EE2029 INTRODUCTION TO ELECTRICAL ENERY SYSTEMS
(Tutorial #2 Transmission Lines)

1. What are the advantages and disadvantages of both overhead transmission lines and underground cables?

2. Consider a three-phase transmission line operating at the sending end voltage of 500kV. The series impedance is z= 0.045+j0.4 Ω per phase per km and the shunt admittance is y= j4×10-6 Siemens per phase per km. For the following cases, evaluate transmission matrix for short length, medium length and long length models, and find the receiving end voltage (per phase) at no load using all three models. What can you conclude from this exercise?
   a) When the length of the transmission line is 50 km
   b) When the length of the transmission line is 100 km
   c) When the length of the transmission line is 250 km

Answer: Per phase receiving end voltage at no load (kV)

	short length model	medium length model	long length model
l=50 km	$288.7 \angle 0^0$	$289.28 \angle -0.01^0$	$289.3 \angle -0.01^0$
l=100 km	$288.7 \angle 0^0$	$291 \angle -0.052^0$	$291.03 \angle -0.05^0$
l=250 km	$288.7 \angle 0^0$	$303.9 \angle -0.34^0$	$303.76 \angle -0.33^0$

3. A three phase 765 kV, 60 Hz, 300 km line has the following positive sequence series impedance and shunt admittance; z= 0.0165+j0.3306 Ω/km and y= j4.674×10-6 S/km. Calculate ABCD parameters in a nominal Π circuit.
   (Answer: $= D = \ 0.9305 + j0.0035, B = 4.95 + j99.18\Omega, \mathrm{C} = (-2.4 \times 10^{-6} + j0.0014\ )S$)

4. A 69 kV three-phase short transmission line is 16 km long. The line has a per phase series impedance of 0.125+j0.4375 Ω/km. Determine the sending end voltage per phase, voltage regulation, and the transmission efficiency when the line delivers 70 MVA, 0.8 lagging power factor at 64 kV.

(Answer: 40.71 kV, 10.17%, and 95.91%)

5. A 200 km, 230 kV, 60 Hz three phase line has a per phase series impedance, z= 0.08+j0.48 Ω/km and a per phase shunt admittance y= j3.33×10-6 S/km. At full load, the line delivers 250 MW at 0.99 p.f. lagging and at 220 kV. Using the nominal Π circuit, find sending end voltage and current per phase.

(Answer: $155.40\angle 23.58°\ kV, 635.38\angle -0.34°\ A$)

6. A 345 kV, 50 Hz, three-phase transmission line is 130 km long. The resistance per phase is 0.036 Ω/km and the inductance per phase is 0.8 mH/km. The shunt capacitance is 0.0112 μF/km where shunt conductance is negligible. The receiving end load is 270 MVA with 0.8 lagging power factor at 325 kV. Use nominal Π model, determine the voltage regulation and transmission line efficiency. If the power factor is corrected to 0.95 lagging, keeping the receiving end MVA constant, what will be the new voltage regulation and transmission line efficiency? What can you conclude from this problem?

(Answer: 6.19%, 98.7%, 4.06%, 98.8%)

1.

Overhead Lines:

(+) Less costly

(+) Air can cool & insulate TLs

(+) Easier maintenance work

(−) Vulnerable to severe weather

(−) Negative visual impact

Underground Cables:

(−) More costly

(−) Expensive insulation at higher voltage

(−) Harder & hence less frequent maintenance

(+) Immune to severe weather

(+) Environment & aesthetic advantage

2.

$$V_{LL} = 500\,kV$$

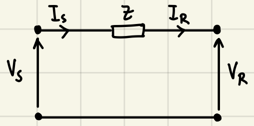
$$\Rightarrow V_{LN} = \frac{500}{\sqrt{3}} \times 10^3\,V$$ (per phase per km)

remember to convert!

For L = 50km: < 80km (short-line model)

$$Z_{total} = 50\,(0.045 + j0.4) = 2.25 + j20\,\Omega$$
$$Y_{total} = 50\,(j4\times10^{-6}) = j2\times10^{-4}\,S$$

per phase



Matrix:

$$\boxed{\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}\begin{bmatrix} V_R \\ 0 \end{bmatrix}}$$ short-line model.

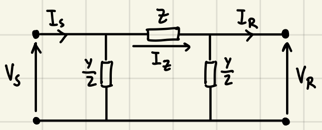
$I_R = 0$ at no load!

Find ABCD: $A = D = 1$, $B = Z = 2.25 + j20$, $C = 0$.

$$\Rightarrow V_s = A\cdot V_R + B\cdot\overset{0}{\cancel{I_R}} = V_R$$
$$\therefore V_R = V_s = \frac{500}{\sqrt{3}} = 288.67^\circ\,kV$$

For L = 100km: > 80km (medium-line model)

$$Z_{total} = 100\,(0.045 + j0.4) = 4.5 + j40\,\Omega$$
$$Y_{total} = 100\,(j4\times10^{-6}) = j4\times10^{-4}\,S$$



Matrix:

$$\boxed{\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} \frac{ZY}{2}+1 & Z \\ Y(1+\frac{ZY}{4}) & \frac{ZY}{2}+1 \end{bmatrix}\begin{bmatrix} V_R \\ 0 \end{bmatrix}}$$ nominal-π model.

Find ABCD: $A = D = \frac{ZY}{2} + 1 = 1 + \frac{1}{2}(4.5 + j40)(j4\times10^{-4}) = 0.992 + j(9\times10^{-4})$

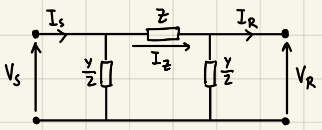
$B = Z = 4.5 + j40$

$C = Y(1 + \frac{ZY}{4}) = j4\times10^{-4}\left[1 + \frac{1}{4}(4.5 + j40)(j4\times10^{-4})\right] = (-1.8\times10^{-7}) + j(3.984\times10^{-4})$

$$\Rightarrow V_s = A\cdot V_R + B\cdot\overset{0}{\cancel{I_R}} = AV_R$$
$$\therefore V_R = \frac{V_s}{A} = \frac{\frac{500}{\sqrt{3}}}{0.992 + j(9\times10^{-4})}$$
$$= 291\angle -0.05^\circ\,V$$

For L = 250km: > 80km (medium-line model)

$$Z_{total} = 250\,(0.045 + j0.4) = 11.25 + j100\,\Omega$$
$$Y_{total} = 250\,(j4\times10^{-6}) = j0.001\,S$$



Matrix:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} \frac{ZY}{2}+1 & Z \\ Y(1+\frac{ZY}{4}) & \frac{ZY}{2}+1 \end{bmatrix}\begin{bmatrix} V_R \\ 0 \end{bmatrix}$$

Find ABCD: $A=D=\frac{ZY}{2}+1 = 1+\frac{1}{2}(11.25+j100)(j0.001) = 0.95+j0.00563$

$B=Z=11.25+j100$

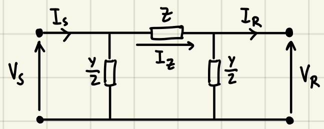
$C=Y(1+\frac{ZY}{4}) = j0.001\left[1+\frac{1}{4}(11.25+j100)(j0.001)\right] = (-2.813\times10^{-6})+j(9.75\times10^{-4})$

$$\Rightarrow V_S = A\cdot V_R + B\cdot \cancel{I_R}^{\,0} = AV_R$$

$$\therefore V_R = \frac{V_S}{A} = \frac{\frac{500}{\sqrt{3}}}{0.95+j0.00563}$$

$$= 303.86\angle -0.34^\circ \text{ V}$$

3.



$$|V_{LL}| = \sqrt{3}\,|V_{LN}|$$

$$\Rightarrow V_{LN} = \frac{765}{\sqrt{3}}\times10^3 \text{ V}$$

$$Z_{total} = 300(0.0165+j0.3306) = 4.95+j99.18\,\Omega$$

$$Y_{total} = 300(j4.674\times10^{-6}) = j0.0014\text{ S}$$

Matrix:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} \frac{ZY}{2}+1 & Z \\ Y(1+\frac{ZY}{4}) & \frac{ZY}{2}+1 \end{bmatrix}\begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

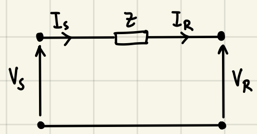
Find ABCD: $A=D=\frac{ZY}{2}+1 = 1+\frac{1}{2}(4.95+j99.18)(j0.0014) = 0.9306+j0.0035$

$B=Z=4.95+j99.18$

$C=Y(1+\frac{ZY}{4}) = j0.0014\left[1+\frac{1}{4}(4.95+j99.18)(j0.0014)\right] = (-2.426\times10^{-6})+j(0.00135)$

4.

Since $L=16\text{km} < 80\text{km}$, we use short-line model.



$$|V_{LL}| = \sqrt{3}\,|V_{LN}|$$

$$\Rightarrow V_{LN} = \frac{64}{\sqrt{3}}\times10^3 \text{ V}$$

$$Z_{total} = 16(0.125+j0.4375) = 2+j7\,\Omega$$

Matrix:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}\begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Find ABCD: $A=D=1$, $B=Z=2+j7$, $C=0$.

$$\Rightarrow V_S = A\cdot V_R + B\cdot I_R = V_R + Z\cdot I_R \text{ —— (1)}$$

$$|S|_{3\phi} = 3|V_{LN}||I_{LN}|$$

$$\Rightarrow |I_R| = \frac{70\times10^6}{3(\frac{64}{\sqrt{3}}\times10^3)} = 631.477\text{A}$$

Since pf $= 0.8$ lagging,

$$\theta = -\cos^{-1}(0.8) = -36.87^\circ$$

$$\therefore I_R = 631.477\angle -36.87^\circ \text{A}$$

Sub. into (1):

$$V_S = \left(\frac{64}{\sqrt{3}} \times 10^3\right) + 631.477\angle -36.87^\circ (2+j7)$$
$$= 40.71\angle 3.91^\circ \text{ kV}$$

Voltage Regulation:

$$\boxed{\text{Voltage Regulation} = \frac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|} \times 100\% = \frac{\left|\frac{V_{S,FL}}{A}\right| - |V_{R,FL}|}{|V_{R,FL}|} \times 100\%}$$

$$= \frac{40.71 - \frac{64}{\sqrt{3}}}{\frac{64}{\sqrt{3}}} \times 100\%$$

$$= 10.17\%$$

Transmission Efficiency:

$$\boxed{\eta = \frac{P_{R,3\phi}}{P_{S,3\phi}} \times 100\% = \frac{Re(S_{R,3\phi})}{Re(S_{S,3\phi})} \times 100\% = \frac{Re[V_R \cdot I_R^*]}{Re[V_S \cdot I_S^*]} \times 100\%} \quad \text{—— (2)}$$

From Matrix:

$$I_S = \cancel{C}^{\,0} V_R + D \cdot I_R = I_R$$
$$\therefore I_S = I_R = 631.477\angle -36.87^\circ \text{ A}$$

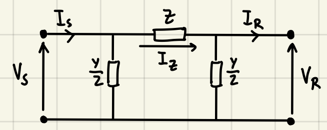
Sub. into (2):

$$\eta = \frac{Re\left[\frac{64}{\sqrt{3}} \times 10^3 \angle 0^\circ (631.477\angle 36.87^\circ)\right]}{Re[40.71 \times 10^3 \angle 3.91^\circ (631.477\angle 36.87^\circ)]} \times 100\%$$

$$= \frac{18\,666\,645.89}{19\,466\,258.74} \times 100\%$$

$$= 95.89\%$$

5.



$$|V_{LL}| = \sqrt{3}\,|V_{LN}|$$
$$\Rightarrow V_R = \frac{220}{\sqrt{3}} \times 10^3 \angle 0^\circ \text{ V}$$
$$Z_{total} = 200(0.08 + j0.48) = 16 + j96\,\Omega$$
$$Y_{total} = 200(j3.33 \times 10^{-6}) = j6.66 \times 10^{-4}\,\text{S}$$

Matrix:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} \frac{ZY}{2}+1 & Z \\ Y\left(1+\frac{ZY}{4}\right) & \frac{ZY}{2}+1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Find ABCD: $A = D = \frac{ZY}{2} + 1 = 1 + \frac{1}{2}(16+j96)(j6.66 \times 10^{-4}) = 0.968 + j0.00533$

$B = Z = 16 + j96$

$C = Y\left(1+\frac{ZY}{4}\right) = (j6.66 \times 10^{-4})\left[1 + \frac{1}{4}(16+j96)(j6.66 \times 10^{-4})\right] = (-1.774 \times 10^{-6}) + j(6.554 \times 10^{-4})$

$$\Rightarrow V_S = A \cdot V_R + B \cdot I_R \quad \text{—— (1)}$$

$$P_{3\phi} = 3VI\cos\theta$$
$$\Rightarrow |I_R| = \frac{250 \times 10^6}{3\left(\frac{220}{\sqrt{3}} \times 10^3\right)(0.99)} = 662.707\,\text{A}$$

Since pf = 0.99 lagging,

$$\theta = -\cos^{-1}(0.99) = -8.11^\circ$$
$$\therefore I_R = 662.707\angle -8.11^\circ \text{ A}$$

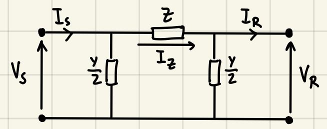
Sub. into (1):

$$V_S=(0.968+j0.00533)\left(\frac{220}{\sqrt{3}}\times10^3\angle0^\circ\right)+(16+j96)(662.707\angle-8.11^\circ)$$
$$=122\ 954.38\angle0.32^\circ+64\ 497.429\angle72.43^\circ$$
$$=155.4\angle23.58^\circ\,V_{//}$$

From Matrix:

$$I_S=C\cdot V_R+D\cdot I_R$$
$$=[(-1.774\times10^{-6})+j(6.554\times10^{-4})]\left(\frac{220}{\sqrt{3}}\times10^3\angle0^\circ\right)+(0.968+j0.00533)(662.707\angle-8.11^\circ)$$
$$=83.247\angle90.16^\circ+641.51\angle-7.79^\circ$$
$$=635.37\angle-0.33^\circ\,A_{//}$$

6.



$$|V_{LL}|=\sqrt{3}\,|V_{LN}|$$
$$\Rightarrow V_R=\frac{325}{\sqrt{3}}\times10^3\angle0^\circ\,V$$

$$R_{total}=130(0.036)=4.68\,\Omega$$
$$L_{total}=130(0.8\times10^{-3})=0.104\,H$$
$$\Rightarrow \boxed{Z_{total}=R+j\omega L}=4.68+j(2\pi)(50)[0.104]=4.68+j32.67\,\Omega$$
$$C_{total}=130(0.0112\times10^{-6})=1.456\times10^{-6}\,F$$
$$\Rightarrow \boxed{Y_{total}=\cancel{G}+j\omega C}=j(2\pi)(50)[1.456\times10^{-6}]=j4.574\times10^{-4}\,S$$

Matrix:

$$\begin{bmatrix}V_S\\I_S\end{bmatrix}=\begin{bmatrix}A&B\\C&D\end{bmatrix}\begin{bmatrix}V_R\\I_R\end{bmatrix}=\begin{bmatrix}\frac{ZY}{2}+1&Z\\Y(1+\frac{ZY}{4})&\frac{ZY}{2}+1\end{bmatrix}\begin{bmatrix}V_R\\I_R\end{bmatrix}$$

Find ABCD: $A=D=\frac{ZY}{2}+1=1+\frac{1}{2}(4.68+j32.67)(j4.574\times10^{-4})=0.99253+j0.00107$

$B=Z=4.68+j32.67$

$C=Y(1+\frac{ZY}{4})=j4.574\times10^{-4}\,[1+\frac{1}{4}(4.68+j32.67)(j4.574\times10^{-4})]=(-2.448\times10^{-7})+j(4.557\times10^{-4})$

$$\Rightarrow V_S=A\cdot V_R+B\cdot I_R \text{ ——— (1)}$$

$$|S|_{3\phi}=3|V_{LN}||I_{LN}|$$
$$\Rightarrow |I_R|=\frac{270\times10^6}{3\left(\frac{325}{\sqrt{3}}\times10^3\right)}=479.645\,A$$

Since pf = 0.8 lagging,

$$\theta=-\cos^{-1}(0.8)=-36.87^\circ$$
$$\therefore I_R=479.645\angle-36.87^\circ\,A$$

Sub. into (1):

$$V_S=(0.9925+j0.0107)\left(\frac{325}{\sqrt{3}}\times10^3\angle0^\circ\right)+(4.68+j32.67)(479.645\angle-36.87^\circ)$$
$$=197.76\angle3.30^\circ\,kV$$

$$\text{Voltage Regulation}=\frac{|V_{R,NL}|-|V_{R,FL}|}{|V_{R,FL}|}\times100\%$$
$$=\frac{\left|\frac{V_{S,FL}}{A}\right|-|V_{R,FL}|}{|V_{R,FL}|}\times100\%$$
$$=\frac{\left|\frac{197.76\times10^3\angle3.3^\circ}{0.9925+j0.0107}\right|-\left(\frac{325}{\sqrt{3}}\times10^3\right)}{\left(\frac{325}{\sqrt{3}}\times10^3\right)}\times100\%$$
$$=6.19\%_{//}$$

$$\eta=\frac{P_{R,3\phi}}{P_{S,3\phi}}\times100\%=\frac{Re[V_R\cdot I_R^*]}{Re[V_S\cdot I_S^*]}\times100\% \text{ ——— (2)}$$

From Matrix: $I_S = C \cdot V_R + D \cdot I_R$

$$= [(-2.448\times10^{-7}) + j(4.557\times10^{-4})]\left(\frac{325}{\sqrt{3}}\times10^3\angle0^\circ\right) + (0.99253+j0.00107)(479.645\angle-36.87^\circ)$$

$$= 85.507\angle90.03^\circ + 476.06\angle-36.81^\circ$$

$$= 430.27\angle-27.66^\circ \text{A}$$

Sub. into (2):

$$\eta = \frac{Re\left[\frac{325}{\sqrt{3}}\times10^3\angle0^\circ(479.645\angle36.87^\circ)\right]}{Re\left[197.76\times10^3\angle3.30^\circ(430.27\angle27.66^\circ)\right]}\times100\%$$

$$= 98.67\%$$

When pf' = 0.95 lagging, w $|S|_R$ same:

$$|S|_{3\phi} = 3|V_{LN}||I_{LN}|$$

$$\Rightarrow |I_R| = \frac{270\times10^6}{3\left(\frac{325}{\sqrt{3}}\times10^3\right)} = 479.645\text{A}$$

Since pf = 0.95 lagging,

$$\theta = -\cos^{-1}(0.95) = -18.19^\circ$$

$$\therefore I_R = 479.645\angle-18.19^\circ \text{A}$$

Sub. into (1):

$$V_S = (0.9925+j0.0107)\left(\frac{325}{\sqrt{3}}\times10^3\angle0^\circ\right) + (4.68+j32.67)(479.645\angle-18.19^\circ)$$

$$= 193.79\angle4.26^\circ \text{kV}$$

$$\text{Voltage Regulation} = \frac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|}\times100\%$$

$$= \frac{\left|\frac{V_{S,FL}}{A}\right| - |V_{R,FL}|}{|V_{R,FL}|}\times100\%$$

$$= \frac{\left|\frac{193.79\times10^3\angle4.26^\circ}{0.9925+j0.0107}\right| - \left(\frac{325}{\sqrt{3}}\times10^3\right)}{\left(\frac{325}{\sqrt{3}}\times10^3\right)}\times100\%$$

$$= 4.05\%$$

$$\eta = \frac{P_{R,3\phi}}{P_{S,3\phi}}\times100\% = \frac{Re[V_R\cdot I_R^*]}{Re[V_S\cdot I_S^*]}\times100\% \quad \text{—— (2)}$$

From Matrix: $I_S = C \cdot V_R + D \cdot I_R$

$$= [(-2.448\times10^{-7}) + j(4.557\times10^{-4})]\left(\frac{325}{\sqrt{3}}\times10^3\angle0^\circ\right) + (0.99253+j0.00107)(479.645\angle-18.19^\circ)$$

$$= 85.507\angle90.03^\circ + 476.06\angle-18.13^\circ$$

$$= 456.70\angle-7.88^\circ \text{A}$$

Sub. into (2):

$$\eta = \frac{Re\left[\frac{325}{\sqrt{3}}\times10^3\angle0^\circ(479.645\angle18.19^\circ)\right]}{Re\left[193.79\times10^3\angle4.26^\circ(456.70\angle7.88^\circ)\right]}\times100\%$$

$$= 98.82\%$$