EE2029 INTRODUCTION TO ELECTRICAL ENERY SYSTEMS
(Solution for Tutorial #1 Transmission Lines)

4.

Using the short length model, $l = 16\,km$, $Z = 16 \times (0.125 + \mathrm{j}0.4375) = 2 + j7\,\Omega$. Per phase equivalent model is shown in Fig. 1.

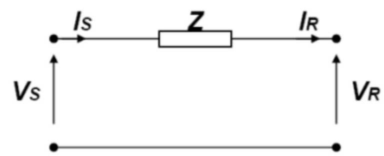

Fig.1

The voltage and current at sending and receiving end can be found from the following.

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & 2+j7 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

The receiving end voltage is given as 64 kV line-to-line. Let the voltage at the load be reference angle, we can find the voltage per phase as follows.

$$V_R = \frac{64 \times 10^3}{\sqrt{3}} \angle 0° = 36.95\angle 0°\ kV.$$

The receiving end current, I_R, can be found from the complex power consumed by the load at the receiving end, denoted by $S_{3\Phi,\mathrm{R}}$, of 70 MVA, 0.8 lagging.

$$|I_R| = \frac{|S_{1\Phi,\mathrm{R}}|}{|V_R|} = \frac{|S_{3\Phi,\mathrm{R}}|}{3|V_R|} = \frac{70 \times 10^6}{3 \times 36.95 \times 10^3} = 631.48\ A$$

The angle of receiving end current is found from power factor. The angle is negative because power factor is lagging.

$$\angle I_R = -\cos^{-1}(0.8) = -36.87°$$

Sending end voltage is found below.

$V_S = V_R + (2 + j7) \times\ I_R = 36950.42\angle\ 0° + (2 + j7) \times (631.48\angle\ -36.87°) = 40.71\ \angle\ 3.91°\ kV.$
Note that the no load voltage for short transmission line model is the same as sending end voltage at full load.

$$Voltage\ regulation = \frac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|} \times 100\% = \frac{40.71 - 36.95}{36.95} \times 100\% = 10.17\%$$

The complex power delivered at the load is given as 70 MVA, 0.8 lagging. Thus, real power delivered at the load is

$$P_{3\Phi,\mathrm{R}} = |S_{3\Phi}| \times p.f. = 70 \times 0.8 = 56\ MW.$$

We then find real power at the sending end. For a short transmission line, sending end current is the same as receiving end current, $I_S = I_R = 631.48\angle -36.87°$.
The sending end three-phase complex power is found from $S_{3\Phi,S} = 3V_s{I_S}^*$,

$$S_{3\Phi,S} = 3 \times 40707.94\angle\ 3.91° \times 631.48\angle\ 36.87° = 58.39 + j50.37\ MVA$$

Three-phase real power at sending end $P_{3\Phi,S} = 58.39\ MW$. The transmission efficiency is found below.

$$\eta = \frac{P_{R,3\Phi}}{P_{S,3\Phi}} \times 100\% = \frac{56}{58.39} \times 100\% = 95.91\ \%$$

5.

The nominal pi-circuit is a medium-length line model, which is shown in Fig. 2.

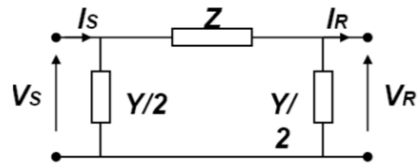

Fig.2

Given per phase series impedance per km and shunt admittance per km, the total impedance and admittances are found.

$$Z = 200 \times (0.08 + \mathrm{j}0.48\ ) = 16 + j96\ \Omega$$
$$Y = 200 \times (\mathrm{j}3.33 \times 10^{-6}) = (\mathrm{j}6.66 \times 10^{-4})\text{siemens}$$

The ABCD parameters in the medium-length model can be found from below.

$A = D = \frac{ZY}{2} + 1 = 0.9680 + j0.0053$

$B = Z = 16 + j96\ \ \Omega$

$C = Y\left(1 + \frac{ZY}{4}\right) = (-1.7742 \times 10^{-6} + j6.5535 \times 10^{-4}\ )S$

At full load, the line delivers 250 MW at 0.99 p.f. lagging at 220 kV (line-to-line). We first find the receiving end voltage per phase.

$$V_R = \frac{220 \times 10^3}{\sqrt{3}} \angle 0° = 127.02\angle 0°\ kV.$$

The receiving end current, I_R, can be found from the complex power consumed by the load at the receiving end, denoted by $P_{3\Phi,\mathrm{R}}$, of 250 MW, 0.99 lagging.

$$|I_R| = \frac{|P_{3\Phi,\mathrm{R}}|}{3|V_R| \times \mathrm{p.f.}} = \frac{250 \times 10^6}{3 \times 127.02 \times 10^3 \times 0.99} = 662.69\ A.$$

The angle of receiving end current is found from power factor. The angle is negative because power factor is lagging.

$$\angle I_R = -\cos^{-1}(0.99) = -8.11°$$

Using ABCD parameters found earlier, together with receiving end voltage and current, the sending end voltage and current is found.

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 155.40\angle 23.58°\ kV \\ 635.38\angle -0.34°\ A \end{bmatrix}$$

6.

This is a medium length line as shown in Fig. 2.

Given per phase series impedance per km and shunt admittance per km, the total impedance and admittances are found.

$$Z = 130 \times (0.036 + \mathrm{j} \times 100\,\Pi\ \times 0.8 \times 10^{-3}) = 4.68 + j32.67\ \Omega$$

$$Y = 130 \times (\mathrm{j} \times 100\,\Pi\ \times 0.0112\ \times 10^{-6}) = (\mathrm{j}4.57 \times 10^{-4})\ \text{Siemens}$$

The ABCD parameters in the medium-length model are found below.

$A = D = \frac{ZY}{2} + 1 = 0.9925 + j0.0011$

$B = Z = 4.68 + j32.67$

$C = Y\left(1 + \frac{ZY}{4}\right) = (-0.2448 \times 10^{-8} + j0.4557 \times 10^{-5})$

The receiving end load is 270 MVA with 0.8 lagging power factor at 325 kV. $V_R = \frac{325}{\sqrt{3}}\angle 0°$ kV,

Find the receiving end current, I_R,

$$|I_R| = \frac{|S_{3\Phi,\mathrm{R}}|}{3|V_R|} = \frac{270 \times 10^6}{3 \times 187.64 \times 10^3} = 479.65\ A.$$

<u>When power factor is 0.8, lagging.</u>

$I_R = |I_R|\angle - cos^{-1}0.8 = 479.64\angle - 36.87°A$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 187.64 \times 10^3\angle 0° \\ 479.64\angle - 36.87° \end{bmatrix} = \begin{bmatrix} 197764\angle 3.30°\ V \\ 430.27\angle - 27.66°\ A \end{bmatrix}$$

At full load, $|V_{R,FL}| = 187.64\ kV$.

At no load, $I_R = 0$, $|V_{R,NL}| = \left|\frac{V_S}{A}\right| = \left|\frac{197764\angle 3.30°}{(0.9925 + j0.0011)}\right| = 199.25\ kV$

$$\%\ Regulation = \frac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|} \times 100\% = \frac{199.25 - 187.64}{187.64} \times 100\% = 6.19\%.$$

Transmission line efficiency:

$$S_{3\Phi,S} = 3V_s I_s^* = 3 \times 197764\angle 3.30° \times 430.26\angle - 27.66° = 218.9 + j131.3\ MVA.$$

$$\eta = \frac{P_{R,3\Phi}}{P_{S,3\Phi}} \times 100\% = \frac{270 \times 0.8}{218.9} \times 100\% = 98.7\%$$

<u>When power factor is 0.95, lagging.</u>

$I_R = |I_R|\angle - cos^{-1}p.f. = 479.64\angle - 18.19°A$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 187.64 \times 10^3\angle 0° \\ 479.64\angle - 18.19° \end{bmatrix} = \begin{bmatrix} 193796\angle 4.26°\ V \\ 456.70\angle - 7.88°\ A \end{bmatrix}$$

At full load, $|V_{R,FL}| = 187.64\ kV$.

At no load, $I_R = 0$, $|V_{R,NL}| = \left|\frac{V_S}{A}\right| = \left|\frac{193796\angle 4.26°}{(0.9925 + j0.0011)}\right| = 195.26\ kV$

$$\%\ Regulation = \frac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|} \times 100\% = \frac{195.26 - 187.64}{187.64} \times 100\% = 4.06\%.$$

Transmission line efficiency:

$$S_{3\Phi,S} = 3V_s I_s^* = 3 \times 193796\angle 4.26° \times 456.70\angle 7.88° = 259.58 + j55.84\ MVA.$$

$$\eta = \frac{P_{R,3\Phi}}{P_{S,3\Phi}} \times 100\% = \frac{270 \times 0.95}{259.58} \times 100\% = 98.8\%$$

When the power factor is close to one, the transmission line efficiency and voltage regulation is improved.