



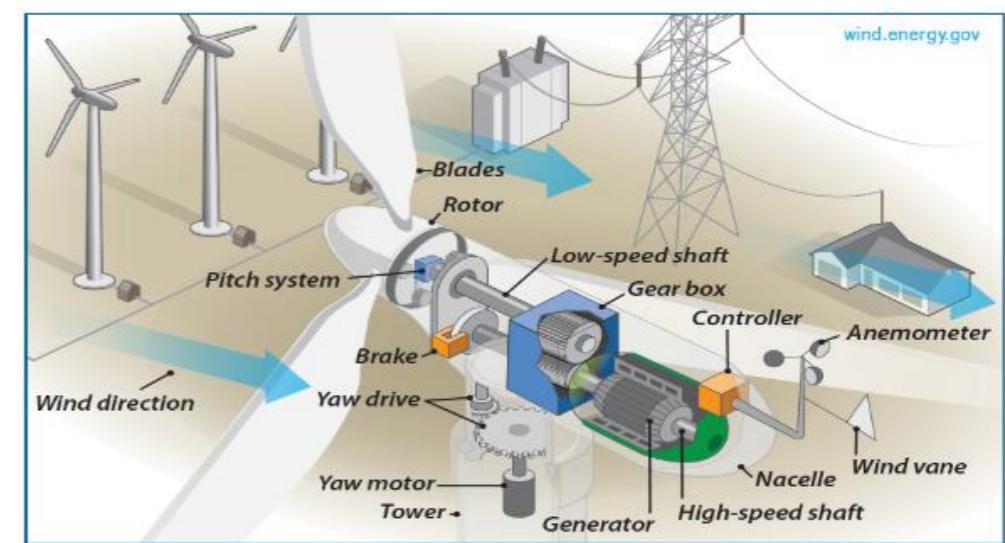
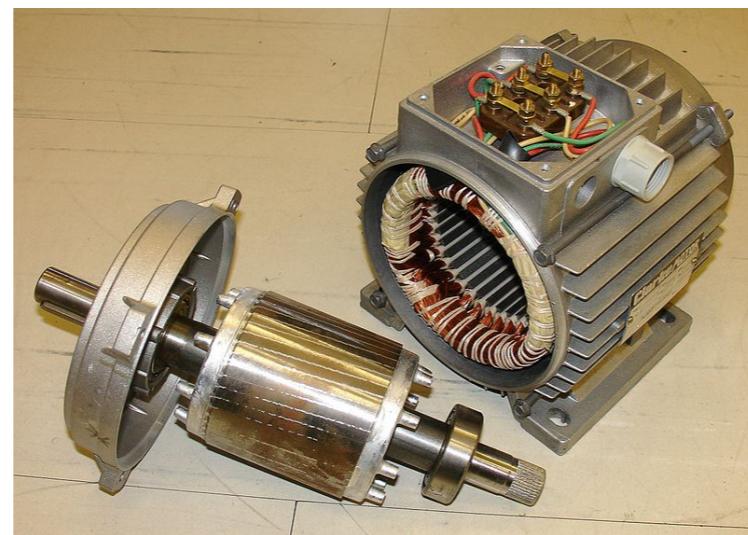
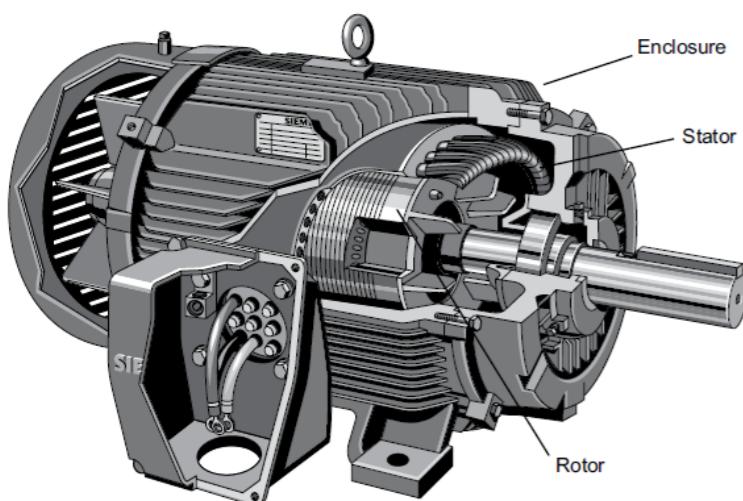
EE2029 - Introduction to Electrical Energy Systems

# Induction Motors

Fundamental Background  
Types of Induction Motor  
Equivalent Circuit  
Torque-Speed Characteristic

# AC Motor

- Consists of stator and rotor:
  - Stator: stationary electrical component
  - Rotor: rotates the motor shaft
- There are two types of AC motors: synchronous and induction.
  - Main difference between synchronous and induction motor is that the rotor of the induction motor travels at a varying speed as the rotating magnetic field.



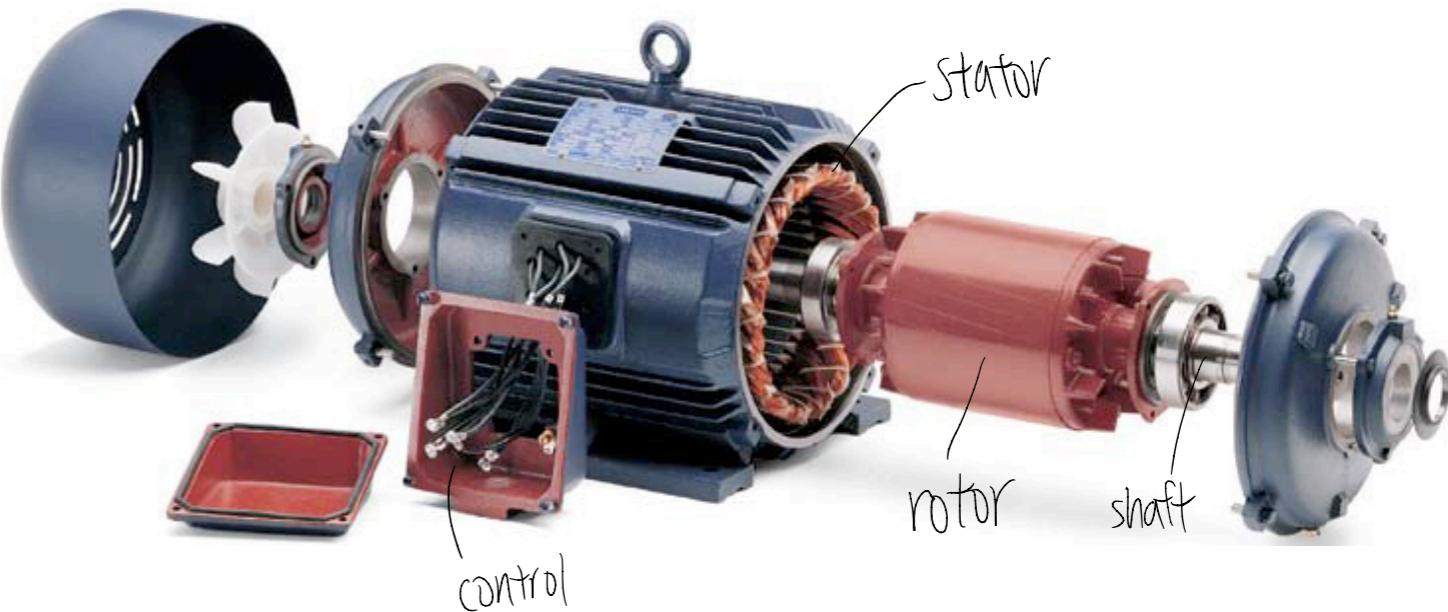
# Induction Motor Advantages

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- Over 70% of our industry uses induction motor.
- Its advantages include:
  - Simple design
  - Inexpensive to construct
  - High power to weight ratio
  - Easy to maintain
  - Direct connection to AC power source

# Operating Principles (Brief)

1. Electricity is supplied to stator
  - Rotating magnetic field established
2. Generated magnetic field moves around the rotor
3. Current is induced in rotor
  - Produces a second magnetic field opposing the stator magnetic field
4. Rotor begins to rotate



# Operating Principles (Detailed)

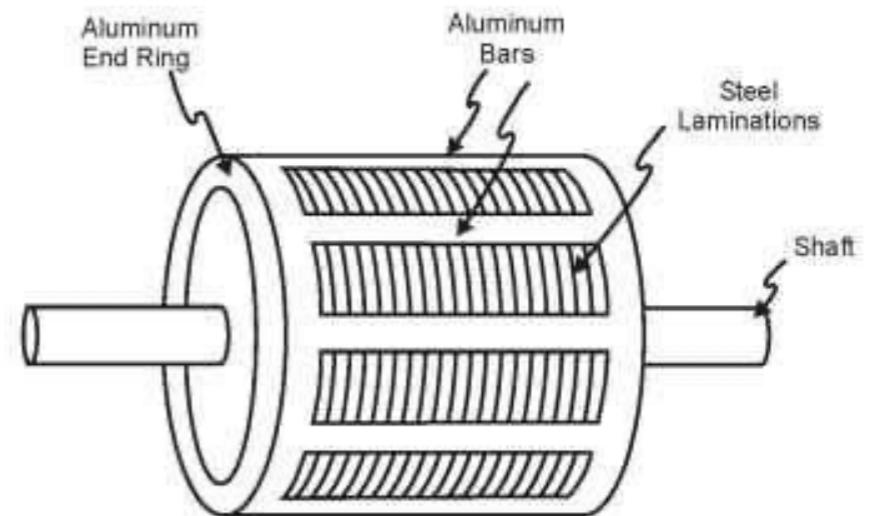
1. Three-phase stator is supplied by balanced three-phase voltage that drives an AC magnetising current through each phase winding.
2. Magnetising current in each phase generates a pulsating AC flux.
3. Total flux in the machine is the sum of the three fluxes.
4. Summation of the three AC fluxes results in a rotating flux, which turns with constant speed and has constant amplitude.
5. Rotating flux induces a voltage in the short-circuited bars of the rotor. This voltage drives current through the bars.
6. Induced voltage is proportional with the difference of motor and synchronous speed. Consequently the motor speed is less than the synchronous speed
7. Interaction of the rotating flux and the rotor current generates a force that drives the motor.

rotor will be  
chasing stator.

# Single-Phase Induction Motor

# Single-Phase Induction Motor

- Single-phase power supply:
  - E.g. Squirrel cage rotor.
- Require a device to start motor:
  - Up to 3 to 4 HP
- Commonly use in household appliances:
  - E.g. fans, washing machines, dryers.



Rotor Design :

1. Wound rotor

↳ higher starting torque (for heavy-duty)  
↳ require higher maintenance

2. Squirrel cage

↳ lower starting torque

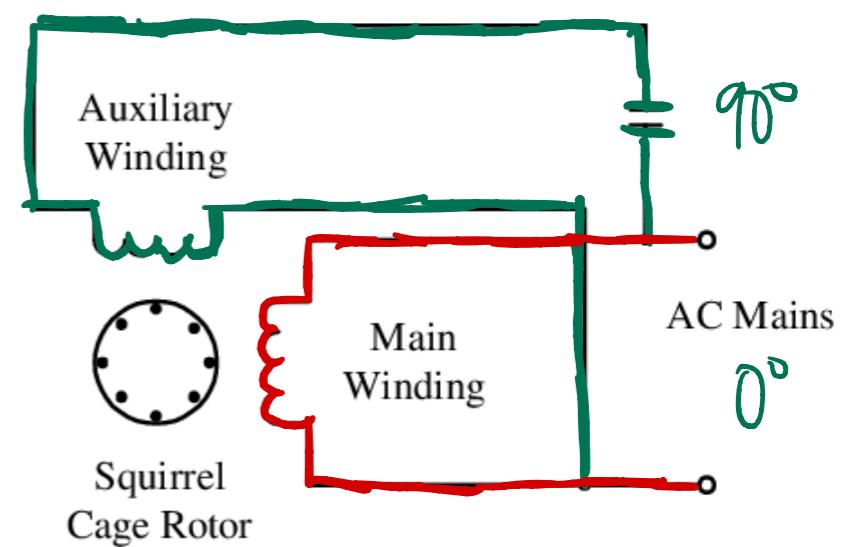


# Three Types of Capacitor-Start Motor

1. **Capacitor-Start** (disconnects capacitor after motor speed picks up)
2. **Capacitor-Run** (Keeps the capacitor connected during the operation of the motor, in order to keep the electric power consumption low)
3. **Capacitor-Start-Run** (uses two capacitors, one for starting and one for running. This further improves Power Consumption)

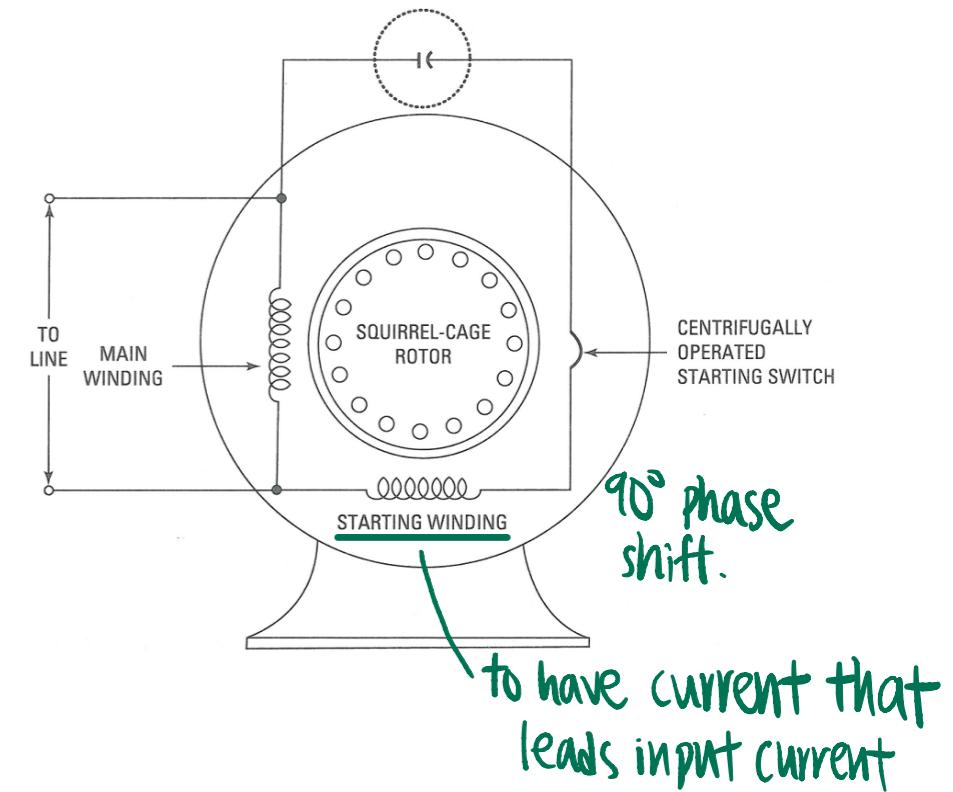
Goal: to mimic a multi-phase system w single-phase supply.

Turn single-phase motors into two-phase motors to produce rotating



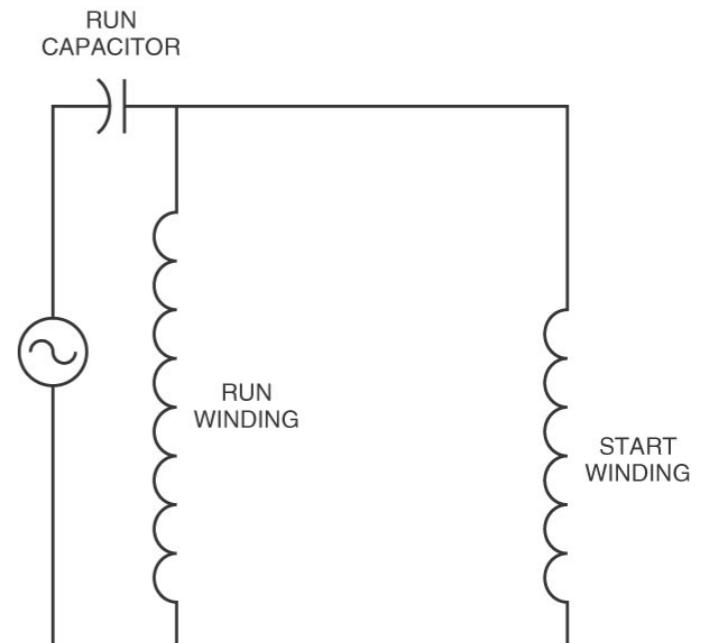
# Capacitor-Start Circuit

- Start circuit has:
  - Centrifugal switch
  - Start winding
  - Start capacitor (produces a higher starting torque)
- Run winding
- Reverse direction of rotation by interchanging run winding or start winding connections (preferred).



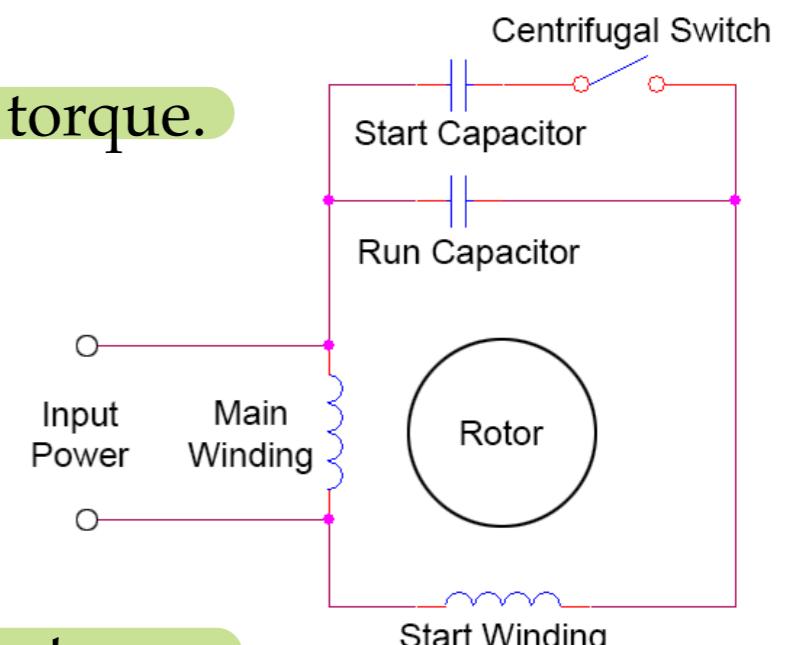
# Capacitor-Run Circuit

- Capacitor shifts the phase of one of the windings.
  - Voltage across the winding is at  $90^\circ$  from other winding.
- Run capacitor produces higher running torque.
- Start-winding stays as part of the circuit.
- Reverse the direction of rotation by interchanging run-winding with start-winding connections (preferred).



# Capacitor-Start-Run Circuit

- Start circuit:
  - Centrifugal switch: disconnect the start winding once the motor approaches its normal operating speed
  - Start capacitor: larger value produces higher starting torque.  
  
*Starting torque >> running torque*
- Run winding: 
  - Run capacitor: smaller value produces higher running torque.
  - Reverse the direction of rotation by interchanging run winding or start winding connections (preferred).

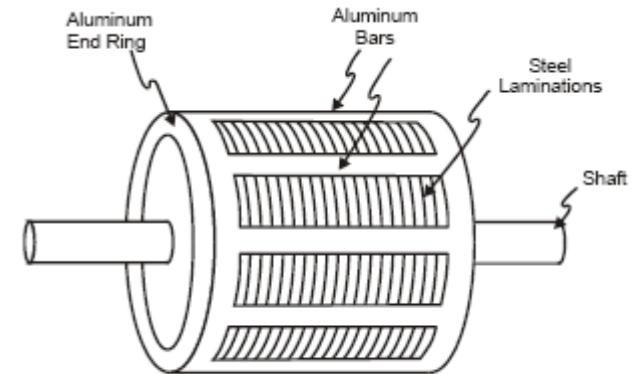


# Three-Phase Induction Motor

# Induction Motor Design

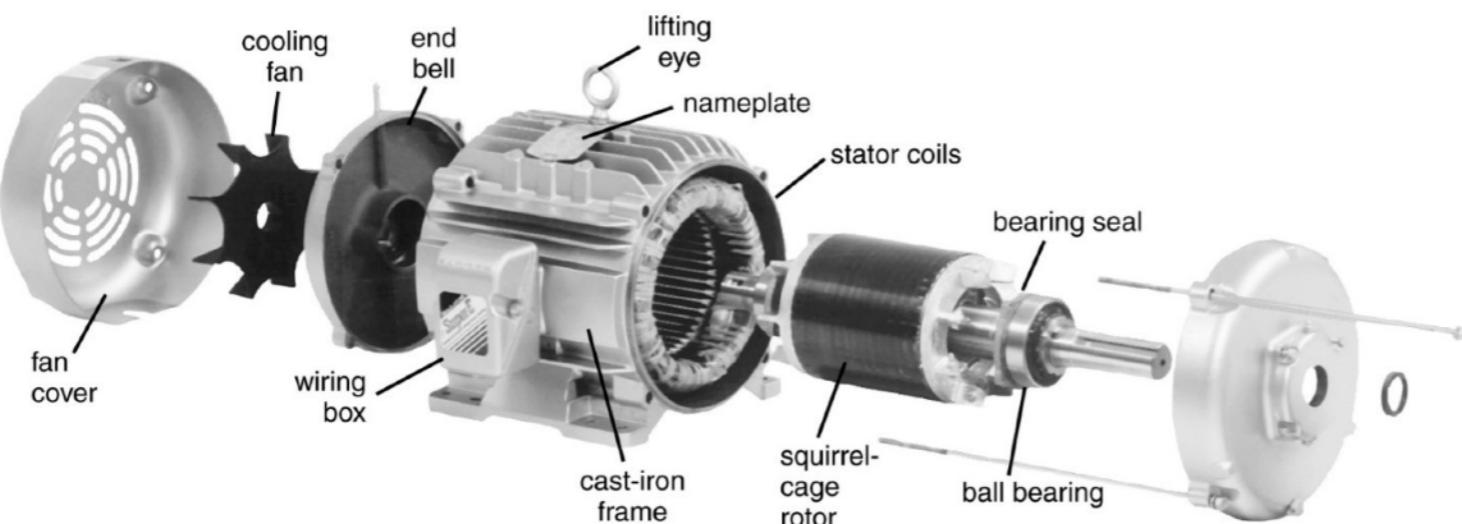
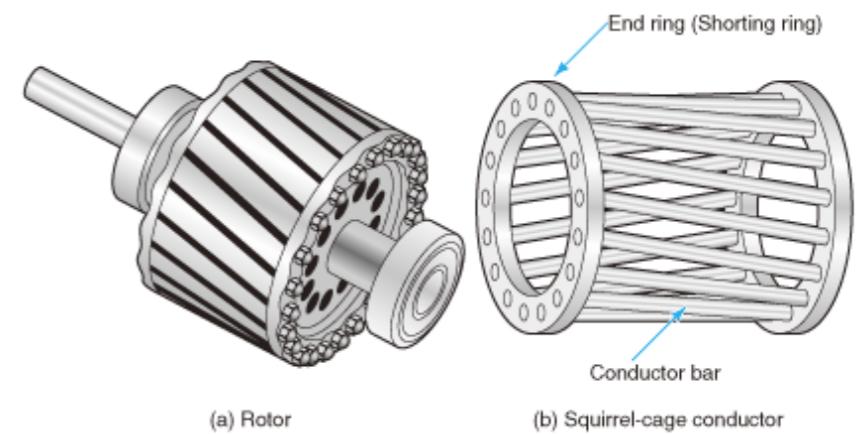
- **Rotor:**

- Squirrel cage: conducting bars in parallel slots.
- Wound rotor: 3-phase, double-layer, distributed winding.



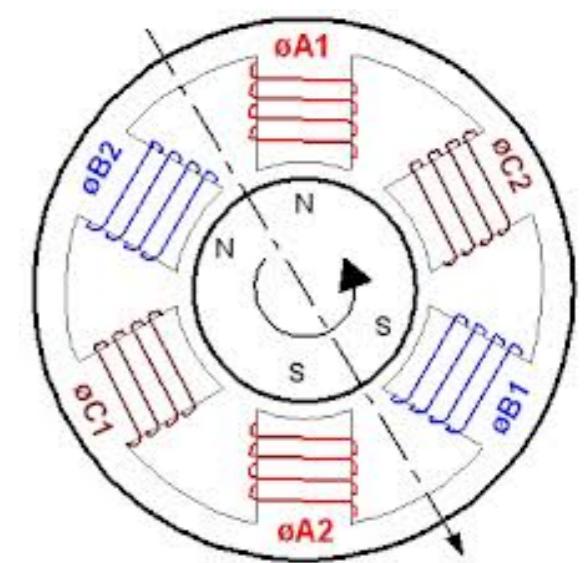
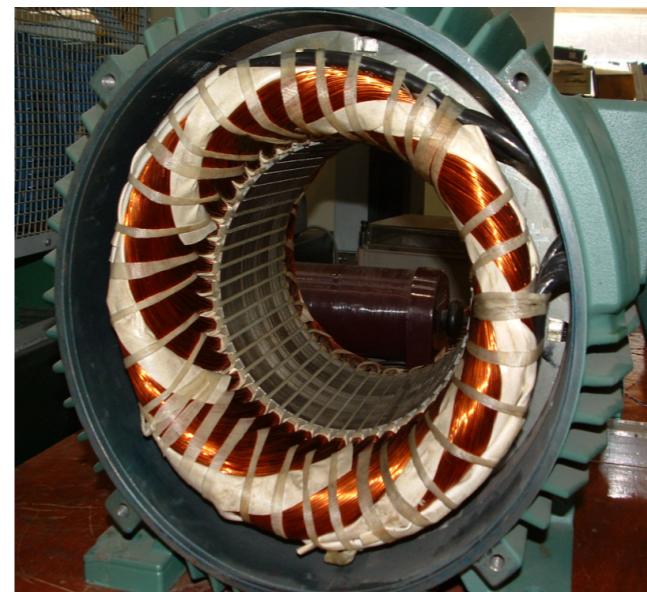
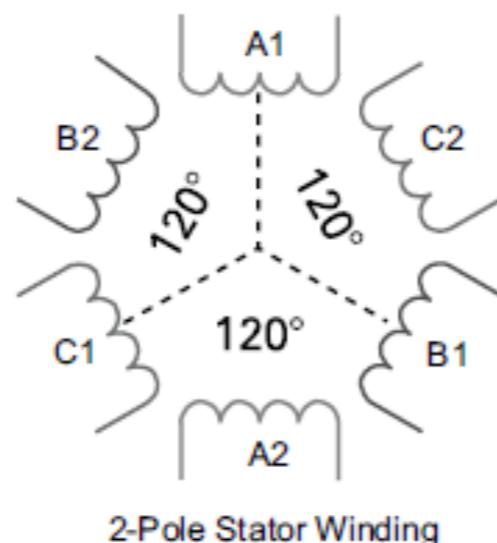
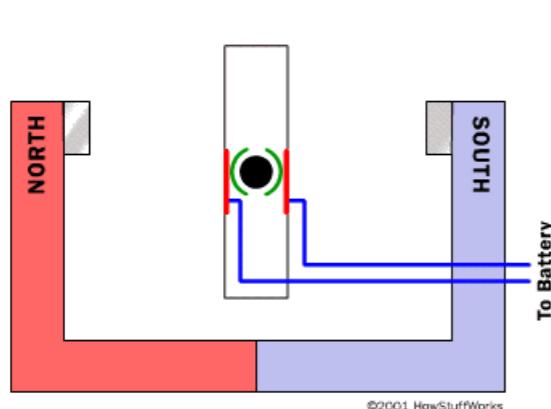
- **Stator:**

- Stampings with slots to carry 3-phase windings.
- Wound for definite number of poles.



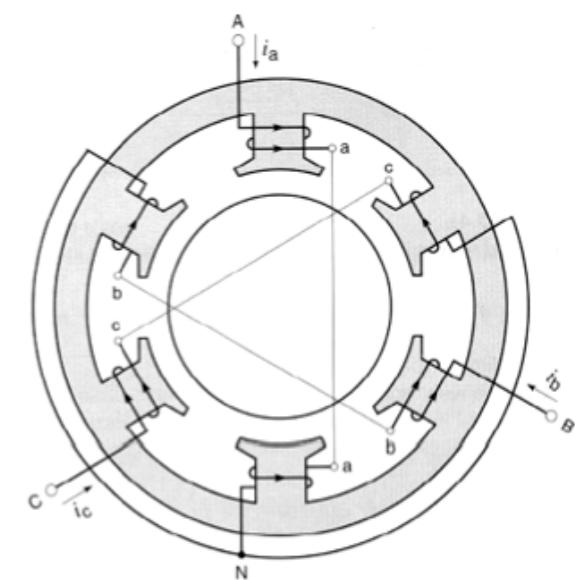
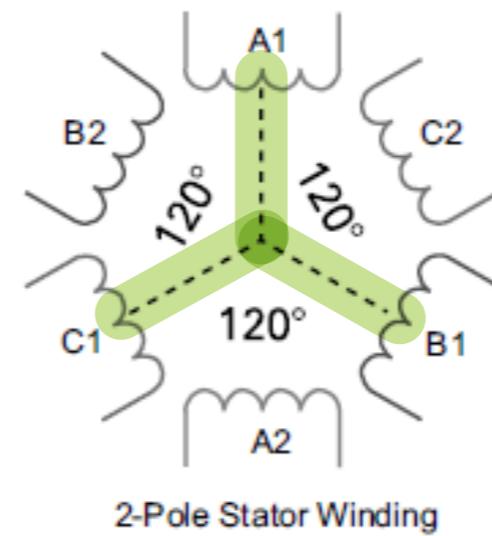
# What are Poles in a Motor?

- Winding(s) that produce the magnetic field(s) necessary to cause the rotor to turn.



# Three-Phase Rotating Field

- AC currents  $I_a$ ,  $I_b$  and  $I_c$  will flow in the windings, but will be displaced in time by  $120^\circ$ .
- Each winding produces its own MMF, which creates a flux across the hollow interior of the stator.
- The 3 fluxes combine to produce a magnetic field that rotates at the same frequency as the supply.



# Rotating Field - Direction of Rotation

- Phase current waveforms follow each other in the sequence A-B-C.
  - This produces a clockwise rotating magnetic field.
- If we interchange any two of the lines connected to the stator, the new phase sequence will be A-C-B.
  - This will produce a counterclockwise rotating field, reversing the motor direction.

# Number of Poles – Synchronous Speed

- The rotating speed of the revolving stator flux can be reduced by increasing the number of poles (in multiples of two).
  - In a four-pole stator, the phase groups span an angle of  $90^\circ$ .
  - In a six-pole stator, the phase groups span an angle of  $60^\circ$ .
- This leads to the definition of synchronous speed, i.e. the speed of the rotating stator flux:

$$N_s = 120 f/p$$

- $N_s$  = synchronous speed (rpm)
- $f$  = frequency of the supply (Hz)
- $p$  = number of poles. ( $\uparrow$  poles,  $\uparrow$  control)

# Induction Motors Slip

- Difference between synchronous speed and rotor speed can be expressed as a percentage of synchronous speed, known as the slip:

$$s = \frac{N_s - N}{N_s} \times 100\%$$

how much motor is converting electrical energy to mechanical power.

- where  $s$  = slip,  $N_s$  = synchronous speed (rpm),  $N$  = rotor speed (rpm)
- At no-load, the slip is nearly zero ( $<0.1\%$ ).  $N_s \approx N$ .
- At full load, the slip for large motors rarely exceeds 0.5%.
- For small motors at full load, it rarely exceeds 5%.
- Slip is 100% for locked rotor.

# Frequency Induced In the Rotor

- The induced frequency in the rotor depends on the slip:

$$f_R = sf$$

$$f_R = \frac{N_s - N}{N_s} f$$

✓ slip frequency.

- $f_R$  = frequency of voltage and current in the rotor, i.e. slip frequency
- $f$  = frequency of the supply and stator field
- $s$  = slip.
- Induction motor is a common form of an asynchronous motor.
  - Basically an AC transformer with a rotating secondary side.

# Example 1

convert to W:  $1 \text{ hp} = 745.7 \text{ W}$ .

A 208 V, 10 hp, four pole, 60 Hz, Y-connected induction motor has a full-load slip of 5 percent. Answer the following questions:

1. What is the synchronous speed of this motor?
2. What is the rotor speed of this motor at rated load?
3. What is the rotor frequency of this motor at rated load?

4. What is the shaft torque of this motor at rated load?

$$1. N_s = \frac{120f}{P} = \frac{120(60)}{4} = 1800 \text{ rpm},$$

$$2. s = \frac{N_s - N}{N_s} \times 100\%.$$

$$\Rightarrow 0.05 = \frac{1800 - N}{1800}$$

$$\therefore N = 1710 \text{ rpm}$$

convert to rad/s.  
 $1 \text{ rpm} = 1 \times \frac{2\pi}{60} \text{ rad/s.}$

$$3. f_R = sf = 0.05(60) = 3 \text{ Hz},$$

$$4. T_{\text{shaft}} = \frac{P_{\text{shaft}}}{\omega_m}$$

$1 \text{ hp} = 745.7 \text{ W}$

$$= \frac{10(745.7)}{1710 \left(\frac{2\pi}{60}\right)}$$

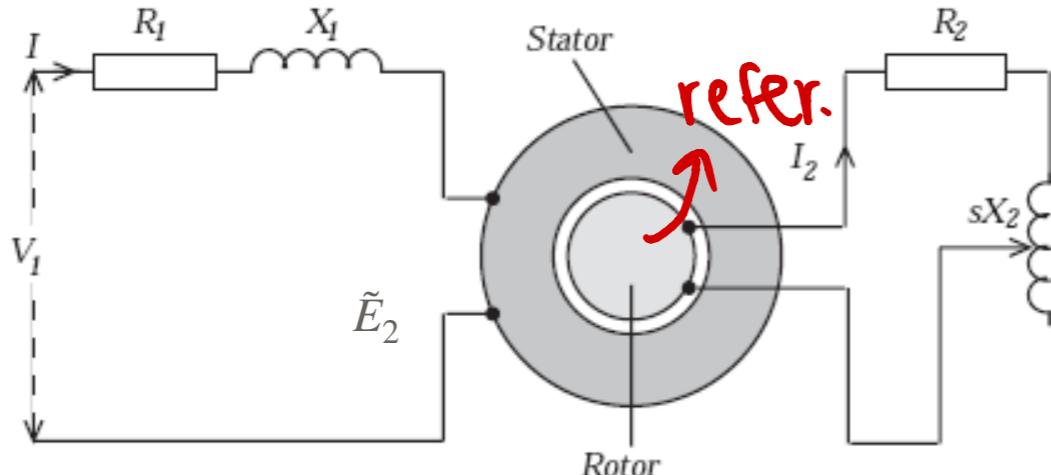
$1 \text{ rpm} = 1 \times \frac{2\pi}{60} \text{ rad/s}$

$$= 41.6 \text{ Nm}$$

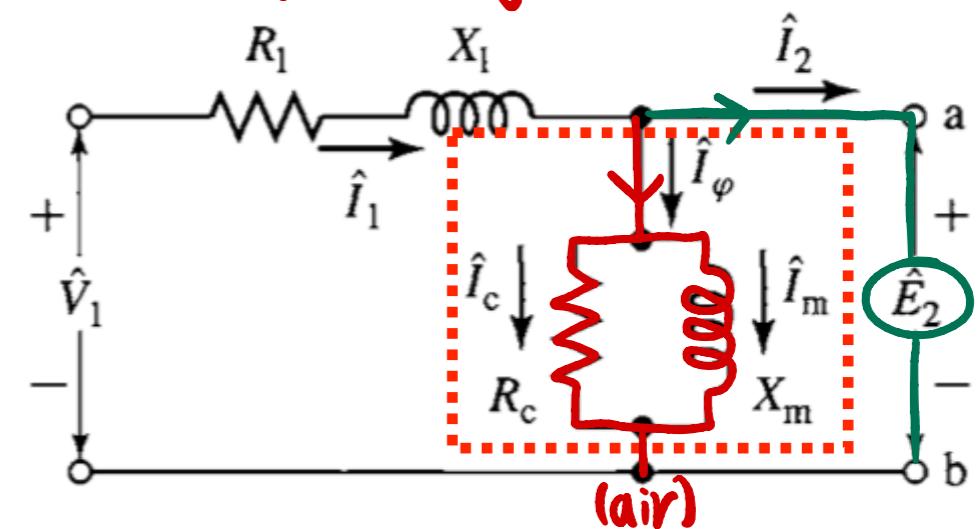
# Equivalent Circuit

# Equivalent Circuit

- Induction motor is an AC transformer with a rotating secondary side.
- Equivalent model can be derived from existing per-phase transformer equivalent circuit.
- View it as a **variable frequency transformer!** magnetizing component → air gap load component → rotor



$$\tilde{V}_1 = \tilde{I}_1(R_1 + jX_1) + \tilde{E}_2$$



$$Z_2 = \frac{E_2}{I_2}$$

$$Z'_2 = \frac{\tilde{E}'_2}{\tilde{I}'_2} = R_2 + jsX_2$$

↳ refer rotor to stator!  
(secondary) (primary)

# Equivalent Circuit

- Because the relative speed of the flux wave with respect to the rotor is governed by the slip-frequency, the relation between  $E_2$  and  $E'_2$  is:

$$E'_2 = sE_2$$

- Phase angle of both voltages can be assumed to be the same such that:

$$\tilde{E}'_2 = s\tilde{E}_2$$

- Rearrange to make slip associated with the impedance:

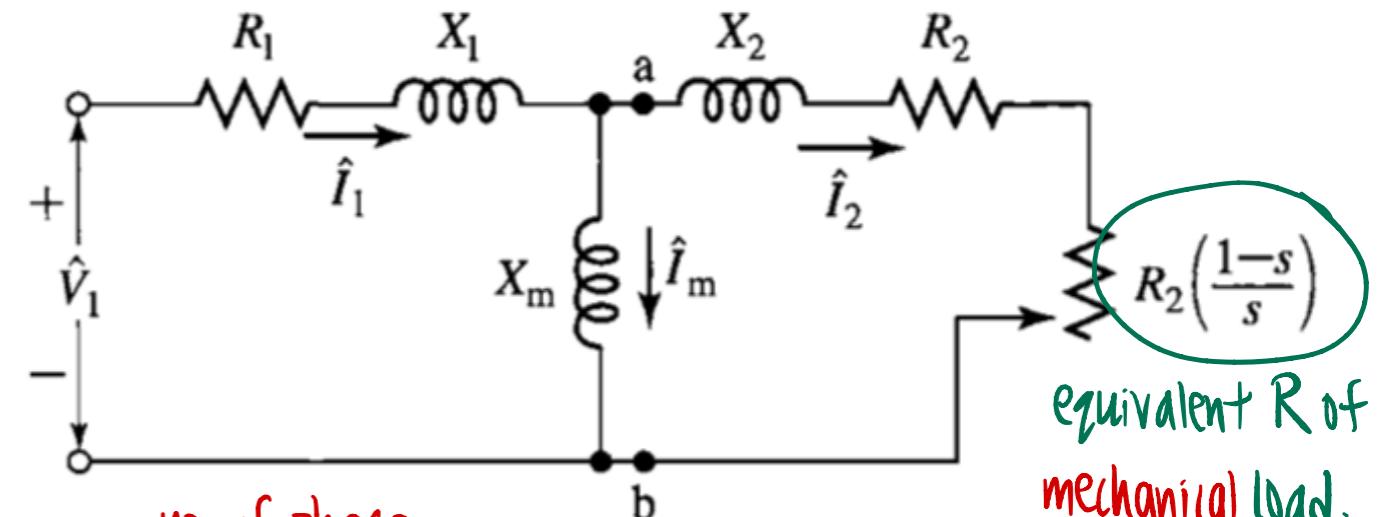
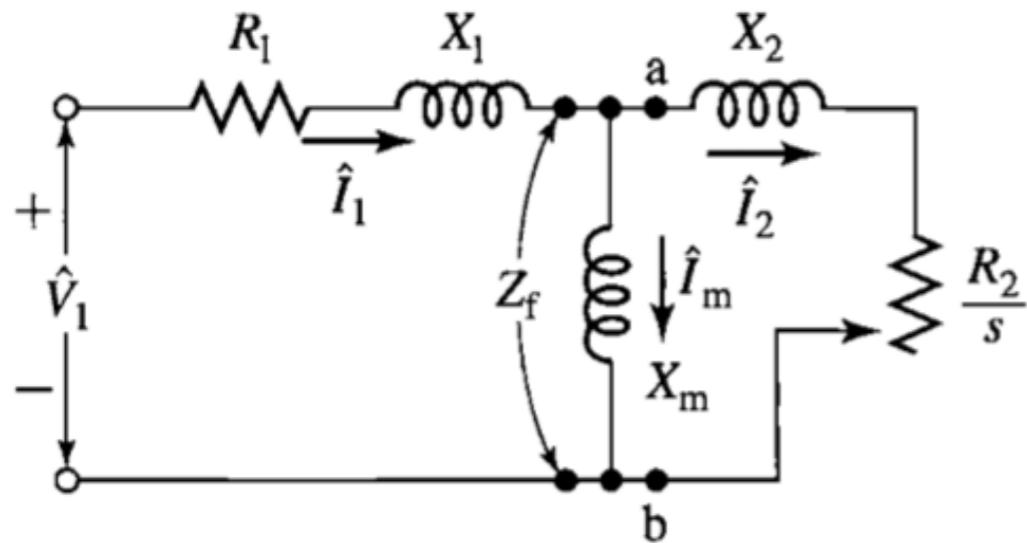
$$\frac{\tilde{E}'_2}{\tilde{I}'_2} = \frac{s\tilde{E}_2}{\tilde{I}_2} = Z'_2 \xrightarrow{\text{referred to stator.}} R_2 + jsX_2$$

- Divide by slip  $s$ :

$$Z_2 = \frac{\tilde{E}_2}{\tilde{I}_2} = \frac{R_2}{s} + jX_2$$

Convert back to  
actual  $Z_2$ .

# Equivalent Circuit Formulation



Total power into rotor:  
(i.e. power transferred from air gap)

$$P_{AG} = n_{ph} \hat{I}_2^2 \left( \frac{R_2}{s} \right) \text{ no. of phases load.}$$

Power loss in rotor:

$$P_{RCL} = n_{ph} \hat{I}_2^2 R_2 \text{ w phase } \varphi.$$

$\uparrow s, \uparrow \text{loss}, \downarrow P_{\text{mech}}$

Mechanical power output:

$$P_{mech} = P_{AG} - P_{RCL} = n_{ph} \hat{I}_2^2 R_2 \left( \frac{1-s}{s} \right)$$

$P_{\text{mech}} = P_{\text{transferred}} - P_{\text{loss}}$

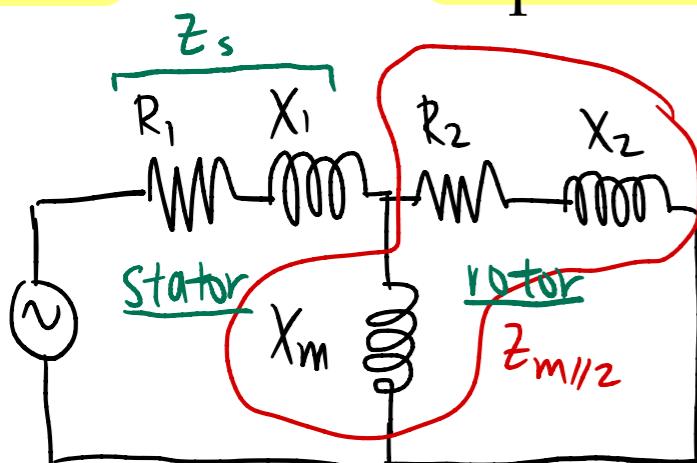
Note.  $n_{ph}$  refers to the number of stator phases

# Example 2

A 460-V, 25-hp, 60 Hz, four-pole, Y-connected induction motor has the following impedances in ohms per phase referred to the stator circuit:

$$R_1 = 0.641 \Omega, R_2 = 0.332 \Omega, X_1 = 1.106 \Omega, X_2 = 0.464 \Omega, X_m = 26.3 \Omega$$

Find the equivalent impedance of the induction motor with respect to the stator side. Assume a slip of 2.2%.



$$N_s = \frac{120f}{P} = \frac{120(60)}{4} = 1800 \text{ rpm}$$

$$N_R = (1-s)N_s = 1760 \text{ rpm}$$

$$Z_2 = \frac{R_2}{s} + jX_2 = \frac{0.332}{0.022} + j0.464 = 15.09 + j0.464 \Omega$$

Since  $X_m \parallel Z_2$ :

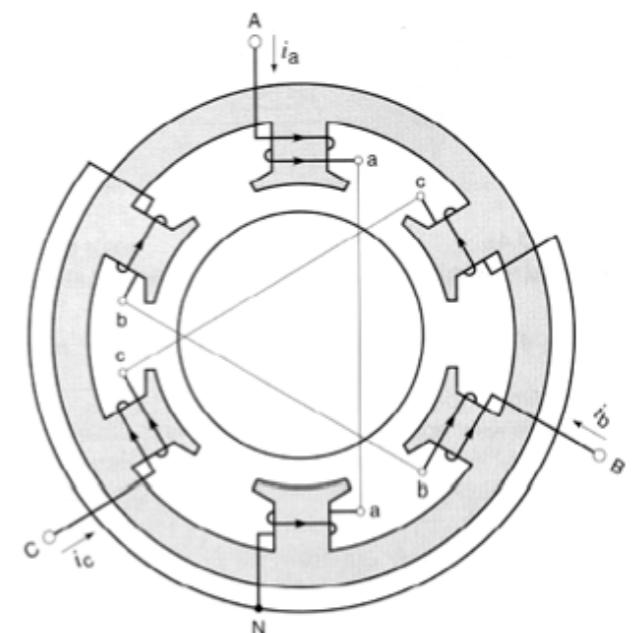
$$Z_{m1/2} = jX_m \parallel Z_2$$

$$Z_{m1/2} = \frac{1}{\frac{1}{jX_m} + \frac{1}{Z_2}} = 11.08 + j6.68 \Omega$$

$$\begin{aligned} Z_{eq} &= Z_s + Z_{m1/2} = R_1 + jX_1 + Z_{m1/2} \\ &= 0.641 + j1.106 + 11.08 + j6.68 \\ &= 11.72 + j7.79 \Omega \end{aligned}$$

# Impact of Air Gap

- Air gap in an induction motor greatly increases the reluctance of flux path.
  - This reduces the coupling between primary and secondary windings.
- Higher reluctance means a higher magnetising reactance,  $X_M$ , in the equivalent circuit than in an ordinary transformer.



# Induction Motor Losses

- Copper losses (conductor losses,  $I^2R$  losses)

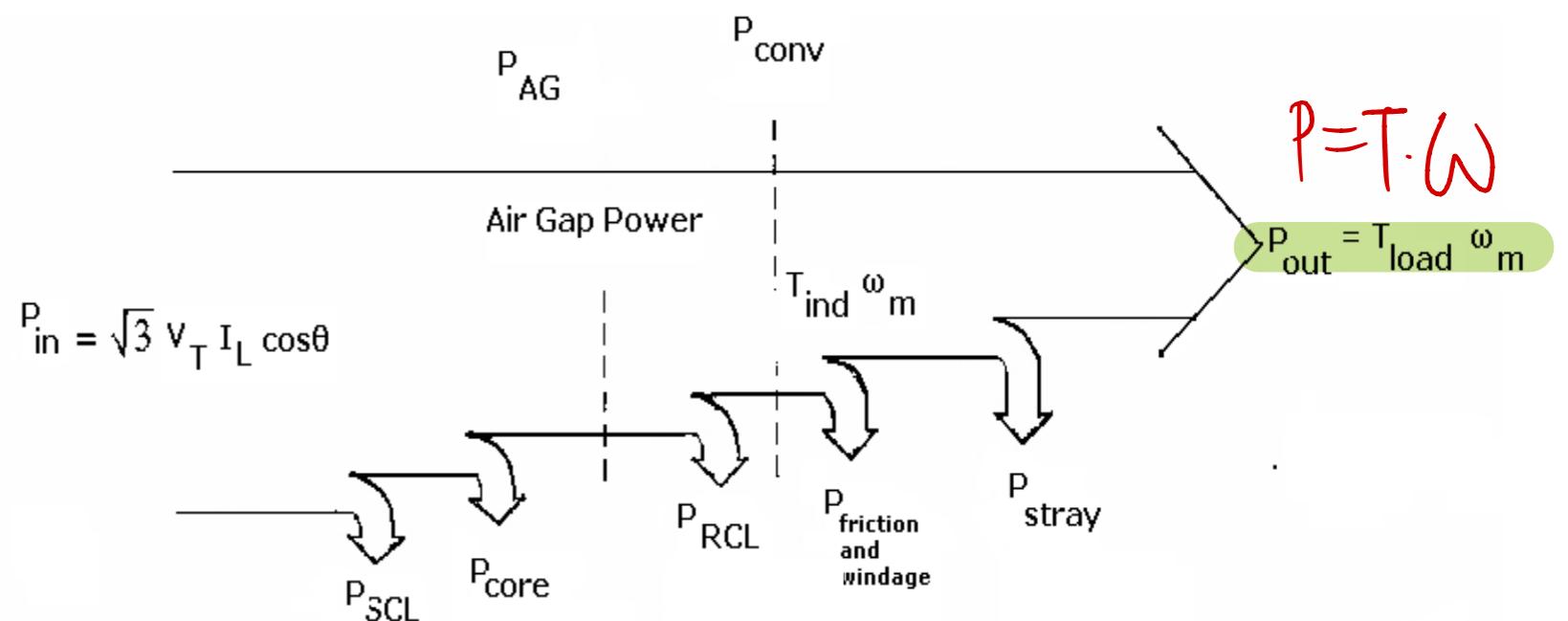
- Stator conductor loss
  - Rotor conductor loss

- Iron losses (core losses)

- Hysteresis
  - Eddy current loss

- Mechanical losses

- Windage loss
  - Friction loss



# Power and Torque in Induction Motors

- The supply power is:

$$P_{in} = \sqrt{3}VI \cos \theta$$

- Power transferred through the air gap by the magnetic coupling is input power ( $P_{in}$ ) minus stator copper loss and magnetising (stator iron) loss.

$$P_{air\ gap} = P_{in} - P_{losses}$$

$$P_{AG} = P_{in} - P_{SCL} - P_{core}$$

$$P_{AG} = 3I_2^2 \frac{R_2}{s}$$

# Power and Torque in Induction Motors

- The stator copper and core losses are determined using the following:

$$P_{SCL} = 3I_1^2 R_1$$

$$P_{core} = \frac{3E_1^2}{R_c}$$

- The electrically developed power ( $P_{dv}$ ) is the difference between air gap power ( $P_{AG}$ ) and rotor copper loss.

$$P_{dv(conv)} = P_{AG} - P_{RCL} = \tau_{ind}\omega_m$$

$$P_{RCL} = 3I_2^2 R_2 \Rightarrow P_{RCL} = sP_{AG}$$

$$P_{dv(conv)} = P_{AG}(1-s) = 3I_2^2 \frac{R_2}{s}(1-s) = \tau_{ind}\omega_m$$

# Induction Motors Output Power

- Subtraction of mechanical losses ( $P_{mloss}$ ) from the developed power gives the mechanical output/shaft power (in hp):  $1 \text{ hp} = 745.7 \text{ W}$

$$P_{out} = P_{dv} - P_{mloss} = \tau_{load} \omega_m$$

- Motor efficiency:

$$\eta = \frac{P_{out}}{P_{in}}$$

- Motor torque:

$$\tau = \frac{P_{out}}{\omega_m}$$

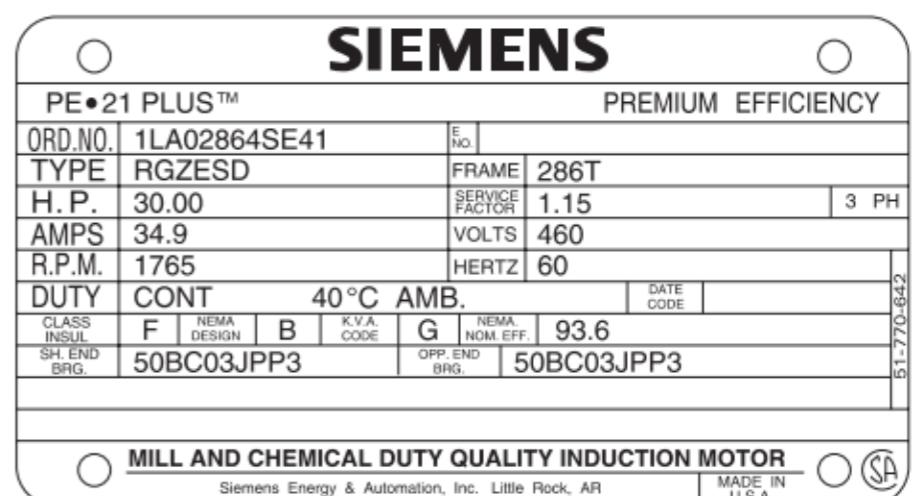
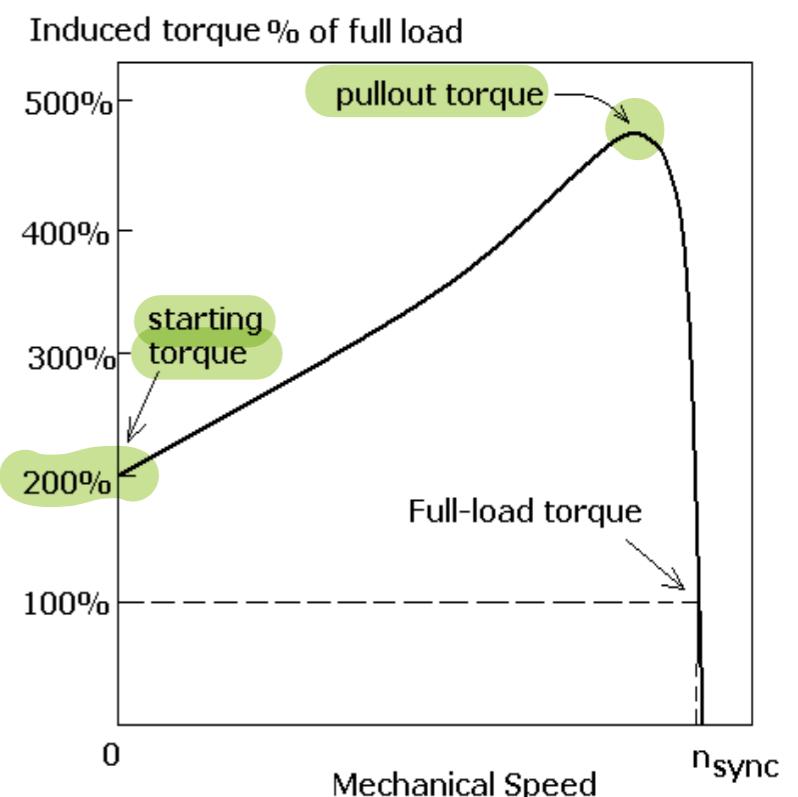
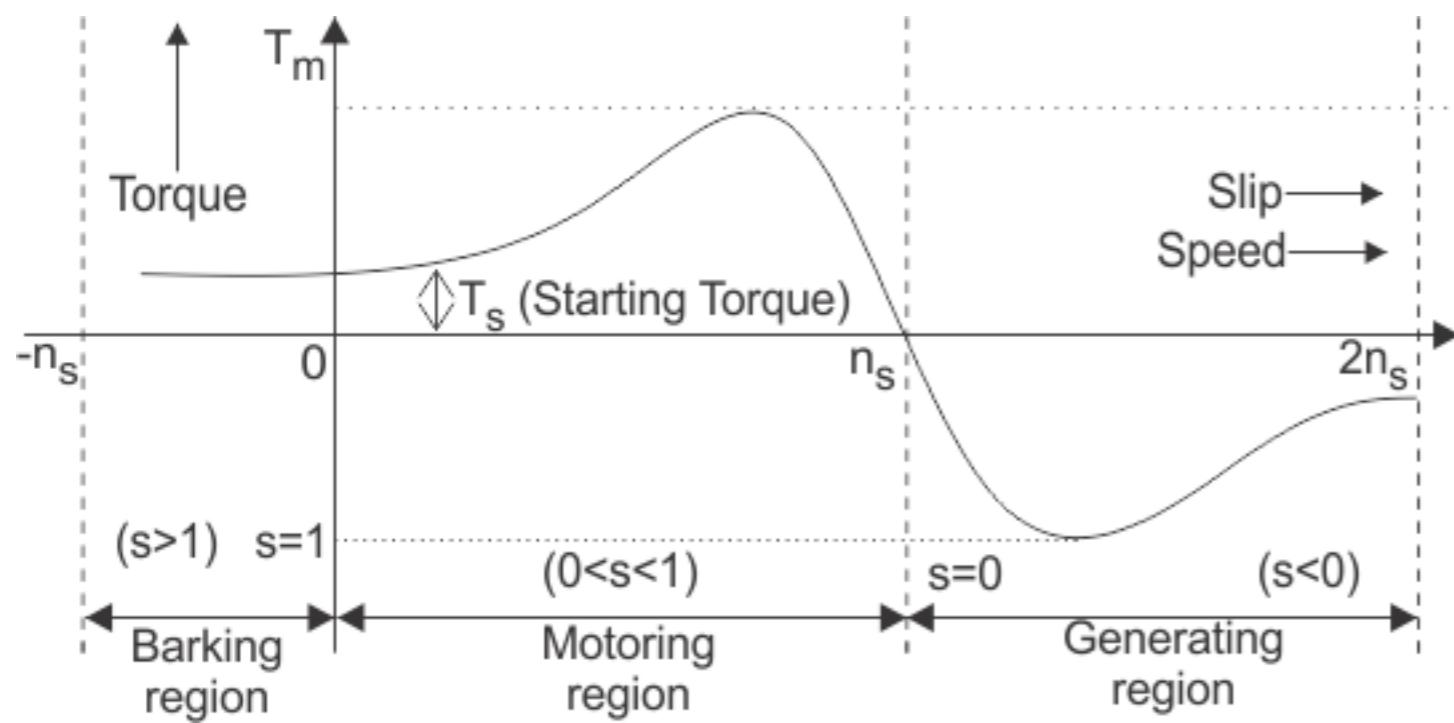
# Torque-Speed Characteristic

# Torque-Speed Characteristic

How does the torque of induction motor change as the load changes?

- At **light loads**: The rotor **slip is very small**, i.e. close to synchronous speed. Never reaches synchronous speed due to **internal impedance**.
- At **heavy loads**: As **load increase, the slip increase**. The rotor speed falls down as a result.  
*requires acceleration.*
- **Starting torque**: About **200-250% of the full load torque** (rated torque).
- **Pullout torque**: Maximum torque the motor can produce at full rated voltage and frequency. (Usually 2 to 3 times of the rated full-load torque of motor.) Motor will **stall beyond this limit**.
- Speed (rpm) of motor on the **nameplate** refers to the **speed at full-load**.  
*rated condition,  
NOT starting condition!*

# Torque-Speed Curve



# Torque-Speed Curve Regions

- **Low slip region:** Slip increases linearly with increased load. Rotor mechanical speed decreases approximately linearly with load.
  - Rotor reactance is negligible, so rotor power factor is approximately unity, while rotor current increases linearly with slip.
  - The entire normal steady-state operating range of an induction motor is included in this linear low-slip region.
- **Moderate slip region:** Rotor current no longer increases as rapidly as before.
  - Power factor starts to drop.
  - Peak torque (pullout torque) of motor occurs at point where, for an incremental increase in load, increase in rotor current is exactly balanced by decrease in rotor power factor.

$\text{pf} \downarrow, P \downarrow, Q \uparrow, n \downarrow$

# Torque-Speed Curve Regions

- **High slip region:** Induced torque decreases with increased load due to decreasing power factor at rotor.
  - Starting torque (at zero speed) is about 150% of full-load torque.
  - Unlike synchronous motor, induction motor can start with a full-load attached to its shaft.

$\Rightarrow$  full/rated  $<$  starting  $<$  pullout torque.

