EE2029 – Introduction to Electrical Energy Systems
Tutorial # 1 Transformers

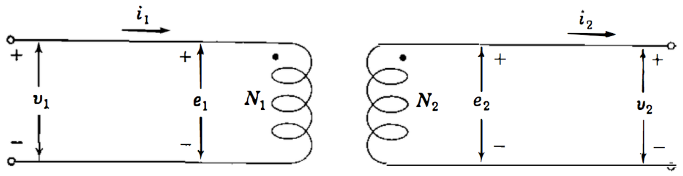
1. Find the turns ratio of a single-phase transformer that transforms the primary voltage 15,000 V of a power line to the secondary voltage 240 V supplied to a house.

(Answer: 62.5)

2. The output stage of an audio system has an output resistance of 2 kΩ. An output transformer provides resistance matching with a 6 Ω speaker. If this transformer has 400 primary turns, how many secondary turns does it have?

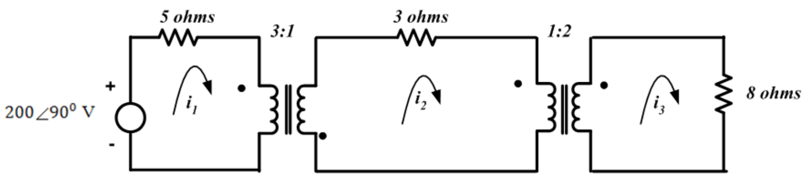
(Answer: 22 turns)

3. Referring to the equivalent circuit of a transformer shown below, N_1 = 2000, and N_2 = 500. Suppose $V_1 = 1200\angle 0°$ V and $I_1 = 5\angle -30°$ A at the primary side with an impedance Z_2 connected across the secondary side. Compute V_2, I_2, Z_2, and Z_2'. Note that Z_2' refers to the impedance Z_2 reflected to the primary side of the transformer.



(Answer: $V_2 = 300\angle 0° V, I_2 = 20\angle -30° A, Z_2 = 15\angle 30° \Omega, Z_2' = 240\angle 30° \Omega$)

4. Find i_1, i_2 and i_3 for the circuit shown below. The transformers are ideal.



(Answer: $i_1 = 4\angle 90°, i_2 = 12\angle 270°, i_3 = 6\angle 270°$)

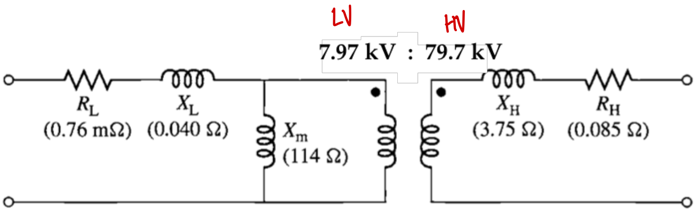
5. Three transformers, each rated at 25 MVA, 66/3.81 kV, have a Wye-Delta connection and is connected to a balanced load of three 0.6 Ω Wye-connected resistors. Choose a base of 75 MVA, 66 kV for the high-voltage side of the transformer, and specify the base for the low-voltage side. Determine the per unit resistance of the load on the base for the low-voltage side. Then, determine the load resistance R_L in ohms referred to the high-voltage side and per unit value of this resistance on the chosen base.

(Answer: $R_{L,LV} = 3.10$ p.u., $R_{L,HV} = 3.10$ p.u.)

6. A Three-phase transformer is rated 400 MVA, and 220/22 (Wye-Delta) kV. The Wye equivalent short-circuit impedance measured on the low-voltage side of the transformer is $0.121\ \Omega$. Determine the per unit reactance of the transformer and the per unit value to represent this transformer in a system whose base values on the high-voltage side is 100 MVA, 230 kV.

(Answer: 0.01 p.u., 0.0228 p.u.)

7. The equivalent circuit for a 100 MVA, 7.97/79.7 kV transformer is shown below. The parameters of the transformers are: $X_L = 0.040\ \Omega$, $X_H = 3.75\ \Omega$, $X_M = 114\ \Omega$, $R_L = 0.76\ m\Omega$, and $R_H = 0.085\ \Omega$. Note the magnetising inductance is referred to the low-voltage side of the equivalent circuit. Convert the equivalent circuit parameters to per unit using the transformer rating as the base.



(Answer: X_L = 0.0630 p.u., X_H = 0.0591 p.u., X_m = 180 p.u., R_L = 0.0012 p.u., R_H = 0.0013 p.u.)

8. A three-phase load is supplied from a 2400/460 V, 250 kVA transformer whose equivalent series impedance is $0.026 + j0.12$ per unit on its own base. The load voltage is observed to be 438 V line-to-line, and is drawing 95 kW at unity power factor. Calculate the voltage at the high-voltage side of the transformer. Perform the calculations on a 460 V at 100 kVA base.

(Answer: V_H = 2313 V, line-to-line)

1.

$$\boxed{a=\frac{N_1}{N_2}=\frac{I_2}{I_1}=\frac{V_1}{V_2}}$$

$$=\frac{15\,000}{240}=62.5$$

2.

$$\boxed{Z_1=a^2Z_2}=\left(\frac{N_1}{N_2}\right)^2Z_2$$

$$\Rightarrow N_2=\sqrt{\frac{N_1^2\cdot Z_2}{Z_1}}=\sqrt{\frac{400^2\,(6)}{2\times10^3}}$$

$$=21.9\approx 22\text{ turns}$$

3.

$$a=\frac{N_1}{N_2}=\frac{V_1}{V_2}=\frac{I_2}{I_1}=\frac{2000}{500}=4$$

$$\Rightarrow V_2=\frac{V_1}{a}=\frac{1200\angle0^\circ}{4}=300\angle0^\circ\,V$$

$$\Rightarrow I_2=a\cdot I_1=4(5\angle-30^\circ)=20\angle-30^\circ A$$

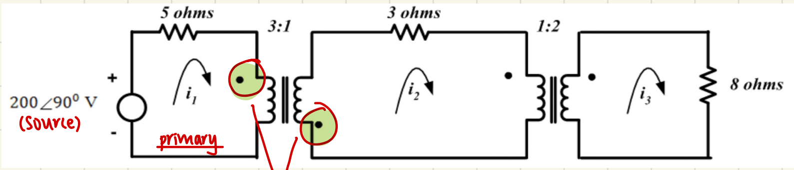
$$Z_2=\frac{V_2}{I_2}=\frac{300\angle0^\circ}{20\angle-30^\circ}$$

$$=15\angle30^\circ\,\Omega$$

$$\boxed{Z_2'=a^2Z_2}$$

$$=4^2(15\angle30^\circ)=240\angle30^\circ\,\Omega$$

4.



change in phase!

$R_{right}'=a_{right}^2R_{right}=\left(\frac{1}{2}\right)^2(8)=2\Omega$ (right referred to mid)

$R_{mid}+R_{right}'=3+2=5\Omega$

$R_{mid}'=a_{left}^2R_{mid}=3^2(5)=45\Omega$ (mid referred to right)

$R_e=R_{left}+R_{mid}'=5+45=50\Omega$

$$\boxed{i_1=\frac{V_s}{R_e}}=\frac{200\angle90^\circ}{50}$$

$$=4\angle90^\circ A$$

$$a_{left}=\frac{N_1}{N_2}=\frac{i_1}{i_2}=3$$

$$\Rightarrow|i_2|=3(4)=12A$$

$$\therefore i_2=12\angle(90+180^\circ)=12\angle270^\circ$$

opposite phase!

$$a_{right}=\frac{N_2}{N_3}=\frac{i_2}{i_3}=0.5$$

$$\Rightarrow|i_3|=0.5(12)=6A$$

$$\therefore i_3=6\angle270^\circ A$$

5.

Transformer Rating: 25MVA, 66Y/3.81△kV

HV Base: 75MVA, 66kV (Y) (primary: HV)

LV Base: 75MVA, 3.81 kV (△)

Same!

$$\boxed{Z_{LV,base} = \frac{V_{base,LV}^2}{|S|_{base}}} = \frac{(3.81\times10^3)^2}{75\times10^6}$$

$$= 0.1935\,\Omega$$

$$\boxed{R_{LV,pu} = \frac{R_{LV,actual}}{Z_{LV,base}}} = \frac{0.6}{0.1935}$$

$$= 3.10 \text{ p.u.}$$

$$R_{actual,LV}' = a^2 R_{actual,LV}$$ (referred to primary)

$$= \left(\frac{V_1}{V_2}\right)^2 R_{actual,LV}$$

$$= \left(\frac{66}{3.81}\right)^2 (0.6) = 180.05\,\Omega$$

$$Z_{HV,base} = \frac{V_{base,HV}^2}{|S|_{base}} = \frac{(66\times10^3)^2}{75\times10^6}$$

$$= 58.08\,\Omega$$

$$\therefore R_{pu,HV} = R_{pu,LV}' = \frac{R_{actual,HV}}{Z_{HV,base}} = \frac{180.05}{58.08}$$

$$= 3.10 \text{ p.u.}$$

6.

Transformer Rating: 400MVA, 220Y/22△kV

HV Base: 400MVA, 220kV (Y) (primary)

LV Base: 400MVA, 22kV (△)

$Z_{actual,LV} = 0.121\,\Omega$

$$Z_{LV,base} = \frac{V_{base,LV}^2}{|S|_{base}} = \frac{(22\times10^3)^2}{400\times10^6}$$

$$= 1.21\,\Omega$$

$$X_{LV,pu} = \frac{Z_{LV,actual}}{Z_{LV,base}} = \frac{0.121}{1.21}$$

$$= 0.10 \text{ p.u.}$$

Old HV Base: 400MVA, 220kV

New HV Base: 100MVA, 230kV

$$Z_{HV,base,old} = \frac{V_{base,HV,old}^2}{|S|_{base,old}} = \frac{(220\times10^3)^2}{400\times10^6}$$

$$= 121\,\Omega$$

$$Z_{HV,base,new} = \frac{V_{base,HV,new}^2}{|S|_{base,new}} = \frac{(230\times10^3)^2}{100\times10^6}$$

$$= 529\,\Omega$$

$$\boxed{X_{pu,new} = X_{pu,old}\left[\frac{Z_{base,old,HV}}{Z_{base,new,HV}}\right]}$$ only works for X & Z!

$$= 0.10\left[\frac{121}{529}\right] = 0.0229 \text{ p.u.}$$

7.

Transformer Rating: 100MVA, 7.97/79.7kV

HV Base: 100MVA, 79.7kV

LV Base: 100MVA, 7.97kV (primary)

At the LV-Side:

$$Z_{base,LV} = \frac{V_{LV,base}^2}{|S|_{base}} = \frac{(7.97\times10^3)^2}{100\times10^6}$$

$$= 0.635\,\Omega$$

$$X_{L,pu} = \frac{X_{L,actual}}{Z_{LV,base}} = \frac{0.040}{0.635}$$

$$= 0.0630\text{ p.u.}$$

$$R_{L,pu} = \frac{R_{L,actual}}{Z_{LV,base}} = \frac{0.76\times10^{-3}}{0.635}$$

$$= 0.00120\text{ p.u.}$$

$$X_{m,pu} = \frac{X_{m,actual}}{Z_{LV,base}} = \frac{114}{0.635}$$

$$= 179.53\text{ p.u.}$$

At the HV-Side:

$$Z_{base,HV} = \frac{V_{HV,base}^2}{|S|_{base}} = \frac{(79.7\times10^3)^2}{100\times10^6}$$

$$= 63.5\,\Omega$$

$$X_{H,pu} = \frac{X_{H,actual}}{Z_{HV,base}} = \frac{3.75}{63.5}$$

$$= 0.0591\text{ p.u.}$$

$$R_{H,pu} = \frac{R_{H,actual}}{Z_{HV,base}} = \frac{0.085}{63.5}$$

$$= 0.0134\text{ p.u.}$$

8.

$Z_{pu,old} = 0.026 + j0.12$ p.u.

$V_{L,actual} = 438V$ → actual values remain constant regardless of base!!

$P_{L,actual} = 95kW$

Old LV Base: 250kVA, 460V

New LV Base: 100kVA, 460V

$$Z_{LV,base,old} = \frac{V_{LV,base,old}^2}{|S|_{base,old}} = \frac{460^2}{250\times10^3}$$

$$= 0.846\,\Omega$$

$$Z_{LV,base,new} = \frac{V_{LV,base,new}^2}{|S|_{base,new}} = \frac{460^2}{100\times10^3}$$

$$= 2.116\,\Omega$$

$$V_{L,HV,pu,new} = V_{L,LV,pu,new} + V_{losses,pu,new}$$

$$= V_{L,LV,pu,new} + I_{L,pu,new}\cdot Z_{pu,new} \quad \text{— (1)}$$

$$V_{L,pu,new} = \frac{V_{L,actual}}{V_{base,LV,new}} = \frac{438}{460} = 0.9522\text{ p.u.} \quad \text{— (2)}$$

$$P_{L,pu,new} = \frac{P_{L,actual}}{|S|_{base,new}} = \frac{95\times10^3}{100\times10^3} = 0.95\text{ p.u.} \quad \text{— (3)}$$

$$I_{L,pu,new} = \frac{P_{L,pu,new}}{V_{L,pu,new}} = \frac{0.95}{0.9522} = 0.9977 \text{ p.u.} \quad \text{—— (4)}$$

$$\boxed{Z_{pu,new} = Z_{pu,old}\left(\frac{Z_{base,LV,old}}{Z_{base,LV,new}}\right)} = (0.026+j0.12)\left[\frac{0.846}{2.116}\right]$$

only works for X & Z!

$$= 0.0104 + j0.048 \text{ p.u.} \quad \text{—— (5)}$$

Sub. into (1):

$$V_{L,HV,pu,new} = V_{L,LV,pu,new} + I_{L,pu,new} \cdot Z_{pu,new}$$

$$= 0.9522 + 0.9977(0.0104 + j0.048)$$

$$= 0.9626 + j0.0479 \text{ p.u.}$$

$$V_{L,HV,actual,new} = V_{L,HV,pu,new} \cdot V_{L,HV,base,new}$$

$$= (0.9626 + j0.0479)(2400)$$

$$= 2313\angle 2.85^\circ \text{ V}$$