National University of Singapore

Department of Electrical & Computer Engineering

EE-4502:Electric Drives and Control

Tutorial - 2 (Sizing of Adjustable Speed Drives - Solution)

Year 2022-2023

1. The following parameters are given:

$$\tau_r = 90\,min, \tau_s = 120\,min, t_r = 30\,min, t_s = 30\,min \quad \text{and} \quad \theta_{min} = 30^0C$$

The temperature-rise curve during the heating part is given by

$$\theta(t) = \theta_{ss}(1 - exp(-t/\tau_r)) + \theta_{min}exp(-t/\tau_r)$$

At $t = 30\,min$, we have

$$\theta(t = 30\,min) = \theta_{max} = \theta_{ss}(1 - exp(-30/90)) + 30exp(-30/90) = 0.28\theta_{ss} + 21.5.......(a)$$

The temperature-rise curve during the cooling part is given by

$$\theta(t') = \theta_{ss'}(1 - exp(-t'/\tau_s)) + \theta_{max}exp(-t'/\tau_s)$$

At $t' = 30\,min$, we have

$$\theta(t' = 30\,min) = \theta_{min} = 30^0C = \theta_{max}(= 0.28\theta_{ss} + 21.5)exp(-30/120) \Rightarrow \theta_{ss} = 60.8^0C$$

Substituting in the above equation (a) we get

$$\theta_{max} = 38.5^0C$$

2. The following parameters are given:

$$\tau_r = 80\,min, \theta_{ss} = 100^0C \quad \text{and} \quad \theta(t = 0) = 0^0C$$

The temperature-rise curve is given by

$$\theta(t) = \theta_{ss}(1 - exp(-t/\tau_r))$$

At $(t = 2.2hrs)$, we have

$$\theta(2.2hrs) = 100(1 - exp(-132/80)) = 80.8^0C$$

Let θ_{max} be the steady-state temperature-rise when the machine is overloaded by a factor "k". The temperature-rise curve is given by

$$\theta(t) = \theta_{max}(1 - exp(-t/\tau_r))$$

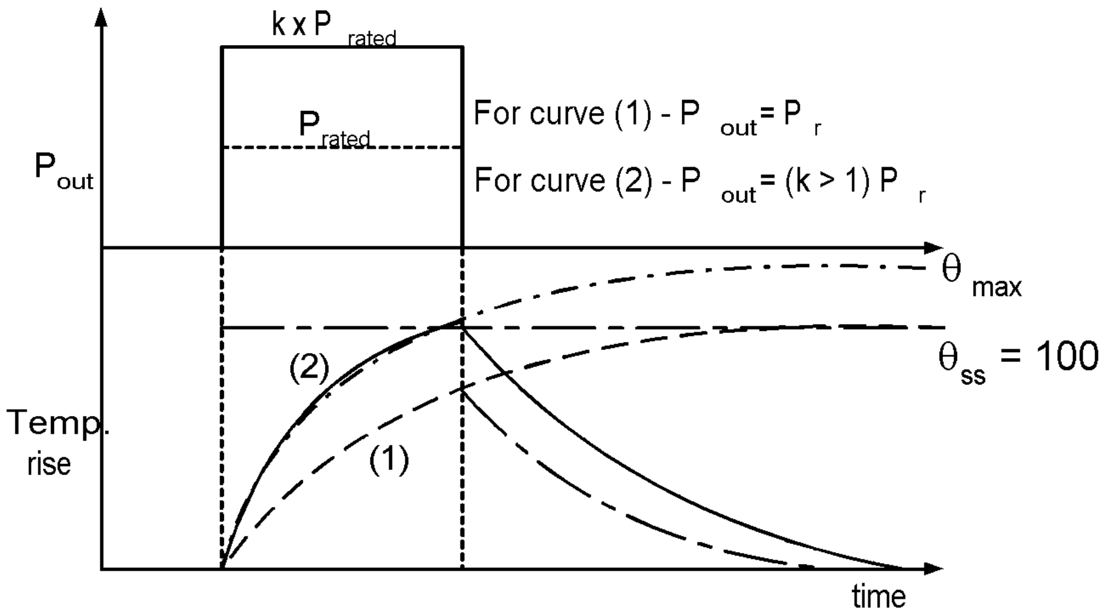

Figure 1:

At $(t = 2.2hrs)$, we have

$$\theta(2.2\,hrs) = \theta_{max}(1 - exp(-132/80)) = 100^0C \Rightarrow \theta_{max} = 123.8^0C$$

Let p_{1s} be the total power loss when the machine is overloaded by a factor of "k" and p_{1r} be the total power loss at rated load.

$$p_{1s} = p_c + k^2 p_{cu} \quad \text{and} \quad p_{1r} = p_c + p_{cu}$$
$$\frac{p_{1s}}{p_{1r}} = \frac{p_c + k^2 p_{cu}}{p_c + p_{cu}} = \frac{(\theta_m = 123.8^0C)}{(\theta_{ss} = 100^0C)}$$

Let $\frac{p_c}{p_{cu}} = \alpha$ and is given to be equal to 0.5.
We have

$$\frac{\alpha + k^2}{\alpha + 1} = \frac{(\theta_m = 123.8^0C)}{(\theta_{ss} = 100^0C)} \Rightarrow k^2 - 0.238 \times 0.5 - 1.238 = 0 \Rightarrow k = 1.165$$

Thus, the machine can be overloaded by 16.5% more than the rated load.

Alternatively, using the overloading factor eqn. directly, we can have get,

$$k = \sqrt{\frac{(1)}{(1 - exp(-132/80)))}} = 1.1$$

3. The parameters given are:

$$P_{r'} = 200\,W, t_r = 10\,min., \tau_r = 40\,min., \eta_{max}\text{takes place at 100\% of F.L.}$$

Thus,

$$\alpha = \frac{p_c}{p_{cu}} = \frac{p_{cu}}{p_{cu}} = 1.00$$

The overloading factor, k for short-term load is given by:

$$k = \sqrt{\frac{(\alpha + 1)}{(1 - exp(-(t_r/\tau_r)))} - \alpha}$$

Substituting all the variables in the above equation we get,

$$k = \sqrt{\frac{(1 + 1)}{(1 - exp(-10/40)))} - 1} = 2.84$$

Thus, $P_r = \frac{P_r}{k} = \frac{200}{2.84} = 70.5\,W$.

4. The expression for instantaneous power as a function of time is given by:

$$p(t) = (P_b - P_a)\frac{t}{T} + P_a$$

Thus, the rms value of the output power can be computed as:

$$P_{rms} = \sqrt{\frac{1}{T}\int_0^T \left[(P_b - P_a)\frac{t}{T} + P_a\right]^2 dt} = \sqrt{\frac{1}{3}\left[{P_a}^2 + P_aP_b + {P_b}^2\right]}$$

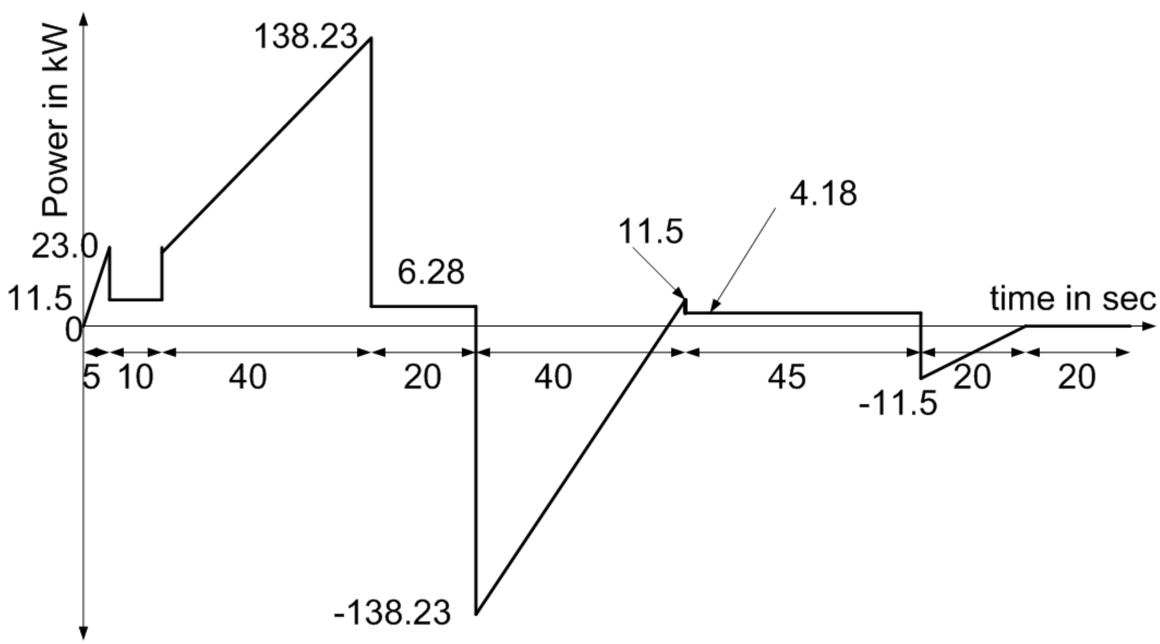

Figure 2:

During $0 < t < 5\;sec$

$$P_{1,rms} = \sqrt{\frac{1}{3}\left[0 + 0 \times 23 + 23^2\right]} = 13.28kW$$

During $< t < 15\;sec$

$$P_{2,rms} = 11.5kW$$

During $15 < t < 55\,sec$

$$P_{3,rms} = \sqrt{\frac{1}{3}\left[23^2 + 23 \times 138.23 + 138.23^2\right]} = 87.2kW$$

During $55 < t < 75\,sec$

$$P_{4,rms} = 6.28kW$$

During $75 < t < 115\,sec$

$$P_{5,rms} = \sqrt{\frac{1}{3}\left[(-138.23)^2 + (-138.23) \times 11.5 + 11.5^2\right]} = 76.78\,kW$$

During $115 < t < 160\,sec$

$$P_{6,rms} = 4.18kW$$

During $160 < t < 180\,sec$

$$P_{7,rms} = \sqrt{\frac{1}{3}[(-11.5)^2 + (-11.5) \times 0 + 0^2]} = 6.63kW$$

During $180 < t < 200\,sec$

$$P_{8,rms} = 0kW$$

Thus, we have

$$P_{rms} = \sqrt{\frac{13.28^2 \times 5 + 11.5^2 \times 10 + 87.2^2 \times 40 + 6.28^2 \times 20 + 76.78^2 \times 40 + 4.18^2 \times 40 + 6.63^2 \times 20}{200}}$$

$$= 52.18 \;\simeq 53\,kW$$

5. The expression for the temp-rise is given by

$$\theta(t) = \theta_{ss}(1 - exp(-t_r/\tau_r)) + \theta_{min}(exp(-t_r/\tau_r))$$

In this specific case we have, $\theta_{min} = 0$ and substituting for $t_{1r} = 25\,min$ and $t_{2r} = 50\,min$ respectively, we have

$$\begin{aligned}\theta(t_{1r} = 25\,min.) &= 25^0C = \theta_{ss}(1 - exp(-25/\tau_r))\\ \theta(t_{2r} = 50\,min.) &= 50^0C = \theta_{ss}(1 - exp(-50/\tau_r))\end{aligned}$$

Solving eqns. (5) and (6), we get $\theta_{ss} = 62.5^0C$ and $\tau_r = 48.94\,min.$

6. Fig. 3 shows the losses as well as temperature-rise curves.

During the interval $0 < t < t_{r1} = 2$ hours the temperature-rise can be expressed as:

$$\theta(t) = \theta_{ss}(1 - exp(-t_r/\tau_r)) + \theta_{min}(exp(-t_r/\tau_r))$$

Substituting $\theta_{ss} = 65^0C$, $\tau_r = 2.2\,hrs.$, $t_r = 2\,hrs.$ and, $\theta_{min} = 0^0C$ we have

$$\theta(t_r = 2hrs.) = 65(1 - exp(-2/2.2)) + 0(exp(-2/2.2)) = 38.81^0C = \theta_1$$

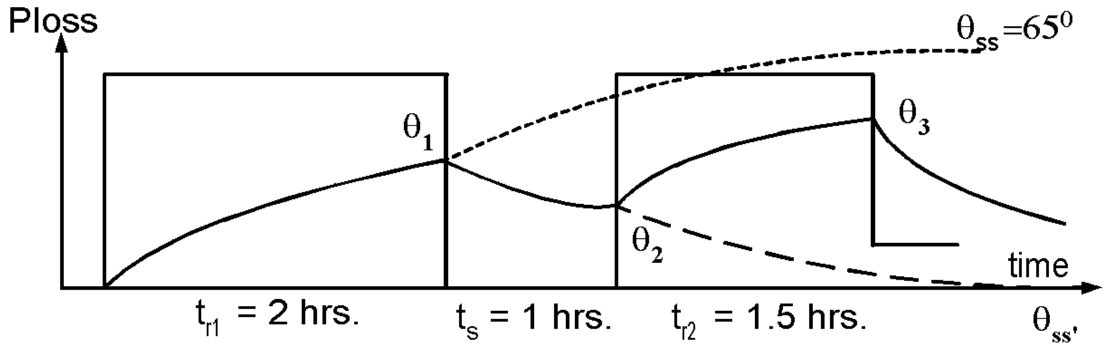

Figure 3:

Similarly, during the cooling period the temperature-rise is given by

$$\theta(t) = \theta_{ss'}(1 - exp(-t_s/\tau_s)) + \theta_{max}(exp(-t_s/\tau_s)) \tag{1}$$

During the interval 2 $hrs. < t < 3\ hrs.$, we have $\theta_{ss'} = 0^0\ C$, $\theta_{max} = 38.81^0\ C$

$$\theta(t_s = 1hrs.) = 0(1 - exp(-1/3.5)) + 38.81(exp(-1/3.5)) = 29.16^0C = \theta_2$$

During the interval 3 $hrs. < t < 4.5\ hrs.$, we have $\theta_{ss} = 65^0\ C$, $\theta_{min} = 29.16^0\ C$

$$\theta(t_{r2} = 1.5hrs.) = 65(1 - exp(-1.5/2.2)) + 29.16(exp(-1.5/2.2)) = 46.9^0C = \theta_3$$

Thus, the temperature-rise at the end of the cycle is $46.9^0\ C$.

7. The parameters given are:

$$P_r = 100\ kW, \tau_r = 50\ min., \tau_s = 70\ min., \eta_{max}\text{takes place at 80\% of full-load (F.L.),}$$

At maximum efficiency (80% of full-load) let the copper-loss be p_{cu} and constant loss be p_c i.e. $p_{cu} = p_c$.

$$\text{At F.L. the cu-loss is:} p_{cu} \times (1.0/0.8)^2 = \frac{p_{cu}}{0.64}$$

Thus,

$$\alpha = \frac{p_c}{p_{cu}} = \frac{p_{cu}}{(p_{cu}/0.64)} = 0.64$$

The overloading factor, k for periodic intermittent-load is given by:

$$k = \sqrt{(\alpha + 1)\frac{(1 - exp(-(t_r/\tau_r + t_s/\tau_s)))}{(1 - exp(-(t_r/\tau_r)))} - \alpha}$$

Substituting all the variables in the above equation we get,

$$k = \sqrt{(0.64 + 1)\frac{(1 - exp(-(10/50 + 10/70)))}{(1 - exp(-(10/50)))} - 0.64} = 1.4093$$

Thus, $P_{r'} = k \times P_r = 1.4093 \times 100\ kW = 140.93\ kW$.

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