

# 5.1 Generators

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## EE2029: Introduction to Electrical Energy System Generators

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Department of Electrical and Computer Engineering



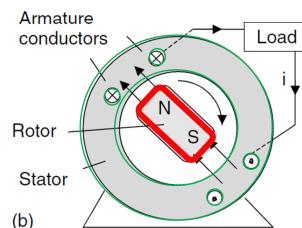
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## Main Components of a Generator

### Rotor

- Moving part that is usually made of electromagnet materials.



### Stator

- Stationary part that contains a set of conductors called 'armature winding'.

- indicates the positive current is directed out of plane of the paper.
- × indicates the positive currents is directed into the plane of the paper.

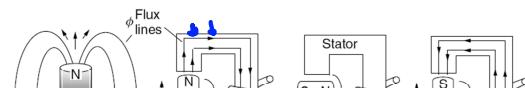


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## Electromagnetic Induction

- Moving a conductor through a magnetic field.

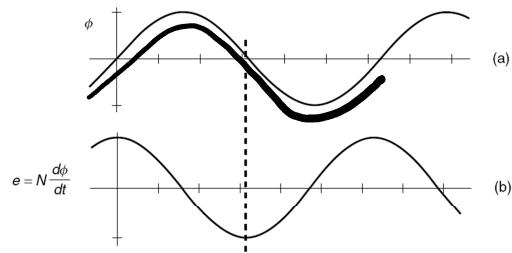
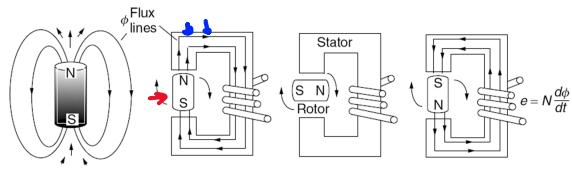
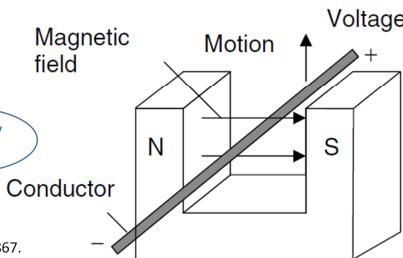


- Moving a conductor through a magnetic field.
- Induced electromotive force (EMF), voltage generated by the magnetic force across wire.
- Faraday's law:  $e = N \frac{d\phi}{dt}$



Electricity rocks!

Michael Faraday,  
English chemist and physicist, 1791-1867.  
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## Rotor Speed

with a p-pole machine.  $\rightarrow \frac{p}{2}$  pole pairs

Speed of the rotor =  $n$  rpm [rotations per minute]

Rotations per second =  $\frac{n}{60}$

Frequency  $\rightarrow$  No. of waves every second

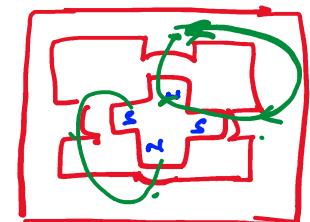
$\frac{p}{2}$  pole pairs  $\rightarrow \frac{p}{2}$  waves every rotation.

$$f = \frac{p}{2} \times \frac{n}{60}$$

$$f = \frac{np}{120}$$

f: voltage frequency (Hz)  
n: rotor speed (rpm)  
p: number of poles

$$\hookrightarrow n = \frac{120f}{p}$$



1 rotation,  
2 waveforms

Any given m/c  $\rightarrow p$



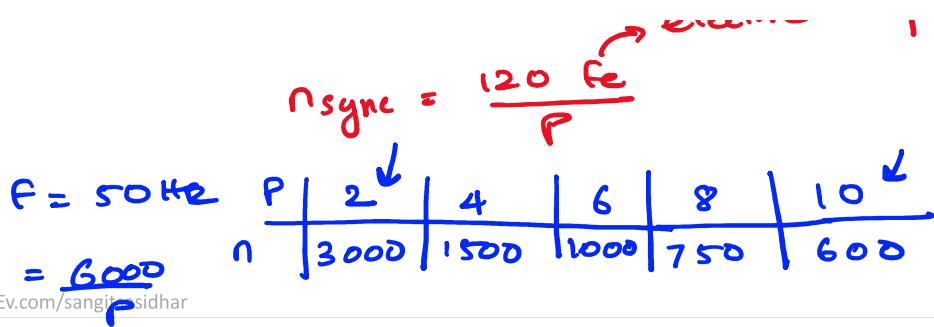
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## Synchronous Speed

- All generators connected to the system must produce AC voltage at the same frequency,  $f_e$ .
- This implies that a generator must run at a constant speed.
- We refer to this rotor speed as 'synchronous speed',  $n_{sync}$ .
- A synchronous speed is found from:

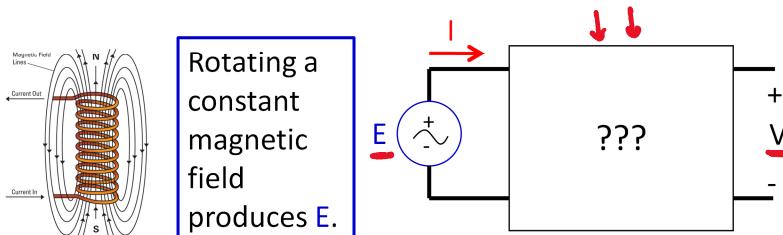
$$n_{sync} = \frac{120 f_e}{p}$$

electrical freq.



## Excitation Voltage (E)

- Excitation voltage (E) or induced EMF is caused by an induced magnetic flux in the air-gap.
- The magnetic flux on the rotor is created by a field winding at the rotor.



Source:  
<http://www.lanl.gov/news/index.php/fuseaction/1663.article/d/20085/d/13276>

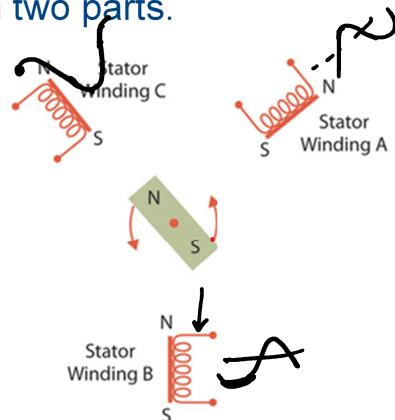
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## Magnetic Flux in the Air-Gap

- Magnetic Flux in the air-gap comes from two parts.

Airgap leakage flux.  
 → leakage reactance. ( $X_L$ )



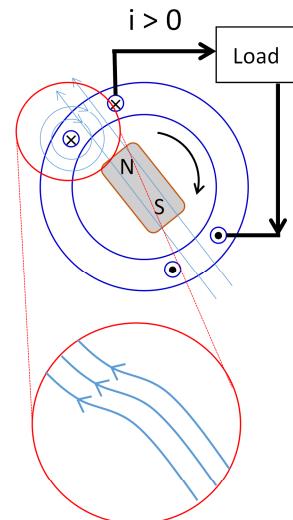
Source: <http://www.ecnmag.com/article-brushless-dc-motor-control-111609.aspx>

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## Armature Reaction (X<sub>a</sub>)

- Magnetic flux created from stator(armature) currents opposes the magnetic flux from field current.
- Flux linkage losses as a result of armature reaction are represented by an inductance called **armature reactance, X<sub>a</sub>**.



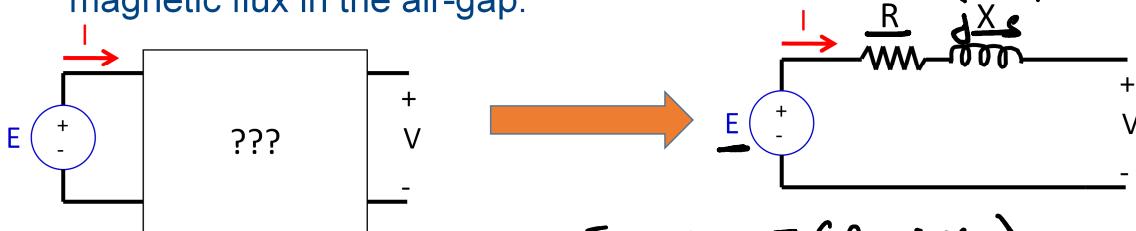
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## An Equivalent Circuit

- R = resistance in the armature winding.
- X = synchronous reactance, representing flux linkage losses with a leakage reactance in the air-gap, X<sub>i</sub> and the armature reaction, X<sub>a</sub>.
- E = Excitation voltage or induced EMF is caused by an induced magnetic flux in the air-gap.

$$x_s = x_a + x_i$$



$$E = V + I(R + jX_s)$$

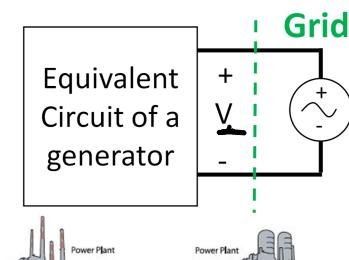
$$V = E - I(R + jX_s)$$

V, E, I → Phasors

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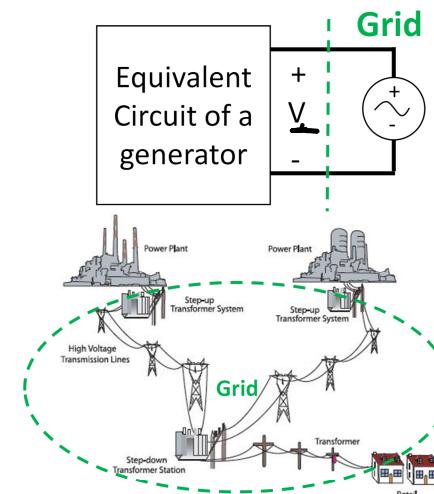
## Connecting A Generator to the Grid

- Four conditions need to be met before connecting a generator to the grid:
  - The three-phase voltage must have the same **frequency** as the grid.
  - The three-phase voltage must have the same **amplitude** at its terminals as the one of the grid voltage.



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  - The three-phase voltage must have the same **amplitude** at its terminals as the one of the grid voltage.
  - The three-phase voltage must have the same **phase sequence** as the grid voltage.
  - The three-phase voltage must be **in phase** with the grid voltage.

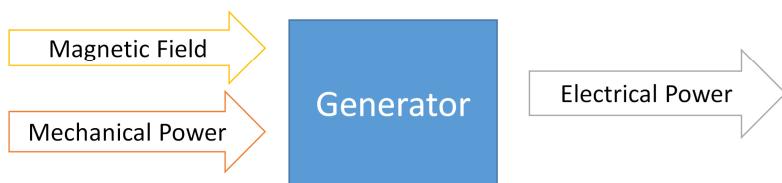


Source:  
[http://www.thermalfluidscentral.org/encyclopedia/index.php/Generation,\\_Transmission,\\_and\\_Distribution\\_of\\_Electricity](http://www.thermalfluidscentral.org/encyclopedia/index.php/Generation,_Transmission,_and_Distribution_of_Electricity)

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## Generator Operation

- In order for a generator to work, we need two inputs:-
  - Magnetic field at the rotor.
  - Mechanical power to turn the rotor.



For large generators, we use AC electrical power output and pass it through a rectifier to create DC currents to supply magnetic field circuit.



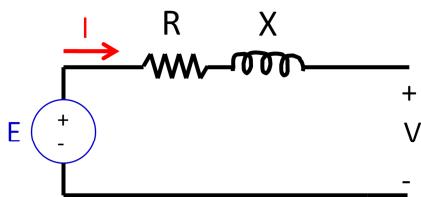
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## An Equivalent Circuit

- $R$  = resistance in the armature winding.
- $X$  = synchronous reactance, representing flux linkage losses with a leakage reactance in the airgap,  $X_l$  and the armature reaction,  $X_a$ .
- $E$  = Excitation voltage or induced EMF is caused by an induced magnetic flux in the air-gap.

- L - Excitation voltage or induced EMF is caused by an induced magnetic flux in the air-gap.



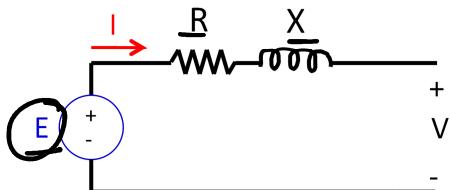
$$E = V + I(R + jX)$$



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## No Load Operation



No load  $\rightarrow I = 0$

$$\bar{E} = V + I(R + jX)$$

$$E = V$$

$E \rightarrow V$

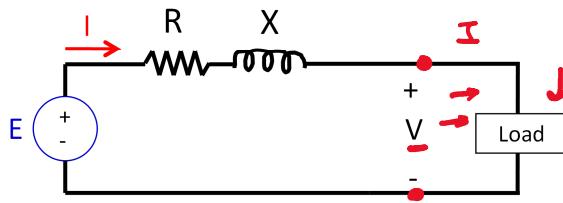
No power exchange with the grid



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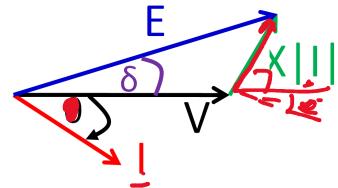
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## Loaded Operation



Generator injects power into the grid

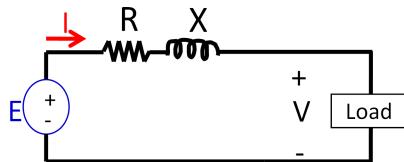
Assumption: Resistance 'R' in the armature winding is negligible.



$\delta$  is called 'power angle', we'll see why.

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## Phasor Diagram for



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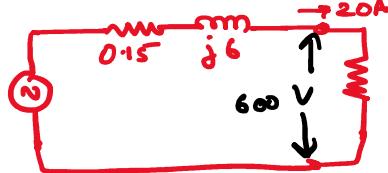
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## Example

Given an equivalent circuit of three-phase wye-connected synchronous generator with a terminal voltage of 600 V per phase, stator reactance of 6  $\Omega$  per phase and an armature resistance of 0.15  $\Omega$  per phase. If the machine is connected to a resistive load that draws 20 A, find the internal EMF and draw phasor diagram.

$$E = V + I(0.15 + j6)$$

$$V = 600 \angle 0^\circ, I = 20 \angle 0^\circ$$

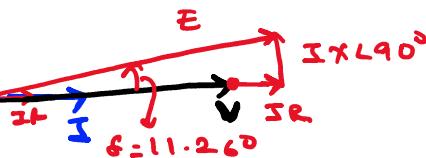


$$V = 600 \angle 0^\circ, I = 20 \angle 0^\circ$$

$$E = 600 \angle 0^\circ + 20 \angle 0^\circ (0.15 + j6)$$

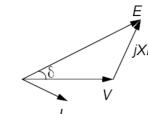
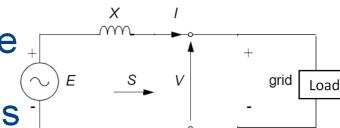
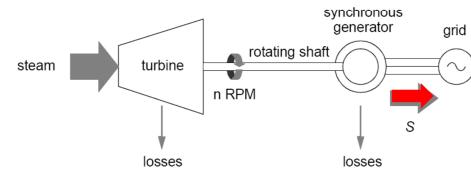
$$= 614.8 \angle 11.26^\circ V$$

$$= 600 \angle 0^\circ + \frac{3 \angle 0^\circ}{IR} + \frac{120 \angle 90^\circ}{IX \angle 90^\circ}$$

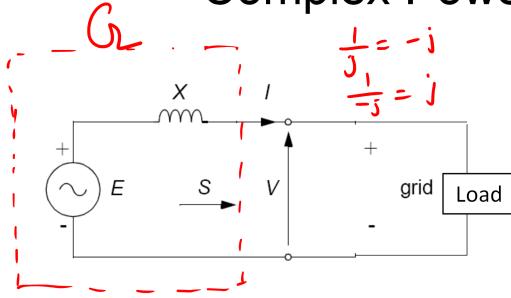


## Complex Power Supplied

- Complex power 'S' supplied by a generator can be calculated.
- Using the equivalent circuit to find complex power in terms of excitation voltage 'E', terminal (grid) voltage 'V', and synchronous reactance 'X'.
- Typically, the resistance 'R' in the armature winding is negligible when compared to a synchronous reactance. **In this analysis, we omit the resistance.**



## Complex Power Supplied Per Phase



$$S_{1\phi} = V I^* = P + j Q$$

$$E L \delta = V + j I j X$$

$$I = \frac{E L \delta - V}{j X}$$

$$\begin{aligned} I^* &= \frac{E L \delta - V}{-j X} \\ S_{1\phi} &= V \left( \frac{E L \delta - V}{-j X} \right) = \frac{|V|||E|L\delta}{-j X} + \frac{|V|^2}{+j X} \\ &= \frac{|V|||E|\cos(-\delta)}{-j X} + j \frac{|V|||E|\sin(-\delta)}{-j X} + \frac{|V|^2}{j X} \\ &= \frac{-j |V|||E|\sin\delta}{j X} + j \frac{|V|||E|\cos\delta}{j X} - j \frac{|V|^2}{X} \\ &= \frac{|V|||E|\sin\delta}{|X|} + j \left( \frac{|V|||E|\cos\delta}{|X|} - \frac{|V|^2}{|X|} \right) \end{aligned}$$

Real power      Reactive Power.



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## Three-Phase Complex Power Supplied

- We have,

$$S_{3\phi} = 3 S_{1\phi} = \frac{3 |V|||E|}{|X|} \sin\delta + j \left( \frac{3 |V|||E|}{|X|} \cos\delta - \frac{3 |V|^2}{|X|} \right)$$

$$P_{3\phi} = \frac{3 |V|||E|}{|X|} \sin\delta$$

$$Q_{3\phi} = \frac{3 |V|||E|}{|X|} \cos\delta - \frac{3 |V|^2}{|X|}$$



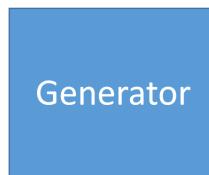
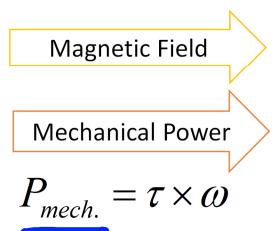
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$$P_{mech} = \underline{\Sigma \tau \omega}$$

$$P_{3\phi} = 3 \frac{|V|||E|}{|X|} \sin\delta$$

## Steady State Operation



$$P_{3\phi} = 3 \frac{|V|||E|}{X} \sin(\delta)$$

- When electrical load is increased, we need to increase mechanical

$$P_{mech} = \underline{I} \times \underline{\omega}$$

$$P_{3\Phi} = \frac{3 |V| |E|}{X} \sin \delta$$

## Steady State Operation



- When electrical load is increased, we need to increase mechanical power input.
- The speed of the rotor ( $\omega$ ) needs to be constant because **rotor speed determines the voltage frequency and the frequency needs to be kept constant**.
- We can only increase the **mechanical 'Torque'** to supply additional electrical load while maintaining the same speed.



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## Real Power



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## Power Angle

- From

$$P_{3\Phi} = 3 \frac{|V||E|}{X} \sin(\delta)$$

- Consider three cases:

$$P_{3\Phi} = \frac{3 |V| |E|}{X} \sin \delta$$

$E = N \frac{d\phi}{dt}$

Rotor magnetic field,

The diagram shows a circular arc with a central angle labeled  $\delta$ . A vector labeled  $E$  is shown at the top, and a vector labeled  $\phi$  is shown at the bottom. A small arrow indicates the direction of rotation. To the right, the equation  $E = N \frac{d\phi}{dt}$  is written, with an arrow pointing to the derivative term  $\frac{d\phi}{dt}$ .

## Power Angle

- From

$$P_{3\Phi} = \frac{3|V||E|}{X} \sin \delta$$

$E = \frac{N \cdot \text{Area}}{\Delta t}$   
Rotor magnetic field.

- Consider three cases:

Power angle	Real power output	Operation mode
$\delta > 0$	$P > 0$	Supply power as generator
$\delta = 0$	$P = 0$	No power exchange
$\delta < 0$	$P < 0$	Absorb power as a motor

This is why  $\delta$  is called 'power angle'!



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$$\text{P_mech} = c \cdot \omega$$

$\text{P_mech}$  increases  $\omega$  decreases

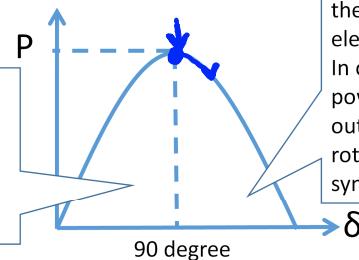
$$P_{3\Phi} = \frac{3|V||E|}{X} \sin \delta$$

## Maximum Power Transfer

- In theory, the power angle  $\delta \leq 90$  degree.
- This limitation is called "Steady-state stability limit".
- Above 90 degree, generator will lose synchronism.
- The maximum power transfer is the real power output when the power angle is 90 degrees.

$$P_{3\Phi} = \frac{3|V||E|}{X} \sin(\delta)$$

Under this region, when the mechanical power input increases, the power angle increase and as a result the electrical power output increases



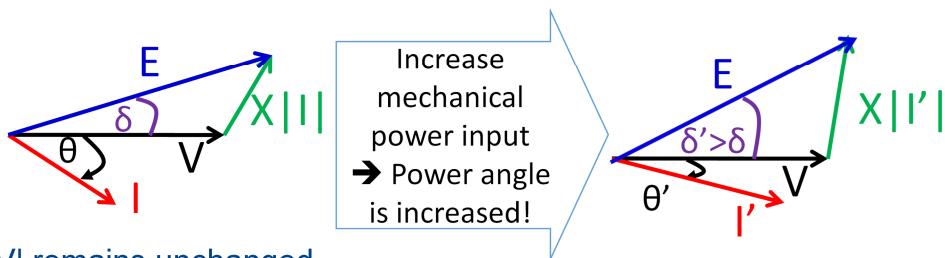
Under this region, when the mechanical power input increases, the power angle increase BUT the electrical power output decreases. In order to balance mechanical power input to electrical power output, the machine will adjust its rotational speed, hence out of synchronism.



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## Control of Real Power Output $\rightarrow \delta$



- $|E|, |V|$  remains unchanged.
- Power angle increases as a result of higher mechanical power input.
- Load current  $|I|$  increases because the electrical power output is increased.
- The power factor will now change because the power angle is increased while the internal voltage magnitude is kept constant.
- We need to adjust the excitation voltage to keep the power factor constant.



## Reactive power



## Reactive Power Output

$$Q_{3\Phi} = \frac{3|V||E|\cos\delta - 3|V|^2}{X} = \frac{3|V|}{X} (E\cos\delta - V)$$

- From  $Q_{3\Phi} = 3\frac{|V||E|}{X}\cos(\delta) - 3\frac{|V|^2}{X} = 3\frac{|V|}{X} \{E\cos(\delta) - |V|\}$

- Reactive power control is done by adjusting  $|E|$ .
- Consider three cases,

Cases	Reactive power	Operation mode
-------	----------------	----------------

## Reactive Power Output

$$Q_{3\Phi} = \frac{3|V||E|}{X} \cos \delta - \frac{3|V|^2}{X}$$

$$= \frac{3|V|}{X} (E \cos \delta - V)$$

- From

$$Q_{3\Phi} = 3 \frac{|V||E|}{X} \cos(\delta) - 3 \frac{|V|^2}{X} = 3 \frac{|V|}{X} \{ |E| \cos(\delta) - |V| \}$$

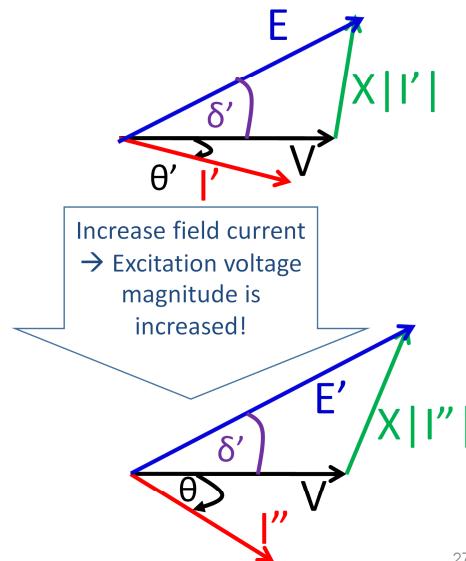
- Reactive power control is done by adjusting  $|E|$ .
- Consider three cases,

Cases	Reactive power output	Operation mode
$ E  \cos \delta >  V $	$Q > 0$	Supply reactive power. This mode is called 'Overexcited'. → lagging - $R_L$
$ E  \cos \delta =  V $	$Q = 0$	No reactive power exchange → $R$
$ E  \cos \delta <  V $	$Q < 0$	Absorb reactive power. This mode is called 'Underexcited'. → leading → $\omega C$



## Control of Reactive Power Output

- $|V|$  and power angle remain unchanged.
- The current magnitude and angle,  $\theta$  (power factor) will change as a result of the change in excitation voltage magnitude.
- We can now adjust the excitation voltage to maintain the power factor of the original load.



## Control of Reactive Power Output



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## Solving generators



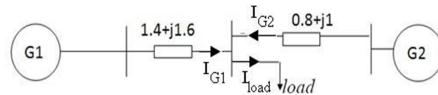
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## Single Line Diagram

- A **one-line diagram** or **single-line diagram (SLD)** is a simplified notation for representing a three-phase power system.



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## Steps for Solving Generators

1. Find the Armature Current from the load parameters
2. Find the Excitation Voltage and power angle from the equivalent circuit of generator  $E = V + I(R + jX)$
3. If real power is changed, keep magnitude of 'E' constant and find the new power angle  $\delta'$ .  $P' = \frac{3|V||E|}{|X|} \sin \delta'$
4. Find the new armature current  $I'$  and the new power factor.

$$I' = \frac{E \angle \delta' - V}{jX}$$



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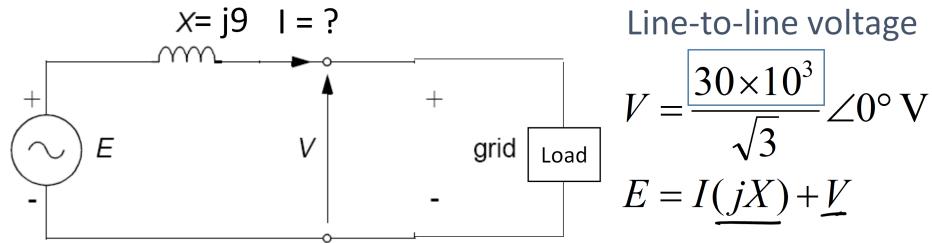
## Example

This voltage is line-to-line voltage.

- A 50-MVA, 30kV, three-phase, wye-connected 60 Hz synchronous generator has a synchronous reactance of 9 Ω per phase and a negligible resistance. The generator is supplying to the system a rated power at 0.8 p.f. lagging at the rated terminal voltage.
  - (a) Determine the excitation voltage per phase ( $E$ ) and the power angle ( $\delta$ ).
  - (b) With the excitation held constant at the value found in (a), the driving torque is reduced until the generator is delivering 25 MW. Determine the armature current and the new power factor.
  - (c) what is the maximum power transfer by this generator at the current excitation?

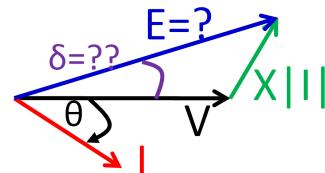


## Example (a) Excitation Voltage



→ Need to find the current !!!

Hint: The generator is delivering rated power to the system at a 0.8 power factor lagging.



## **Step 1:** Armature Current



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## **Step 2:** Excitation Voltage



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## Step 3: New Power Angle



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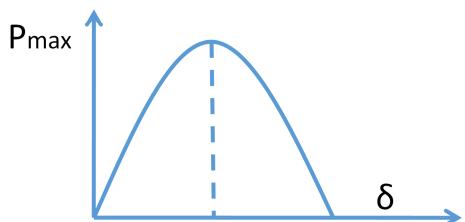
## Step 4: New Armature Current



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## Step 5 (Sometimes): Maximum Power Transfer



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## Example : Points to Note

- From (b), when we adjust *only* mechanical power input, we see that the current magnitude and power factor has changed.

$$I = 962.25 \angle -36.87^\circ \quad \rightarrow \quad I = 807 \angle -53.4^\circ$$

- If we want to keep the power factor constant, we need to adjust the excitation voltage magnitude too!



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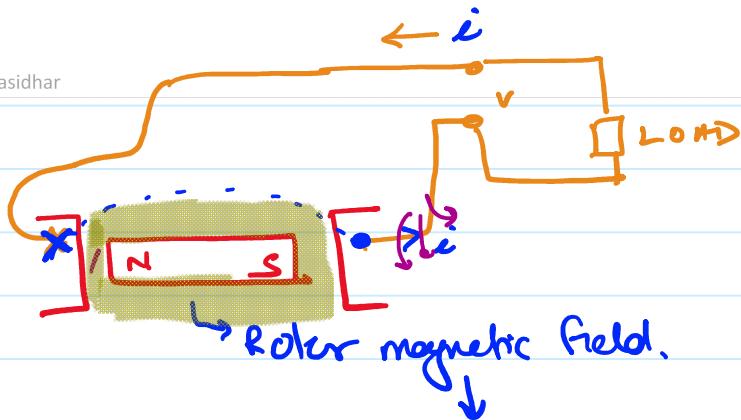
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## Phasor Diagrams



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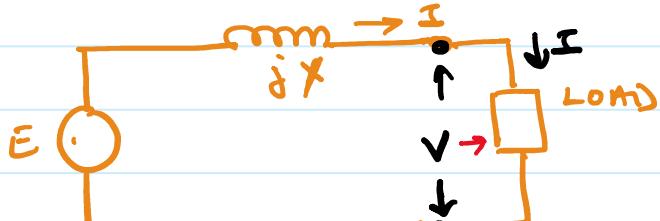
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Induces a current in the  
stator conductor ( $i$ )

Stator magnetic field,  
Oppose the rotor magnetic  
field,

→ Loaded Operation of a Generator. [Neglecting  $r$ ]





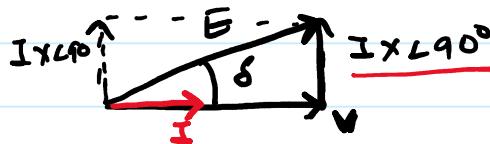
Unity power factor load  $\rightarrow$  R load,

$$E = V + I(jx)$$

$$E = V + I(x \angle 90^\circ)$$

| I is in phase with V

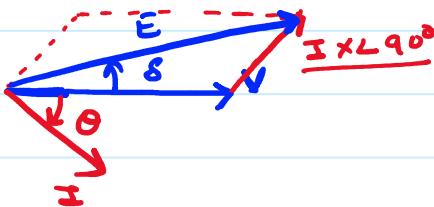
$$\Rightarrow |E| \angle \delta = |V| \angle 0^\circ + (|I| \angle 0^\circ) (x \angle 90^\circ)$$



$\rightarrow$  RL load  $\rightarrow$  Lagging p.f. load.

$$I = |I| \angle \theta \quad -90^\circ < \theta < 0^\circ$$

$$E = V + I(x \angle 90^\circ)$$



$\rightarrow$  Leading p.f. load (RC load)

$$I = |I| \angle \theta \quad 90^\circ < \theta < 0$$

$$E = V + I(x \angle 90^\circ)$$

