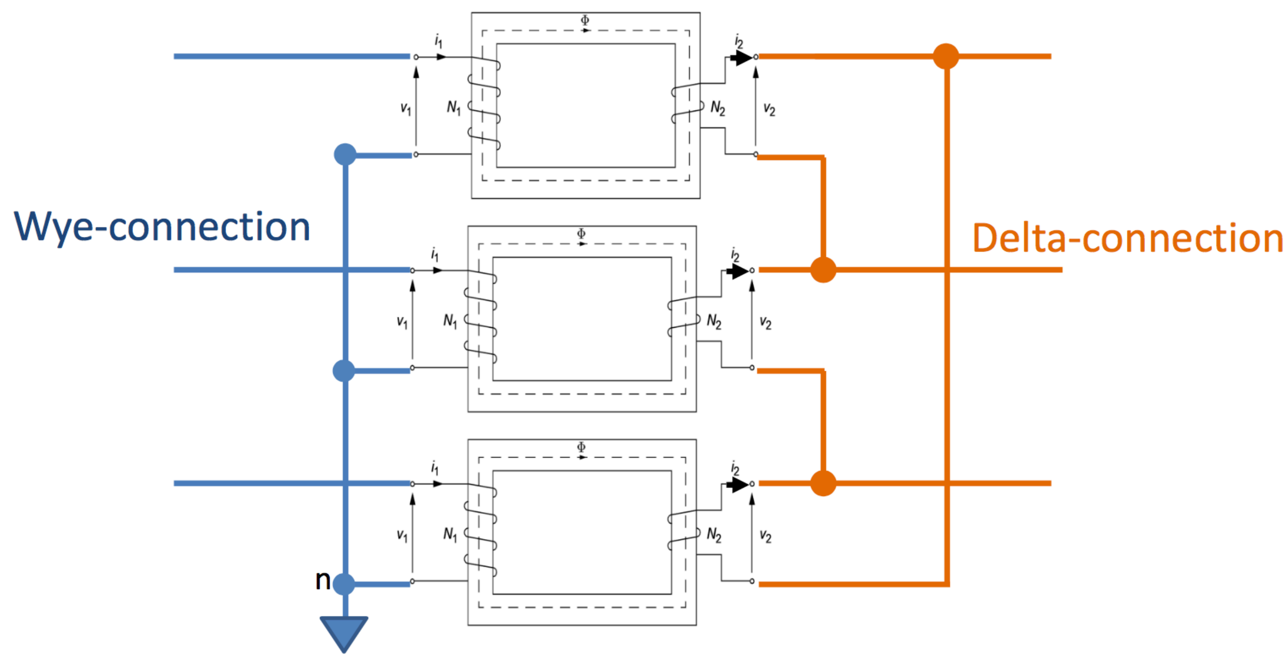
Example: A 350 MVA, 230/20 kV transformer has an equivalent leakage reactance of 10%. What is p.u. value on 100 MVA base? What is value in ohms (230 kV)?

$$X_e = 0.10 \times \frac{100}{350} = 0.0286 \text{ p.u.}$$

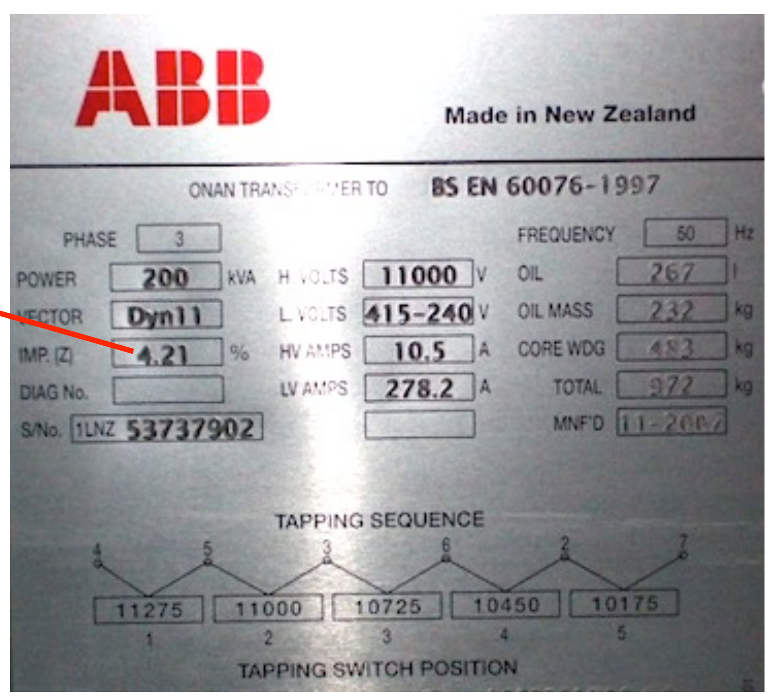
$$\mathrm{X}_e = 0.0286 \times \frac{(230\text{kV})^2}{100\text{MVA}} = 15.1\ \Omega$$

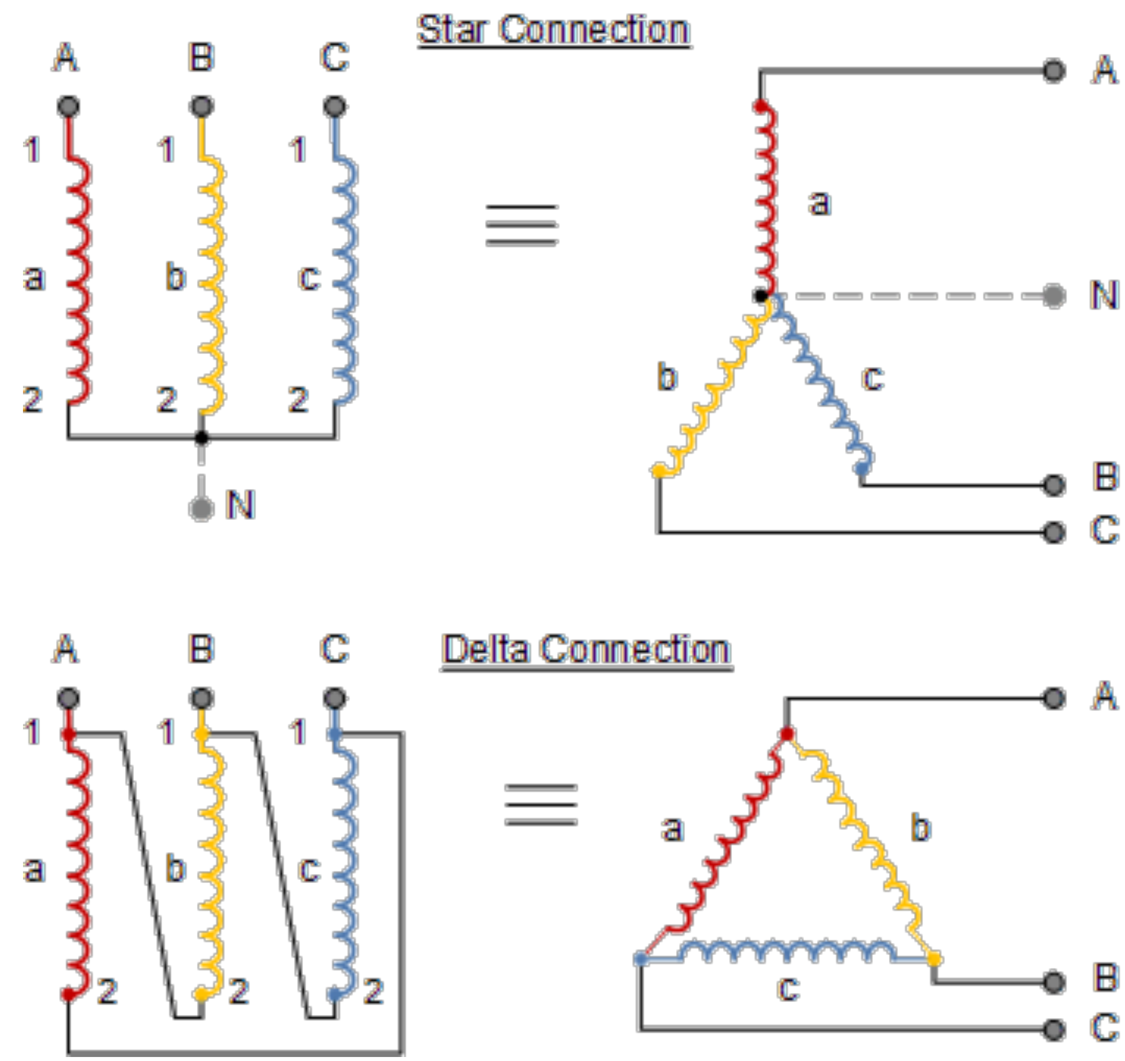
Three-Phase Transformer

Three-Phase Transformer



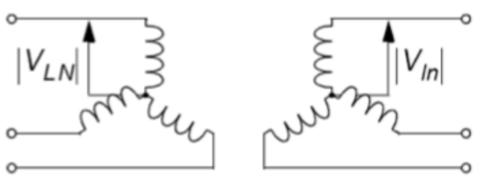
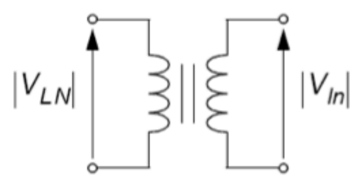
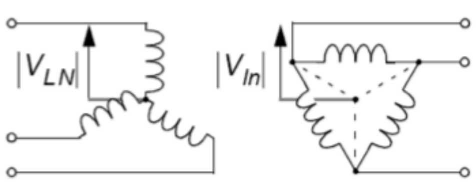
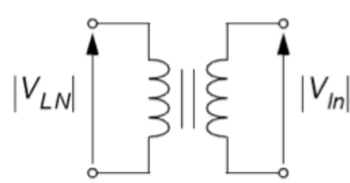
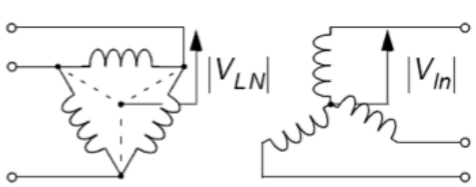
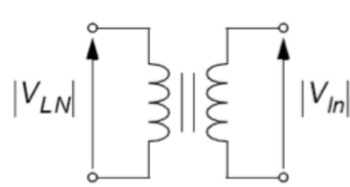
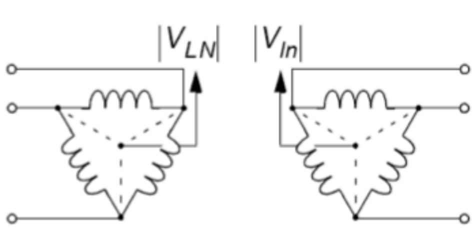
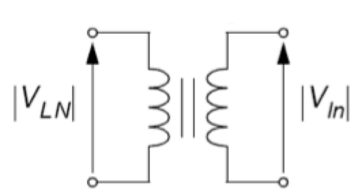
4.21 % in per unit





Transform Nameplate
HV & LV voltage values are line-to-line rating.

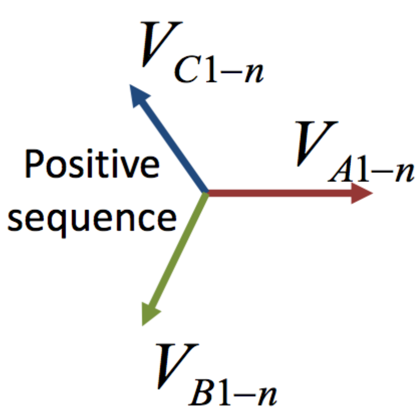
Three-Phase Transformer

designation	winding connection	single-phase equivalent
Yy		
Yd		
Dy		
Dd		

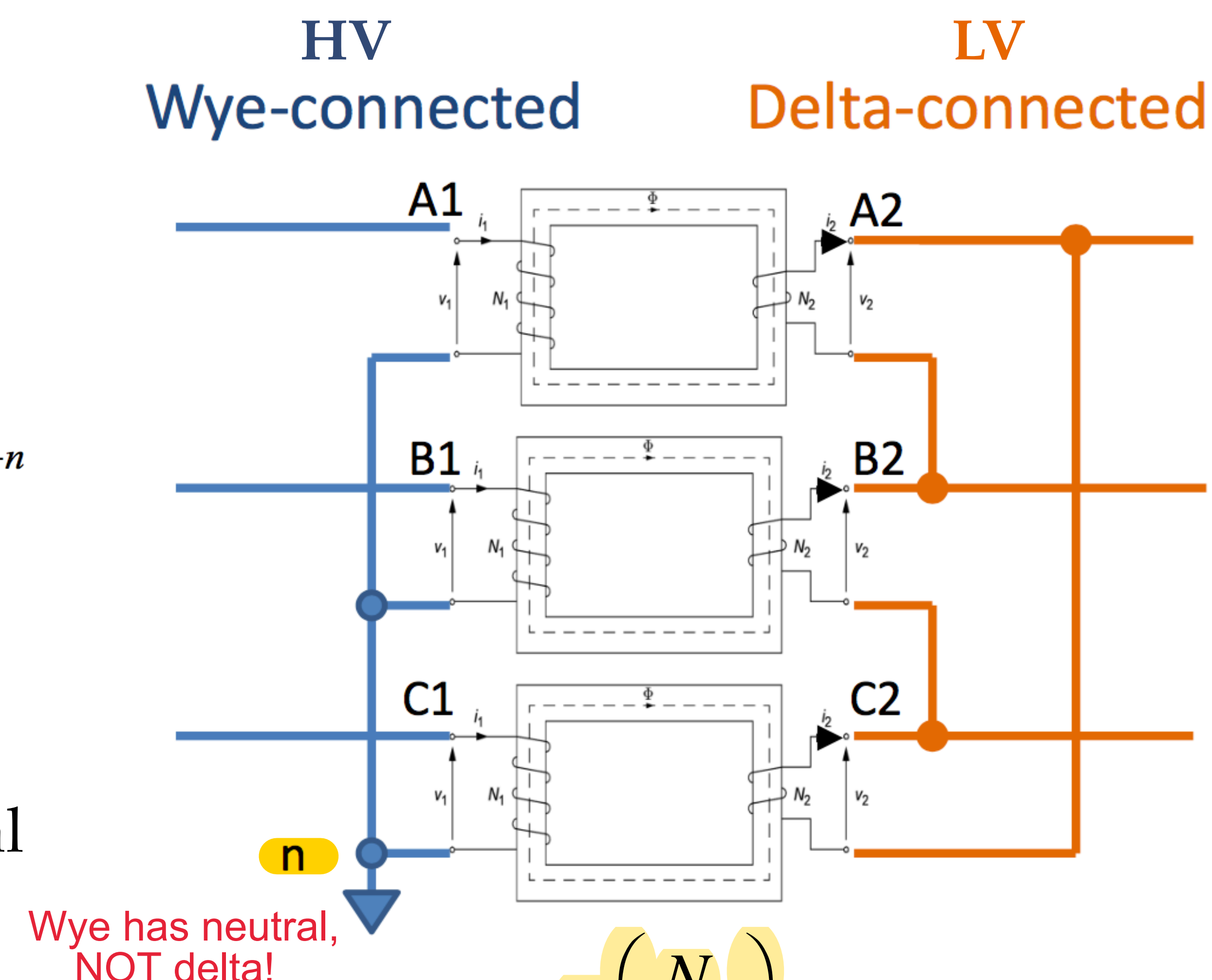
- Voltage rating of a three -transformer is the ratio between line-to-line voltage at the primary side and line-to-line voltage at the secondary side.
- Single-phase equivalent shows *line-to-neutral voltage.*
- For Y-Y and Δ-Δ transformers, voltage and current in primary and secondary are in-phase.
    - The ratio of the voltage and current follows the turn ratio of the transformer.
    - The same does not apply to Y-Δ and Δ-Y connections.

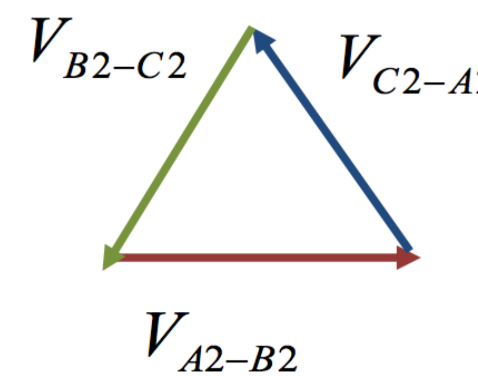
Wye-Delta Transformer

- Yd introduces <u>magnitude variation</u> and <u>phase shift</u> due to the line-to-line voltage.



line-to-neutral





line-to-line

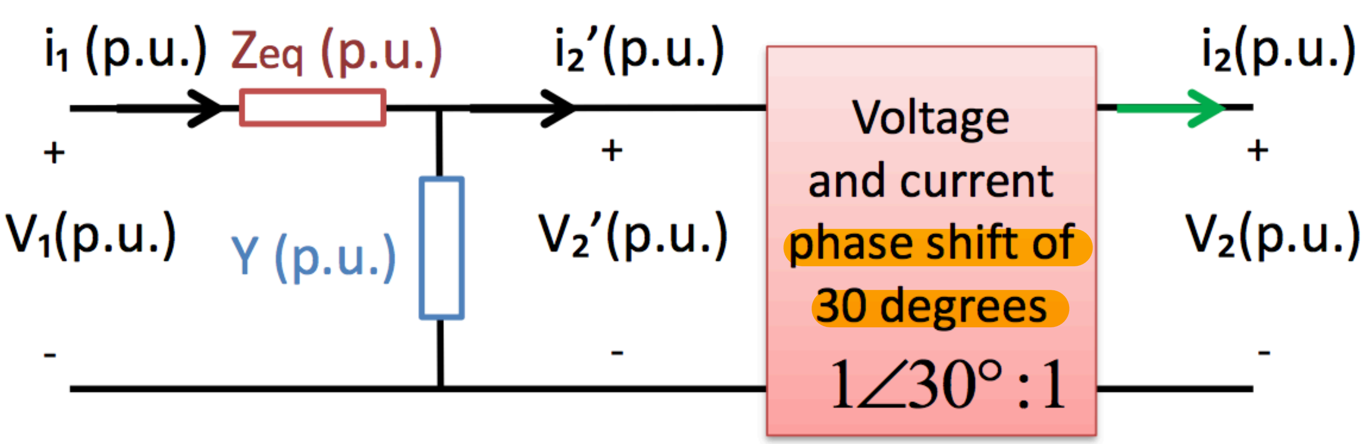
$$V_{Al-n} = \left(\frac{N_1}{N_2}\right) V_{A2-B2}$$

Wye-Delta Transformer

- For a positive voltage sequence source:

$$V_{A1-n} = \left(\frac{N_1}{N_2}\right)\sqrt{3}V_{A2-n}\angle 30°$$

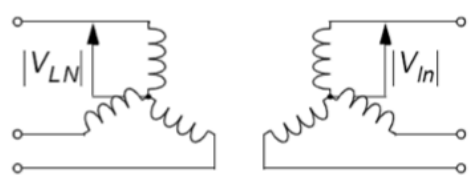
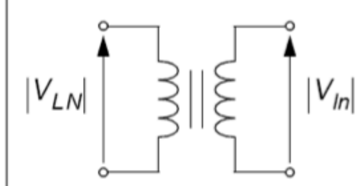
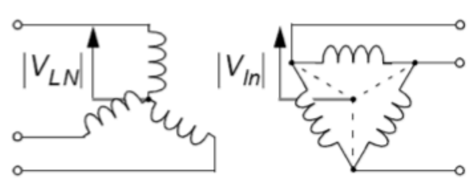
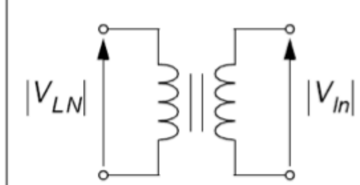
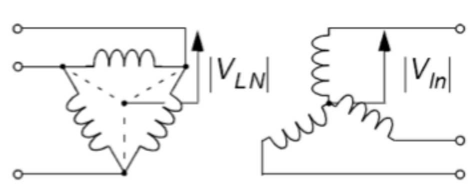
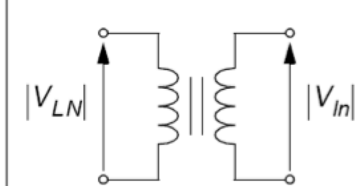
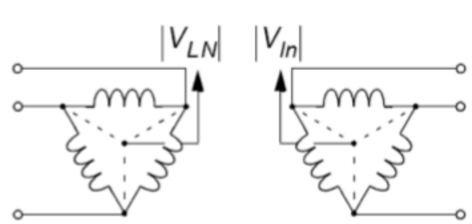
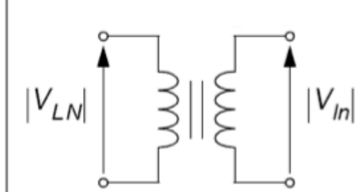
$$\angle V_{A1-n} : \angle V_{A2-n} = 1\angle 30° : 1$$



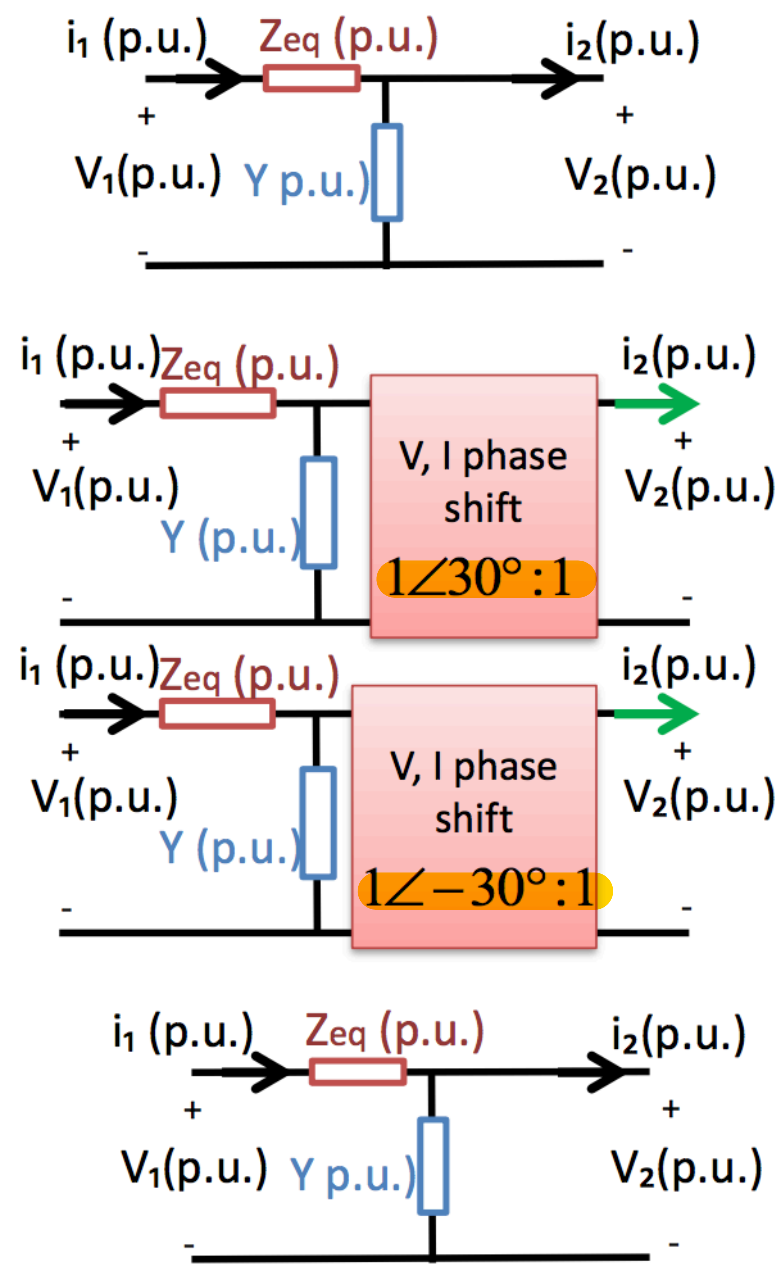
$$i_2 = i_2' \angle -30°$$

$$V_2 = V_2' \angle -30°$$

Summary of Three-Phase Transformer

designation	winding connection	single-phase equivalent
Yy		
Yd		
Dy		
Dd		

Per phase
per unit
circuit



Example - Three-phase Transformer

A three-phase transformer rated at 25MVA, 66/3.81 kV (YNd) with a balanced load of three 0.6Ω wye connected resistors. Choose a base of 75MVA, 66kV at the HV side of the transformer and specify a base for the LV side. Determine the per unit resistance of the load on the selected base for the LV side. Then calculate the load resistance R_L in ohms referred to the HV side and the per unit resistance on the HV base.

Example - Phase Shift

A three-phase generator is rated at 300MVA, 23kV supplying a load of 240MVA, 0.9 lagging power factor at 230kV through a 330MVA 23/230kV (delta-wye) step-up transformer of leakage reactance 11%. Neglecting magnetising current and choosing base values at the load of 100MVA and 230kV, find I_a, I_b, and I_c at the LV side Assume a balanced system.