1.2 Pre-requisites Primer: Complex Numbers

Wednesday, January 12, 2022 10:33 PM

$z = \underset{\text{Real}}{\bar{a}} + j\underset{\text{Real.}}{\bar{b}} \rightarrow$ Cartesian form

PollEv.com/sangitsasidhar

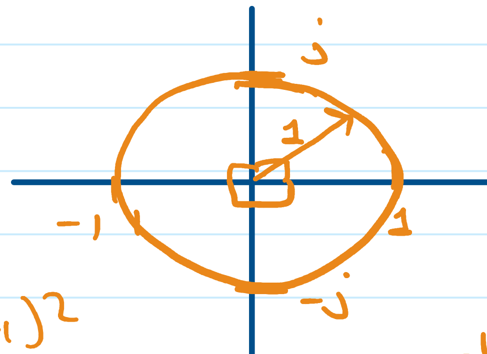
$\neq i \rightarrow$ instantaneous current

$j = \sqrt{-1}$

$F(x^n) \rightarrow$ n roots.

$x^2 - 1 = 0 \rightarrow 1, -1$

$x^4 - 1 = 0 \rightarrow 1, -1$



$j^4 = j^2 \cdot j^2 = (\sqrt{-1})^2 (\sqrt{-1})^2$
$= -1 \times -1 = 1$

$(-j)^4 = (-\sqrt{-1})^4 = 1$

$x^6 - 1 = 0 \rightarrow$ 6 roots.

$\frac{360^\circ}{6} \rightarrow 60^\circ$

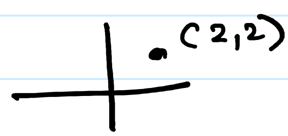
$1\angle 0^\circ, 1\angle 60^\circ, 1\angle 120^\circ$
$1\angle 180^\circ, 1\angle 240^\circ, 1\angle 300^\circ$

$j^2 = -1$

$j^3 = -1 \cdot j = -j$

$j^4 = (-1)(-1) = 1$

$j^5 = j^4 \cdot j = j$

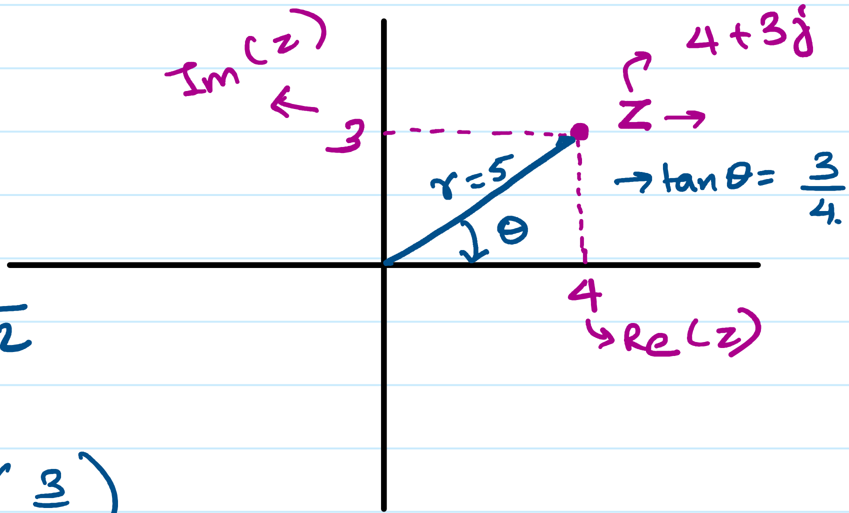


★ Properties of complex numbers.

$Im(z)$

$4+3j$

$z = 4 + 3j$
$= r\angle\theta$



$r = \sqrt{4^2 + 3^2}$
$= 5$

$\theta = \tan^{-1}\left(\frac{3}{4}\right)$

$z_1 = r_1\angle\theta_1$ $\qquad$ $z_2 = r_2\angle\theta_2$,

$z_1 = a_1 + jb_1$ $\qquad$ $z_2 = a_2 + jb_2$,

→ Multiplication,

$z_1 \cdot z_2 = (a_1 + jb_1)(a_2 + jb_2)$
$= a_1a_2 + jb_1a_2 + jb_2a_1 - b_1b_2$

$z_1 \cdot z_2 = r_1\angle\theta_1 \cdot r_2\angle\theta_2$
$= r_1 \cdot r_2 \angle(\theta_1 + \theta_2)$

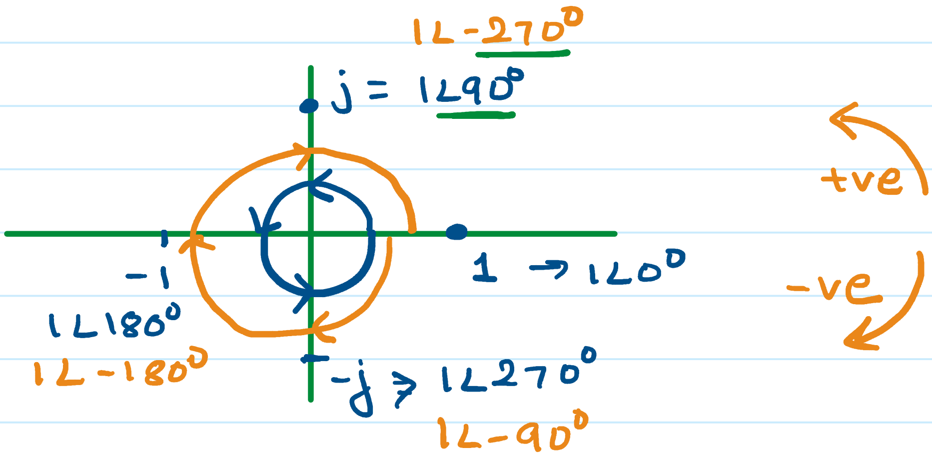
$1 \rightarrow 1\angle 0^\circ$
$-1 \rightarrow 1\angle 180^\circ$

→ Division

$$\frac{z_1}{z_2} = \frac{a_1 + jb_1}{a_2 + jb_2} = \frac{(a_1+jb_1)(a_2-jb_2)}{a_2^2 + b_2^2}$$

$$\frac{z_1}{z_2} = \frac{r_1\angle\theta_1}{r_2\angle\theta_2} = \frac{r_1}{r_2}\angle(\theta_1 - \theta_2)$$

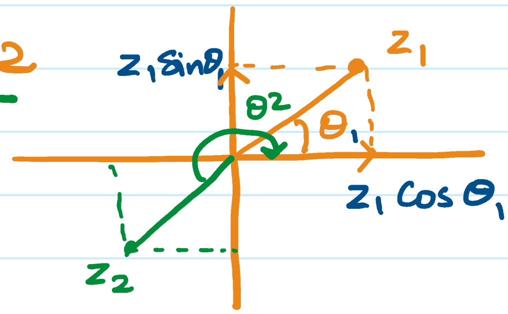
$1\angle -270^\circ$



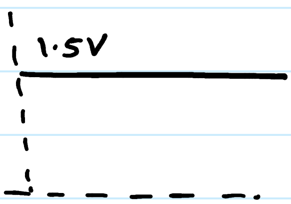
Addition

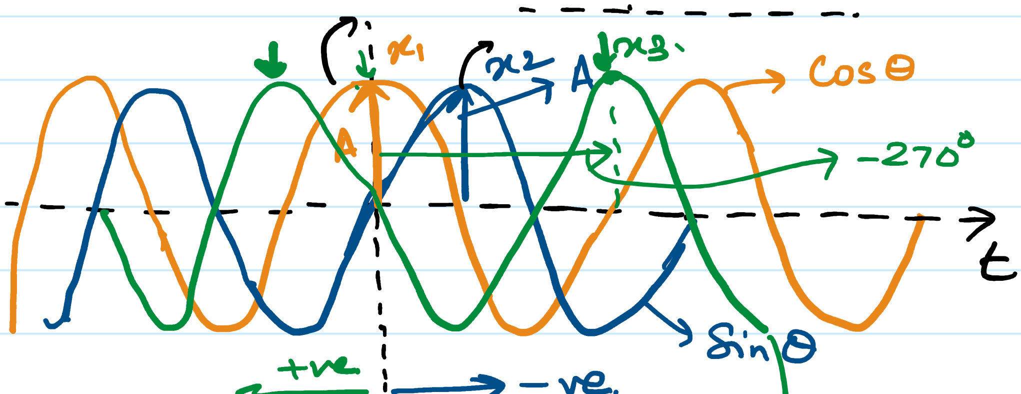
$$\underline{z_1\angle\theta_1 \;-\; z_2\angle\theta_2}$$

$$(a_1+jb_1)-(a_2+jb_2)$$



→ AC vs DC.





Reference wave → cos wave.

$-\sin\theta$.

$-\sin\theta$.
$\cos(\theta + 90^\circ)$

$x_1 = A\cos(2\pi f t)$
$= A\cos\omega t$

$\rightarrow \omega \rightarrow$ Angular frequency
$= 2\pi f$
$\rightarrow$ rad/sec

$$x_2 = A\cos\left(\omega t - \frac{\pi}{2}\right) = A\cos(\omega t - 90^\circ)$$

$$x_3 = A\cos\left(\omega t + \frac{\pi}{2}\right) = A\cos(\omega t + 90^\circ)$$

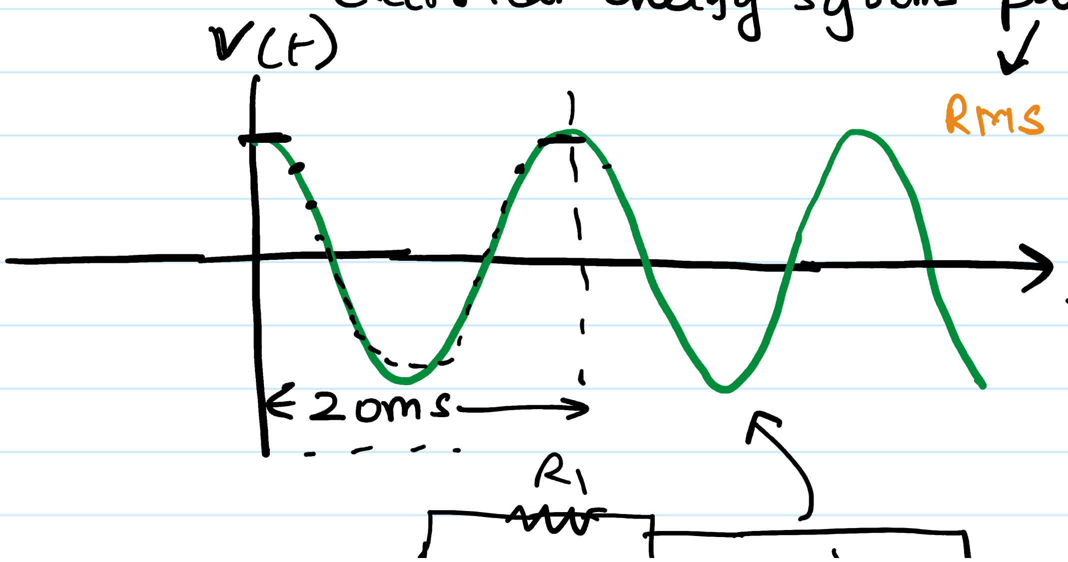
$$= A\cos(\omega t - 270^\circ)$$

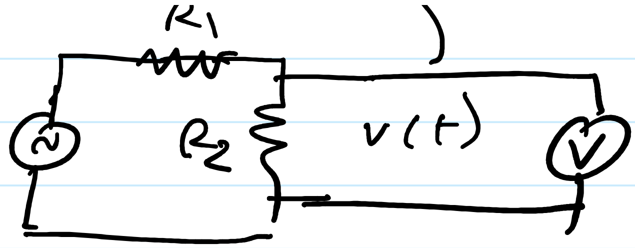
$$x(t) = A\cos(\omega t + \theta)$$

$A \rightarrow$ Amplitude
$\omega \rightarrow$ Frequency
$\theta \rightarrow$ Phase.

Frequency $\rightarrow$ Singapore $\rightarrow$ 50Hz.
Frequency $\rightarrow$ US $\rightarrow$ 60Hz.
Frequency $\rightarrow$ Constant for a system

$\rightarrow$ Phasors $\rightarrow$ Mathematical representation of electrical energy systems parameters
parameters $\rightarrow$ RMS
parameters $\rightarrow$ Phase.



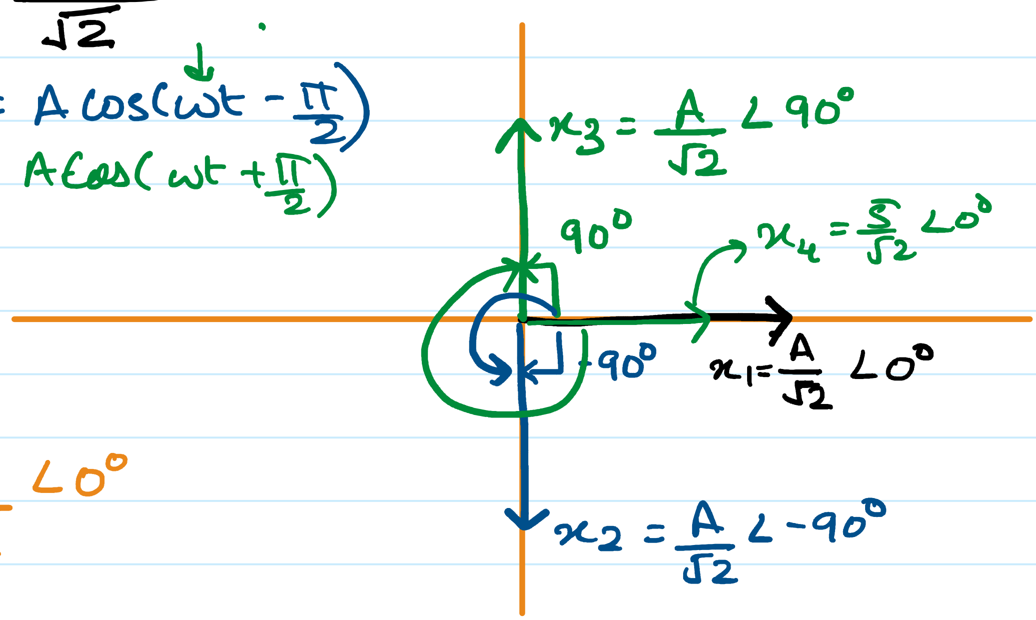


$$\text{RMS} = \frac{\text{Peak}}{\sqrt{2}}$$

$$x_1 = A\cos(\omega t + 0^\circ)$$

$$x_2 = A\cos(\omega t - \tfrac{\pi}{2})$$

$$x_3 = A\cos(\omega t + \tfrac{\pi}{2})$$



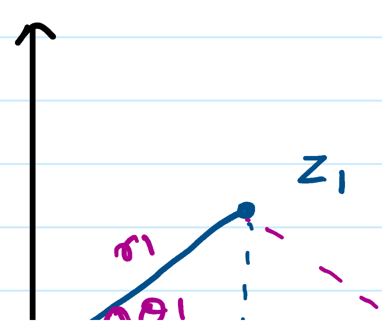
$$x_1 = \frac{A}{\sqrt{2}} \angle 0^\circ$$

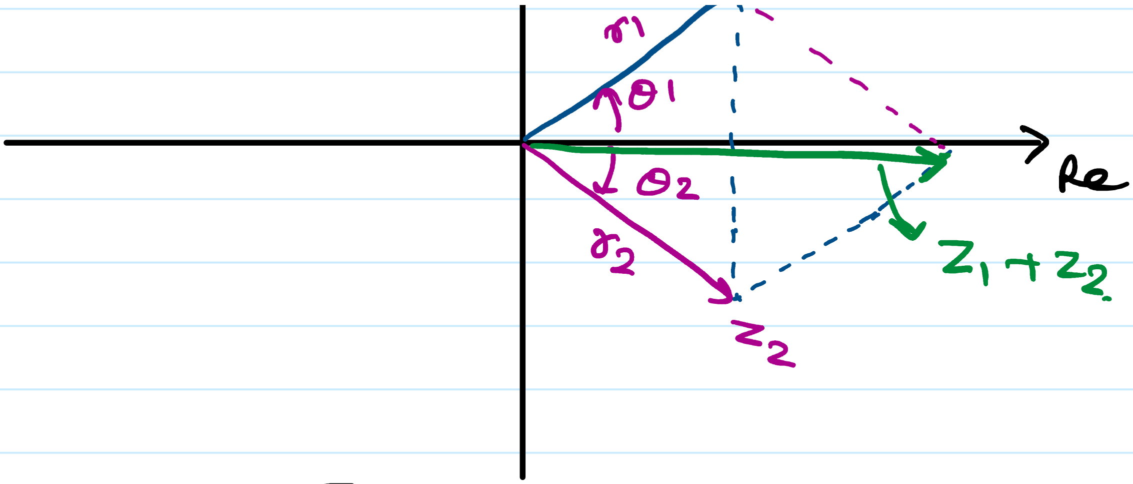
$$x_2 = \frac{A}{\sqrt{2}} \angle -90^\circ = \frac{A}{\sqrt{2}} \angle 270^\circ$$

$$x_3 = \frac{A}{\sqrt{2}} \angle +90^\circ = \frac{A}{\sqrt{2}} \angle -270^\circ$$

$$x_4 = 5\sin(\omega t + 90^\circ) \rightarrow 5\cos(\omega t - \cancel{90} + \cancel{90})$$
$$= \frac{5}{\sqrt{2}} \angle 0^\circ$$

*





$z_1 + z_2$