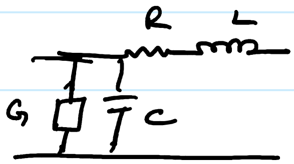
9.1 Line Conductors

Tuesday, 15 March 2022 11:37 am

⇒ R, L, C, G



→ Magnetic Flux Density.

$B = \mu H$ → magnetic flux intensity

μ → permeability $\mu = \mu_0 \mu_r$

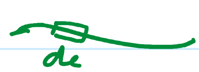
μ_0 → " of freespace $= 4\pi \times 10^{-7}$ H/m

μ_r → relative permeability of the material wrt free space $= \frac{\mu}{\mu_0}$

AIR → μ_r → 1

→ Magnetic Flux. ϕ → B

$\oint H \, dl$



$\phi = \oint B \, da$

→ Ampere's Circuital law.

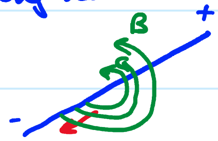
Current through a conductor

↳ magnetic flux around it

$\oint H \, dl = I_{enclosed}$.

H = Magnetic field intensity.

dl → differential length



$B = \mu H$

→ Conductor of infinite length,

conductor closes @ infinity

↳ $N = 1$

→ Faraday's law of induction.

$$e = \frac{d\lambda}{dt}$$

λ → Flux linkages → amount of flux linking

an N turn coil. → $\lambda = N\Phi$

$$e = \frac{d(N\Phi)}{dt} = N\frac{d\Phi}{dt}$$

→ Inductance $L = \dfrac{\text{Flux linkages}}{\text{current}} = \dfrac{\lambda}{I}$

→ Flux linkages of a single conductor.

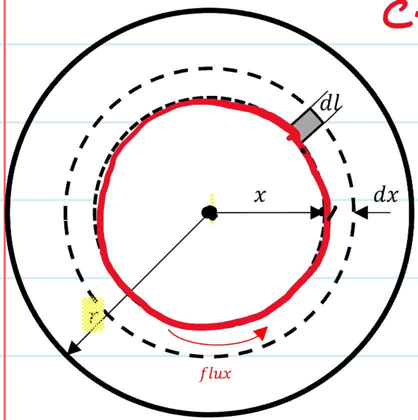
→ Single, infinitely long wire → $N = 1$

→ Flux linkage inside the conductor

→ Flux linkage outside the conductor

$$H \xrightarrow{B=\mu H} B \xrightarrow{\Phi=\int B\,da} \Phi \xrightarrow{N\Phi} \lambda \xrightarrow{\lambda/I} L$$

→ Flux linkage inside the conductor



Cross Sectional Area of a conductor.

→ Current density → uniform

$$I_x = \oint H_x \, dl = H_x \oint dl$$

$$I_x = H_x [2\pi x]$$

$$H_x = \frac{I_x}{2\pi x} \quad \text{— (1)}$$

Total current $= I$

Current $I_x = \frac{I}{\pi r^2} \cdot \pi x^2 \Rightarrow I_x = I \cdot \frac{x^2}{r^2}$

current density.

$$I_x = \frac{x^2}{r^2} \cdot I \quad \text{— (2)}$$

(2) in (1) $\Rightarrow H_x = \frac{x^{\cancel{2}}}{r^2} \cdot I \cdot \frac{1}{2\pi \cancel{x}} = \frac{x I}{2\pi r^2}$

$$H_x = \frac{x I}{2\pi r^2}$$

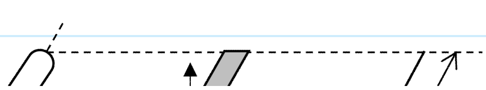
→ $B_x = \mu H_x = \frac{\mu x I}{2\pi r^2}$

→ $\lambda = N\phi = \oint B_x \, da$ [Since $N = 1$ $\Rightarrow \lambda = \phi$]

$$\lambda_{inside} = \frac{\mu}{8\pi} \cdot I^2$$

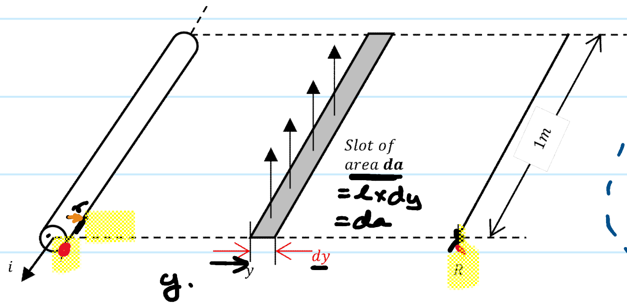
→ Flux Linkage outside the conductor

$N = 1$



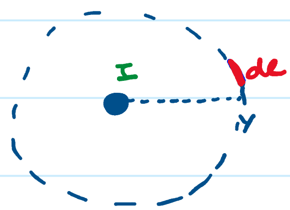
$I_{enclosed} = \oint H \, dl$

$N=1$



$y.$

$I_{enclosed} = \oint H\, dl$



$I_{enclosed}$ = Total current I

$$I = \oint H\, dl = H \cdot \oint dl$$

$$= H\, 2\pi y.$$

$$H = \frac{I}{2\pi y},$$

Flux linkage @ R from the centre of conductor/wire.

$$\phi = \oint B\, da$$

$$H = \frac{I}{2\pi y}$$

$$B = \mu_0 \underbrace{\mu_r}_{1 \text{ for air.}} H = \mu_0 \frac{I}{2\pi y}.$$

$$\phi = \oint \mu_0 \frac{I}{2\pi y} (l \cdot dy)$$

$$= l \oint \frac{\mu_0 I}{2\pi y} dy.$$

$$\lambda = N\phi = \phi = l \oint \frac{\mu_0 I}{2\pi y} dy.$$

$$\lambda = l \cdot \frac{\mu_0 I}{2\pi} \left[\ln y \right]_r^R$$

$$= l \frac{\mu_0 I}{2\pi} \ln \left[\frac{R}{r} \right]$$

→ Flux linkage per unit length. $l = 1m$

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$$\lambda_{outside} = \frac{\lambda}{1} = \frac{\mu_0 I}{2\pi} \ln\left[\frac{R}{r}\right]$$

$$\lambda = \lambda_{inside} + \lambda_{outside}$$

$$= \frac{\mu_0 \mu_r}{8\pi} I + \frac{\mu_0 I}{2\pi} \ln\left[\frac{R}{r}\right]$$

$$= \frac{\mu_0}{2\pi} I \left[\frac{\mu_r}{4} + \ln\left[\frac{R}{r}\right]\right]$$

$$= \frac{\mu_0}{2\pi} I \left[\ln e^{\frac{\mu_r}{4}} + \ln|R| - \ln|r|\right]$$

$$= \frac{\mu_0}{2\pi} I \left[\ln|R| - \left[\ln|r| - \ln e^{\frac{\mu_r}{4}}\right]\right]$$

$$= \frac{\mu_0}{2\pi} I \left[\ln|R| - \left[\ln|r| + \ln e^{-\frac{\mu_r}{4}}\right]\right]$$

$$= \frac{\mu_0}{2\pi} I \left[\ln|R| - \ln|r \cdot e^{-\frac{\mu_r}{4}}|\right]$$

Let $r' = r \cdot e^{-\frac{\mu_r}{4}}$

$$\lambda = \frac{\mu_0}{2\pi} I \left[\ln|R| - \ln|r'|\right]$$

$$\lambda = \frac{\mu_0}{2\pi} I \ln\left|\frac{R}{r'}\right| \quad \text{— (A)}$$

$$= \frac{4\pi \times 10^{-7}}{2\pi} I \ln\left|\frac{R}{r'}\right|$$

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$$\lambda_{total} = 2\times10^{-7} I \ln\left|\frac{R}{r'}\right|$$

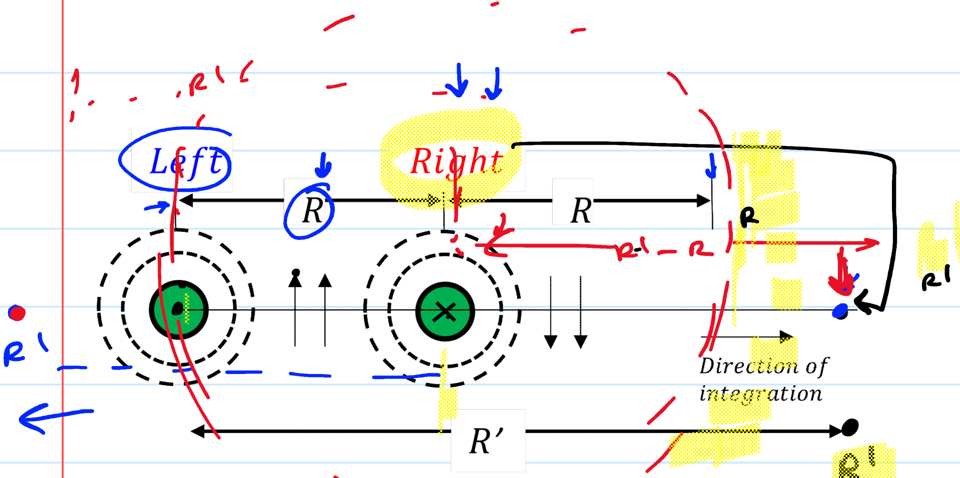
$$\rightarrow L = \frac{\lambda_{total}}{I} = 2\times10^{-7} \ln\left|\frac{R}{r'}\right|$$

★ Two conductor line inductance,

→ Two wires/conductors

→ Same current I but opposite direction

→ Distance between conductors is R



→ Some field cancellation from the other conductor due to opposite current

$$\lambda_{left} = \frac{\mu_0 I}{2\pi} \ln\left|\frac{R'}{r'}\right| - \frac{\mu_0}{2\pi} I \ln\left|\frac{R'-R}{R}\right|$$

$$r' = re^{-\frac{\mu_r}{4}}$$

$$\lambda_{left} = \frac{\mu_0}{2\pi} I \left[\ln \frac{R'}{r'} - \ln \frac{R'-R}{R} \right]$$

$$= \frac{\mu_0}{2\pi} I \left[\ln R' - \ln r' - \ln |R'-R| + \ln |R| \right]$$

$$= \frac{\mu_0}{2\pi} I \left[\ln \left| \frac{R}{r'} \right| + \ln \frac{R'}{R'-R} \right]$$

$\hookrightarrow 0$

If $R' \to \infty$

$$\frac{R'}{R'-R} = \frac{1}{1 - \frac{R}{R'}} = \frac{1}{1 - \frac{R}{\infty}} = 1$$

$$\Rightarrow \ln \left| \frac{R'}{R'-R} \right| = \ln |1| = 0$$

$$\lambda_{left} = \frac{\mu_0}{2\pi} I \left[\ln \left| \frac{R}{r'} \right| + 0 \right]$$

$$= \frac{\mu_0}{2\pi} I \left[\ln \left| \frac{R}{r'} \right| \right]$$

$$\lambda_{left} = \frac{\mu_0}{2\pi} I \ln \left| \frac{R}{r'} \right|$$

$$\rightarrow L_{left} = \frac{\mu_0}{2\pi} I \ln \left| \frac{R}{r'} \right| / I$$

$$= \frac{\mu_0}{2\pi} \ln \left| \frac{R}{r'} \right|$$