EE2029 Introduction to Electrical Energy Systems

(Solution for Tutorial 4 on Generator Modelling)

Solution Q.1

A 60kVA three-phase wye-connected 440V, 60 Hz synchronous generator

Given $X_s = 3.5\ \Omega, R_a = 0.15\ \Omega$

Rated load with unity power factor, $|S_{3\Phi}| = 60\ kVA$

$$V = \frac{440}{\sqrt{3}}\angle 0^o = 254.034\angle 0^o\ V$$

$$\boldsymbol{I} = \frac{60000}{\sqrt{3}\times 440}\angle \cos^{-1}(1) = 78.73\angle 0^o\ A$$

$$E = V + I\times(R_a + jX_a) = 254.034\angle 0^o + 78.73\angle 0^o \times (0.15 + j3.5) = 382.89\angle 46.03^o\ V$$

Power angle $\delta = 46.03^o$.

Solution Q.2

1 MVA, 11kV, 3-phase Y-connected Synchronous Generator

Given $X_s = 24\ \Omega, R_a = 0$

$V_{LL} = 11kV, V_{LN} = \frac{11000}{\sqrt{3}}$ V

Load is 600 kW, 0.8 power factor leading

$$\boldsymbol{I} = \frac{600000}{\sqrt{3}\times 11000 \times 0.8}\angle \cos^{-1}(0.8) = 39.365\angle 36.87^o\ A$$

$$V = \frac{11000}{\sqrt{3}}\angle 0^o = 6350.853\angle 0^o\ V$$

$$E = V + I\times(R_a + jX_a) = 6350.853\angle 0^o + 39.365\angle 36.87^o \times (j24) = 5833.168\angle 7.44^o\ V$$

Power angle $\delta = 7.44^o$.

Solution Q.3

3-phase 2500kVA, 6.6kV Synchronous generator

Given $R_a = 0\ \Omega, X_S = 4\ \Omega$ (Per phase)

Load 1: Full load $|S_{3\Phi}| = 2500\ kVA$ i.e. at 0.9 power factor lagging

$$\boldsymbol{I} = \frac{2500000}{\sqrt{3} \times 6600} \angle - \cos^{-1}(0.9) = 218.693\angle - 25.84^o\ A$$

$$V = \frac{6600}{\sqrt{3}} \angle 0^o = 3810.512\angle 0^o\ V$$

$$E = V + I \times (R_a + jX_a) = 3810.512\angle 0^o + 218.693\angle - 25.84^o \times (j4) = 4265.09\angle 10.64^o\ V$$

As $E \cos\delta = 4265.09 \times \cos(10.64^o) = 4191.76\ V$ is greater than V, the machine is over excited.

Load 2: Full load $|S_{3\Phi}| = 2500\ kVA$ i.e. at 0.9 power factor leading

$$\boldsymbol{I} = \frac{2500000}{\sqrt{3} \times 6600} \angle \cos^{-1}(0.9) = 218.693\angle 25.84^o\ A$$

$$V = \frac{6600}{\sqrt{3}} \angle 0^o = 3810.512\angle 0^o\ V$$

$$E = V + I \times (R_a + jX_a) = 3810.512\angle 0^o + 218.693\angle 25.84^o \times (j4) = 3518.45\angle 12.93^o\ V$$

As $E \cos\delta = 3518.45 \times \cos(12.93^o) = 3429.24\ V$ is lesser than V, the machine is under excited.

Solution Q.4

75 kVA, 2.2kV, 60Hz, 3-Phase,Y-connected Synchronous Generator

Given $R_a = 0.2\ \Omega, X_S = 6\ \Omega$ (Per phase)

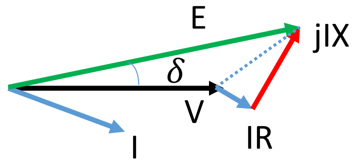
(i) Operating at full load i.e. $|S_{3\Phi}| = 75\ kVA$ at power factor of 0.85 lagging

$$I = \frac{75000}{\sqrt{3} \times 2200} \angle - \cos^{-1}(0.85) = 19.682\angle - 31.79^o\ A$$

$$V = \frac{2200}{\sqrt{3}} \angle 0^o = 1270.171\angle 0^o\ V$$

$E = V + I \times (R_a + jX_a) = 1270.171\angle 0^o + 19.682\angle - 31.79^o \times (0.2 + j6) =$

$= 1339.34\angle 4.21^o V$

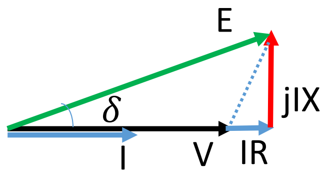


(ii) Operating at full load i.e. $|S_{3\Phi}| = 57kVA$ at unity power factor

$$I = \frac{75000}{\sqrt{3} \times 2200}\angle 0^o = 19.682\angle 0^o\ A$$

$E = V + I \times (R_a + jX_a) = 1270.171\angle 0^o + 19.682\angle 0^o \times (0.2 + j6) =$

$= 1279.57\angle 5.29^o V$



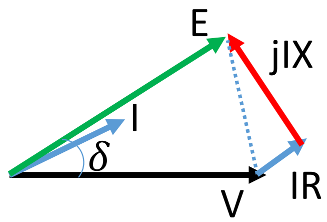
(iii) Operating at full load i.e. $|S_{3\Phi}| = 57kVA$ at power factor of 0.85 leading

$$I = \frac{75000}{\sqrt{3} \times 2200}\angle \cos^{-1}(0.85) = 19.682\angle 31.79^o\ A$$

$$V = \frac{2200}{\sqrt{3}}\angle 0^o = 1270.171\angle 0^o\ V$$

$E = V + I \times (R_a + jX_a) = 1270.171\angle 0^o + 19.682\angle 31.79^o \times (0.2 + j6) =$

$= 1215.63\angle 4.83^o V$



Solution Q.5

Three-phase synchronous generator has synchronous reactance, $jX = j5\ \Omega$. The voltage at three phase load is given as line-to-line voltage of 11 kV. The terminal voltage can be found.

$$V_{T,line-to-neutral} = \frac{11 \times 10^3}{\sqrt{3}} \angle 0° = 6350.85\angle 0°\, V$$

Line current at the load is found below.

$$|I_{load}| = \frac{|S_{3\Phi}|}{\sqrt{3}\left|V_{T,line-to-line}\right|} = \frac{1 \times 10^6}{\sqrt{3} \times 11 \times 10^3} = 52.49\ A$$

Load current angle is found from power factor. Use voltage at load as a reference angle.

$$\angle I_{load} = -\cos^{-1}(p.f.) = -25.84°$$

The excitation voltage is found from a generator equivalent circuit model,

$$E = V_T + jXI_{load} = 6350.85\angle 0° + j5 \times 52.49\angle -25.84° = 6469.55\angle 2.09°\, V.$$

When the excitation remains constant, this implies that the magnitude of excitation voltage remains constant at 6469.55 V. The input torque is reduced such that the real power output is half the previous case. This means that now the real power output can be found from,

$$P_{3\Phi,\text{new}} = \frac{1}{2} P_{3\Phi,\text{old}} = \frac{1}{2} S_{3\Phi,\text{old}} \times p.f. = \frac{1}{2} \times 1 \times 10^6 \times 0.9 = 0.45\ MW.$$

With this new real power output, the power angle has changed according to

$$P_{3\Phi,\text{new}} = \frac{3\left|V_{T,line-to-neutral}\right||E|}{X} \sin\delta.$$

The new power angle is, $\delta = \sin^{-1}\left(\frac{5 \times 0.45 \times 10^6}{3 \times \frac{11 \times 10^3}{\sqrt{3}} \times 6469.55}\right) = 1.046°$. The armature current is found below.

$$I_{load} = \frac{E - V_T}{jX} = \frac{6469.55\angle 1.046° - 6350.85\angle 0°}{j5} = 33.34\angle -44.88°\, A$$

The power factor is $cos\big(0 - (-44.88°)\big) = 0.71$ lagging.

Solution Q.6

The terminal voltage is given to be 11kV (line-to-line) and the excitation voltage at the same condition is 12 kV(line-to-line). At the maximum power, the power angle is **90** degrees. Therefore, the maximum power can be found from,

$$P_{3\Phi} = \frac{3|V_{T,line-to-neutral}||E|}{X}\sin\delta = \frac{3 \times \frac{12\times10^3}{\sqrt{3}} \times \frac{11\times10^3}{\sqrt{3}}}{6}\sin 90 = 22\ MW.$$

The armature current can be found from,

$$I_{load} = \frac{E - V_T}{jX} = \frac{\frac{12\times10^3}{\sqrt{3}}\angle 90° - \frac{11\times10^3}{\sqrt{3}}\angle 0°}{j6} = 1566.43\angle 42.51°\,A$$

The power factor at the maximum power condition is $cos\big(0-(42.51°)\big) = 0.74$ leading.

Solution Q.7

Generator 1 and 2 have a synchronous reactance of 3 per phase and a negligible armature resistance.

We can draw the per phase circuit as shown in Fig. 1.

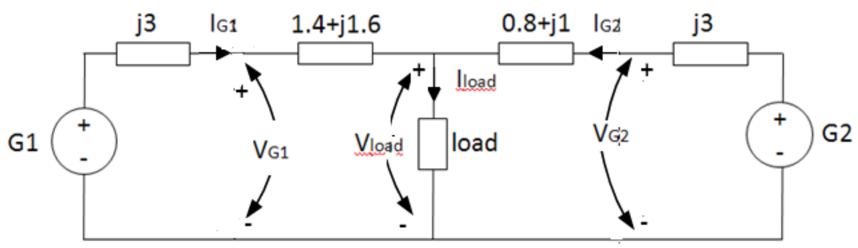


Fig. 1 Per-phase equivalent circuit of the system in Fig. 1.

In order to find internal excitation voltage of both generators, we need to find the currents supplied by both generators I_{G1} and I_{G2} and terminal voltage of both generators, V_{G1} and V_{G2}.

<u>Consider G1</u>:

It is given that generator 1 supplies 15 kW at 0.8 power factor lagging with terminal voltage of 460 line-to-line. This means that $V_{G1,line-to-line} = 460\ V$ and $V_{G1,line-to-neutral} = \frac{460}{\sqrt{3}} = 265.58\ V$.

We can find the current supplied by this generator as follows.

$$|I_{G1}| = \frac{|S_{3\Phi,\text{supplied } by\ G1}|}{\sqrt{3}|V_{G1,line-to-line}|} = \frac{P_{3\Phi,\text{supplied } by\ G1}/p.f.}{\sqrt{3}|V_{G1,line-to-line}|} = \frac{15000/0.8}{\sqrt{3}\times 460} = 23.53\ A$$

Load current angle is found from power factor. Use terminal voltage of generator 1 as a reference angle.

$$\angle I_{G1} = -\cos^{-1}(0.8) = -36.87°$$

The excitation voltage of generator 1 can be found below.

$$E_{G1} = V_{G1} + jXI_{G1} = 265.58\angle 0° + j3 \times 23.53\angle -36.87° = 313.07\angle 10.39°\, V.$$

<u>Consider G2:</u>

In order to find the excitation voltage of generator 2, we need to know the current supplied by generator 2. The current can be found from KCL at the load.

$$I_{G1} + I_{G2} = I_{Load}$$

It is given that the three-phase load absorbs 30 kW at 0.8 power factor lagging. We need to find the voltage at the load so that the load current can be found. Using KVL at generator 1, line-to-neutral voltage at the load is found.

$$V_{Load} = V_{G1} - I_{G1} \times (1.4 + j1.6) = 265.58\angle 0° - 23.53\angle -36.87° \times (1.4 + j1.6) = 216.88\angle -2.74°$$

Given that the three-phase load absorbs 30 kW at 0.8 power factor lagging, the three-phase power absorbs reactive power of,

$$Q_{3\Phi} = P_{3\Phi}\tan(\cos^{-1}\text{p.f.}) = 30000 \times \tan(\cos^{-1}0.8) = 22500\, VAR.$$

Note that the load consumes reactive power (lagging power factor) and thus the reactive power is positive. From $S_{3\Phi} = 3V_{Load,line-to-neutral}I_{Load}^{*}$, we can find the load current per phase.

$$I_{Load} = \left(\frac{S_{3\Phi}}{3V_{Load}}\right)^{*} = \left(\frac{P_{3\Phi} + jQ_{3\Phi}}{3V_{Load}}\right)^{*} = \left(\frac{30000 + j22500}{3 \times 216.88\angle 2.74°}\right)^{*} = 57.64\angle -39.56°$$

This means that the current supplied by generator 2 can be found from,

$$I_{G2} = I_{Load} - I_{G1} = 57.64\angle -39.56° - 23.53\angle -36.87° = 34.15\angle -41.41°\, A.$$

The excitation voltage per phase at the generator 2 is found from below.

$$E_{G2} = I_{G2} \times (j3 + 0.8 + j1) + V_{Load} = 34.15\angle -41.41° \times (0.8 + j4) + 216.88\angle -2.74°$$
$$= 335.73\angle 12.74°\, V.$$