

National University of Singapore
Department of Electrical & Computer Engineering
EE-4502: Electric Drives and Control
Tutorial - 2 (Sizing of Adjustable Speed Drives - Solution)
Year 2021-2022

1. The following parameters are given:

$$\tau_r = 90 \text{ min}, \tau_s = 120 \text{ min}, t_r = 30 \text{ min}, t_s = 30 \text{ min} \quad \text{and} \quad \theta_{min} = 30^0C$$

The temperature-rise curve during the heating part is given by

$$\theta(t) = \theta_{ss}(1 - \exp(-t/\tau_r)) + \theta_{min}\exp(-t/\tau_r)$$

At $t = 30 \text{ min}$, we have

$$\theta(t = 30 \text{ min}) = \theta_{max} = \theta_{ss}(1 - \exp(-30/90)) + 30\exp(-30/90) = 0.28\theta_{ss} + 21.5 \dots (a)$$

The temperature-rise curve during the cooling part is given by

$$\theta(t') = \theta_{ss'}(1 - \exp(-t'/\tau_s)) + \theta_{max}\exp(-t'/\tau_s)$$

At $t' = 30 \text{ min}$, we have

$$\theta(t' = 30 \text{ min}) = \theta_{min} = 30^0C = \theta_{max}(\exp(-30/120)) \Rightarrow \theta_{ss} = 60.8^0C$$

Substituting in the above equation (a) we get

$$\theta_{max} = 38.5^0C$$

2. The following parameters are given:

$$\tau_r = 80 \text{ min}, \theta_{ss} = 100^0C \quad \text{and} \quad \theta(t = 0) = 0^0C$$

The temperature-rise curve is given by

$$\theta(t) = \theta_{ss}(1 - \exp(-t/\tau_r))$$

At $(t = 2.2 \text{ hrs})$, we have

$$\theta(2.2 \text{ hrs}) = 100(1 - \exp(-132/80)) = 80.8^0C$$

Let θ_{max} be the steady-state temperature-rise when the machine is overloaded by a factor "k". The temperature-rise curve is given by

$$\theta(t) = \theta_{max}(1 - \exp(-t/\tau_r))$$

At $(t = 2.2 \text{ hrs})$, we have

$$\theta(2.2 \text{ hrs}) = \theta_{max}(1 - \exp(-132/80)) = 100^0C \Rightarrow \theta_{max} = 123.8^0C$$

$$k = \sqrt{\frac{1 + \alpha}{1 - e^{-\frac{t_r}{\tau_r}}}} - \alpha = \sqrt{\frac{1 + 0.5}{1 - e^{-\frac{-2.2 \times 60}{132}}}} - 0.5 = 1.165$$

Thus, the machine can be overloaded by 16.5 % more than the rated load.

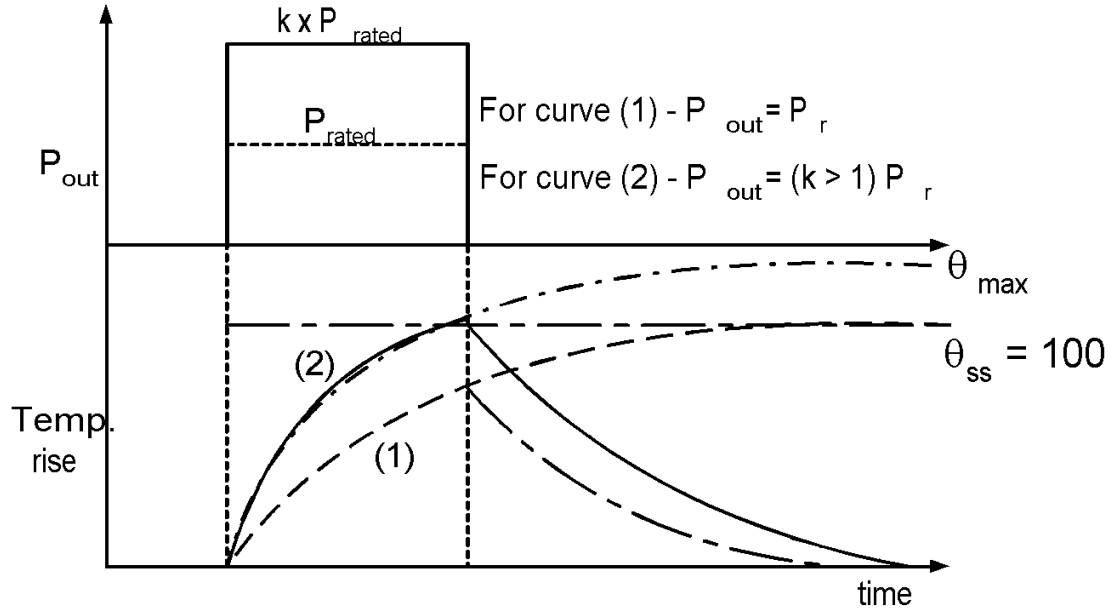


Figure 1:

3. The parameters given are:

$$P_{r'} = 200 \text{ W}, t_r = 10 \text{ min.}, \tau_r = 40 \text{ min.}, \eta_{max} \text{ takes place at 100\% of F.L.}$$

Thus,

$$\alpha = \frac{p_c}{p_{cu}} = \frac{p_{cu}}{p_{cu}} = 1.00$$

The overloading factor, k for short-term load is given by:

$$k = \sqrt{\frac{(\alpha + 1)}{(1 - \exp(-(t_r/\tau_r)))}} - \alpha$$

Substituting all the variables in the above equation we get,

$$k = \sqrt{\frac{(1 + 1)}{(1 - \exp(-10/40))}} - 1 = 2.84$$

$$\text{Thus, } P_r = \frac{P_c}{k} = \frac{200}{2.84} = 70.5 \text{ W.}$$

4. The expression for instantaneous power as a function of time is given by:

$$p(t) = (P_b - P_a) \frac{t}{T} + P_a$$

Thus, the rms value of the output power can be computed as:

$$P_{rms} = \sqrt{\frac{1}{T} \int_0^T \left[(P_b - P_a) \frac{t}{T} + P_a \right]^2 dt} = \sqrt{\frac{1}{3} [P_a^2 + P_a P_b + P_b^2]}$$

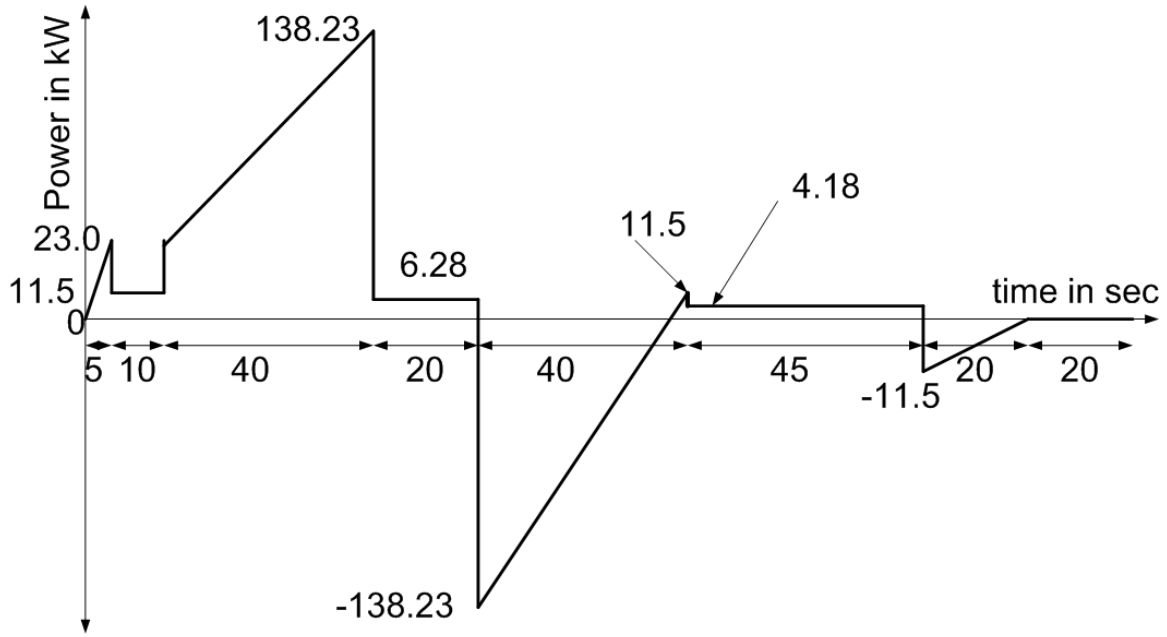


Figure 2:

During $0 < t < 5 \text{ sec}$

$$P_{1,rms} = \sqrt{\frac{1}{3} [0 + 0 \times 23 + 23^2]} = 13.28 \text{ kW}$$

During $5 < t < 15 \text{ sec}$

$$P_{2,rms} = 11.5 \text{ kW}$$

During $15 < t < 55 \text{ sec}$

$$P_{3,rms} = \sqrt{\frac{1}{3} [23^2 + 23 \times 138.23 + 138.23^2]} = 87.2 \text{ kW}$$

During $55 < t < 75 \text{ sec}$

$$P_{4,rms} = 6.28 \text{ kW}$$

During $75 < t < 115 \text{ sec}$

$$P_{5,rms} = \sqrt{\frac{1}{3} [(-138.23)^2 + (-138.23) \times 11.5 + 11.5^2]} = 76.78 \text{ kW}$$

During $115 < t < 160 \text{ sec}$

$$P_{6,rms} = 4.18 \text{ kW}$$

During $160 < t < 180 \text{ sec}$

$$P_{7,rms} = \sqrt{\frac{1}{3} [(-11.5)^2 + (-11.5) \times 0 + 0^2]} = 6.63 \text{ kW}$$

During $180 < t < 200 \text{ sec}$

$$P_{8,rms} = 0 \text{ kW}$$

Thus, we have

$$P_{rms} = \sqrt{\frac{13.28^2 \times 5 + 11.5^2 \times 10 + 87.2^2 \times 40 + 6.28^2 \times 20 + 76.78^2 \times 40 + 4.18^2 \times 40 + 6.63^2 \times 20}{200}}$$

$$= 52.18 \simeq 53 \text{ kW}$$

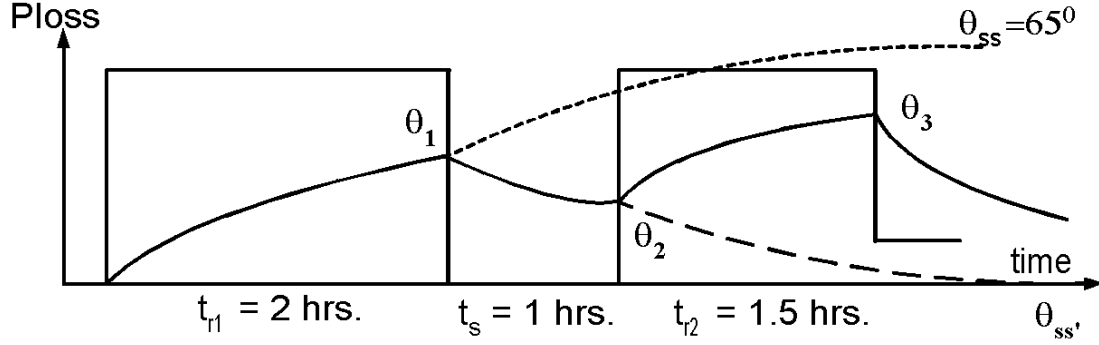


Figure 3:

5. Fig. 3 shows the losses as well as temperature-rise curves.

During the interval $0 < t < t_{r1} = 2$ hours the temperature-rise can be expressed as:

$$\theta(t) = \theta_{ss}(1 - \exp(-t_r/\tau_r)) + \theta_{min}(\exp(-t_r/\tau_r))$$

Substituting $\theta_{ss} = 65^\circ\text{C}$, $\tau_r = 2.2 \text{ hrs.}$, $t_r = 2 \text{ hrs.}$ and, $\theta_{min} = 0^\circ\text{C}$ we have

$$\theta(t_r = 2 \text{ hrs.}) = 65(1 - \exp(-2/2.2)) + 0(\exp(-2/2.2)) = 38.81^\circ\text{C} = \theta_1$$

Similarly, during the cooling period the temperature-rise is given by

$$\theta(t) = \theta_{ss'}(1 - \exp(-t_s/\tau_s)) + \theta_{max}(\exp(-t_s/\tau_s)) \quad (1)$$

During the interval $2 \text{ hrs.} < t < 3 \text{ hrs.}$, we have $\theta_{ss'} = 0^\circ\text{C}$, $\theta_{max} = 38.81^\circ\text{C}$

$$\theta(t_s = 1 \text{ hrs.}) = 0(1 - \exp(-1/3.5)) + 38.81(\exp(-1/3.5)) = 29.16^\circ\text{C} = \theta_2$$

During the interval $3 \text{ hrs.} < t < 4.5 \text{ hrs.}$, we have $\theta_{ss} = 65^\circ\text{C}$, $\theta_{min} = 29.16^\circ\text{C}$

$$\theta(t_{r2} = 1.5 \text{ hrs.}) = 65(1 - \exp(-1.5/2.2)) + 29.16(\exp(-1.5/2.2)) = 46.9^\circ\text{C} = \theta_3$$

Thus, the temperature-rise at the end of the cycle is 46.9°C .

6. The parameters given are:

$$P_r = 100 \text{ kW}, \tau_r = 50 \text{ min.}, \tau_s = 70 \text{ min.}, \eta_{max} \text{ takes place at 80\% of full-load (F.L.),}$$

At maximum efficiency (80% of full-load) let the copper-loss be p_{cu} and constant loss be p_c i.e. $p_{cu} = p_c$.

At F.L. the cu-loss is: $p_{cu} \times (1.0/0.8)^2 = \frac{p_{cu}}{0.64}$

Thus,

$$\alpha = \frac{p_c}{p_{cu}} = \frac{p_{cu}}{(p_{cu}/0.64)} = 0.64$$

The overloading factor, k for periodic intermittent-load is given by:

$$k = \sqrt{(\alpha + 1) \frac{(1 - \exp(-(t_r/\tau_r + t_s/\tau_s)))}{(1 - \exp(-(t_r/\tau_r)))} - \alpha}$$

Substituting all the variables in the above equation we get,

$$k = \sqrt{(0.64 + 1) \frac{(1 - \exp(-(10/50 + 10/70)))}{(1 - \exp(-(10/50)))} - 0.64} = 1.4093$$

Thus, $P_{r'} = k \times P_r = 1.4093 \times 100 \text{ kW} = 140.93 \text{ kW}$.

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