2.1 AC Fundamentals

Tuesday, January 18, 2022 10:33 AM

EE2029: Introduction to Electrical Energy System

AC Fundamentals

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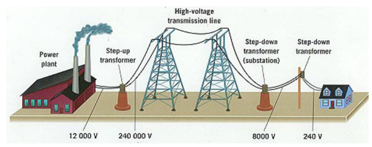
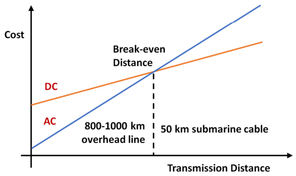
Why AC and not DC???

- Transformers allow easy transformation of voltage



- Break-even distance for high voltage direct current (HVDC)



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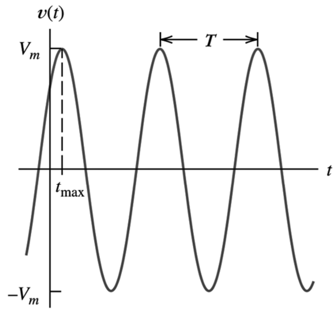
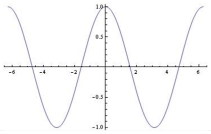
Why a Sinusoidal Alternating Voltage?

- Easily generated using the synchronous generator
- Basic operations: +, -, x, division, differentiation, integration
  - These operations will result in another sinusoid of same frequency and shape
  - Any periodic signal can be represented by a linear combination of sinusoidal waveforms (remember Fourier Series!!!)





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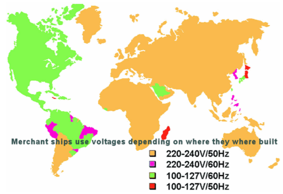
Choice of Supply Frequency

- 50 Hz and 60 Hz
- Today :
  - 60 Hz in North America, Brazil and Japan (which also uses 50 Hz!!) etc
  - 50 Hz in other countries
- Exceptions:
  - 25 Hz Railways (Amtrak)
  - 16$\frac{2}{3}$ Hz Railways
  - 400 Hz Oil rigs, ships and airplanes

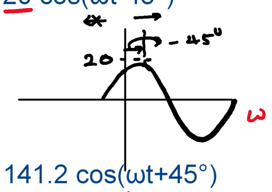
- A too low frequency like 10 Hz or 20 Hz causes flicker
- A too high frequency
  - Increases cable and line impedance
  - Increases the hysteresis losses
  - Increases eddy current losses



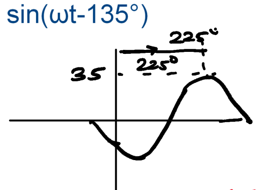
Plot the following curves

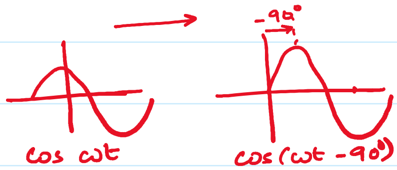
$\cos(\omega t-90-135)$
$\cos(\omega t-225^\circ)$

- 20 cos(ωt-45°) $\omega t+(-45)$

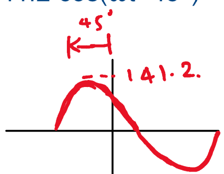


- 35 sin(ωt-135°)



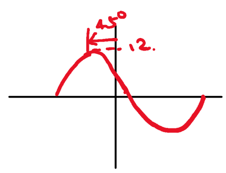


- 141.2 cos(ωt+45°)



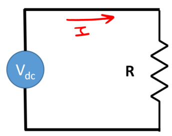
- 12 sin(ωt+135°)

$12\cos(\omega t-90+135^\circ)$
$12\cos(\omega t+45^\circ)$



How do we represent AC Signals? SINUSOID - NO SINGLE VALUE

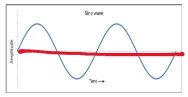
- It is desirable to have same form of equation for power in both a.c. and d.c. circuits mainly because of
  - Convenience
  - Consistence
- For a DC Circuit with a resistance R and voltage source V

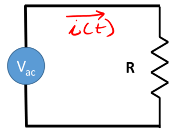


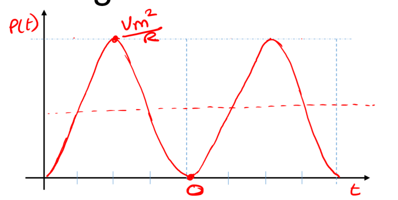
$I_{dc} = \frac{V_{dc}}{R}$ — (1)

$P_{dc} = V_{dc} \cdot I_{dc} \Rightarrow P_{dc} = \frac{(V_{dc})^2}{R}$ — (2)

How do we represent AC Signals?







AVERAGE VALUE = 0

$i(t) = \frac{V_{ac}}{R}$

$V_{ac} = V_m \sin\omega t$

$\Rightarrow \; i(t) = \frac{V_m \sin\omega t}{R}$

$= \left(\frac{V_m}{R}\right) \sin\omega t$ — (3)

$P_{ac} = V_{ac} \cdot i(t) = \frac{V_m^2}{R} \sin^2\omega t$ — (4)

$P_{avg} = \left(\frac{V_m^2}{R} + 0\right)/2 = \frac{V_m^2}{2R}$

$P_{avg} = \frac{V_m}{\sqrt{2}} \cdot \frac{V_m}{\sqrt{2}R} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$

ROOT MEAN SQUARE OF A SINUSOID

$\Rightarrow \boxed{P_{ac} = V_{rms} \cdot I_{rms}}$

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RMS values in AC circuits

- The use of the RMS value gives us the DC equivalent AC power equation i.e. (Resistor)

  $P_{ac} = V_{rms} \cdot I_{rms}$

- An AC voltage source's value is the RMS value by default
  - In Singapore, the voltage supply at households is mentioned as 230V/50Hz

$V_{rms} = 230V$

$V_{max} = \sqrt{2} V_{rms} = \sqrt{2} \cdot 230$

$= 325V$

325V

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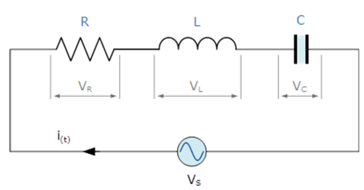
A Typical AC circuit Analysis



$V_R = i \cdot R$ $\qquad V_s = V_m \cos(\omega t + \theta)$

$V_L = L\frac{di}{dt}$

$V_C = \frac{1}{C}\int i\,dt$

KVL FOR THIS CIRCUIT

$V_s = V_R + V_L + V_C$

$V_m \cos(\omega t + \theta) = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$

DIFFERENTIAL EQUATIONS?

COMMON PARAMETER – FREQUENCY

PHASE AND AMPLITUDE CHANGE

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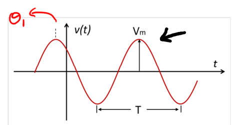
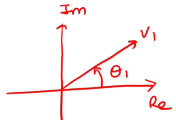
Phasor Representation of a Sinusoid

- Time Function: $v_1(t) = V_m\cos(\omega t + \theta_1)$
- Phasor: $\mathbf{V_1}=V_1\angle\theta_1$, here $\bar{V_1}$ is the RMS value of the voltage



$v_1 = \frac{v_m}{\sqrt{2}}$



+ COUNTER CLOCKWISE

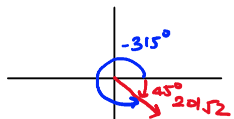
− CLOCKWISE

- Rotating Vector with
  - Length representing the rms value of the waveform
  - Angle representing the phase of the waveform
- The phasor for a sinusoid is a snapshot of the corresponding rotating vector at t=0 with its rms values
- Time domain signal is expressed as cosine function

Find and draw the phasors of the following curves

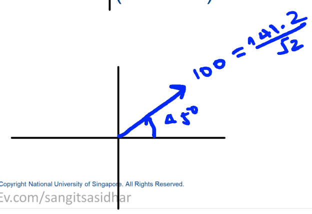
- 20 cos(ωt-45°)

II | I
III | IV

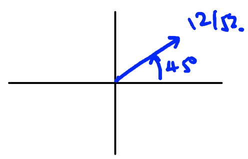


- 35 sin(ωt-135°) 35 cos(ωt -90-135)
  35 cos(ωt -225)

- 141.2 cos(ωt+45°)



- 12 sin(ωt+135°) = 12 cos(ωt -90 +135)
  = 12 cos(ωt +45)

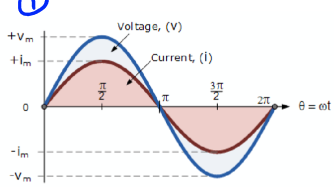


20 cos (ωt - 45)

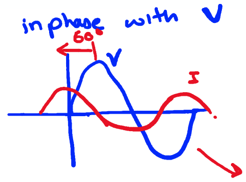
RMS value is 20V → 20√2 cos(ωt - 90°)
Phase is -90°

Phase Relationships between Sinusoids

1

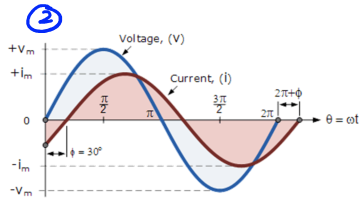


I is in phase with V



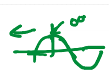
I leads V by 60°

2



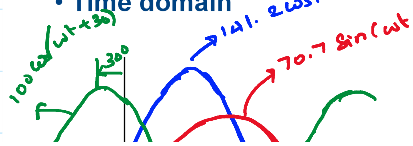
-30°

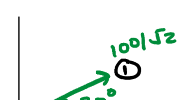
I lags V by 30°

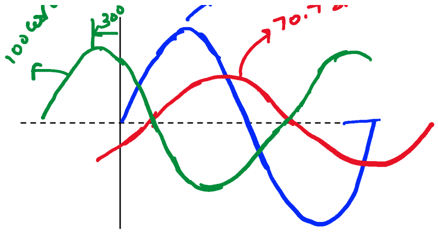


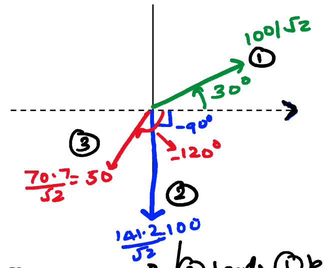
Find and draw the time domain and the phasor of the 141.2 cos(ωt-90°), 70.7 sin(ωt-30°) &100 cos(ωt+30°) curves on the same axis!!!

- **Time domain**
- **Phasors!!!**









② lags ① by 120°
③ lags ① by 150°

③ leads ① by 210°

Impedance ➔ Complex Resistance

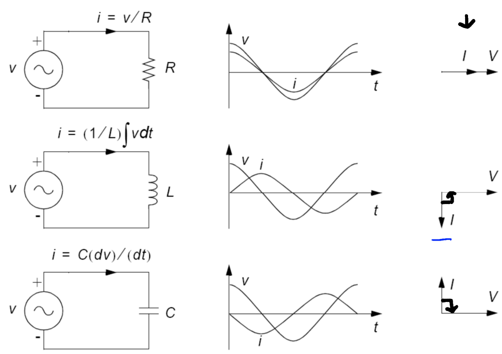
- $DC: \ Resistance\ (R) \ = \dfrac{Voltage\ (V)}{Current\ (I)}$

- $AC: Impedance\ (Z) \ = \dfrac{Voltage\ Phasor(V)}{Current\ Phasor(I)}$

DC : Resistance (R) = $\dfrac{\text{Voltage}}{\text{Current}}$

AC : Impedance (Z) = $\dfrac{\text{Voltage Phasor (V)}}{\text{Current Phasor (I)}}$

Time ⟷ Phasor



Element	Voltage	Current	Impedance
R	$V\angle 0°$	$\frac{V}{R}\angle 0°$	$R\angle 0°$
L	$V\angle 0°$	$\frac{V}{\omega L}\angle -90°$	$\omega L\angle 90°$ $j\omega L$
C	$V\angle 0°$	$\omega C V\angle 90°$	$\frac{1}{\omega C}\angle -90°$ $\frac{-j}{\omega C}$

$\omega = 2\pi f$

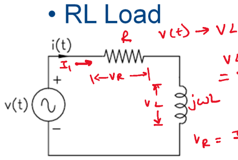
$|I_L| = 50A.$ $\quad I_L = 50\angle -90°$

$V_L = 250\angle 0°$

$\rightarrow 250\angle -90° \rightarrow I_L = 50\angle -180°$

Complex Impedance

- RL Load



$v(t) \rightarrow V\angle 0°$

$V\angle 0° = V_R + V_L$

$V_R = I_1 \cdot R$

$V_L = I_1 (j\omega L)$
$= I_1 \cdot (\omega L\angle 90°)$

$Z_1 = R + j\omega L$

$Z_1 = |Z_1|\angle\theta_1$

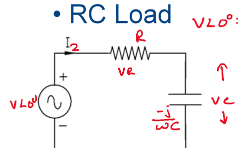
$|Z_1| = \sqrt{R^2 + \omega^2 L^2}$

$\theta_1 = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$I_1 = \frac{V\angle 0°}{|Z_1|\angle\theta_1} = \left|\frac{V}{Z_1}\right|\angle 0° - \theta_1$

$0° \leftarrow \angle I_1 \rightarrow -90°$

- RC Load



$V\angle 0° = V_R + V_C$

$Z_2 = R + \left(-\frac{j}{\omega C}\right)$

$= |Z_2|\angle\theta_2$

$|Z_2| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

$\theta_2 = \tan^{-1}\left(\frac{-1/\omega C}{R}\right)$

$I_2 = \frac{V\angle 0}{|Z_2|\angle\theta_2} = \left|\frac{V}{Z_2}\right|\angle 0 - \theta_2.$

$0 \leftarrow \angle I_2 \rightarrow 90°$

$I_1 = \frac{V\angle 0^\circ}{|Z_1|\angle\theta_1} = \left|\frac{V}{Z_1}\right|\angle 0^\circ - \theta_1$

$0^\circ \leftarrow \angle I_1 \rightarrow -90^\circ$

I_1 lags V by θ_1

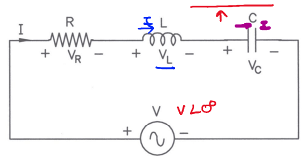
$I_2 = \frac{V\angle 0}{|Z_2|\angle\theta_2} = \left|\frac{V}{Z_2}\right|\angle 0 - \theta_2.$

$0 \leftarrow \angle I_2 \longrightarrow 90^\circ$

I_2 leads V by θ_2

Series RLC Circuit

• Find the impedance of the following circuit. Draw the impedance diagram. What is the current if $|Z_L| = |Z_C|$?



$Z_L \rightarrow j\omega L \rightarrow \omega L\angle 90^\circ$

$Z_C = \frac{-j}{\omega C} \rightarrow \frac{1}{\omega C}\angle -90^\circ$

$Z_R = R \rightarrow R\angle 0^\circ$

$Z = Z_R + Z_L + Z_C$

$= R + j\omega L - \frac{j}{\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$

$= R + \omega L\angle 90^\circ + \frac{1}{\omega C}\angle -90^\circ$

Series RLC Circuit

$Z = |Z|\angle\theta$

$\angle Z = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$

IF $\omega L > \frac{1}{\omega C} \rightarrow \angle Z \rightarrow$ +ve.

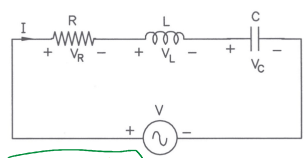
IF $\omega L < \frac{1}{\omega C} \rightarrow \angle Z \rightarrow$ -ve.

$\angle Z$	$\angle I$
+ve	-ve.
-ve	+ve.

$I = \frac{|V|\angle 0^\circ}{|Z|\angle\theta^\circ} = \frac{|V|}{|Z|}\angle -\theta^\circ$

IF $\omega L > \frac{1}{\omega C} \rightarrow \angle I \rightarrow$ -ve.

IF $\omega L < \frac{1}{\omega C} \rightarrow \angle I \rightarrow$ +ve.



$\boxed{\omega L = \frac{1}{\omega C}}$

$\omega^2 = \frac{1}{LC} \rightarrow \omega = \frac{1}{\sqrt{LC}}$

$f = \frac{1}{2\pi\sqrt{LC}}$

$Z = R + j\left(\cancel{\omega L} - \cancel{\frac{1}{\omega C}}\right) = R\angle 0^\circ$

$I = \frac{V\angle 0^\circ}{R\angle 0^\circ} = \left|\frac{V}{R}\right|$

Resonance.

$V = |V|\angle 0^\circ$

Voltage Phasors and KVL

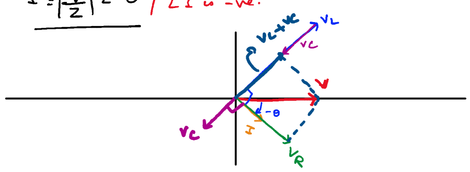
$V = V_R + V_L + V_C$

$V_R = I.R. = \frac{|V|}{|Z|}.R\angle -\theta^\circ$

$V_L = I.\omega L\angle 90^\circ = \left|\frac{V}{Z}\right|.\omega L\angle 90^\circ - \theta^\circ$

$V_C = I.\frac{1}{\omega C}\angle -90^\circ = \left|\frac{V}{Z}\right|.\frac{1}{\omega C}\angle -90^\circ - \theta^\circ$

$I = \left|\frac{V}{Z}\right|\angle -\theta^\circ$ / $\angle I$ is -ve.



$|Z_L| = |Z_C| \Rightarrow \omega L = \frac{1}{\omega C}$

$I = \left|\frac{V}{R}\right|\angle 0^\circ$

$\frac{V\omega L}{R} = \frac{V}{\omega RC}$

$V_R = I.R$

$= \left|\frac{V}{R}\right|.R\angle 0^\circ$

$= V\angle 0^\circ$

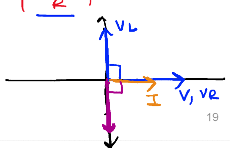
$V_L = I.\omega L\angle 90^\circ$

$= \frac{V}{R}.\omega L\angle 90^\circ$

$= \left|\frac{\omega LV}{R}\right|\angle 90^\circ$

$V_C = I.\frac{1}{\omega C}\angle -90^\circ$

$= \left|\frac{V}{\omega RC}\right|\angle -90^\circ$



$V = V_R + V_L + V_C$

Fourier Series Representation of a Square wave signal

3rd.

5th.

Fourier Series Representation of a Square Wave signal