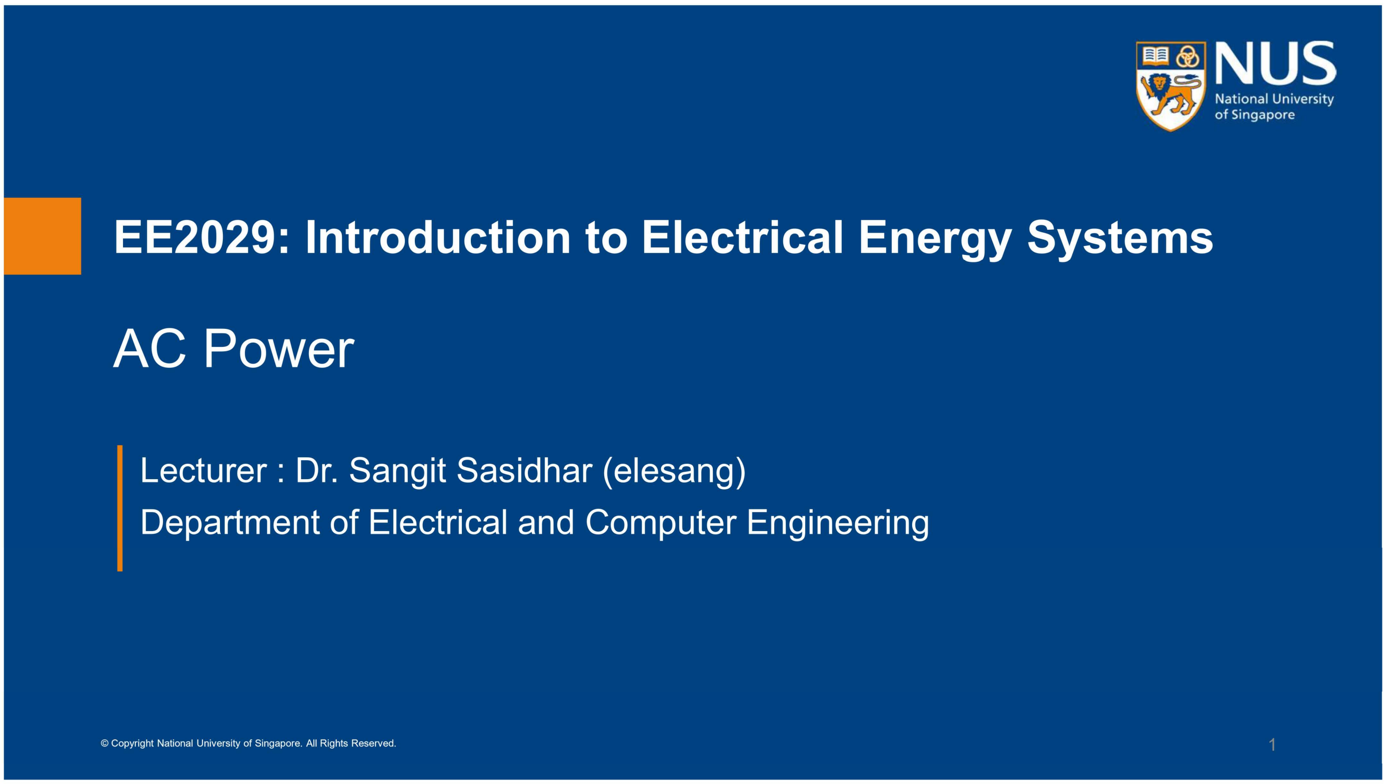
3.1 AC Power

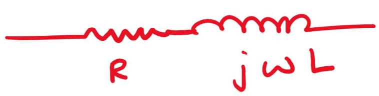
Tuesday, January 25, 2022 11:27 AM

3.0 In-Class Lecture - AC Power - EE2029



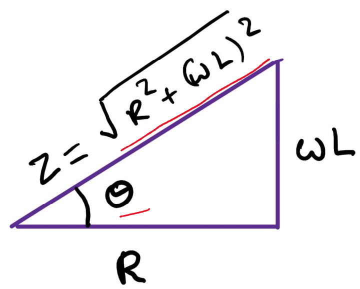
Impedance Triangle

RL



$$Z_{RL} = R + j\omega L$$

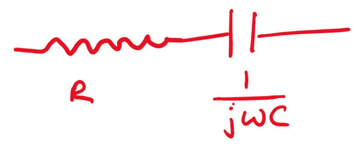
$$\theta = \tan^{-1}\frac{\omega L}{R}$$



θ +ve

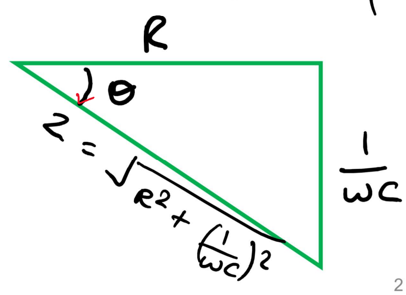
−ve.

RC

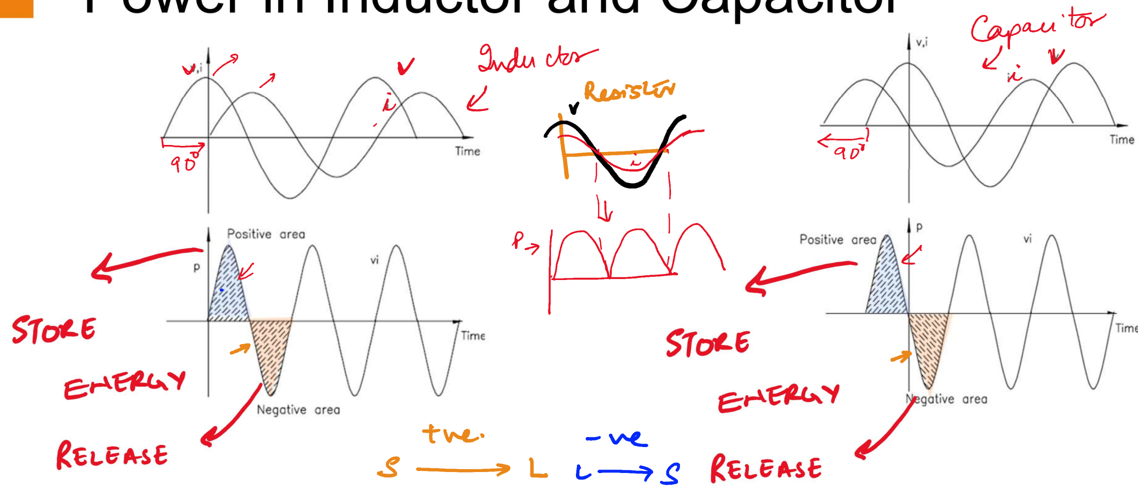


$$Z_{RC} = R - \frac{j}{\omega C}$$

$$\theta = \tan^{-1}\left(\frac{1/\omega C}{R}\right)$$



Power in Inductor and Capacitor



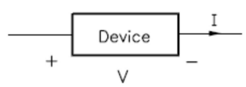
Both inductor and capacitor store and release energy during the AC cycle. When both the voltage and current are of the same sign, the element (L or C) draws energy equivalent to the area under the positive half cycle of p(t) from the source and stores it. When they are of opposite signs, it is returning the energy to the source.

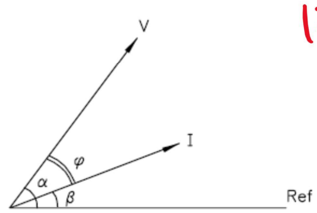
$P_{dc} \to V_{dc} I_{dc}.$

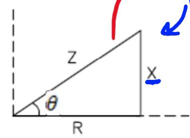
$\to P_{ac}$ (e) $\to V_{rms}\, I_{rms}$

$X \to$ Reactive component
L or C

Power in AC Circuits







$$\boxed{\cos\theta = \frac{R}{\sqrt{R^2+X^2}}}$$

RL

Z → IMPEDANCE OF DEVICE

$$Z = R + jX = |Z|\angle\theta \quad — (1)$$

$$|Z| = \sqrt{R^2+X^2}$$

$$Z = \frac{V}{I} = \frac{|V|\angle\alpha}{|I|\angle\beta} = \left|\frac{V}{I}\right|\angle(\alpha-\beta) = \left|\frac{V}{I}\right|\angle\phi \quad — (2)$$

$$\Rightarrow \theta = \phi = \alpha - \beta = \text{IMPEDANCE ANGLE}$$

$$V = |V|\angle\alpha$$

$$I = |I|\angle\beta$$

REAL POWER

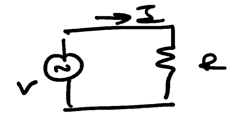
$$P = |I|^2 R$$

$$= |I||I| R$$

$$P = \frac{|V|}{|Z|}|I| R = \frac{|V|}{\sqrt{R^2+X^2}} \cdot |I| R$$

$$P = |V||I| \frac{R}{\sqrt{R^2+X^2}} = |V||I| \cos\theta$$

4



$\theta = 0°$

I V

$V I \cos\theta$

↓

1

$V_{rms} \times I_{rms}$ (R)

Apparent Power

- We express power in d.c. and a.c. circuits as follows:

$$P_{dc} = V_{dc} I_{dc} \text{ WATTS}$$

$$P_{ac} = V_{rms} I_{rms} \cos\theta \text{ WATTS}$$

- In a.c. circuits an additional term cos θ has occurred in the expression for **Real** power
- Define Apparent Power ➔ $|S| = V_{rms} I_{rms}$
- Units of Apparent Power ➔ VA (VOLT AMPERES)
- So, the Real power in a.c. circuit may also be expressed in terms of apparent power as

$$|P| = |S| \cos\theta$$

5

Reactive Power

- Real Power ➔ Power in the Resistor
- Average power in inductor or capacitor =0
- Define Reactive Power (Q) ➔ Power stored/released by the inductor or capacitor.

REACTIVE POWER

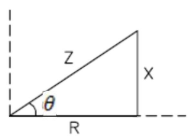
$$Q = I^2 X$$
$$= |I||I| X$$
$$= \frac{|V|}{|Z|} |I| X$$
$$= |V||I| \frac{X}{\sqrt{R^2+X^2}}$$

If $X = j\omega L$

$$\frac{X}{\sqrt{R^2+X^2}} = \sin\theta$$

If $X = -\frac{j}{\omega C}$

$$\frac{X}{\sqrt{R^2+X^2}} = -\sin\theta$$



$$Q_{RL} = V_{rms} I_{rms} \sin\theta$$

$$Q_{RC} = -V_{rms} I_{rms} \sin\theta$$

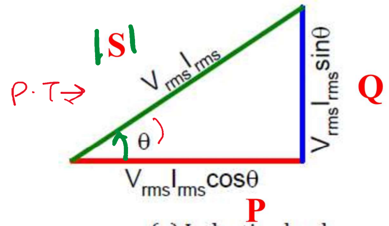
6

P, Q, |S|

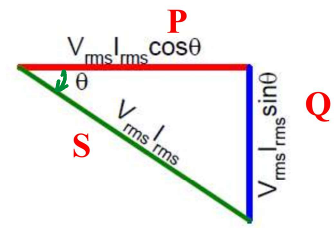
Power Triangle

- P – Real (or active) Power: Power consumed in the resistive part of the circuit
- Q – Reactive Power: Power stored in the inductor or capacitor
- |S| – Apparent Power: Combines P and Q in one quantity
  - Indication of Current the System can support

$$P = V_{rms} I_{rms} \cos\theta$$
$$S = V_{rms} I_{rms}$$
$$Q = V_{rms} I_{rms} \sin\theta$$



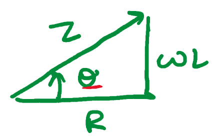
(a) Inductive load,

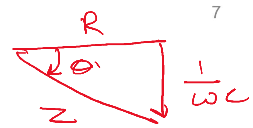


(b) Capacitive load

7

IT →





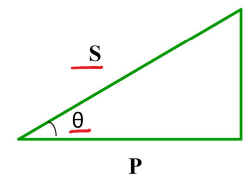
Complex Power

- Define Complex Power

IMPEDANCE

$Z = R + jX = |Z| \angle \theta$

$S = |S| \angle \theta$



$S = V_{rms} I_{rms} \angle \theta \Rightarrow S = V_{rms} I_{rms} \angle \alpha - \beta$

$\Rightarrow S = V_{rms} \angle \alpha \cdot I_{rms} \angle -\beta$

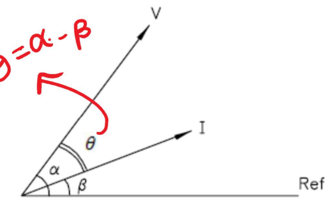
CONJUGATE

$I^* =$ COMPLEX CONJUGATE OF $I = I_{rms} \angle -\beta$

$\Rightarrow \boxed{S = VI^*} = P + jQ$

COMPLEX POWER

$\theta = \alpha - \beta$



$V = |V| \angle \alpha$

$I = |I| \angle \beta$

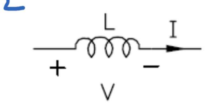
$I \rightarrow |I| \angle \beta$

$\rightarrow I^* \rightarrow |I| \angle -\beta$

8

Power in an Inductor

IMPEDANCE OF AN INDUCTOR = $Z_L = j\omega L = \omega L \angle 90^\circ \,\Omega$



$V = |V| \angle 0$

$$I = \frac{V}{Z_L} = \frac{|V|\angle 0^\circ}{\omega L \angle 90^\circ} = \left|\frac{V}{\omega L}\right| \angle -90^\circ = |I| \angle -90^\circ$$

COMPLEX POWER IN AN INDUCTOR $S_L = VI^* = V\angle 0 \cdot I \angle 90^\circ$

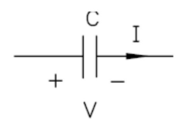
$= Z_L I \cdot I^* = j\omega L |I|^2$

REAL POWER CONSUMED BY INDUCTOR $= |V||I| \cos 90^\circ = \underline{0}$

REACTIVE POWER CONSUMED BY INDUCTOR $= |V||I| \sin 90^\circ = |V||I| = \omega L |I|^2$

Power in a Capacitor

IMPEDANCE OF A CAPACITOR $= Z_C = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90^\circ$



$$I = \frac{V}{Z_C} = \frac{|V|\angle 0^\circ}{\frac{1}{\omega C}\angle -90^\circ} = |\omega C V|\angle 90^\circ = \underline{|I|\angle 90^\circ}$$

$V = |V|\angle 0^\circ$

COMPLEX POWER IN A CAPACITOR $S_C = VI^* = Z_C I \cdot I^* = -j\frac{|I|^2}{\omega C}$

REAL POWER CONSUMED BY CAPACITOR $= |V||I|\cos(-90^\circ) = 0$

REACTIVE POWER CONSUMED BY CAPACITOR $= |V||I|\sin(-90^\circ) = -|V||I| = -\frac{|I|^2}{\omega C}$

$= \underline{-\omega C|V|^2}$

INDUCTANCE $\rightarrow$ +ve REACTIVE POWER

CAPACITANCE $\rightarrow$ -ve REACTIVE POWER

10

Complex Power of Series and Parallel Connected Loads

- Series Connected Loads

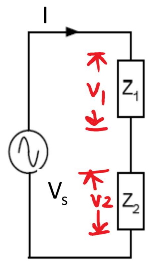
$S = V_s I^*$

$V_s = V_1 + V_2$

$\Rightarrow S = (V_1 + V_2) I^*$

$= V_1 I^* + V_2 I^*$

$= S_1 + S_2$



$\boxed{|S| \neq |S_1| + |S_2|}$

- Parallel Connected Loads

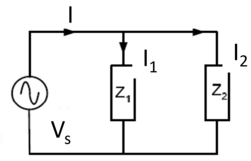
$S = V I^*$

$I = I_1 + I_2$

$\Rightarrow S = V (I_1 + I_2)^*$

$= V I_1^* + V I_2^*$

$= S_1 + S_2$



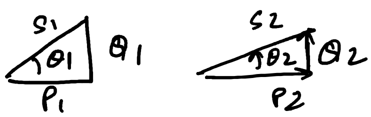
$|S| \neq |S_1| + |S_2|$

$P_1 + P_2 = P$

$Q_1 + Q_2 = Q$

$S_1 + S_2 = S$

$|S_1| + |S_2| \neq S$



$P = P_1 + P_2$ $\quad Q = Q_1 + Q_2$

Complex Power Summary

		Units	
$\underline{S}$	**Complex power**	VA	$VI^* = P + jQ = \lvert S\rvert\angle\theta$
$\lvert S\rvert$	**Apparent power**	VA	$\lvert S\rvert = \lvert V\rvert\lvert I\rvert = \sqrt{P^2+Q^2}$
P	**Active power** Average power, Real power	W	$P = Re(S) = VI\cos\theta$ $= \lvert S\rvert\cos\theta$
Q	**Reactive power**	VA$\underline{\text{R}}$	$Q = Im(S) = VI\sin\theta$ $= \lvert S\rvert\sin\theta$

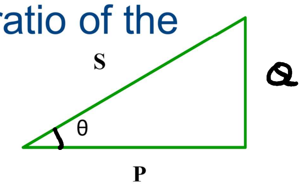
$Q = +ve \rightarrow RL$
$Q = -ve \rightarrow RC$

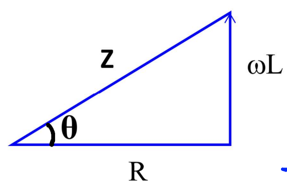
$P = |S| \cos\theta$

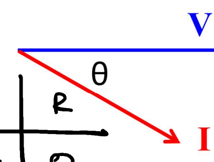
$\cos\theta = \dfrac{P}{|S|} \rightarrow$ Power Factor.

Power Factor

- Let us define Power Factor of an AC circuit as the ratio of the real power to the apparent power
  - Power Factor $= \dfrac{P}{|S|} = \dfrac{P}{|V||I|} = \cos\theta$
  - If 'θ' is larger, power factor (cosθ) is smaller
- Power factor angle (θ) is the same in the power triangle, impedance triangle and the angle between the voltage and the current







θ	RL	RC	R
→ I.T	+ve.	-ve	0
→ P.T	+ve	-ve	0
→ ∠I wrt V	-ve	+ve	0

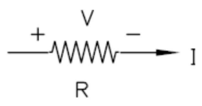
I wrt V → -30° → (lagging)

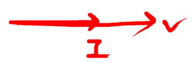
RL → P.T → $+30^\circ$

13

RL RC

Power Factor for Resistive, Inductive and Capacitive Load





$|S| = VI$

$P = VI\cos\theta$

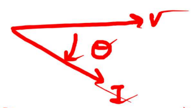
$\theta = 0°$

$S = VI, P = VI$

Pw factor $= \frac{P}{S} = 1$

UNITY POWER FACTOR

$$Z = R + j\omega L = R + jX_L = |Z| \angle \theta$$



$0 > \theta > -90°$

$|S| = VI$

$P = VI\cos\theta$

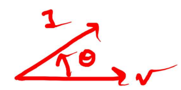
Power factor $= \cos\theta$

Range (0 to 1)

Lagging

I wrt V

$$Z = R - j\frac{1}{\omega C} = R - jX_C = |Z| \angle -\theta$$



$0 < \theta < 90°$

$|S| = VI$

$P = VI\cos\theta$

Power Factor $= \cos\theta$

Range (0 to 1)

Leading

I wrt V

How do we Improve Poor Power Factor?

$I \rightarrow$ Heat (R)

$I \rightarrow$ Charging (L or C)

P

Unity P.F $\rightarrow$ Current $\rightarrow$ R (Heat)

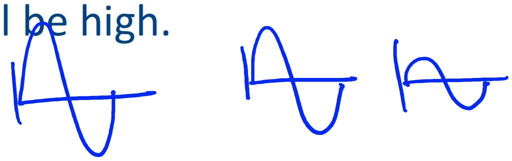
$0.9 \rightarrow R + jX \rightarrow L,\ \rightarrow C$

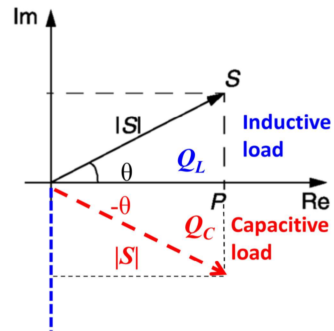
$0.8 \rightarrow$

0.7

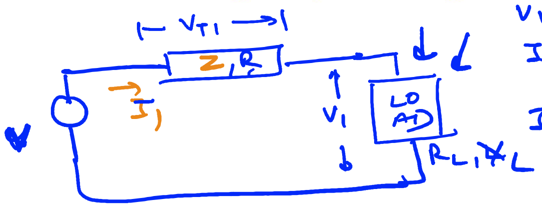
Poor Power Factor

- If the power factor is poor (i.e., very low), current drawn by the load will be high.





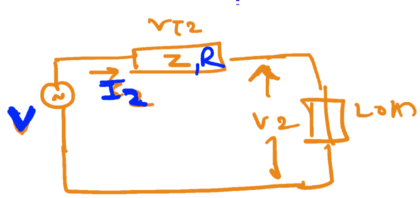
  - Poor voltage regulation at the load
  - Heavy transmission lines losses
  - High operating cost



V_1 | V_2

I_1 | I_2

$I_2 >> I_1$



16

$V_{T1} << V_{T2}$

$V_2 << V_1$

$I_1^2 \cdot R \rightarrow$ T.L. Loss.

$I_2^2 \cdot R \rightarrow$ T.L. Losses

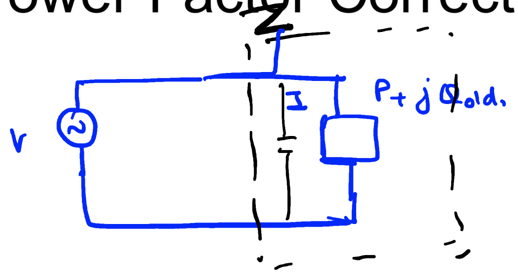
$V_1 = V - V_{T1}$

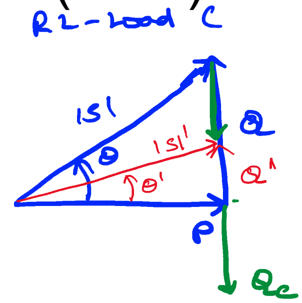
$V_{T1} = I_1 \cdot Z$

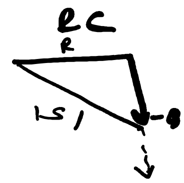
$V_{T2} = I_2 \cdot Z$

$V_2 = V - V_{T2}$

Power Factor Correction (PFC)







Improve Power factor →

cos θ is not low → Reduce angle θ

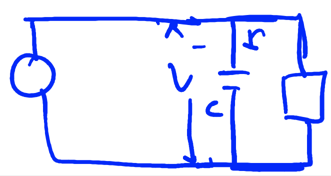
→ $R + j\left[\omega L - \frac{1}{\omega C}\right]$ → Series Impedance.

Aim → Reduce θ so that cos θ is larger → Reduce Q

RL → we add a capacitor to improve p.f.

RC → we add an inductor to improve p.f.

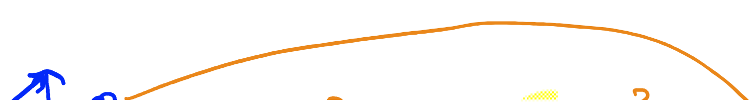
17

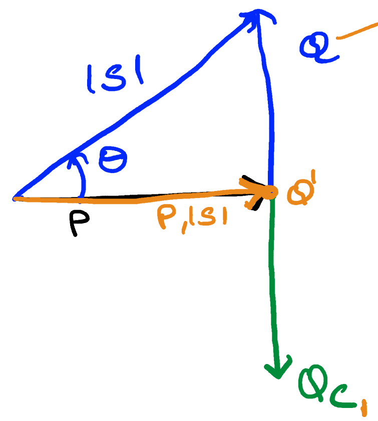


PFC: Fixed Capacitive Load ➔ New Power Factor

a) low power factor lagging → unity power factor

↳ $\theta' = 0°$





$Q_{c1} = -\omega C|V|^2$

$Q' = 0$

$Q' = Q + Q_{c1} = 0$

$Q_{c1} = -Q.$

$-\omega C|V|^2 = -Q$

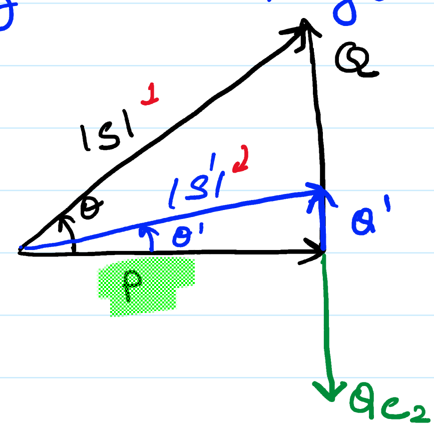
$C = \dfrac{Q}{\omega |V|^2}.$

18

b) low power factor lagging $\rightarrow$ better power factor lagging.

0.4 lag $\rightarrow$ 0.9 lag. $\rightarrow \cos\theta'$

$\cos\theta$



$Q' = Q + Q_{c2}$

$Q_{c2} = Q' - Q$

$\downarrow$

C

$P = S\cos\theta$

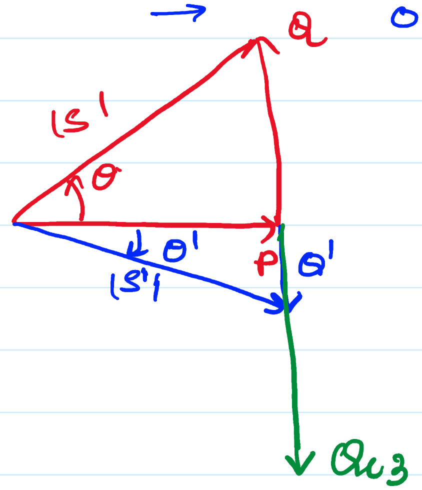
$S = \dfrac{P}{\cos\theta}$

$\downarrow$

$Q = S\sin\theta = P\tan\theta$

c) low power factor lagging $\rightarrow$ better powerfactor leading.

0.4 lag $\rightarrow$ 0.9 leading



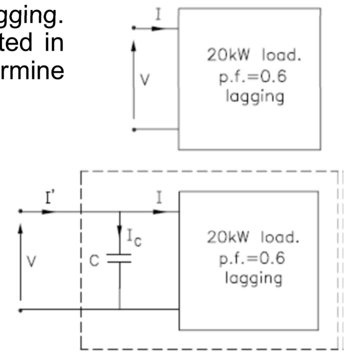
$Q' = Q + Q_{c3}$

$Q_{c3} = Q' - Q$

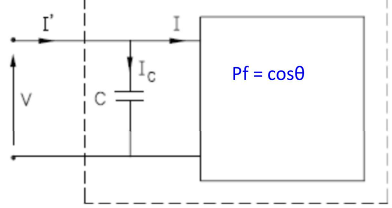
$\downarrow$

C

Example: A load connected across a 230V, 50HZ line draws 20kW at 0.6pf lagging. Determine the current drawn by the load. If a capacitor of 800 μF is connected in parallel with the load, what will be the current drawn from the source? Also determine the overall power factor of the system as seen by the source

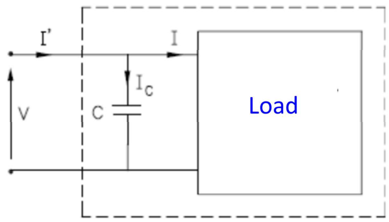


PFC: Required Power Factor → Capacitive Load



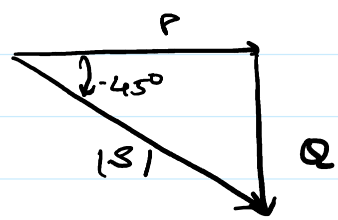
20

Example :A load connected across a 200 V, 50Hz line draws 10 kW at 0.5 power factor lagging. Determine the current drawn by the load. A capacitor C is now connected in parallel with the load to improve the power factor. What must be the value of C to make the overall power factor 0.9 lagging



Thank You

RC → p.f →



$S = |S| \angle -45^\circ$

$\underline{Z \rightarrow 100\,\Omega.}$

$V \rightarrow$ Same

$R \rightarrow 100\,\Omega.$

$R'L.$

$R' = \sqrt{100^2 - \omega^2 L^2}$

$\underline{R' < R.}$

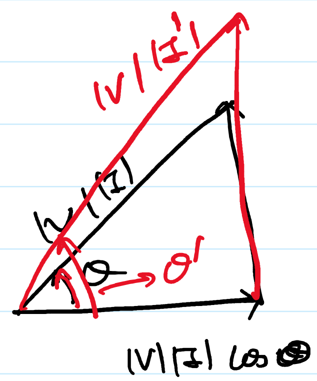
$K < K,$

$$|I| = \frac{|V|}{|Z|} = \text{same.}$$

$$\left.\begin{aligned} P' &= |I|^2 R' \\ P &= |I|^2 R \end{aligned}\right\} \quad P' < P$$

$P \rightarrow |V||I| \cos\theta$

$S \rightarrow |V||I|$



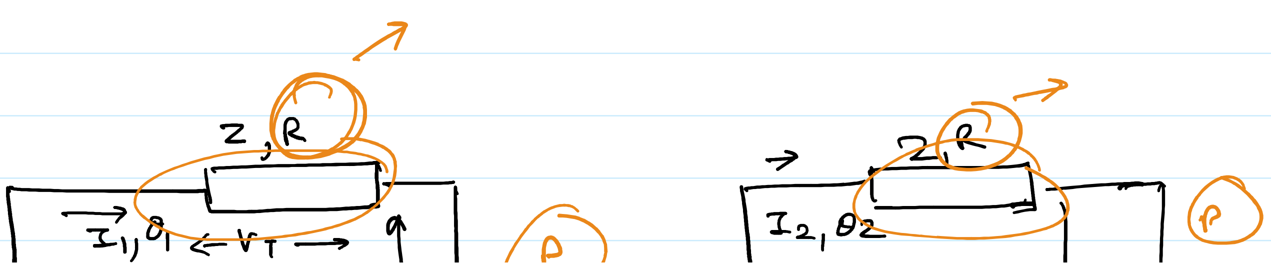
Projection of $|V||I|$ on x-axis is P

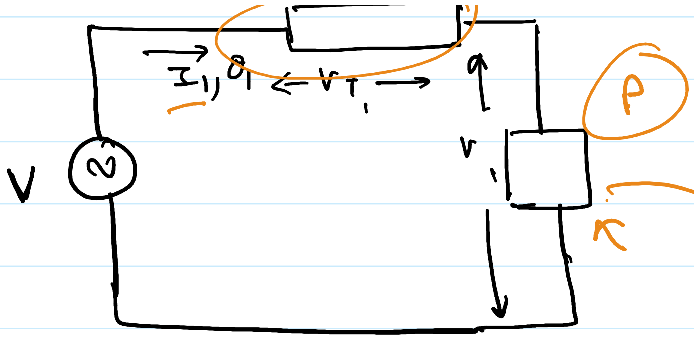
$|V| \; / \; |I| \cos\theta.$

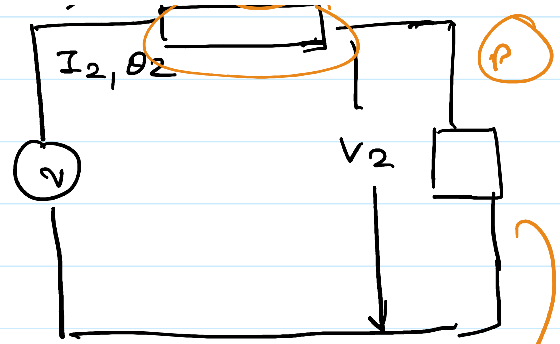
$|V||I'| \cos\theta' \rightarrow$ projection of $|V||I'|$ on x-axis

$|I'| \cos\theta' = |I| \cos\theta$

Tr 1 $\rightarrow$	Tr 2 $\rightarrow$
1000 W	1000 W
p.f $\rightarrow$ 0.5 lag.	p.f. $\rightarrow$ 0.9 lag







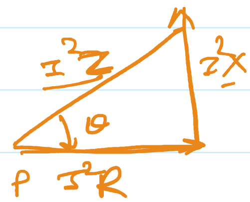
$\theta_2 > \theta_1$

$\Rightarrow \cos\theta_2 < \cos\theta_1$

$\rightarrow |I_2| > |I_1|$

$|I_1|^2$ $|I_2|^2$

$P = |V||I_1|\cos\theta_1 \quad = \quad P = |V||I_2|\cos\theta_2$



$|I_1|\cos\theta_1 = |I_2|\cos\theta_2$

$\rightarrow V_{T_1} I_1^* = P_{T_1} + j\,Q_{T_2}$ $\qquad V_{T_2} I_2^* = P_{T_2} + j\,Q_{T_2}.$

$V_{T_1} \not\approx V_{T_2}$

$P_{T_1} = V_{T_1} I_1 \cos\theta_T$ $\qquad P_{T_2} = V_{T_2} I_2 \cos\theta_T$