EE2029 Introduction to Electrical Energy Systems

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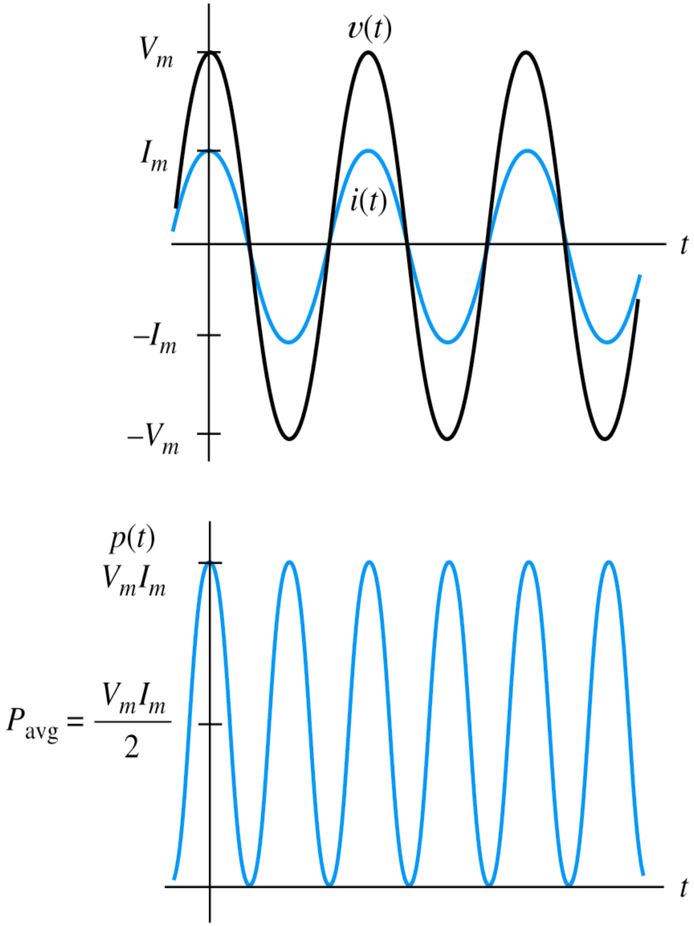
AC Power

DC Power vs AC power

- Electrical power, $P = VI$
- In DC circuits, both voltage and current are constant. Hence DC power is constant.
- However in AC circuits, both voltage and current keep changing with time. So the power also keeps changing with time: $p(t) = v(t) \times i(t)$.
- AC Power can be both positive as well as negative. Positive power means the source is supplying power to the load. Negative power means the load is returning power to the source.
- The average power is important as it indicates the actual work being done as in case of a motor moving a loaded conveyor belt.

AC power in a Resistor



Current, voltage, and power versus time for a purely resistive load.

v(t) = V_m Cos ωt

$i(t) = \frac{V_m}{R} \cos \omega t = I_m \cos \omega t$

Instantaneous power: *p(t)* = $V_m I_m \cos^2 \omega t$

The instantaneous power in the resistor is square of the cosine function, hence always positive.

Average power in the resistor, $P_{avg} = \frac{1}{T}\int_0^T p(t)dt$

$$P_{avg} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = V_{rms} I_{rms}$$

Example

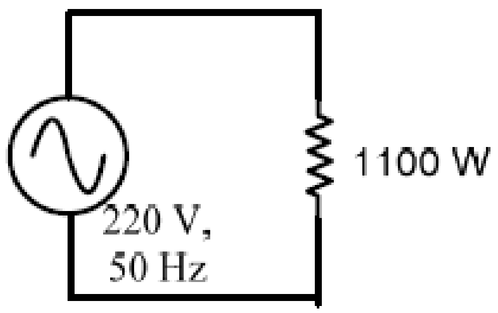
A resistive load is rated as 1100W, 220V, 50 Hz. Calculate the value of load resistance and the amount of current that the load draws from the source.

$$P_{R,avg} = \frac{V_{rms}^2}{R}$$

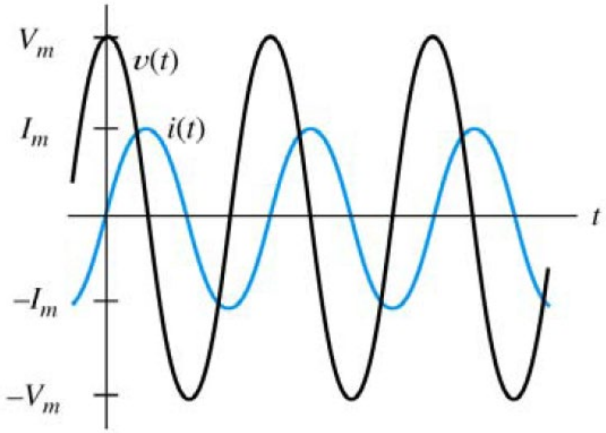
$$1100 = \frac{V_{rms}^2}{R} = \frac{220^2}{R}$$

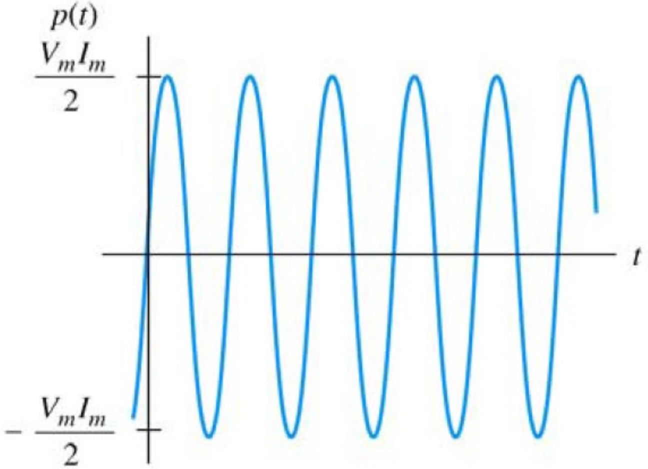
$$R = \frac{220 \times 220}{1100} = 44\ \Omega$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{220}{44} = 5\ \text{A}$$



AC Power in an Inductor





In inductor, the current lags the voltage by 90^o.

$$v(t) = V_m \cos(\omega t) \qquad i(t) = I_m \cos(\omega t - 90°) = I_m \sin(\omega t)$$

The instantaneous power: $$p(t) = V_m I_m \cos(\omega t)\sin(\omega t)$$

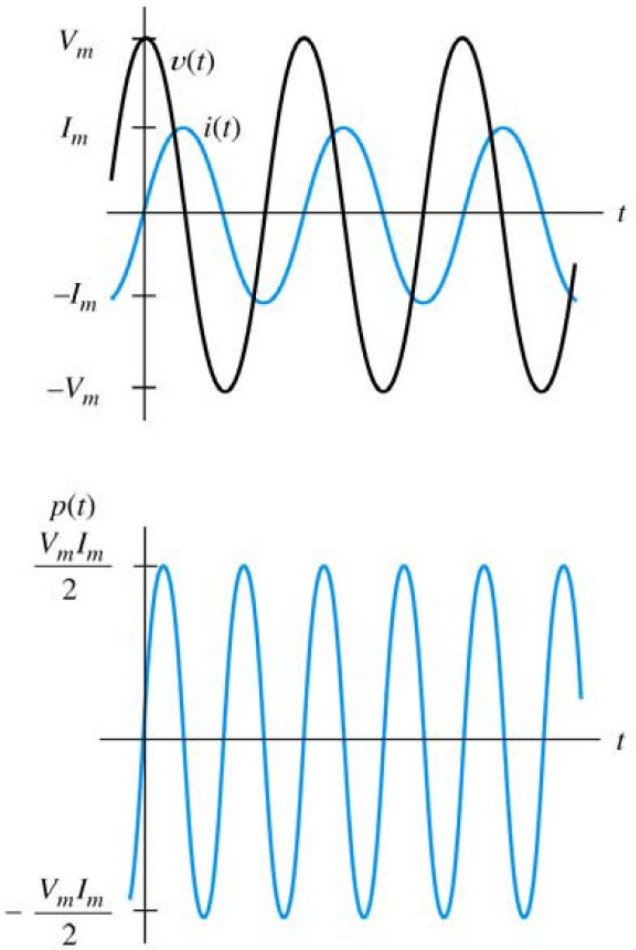
$$= \frac{V_m I_m}{2} \sin(2\omega t)$$

$$= V_{rms} I_{rms} \sin(2\omega t)$$

Instantaneous power in an inductor is sinusoidal with a frequency twice that of the frequency of voltage and current.

The average power for inductor, $P_{avg} = 0.$

AC Power in a Capacitor



In capacitor, the current leads the voltage by 90^o.

$v(t) = V_m \cos(\omega t)$ $\quad$ $i(t) = I_m \cos(\omega t + 90^o) = -I_m \sin(\omega t)$

The instantaneous power: $p(t) = -V_m I_m \cos(\omega t)\sin(\omega t)$

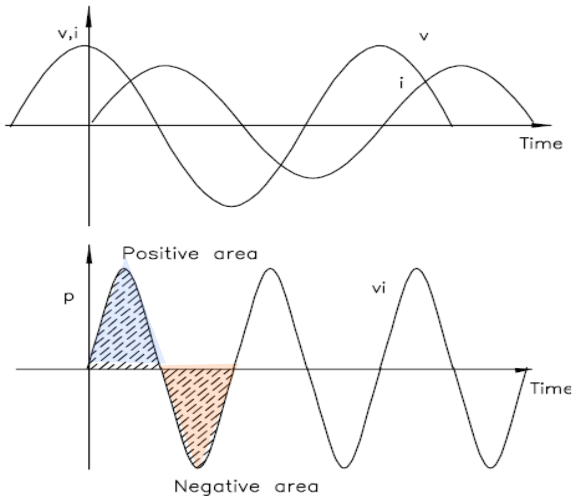
$$= -\frac{V_m I_m}{2}\sin(2\omega t)$$

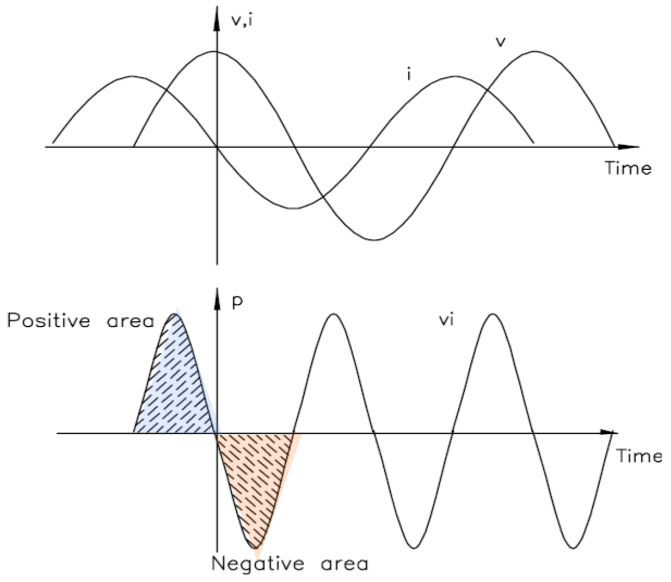
$$= -V_{rms} I_{rms} \sin(2\omega t)$$

Instantaneous power in a capacitor is sinusoidal with a frequency twice that of the frequency of voltage and current. It is of opposite sign as compared to an inductor.

The average power for capacitor, $P_{avg} = 0$.

Power in Inductor and Capacitor

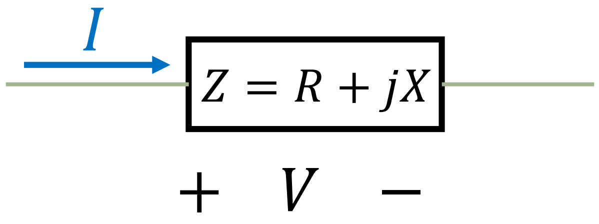


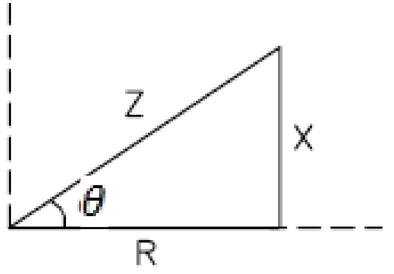


Both inductor and capacitor store and release energy during the AC cycle. When both the voltage and current are of the same sign, the element (L or C) draws energy equivalent to the area under the positive half cycle of p(t) from the source and stores it. When they are of opposite signs, it is returning the energy to the source.

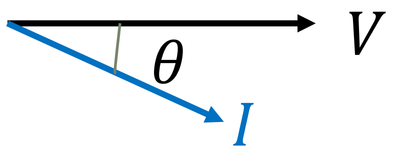
Real, Reactive and Apparent Power

Power in general AC load





- A typical AC load can be considered as a series combination of a resistive part and reactive part (inductive or capacitive).



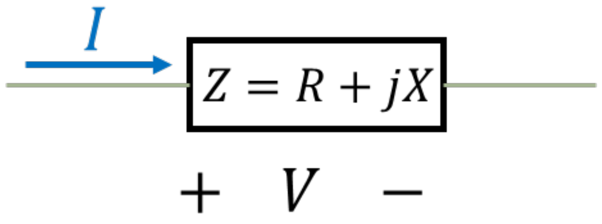
- Load impedance $Z = R + jX = \sqrt{R^2 + X^2}\,\angle\theta$
- Current will lag voltage by the angle θ: $\mathrm{I} = \frac{V}{\sqrt{R^2+X^2}} \angle -\theta$
- Average power is equal to the power in the resistive part: $P_{avg} = I^2 R$

$$P_{avg} = \frac{V^2}{R^2 + X^2} R = V \frac{V}{\sqrt{R^2 + X^2}} \frac{R}{\sqrt{R^2 + X^2}} = V\ I \cos\theta$$

- We shall see that power associated with the reactive component, X

$$Q = I^2 X = \frac{V^2}{R^2 + X^2} X = V \frac{V}{\sqrt{R^2 + X^2}} \frac{X}{\sqrt{R^2 + X^2}} = V\ I\ sin\,\theta$$

Real power and Reactive power



- In a general AC load, the average power (I^2R) is associated with the resistive element (R). This power is also known as real power (P) and it is in watts.

- The average power associated with reactive element (X) is zero. However, I^2X is known as reactive power (Q) and its unit is VAr.

Apparent Power

- We express average power in d.c. and a.c. circuits as follows:

$$P_{dc} = V_{dc}I_{dc} \text{ Watts}$$

$$P_{ac} = V_{rms}\, I_{rms} \cos\theta \text{ Watts}$$

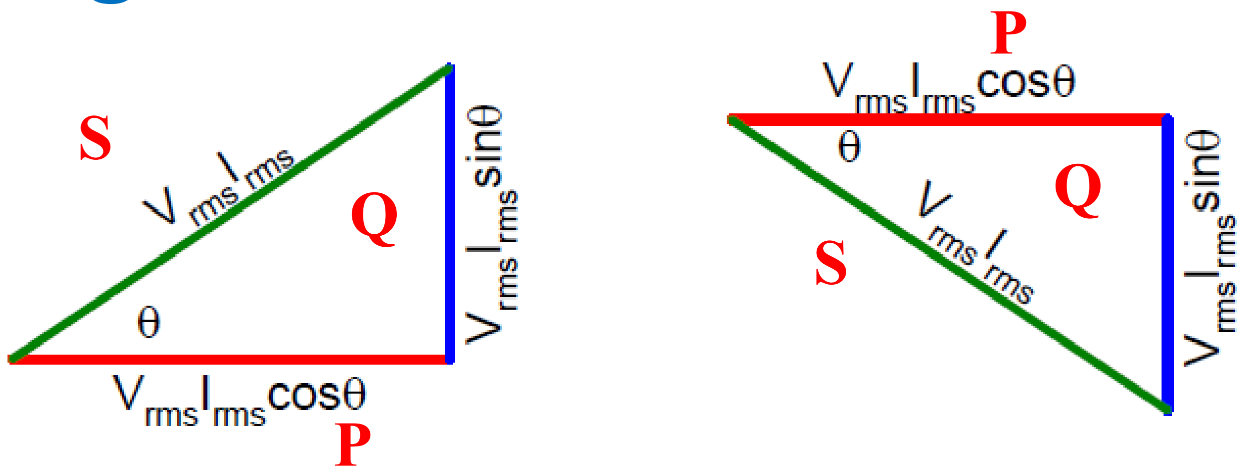
- In d.c. circuit, the product ($V_{dc}\, I_{dc}$) is the real power expressed in watts.
- But in a.c. circuits, an additional term cos θ is multiplied to obtain average power.
- So in a.c. circuits, the product ($V_{rms}\, I_{rms}$) is known as Apparent Power, i.e.

$$|S| = V_{rms}\, I_{rms} \quad \text{with unit of VA (volt-amperes).}$$

- Average power in a.c. circuit may also be expressed in terms of apparent power as follows:

$$P = |S| \cos\theta$$

Power Triangle



(a) Inductive load, (b) Capacitive load

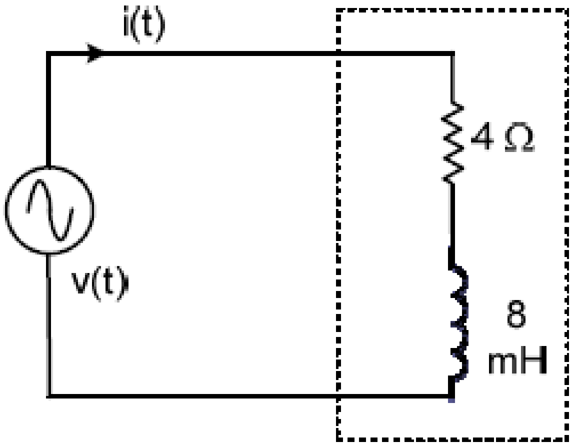
P – Real (or active) power; power consumed in the resistive part of the circuit

Q – Reactive power consumed by the device, due to inductor or capacitor

Reactive power is important because transmission lines, transformers, and other elements must be able to withstand the current associated with reactive power.

Example

Compute the instantaneous, average, real and reactive powers in the following circuit if *v(t)=14.14 sin (377t)*



$\omega = 377$ rad/s

$Z = R + j\omega L = 4 + j3\ \Omega = 5\angle 36.9°$

$X_L = \omega L = 3\ \Omega$

$$v(t) = 14.14\sin(377t) = 14.14\cos(377t - 90°)$$

$$V = 10\angle -90°$$

$$I = \frac{10\angle -90°}{5\angle 36.9°} = 2\angle -126.9°$$

$$i(t) = 2\sqrt{2}\cos(377t - 126.9°) = 2.28\cos(377t - 126.9°)\ \text{A}$$

Instantaneous power, $p(t) = v(t)i(t) = (14.14\sin(377t)) \times (2.28\cos(377t - 126.9°))$ W

Average power, $P_{avg} = V_{rms} I_{rms} \cos\theta = 10 \times 2 \times \cos(36.9°) = 16$ W

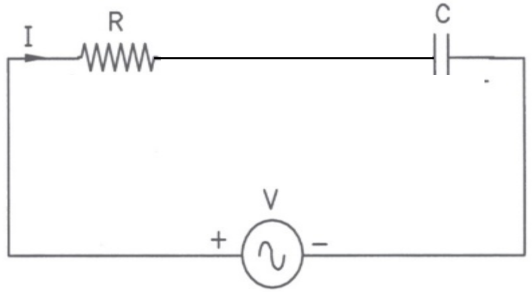
Real power, $P = I_{rms}^2 R = 2^2 \times 4 = 16$ W

Reactive power, $Q = I_{rms}^2 X = 2^2 \times 3 = 12$ VAR

Example

a) A resistive lamp is rated as 200 V, 120 W. (a) Calculate the current drawn by the lamp when connected across 200V supply.

b) We now want to use this lamp with a 240 V, 50 Hz voltage source. A capacitor bank is connected in series with the lamp so that the voltage across lamp is reduced to the rated 200 V. What is the value of capacitor to be used?



$$P = V_{rms} I_{rms}$$

$$I_{rms} = \frac{120\ W}{200\ V} = 0.6\ \ A$$

$$R_{lamp} = \frac{200\ V}{0.6\ A} = 333.33\ \Omega$$

With the capacitor connected in series, the total impedance is $Z = R_{lamp} - jX_C$

$$|Z| = \frac{240\ V}{0.6\ A} = 400\ \Omega$$

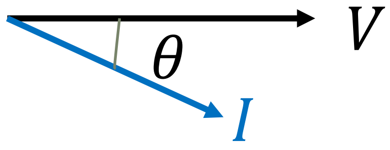
$$Z = \sqrt{R_{lamp}^2 + X_C^2} \quad \Rightarrow \quad X_C^2 = Z^2 - R_{lamp}^2$$

$$X_C = \frac{1}{\omega C} \quad \Rightarrow \quad C = \frac{1}{\omega X_C} = \frac{1}{(2\pi \times 50)(221.11)} = 14.4\ \mu\text{F}$$

Complex Power

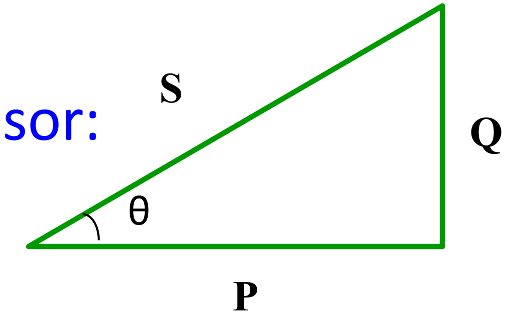
Complex Power

Apparent power, $|S| = V_{rms} I_{rms}$



$$V = V_{rms}\angle 0^o, \mathrm{I} = I_{rms}\angle - \theta$$

Complex power can be written in terms of voltage and current phasor:



Complex power, $S = |S|\angle\theta = V_{rms} I_{rms} \angle \theta$

Defining I* as the complex conjugate of I, $I^* = I_{rms}\angle\theta$

Thus, complex power S = V I*=P+jQ

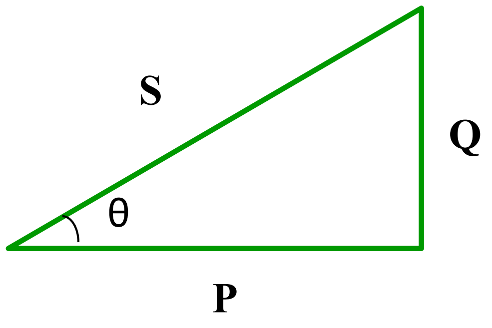
Complex power is useful for computation of real power P and reactive power Q from the voltage and current phasors.

Complex Power for ease of calculation

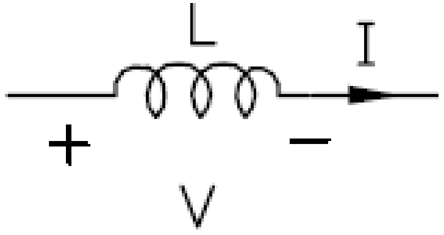
If we have voltage (V) and current phasor (I), we can calculate complex power first. From there, we can obtain apparent power, active power and reactive power easily.

CONJUGATE!!!

S	**Complex power**	VA	$S = \|S\|\angle\theta = P + jQ = VI^* = \|V\|\|I\|\angle\theta = \|V\|\|I\|(\cos\theta + j\sin\theta)$
$\|S\|$	**Apparent power**	VA	$\|S\| = \|V\|\|I\| = \sqrt{P^2 + Q^2}$
P	**Active power** Average power, Real power	W	$P = \mathrm{Re}(S) = \|S\|\cos(\theta) = \|V\|\|I\|\cos(\theta)$
Q	**Reactive power**	var	$Q = \mathrm{Im}(S) = \|S\|\sin(\theta) = \|V\|\|I\|\sin(\theta)$



Power in an Inductor



The impedance of a pure inductor is $Z_L = j\omega L = \omega L \angle 90^\circ$

Taking the voltage V as reference phasor,

$$V = |V| \angle 0^\circ$$

$$I = \frac{V}{Z_L} = \frac{|V|\angle 0^\circ}{j\omega L} = \frac{|V|}{\omega L}\angle -90^\circ = |I| \angle -90^\circ$$

So, the complex power in an inductor is $S_L = P_L + jQ_L = V\, I^*$

$$= Z_L I \times I^* = j\omega L |I|^2$$

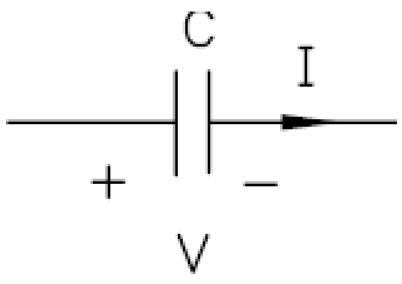
real and reactive power consumed by the inductor

$$P_L = |V||I|\, cos\, 90^\circ = 0$$

$$Q_L = |V||I|\, sin\, 90^\circ = |V||I| = \omega L |I|^2$$

Note: The magnitude of phasor |V| or |I| is the same as its rms value, and is used interchangeably

Power in a Capacitor



The impedance of a capacitor

$$Z_C = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle{-90^\circ}$$

Taking the voltage V as reference phasor, $V = |V|\angle 0^\circ$

$$I = \frac{V}{Z_C} = \omega C|V|\angle 90^\circ = |I|\angle 90^\circ$$

complex power in a capacitor $S_C = P_C + jQ_C = V\,I^*$

$$= Z_C I \times I^* = -j\frac{|I|^2}{\omega C}$$

real and reactive power consumed by the capacitor

$$P_C = |V||I|\,cos\,(-90^\circ) = 0$$

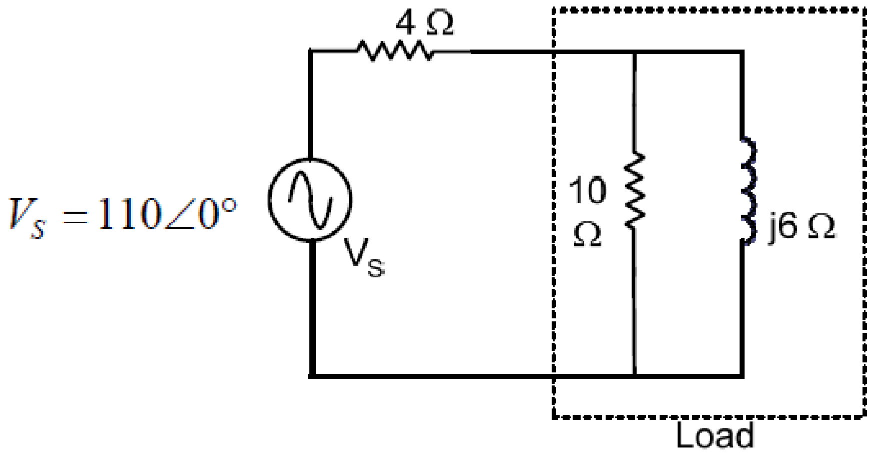
$$Q_C = |V||I|\,sin\,(-90^\circ) = -|V||I| = -\frac{|I|^2}{\omega C} = -\omega C|V|^2$$

Reactive power drawn by a capacitor is negative. So, as a convention, it is assumed that a capacitor generates reactive power while an inductor consumes reactive power.

Example

A voltage source with series resistor is connected to a parallel combination of inductor and resistor. Find the load voltage and load current. Use these values to find complex power and hence real power and reactive power. Also find power delivered to the load.

$$Z_L = (10\Omega) \parallel (j6\Omega) = \frac{10\times j6}{10+j6} = 5.145\angle 59°\Omega$$



Voltage across the load,

$$V_L = \frac{Z_L}{4+Z_L}(110\angle 0°) = \frac{5.145\angle 59°}{4+5.145\angle 59°}(110\angle 0°) = 70.9\angle 25.44°\,\mathrm{V}$$

Current flowing through the load, $I_L = \frac{70.9\angle 25.44°}{5.145\angle 59°} = 13.8\angle -33.6°$

Complex power S = $V_L I^*_L = (70.9\angle 25.44°)(13.8\angle 33.6°) = 978\angle 59°$

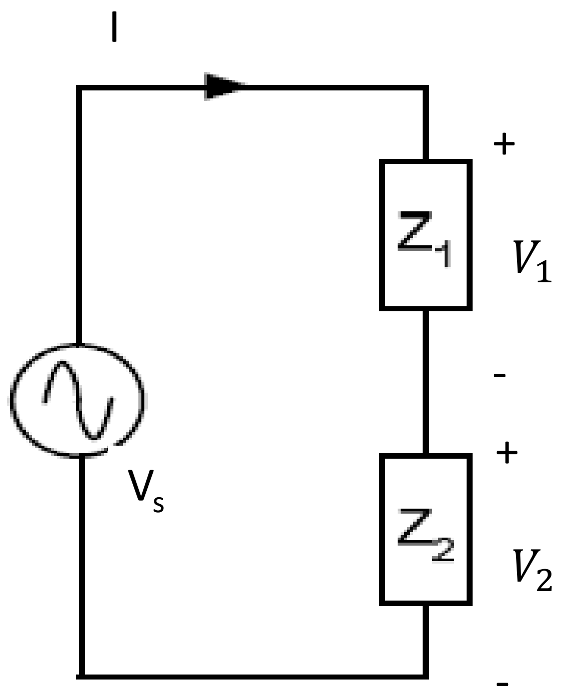
Complex power S = 503 + j839 W

Real power, P = 503 W, Reactive power Q = 839 VAr

Complex power delivered by the source = $V_s I^*_s = (110\angle 0°)(13.8\angle 33.6°) = 1518\angle 33.6°$

$$= 1264 + j838$$

Complex Power of Series and Parallel-connected loads



Series connected load:

If two loads Z_1 and Z_2 are connected in **series** across a voltage source,

For load 1: $S_1 = V_1 I^* = P_1 + jQ_1$

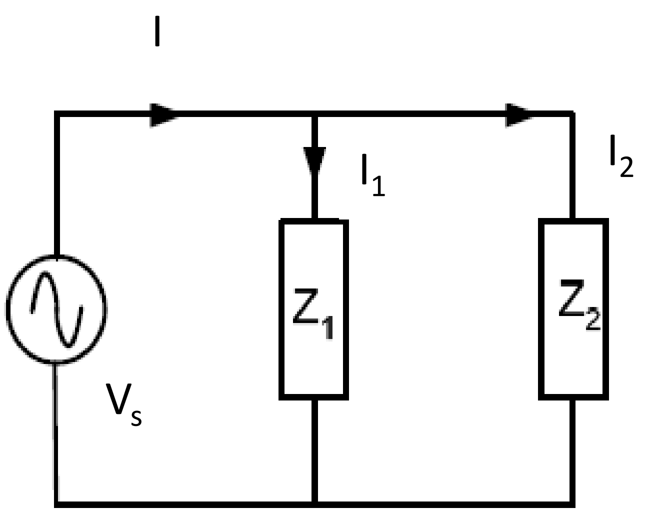
For load 2: $S_2 = V_2 I^* = P_2 + jQ_2$

Total complex power $S = V_s I^* = (V_1 + V_2)\, I^* = V_1 I^* + V_2 I^*$

Or, $S = P_1 + jQ_1 + P_2 + jQ_2 = (P_1 + P_2) + j(Q_1 + Q_2)$

- Complex power of a network of interconnected loads is equal to the sum of complex powers of individual loads.
- Real (or Reactive) power of a network of interconnected loads is equal to the sum of real (or reactive) powers of individual loads.

Complex Power of Series and Parallel-connected loads



Parallel connected load:

If two loads Z_1 and Z_2 are connected in **parallel**,

For load 1: $S_1 = V_s I_1^* = P_1 + jQ_1$

For load 2: $S_2 = V_s I_2^* = P_2 + jQ_2$

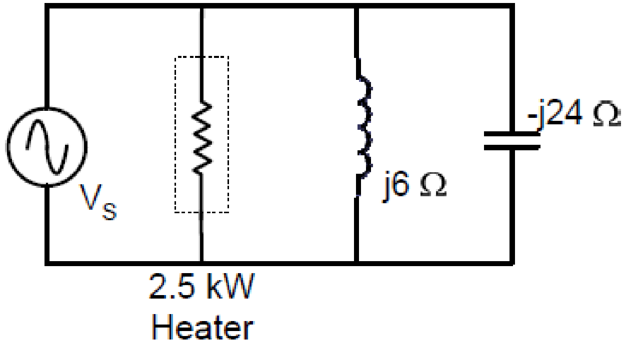
Total complex power $S = V_s I^* = V_s (I_1^* + I_2^*)$

Or, $S = P_1 + jQ_1 + P_2 + jQ_2 = (P_1 + P_2) + j(Q_1 + Q_2)$

- Complex power of a network of interconnected loads is equal to the sum of complex powers of individual loads.
- Real (or Reactive) power of a network of interconnected loads is equal to the sum of real (or reactive) powers of individual loads.

Example

A voltage source supplies power to an electric heater (with resistance R), an inductive element and a capacitor as shown below. The supply voltage is 120 V , 50 Hz and the power consumed in the heater is 2.5 kW. Find the reactive power and complex power provided by the source, and the current *i(t)* drawn from the supply.



Find power in each load –

Heater: $P_H = 2.5$ kW $\quad Q_H = 0$ VAR

Inductor: $P_L = 0$ W $\quad Q_L = \frac{120^2}{6} = 2.4$ kVAR

Capacitor: PC = 0 W $\quad Q_C = -\frac{120^2}{24} = -600$ VAR

Total real power: $P = P_H + P_L + P_C = 2.5$ kW

Total reactive power: $Q = Q_H + Q_L + Q_C = 0 + 2400 - 600 = 1.8$ kVAR

Total complex power: $S = P + jQ = 2500 + j1800$

$$|S| = \sqrt{P^2 + Q^2} = 3081 \text{ VA}$$

$$I_{rms} = \frac{|S|}{V_{rms}} = \frac{3081}{120} = 25.7 \text{ A}$$

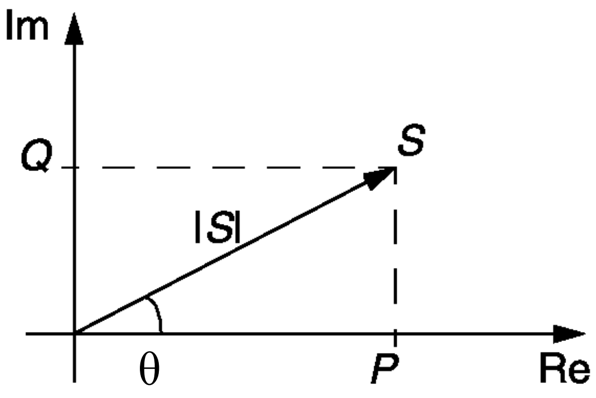
$$i(t) = 25.7\sqrt{2}\cos(314t - 35.75°) \text{ A}$$

Power Factor

Power Factor of an AC circuit

Power Factor of an AC circuit as the ratio of real power to the apparent power.

$$\text{Power Factor} = \frac{P}{|s|}$$

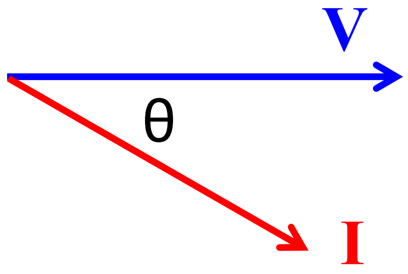


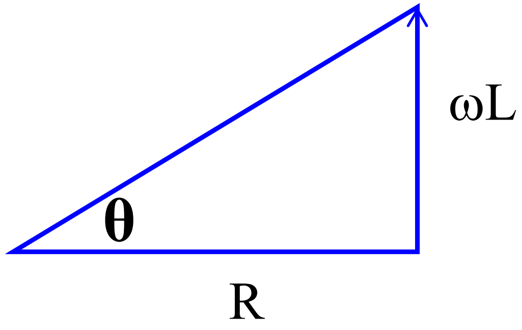
$$P = |S|\cos(\theta) = |V||I|\cos(\theta)$$

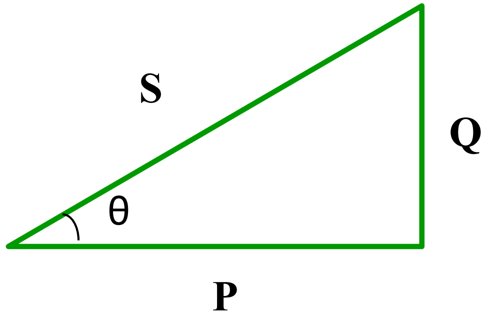
$$\cos(\theta) = \frac{P}{|S|} = \frac{P}{\sqrt{P^2 + Q^2}}$$

$$\cos(\theta) = \cos\left(\tan^{-1}\left(\frac{Q}{P}\right)\right)$$

Power factor angle (θ) is the same as power triangle, impedance triangle, and the angle between voltage and current.

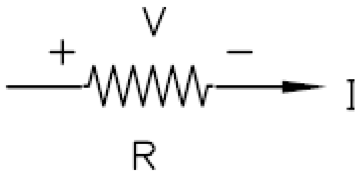




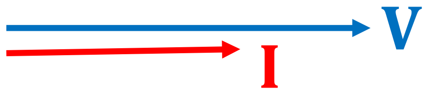


Unity Power Factor

Consider a device that is purely resistive, i.e. Z = R.



Taking voltage V as reference phasor, we have $V = |V|\angle 0^{o}$



Both the voltage and current are in phase. So, the power in a resistor is

$$P = |V|\,|I|\cos 0^{\circ} = |V|\,|I|$$

The power factor of a purely resistive network is unity.

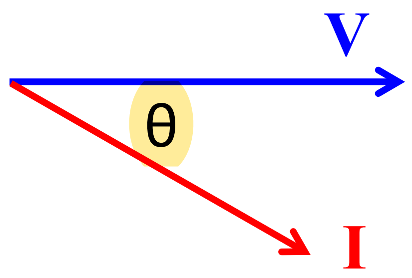
Lagging Power Factor

Consider a load with both resistive and inductive parts, $Z = R + j\omega L = |Z|\angle\theta$

Taking voltage V as reference phasor, $V = |V|\angle 0^o$

Current in the load, $I = \frac{V}{Z} = \frac{|V|\angle 0^o}{|Z|\angle\theta} = \frac{|V|}{|Z|}\angle - \theta = |I|\angle - \theta$

As the impedance angle is positive, the current is lagging behind the voltage as shown in the phasor diagram :



The real power in the load is $P = |V||I|\cos\theta$

Power factor is said to be lagging, if the current lags behind the voltage. In other words, p.f. is lagging when the impedance angle θ is positive, i.e. when the load or device is inductive in nature.

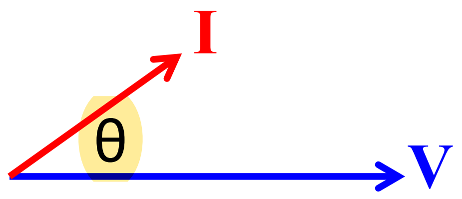
Leading Power Factor

Consider a load with resistive and capacitive component, the load impedance is given by:

$$Z = R - j\frac{1}{\omega C} = R - jX_C = |Z| \angle -\theta$$

Taking the voltage V as reference phasor, $V = |V|\angle 0^0$

Current in the load, $I = \frac{V}{Z} = \frac{|V|\angle 0^o}{|Z|\angle -\theta} = \frac{|V|}{|Z|}\angle\theta = |I|\angle\theta$



As the impedance angle is negative, the current is leading the voltage as shown in the phasor diagram below:

The real power in the device is $P = |V||I|\cos\theta$

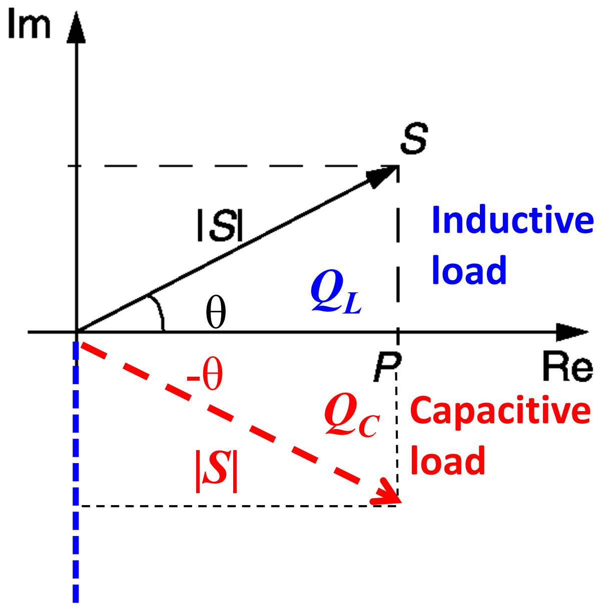
Power factor is said to be leading, if the current leads the voltage. In other words, p.f. is leading when the impedance angle θ is negative, i.e. when the load or device is capacitive in nature.

Power Factor

$$P = |S|\cos(\theta) = |V||I|\cos(\theta)$$

$-90^\circ \le \theta \le 90^\circ$

Hence, power factor lies between: $0 \le$ p.f. ≤ 1



Since cos (+θ) = cos (-θ), we identify the power factor as leading or lagging.

For inductive load: Current lags behind voltage; hence, power factor is lagging p.f.

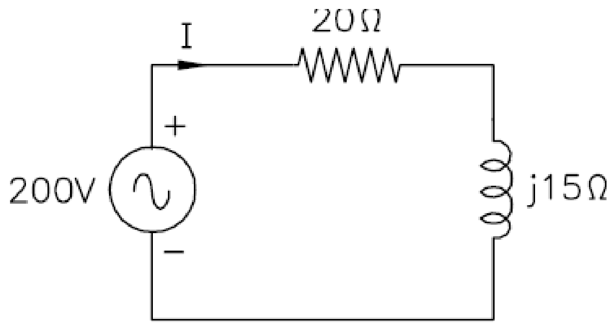
For capacitive load: Current leads the voltage; hence, power factor is leading p.f.

For example: If the load is $10\angle 25^\circ$ the power factor is 0.906 lagging.

If the load is $20\angle -30^\circ$ the power factor is 0.87 leading.

Example

Consider the simple AC circuits shown below. Determine the apparent, real and reactive power delivered by the source. What is the power factor of the circuit?



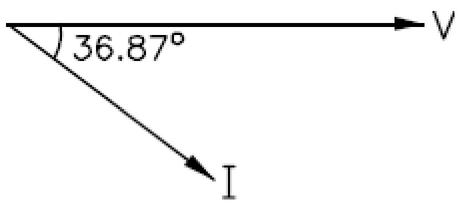
$$V = 200\angle 0^\circ$$

$$Z = 20 + j15 = 25\angle 36.87^\circ$$

$$I = \frac{V}{Z} = \frac{200}{25\angle 36.87^\circ} = 8.0\angle -36.87^\circ \ \text{A}$$

$$S = VI^* = 200 \times 8.0\angle 36.87^\circ$$

$$= 1600\angle 36.87^\circ = 1280 + j960$$



$\text{Apparent Power} = 1600 \ \text{VA}$

$\text{Real Power} = 1280 \ \text{W}$

$\text{Reactive Power} = 960 \ \text{VAr}$

$\text{Power factor} = cos\, 36.87^\circ = 0.8$ lagging

Example

In the parallel R-C circuit shown below, determine the apparent power, real power and reactive power delivered by the source. What is the power factor of the circuit?

$$V = 500\angle 0^\circ$$

$$Y = Y_1 + Y_2 = \frac{1}{40} + \frac{1}{-j80}$$

$$= 0.025 + j0.0125 = 0.02795\angle 26.57^\circ$$

$$I = YV = 13.975\angle 26.57^\circ \ \text{A}$$

$$S = VI^* = 500 \times 13.975\angle -26.57^\circ$$

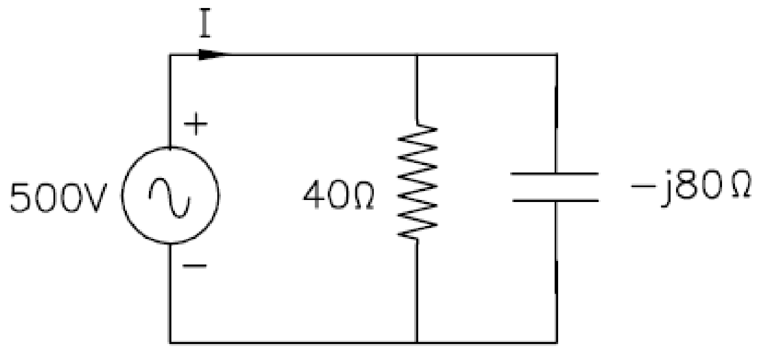
$$= 6987.5\angle -26.57^\circ \quad = 6250 - j3125$$

$$\text{Apparent Power} = 6987.5 \ \text{VA}$$

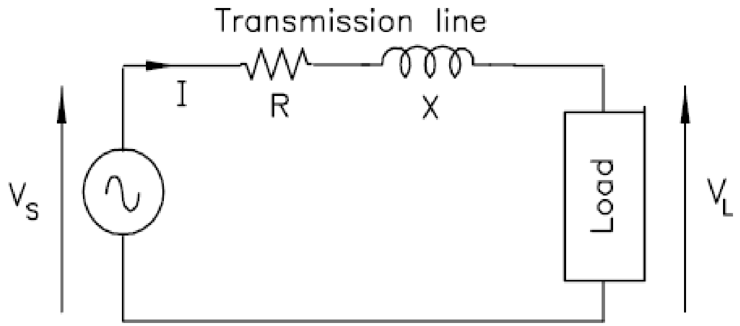
$$\text{Real Power} = 6250 \ \text{W}$$

$$\text{Reactive Power} = -3125 \ \text{VAr}$$

$$\text{Power factor} = cos\,(-26.57^\circ) \quad = 0.894 \ \text{leading}$$



Power factor and transmission Line Loss



- Industrial load connected to a substation
- V_S = Substation or sending end voltage.
- V_L = Voltage at the load.
- I = Current drawn by the load.
- R = Resistance of the transmission line.
- X = Reactance of the transmission line.
- The transmission line loss is given by: $P_{loss} = |I|^2 R$
- For the same real power demand, if the power factor of the load decreases, then |I| increases as shown by the following equation. This results in heavy line losses.

$$|I| = \frac{P}{|V|cos\theta}$$

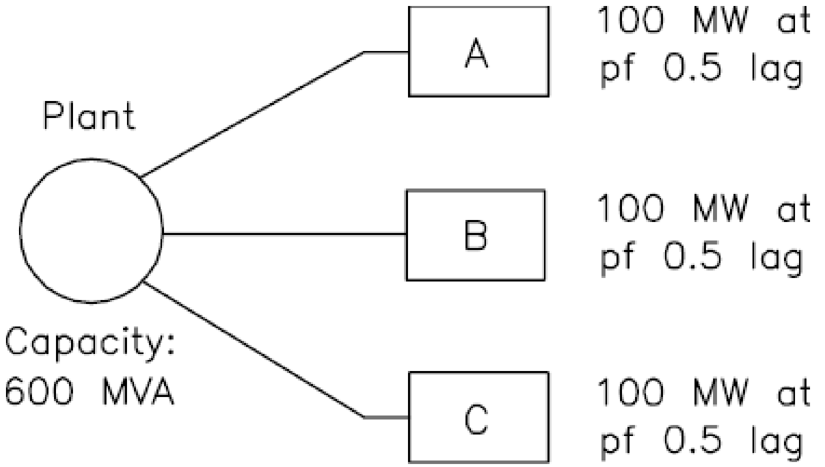
A poor power factor results in higher current and hence higher power loss. Power factor correction is done to reduce this loss.

Power factor and installed capacity

Three industrial customers A, B and C are drawing power from the Power Plant. Let the load at each of the three industries be 100 MW at a p.f. of 0.5 lagging. The maximum demand of each consumer is

$$\text{Max. Demand} = |V||I| = \frac{P}{\cos\theta} = \frac{100}{0.5} = 200\ \text{MVA}$$

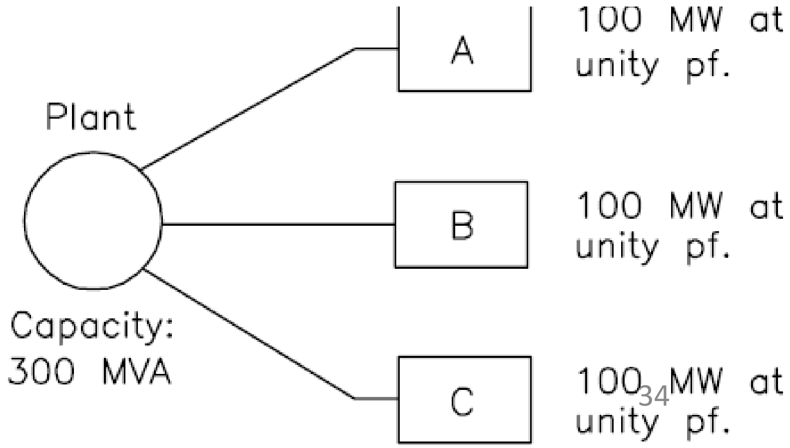
So the installed capacity of the plant should be $\sum \text{Max. Demand} = 600\ \text{MVA}$



If the customers had unity power factor, then the maximum demand of each consumer is

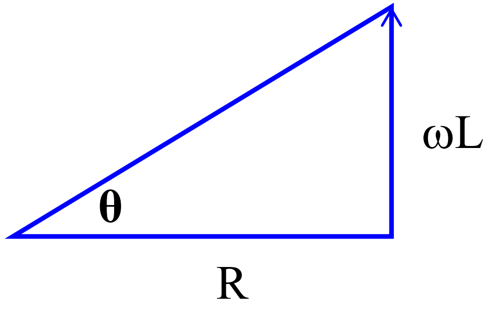
$$\text{Max. Demand} = |V||I| = \frac{P}{\cos\theta} = \frac{100}{1.0} = 100\ \text{MVA}$$

So the installed capacity of the plant should be $\sum \text{Max. Demand} = 300\ \text{MVA}$

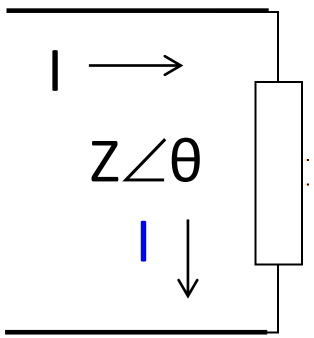


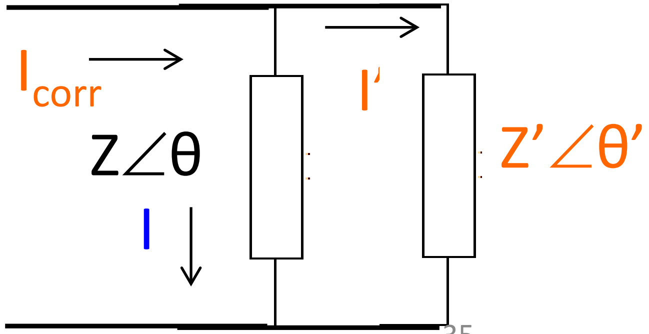
Power Factor Correction

$$P = V_{rms} I_{rms}\,(Power\ factor) \Rightarrow I_{rms} = \frac{P}{V_{rms} \times (Power\ factor)}$$

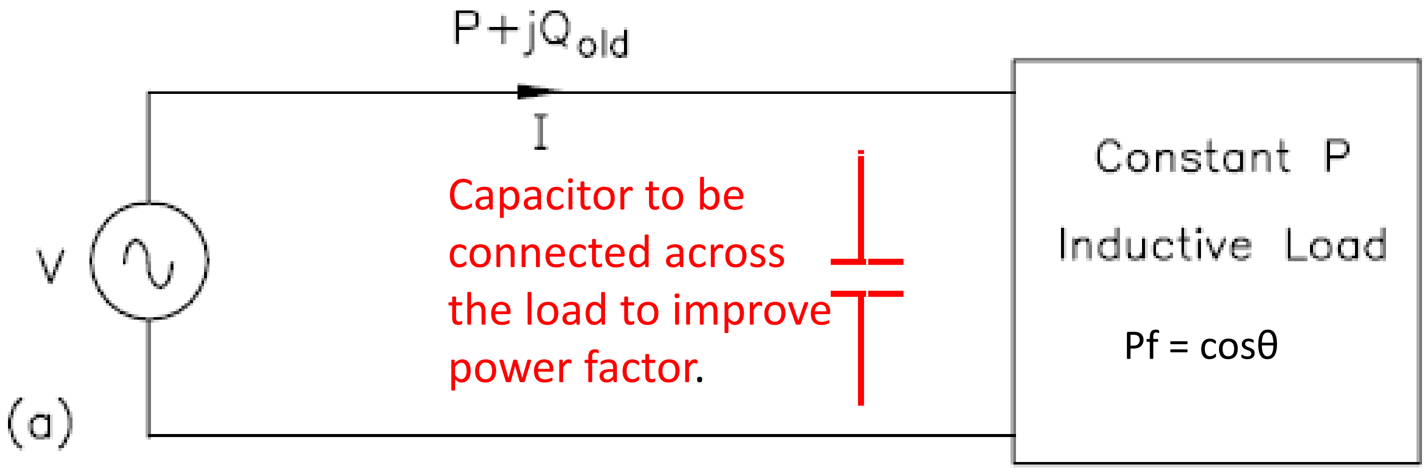


- Loads are usually connected to a fixed voltage supply, e.g. 220V, 50 Hz in Singapore, hence V_{rms} is fixed.
- To deliver a certain amount of power to the load, current will be larger if power factor is smaller. Larger current means higher loss in the cables. Hence it is desirable that power factor be as close to 1 as possible. i.e. θ should be as small as possible.
- Once the load is connected, its θ cannot be changed. Another reactive element can be added parallel to the load to improve power factor.
  - If the load is originally inductive, choose a capacitor.
  - If the load is originally capacitive, choose an inductor as correcting device.





Power Factor Correction

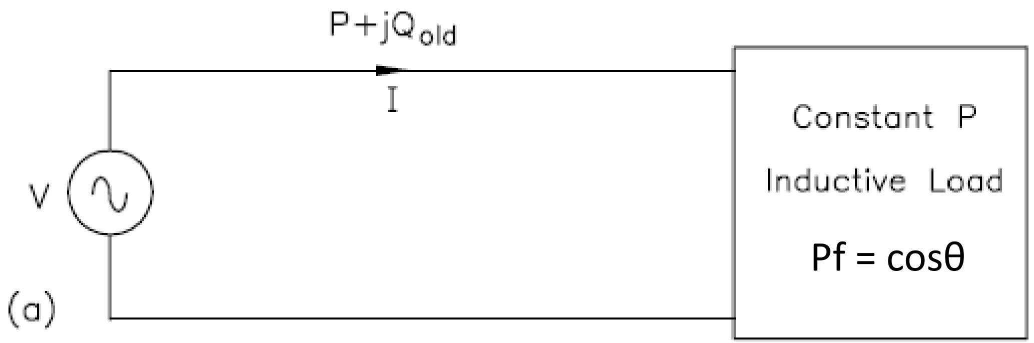


Problem statement:

- Given power factor, voltage and active power consumption of a load.
- To determine the size of the capacitor to be connected across load to improve power factor to new value.

- If voltage, power factor and active power consumption (P) are given, the reactive power demand (Q_{old}) by the load can be calculated.
- From the target new power factor, we can determine the new reactive power demand (Q_{new}) of the system. The new reactive power demand will be lesser if power factor is to be improved.
- The difference between the old and new reactive power ($Q_c=Q_{new}-Q_{old}$), can be supplied by connecting a capacitor across the load.

Power Factor Correction



Load P, V and power factor given (old)

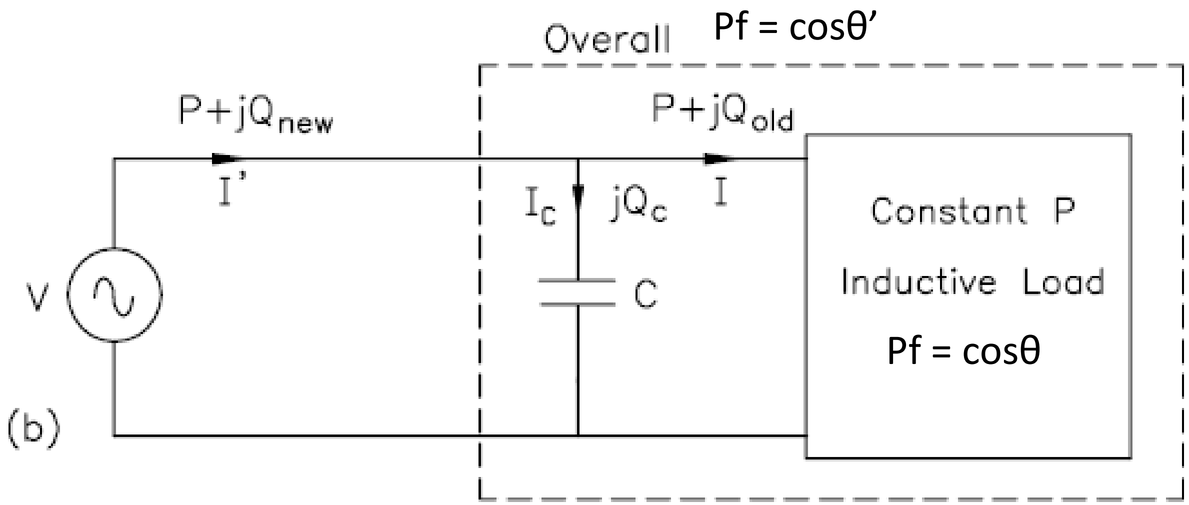
P=|V|.|I|cosθ

$$|I| = \frac{P}{|V|cos\theta}$$

$$I = |I|\angle - \theta$$

Complex power (load): S_{old} = VI* = P+jQ_{old}

When a capacitor is connected to improve the power factor, the new complex power will be :



Desired power factor given: $Pf_{new} = \cos\theta'$

$$|I'| = \frac{P}{|V|cos\theta'} \qquad I' = |I'|\angle - \theta'$$

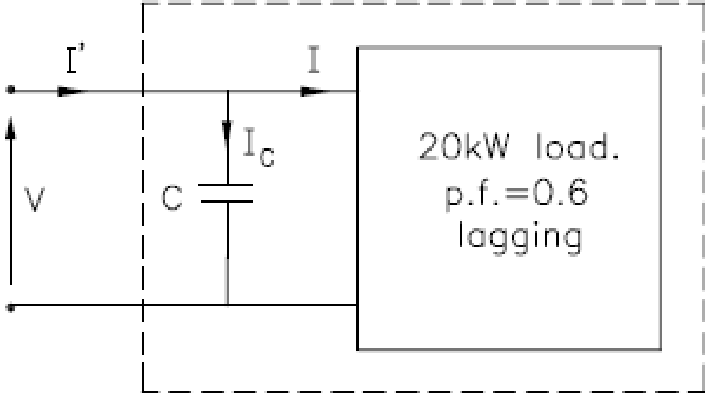
S_{new} = VI'* = P+jQ_{new}

$Q_{new} = Q_{old} + Q_c$ Or, $Q_c = Q_{new} - Q_{old}$

So, the capacitance C needed to improve the power factor of the load can be calculated from:

$$Q_c = -\omega C |V|^2$$

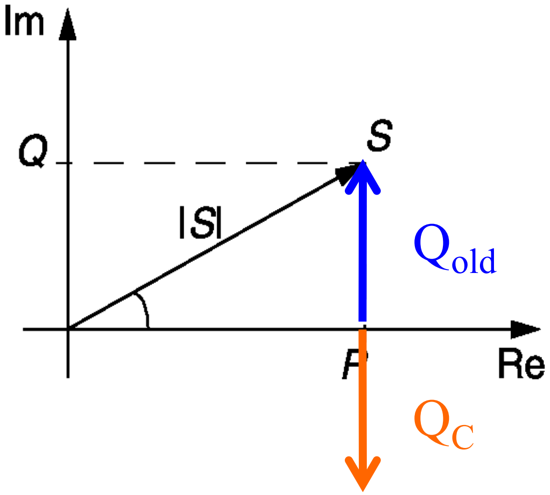
To improve power factor to unity



$S_{new} = P+jQ_{new} = P+ jQ_{old} +jQ_c$

$Q_{new} = Q_{old} + Q_c$

Or, $Q_c = Q_{new} - Q_{old}$



If the overall power factor is improved to unity,

$$\cos\theta' = 1$$

$$S_{new} = |V|.|I'|\cos\theta'+j\,|V|.|I'|\sin\theta' = P+jQ_{new}$$

$$Q_{new} = |V|.|I'|\sin\theta' = 0$$

So, the capacitance C needed to improve the power factor of the load can be calculated as

$$Q_c = -Q_{old} = -\omega C|V|^2$$

$$C = \frac{Q_{old}}{\omega|V|^2}$$

Then capacitor supplies 100% of the reactive power required by the load.

Example

A load connected across a 200 V, 50Hz line draws 10 kW at 0.5 power factor lagging. Determine the current drawn by the load. A capacitor C is now connected in parallel with the load to improve the power factor. What must be the value of C to make the overall power factor (i) 0.9 lagging, (ii) unity and (iii) 0.8 leading?

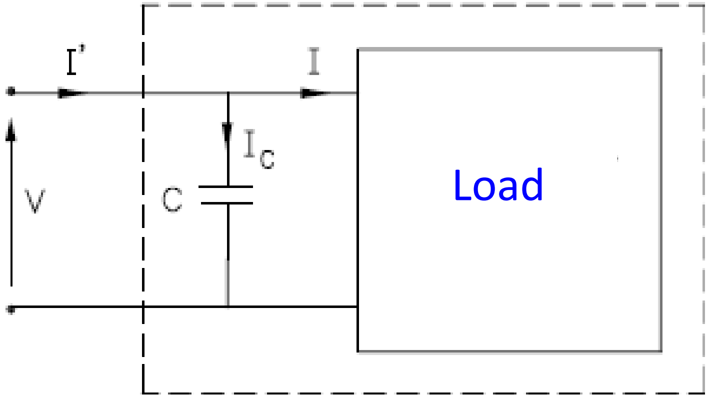
Solution: Take source voltage as reference. So, current drawn at 0.5 p.f. lagging is

$$V = 200\angle 0^\circ$$

$$|I| = \frac{P}{|V|cos\theta} = \frac{10000}{200 \times 0.5} = 100 \text{ A}$$

$$\theta = cos^{-1}0.5 = 60^\circ$$

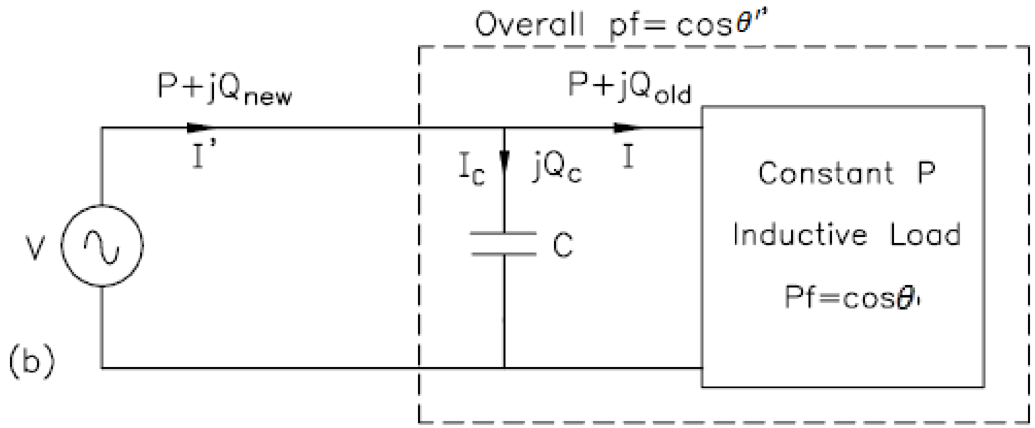
$$I = 100\angle{-60^\circ}$$



$$S_{old} = P + jQ_{old} = V\ I^* = 200 \times 100\angle 60^\circ = 10000 + j17320.5$$

Solution

Case 1: 0.9 lagging



$$|I'| = \frac{P}{|V|\,cos\,\theta'} = \frac{10000}{200 \times 0.9} = 55.56 \ \text{ A}$$

$$\theta' = cos^{-1}0.9 = 25.84^{\circ}$$

$$I' = 55.56\angle{-25.84^{\circ}}$$

$$S_{new} = P + jQ_{new} = V\ I'^{*}$$

$$= 200 \times 55.56\angle 25.84' = 10000 + j4843.3$$

Hence, Q_C and C may be obtained from

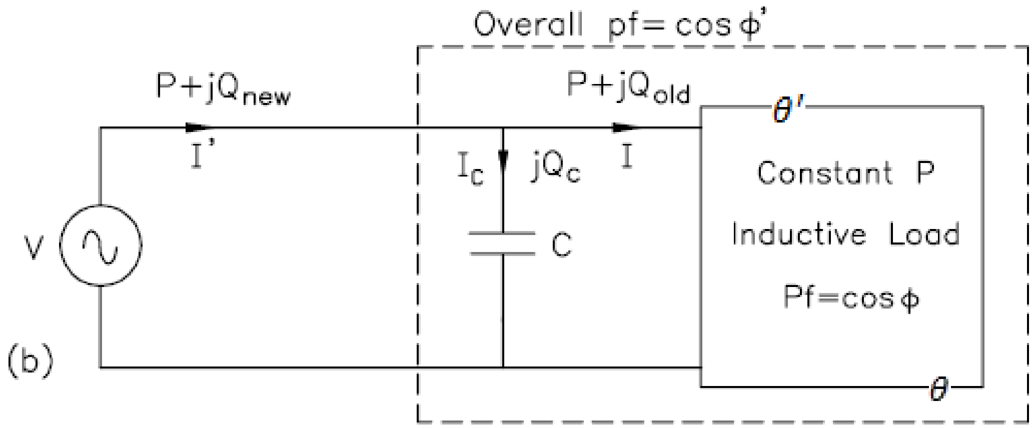
$$Q_C = Q_{new} - Q_{old}$$

$$Q_C = -\omega C|V|^2 = -12476.7 \ \text{ VAr}$$

$$C = \frac{12476.7}{314 \times 200^2} = 993.4 \ \mu F$$

Solution

Case 2: Unity p.f.



$$S_{new} = P + jQ_{new} = 10000 + j0$$

$$Q_{new} = 0$$

$$Q_C = Q_{new} - Q_{old} = -Q_{old}$$

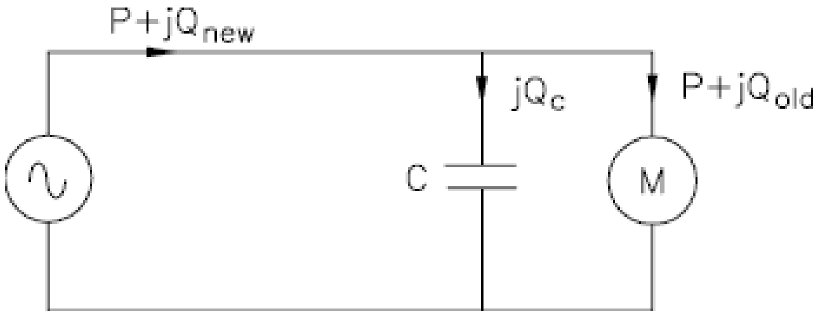
$$= -\omega C|V|^2$$

$$C = \frac{17320.5}{314 \times 200^2} = 1378.3 \ \ \mu F$$

Example

A 5000 W electric motor is connected to a source of 230 V, 50 Hz and the result is a lagging power factor of 0.8. To correct the power factor to 0.95 lagging, a capacitor is placed in parallel with the motor. Calculate the current drawn from the source with and without capacitor. Determine the value of the capacitor required to make the correction.

Solution



$$|I| = \frac{P}{|V|\,cos\,\theta} = \frac{5000}{230 \times 0.8} = 27.174 \;\; \text{A}$$

$$\theta = cos^{-1}0.8 = 36.87^{\circ}$$

$$I = 27.174\angle{-36.87^{\circ}}$$

$$S_{old} = V\,I^* = P + jQ_{old}$$

$$= 230 \times 27.174\angle 36.87^{\circ} \;\; = 5000 + j3750$$

$$|I'| = \frac{P}{|V|\,cos\,\theta'} = \frac{5000}{230 \times 0.95} = 22.883 \;\; \text{A}$$

$$\theta' = cos^{-1}0.95 = 18.19^{\circ}$$

$$I' = 22.883\angle{-18.19^{\circ}}$$

$$S_{new} = V\,I'^* = P + jQ_{new} \;\; = 230 \times 22.883\angle 18.19^{\circ} \;\; = 5000 + j1643$$

$$Q_C = Q_{new} - Q_{old}$$

$$Q_C = -\omega C|V|^2 = -2107 \;\; \text{VAr}$$

$$C = \frac{2107}{100\pi \times 230^2} = 126.85 \;\; \mu F$$