7.1 Transformers

Tuesday, 1 March 2022 10:29 am

EE2029: Introduction to Electrical Energy System

Transformers

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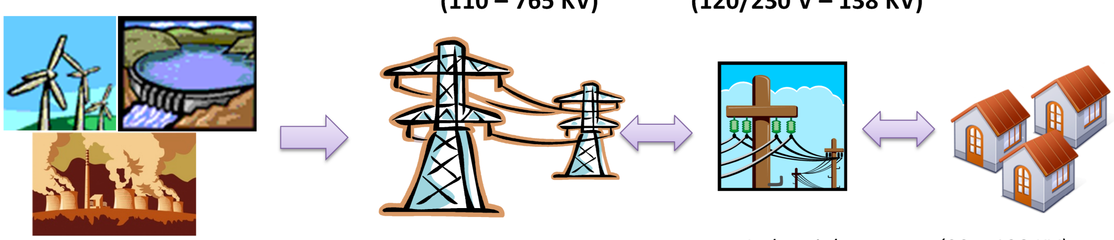
Electrical Energy Systems

Generation (11 – 36 KV)

Transmission (110 – 765 KV)

Distribution (120/230 V – 138 KV)



Large-Scale Power Plants: Coal-fired, Hydro, Wind, Solar, Nuclear

Industrial customer (23 – 138 KV)
Commercial customer (4.16 – 34.5 KV)
Residential customer (120 – 240 V)

How to connect generation (medium voltage) to transmission (high voltage) to the point of consumption (low voltage)?

Transformers

- Transformers used to step up voltages from generation units to transmission-line and to step down voltage from transmission to end users.
- Transformers are important in electrical energy systems. With transformers, the overall efficiency can be improved.

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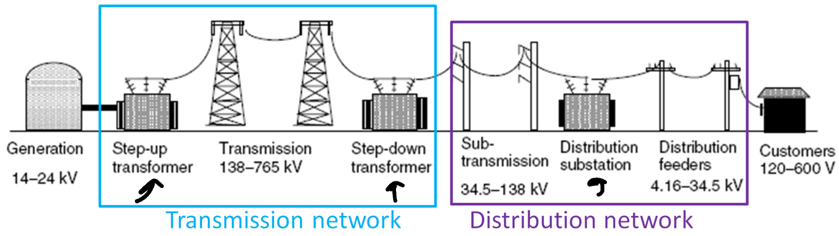
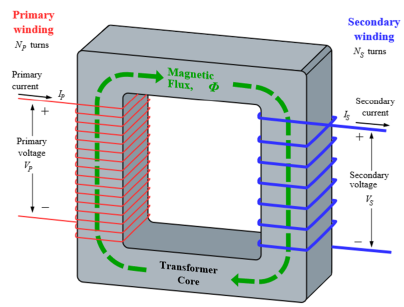
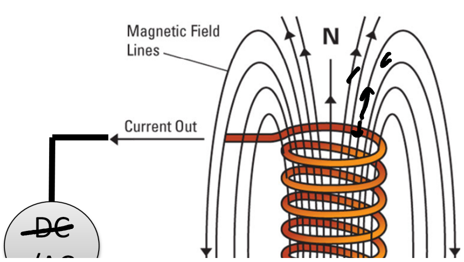




Electromagnetism
Electromagnetic induction
Dot notation
Faraday's Law
Ampere's Law

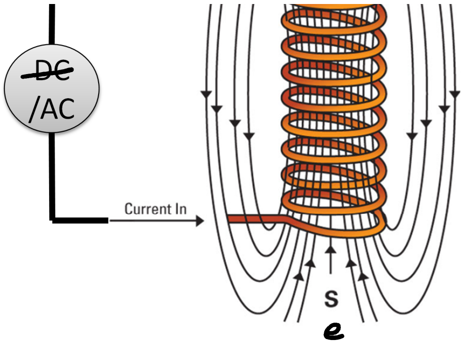
PRINCIPLE OF TRANSFORMERS

Electromagnetism and Electromagnetic Induction



$\frac{d\Phi}{dt} \to 0$ Φ DC

- DC source → Constant magnetic flux
- AC source → $\boxed{\text{Varying magnetic flux}}$ Φ t
- *What will happen if we have another coil to link the varying magnetic flux?*



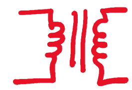
- *What will happen if we have another coil to link the varying magnetic flux?*
- **Faraday's Law:** $e = N\frac{d\Phi}{dt}$

$e = N\frac{d\phi}{dt}$

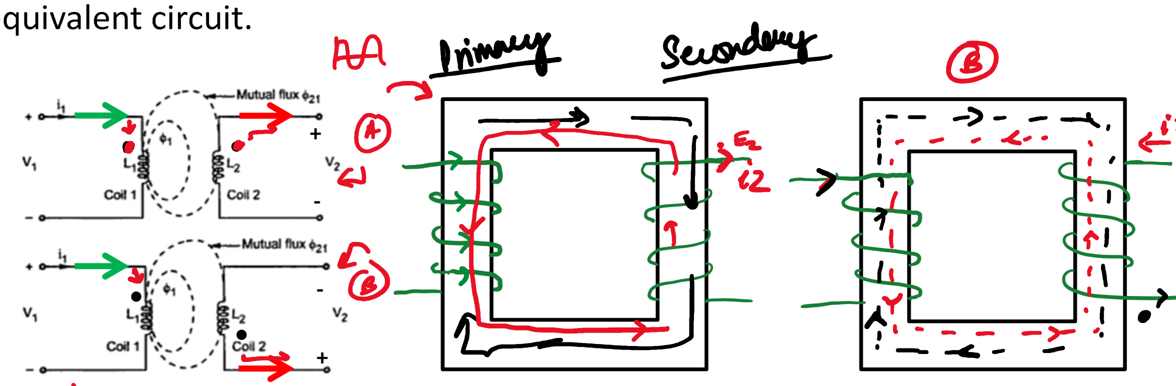
No of turns

Source: http://www.lanl.gov/news/index.php/fuseaction/1663.article/d/20085/id/13276

Dot Notation in Transformers



- **Dot notation** is used to indicate the direction of current out of Coil 2 in the equivalent circuit.

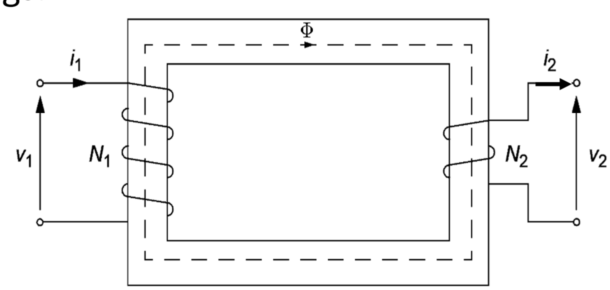


i_1 & i_2 are out of phase by 180°

Lenz's law

Magnetic Core

- We can better link the magnetic flux by using magnetic core.
- Magnetic flux "Φ" is now confined in the core and links both windings.



Note that 'N' refers to number of turns.

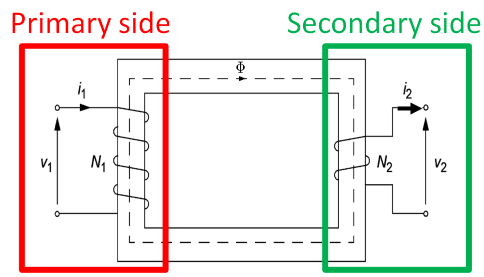
We are now interested to relate V1 and V2, and relate i1 and i2.

Ideal Transformer

$R_p \cdot R_s = 0$

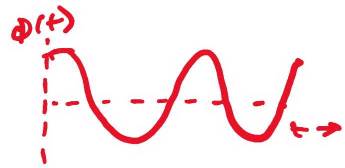
1. No resistance in both windings.
2. No leakage flux around the core.
3. No core resistive loss. → Heat = 0
4. Core permeability is infinite.

$\mu \to \infty$



Faraday's Law

- Recall that: $$e = N\frac{d\phi}{dt}$$



$\phi(t) = \sqrt{2}\,\phi \cos \omega t$

rms → ϕ; ω → $2\pi f$ → supply frequency

$e = N \frac{d}{dt}(\sqrt{2}\phi \cos \omega t)$

$= N(\sqrt{2}\phi\omega)(-\sin\omega t)$

$e(t) = N\omega(\sqrt{2}\phi)(\cos(\omega t + 90°))$

$= \sqrt{2} N\omega\phi \cos(\omega t + 90)$

→ phasor

$\bar{E} = N\omega\phi \angle 90° = N\Phi(j\omega)$

$\boxed{E = N\phi(j\omega)}$ — Ⓐ

(Wb) ← $\phi = B \cdot A$ → Area (m²)

B → Fluxdensity (wb/m²)

Voltage Relationship

- For **ideal** transformer, we assume that the flux linkage at coil 1 and coil 2 is the same i.e. there is no flux linkage loss. $(\phi_1 = \phi_2)$
- We can now find a relationship between the voltages at two sides of the transformer as follows.

N_2 → Turns

Using A

We can now find a relationship between the voltages at two sides of the transformer as follows.

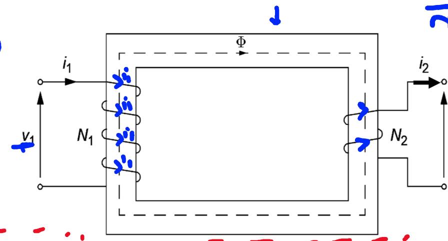
Using A

$V_1 = N_1 \Phi_1 (j\omega)$; $V_2 = N_2 \Phi_2 (j\omega)$

$\phi_1 = \frac{V_1}{N_1 (j\omega)}$; $\Phi_2 = \frac{V_2}{N_2 (j\omega)}$

Since $\phi_1 = \phi_2$

$\Rightarrow \frac{V_1}{N_1 \cancel{(j\omega)}} = \frac{V_2}{N_2 \cancel{(j\omega)}} \Rightarrow \boxed{\frac{V_1}{V_2} = \frac{N_1}{N_2}} \rightarrow \boxed{\frac{V_2}{V_1} = \frac{N_2}{N_1}}$



$\frac{N_2}{N_1} \rightarrow$ Turns Ratio

$\frac{N_2}{N_1} > 1$ ↳ step up

$\frac{N_2}{N_1} < 1$ ↳ step down

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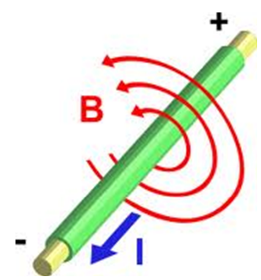
Ampere's Law

$\oint H dl = I_{enclosed}$

$N_1 I_1 - N_2 I_2$

- "Current passing through a conductor creates magnetic field around it "

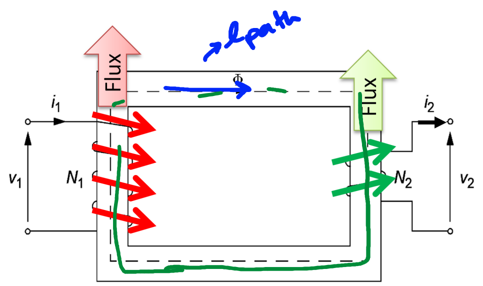
$$\oint H dl = I_{enclosed}$$



- B = µH
- B = Magnetic flux density (Weber/m² or Tesla)
- H = Magnetic field intensity (A/m)
- µ = Magnetic core permeability (H/m)

Ampere's Law Applied to Transformers

- "Magnetic flux along the path equals the net current enclosed by the path"



i_1N_1 $\quad$ $-i_2N_2$

i_1N_1 $\quad$ $-i_2N_2$

$$\oint H\,dl = H \cdot l_{path} = I_{enclosed}$$

$$= I_1N_1 - I_2N_2$$

$$B = \mu H$$

$$H = \frac{B}{\mu}$$

$$I_1N_1 - I_2N_2 = \frac{B}{\mu} \cdot l_{path},$$

Current Relationship

- Note that magnetic core permeability (μ) represents the degree of magnetization that the magnetic core will allow the magnetomotive force to pass through.
- For **ideal transformer**, the value of the permeability is infinity.

$$i_1N_1 - i_2N_2 = \frac{B}{\mu} \cdot l_{path}.$$

If $\mu = \infty \Rightarrow RHS = 0$

$$i_1N_1 - i_2N_2 = 0$$

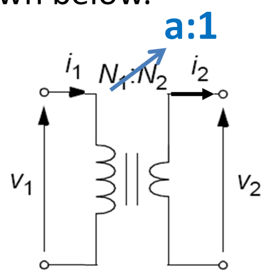
$$I_1N_1 = I_2N_2$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

$$\frac{I_2}{I_1} = \frac{N_1}{N_2}$$

Transformer Equivalent Circuit

- We represent an equivalent circuit of an ideal transformer as shown below.



$a = \frac{N_1}{N_2}$

- Define turns ratio as:

$$a \equiv \frac{N_1}{N_2}$$

- From Faraday's and Ampere's Law:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \qquad i_1 N_1 = i_2 N_2$$

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$, $i_1 N_1 = i_2 N_2$.

Complex Power

- Complex power *at* *primary* *side*,

$S_p = V_1 I_1^*$

$\frac{V_1}{V_2} = \frac{N_1}{N_2} = a = \frac{I_2}{I_1}$

$V_2 = \frac{V_1}{a}$

$I_2 = a \cdot I_1$

- Complex Power at *secondary* side

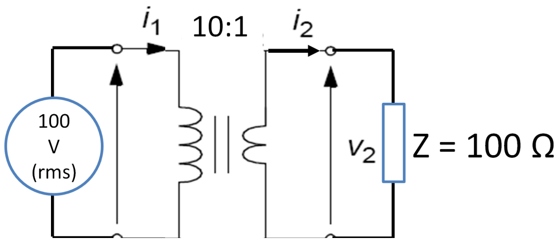
$S_s = V_2 I_2^*$

$= \left(\frac{V_1}{\cancel{a}}\right) \cdot (\cancel{a} I_1)^*$

$= V_1 I_1^* = S_p$.

Example 1

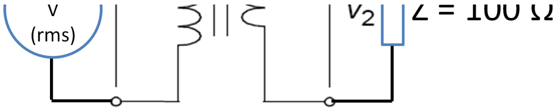
From the circuit below, what is the current at the secondary and primary side?



$\frac{N_1}{N_2} = \frac{10}{1} = a$

$V_1 = 100V$

$V_2 = \frac{V_1}{a} = \frac{100}{10} = 10V$



$$V_2 = \frac{V_1}{a} = \frac{100}{10} = 10V$$

$$I_2 = \frac{V_2}{Z} = \frac{10}{100} = 0.1A.$$

$$I_1 = \frac{I_2}{a} = \frac{0.1}{10} = 0.01A.$$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1} = a$$

v_1 v_L

100V ↔ 500V.

$$a = \frac{V_1}{V_L} = \frac{1}{5} = 0.2.$$

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Reflected load

Impedance matching

APPLICATION OF TRANSFORMERS

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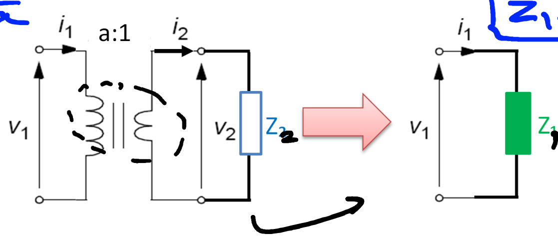
Reflected Load

- We can reflect a load from one side of a transformer to the other side of the transformer.
- This trick allows us to combine the two separate circuits into one for easy(?) calculation. $I_2 = \frac{V_2}{Z_2}$ $Z_2 = \frac{V_2}{I_2}$ $Z_1 = \frac{V_1}{I_1}$

$$Z_1 = \frac{aV_2}{\frac{I_2}{a}} = a^2 \cdot \frac{V_2}{I_2} = a^2 Z_2$$

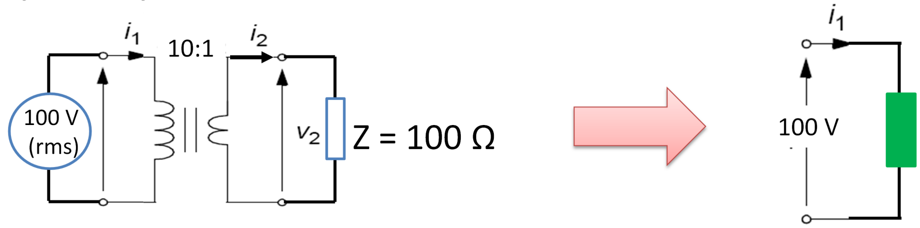
$$\boxed{Z_1 = a^2 Z_2}$$

Interest to find reflected load "Z_1"



Example 2

- Find the reflected load of impedance 100 Ω seen from the primary side of the transformer.



$Z_1 = a^2 . Z_2$
$= 10^2 \times 100$
$= 10 K\Omega$

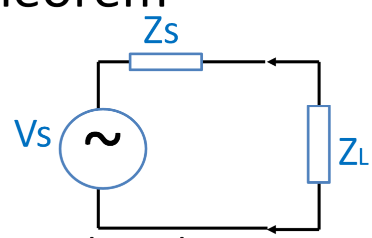
$I_1 = \frac{V_1}{Z_1} = \frac{100}{10 \times 10^3} = 0.01 A$

Maximum Power Transfer Theorem

- From real power expression at the load,

$P_L = \frac{|V_S|^2}{|Z_S + Z_L|^2} Re(Z_L)$

$$P_L = \frac{|V_S|^2}{|Z_S + Z_L|^2} \mathrm{Re}\{Z_L\}$$



- This means that the maximum power will occur when the denominator is minimum

$|Z_S + Z_L|^2 \rightarrow$ minimum

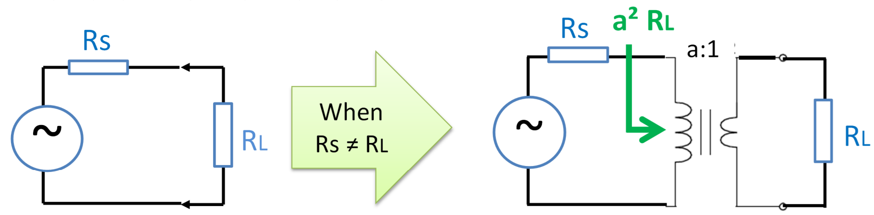
Maximum Power Transfer Theorem

$Z_s = Z_L^*$

$|R_s| = |R_L| \qquad |X_s| = |X_L|$

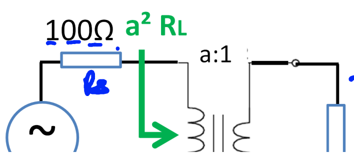
Impedance Matching for ‘R’

- Maximum power transfer occurs when Rs = RL.
- In the case that we need to connect the voltage source that has internal impedance of Rs to a load RL that does not satisfy the above condition, we can **design** a **transformer** to *match* impedance for maximum power transfer.
- To find an appropriate transformer, we let **Rs = a²RL** and find a transformer turns ratio.



Example 3

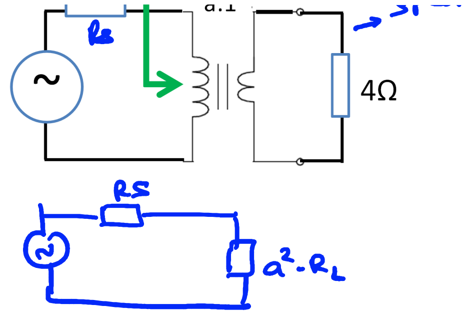
Given the voltage source with an internal resistance of 100 Ω. A transformer is used to connect this voltage source to the load of 4 Ω to achieve maximum power transfer at the load. What should be the turns ratio of a transformer?



→ speaker.

$R_s \neq R_L$

$R_s = a^2 R_L$



$R_S = a^2 R_L$

$a^2 = \frac{R_S}{R_L} = \frac{100}{4} = 25$

$a = 5$

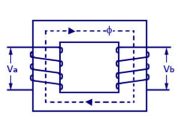
Transformer components
Losses in Transformers
Magnetizing current losses
A practical transformer model
A simplified model

PRACTICAL TRANSFORMERS

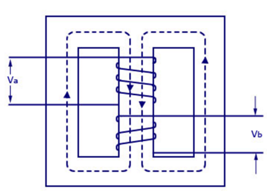
Transformer Components

Magnetic core

- Mainly two types of design.
  1. Core type – the magnetic core is surrounded by the windings.
  2. Shell type – the windings are surrounded by the core.



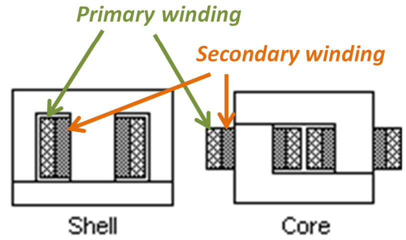
1. Core type



2. Shell type

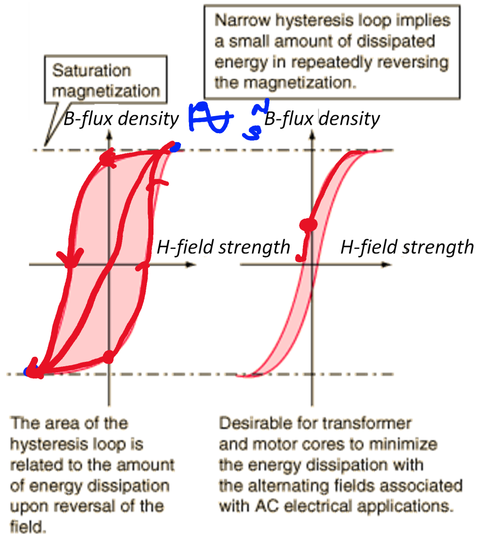
Primary/Secondary winding

- Windings are placed on top of each other to reduce the amount of flux leakage losses.



Losses in Transformers: Hysteresis Losses

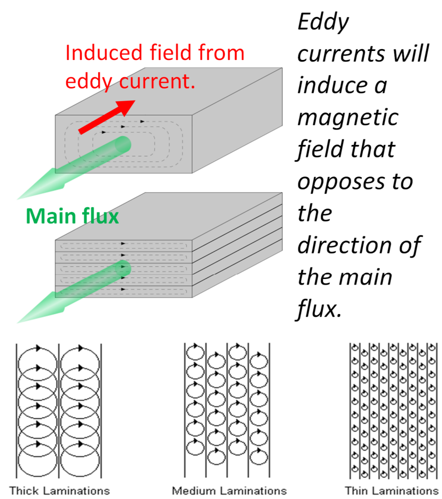
- Hysteresis loop is a characteristic of how a ferromagnetic material is magnetized.
- Each time the direction of magnetic field is reversed, some amount of energy is dissipated in the core.
- This means that this loss is proportional to the frequency of electricity.
- Hysteresis loss produces heat and is represented as a **resistance parallel to** the ideal transformer.



Source: http://hyperphysics.phy-astr.gsu.edu/hbase/solids/hyst.html

Losses in Transformers: Eddy Current Losses

- Eddy current refers to small circular current in the magnetic core caused by the flux that passes through the core.
- The magnitude of eddy current losses depends on the strength of the main flux, thus the voltage supplied.
- Eddy current loss produces heat and is represented as a **resistance parallel to** the ideal transformer.
- Eddy current loss can be reduced by making the cores from thin sheets of steel i.e. the core is laminated. The thinner the sheets, the smaller are the eddy current losses.

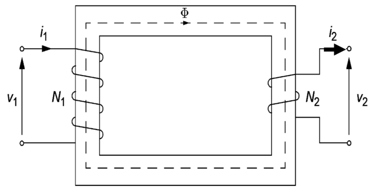


Source: http://sound.westhost.com/xfmr2.htm

Ideal VS Practical Transformer

Ideal transformer

1. Zero resistance in the both windings.
2. No leakage flux around the core.
3. No core resistive loss.
4. Core permeability is infinite



Practical transformer

1. Winding losses (copper losses) represented as ***resistance in both windings***.
2. Leakage flux around the core represented as ***inductance in both windings***.
3. Core resistive losses (hysteresis loss + eddy current loss) represented as ***resistance in parallel to the core***.
4. Magnetic core permeability is finite.

How to represent this effect?

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Finite Core Permeability

$\frac{B l_{path}}{\mu} = i_1N_1 - i_2N_2$,

- From Ampere's law applied to transformers,

$$\frac{B l_{path}}{\mu} = i_1 N_1 - i_2 N_2$$

Recall that in ideal transformer, μ is infinite so $i_1 N_1 = i_2 N_2$

- When the core material has finite core permeability ($\mu \neq \infty$), and $V_1 = N_1 (j\omega) BA$ (Faraday's law), **assuming a constant μ**,

$V_1 = N_1 (j\omega) BA$.

$$\frac{B l_{path}}{\mu} = i_1N_1 - i_2N_2.$$

$$\frac{V_1 l_{path}}{\mu N_1 (j\omega) A} = i_1N_1 - i_2N_2$$

$$\frac{V_1 l_{path}}{N_1^2 (j\omega) A\mu} = i_1 - i_2 \frac{N_2}{N_1}$$

$$\frac{V_1}{j\omega \left(\frac{\mu N_1^2 A}{l_{path}}\right)} = i_1 - i_2 \frac{N_2}{N_1}$$

← Impedance.

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Magnetizing Current Losses

$i_m = -j\left(\frac{l_{path}}{\omega N_1^2 A\mu}\right) \cdot V_1$

- From the magnetizing current,

$i_1 - i_2 \frac{N_2}{N_1} =$

$$i_m = -j\left(\frac{l_{path}}{\omega N_1^2 A\mu}\right) V_1$$

- We find that the current lags the voltage V_1 by 90°(-j).
- As such, we can use an **inductor** to represent the effect of finite

Magnetizing Current Losses

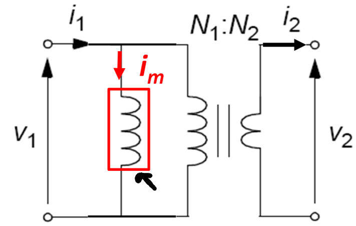
$$i_m = -j\left(\frac{l_{path}}{\omega N_1^2 A\mu}\right)\cdot V_1$$

- From the magnetizing current,

$$i_1 - i_2\frac{N_2}{N_1} = \qquad i_m = -j\left(\frac{l_{path}}{\omega N_1^2 A\mu}\right)V_1$$

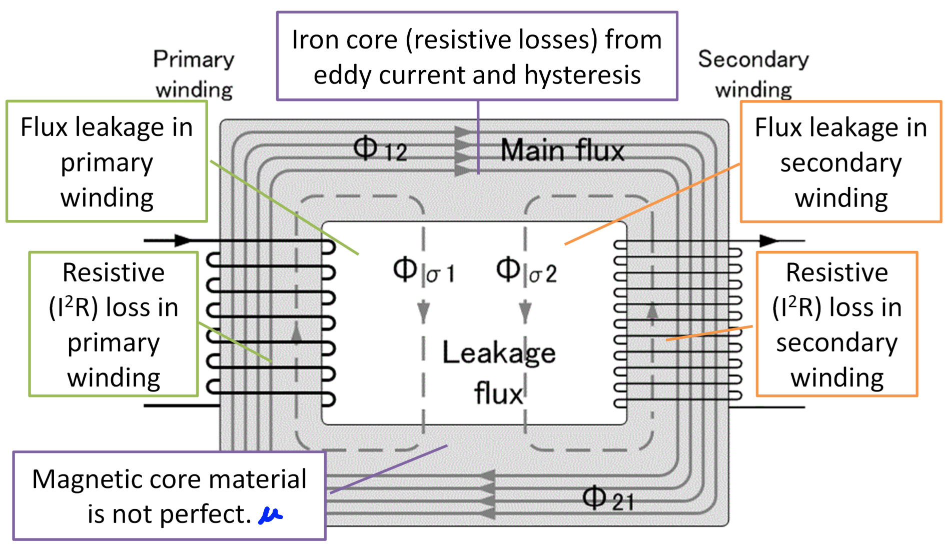
- We find that the current lags the voltage V_1 by 90°(-j).
- As such, we can use an **inductor** to represent the effect of finite magnetic core permeability in the equivalent circuit.

$$i_1 - \left(\frac{N_2}{N_1}\right)i_2 = i_m$$

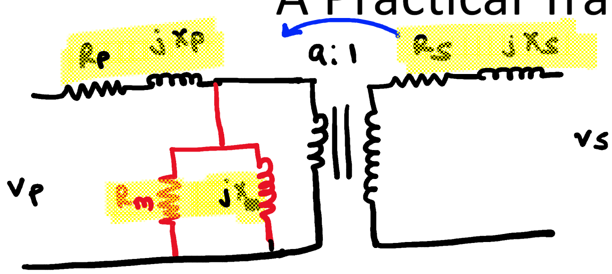


$$i_1 - i_2\frac{N_2}{N_1} = i_m .$$

Practical Transformers



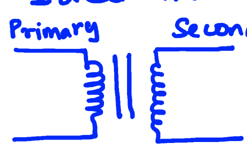
A Practical Transformer Model



$z_1 = R_p + jX_p$

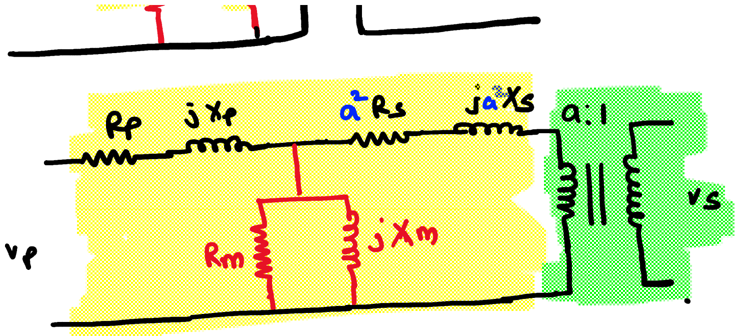
$z_2 = R_s + jX_s$

Ideal Transformer



Assumptions

a) No Resistance



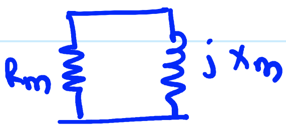
Assumptions

a) No Resistance

b) No leakage Flux

c) No core losses

d) $\mu \to \infty$



$$Y = \frac{1}{Z}$$

$$Z = \left(\frac{1}{R_m} + \frac{1}{jX_m}\right)^{-1}$$

$$\Rightarrow Y = \frac{1}{Z} = \frac{1}{R_m} + \frac{1}{jX_m}$$

$$Y = \frac{1}{R_m} - \frac{j}{X_m}$$

$$X_m = \omega L_m$$

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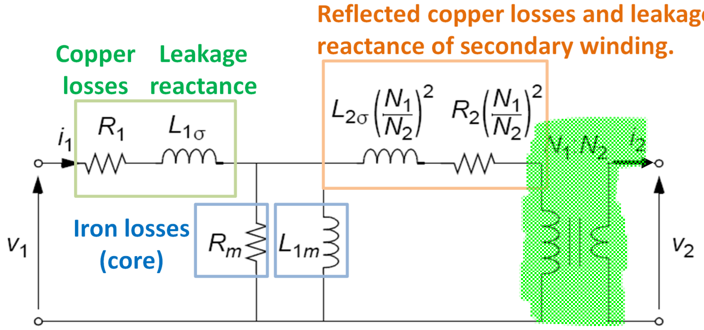
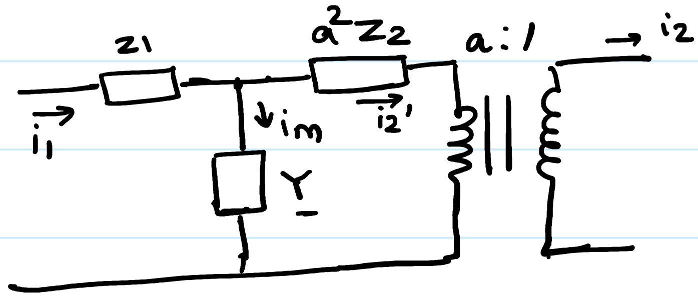
A Practical Transformer Model

By convention, the **primary side** of transformer is the side with a **higher number** of turns.



Note that in Chapter 3 [Glover, Sarma, and Overbye, "Power System Analysis and Design"], the core losses are represented as 'shunt admittance', Y = G –jB where G and B are positive. The imaginary part is negative to represent inductive property.



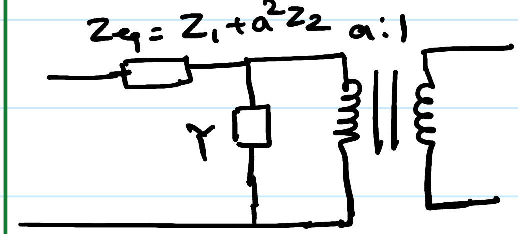
Current i_m is small

→ Y is small or

$$Z = \left(\frac{1}{R_m} + j\frac{1}{X_m}\right)^{-1} = \frac{1}{Y}$$

is very large.

Since $i_1 \approx i_2'$

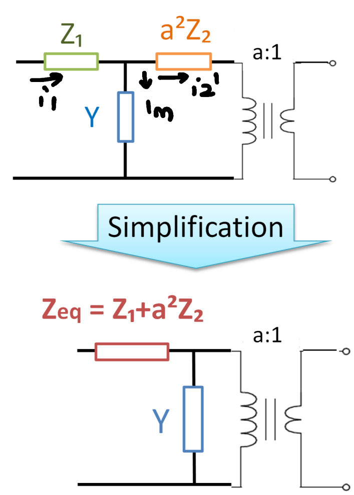


Z → ohm → Ω

Y → mho → ℧ → Siemens

A Simplified Model

- Typically the admittance, Y, is very small i.e. resistance is very large.
- This means that the currents flowing through Z_1 and a^2Z_2 are almost the same. ($i_2' \approx i_1$)
- We can simply combine Z_1 and a^2Z_2 to "Z_{eq}", the equivalent series impedance.



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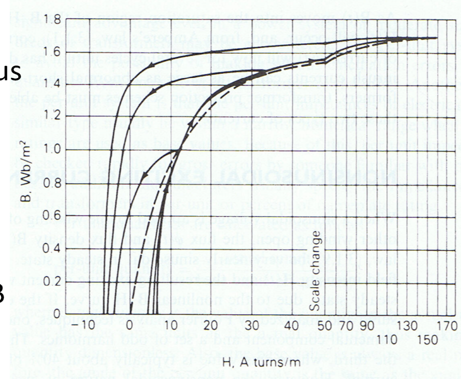
Saturation

- In practical transformer model, we assume **a constant core permeability** thus linear relationship between B and H.
- In fact, the B-H curve for ferromagnetic materials used for transformer core is nonlinear and has multiple values.
- As H increases, the core become **saturated** i.e. the magnetic flux density B increase at a much lower rate.
- This means that there will be high magnetizing current flow making the transformer to heat up.
- This effect is **NOT included** in the equivalent circuit.



B-H curve is approximated by a dashed line.

$B = \mu H$

B = Magnetic flux density (Weber/m² or Tesla)

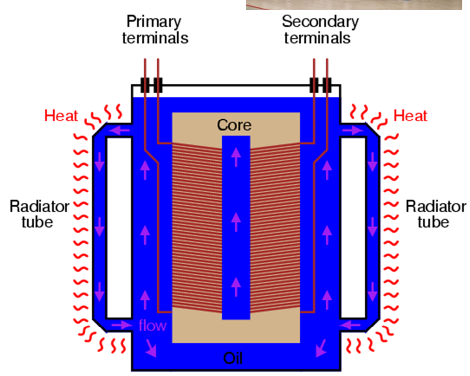
H = Magnetic field intensity (A/m)

μ = Magnetic core permeability (H/m)

Transformer Heating

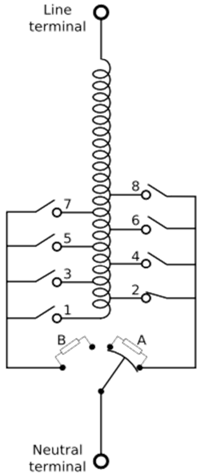
- Heating is caused by high loading of the transformer, eddy current losses, saturation.
- Heating can lead to winding insulation damage, short circuit, and even explosion.
- In order to prevent overheating, transformers are usually cooled by a fan and a convection oil to reduce heat inside the iron core.

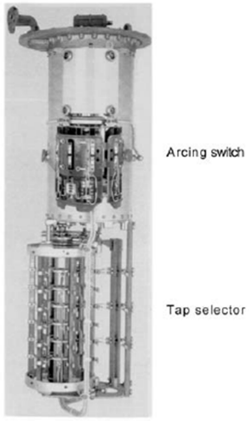
Source: abb.com



Source: http://openbookproject.net/electricCircuits/AC/AC_9.html

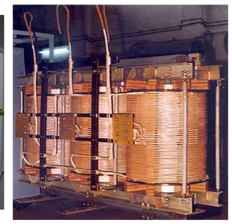
Tap Changing Transformer





Online

- Transformers with tap changer are used to adjust the voltage by changing the turns ratio.
- Tap-changing transformers are used to regulate the voltage at the end users to be at the desired value.
- This can be done both off-line and on-line.



Off-line

Source: http://en.wikipedia.org/wiki/Tap_%28transformer%29
http://www.powertransformerdesign.net

Short-circuit test

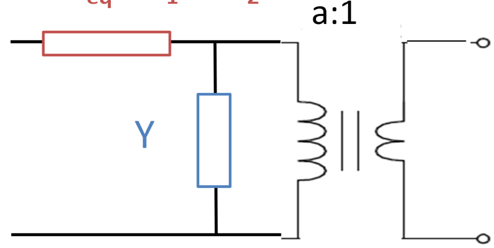
Open-circuit test

TRANSFORMER PARAMETER TESTS

Transformer Parameters

- Series impedance
  - Z_1 and Z_2 are series impedances representing the resistive loss and flux linkage loss in the two windings.
- Shunt admittance
  - Y is a shut admittance representing iron core loss and magnetizing susceptance.

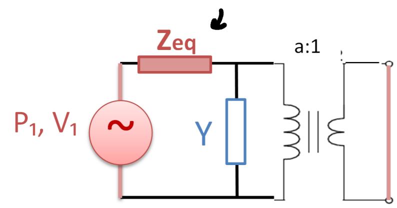
$Z_{eq} = Z_1 + a^2 Z_2$



Note that by convention, the primary side of a transformer is the side with a higher number of turns. This means that a > 1.

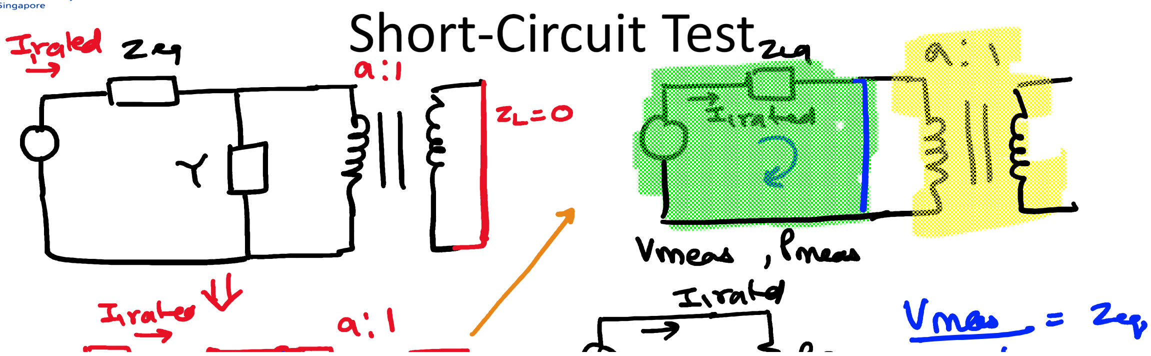
Short-Circuit Test

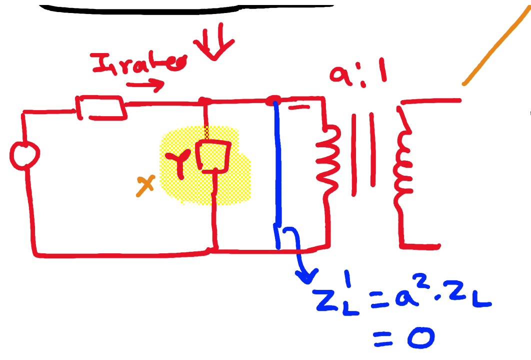
- To find equivalent series impedance.
- Short circuit the *secondary* side.
- Apply **rated current** (implies small voltage) at the primary side.
- Measure real power and voltage at the primary side.

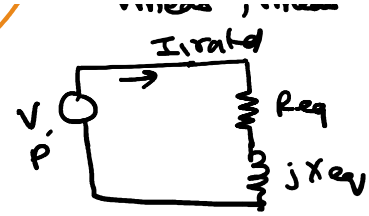


Note that we want small voltage applied to the primary side so that there will be large amount of current passing through the impedance. This will allow more accurate calculation of the series impedance

Short-Circuit Test







$$\frac{V_{meas}}{I_{1rated}} = Z_{eq}$$

$$P_{meas} = I_{1rated}^2 \times R_{eq}$$

$$R_{eq} = \frac{P_{meas}}{I_{1rated}^2}.$$

$$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2}$$

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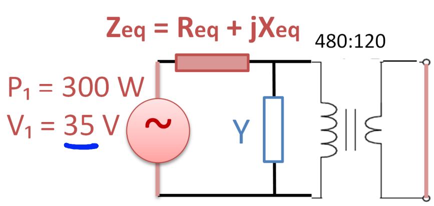
Example 4: Short Circuit Test

Consider a single-phase 20kVA 480/120 V 60 Hz transformer. During short circuit test, rated current is applied to the primary side. The voltage of **35 V** and real power of **300 W** are measured. Find equivalent series impedance of this transformer.

(S rated)



$$I_{1rated} = \frac{20000}{480} = 41.667A$$

$$Z_{eq} = \frac{35}{41.667} = 0.84\Omega.$$

$$R_{eq} = \frac{P}{I_{1rated}^2} = \frac{300}{41.667^2} = 0.1728\,\Omega$$

$$X_{eq} = \sqrt{0.84^2 - 0.1728^2} = 0.822\,\Omega.$$

$$Z_{eq} = 0.1728 + j\,0.822\,\Omega$$

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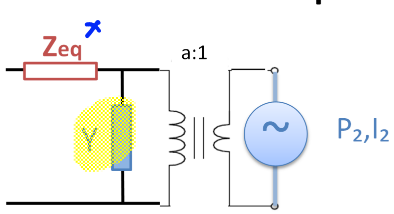
Open-Circuit Test



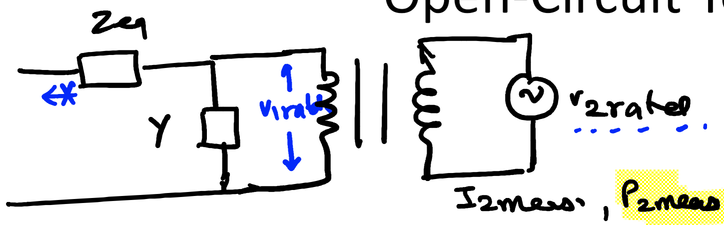
Note that we want rated voltage at the secondary side so that there will be large amount of current passing through the admittance Y (I=YV). This will allow more accurate calculation of the shunt admittance.

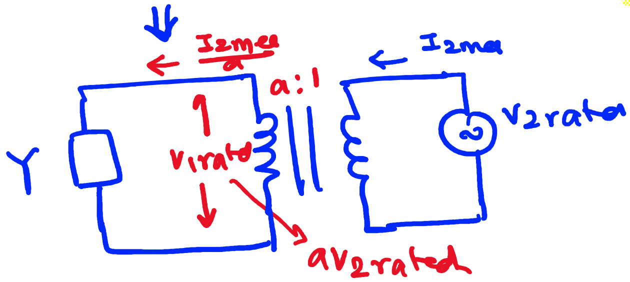
- To find equivalent shunt admittance.
- Open circuit the *primary* side.
- Apply **rated voltage** at the secondary side.
- Measure real power and current at the secondary side.

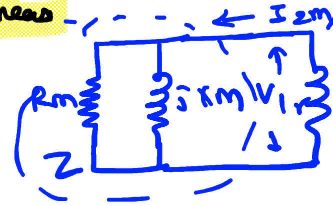
Open-Circuit Test

$Y = \frac{1}{R_m} - \frac{j}{X_m}$, where $\frac{1}{R_m} \rightarrow G$ and $\frac{j}{X_m} \rightarrow B$

$Y = G - jB$







$Z = \frac{V_{1rated}}{(I_{2m}/a)}$

$$\boxed{Y = \frac{I_{2m}/a}{V_{1rated}}}$$

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$$P_{2meas} = \frac{V_{1rated}^2}{R_m} = V_{1rated}^2 G$$

$$G = \frac{P_{2meas}}{V_{1rated}^2}$$

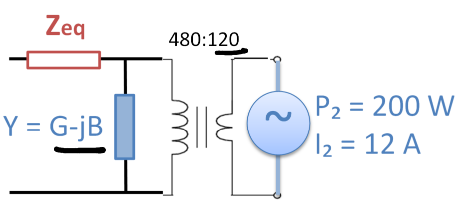
$$B = \sqrt{Y^2 - G^2}$$

$$Y = G - jB$$

$$X_m = \frac{1}{B}$$

Example 5: Open Circuit Test

Consider the same transformer as Example 4. During open circuit test: rated voltage applied to secondary side, then **I_2 = 12 A** and **P_2 = 200 W**. Find equivalent shunt admittance Y of this transformer.



480/120 V — V_{1rated}

$I_2 = 12A$

$I_1 = \frac{12}{a} = \frac{12}{4} = 3A$

$$|Y| = \frac{I_1}{V_{1rated}} = \frac{3}{480} = 0.00625 ℧$$

$P_1 = P_2 = 200W = V_{1rated}^2 \cdot G$

$B = \sqrt{Y^2 - G^2}$

$P_1 = P_2 = 200W = V_1^2{}_{true} \cdot G$

$G = \frac{200}{480^2} = 0.000868\ \mho$

$B = \sqrt{Y^2 - G^2}$

$= 0.00619\ \mho$

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$R_m = \frac{1}{G} = 1152\ \Omega$

$X_m = \frac{1}{B}$

$= 161.5\ \Omega$

QUESTIONS

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