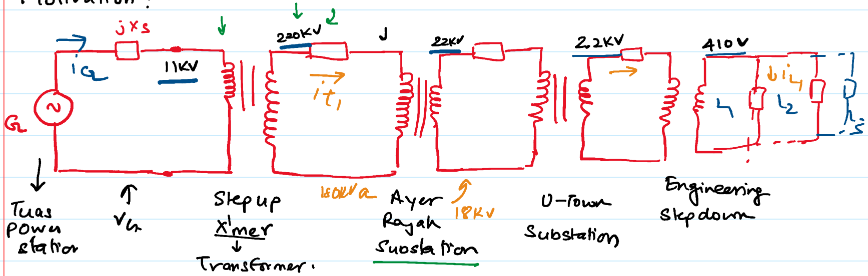
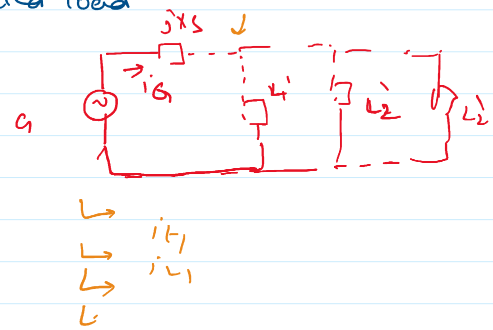
10.1 Per Unit Analysis

Tuesday, 22 March 2022 11:48 am

→ Motivation?



→ Reflected load



→ Difficult to observe the current situation of the system

→ Decoupling the dependence on transformer ratios

★ Per Unit Quantity

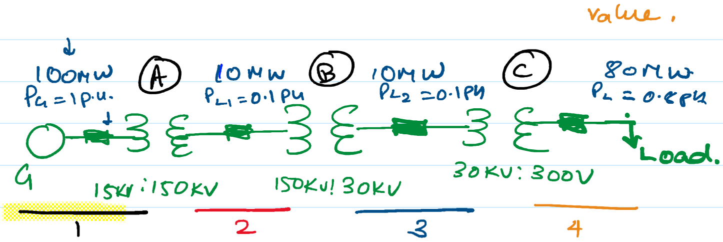
$$\text{per unit quantity} = \frac{\text{Actual Quantity}}{\text{Base Value of Quantity}}.$$

→ p.u.

→ All Base values → Real Quantities S_B

Denotes Base value.

Ex)



→ Use transformers to differentiate the zones of the system

→ Power Base → Single Base for the whole system

→ Voltage Base →

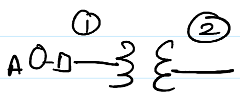
~~Example of single voltage Base for the whole system~~

~~→ 150kV.~~

~~0.1 : 1 1 : 0.2 0.2 : 0.02~~

~~→ Different voltage levels in each zone~~

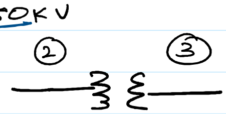
At substation A



$V_{1B} \rightarrow 15kV$

$V_{2B} \rightarrow 150kV$

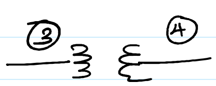
At substation B



$V_{3B} \rightarrow 30kV$

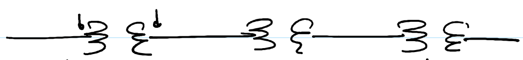
$V_{2B} \rightarrow 150kV$

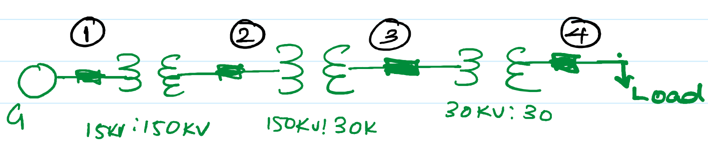
At substation C



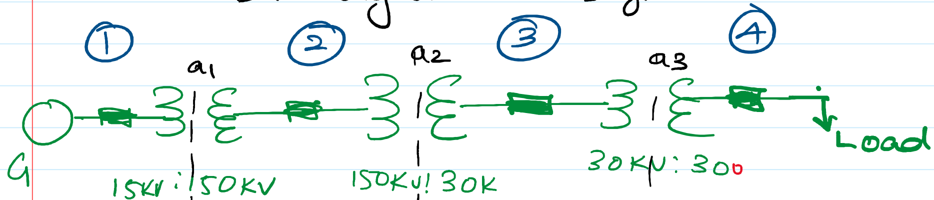
$V_{3B} \rightarrow 30kV$

$V_{4B} \rightarrow 300V$





① Choose V_B for any one zone.
or V_B for any one zone is given.



$V_{2B} = 150kV$

$\boxed{V_{1B} = a_1 V_{2B}}$

$= \frac{15}{150} \times 150$

$= 15kV$

$\boxed{V_{3B} = \frac{V_{2B}}{a_2}}$

$= 150kV \times \frac{30}{150}$

$= 30kV$

$\boxed{V_{4B} = \frac{V_{3B}}{a_3}}$

$= \frac{30kV}{30kV} \times 300$

$= 300V$

$= 15KV$ | $= 150KV \times \frac{30}{150}$ $= 30KV$ | $= 300V$

$V_{1B} = a_1 \cdot V_{2B}$ $= \frac{1}{10} \times 500$ $= 50KV$ | $V_{2B} = a_2 \cdot V_{3B}$ $= 500KV$ | $V_{3B} = 100KV$ | $V_{4B} = \frac{V_{3B}}{a_3}$ $= 100KV \cdot \frac{300}{30KV}$ $= 1000V$

→ Given or choose one zone voltage Base
↳ Use transformer ratios to find Base for other zones

3) Current Bases $S_B = V_B I_B$ → $I_B = \frac{S_B}{V_B}$

$I_{1B} = \frac{S_B}{V_{1B}}$ | $I_{2B} = \frac{S_B}{V_{2B}}$ | $I_{3B} = \frac{S_B}{V_{3B}}$ | $I_{4B} = \frac{S_B}{V_{4B}}$

4) Impedance Base $Z_B = \frac{V_B}{I_B} = \frac{V_B^2}{S_B}$

$Z_{1B} = \frac{V_{1B}^2}{S_B}$ | $Z_{2B} = \frac{V_{2B}^2}{S_B}$ | $Z_{3B} = \frac{V_{3B}^2}{S_B}$ | $Z_{4B} = \frac{V_{4B}^2}{S_B}$

$S_B = V_B I_B$
$V_B = Z_B I_B$

$$\text{P.u value} = \frac{\text{Actual value}}{\text{Base value.}}$$

→ S_{pu} → V_{pu} → I_{pu} → Z_{pu}

$S_{pu} = V_{pu} \cdot I_{pu}^*$

$S_{pu} = P_{pu} + jQ_{pu}$
$= \frac{P + jQ}{S_B}$
$= \frac{P}{S_B} + j\frac{Q}{S_B}$

$S = V \cdot I^*$

$$\frac{S}{S_B} = \frac{V \cdot I^*}{S_B} = \frac{V \cdot I^*}{V_B \cdot I_B}$$

↓

$$S_{pu} = \frac{V}{\ } \cdot \frac{I^*}{\ } = \frac{V}{\ } \cdot \left(\frac{I}{\ }\right)^*$$

$$\downarrow$$

$$S_{pu} = \frac{V}{V_B}\cdot\frac{I^*}{I_B} = \frac{V}{V_B}\cdot\left(\frac{I}{I_B}\right)^*$$

$$\boxed{S_{pu} = V_{pu}\cdot I_{pu}^*}$$

→ $V = I\cdot Z$

$$\frac{V}{V_B} = \frac{I\cdot Z}{V_B} = \frac{I\cdot Z}{I_B\cdot Z_B}$$

$$\boxed{V_{pu} = I_{pu}\cdot Z_{pu}}$$

→ P, Q, S all have the same power base.

↳ $S = VI^*$

$$\boxed{\frac{S}{S_B}} = \frac{VI^*}{V_B\cdot I_B}$$

$$\downarrow$$

$$\frac{P+jQ}{S_B} \rightarrow \frac{P}{S_B} + j\frac{Q}{S_B}$$

$$\rightarrow P_{pu} + jQ_{pu}$$

→ 10MW. $\cos\theta = 0.8$ lag

→ 12.5MVA.

$S_B \rightarrow$ 20MVA.

$$P_{pu} = \frac{10}{20} = 0.5\ pu.$$

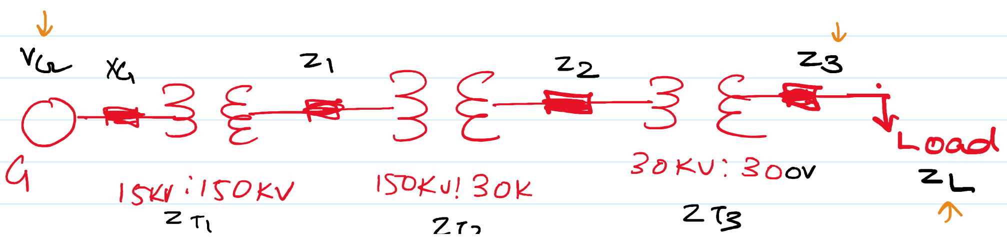
$$S_{pu} = \frac{12.5}{20}\angle 36.87 = 0.625\angle 36.87\ pu$$

$$Q_{pu} = \frac{Q_{actual}}{S_B} = \frac{10\tan 36.87}{20} = \frac{7.5}{20} =$$

$$= 0.375\ pu.$$

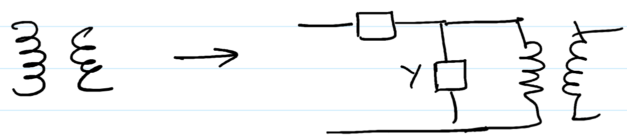
→ KVL & KCL for per unit values

→ Actual quantity is found by multiplying the pu value with the corresponding base value.



G 15kV : 150kV 150kV : 30k 30kV : 300V Load

Z_{T1} Z_{T2} Z_{T3} Z_L



$\leftarrow S_B \rightarrow$ 10 MW

$V_{1B} = 15kV$	$V_{2B} = 150kV$	$V_{3B} = 30kV$	$V_{4B} = 300V$
$I_{1B} = \frac{S_B}{V_{1B}}$	$I_{2B} = \frac{S_B}{V_{2B}}$	$I_{3B} = \frac{S_B}{V_{3B}}$	$I_{4B} = \frac{S_B}{V_{4B}}$
$Z_{1B} = \frac{V_{1B}^2}{S_B}$	$Z_{2B} = \frac{V_{2B}^2}{S_B}$	$Z_{3B} = \frac{V_{3B}^2}{S_B}$	$Z_{4B} = \frac{V_{4B}^2}{S_B}$
$V_{Gpu} = \frac{V_G}{V_{1B}}$	$Z_{1pu} = \frac{Z_1}{Z_{2B}}$	$Z_{2pu} = \frac{Z_2}{Z_{3B}}$	$Z_{3pu} = \frac{Z_3}{Z_{4B}}$
$X_{Gpu} = \frac{X_G}{Z_{1B}}$			$Z_{Lpu} = \frac{Z_L}{Z_{4B}}$

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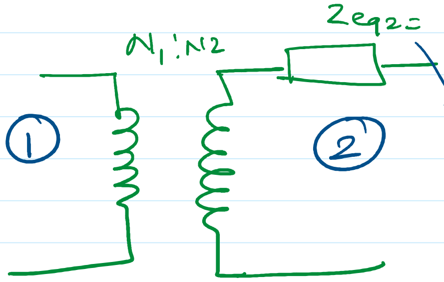
eg →

$Z_{eq2} = 0.0525 \angle 78.13^\circ$



Z_{pu} ?

480/120V , S = 20kVA , Z in p.u.

Transformer rating as Base values.

$S_B = 20kVA$

$V_{1B} = 480V$ | $V_{2B} = \frac{120}{\cancel{480}} \times \cancel{480}$

$= 120V$

$Z_{1B} = \frac{V_{1B}^2}{}$ 2

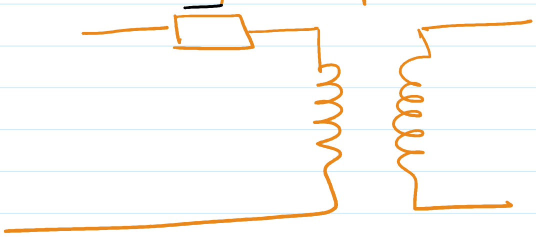
$= 120V$

$$Z_{1B} = \frac{V_{1B}^2}{S_B} = 11.52\,\Omega$$

$$Z_{2B} = \frac{V_{2B}^2}{S_B} = 0.72\,\Omega$$

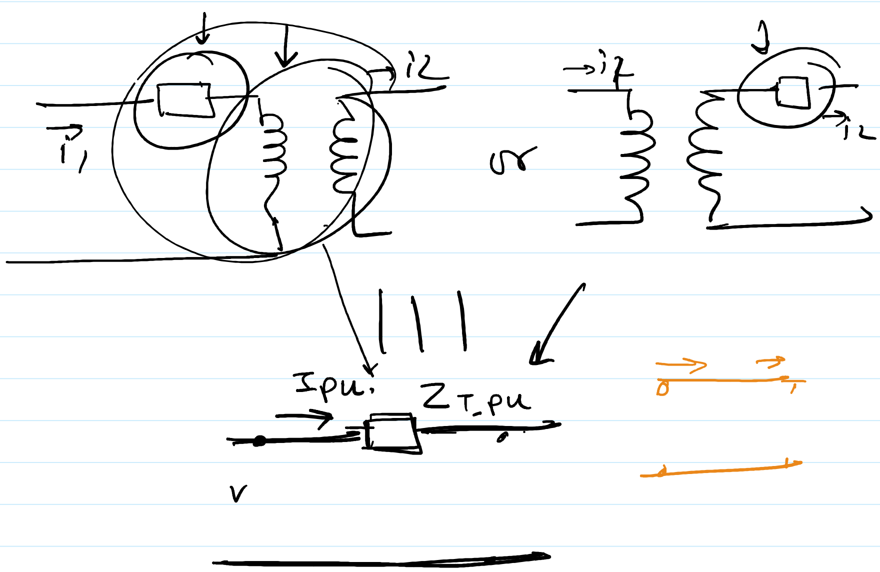
$$Z_{pu} = \frac{0.0525\angle 78.13^\circ}{Z_{2B}} = \underline{0.0729\angle 78.13^\circ \text{ p.u.}}$$

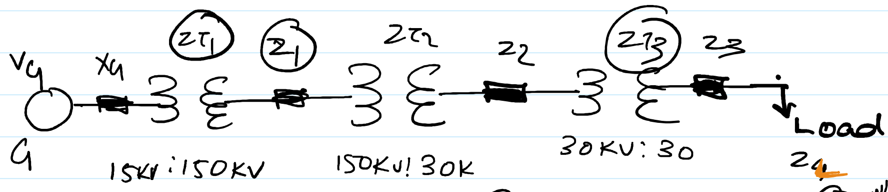
$\underline{Z_1} = a^2 Z_{eq2}$

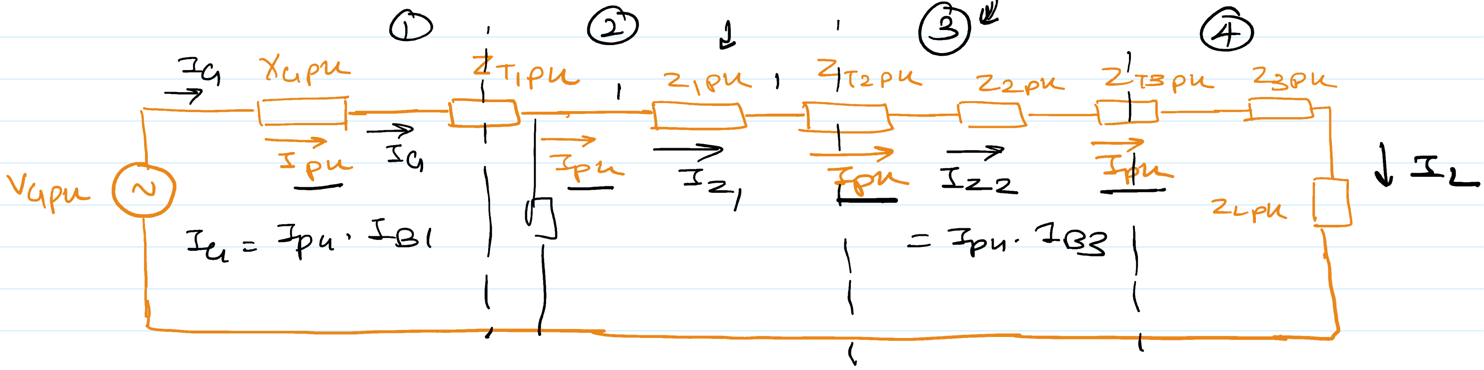


$$Z_1 = \left(\frac{480}{120}\right)^2 \times 0.0525\angle 78.13 = \underline{0.84}\angle 78.13^\circ\,\Omega$$

$$\underline{Z_{1pu}} = \frac{Z_1}{Z_{1B}} = \frac{0.84\angle 78.13}{11.52} = \underline{0.0729\angle 78.13^\circ \text{ p.u.}}$$







per unit impedance diagram

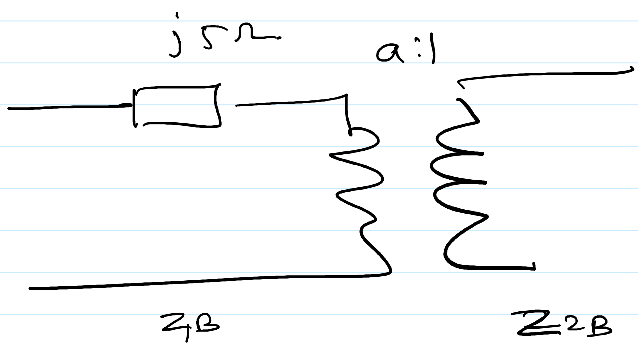
$$I_{pu} = \frac{V_{G\,pu}}{Z_{Total\,pu}}.$$

$$I_G = I_{pu} \times I_{B1}$$

$$I_{Z_1} = I_{pu} \times I_{B2}$$

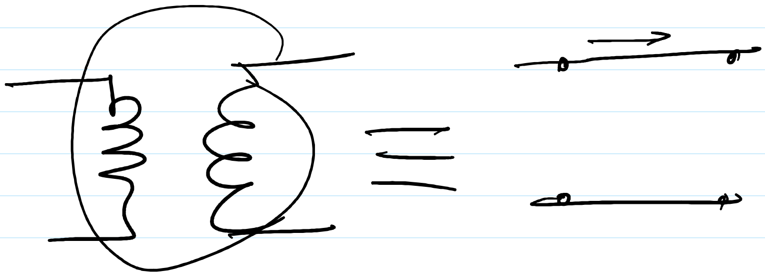
$$I_{Z_2} = I_{pu} \times I_{B3}$$

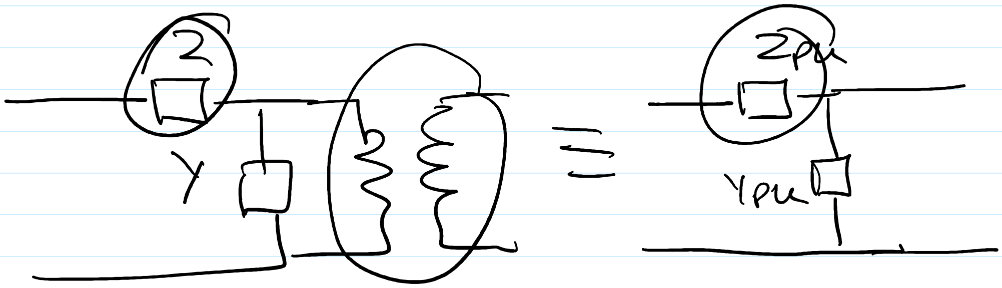
$$I_L = I_{pu} \times I_{B4}$$



$$Z_{T\,pu} = \frac{j5}{Z_{1B}}$$

$$Z_{T\,pu} = \frac{j5/a^2}{Z_{2B}}$$




Z
Y
Zpu
Ypu