9.2 Line Conductors

Tuesday, 15 March 2022 2:37 PM

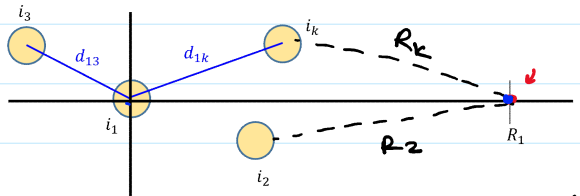
★ **Multiple conductors**

→ K conductors → i_K

→ Each conductor's flux linkage
- ↳ own current
- ↳ current in all other conductors

d_{1K} → distance between conductors 1 & K

i_K → current in conductor K



conductor 2 effect on 1

$$\lambda_1 = \frac{\mu_0}{2\pi} i_1 \ln \frac{R_1}{r_1'} + \frac{\mu_0}{2\pi} i_2 \ln \frac{R_2}{d_{12}} + \cdots$$

$$+ \cdots + \frac{\mu_0}{2\pi} i_K \ln \frac{R_K}{d_{1K}}$$

conductor K effect on 1

$$\lambda_1 = \frac{\mu_0}{2\pi}\left[i_1 \ln \frac{R_1}{r_1'} + i_2 \ln \frac{R_2}{d_{12}} + \cdots + i_K \ln \frac{R_K}{d_{1K}} \right]$$

$$\lambda_1 = \frac{\mu_0}{2\pi}\left[i_1 \ln \frac{1}{r_1'} + i_2 \ln \frac{1}{d_{12}} + \cdots + i_K \ln \frac{1}{d_{1K}} \right]$$
$$+ \frac{\mu_0}{2\pi}\left[i_1 \ln R_1 + i_2 \ln R_2 + \cdots + i_K \ln R_K \right]$$

As $R_1 \to \infty \Rightarrow R_1 = R_2 = R_3 \cdots = R_K \to \infty$

$$\lambda_1 = \frac{\mu_0}{2\pi}\left[i_1 \ln \frac{1}{r_1'} + i_2 \ln \frac{1}{d_{12}} + \cdots + i_K \ln \frac{1}{d_{1K}} \right]$$

$$+ \frac{\mu_0}{2\pi} \ln R_1 \left[i_1 + i_2 + \cdots + i_K \right]$$

For a balanced system

$i_1 + i_2 + \cdots + i_K = 0$

$$\lambda_1 = \frac{\mu_0}{2\pi}\left[i_1 \ln\frac{1}{r_1'} + i_2 \ln\frac{1}{d_{12}} + \dots + i_k \ln\frac{1}{d_{1k}} \right]$$

$\rightarrow$ Inductance for conductor 1 $L = \frac{\lambda}{I}$

$$\lambda = IL$$

$$\lambda_1 = \underbrace{i_1 \cdot \frac{\mu_0}{2\pi} \ln\frac{1}{r_1'}}_{\text{Flux linkage of conductor 1 due to conductor 1}} + i_2 \frac{\mu_0}{2\pi} \ln\frac{1}{d_{12}} + \dots + \underbrace{i_k \frac{\mu_0}{2\pi} \ln\frac{1}{d_{1k}}}_{\text{Flux linkage of 1 due to conductor k}}$$

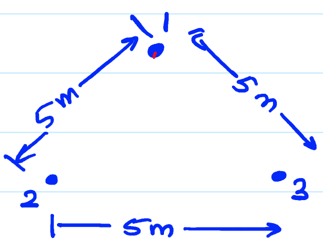
$$\lambda_1 = i_1 \underset{\text{Self inductance}}{L_{11}} + i_2 L_{12} + \dots + i_k \underset{\text{Mutual Inductance}}{L_{1k}}$$

$L_{1k} = \frac{\mu_0}{2\pi} \ln\frac{1}{d_{1k}}$ $\qquad$ $L_{11} = \frac{\mu_0}{2\pi} \ln\frac{1}{r_1'}$

eg Per phase reactance - 3ϕ, 50Hz.

$r = 1.24$ cm,

length $= 5$ km,



$i_1 + i_2 + i_3 = 0$

[Balanced system]

$i_2 + i_3 = -i_1$

$$r_1' = r \cdot e^{-\frac{\mu_r}{4}} = r \cdot e^{-1/4} = 0.967 \text{ cm}$$

$$d_{12} = d_{13} = 5\text{m}$$

$$\lambda_1 = \frac{\mu_0}{2\pi}\left[i_1 \ln\frac{1}{r_1'} + i_2 \ln\frac{1}{d_{12}} + i_3 \ln\frac{1}{d_{13}} \right]$$

$$= \frac{\mu_0}{2\pi}\left[i_1 \ln\frac{1}{r_1'} + i_2 \ln\frac{1}{5} + i_3 \ln\frac{1}{5} \right]$$

$$= \frac{\mu_0}{2\pi}\left[i_1 \ln\frac{1}{r_1'} + \ln\frac{1}{5}(i_2 + i_3) \right]$$

$$= \frac{\mu_0}{2\pi}\left[i_1 \ln\frac{1}{r_1'} + \ln\frac{1}{5}(-i_1) \right]$$

$$= \frac{\mu_0}{2\pi}\left[i_1 \left(\ln \frac{1}{r_1'} - \ln \frac{1}{5} \right) \right]$$

$$= \frac{\mu_0}{2\pi} i_1 \left(\ln \frac{5}{r_1'} \right)$$

$$L_1 = \frac{\lambda_1}{i_1} = \frac{\mu_0}{2\pi}\left(\ln \frac{5}{r_1'} \right)$$

$$L_1 = \frac{\mu_0}{2\pi}\left(\ln \frac{5}{0.967 \times 10^{-2}} \right)$$

$L_1 = 1.25 \times 10^{-6}$ H/m

→ Reactance per m

$X_1 = 2\pi f \times L_1$

$= 2\pi \times 50 \times 1.25 \times 10^{-6}$ Ω/m

$= 3.93 \times 10^{-4}$ Ω/m

Total $X_1 = X_1 \times$ length

$= 3.93 \times 10^{-4} \times 5 \times 10^{3}$

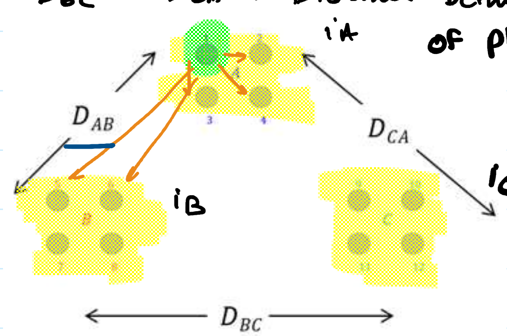
$= 1.965$ Ω. ←

Total reactance per phase

★ Bundled conductors.

→ d_{1k} → Distance between conductor 1 & k

D_{AB} → D_{BC} → D_{CA} → Distance between the bundles of phases



$$\lambda_1 = \frac{\mu_0}{2\pi}\left[\frac{i_A}{4} \ln \frac{1}{r_1'} + \frac{i_A}{4} \ln \frac{1}{d_{12}} + \frac{i_A}{4} \ln \frac{1}{d_{13}} + \frac{i_A}{4} \ln \frac{1}{d_{14}} \right]$$

$$+ \frac{\mu_0}{2\pi}\left[\frac{i_B}{4} \ln \frac{1}{d_{15}} + \frac{i_B}{4} \ln \frac{1}{d_{16}} + \frac{i_B}{4} \ln \frac{1}{d_{17}} + \frac{i_B}{4} \ln \frac{1}{d_{18}} \right]$$

$$\frac{}{2\pi}\left[\frac{}{4} \quad \frac{}{r_1'} \quad \frac{}{4} \quad \frac{}{d_{12}} \quad \frac{}{4} \quad \frac{}{d_{13}} \quad \frac{}{4} \quad \frac{}{d_{14}}\right]$$

$$+\frac{\mu_0}{2\pi}\left[\frac{i_B}{4}\ln\frac{1}{d_{15}}+\frac{i_B}{4}\ln\frac{1}{d_{16}}+\frac{i_B}{4}\ln\frac{1}{d_{17}}+\frac{i_B}{4}\ln\frac{1}{d_{18}}\right]$$

$$+\frac{\mu_0}{2\pi}\left[\frac{i_C}{4}\ln\frac{1}{d_{19}}+\frac{i_C}{4}\ln\frac{1}{d_{1,10}}+\frac{i_C}{4}\ln\frac{1}{d_{1,11}}+\frac{i_C}{4}\ln\frac{1}{d_{1,12}}\right]$$

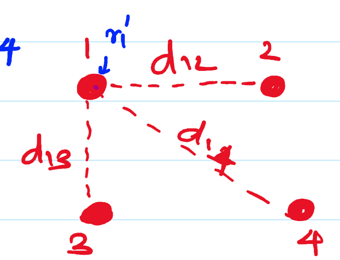
$$\lambda_1=\frac{\mu_0}{2\pi}\frac{i_A}{4}\left[\ln\frac{1}{r_1'}+\ln\frac{1}{d_{12}}+\ln\frac{1}{d_{13}}+\ln\frac{1}{d_{14}}\right]$$
$$+\frac{\mu_0}{2\pi}\frac{i_B}{4}\left[\ln\frac{1}{d_{15}}+\ln\frac{1}{d_{16}}+\ln\frac{1}{d_{17}}+\ln\frac{1}{d_{18}}\right]$$
$$+\frac{\mu_0}{2\pi}\frac{i_C}{4}\left[\ln\frac{1}{d_{19}}+\ln\frac{1}{d_{1,10}}+\ln\frac{1}{d_{1,11}}+\ln\frac{1}{d_{1,12}}\right]$$

$$\lambda_1=\frac{\mu_0}{2\pi}\left[i_A\cdot\frac{1}{4}\ln\frac{1}{r_1'd_{12}d_{13}d_{14}}+i_B\cdot\frac{1}{4}\ln\frac{1}{d_{15}d_{16}d_{17}d_{18}}\right.$$
$$\left.+i_C\cdot\frac{1}{4}\ln\frac{1}{d_{19}d_{1,10},d_{1,11},d_{1,12}}\right]$$

$$\lambda_1=\frac{\mu_0}{2\pi}\left[i_A\ln\frac{1}{(r_1'd_{12}d_{13}d_{14})^{1/4}}\right.$$
$$+i_B\ln\frac{1}{(d_{15}d_{16}d_{17}d_{18})^{1/4}}$$
$$+i_C\ln\frac{1}{(d_{1,9}d_{1,10}d_{1,11}d_{1,12})^{1/4}}$$

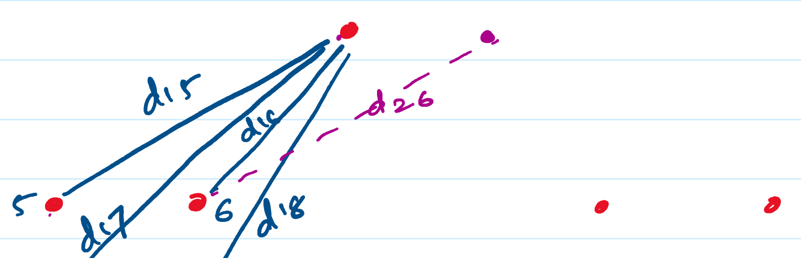
$(r_1'd_{12}d_{13}d_{14})^{1/4}$

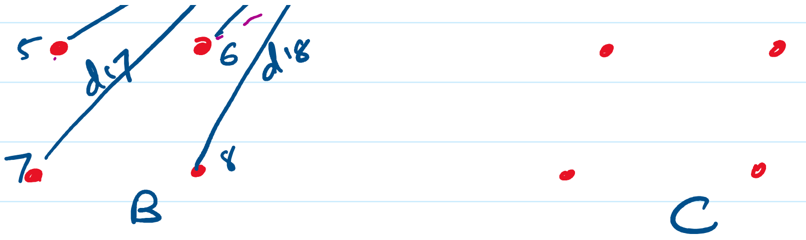
r_1' → Phase A



$(r_2'd_{21}d_{24}d_{23})^{1/4}$

Geometrical Mean Radius [GMR]
→ R_n





$(d_{15} d_{16} d_{17} d_{18})^{1/4}$

→ Geometrical Mean distance GMD

→ D_{AB}

$(d_{19} d_{1,10} d_{1,11} d_{1,12})^{1/4} = D_{AC}$

$D_{AB} = D_{AC} = D$

For a balanced system

$$i_A + i_B + i_C = 0 \Rightarrow i_B + i_C = -i_A$$

$$\lambda_1 = \frac{\mu_0}{2\pi}\left[i_A \ln\frac{1}{R_n} + i_B \ln\frac{1}{D} + i_C \ln\frac{1}{D} \right]$$

$$= \frac{\mu_0}{2\pi}\left[i_A \ln\frac{1}{R_n} + \ln\frac{1}{D}\left(i_B + i_C\right) \right]$$

$$\lambda_1 = \frac{\mu_0}{2\pi}\left[i_A \ln\frac{1}{R_n} - i_A \ln\frac{1}{D} \right]$$

$$\lambda_1 = \frac{\mu_0}{2\pi} i_A \ln\left|\frac{\overset{\text{GMD}}{D}}{\underset{\text{GMR}}{R_n}}\right|$$

$$i_1 = \frac{i_A}{4} \Rightarrow i_A = 4 i_1$$

Inductance $\frac{\lambda_1}{i_1} = \frac{\lambda_1}{\frac{i_A}{4}}$

$$L_1 = 4 \cdot \frac{\mu_0}{2\pi} \ln\left|\frac{D}{R_n}\right|$$

$$L_a = \frac{L_1}{n} = \frac{4\frac{\mu_0}{2\pi} \ln\left|\frac{D}{R_n}\right|}{4}$$

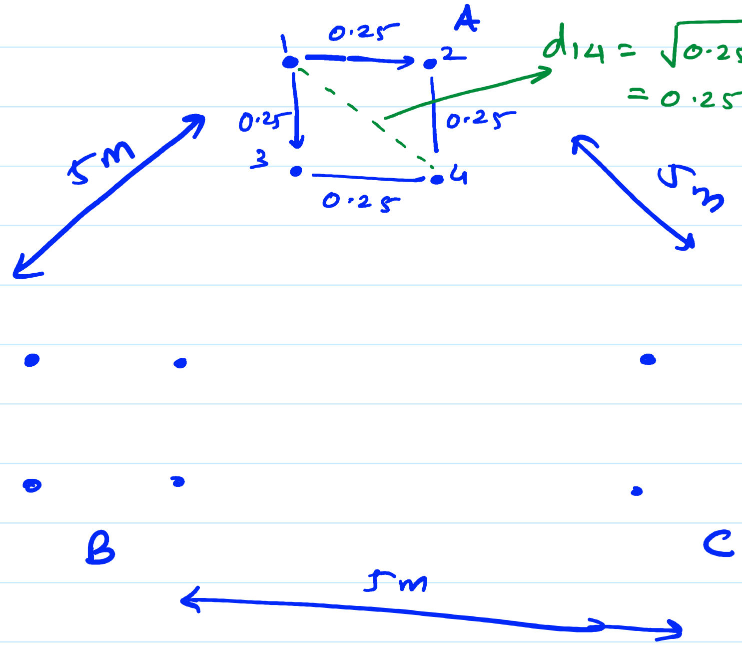
if $n = 4$ as in this case.

$L_a = \mu_0 \ln \left| D \right|$

$$L_a = \frac{\mu_0}{2\pi} \ln\left|\frac{D}{R_n}\right|$$ if n = 4 as in this's case.

eg 3ϕ , $D = 5m$. $d_{12} = d_{13} = 0.25$

$d_{14} = \sqrt{0.25^2 + 0.25^2}$
$= 0.25\sqrt{2}$



Find inductance per meter

$r = 1.24\,cm \rightarrow r' = 1.24 \times e^{-1/4} = r.e^{-\frac{\mu_r}{4}}$
$= 0.967\,cm$
$= 9.67 \times 10^{-3}\,m$

$L_a = \frac{\mu_0}{2\pi} \ln\left|\frac{D}{R_n}\right|$

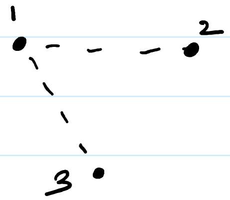
$R_n = (r'.d_{12}.d_{13}.d_{14})^{1/4}$

$= \sqrt[4]{\underset{r_1'}{9.67 \times 10^{-3}} \times \underset{d_{12}}{0.25} \times \underset{d_{13}}{0.25} \times \underset{d_{14}}{0.25\sqrt{2}}}$

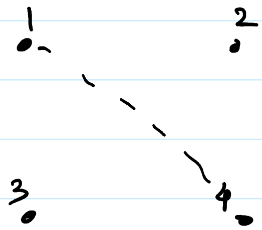
$R_n = 0.12\,m$,

$L_a = \frac{\mu_0}{2\pi} \ln \frac{5}{0.12} = 7.46 \times 10^{-7}\,H/m$

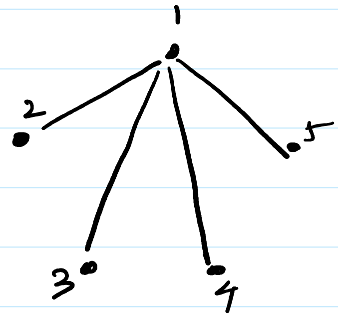
Previous case $L_1 = 1.25 \times 10^{-6}\,H/m$.



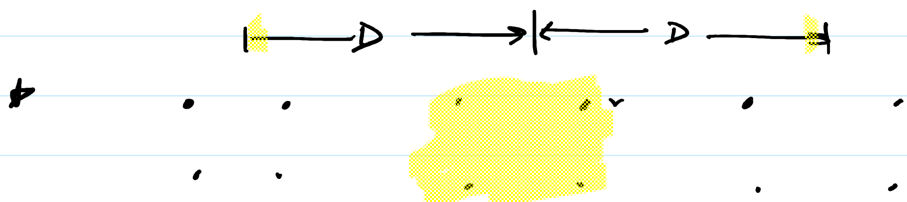
$$GMR = \sqrt[3]{r_1' d_{12} d_{13}}$$



$$GMR = \sqrt[4]{r_1' d_{12} d_{13} d_{14}}$$

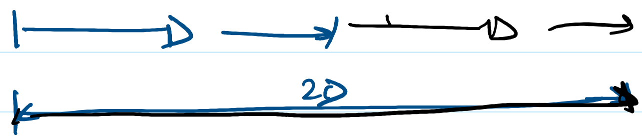


$$GMR = \sqrt[5]{r_1' d_{12} d_{15} d_{13} d_{14}}$$



A B C

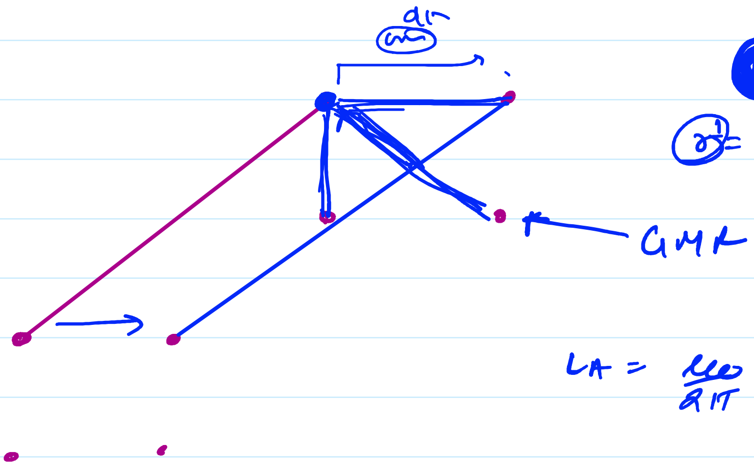
L_A L_B



$$GMD = \sqrt[3]{D \cdot D \cdot 2D}$$

$$i_B \ln \frac{1}{D} + i_C \ln \frac{1}{2D}$$

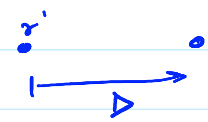
$$i_B \ln \frac{1}{D} + i_C \ln \frac{1}{D}$$

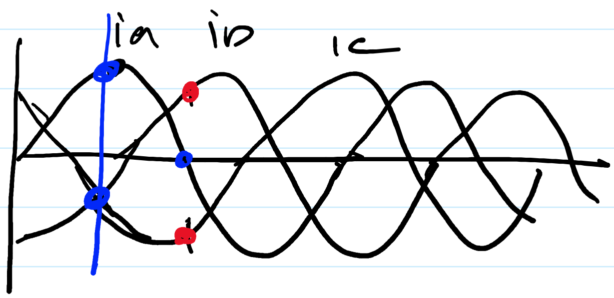


$$r' = r \cdot e^{-1/4}$$

$$L_A = \frac{\mu_0}{2\pi} \ln\left|\frac{GMD}{GMR}\right|$$

$$= \frac{\mu_0}{2\pi} \ln\left|\frac{D}{r'}\right|$$





$i_a + i_a \angle 180°$

$i_a - i_a$

$i_a + i_b + i_c = 0$

$$i_a + \frac{-i_a}{2} + \frac{-i_a}{2} = 0$$

$i_a \quad i_1 \rightarrow |i_1| \angle \phi \;, +$

$-i_1$