EE2029 Introduction to Electrical Energy Systems

Three-Phase Circuit Analysis

Learning Outcomes

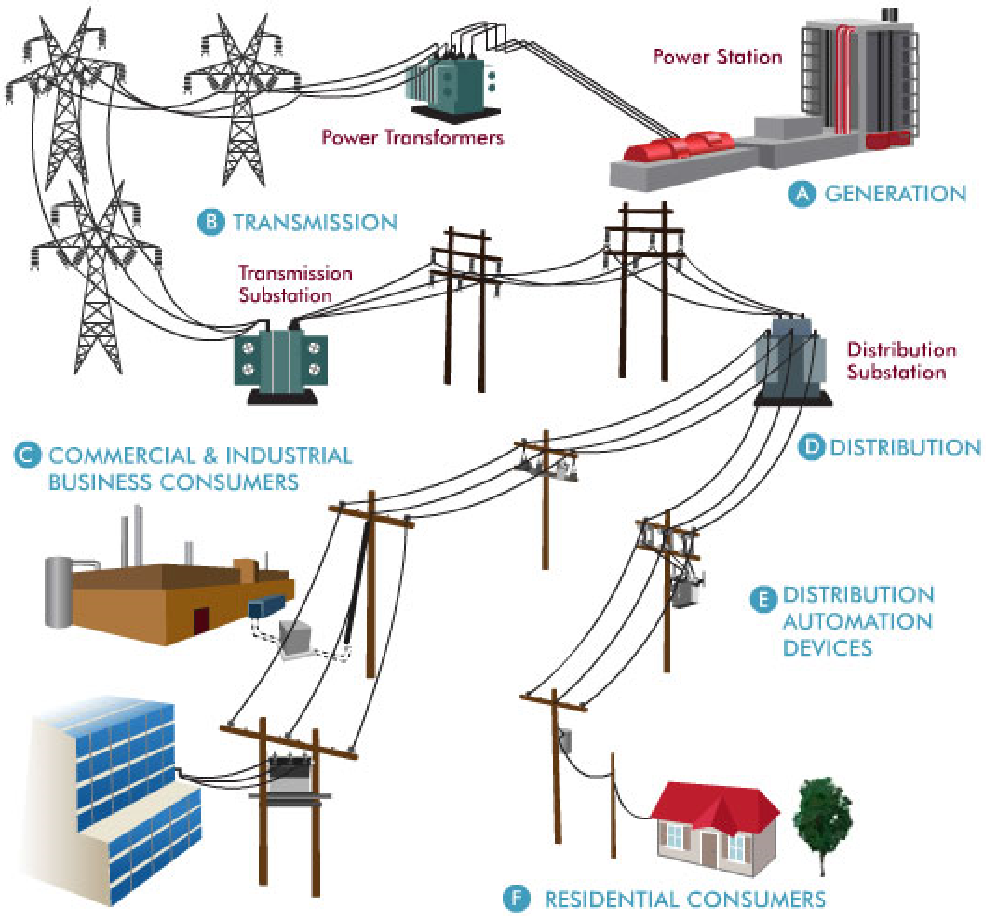
To be able to:

- calculate the complex power, voltages and currents in balanced three-phase AC circuits
- to describe three-phase voltage and currents using Phasor diagrams.

Outline

- Three-Phase Circuit Analysis
  - Generation, Transmission, and Distribution.
  - Three-phase balanced systems.
  - Advantages of three-phase balanced systems.
- Three-Phase voltage and current
  - Line-to-neutral voltage
  - Line-to-Line voltage
  - Line current.
  - Delta/Wye configuration.

Generation, Transmission and Distribution



- Most of bulk generation, transmission and distribution of electricity is three-phase.
- Only small residential loads are single phase.

A Three-Phase Circuit System

Three-phase voltage source

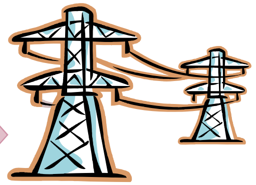
Generation (11 – 36 KV)

3-Phase Generation system.

Generation

Three transmission lines

Transmission (110 – 765 KV)



3-Phase Transmission system.

Transmission and Distribution

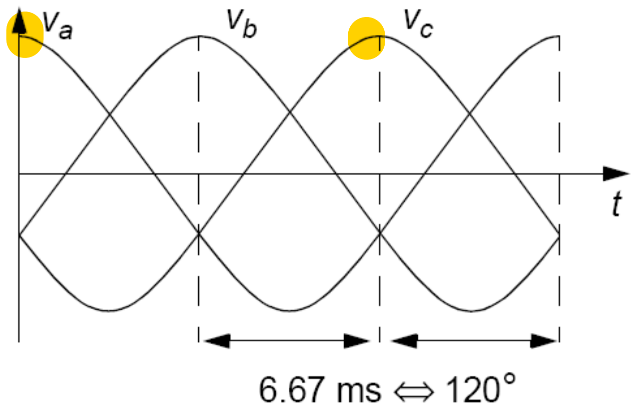
Three-phase load

Industrial customer
(23 – 138 KV)
Commercial customer
(4.16 – 34.5 KV)
Residential customer
(120 – 240 V)

Load

1. same frequency
2. same magnitude
3. phase angle 120 degrees

Three-Phase Voltage Sources

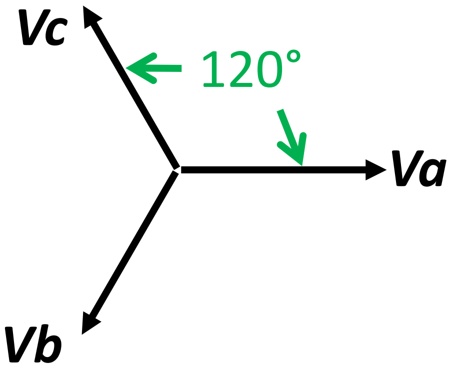


$$v_a = \boxed{\sqrt{2}|V|}\cos(\omega t)$$

$$v_b = \sqrt{2}|V|\cos\left(\omega t - \frac{2\pi}{3}\right)$$

$$v_c = \sqrt{2}|V|\cos\left(\omega t - \frac{4\pi}{3}\right)$$

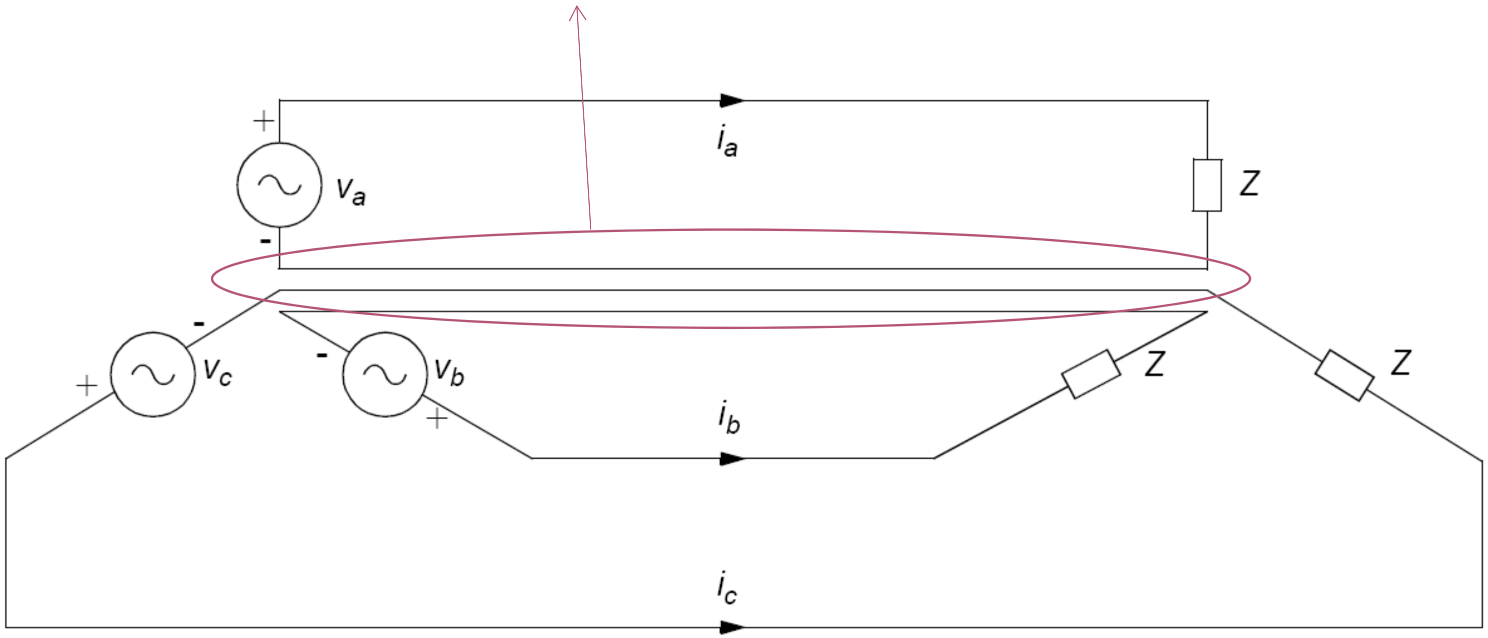
Note that |V| is rms value.



All three voltage sources have the same voltage magnitude, with 120 degrees apart

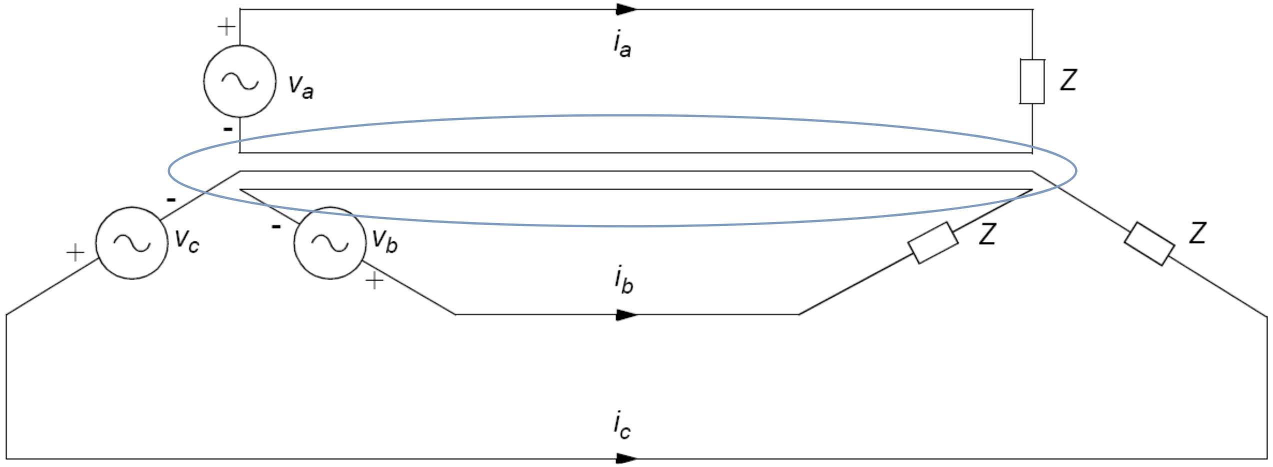
Three Single-Phase Circuits

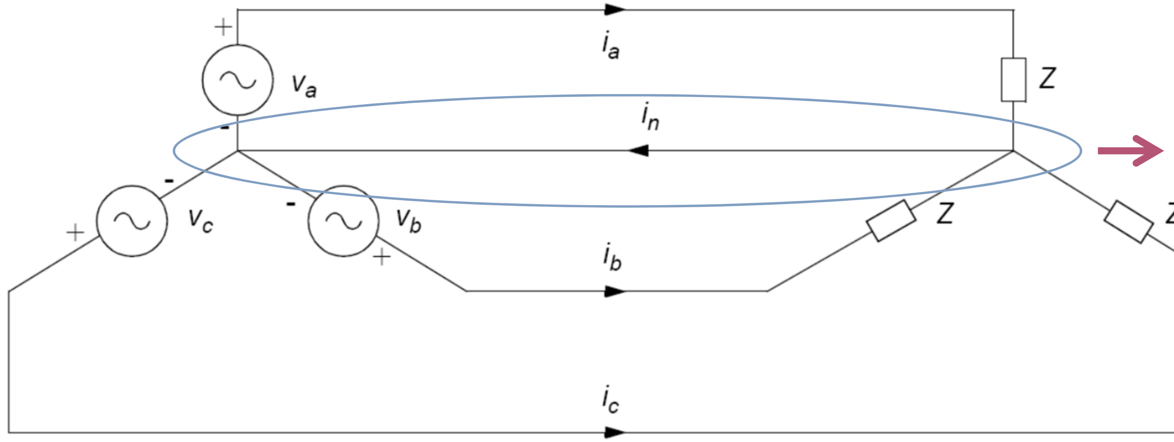
These lines can be combined.



Generation Transmission Load

Three Phase Circuit

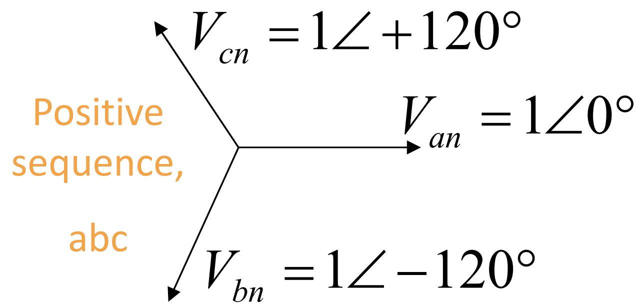


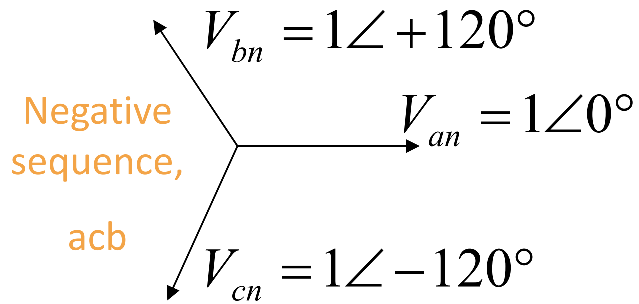


This line is called 'neutral' line. Later on, we'll see why this line can be removed.

Positive vs Negative Sequences

- Voltage sources can be either in positive sequence or negative sequence.
  - Positive sequence i.e. “abc”.
  - Negative sequence i.e. “acb”.





- In practice, phase sequence depends on how we label the wires.

$\angle$

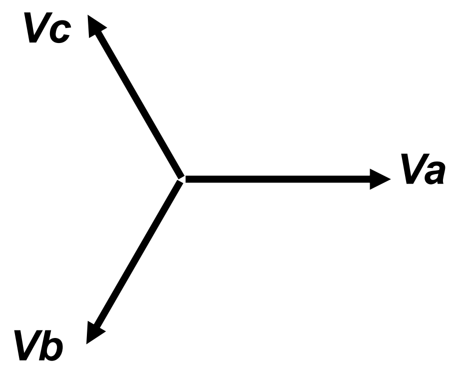
Example

A balanced three-phase Y-connected source has one phase voltage of $\boldsymbol{V_{cn}} = 277\angle 45^{o}\text{V}$. If the phase sequence is negative sequence i.e. ‘acb’, find the line voltage phasors $\boldsymbol{V_{ca}}, \boldsymbol{V_{ab}}$and $\boldsymbol{V_{bc}}$.

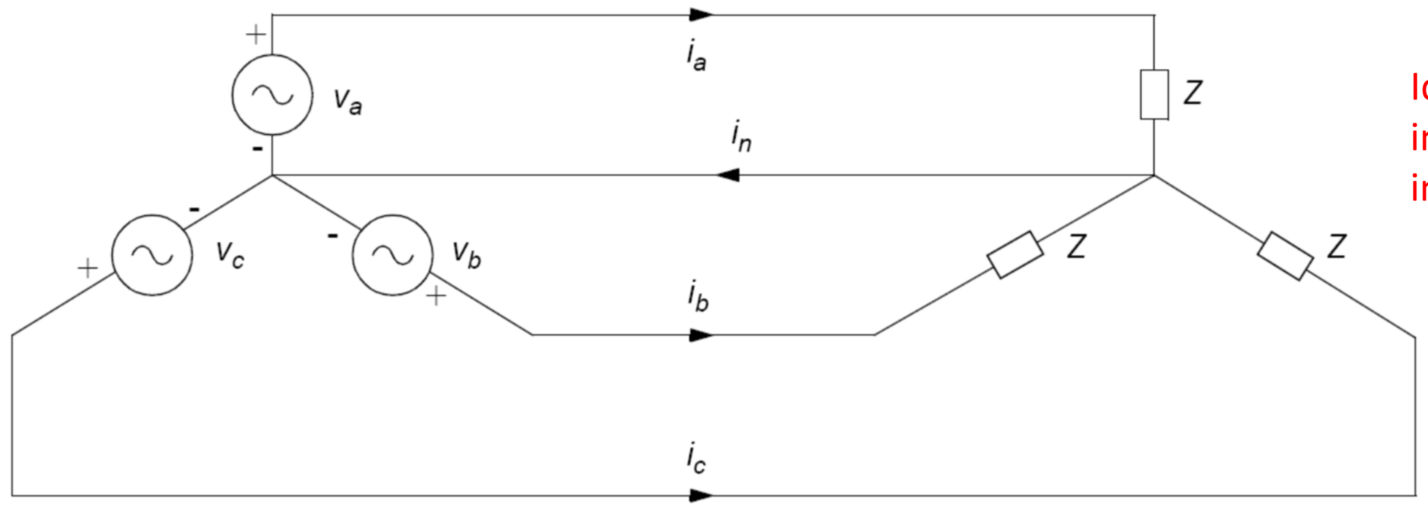
(Answer: $V_{ca} = 480\angle 15^{o}$V, $V_{ab} = 480\angle 135^{o}$V, and $V_{bc} = 480\angle - 105^{o}$V)

Balanced Three-Phase Circuit

Three-phase circuit is said to be balanced when the impedances in the 3 phases are identical.

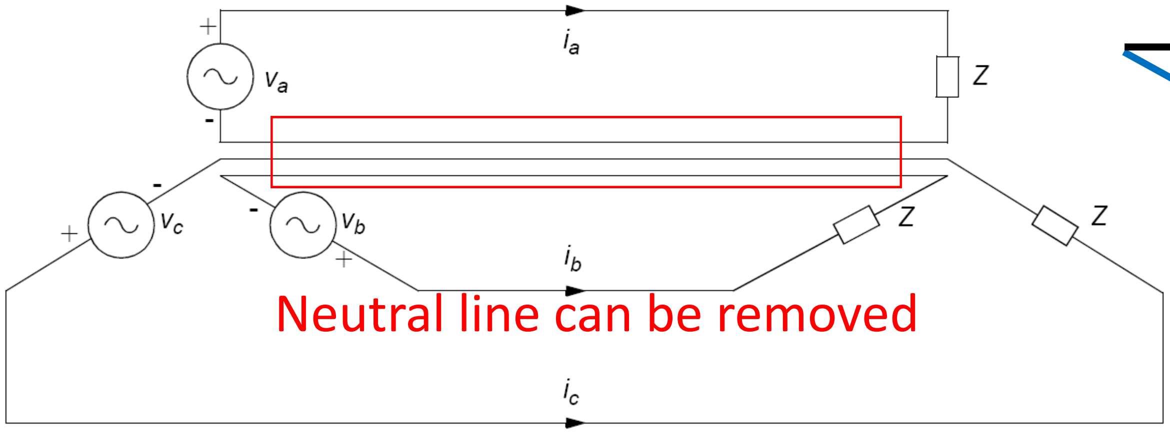


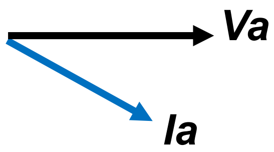
Three-phase voltage source with identical magnitude and 120 phase differences



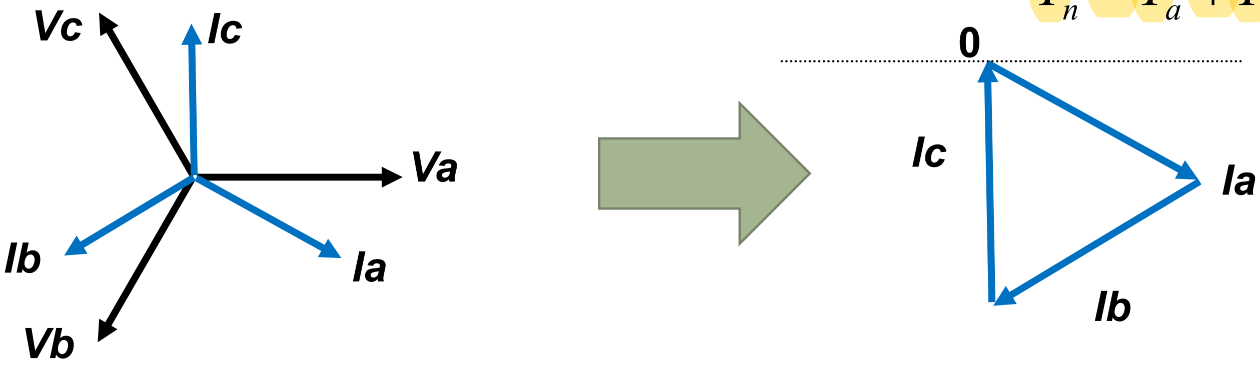
Identical impedances in 3 phases!

Balanced Three-Phase Circuit





$$I_n = I_a + I_b + I_c = 0$$



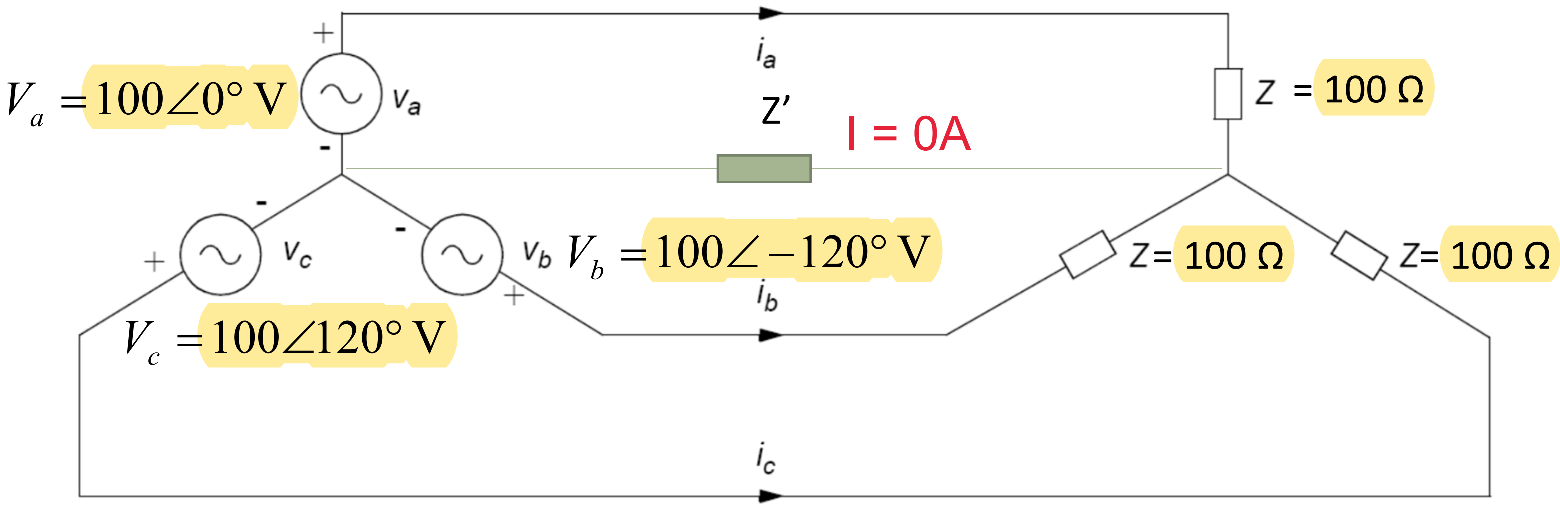
Advantages of Balanced 3-Phase Systems

- When compared to three single-phase circuits, three-phase circuits have better use of equipment and materials
  - 3 phase conductors instead of 6
  - Reduce power losses in phase conductors
- This implies reduced capital and operating costs of transmission and distribution.
- We can calculate voltage and current for only one phase and refer to other phases easily.

Example

Consider the following three-phase system shown below.

Find the current ia when z’ = 10 Ω.



Ans: $i_a = 1\angle 0° \text{ A}, i_b = 1\angle -120° \text{ A}, i_c = 1\angle 120° \text{ A}$

BALANCED

Three-phase Current and Voltage

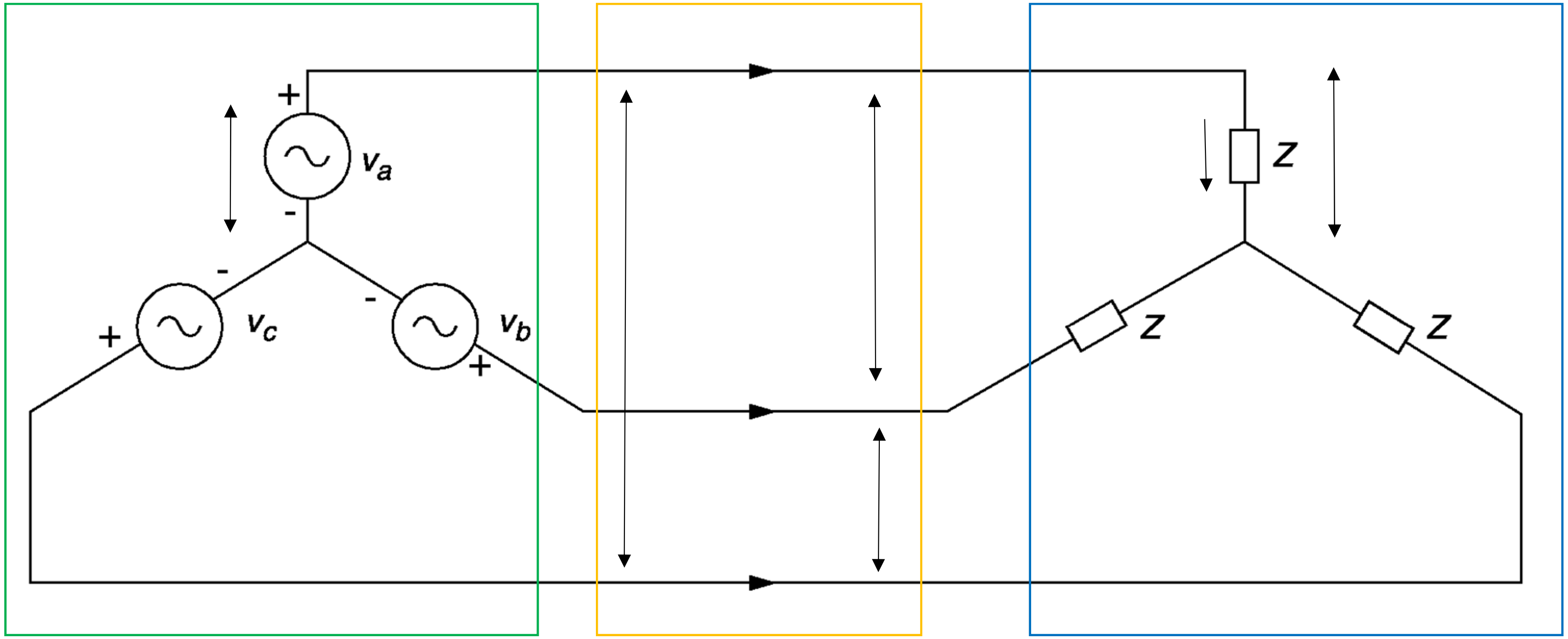
Line-to-Neutral Voltage

Line-to-line voltage

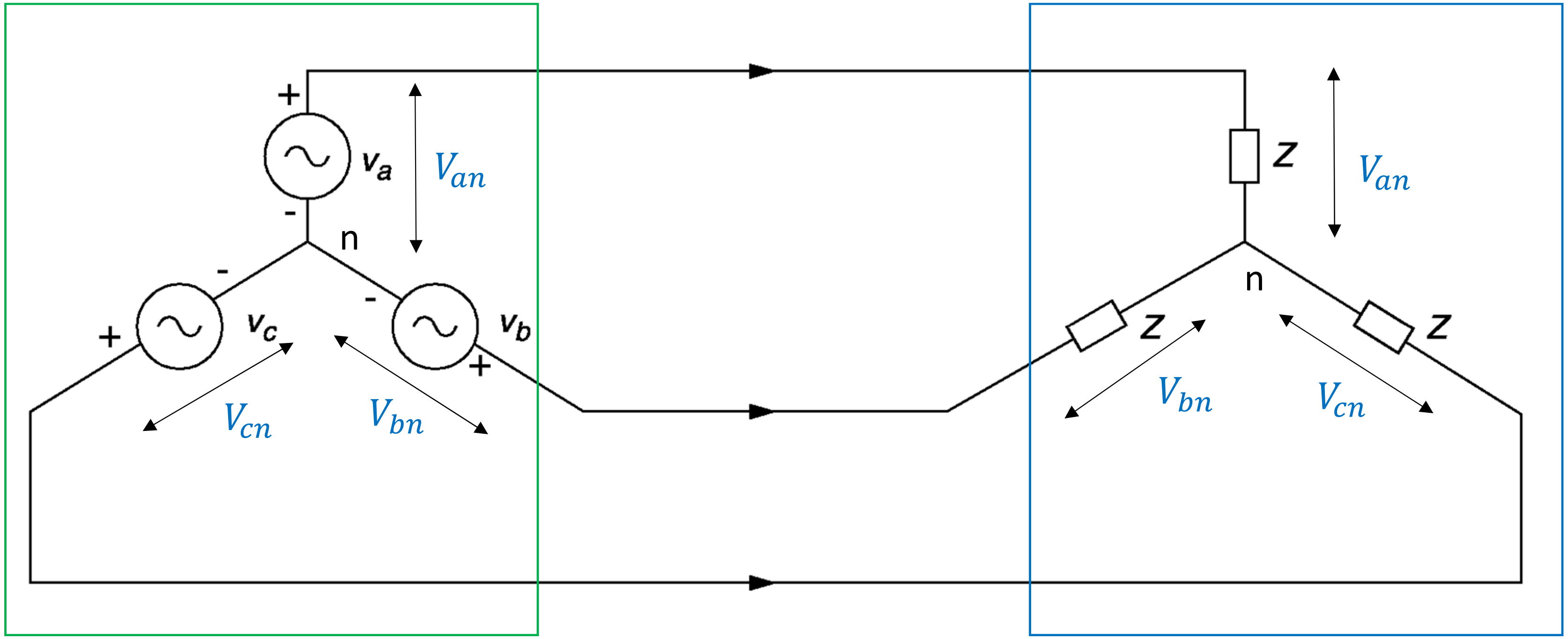
Line current

Wye-Delta connection

How Do We Measure Voltage/Current?

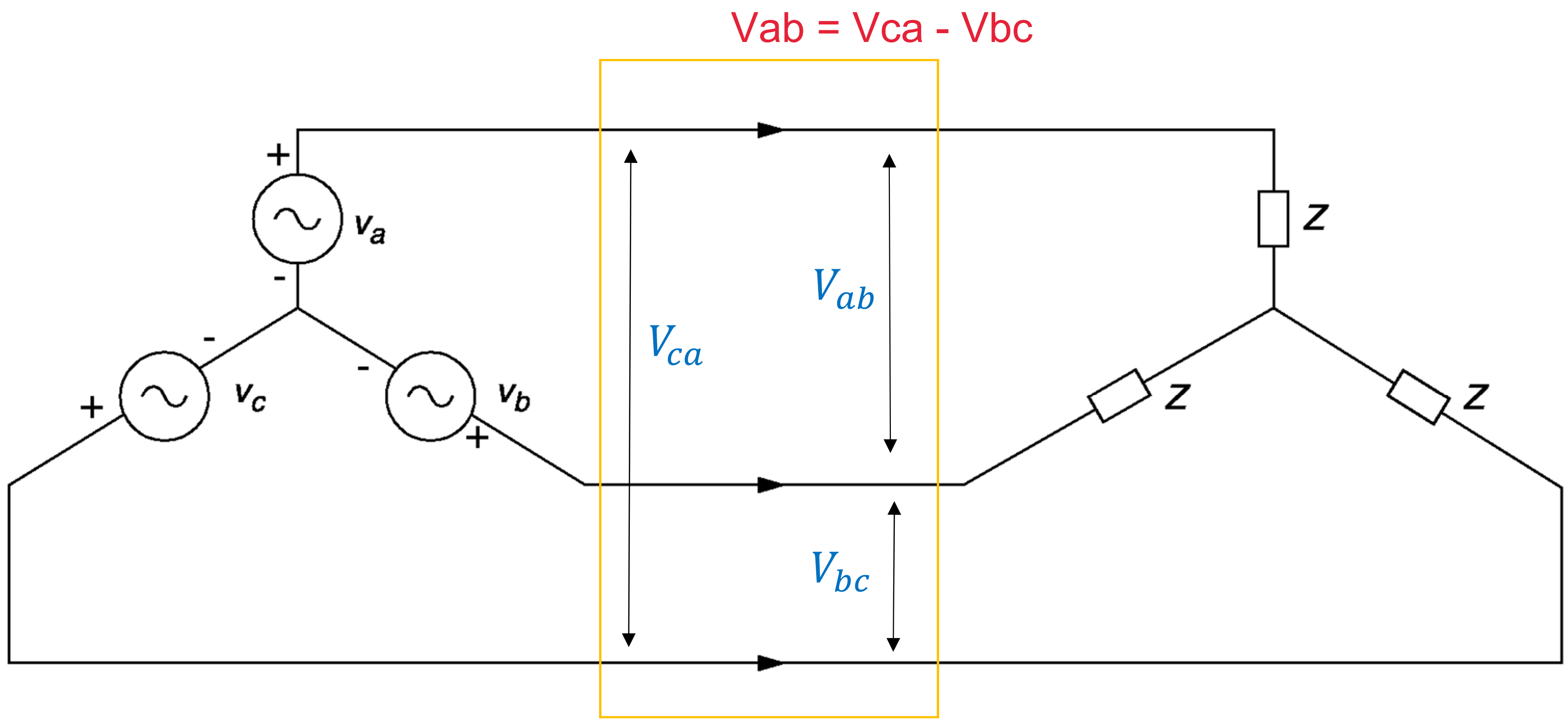


Line-To-Neutral Voltage



- V_{an}, V_{bn}, V_{cn} are called line-to-neutral voltage.
- Also known as ‘phase voltages’.

Line-to-line voltages

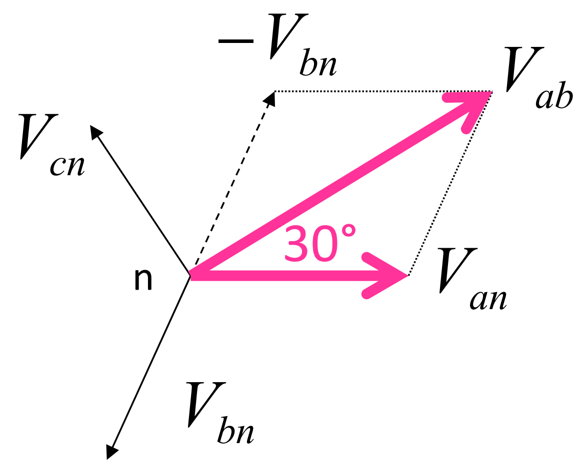


- V_{ab}, V_{bc}, V_{ca} are called line-to-line voltages.
- Also known as ‘line voltages’.

‘Line voltage’ and ‘phase voltage’ relationship

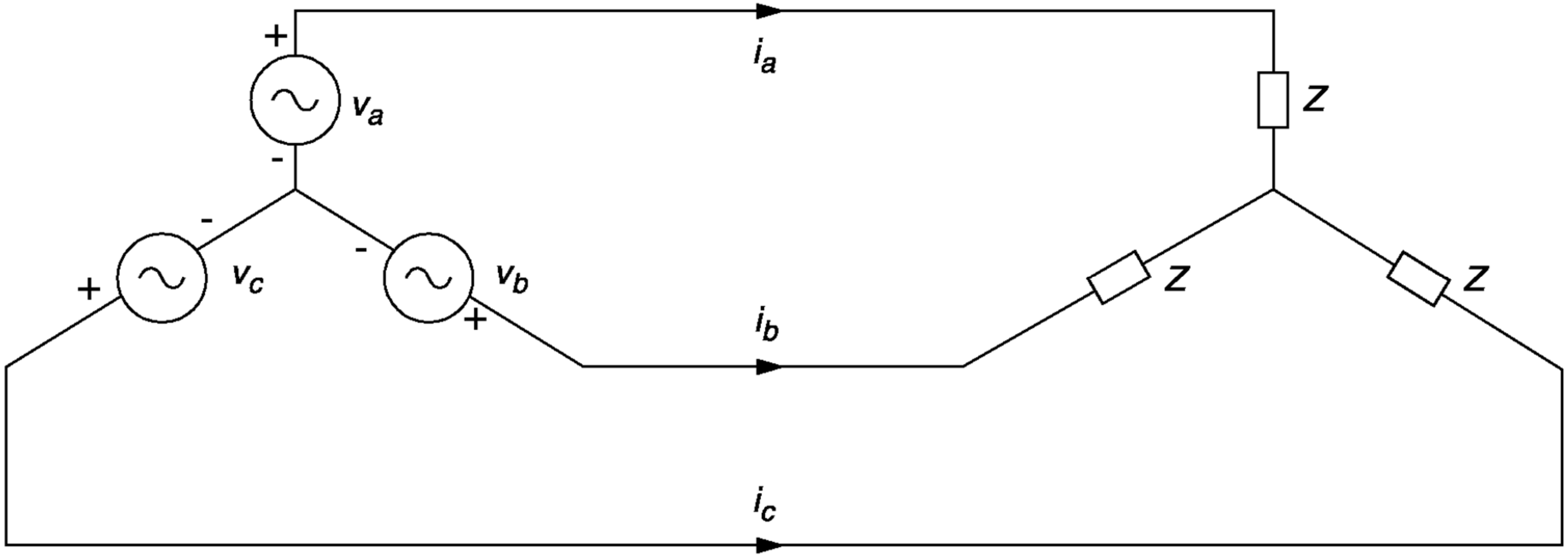
- Three-phase voltage is given as line-to-line voltage by convention.
- KVL: $V_{ab} = V_{an} - V_{bn} = \sqrt{3} V_{an} \angle 30°$

$$\boxed{\left| V_{\text{Line-Line}} \right| = \sqrt{3} \left| V_{\text{Line-neutral}} \right|}$$



Vbc = Vbn - Vcn = rt(3) Vbn angle(30degree)
= rt(3) Van angle (-90degree)

Line Current



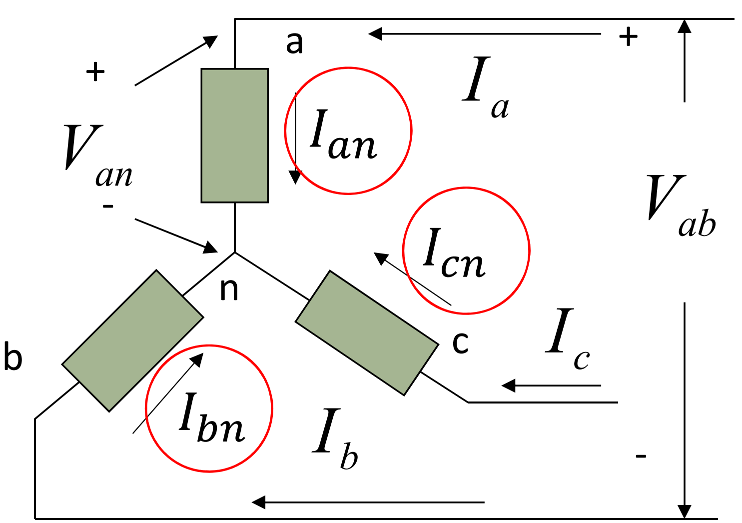
i_a, i_b, i_c are called line currents.

3-Phase Load Connection

Two types of connections apply to both three-phase voltage sources and three-phase loads.

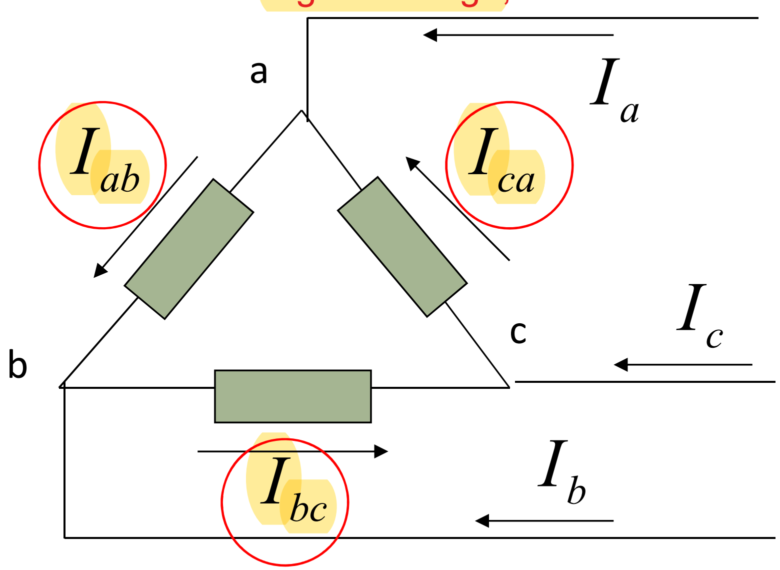
1. Wye Connection

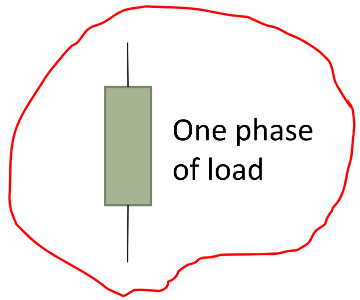
lower voltage, higher curent

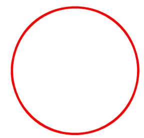


2. Delta Connection

higher voltage, lower current







Phase currents

- I_a, I_b, I_c are line currents

Currents through the three-phase conductor lines are called ‘Line currents’.

Currents carried by the load impedance are called ‘Phase currents’ or ‘Load Current’.

Delta-Connected Load

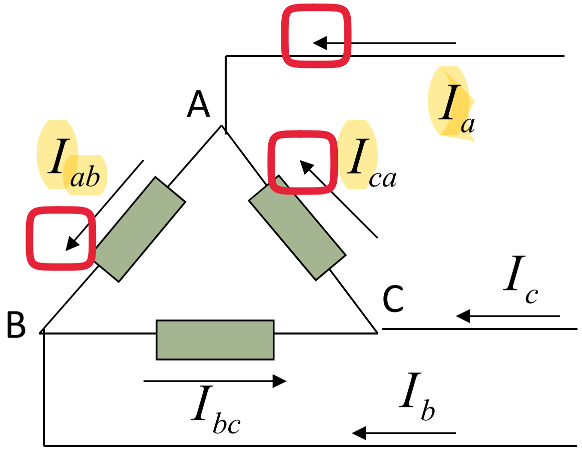
- I_{ab}, I_{bc}, I_{ca} are called Phase currents.

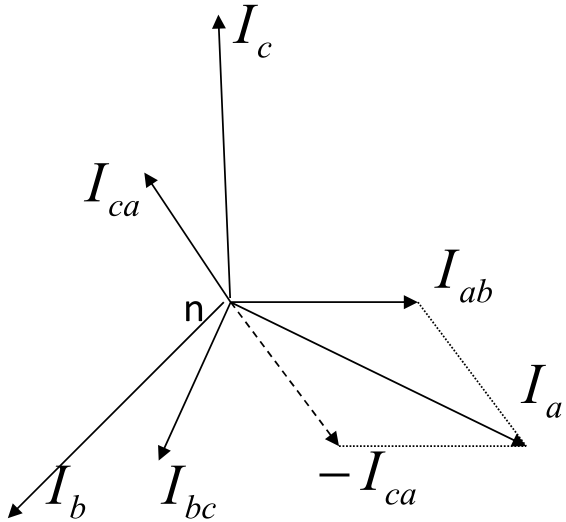
- We can find relationship between line currents and phase currents using KCL,

$$I_a = I_{ab} - I_{ca} = \sqrt{3} I_{ab} \angle -30°$$

$$\boxed{\left| I_{\text{Line}} \right| = \sqrt{3} \left| I_{\text{Phase}} \right|}$$

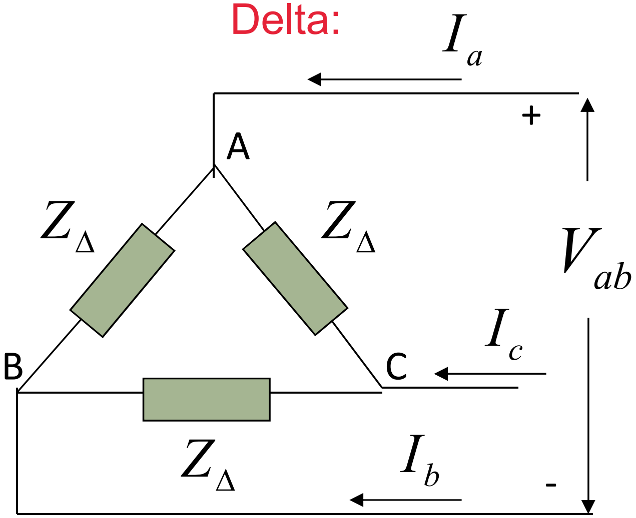
same as voltage -- only the angle is -ve!!!



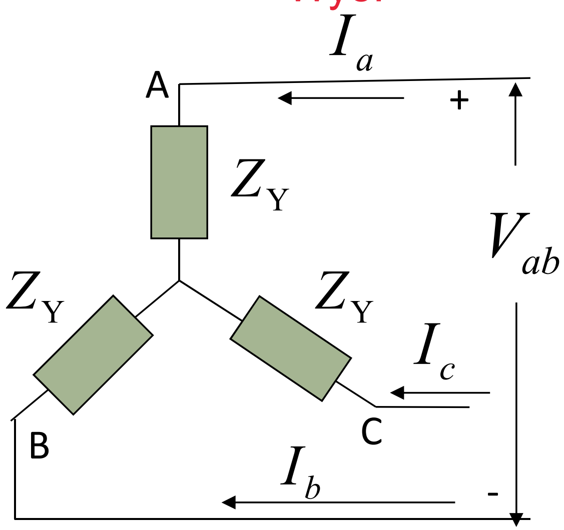


Delta-Wye Load Transformation

Delta:



Wye:



$$I_a = \sqrt{3} I_{ab} \angle \boxed{-} 30^\circ$$

$$V_{ab} = \sqrt{3} V_{an} \angle 30^\circ$$

$$Z_\Delta = \frac{V_{ab}}{I_{ab}} = \frac{\sqrt{3} V_{ab}}{I_a \angle 30^\circ}$$

$$\boxed{Z_\Delta = 3 Z_Y}$$

$$Z_Y = \frac{V_{an}}{I_a} = \frac{V_{ab}}{\sqrt{3} I_a \angle 30^\circ}$$

Delta has higher voltage
--> Z for delta is higher

Example

For a balanced Y-connected three phase voltage source and Y-connected load system with a line voltage of 440 V and three equal resistive loads of 100 Ω per phase, assume positive sequence, what will be the magnitudes of

(a) the line-to-neutral voltage,

(b) the phase current,

(c) the line current?

Example

For a balanced Y-connected three phase generator with the line-to-neutral voltage of 80 V, Δ-connected load of 120 Ω, assume positive sequence, find

(a) the line-to-line voltage,

(b) the voltage across a resistor,

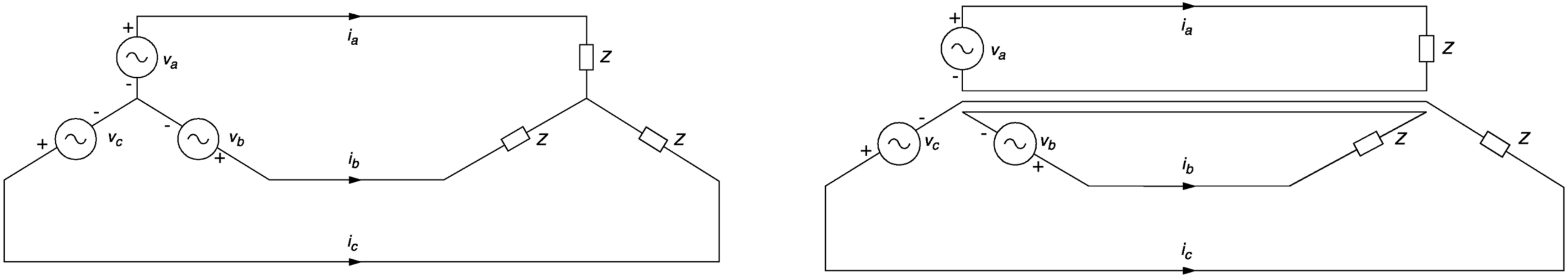
(c) the current through a resistor?

Three-phase Circuit Analysis

Three-phase complex power

Per Phase analysis

Three Phase Power Calculation



Three phase power is found from summation of each phase power.

$$S_{3\Phi} = V_{an} I_a^* + V_{bn} I_b^* + V_{cn} I_c^*$$

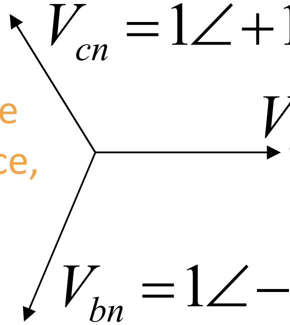
Balanced Three-Phase Power

- From three phase power,

$$S_{3\Phi} = V_{an} I_a^* + V_{bn} I_b^* + V_{cn} I_c^*$$

- When the system is balanced, (assume positive sequence) we can write,

$$S_{3\Phi} = V_{an} I_a^* + V_{an}\angle -120°\left(I_a \angle -120°\right)^* + V_{an}\angle 120°\left(I_a \angle 120°\right)^*$$



$V_{cn} = 1\angle +120°$

$V_{an} = 1\angle 0°$

$V_{bn} = 1\angle -120°$

Positive sequence, abc

$$\boxed{S_{3\Phi} = 3 V_{an} I_a^*}$$

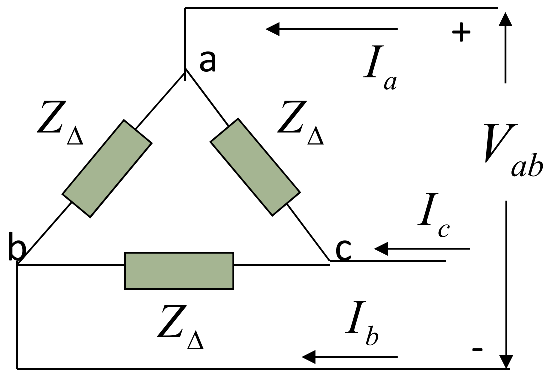
Balanced Three-Phase Load

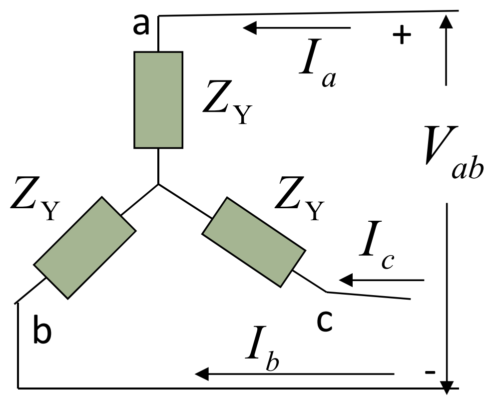
- Three-phase load can be connected in either Wye or Delta connection.
- 3-phase load parameter is given as total apparent power ($|S_{3\Phi}|$) with power factor.
- The voltage given is **Line-to-line voltage**.
- We can find three-phase real and reactive power as follows.

$$P_{3\Phi} = 3P_{1\Phi} = |S_{3\Phi}| \times \text{p.f.}$$

$$Q_{3\Phi} = 3Q_{1\Phi} = |S_{3\Phi}| \times \sin\left(\cos^{-1}(\text{p.f.})\right) = |S_{3\Phi}| \times \sin\phi$$

Delta/Wye Connected 3-Phase Load





$$I_a = \sqrt{3} I_{ab} \angle -30^\circ$$

$$|S_{3\Phi}| = 3|V_{ab} I_{ab}^*| = \sqrt{3} |V_{ab}||I_a|$$

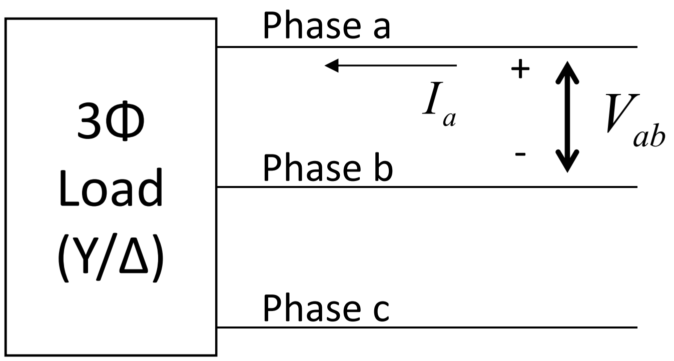
$$V_{ab} = \sqrt{3} V_{an} \angle 30^\circ$$

$$|S_{3\Phi}| = 3|V_{an} I_a^*| = \sqrt{3} |V_{ab}||I_a|$$

$$\boxed{|S_{3\Phi}| = \sqrt{3} |V_{\text{Line-To-Line}}||I_{\text{Line}}|}$$

for Wye and Delta!!!!

Three-Phase Load



No matter what the load connection is (Wye or Delta), we can still compute the apparent power using the same equation.

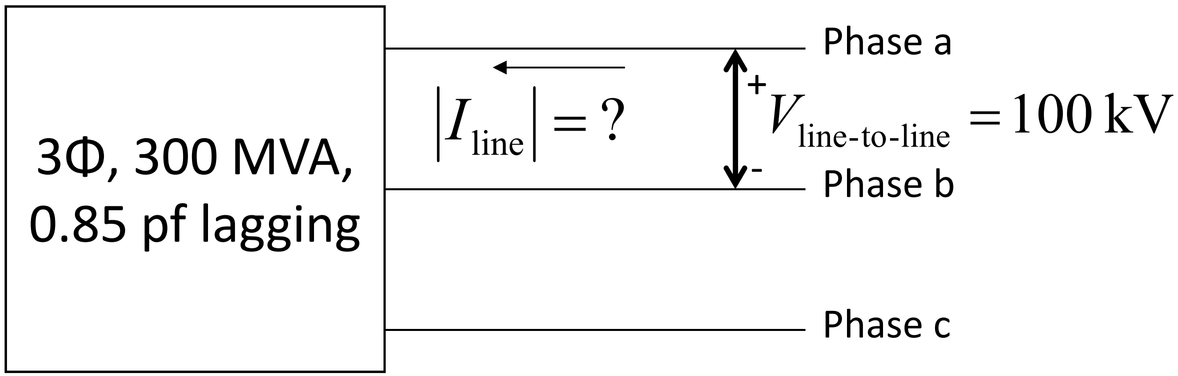
(Given that line-to-line voltage and line currents are known.)

$$\boxed{\left|S_{3\Phi}\right| = \sqrt{3}\left|V_{\text{Line-To-Line}}\right|\left|I_{\text{Line}}\right|}$$

Example

A 3Φ load of 300 MVA, 100 kV at 0.85 p.f. lagging, find

- The magnitude of line current $\left|I_{\text{Line}}\right|$
- Three-phase (real) power P_{Load}



Ans: 1732 A, 255 MW.

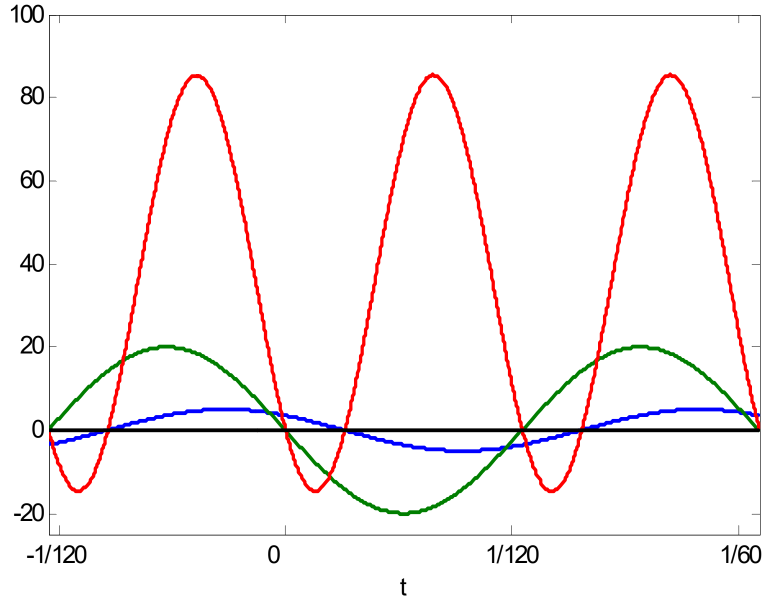
Instantaneous Three-Phase Power

- Given by, $p_{3\Phi}(t) = v_a(t)i_a(t) + v_b(t)i_b(t) + v_c(t)i_c(t)$

- Recall that single phase instantaneous power,

$$v(t) = |V|\cos(\omega t + \theta_v)$$
$$i(t) = |I|\cos(\omega t + \theta_i)$$

$$\begin{aligned} p(t) &= v(t) \times i(t) \\ &= |V||I|\cos(\theta_v - \theta_i) \\ &\quad + |V||I|\cos(2\omega t + \theta_v + \theta_i) \end{aligned}$$



Instantaneous Three-Phase Power

- For a balanced three-phase system,

=0

$$\begin{aligned} p_{3\Phi}(t) &= |V_a||I_a|\cos(\theta_v - \theta_i) + |V_a||I_a|\cos(2\omega t + \theta_v + \theta_i) \\ &+ |V_b||I_b|\cos(\theta_v - \theta_i) + |V_b||I_b|\cos(2\omega t + \theta_v + \theta_i - 240^\circ) \\ &+ |V_c||I_c|\cos(\theta_v - \theta_i) + |V_c||I_c|\cos(2\omega t + \theta_v + \theta_i - 480^\circ) \end{aligned}$$

- We can find three phase instantaneous power as,

$$p_{3\Phi}(t) = 3|V_a||I_a|\cos\phi = 3P$$

- **Constant** power transfer to load.

An Additional Advantage of Balanced 3-Phase Circuit

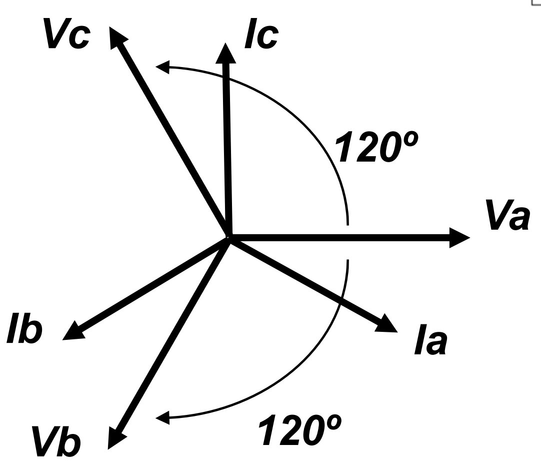
- When compared to three single-phase circuits, three-phase circuits have better use of equipment and materials
  - 3 phase conductors instead of 6
  - Reduce power losses in phase conductors
- This implies reduced capital and operating costs of transmission and distribution.
- We can calculate voltage and current for only one phase and refer to other phases easily.
- Constant power transfer to load.
  - This also implies constant mechanical power input for a generator.
  - When mechanical power input is constant, mechanical shaft torque is also constant.
  - This helps to reduce shaft vibration and noise, extending the machine’s lifetime.

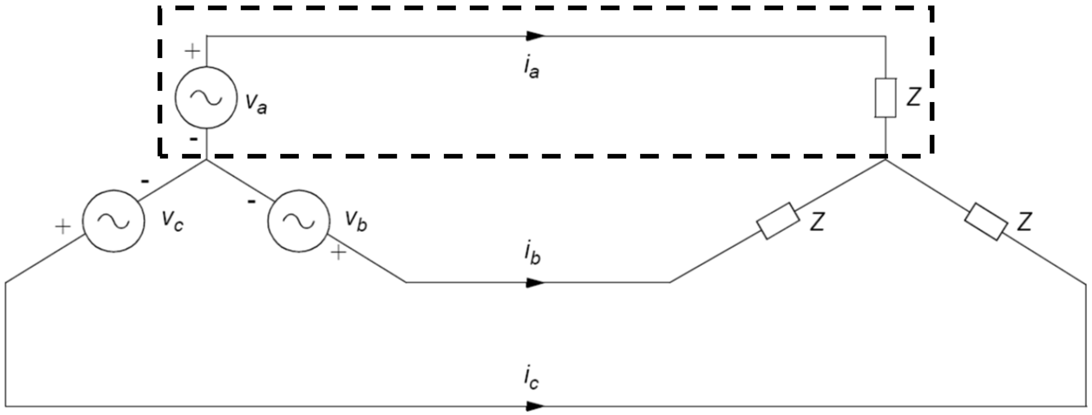
Per phase analysis

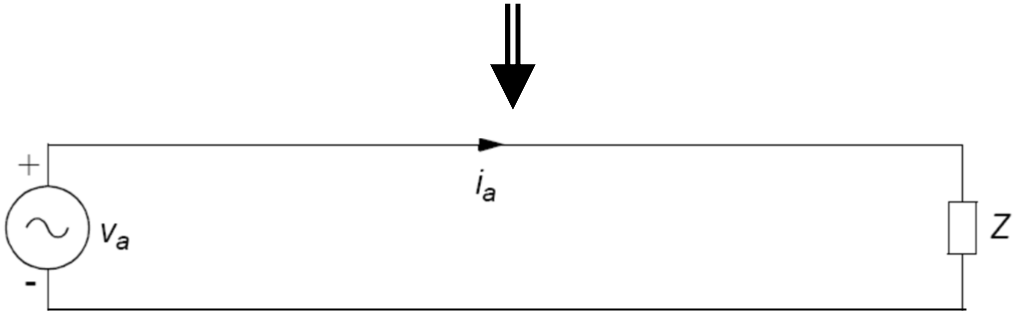
Per Phase Analysis: Assumption

It must be balanced three-phase circuit.

$$I_n = I_a + I_b + I_c = 0$$



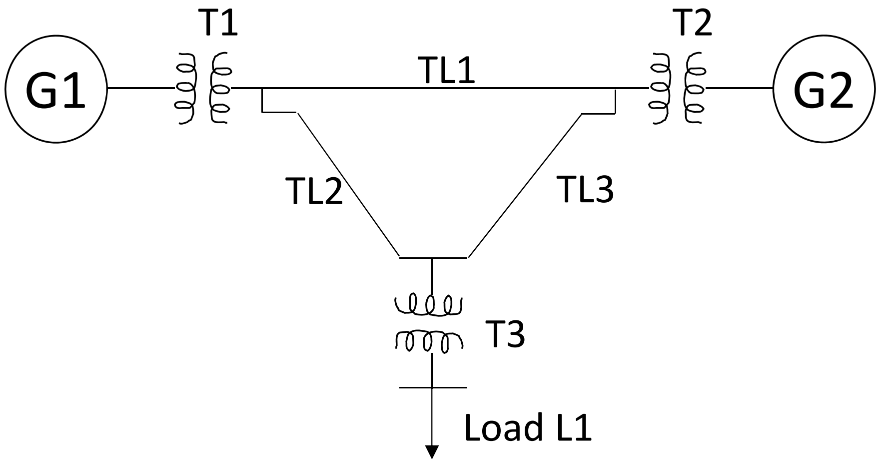




Steps of Per Phase Analysis

- Make sure that the three-phase system is balanced.
  - The three-phase sources need to have the same magnitude with 120 degree phase difference.
  - The three-phase impedances must be of the same value (both phase and magnitude).
- **Convert** all Delta-connected sources/loads to Wye-connected sources/loads.
- Per phase analysis reduce three-phase circuit to single-phase circuit. We can apply the same concept used in single-phase.

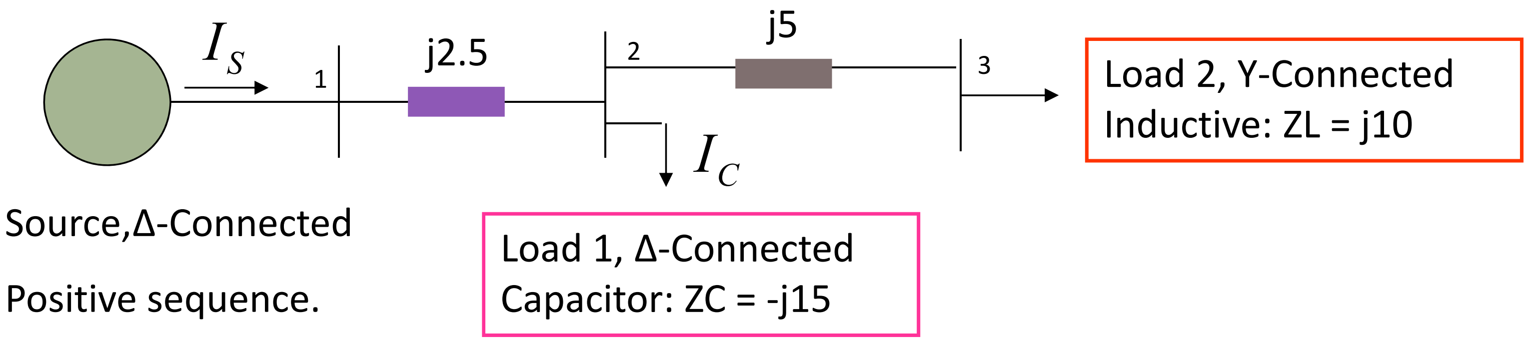
Single-Line Diagram



- Show the interconnections of a transmission system
  - Generator
  - Load
  - Transmission line
  - Transformer
- This is a representation of a 3Φ circuit. Each line represents three conductors in three-phase system.

Example

- Given a one-line diagram,

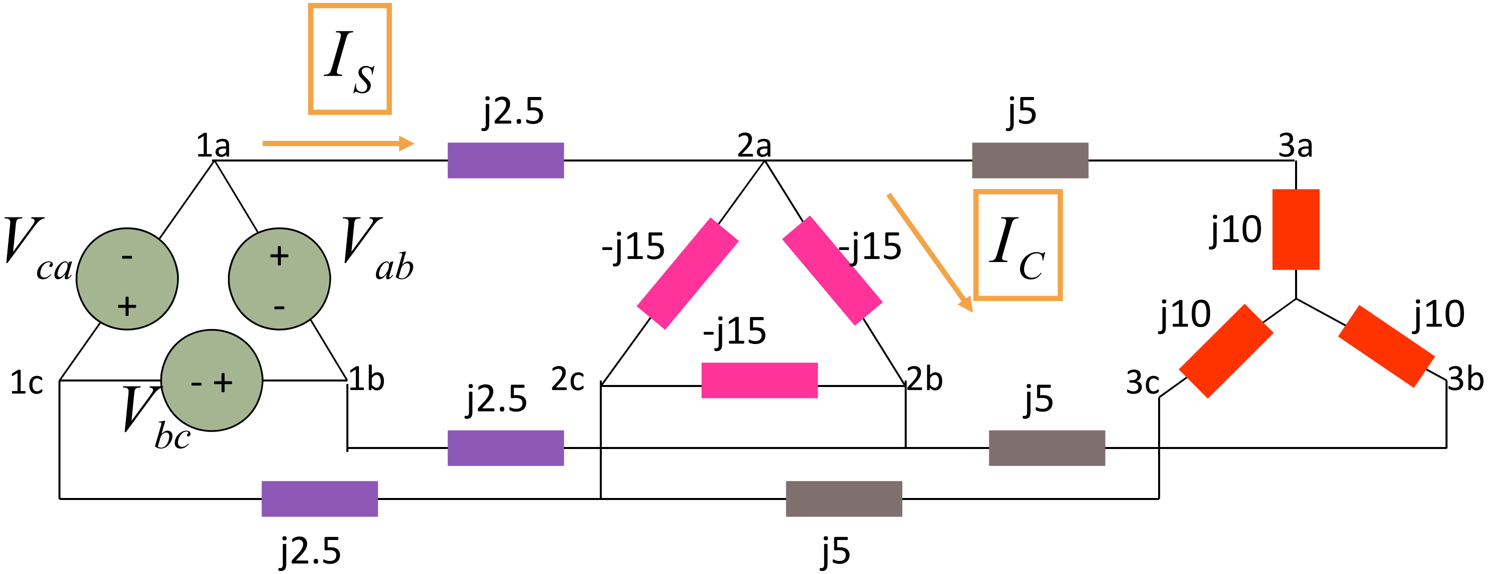


If the voltage source is $\left|V_{\text{Line-Line}}\right| = \sqrt{3}\ \text{V}$. Find,

1. Current magnitude supplied by source, $\left|I_S\right|$, and,
2. Current magnitude through a capacitor, $\left|I_C\right|$.

Example

- Find the source current I_s and capacitor phase-current I_C.

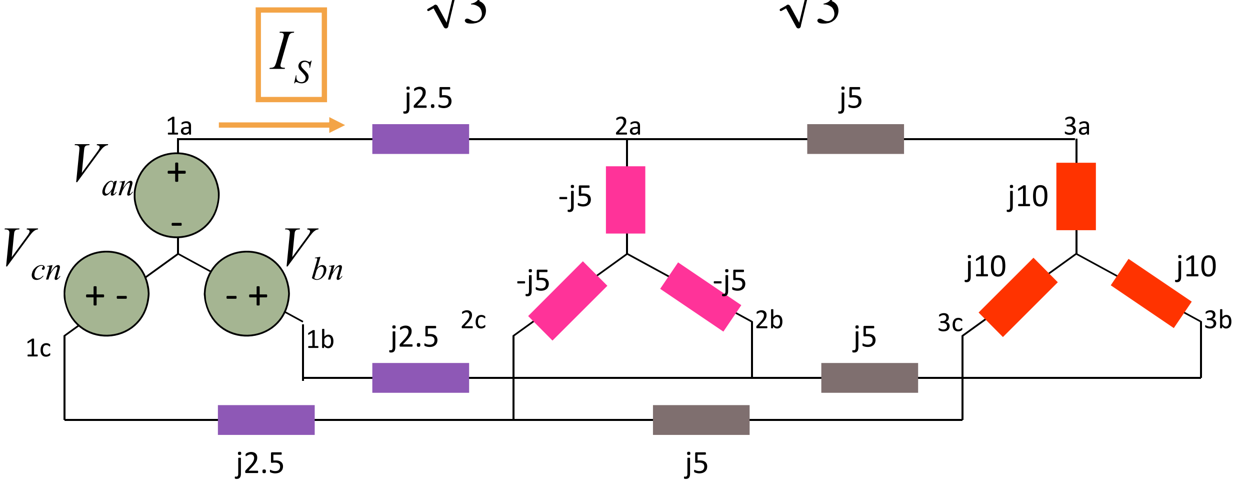


Solution : Convert from Δ → Y

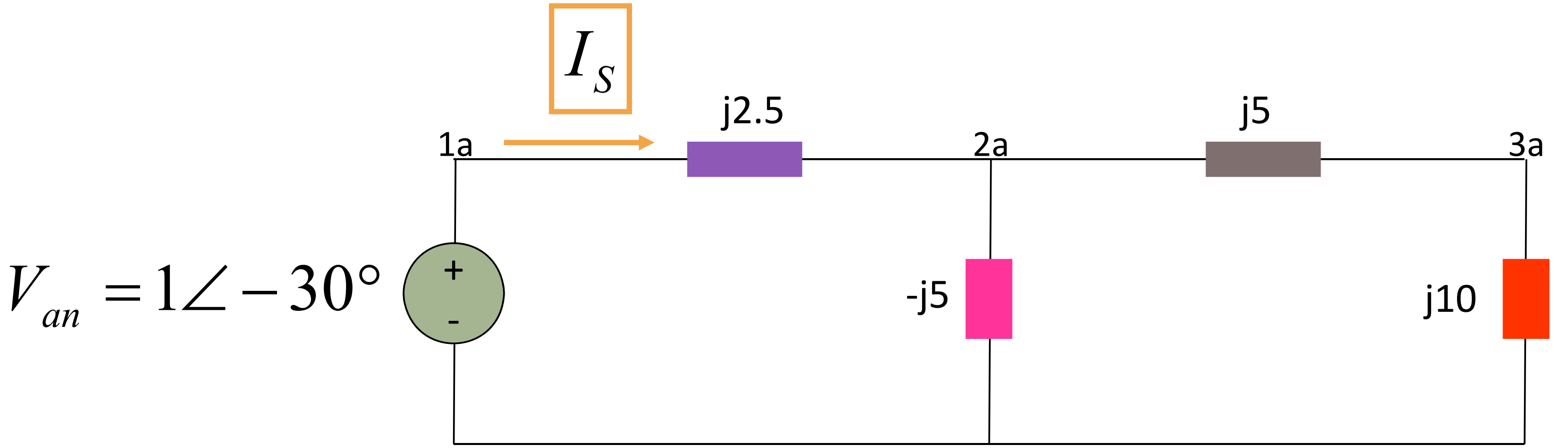
$$Z_Y = \frac{Z_\Delta}{3} = \frac{-j15}{3} = -j5$$

Use line-to-line voltage source as angle reference

$$V_{an} = \frac{V_{ab}}{\sqrt{3}}\angle -30^\circ = \frac{\sqrt{3}\angle 0^\circ}{\sqrt{3}}\angle -30^\circ = 1\angle -30^\circ$$



Solution: 1-Phase diagram

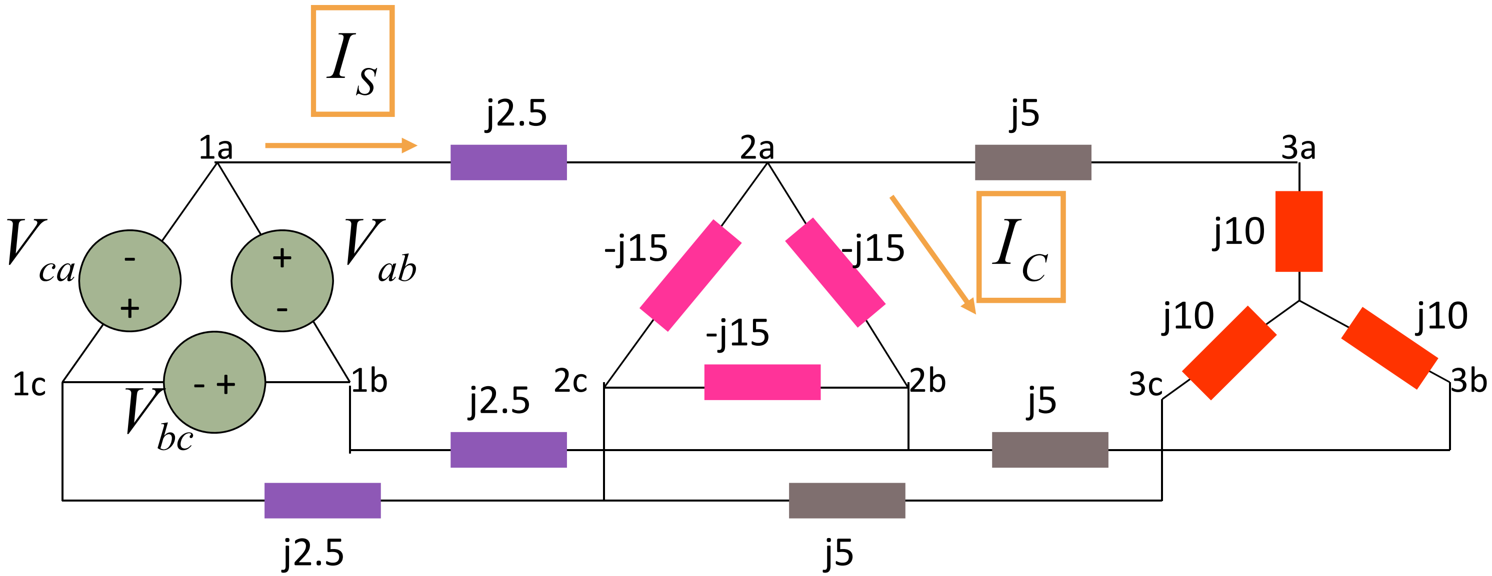


$$Z_{eq} = j2.5 + \frac{(j10 + j5)(-j5)}{(j10 + j5) + (-j5)} = -j5$$

$$I_S = \frac{V_{an}}{Z_{eq}} = \frac{1\angle -30°}{-j5} = \frac{1\angle -30°}{5\angle -90°} = 0.2\angle 60°\ \text{A}$$

$V_{2a} = V_{an} - j2.5 \times I_S = 1.5\angle -30°$ We will use this to find $\boxed{I_C}$

Solution: Final Calculation



$$V_{2b} = V_{2a}\angle -120°$$

$$I_C = \frac{V_{2a} - V_{2b}}{-j15} = \frac{1.5\angle -30° - 1.5\angle(-30° - 120°)}{15\angle -90°} = \frac{\sqrt{3}}{10}\angle 90° \text{ A}$$

Ans: $I_S = 0.2\angle 60°, I_C = 0.1732\angle 90°$