# EE2029: Introduction to Electrical Energy System

## Modelling Line Conductors

Lecturer : Dr. Sangit Sasidhar (elesang)

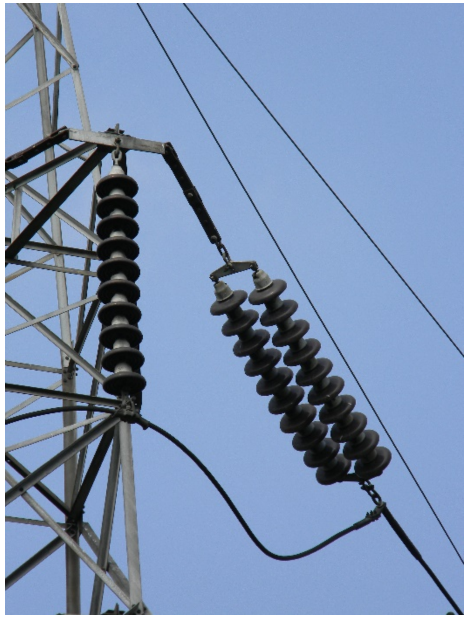
Department of Electrical and Computer Engineering

# Learning Outcome

- Model key components of power systems including transformer, induction motor load, static load, transmission line, cable, and rectifier load

# Types of Transmission Lines

## Overhead Transmission Line



## Underground Cables

Fig. 5. Typical colony of termites found around the cables.

# Magnetic Flux Density

- Magnetic fields are usually measured in terms of flux density (B)

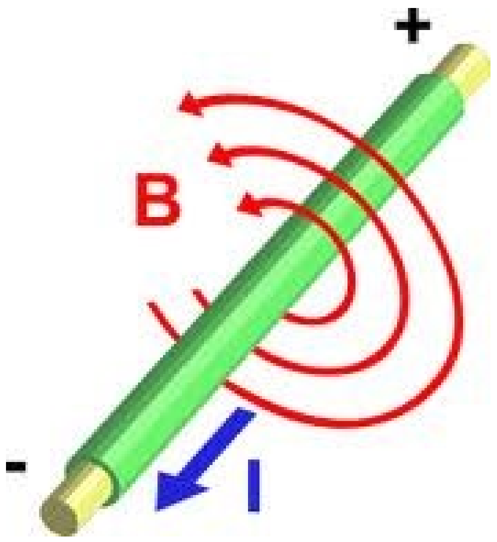
$$B = \mu H$$

  - $\mu$ is the permeability, i.e. $\mu = \mu_0 \mu_r$
  - $\mu_0$ is the permeability of free space (constant) = $4\pi \times 10 - 7\ H/m$
  - $\mu_r$ is the relative permeability of the material to free space, e.g. 1 for air.
- Magnetic flux ($\phi$)is a measurement of total magnetic field passing through a given area.
  - A useful tool for helping to describe effects of magnetic force on something occupying a given area.
  - Total flux ($\phi$) passing through a surface A is:$\phi = \oint Bda$
    - $da$ = vector with direction normal to surface.

# Ampere's Circuital Law

- "Current passing through a conductor creates magnetic field around it "

$$\oint Hdl = I_{enclosed}$$




  - $B = \mu H$
  - $B$ = Magnetic flux density (Weber/m² or Tesla)
  - $H$ = Magnetic field intensity (A/m)
  - $\mu$ = Magnetic core permeability (H/m)
  - $dl$ = vector differential path length

# Faraday’s Law of Induction

- Induced electromotive force ($e$) in any closed circuit is equal to the negative of time rate of change of magnetic flux enclosed by the circuit

$$e = \frac{d\lambda}{dt} = \frac{d(N\phi)}{dt}$$

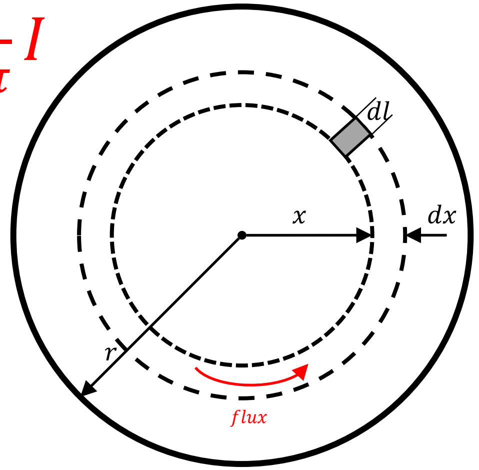
- Flux linkages ($\lambda$) is the amount of flux linking an $N$ turn coil
  - For simplicity, we assume the flux links to all turns and there are no leakages
- Flux linkages can be related to inductance for RLC circuit analysis
  - For a linear magnetic system, i.e. $B = \mu H$, inductance (L) can be defined as the constant relating the current to the flux linkage
  - $L = \lambda / I$

# Flux Linkages of a Single Conductor

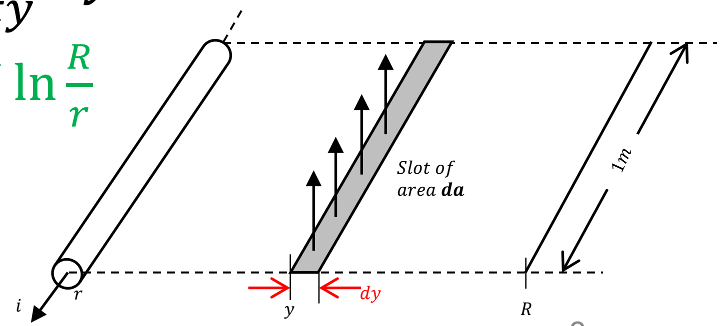
- To develop models of transmission lines, we first need to determine the inductance of a single, infinitely long wire
- To do this we need to determine the wire's total flux linkage, including
  - Flux linkages outside of the wire
  - Flux linkages within the wire

# Flux Linkages of a Single Conductor

- Flux Linkages Inside of a Conductor
  - Assume that the current density within the wire is uniform and that the wire has a radius of $r$
  - $\oint H_x dl = I_x \Rightarrow 2\pi x H_x = I_x$
  - $Now, I_x = \frac{\pi x^2}{\pi r^2} I$
  - $H_x = \frac{x}{2\pi r^2} I \Rightarrow B_x = \mu H_x = \frac{\mu x I}{2\pi r^2}$
  - $\lambda_{inside} = \frac{\mu}{8\pi} I$




- Flux Linkages Outside of Conductor
  - Suppose a wire is a single loop that is closed at infinity $\Rightarrow \lambda = \phi$ as $N = 1$
  - Flux linking the wire out to '$R$' from the center of the wire is:
  - $\phi = \oint B d\boldsymbol{a} = l \cdot \int_r^R \mu_0 \frac{I}{2\pi y} dy$
  - $\lambda = \phi \Rightarrow \lambda = l \cdot \int_r^R \mu_0 \frac{I}{2\pi y} dy$
  - Since length is infinity, we can represent the expression as per unit length
  - $\frac{\lambda}{l} = \int_r^R \mu_0 \frac{I}{2\pi y} dy$
  - $\lambda_{outside} = \frac{\mu_0}{2\pi} I \ln \frac{R}{r}$



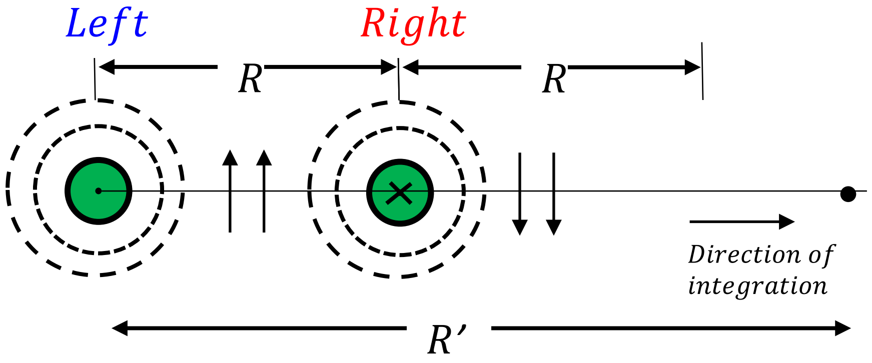
# Single Conductor -Total Flux and Inductance

- Total Flux Linkages
  - $\lambda_{total} = \lambda_{inside} + \lambda_{outside}$
  - $\lambda_{total} = \frac{\mu_0 \mu_r}{8\pi} I + \frac{\mu_0}{2\pi} I \ln \frac{R}{r}$
  - $\lambda_{total} = \frac{\mu_0}{2\pi} I \left( \frac{\mu_r}{4} + \ln \frac{R}{r} \right)$
- Simplified Form:
  - $\lambda_{total} = \frac{\mu_0}{2\pi} I \ln \frac{R}{r'}$
    - $r' = re^{-\frac{\mu_r}{4}}$
- Inductance $L = \lambda / I$

  $\Rightarrow L = \frac{\mu_0}{2\pi} \left( \frac{\mu_r}{4} + \ln \frac{R}{r} \right)$
- Simplified Form:
  - $L = \frac{\mu_0}{2\pi} \ln \frac{R}{r'}$
  - $L = 2 \cdot 10^{-7} \cdot \ln \frac{R}{r'}$
    - $r' = re^{-\frac{\mu_r}{4}}$
    - $r' \approx 0.78r \,;$ $if \;\; \mu_r = 1$

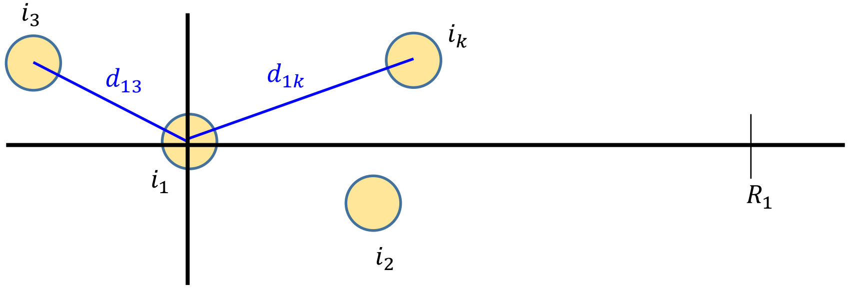
# Two Conductor Line Inductance

- Consider the case of two wires, each carrying the same current (I), but in opposite directions. The wires are separated by distance (R).
  - To determine the inductance of each conductor, apply integration as before. In this case, there will be some field cancellation due to the opposite flow of current
  - Integrate for the left to an arbitrary distance of R', the total flux linkages are:




- $\lambda_{left} = \frac{\mu_0}{2\pi} I \ln \frac{R'}{r'} - \frac{\mu_0}{2\pi} I \ln\left(\frac{R'-R}{R}\right)$
  - $r' = re^{-\frac{\mu_r}{4}}$
- $\lambda_{left} = \frac{\mu_0}{2\pi} I \left(\ln \frac{R'}{r'} - \ln\left(\frac{R'-R}{R}\right)\right)$
- $\lambda_{left} = \frac{\mu_0}{2\pi} I (\ln R' - \ln r' - \ln(R' - R) + \ln R)$
- $\lambda_{left} = \frac{\mu_0}{2\pi} I \left(\ln \frac{R}{r'} + \ln\left(\frac{R'}{R'-R}\right)\right)$
- $\lambda_{left} = \frac{\mu_0}{2\pi} I \left(\ln \frac{R}{r'}\right)$ $as\ R' \rightarrow \infty$ (Flux Linkage)
- $L_{left} = \frac{\mu_0}{2\pi} \left(\ln \frac{R}{r'}\right)$ (Line Inductance)

# Multiple Conductors




- Assume there are k conductors, each with current $i_k$, arranged in an arbitrary geometry. How to find flux linkages of each wire?
  - Each conductor's flux linkage, $\lambda_k$, depends upon its own current and the current in all the other conductors
  - To derive $\lambda_1$, the same approach is taken by integrating from conductor 1 along the right along the axis.

- $\lambda_1 = \frac{\mu_0}{2\pi}\left[i_1 \ln \frac{R_1}{r_1'} + i_2 \ln \frac{R_2}{d_{12}} + \cdots + i_k \ln \frac{R_k}{d_{1k}}\right]$

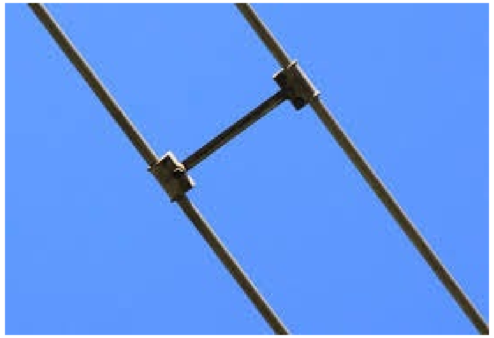
- $\lambda_1 = \frac{\mu_0}{2\pi}\left[i_1 \ln \frac{1}{r_1'} + i_2 \ln \frac{1}{d_{12}} + \cdots + i_k \ln \frac{1}{d_{1k}}\right] + \frac{\mu_0}{2\pi}\left[i_1 \ln R_1 + i_2 \ln R_2 + \cdots + i_k \ln R_k\right]$
- As $R_1$ goes to infinity, $R_1 = R_2 = \ldots = R_k$ yielding the following relation:
- $\lambda_1 = \frac{\mu_0}{2\pi}\left[i_1 \ln \frac{1}{r_1'} + i_2 \ln \frac{1}{d_{12}} + \cdots + i_k \ln \frac{1}{d_{1k}}\right] + \frac{\mu_0}{2\pi}\left[\sum_{j=1}^{k} i_j\right] \ln R_1$
- For a balanced System, $\sum_{j=1}^{k} i_j = 0$
- $\lambda_1 = \frac{\mu_0}{2\pi}\left[i_1 \ln \frac{1}{r_1'} + i_2 \ln \frac{1}{d_{12}} + \cdots + i_k \ln \frac{1}{d_{1k}}\right]$
- $\lambda_1 = i_1 L_{11} + i_2 L_{12} + \cdots + i_k L_{1k}$
- Flux linkages of each conductor will consist of self and mutual inductances (=0)

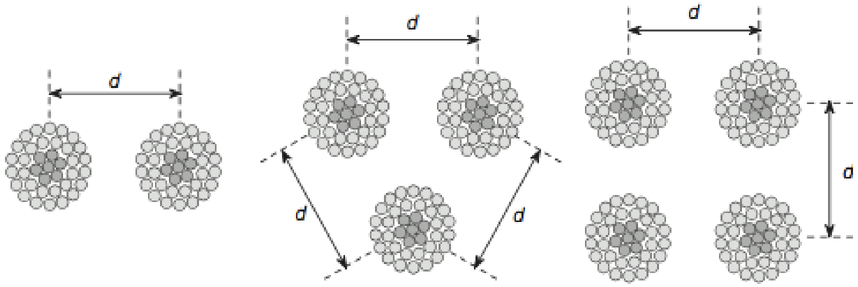
# Example: Line Inductance

- Calculate the per-phase reactance for a balanced three-phase, 50 Hz transmission line with a conductor geometry of an equilateral triangle with D = 5m, r = 1.24 cm (Rook conductor) and a length of 5 km. Assume the system is balanced, i.e., $i_1 + i_2 + i_3 = 0$. Note that D = R for this problem

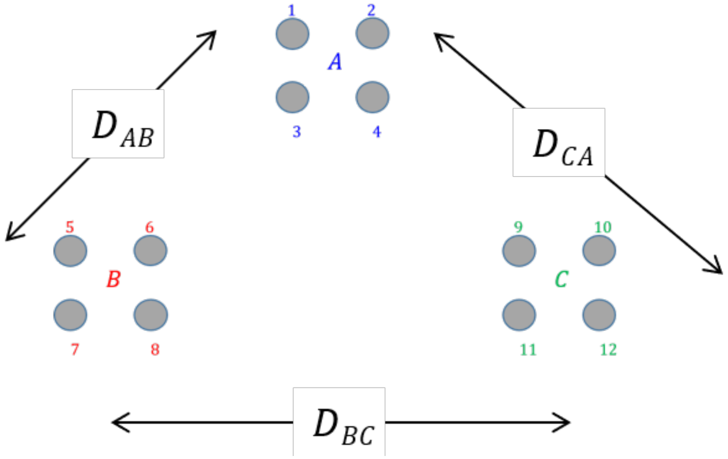
# Bundled Conductors

- To increase the capacity of high voltage transmission lines it is very common to use several conductors per phase
  - This is known as conductor bundling
  - Typical values are two conductors for 345 kV lines, three for 500 kV, and four for 765 kV





# Bundled Conductor Flux Linkages




- Define $d_{ij}$ as the distance between conductors i and j. λ for each conductor can be defined as:

- $$\lambda_1 = \frac{\mu_0}{2\pi}\left[\begin{array}{l} \frac{i_a}{4}\left(\ln\frac{1}{d_{11}} + \ln\frac{1}{d_{12}} + \ln\frac{1}{d_{13}} + \ln\frac{1}{d_{14}}\right) + \\ \frac{i_b}{4}\left(\ln\frac{1}{d_{15}} + \ln\frac{1}{d_{16}} + \ln\frac{1}{d_{17}} + \ln\frac{1}{d_{18}}\right) + \\ \frac{i_c}{4}\left(\ln\frac{1}{d_{19}} + \ln\frac{1}{d_{110}} + \ln\frac{1}{d_{111}} + \ln\frac{1}{d_{112}}\right) \end{array}\right]$$

- $$\lambda_1 = \frac{\mu_0}{2\pi}\left[\begin{array}{l} i_a \ln\left(\boxed{\frac{1}{(d_{11}d_{12}d_{13}d_{14})^{1/4}}}\right) + \\ i_b \ln\left(\boxed{\frac{1}{(d_{15}d_{16}d_{17}d_{18})^{1/4}}}\right) + \\ i_c \ln\left(\boxed{\frac{1}{(d_{19}d_{110}d_{111}d_{112})^{1/4}}}\right) \end{array}\right]$$

GMR (Geometrical Mean Radius)

GMD (Geometrical Mean Distance)

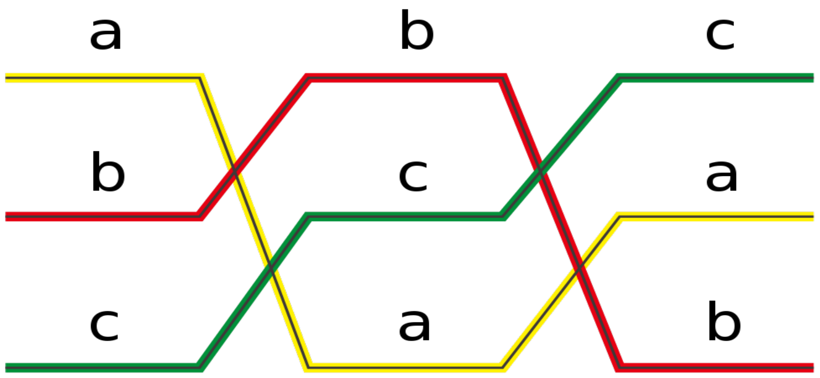
- For a balanced system, $D_{AB} = DBC = DCA = D$
- Also, $i_a + ib + ic = 0 \Rightarrow i_a = -i_b - i_c$
- $\lambda_1 = \frac{\mu_0}{2\pi}\left[i_a \ln\left(\frac{1}{R_n}\right) - i_a \ln\left(\frac{1}{D}\right)\right]$
  - $R_n = (d_{11}d_{12}d_{13}d_{14})^{1/4} = GMR$
  - $D = (d_{15}d_{16}d_{17}d_{18})^{1/4} = GMD = D_{AB} = DBC = DCA$
- $\lambda_1 = \frac{\mu_0}{2\pi}\left[i_a \ln\left(\frac{D}{R_n}\right)\right] = \frac{\mu_0}{2\pi} 4I_1 \left[\ln\left(\frac{D}{R_n}\right)\right]$
- Conductor 1 has an inductance of
  - $L_1 = \frac{\mu_0}{2\pi} 4 \left[\ln\left(\frac{D}{R_n}\right)\right]$
- If each bundle has n conductors ➔
  - $L_a = \frac{L_1}{n} = \frac{\mu_0}{2\pi}\left[\ln\left(\frac{D}{R_n}\right)\right], here\ n = 4$

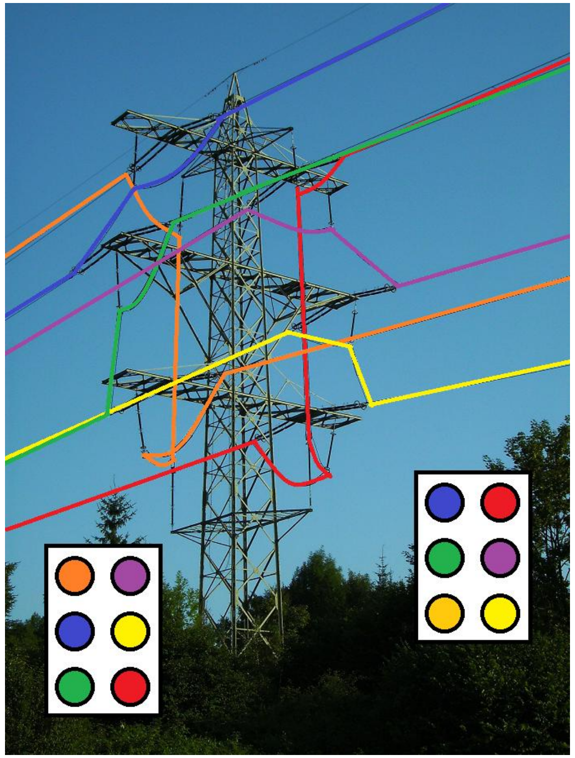
# Example - Bundled Inductance

- Consider the previous example of the three-phase lines that are symmetrically spaced 5 meters apart using conductors with a radius of r = 1.24 cm. However, now assume each phase has 4 conductors arranged in a square bundle and spaced by 0.25 meters apart. What is the new inductance per meter?

# Line Transposition

- To keep system balanced, the conductors are rotated over the length of a transmission line
- Each phase occupies each position on tower for an equal distance
- This is known as Line Transposition



# Questions