AC Machine Fundamentals

- AC machines classification –

  a) **Synchronous machines** – run at synchronous speed – field current is supplied from a separate DC source.

  b) **Asynchronous (induction) machines** – run at speeds other than synchronous speed – field current is induced by transformer action from stator supply.

- DC machines – armature winding is mostly placed on *rotor* – field windings on *stator*.

- AC machines – armature windings are mostly placed on *stator* – field windings on *rotor*.

- In AC machines, if a rotating magnetic field is produced by the rotor field winding then a set of three phase AC voltages are induced on the stator armature windings and the machine is said to operate as a *generator*.

- Synchronous machines are usually used as AC generators.

- Alternatively, a set of three-phase currents in the stator three-phase armature windings produce a rotating magnetic field which then interacts with the rotor magnetic field, producing torque in the machine and the machine is said to operate as a *motor*.

- Rotor magnetic field can be produced by: (1) a dc current in the field winding as in synchronous machines or (2) a permanent magnet on the rotor or (3) inducing three-phase currents in the rotor by transformer action as in the case of induction motors.

- Both **synchronous** as well as **asynchronous** machines can operate as AC motors. However, it is the asynchronous machine (induction) which is widely used as AC motor.

- The synchronous machines with field winding are widely used as AC Generators and permanent magnet synchronous machines are used as AC motors.

- Induction motors (squirrel-cage type) is considered as the work-horse of the modern industry.

A Simple Loop in a Uniform Magnetic Field

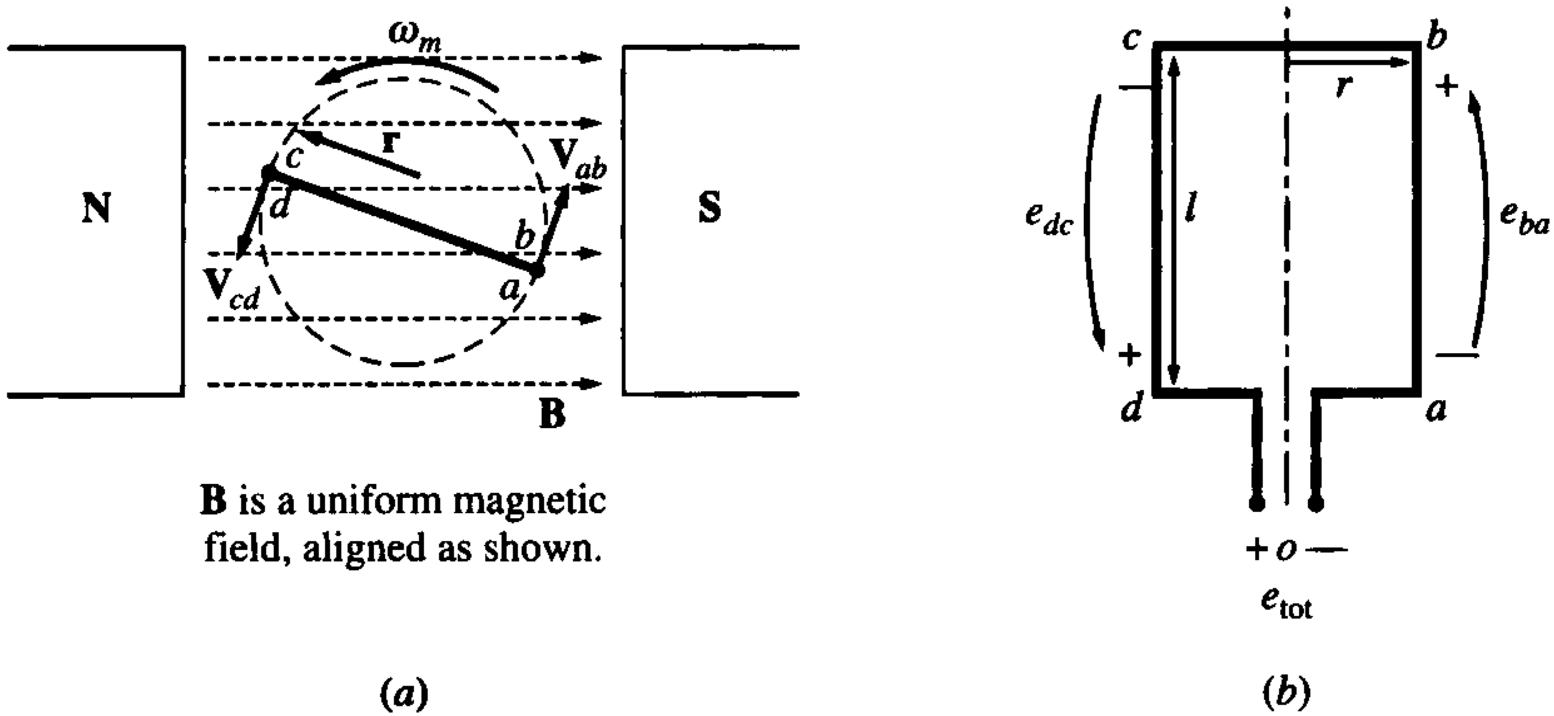


Figure 4.1: A simple rotating loop in a uniform magnetic field (a) front view and (b) top view of the coil.

https://www.youtube.com/watch?v=nt6LFbCRtzY

- The voltage induced in each segment of the loop is given by $e_{ind} = (v \times B) \cdot l$ (5.1)

1. *Segment ab:*

$$e_{ba} = (v \times B) \bullet l = [v B \sin(\theta_{ab})] l \cos(0) \quad \textit{into the page}$$

2. *Segment bc:*

$$e_{cb} = (v \times B) \bullet l = 0 \quad (\because \; v \times \mathbf{B} \textit{ is either into or out of the page whereas } l \textit{ is on the page})$$

3. *Segment cd:*

$$e_{dc} = (v \times B) \bullet l = v B \sin(\theta_{cd}) l \quad \textit{out of the page}$$

4. *Segment da:*

$$e_{ad} = (v \times B) \bullet l = 0 \quad (\because \; v \times \mathbf{B} \textit{ is either into or out of the page whereas } l \textit{ is on the page})$$

▪

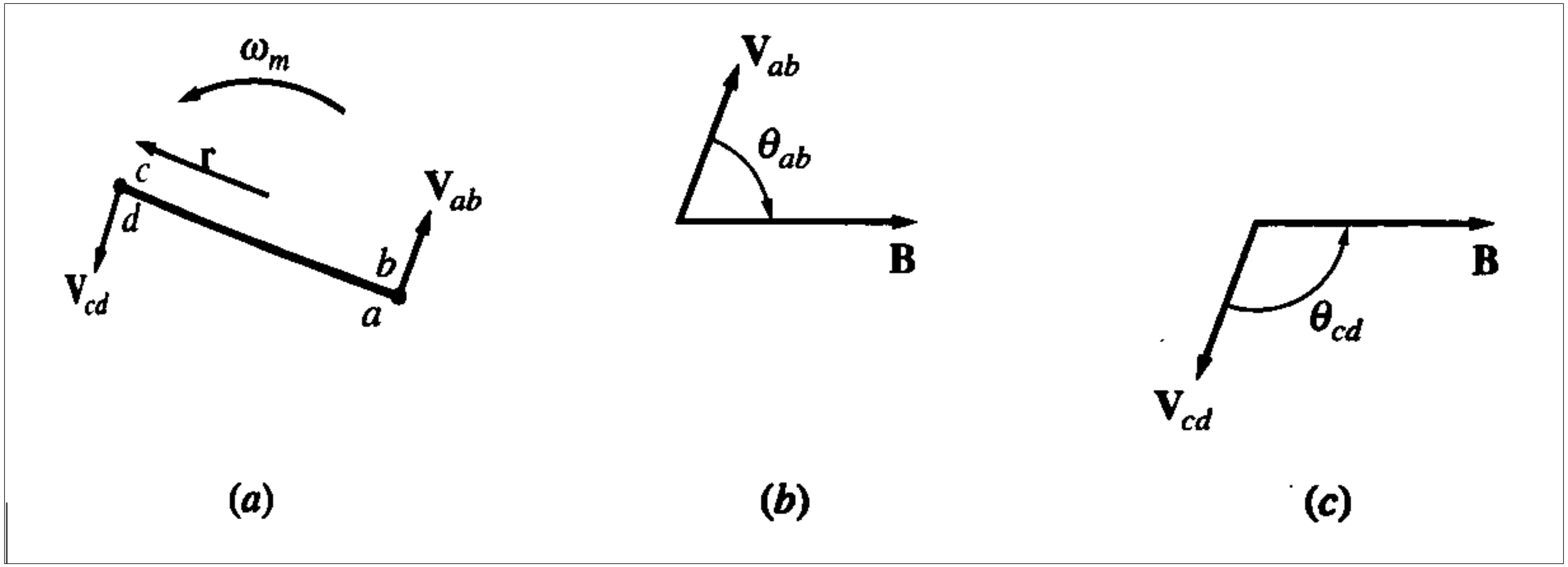


Figure 4.2: (a) Velocities and orientations of the sides of the loop w.r.t the magnetic field. The direction of motion w.r.t Magnetic field for side (b) *ab* and (c) *cd*.

- Total induced voltage on the loop is the sum of voltages induced on each segment

$$e_{ind} = e_{ba} + e_{cb} + e_{dc} + e_{ad}$$
$$= vBl\sin\theta_{ab} + vBl\sin\theta_{cd} = 2vBl\sin\theta \qquad (5.2)$$

Where $\theta_{ab} = 180° - \theta_{cd} = \theta$ *and from trigonometry we know that* $\sin\theta = \sin(180° - \theta)$.

- If we substitute $\theta = \omega t$ and $v = r\omega$ in eqn. 5.2, we get

$$e_{ind} = 2vBl\sin\theta = 2(r\omega)Bl\sin(\omega t) \quad [A = 2rl = \text{area of the loop}]$$
$$= AB\omega\sin(\omega t)$$
$$= \phi_{max}\,\omega\sin(\omega t) \quad [\phi_{max} = AB] \quad (5.3)$$

$$e_{ind} \propto \omega,\ e_{ind} \propto \phi$$

▪

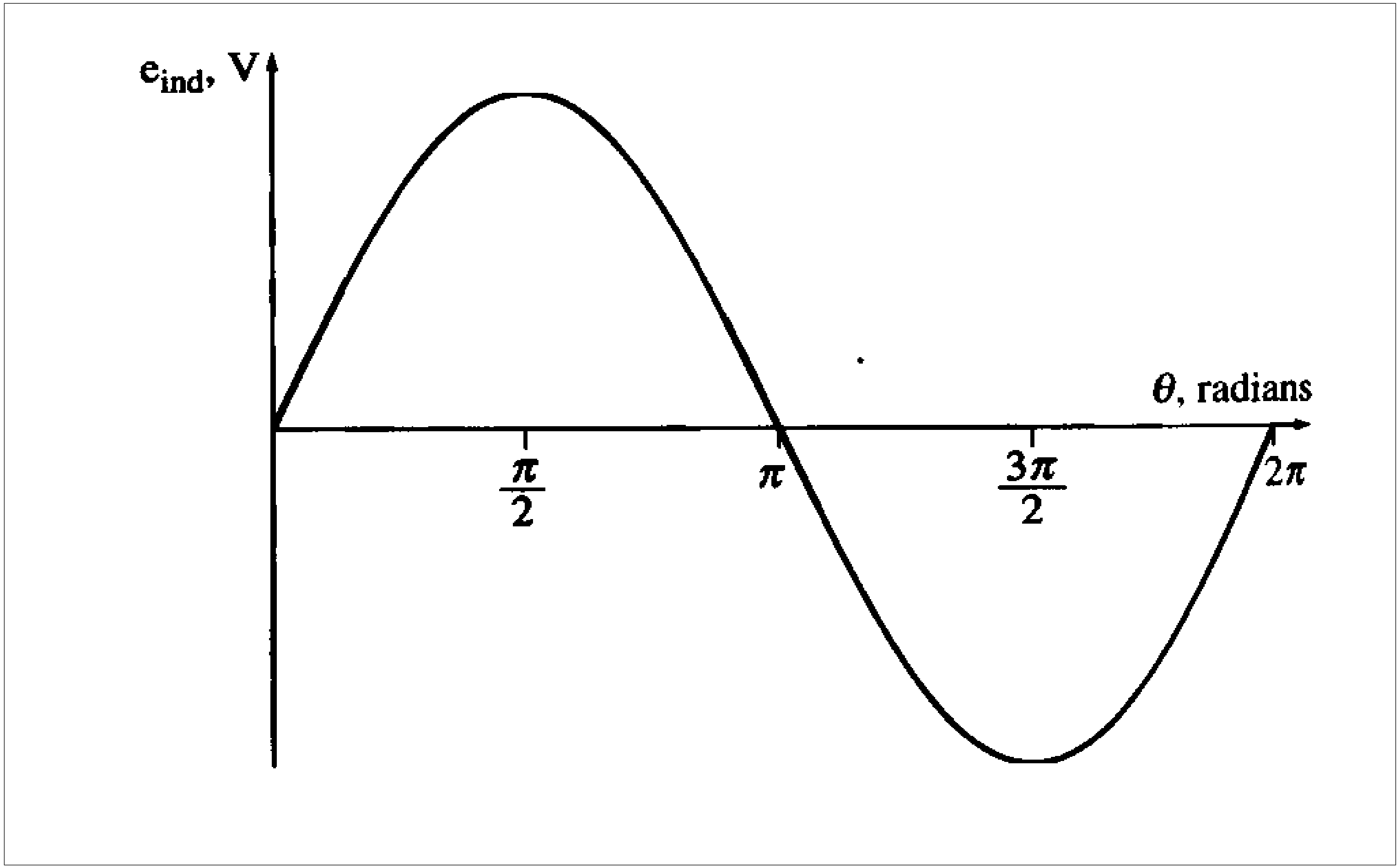


Figure 5.3: Plot of e_{ind} as a function of θ

- Thus, *the voltage generated in the loop is a sinusoid whose magnitude is equal to the product of the flux, ϕ in the machine and the speed of rotation, ω of the machine.*
- This is true for any real machines, in general, the voltage induced will depend on three factors:

1. The flux, ϕ in the machine;
2. the speed of rotation, ω and
3. a constant, *AB* representing the construction of the machine (number of loops etc.).

- *Similar rational to that of a DC machine.*

Torque Induced in a Current-carrying Loop

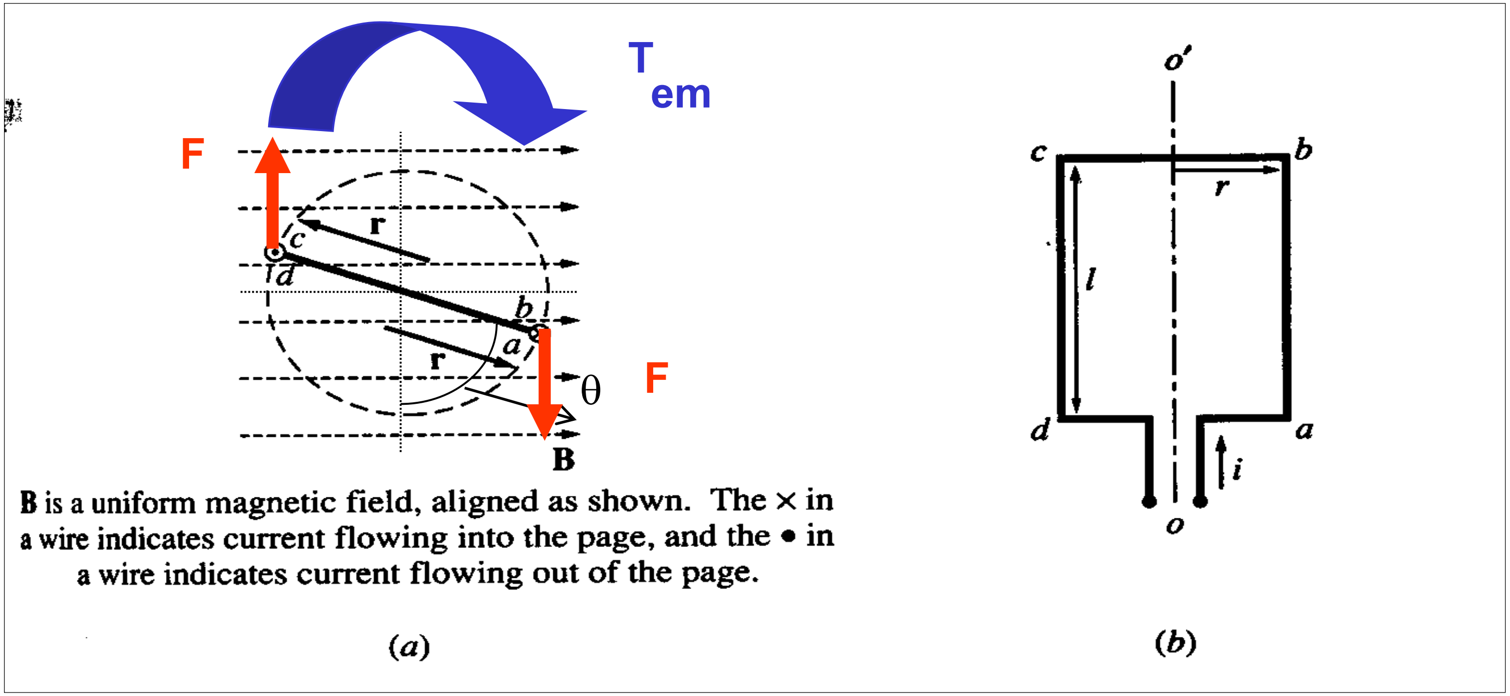


Figure 5.4: A current carrying loop in a uniform magnetic field (a) front view and (b) top view of the coil.

- Assume that the rotor loop is at some arbitrary angle θ, w.r.t the magnetic field, ***B*** and current, *i* is flowing in the loop.

- The force on each segment and torque is given by

$$F = i(l \times B) \qquad and \qquad T = r \times F \times sin(\theta)$$

- The torque induced on the loop can be determined by calculating by torque on each segment of the loop:

1. *Segment ab*: $T_{ab} = (F_{ab}) \times (r) \times (\sin\theta_{ab}) = rBil \sin\theta_{ab} (clockwise)$
2. *Segment bc*: $T_{bc} = (F_{bc}) \times (r) \times (\sin\theta_{bc}) = 0 \quad [\because \theta_{bc} = 0]$
3. *Segment cd*: $T_{cd} = (F_{cd}) \times (r) \times (\sin\theta_{cd}) = rBil \sin\theta_{cd} (clockwise)$
4. *Segment da*: $T_{da} = (F_{da}) \times (r) \times (\sin\theta_{da}) = 0 \quad [\because \theta_{da} = 0]$

- Thus, the total induced torque on the loop is given by

$$\begin{aligned} T_{ind} &= T_{ab} + T_{bc} + T_{cd} + T_{da} \\ &= rBil \sin\theta_{ab} + rBil \sin\theta_{cd} \\ &= 2rBil \sin\theta \qquad [\theta_{ab} = \theta_{cd} = \theta] \quad (5.5) \end{aligned}$$

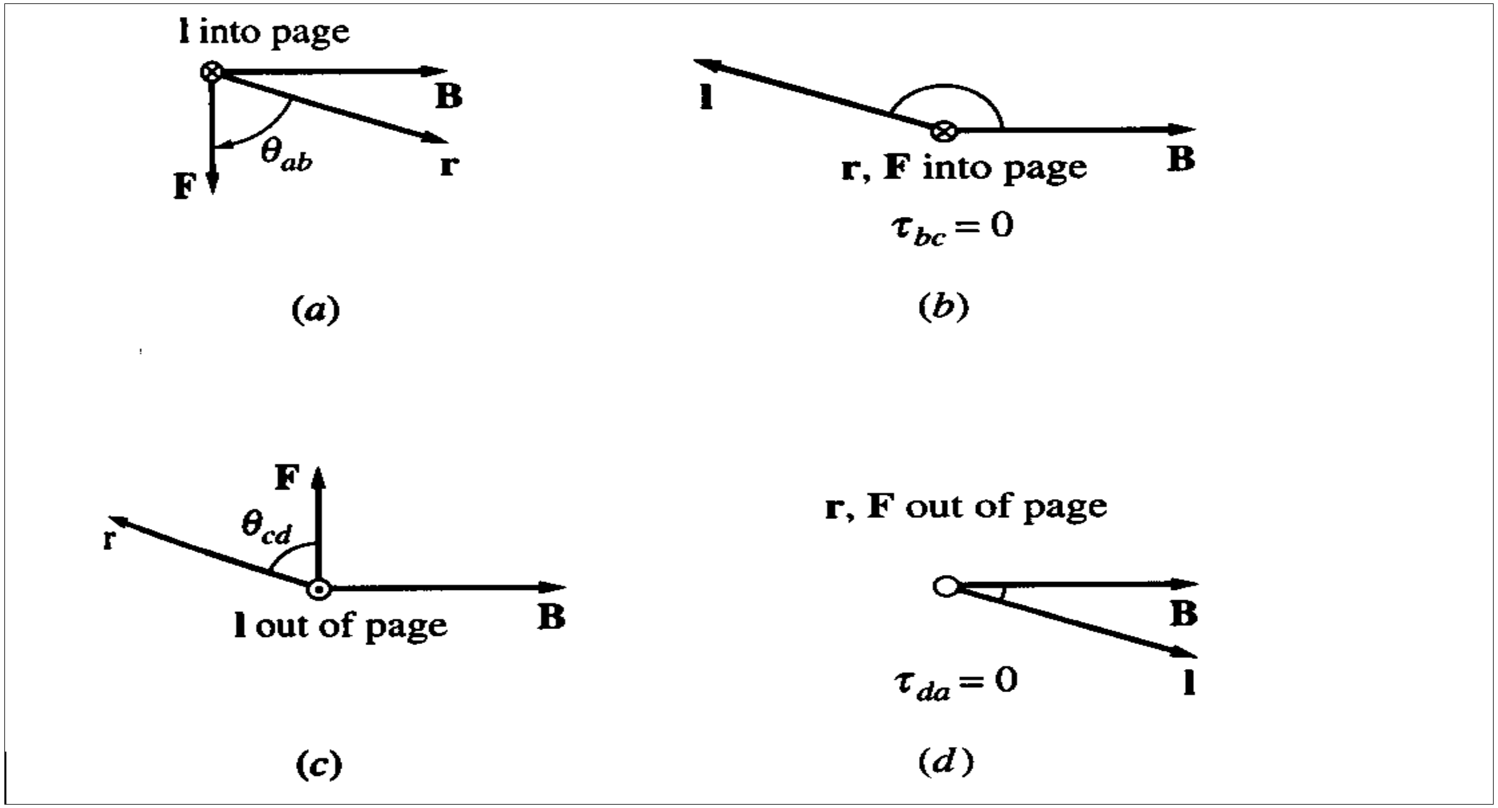


Figure 5.5: (a) Derivation of force and torque on segment *ab*. (b) Derivation of force and torque on segment *bc.* (c) Derivation of force and torque on segment *cd*. (d) Derivation of force and torque on segment *da*.

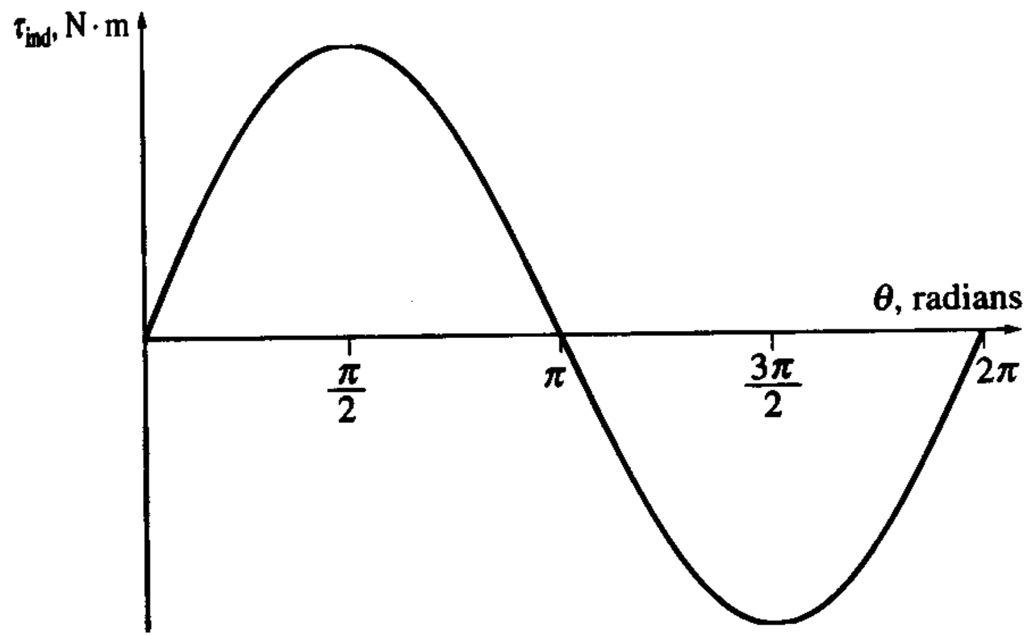


Figure 5.6: Plot of T_{ind} versus θ

- The variations of T_{ind} as a function of θ is shown in Fig. 5.6.
- Note that the induced torque is maximum when the plane of the loop is parallel to the magnetic field (i.e. $\theta = 90^0$) and minimum when it is perpendicular to the magnetic field (i.e. $\theta = 0^0$).

- Alternative way to represent the induced torque in terms of magnetic flux density vectors is as follows: *current in the loop produces* $\mathbf{B}_{loop}$ *and* $\mathbf{B}_s$ *is the stator magnetic flux density vector*.

- From eqn. 5.5 we have

$$T_{ind} = 2rBil \sin\theta$$

$$= \left(\frac{AG}{\mu}\right) B_{loop} B_s \sin\theta \quad \left[A = 2rl,\ B = B_s,\ B_{loop} = \mu H = \frac{\mu i}{G}\right]$$

$$= k\left(B_{loop} B_s \sin\theta\right) = k\left(B_{loop} \times B_s\right) \quad \left[k = \frac{AG}{\mu}\right] \quad (5.6)$$

- Note G is a constant depending on the construction and number of turns of the winding.

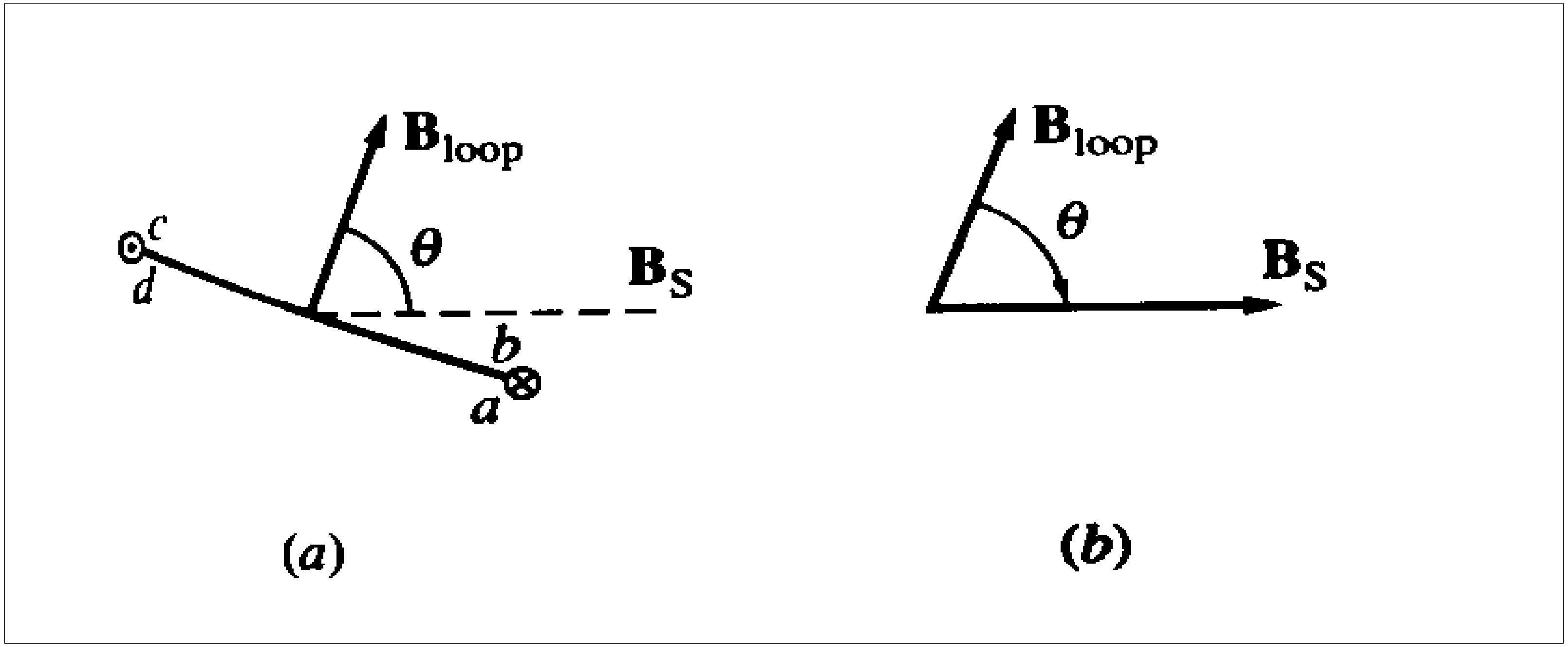


Figure 5.7: (a) Current in the loop produces a magnetic flux density B_{loop}, (b) geometric relationship between B_{loop} and B_s

- Thus, ***the torque induced in the loop is proportional to the strength of the loop (rotor's) magnetic field,*** B_r ***the strength of the external (stator's) magnetic field,*** B_s ***and the sine of the angle,*** θ ***between them***.
- In general the torque in any real machine will depend on the following four factors:
  1. The strength of the rotor magnetic field (B_r).
  2. The strength of the stator magnetic field (B_s).
  3. The sine of the angle between them (θ).
  4. A constant representing the construction of the machine (k).

Rotating MMF in AC Machines

- Consider a single *N*-turn coil as shown in Fig. 5.8.
- Applying Ampere's Law we get

$$\oint H \bullet dl = Ni \Rightarrow H(2g) = Ni \Rightarrow H = \frac{(Ni/2)}{g} \quad (5.7)$$

where, *g* – air-gap length, *N* – number of turns and *i* – current through the coil.

- Half the MMF appears across the top half of the air-gap and the other half appears across the bottom half air-gap as shown in Fig. 5.8.
- The rectangular square-wave MMF waveform (Fig. 5.8 (b)) can be resolved into its Fundamental as well as higher order harmonic components using Fourier Series.

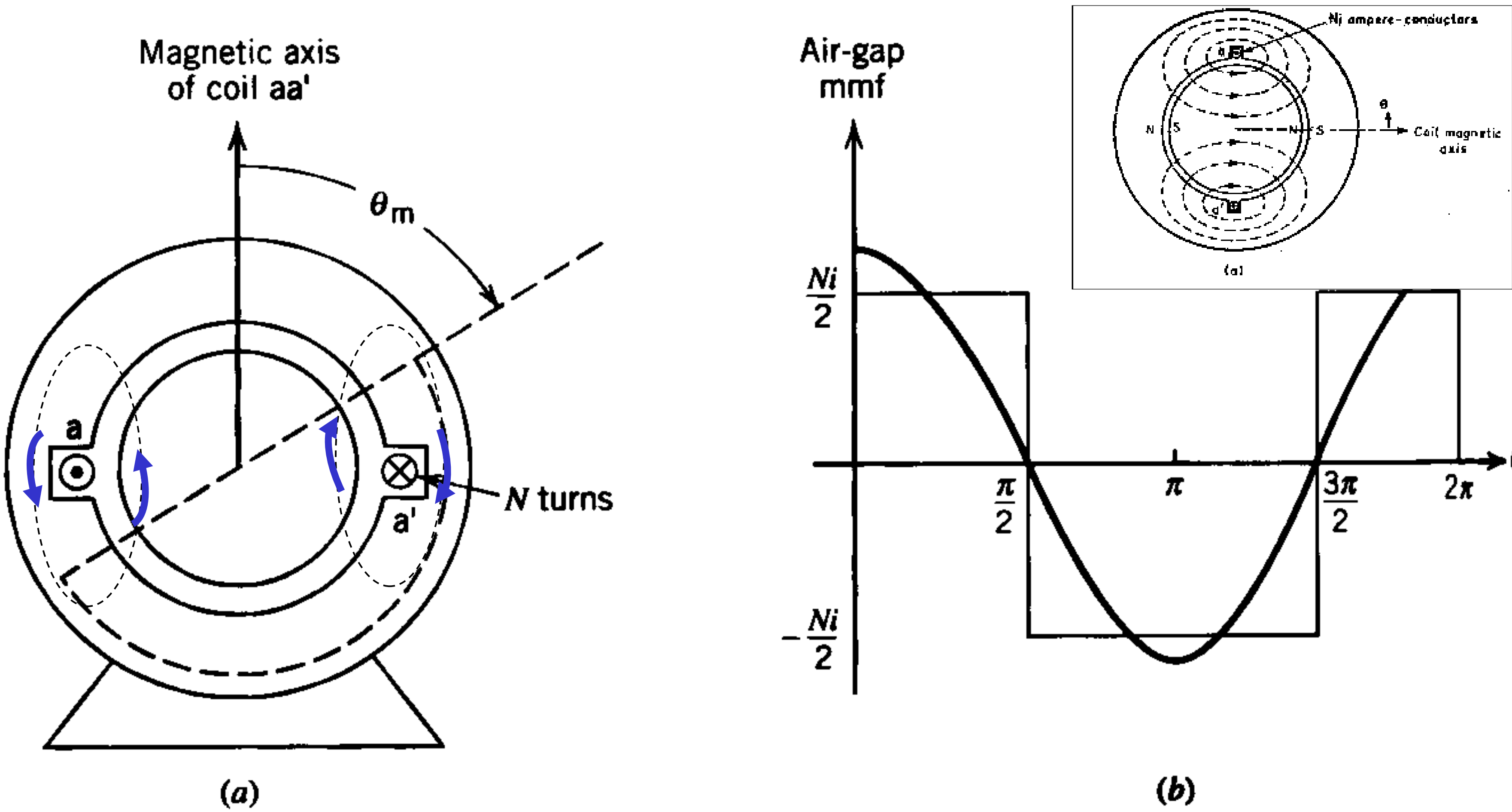


Figure 5.8: Single-coil machine (a) concentrated full-pitch coil; (b) air-gap MMF.

- The MMF, ***F*** can be approximated by its Fundamental component neglecting higher-order harmonics as:

$$F = F_{a1} = \frac{4}{\pi}\frac{Ni}{2}\cos\theta_m = F_m \cos\theta_m \qquad (5.8)$$

where F_{a1} is the fundamental component of the MMF, F and $F_m = (4/\pi)\,(Ni/2)$.

- The magnetic field intensity, ***H*** is equal to the MMF drop ($Ni/2$) across the air-gap divided by the air-gap length, g.

$$H = H_{a1} = \frac{F_{a1}}{g} = \frac{F_m}{g}\cos\theta_m = H_m \cos\theta_m \qquad (5.9)$$

where $H_m = (F_m/g) = (4/\pi)\,(Ni/2g)$.

Concept of Rotating Magnetic Field in three-phase AC Machines

- *If a three-phase set of currents, each of equal magnitude and differing in phase by 120º, flows in a three-phase winding of an AC machine, then it will produce a rotating magnetic field of constant magnitude.*
- Consider a simple stator with three coils as shown in Fig. 5.11.
- Assume that the currents in the three coils are given b

$$
\begin{aligned}
i_{aa'}(t) &= I_M \sin \omega t, \\
i_{bb'}(t) &= I_M \sin(\omega t - 120°), \\
i_{cc'}(t) &= I_M \sin(\omega t - 240°)
\end{aligned}
\qquad (5.7)
$$

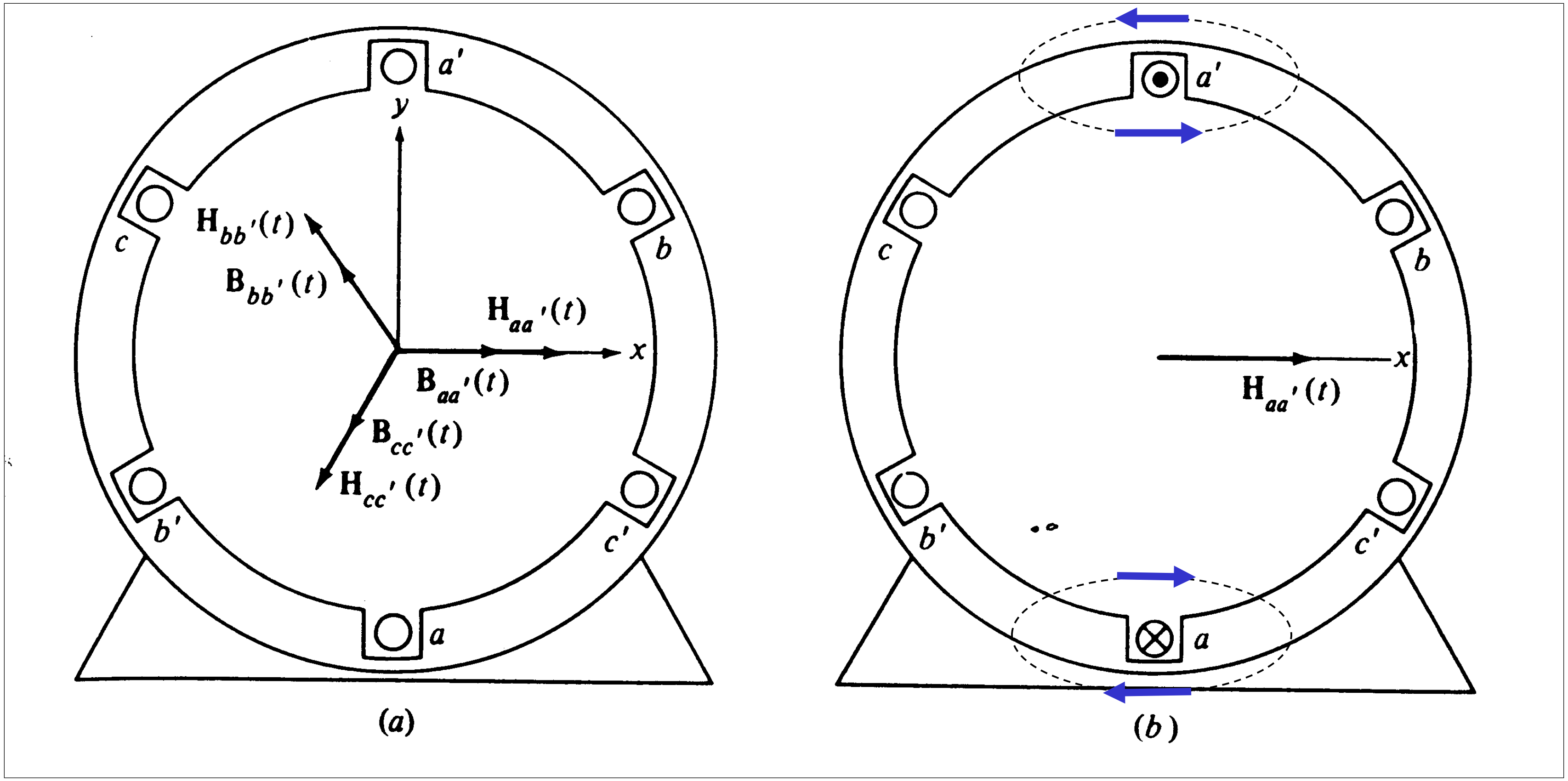


Figure 5.11: (a) A simple three-phase stator and (b) Magnetizing intensity vector produced due to current flowing in coil *aa'* only

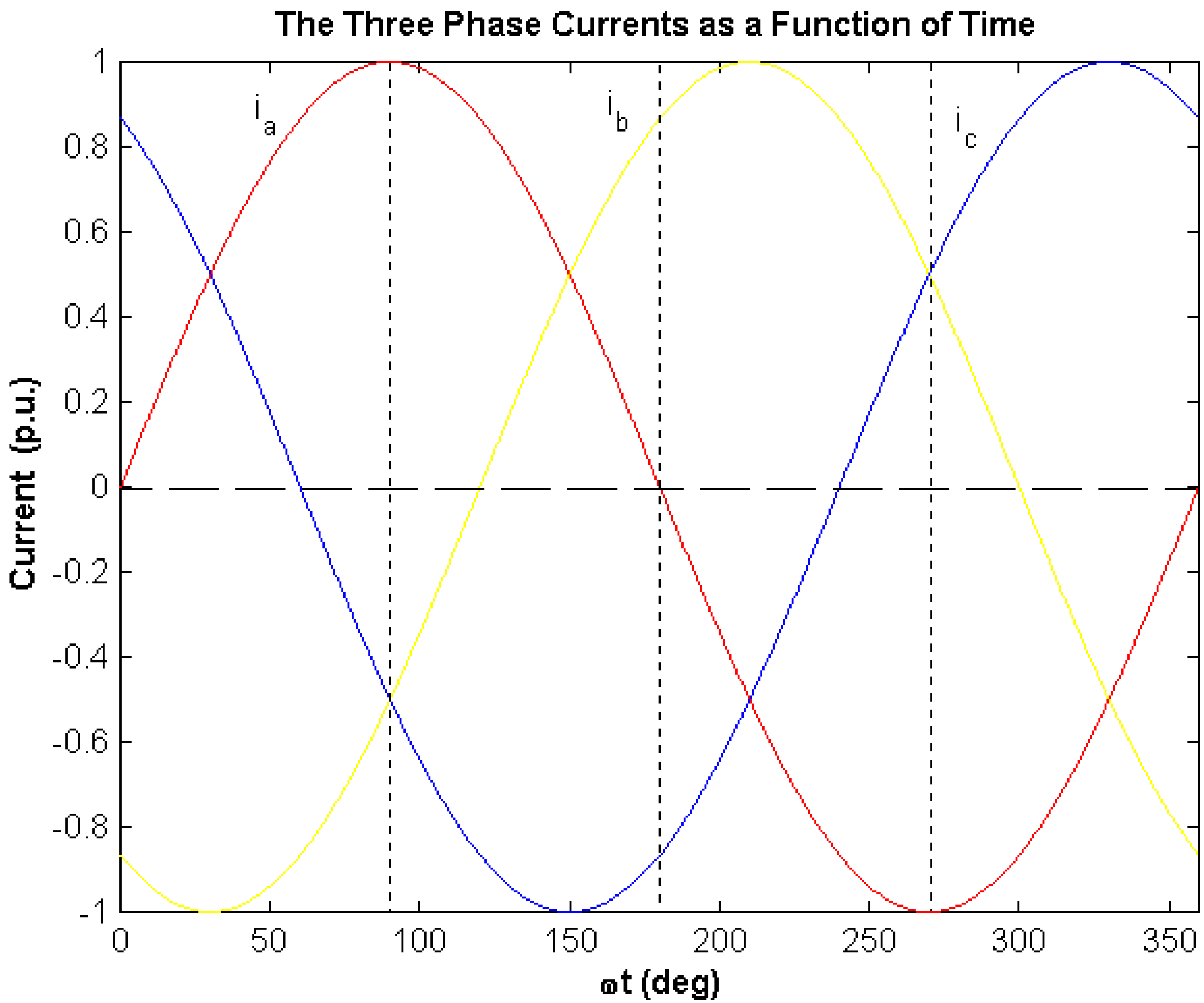


Fig. 5.12: Three phase currents

- The three-phase currents produce magnetic field and their intensities are given by

$$H_{aa'}(t) = H_M \sin \omega t \angle 0^\circ A \cdot turns/m$$
$$H_{bb'}(t) = H_M \sin(\omega t - 120^\circ) \angle 120^\circ A \cdot turns/m$$
$$H_{cc'}(t) = H_M \sin(\omega t - 240^\circ) \angle 240^\circ A \cdot turns/m \quad (5.8)$$

- Notice that the magnitude of the magnetic field intensity vector $\boldsymbol{H}_{aa'}(t)$ varies sinusoidally w.r.t. time, but the direction is always constant. This can be seen from Fig. 5.13.

- Malab Program to demonstrate the variation of magnetic field intensity, ***H*** or flux-density, ***B***.

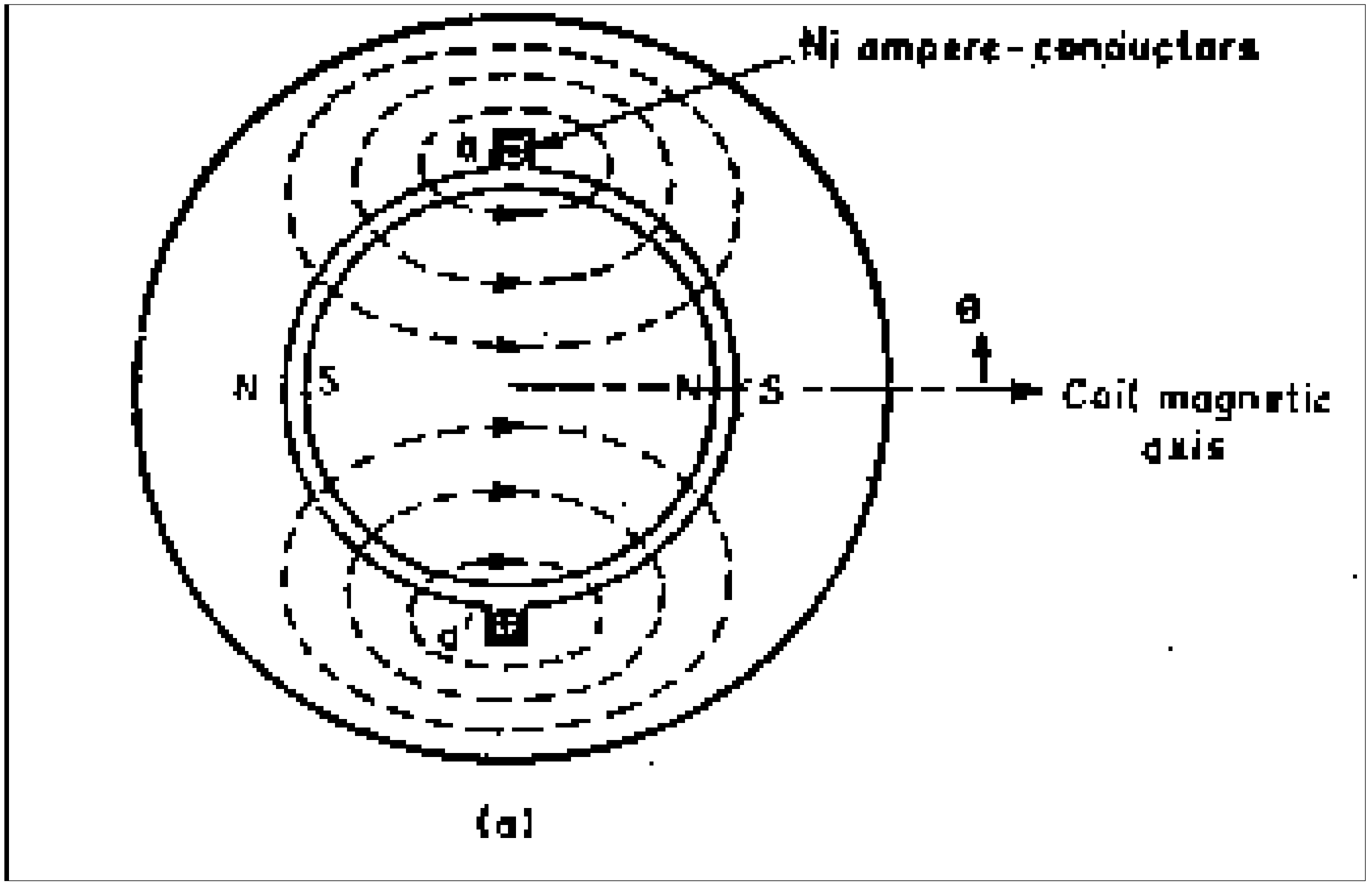


Figure 5.13: (a) MMF space wave of a single coil.

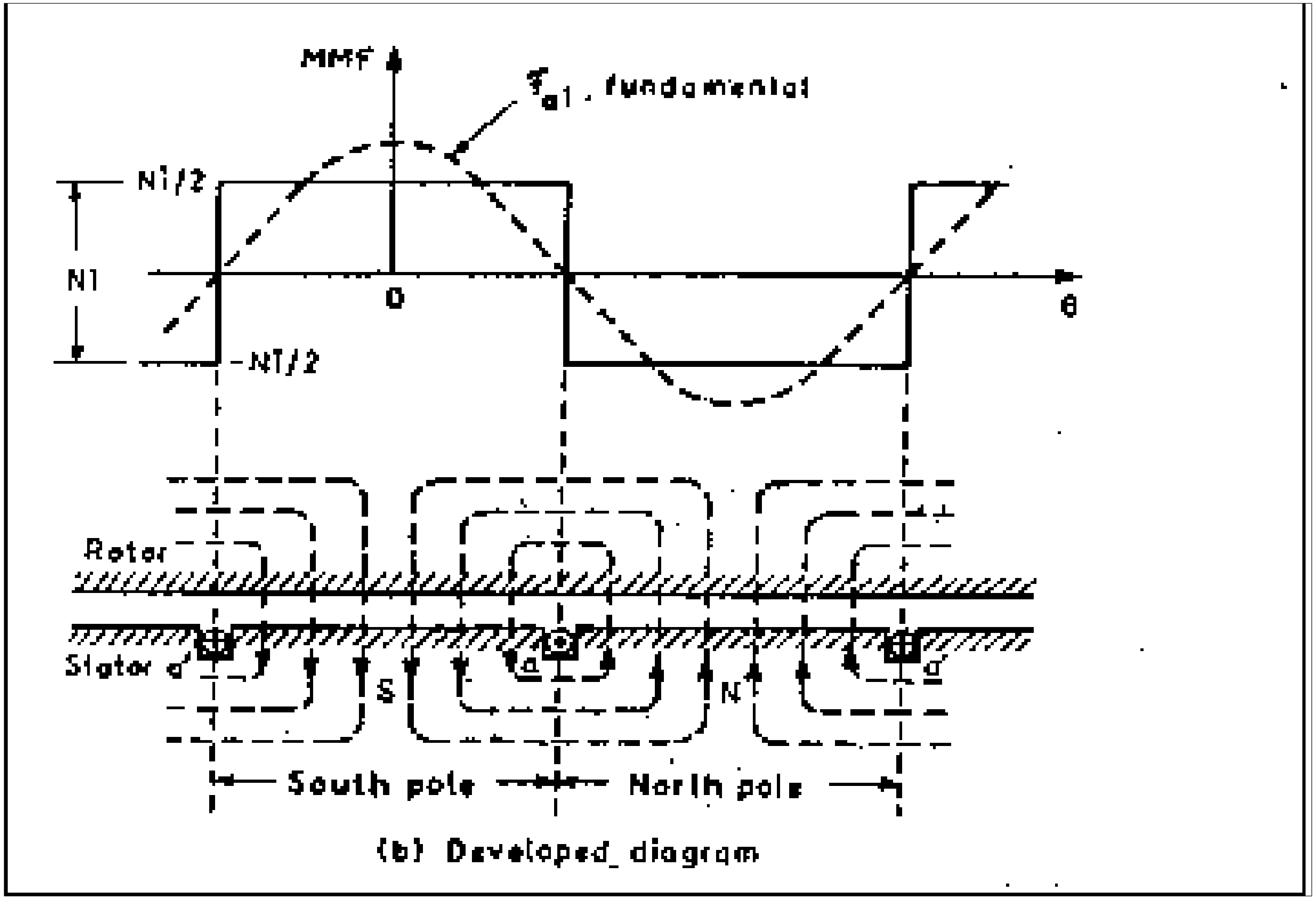


Figure 5.13: (b) MMF space wave of a single coil

- The corresponding flux-densities due to the three-phase currents as given by

$$B_{aa'}(t) = B_M \sin \omega t \angle 0^\circ \ T$$
$$B_{bb'}(t) = B_M \sin(\omega t - 120^\circ) \angle 120^\circ \ T$$
$$B_{cc'}(t) = B_M \sin(\omega t - 240^\circ) \angle 240^\circ \ T \qquad (5.9)$$

where $B_M = \mu H_M$.

- The current and their corresponding flux densities can be examined at specific instant of time in order to determine the resultant magnetic field in the stator.

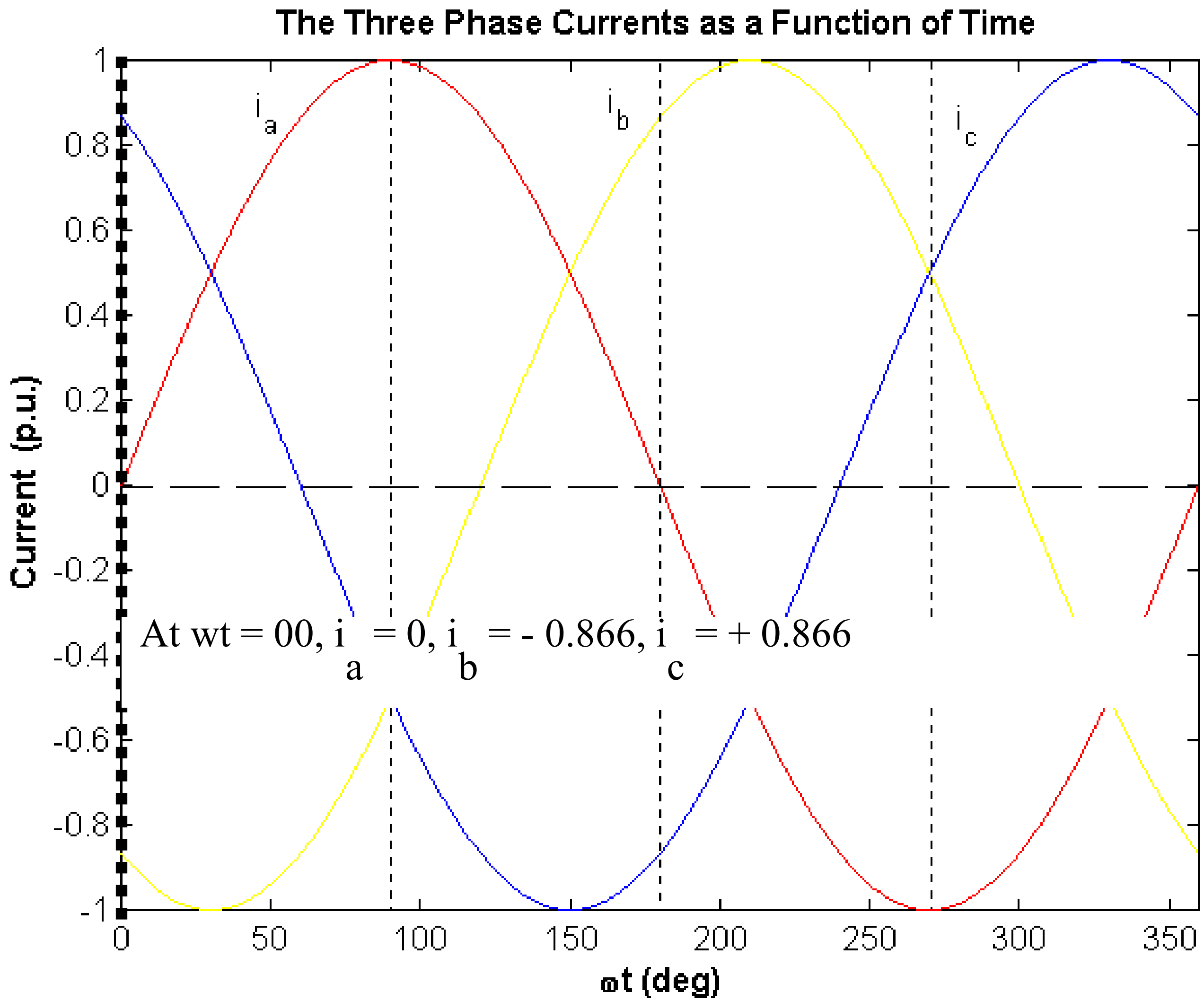
- For example for $\omega t = 0^0$, we have

$$i_{aa'}(t) = I_M \sin(0°), \Rightarrow B_{aa'(t)} = B_M \sin(0°)\angle 0° = 0\angle 0°$$
$$i_{bb'}(t) = I_M \sin(-120°), \Rightarrow B_{bb'(t)} = B_M \sin(-120°)\angle 120° = -0.866 B_M \angle 120°$$
$$i_{cc'}(t) = I_M \sin(-240°) \Rightarrow B_{cc'(t)} = B_M \sin(-240°)\angle 240° = 0.866 B_M \angle 240°$$

- *The resulting magnetic field from all three coils will be*

$$B_{net} = B_{aa'} + B_{bb'} + B_{cc'}$$
$$= 0\angle 0° + (-0.866) B_M \angle 120° + 0.866 B_M \angle 240°$$
$$= 1.5 B_M \angle -90°$$

- The resulting net magnetic field is shown in Fig. 5.14.



The Three Phase Currents as a Function of Time
Current (p.u.)
ωt (deg)
i_a
i_b
i_c
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
50
100
150
200
250
300
350
At wt = 00, i_a = 0, i_b = - 0.866, i_c = + 0.866

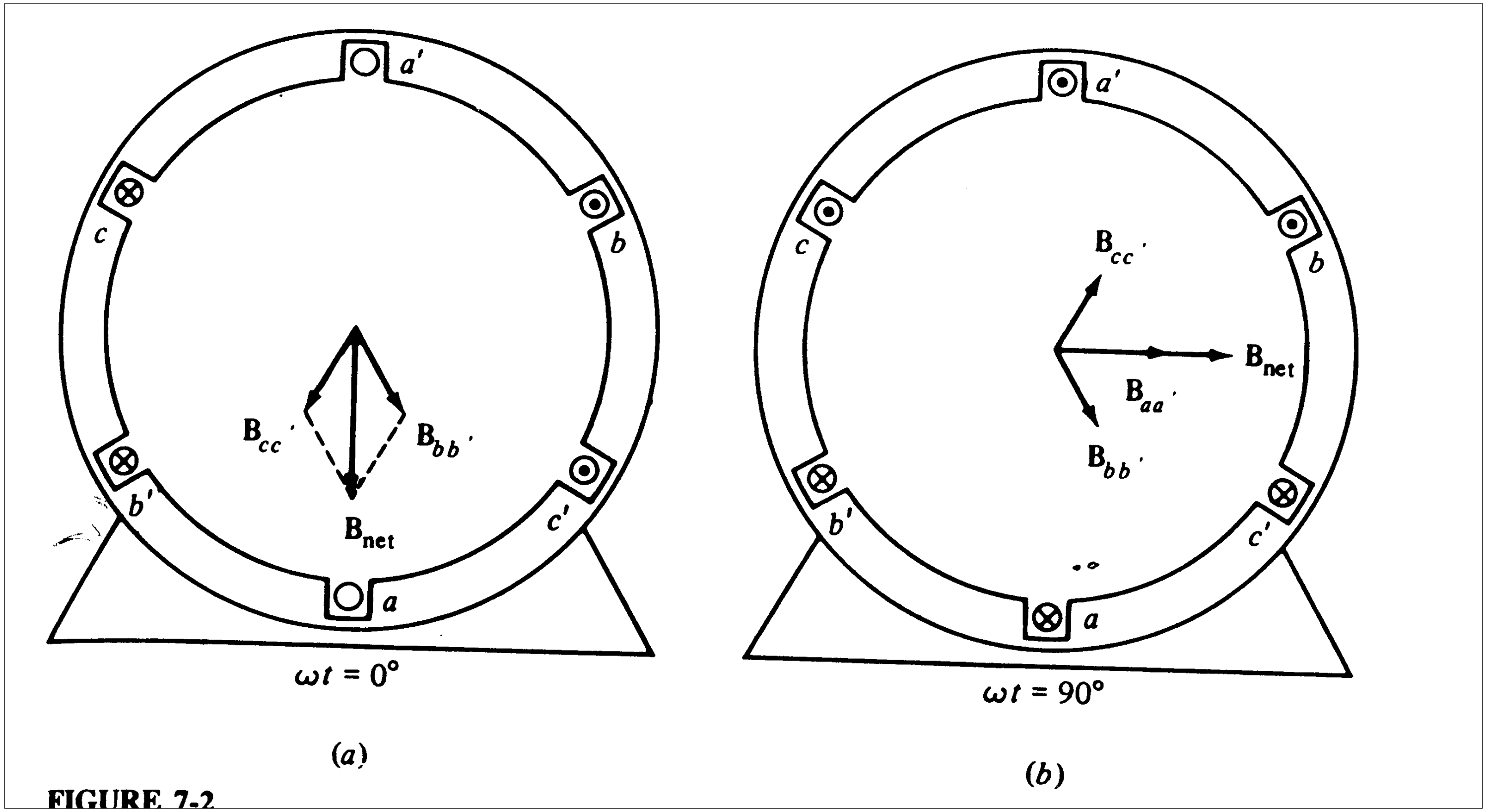


Figure 5.14: The net magnetic field vector for (a) $\omega t = 0^\circ$ and (b) $\omega t = 90^\circ$

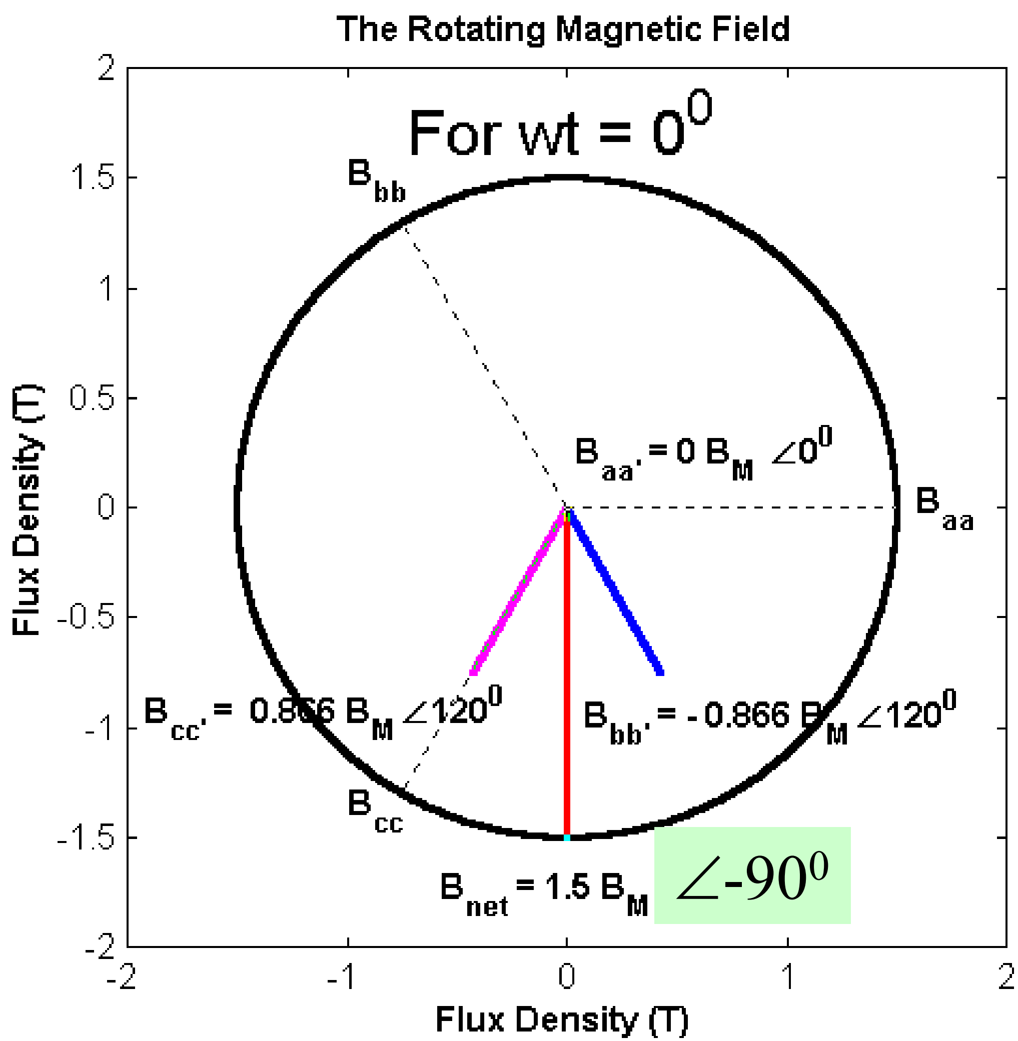


Figure 5.14(a): The net magnetic field vector for $\omega t = 0°$

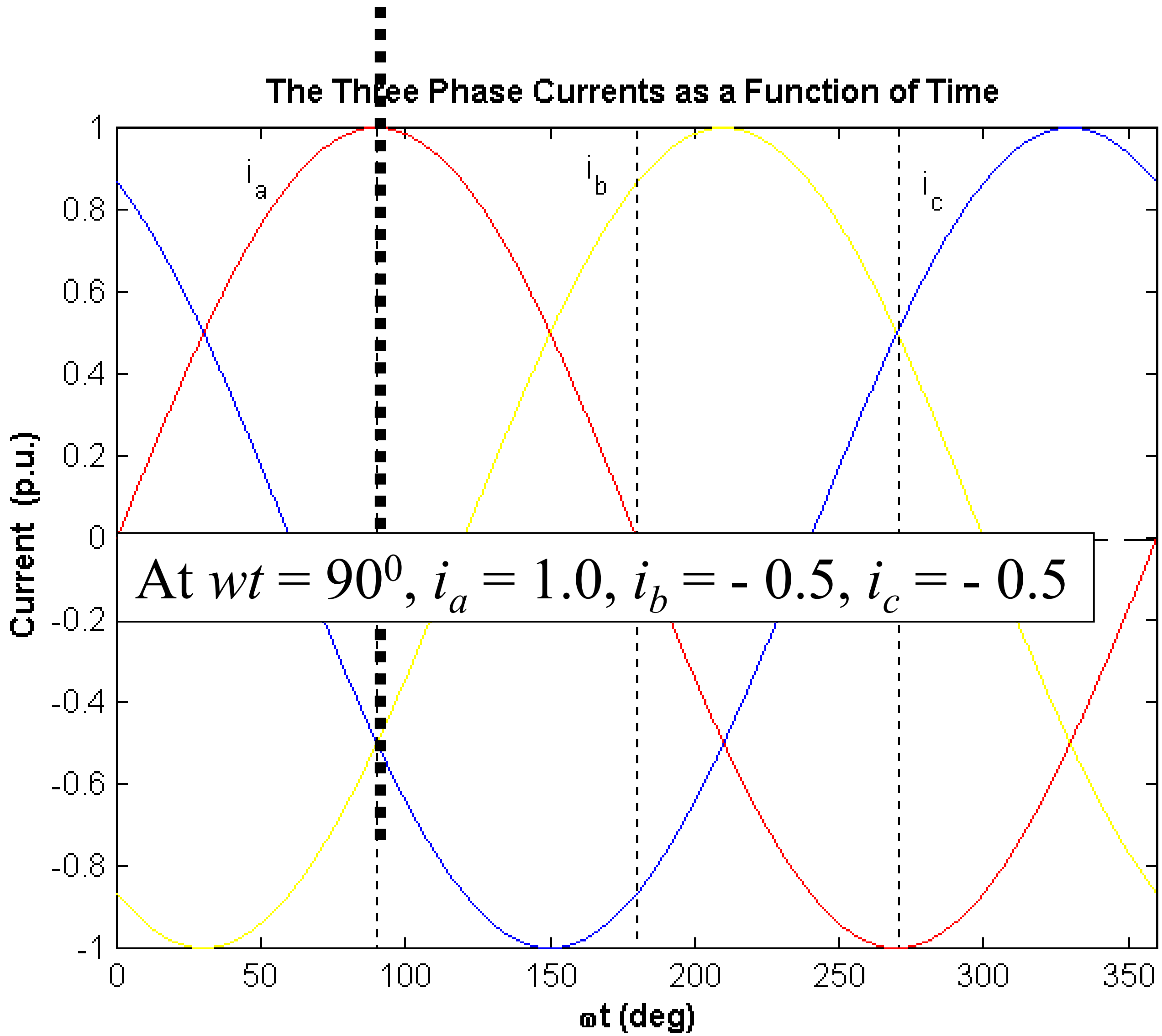
- For example for ωt = $90^{0,}$ we have

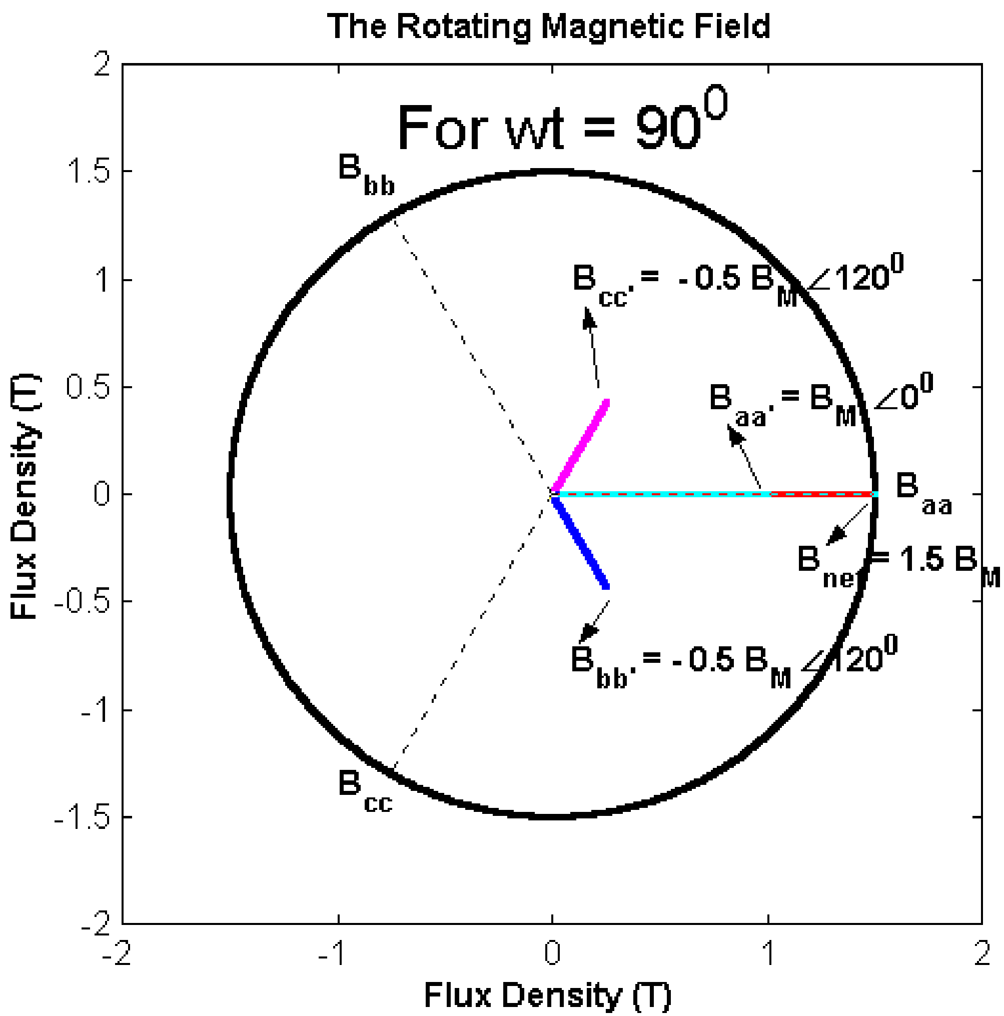
$$i_{aa'}(t) = I_M \sin(90^o),\Rightarrow B_{aa'(t)} = B_M \sin(90°)\angle 0° = B_M \angle 0°$$
$$i_{bb'}(t) = I_M \sin(90^o - 120^o),\Rightarrow B_{bb'(t)} = B_M \sin(-30°)\angle 120° = -0.5B_M \angle 120°$$
$$i_{cc'}(t) = I_M \sin(90^o - 240°) \Rightarrow B_{cc'(t)} = B_M \sin(-150°)\angle 240° = -0.5B_M \angle 240°$$

- *The resulting magnetic field from all three coils will be*

$$B_{net} = B_{aa'} + B_{bb'} + B_{cc'}$$
$$= B_M \angle 0° + (-0.5)B_M \angle 120° + (-0.5)B_M \angle 240°$$
$$= 1.5B_M \angle 0°$$

- The resulting net magnetic field is shown in Fig. 5.14(b).


The Three Phase Currents as a Function of Time
Current (p.u.)
ωt (deg)
i_a
i_b
i_c
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
50
100
150
200
250
300
350
At $wt = 90^0$, $i_a = 1.0$, $i_b = -0.5$, $i_c = -0.5$



$\angle 00$

Figure 5.14(b): The net magnetic field vector for $\omega t = 90°$

- For example for ωt = 180^{0,} we have

$$i_{aa'}(t) = I_M \sin(180^o), \Rightarrow B_{aa'(t)} = B_M \sin(180°)\angle 0° = 0\angle 0°$$

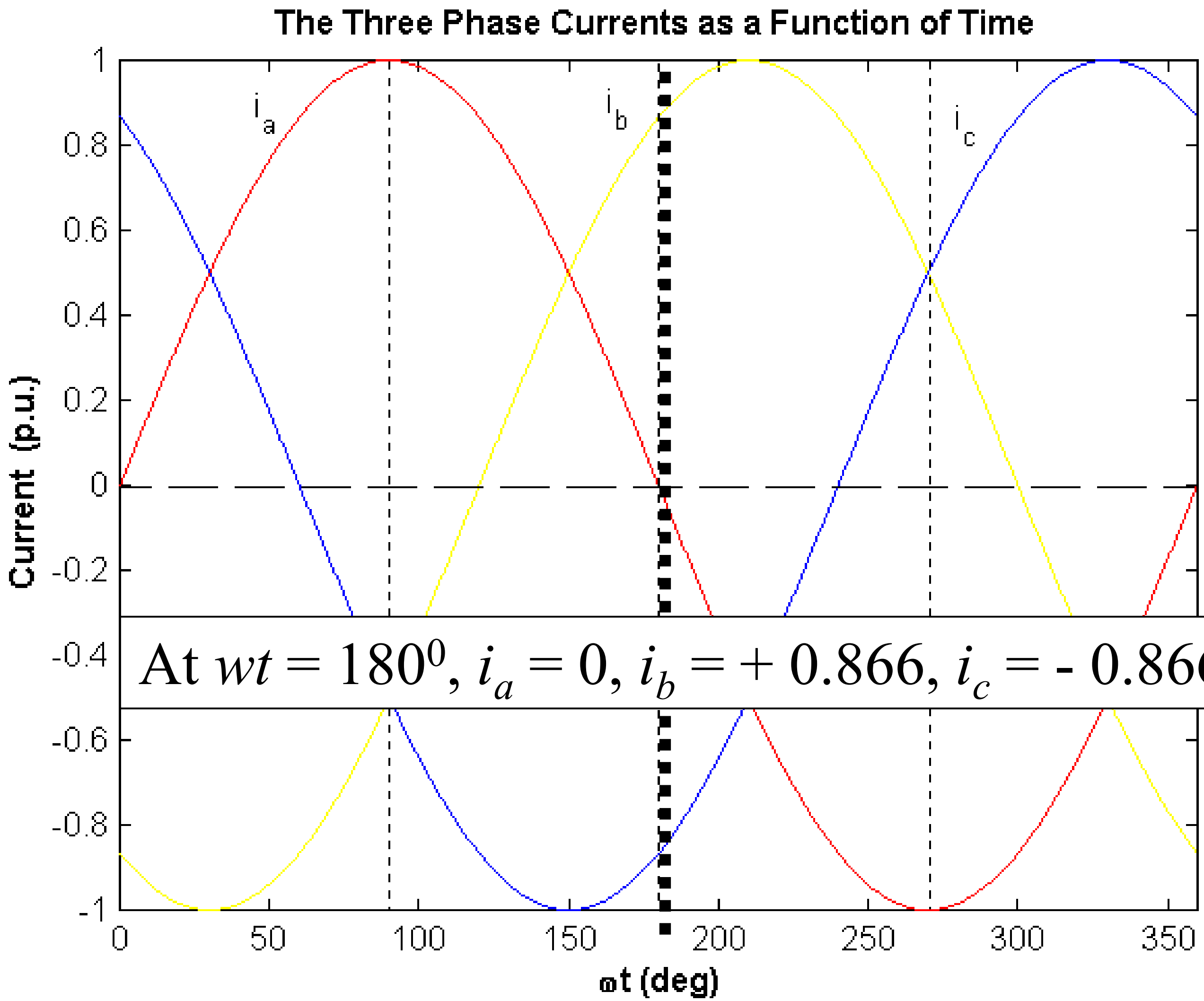
$$i_{bb'}(t) = I_M \sin(180^o - 120^o), \Rightarrow B_{bb'(t)} = B_M \sin(60°)\angle 120° = 0.866 B_M \angle 120°$$

$$i_{cc'}(t) = I_M \sin(180^o - 240°) \Rightarrow B_{cc'(t)} = B_M \sin(-60°)\angle 240° = -0.866 B_M \angle 240°$$

- The resulting magnetic field from all three coils will be

$$B_{net} = B_{aa'} + B_{bb'} + B_{cc'}$$
$$= 0\angle 0° + 0.866 B_M \angle 120° + (-0.866) B_M \angle 240°$$
$$= 1.5 B_M \angle 90°$$

- The resulting net magnetic field is shown in Fig. 5.14(c).



At $wt = 180^0$, $i_a = 0$, $i_b = + 0.866$, $i_c = - 0.866$

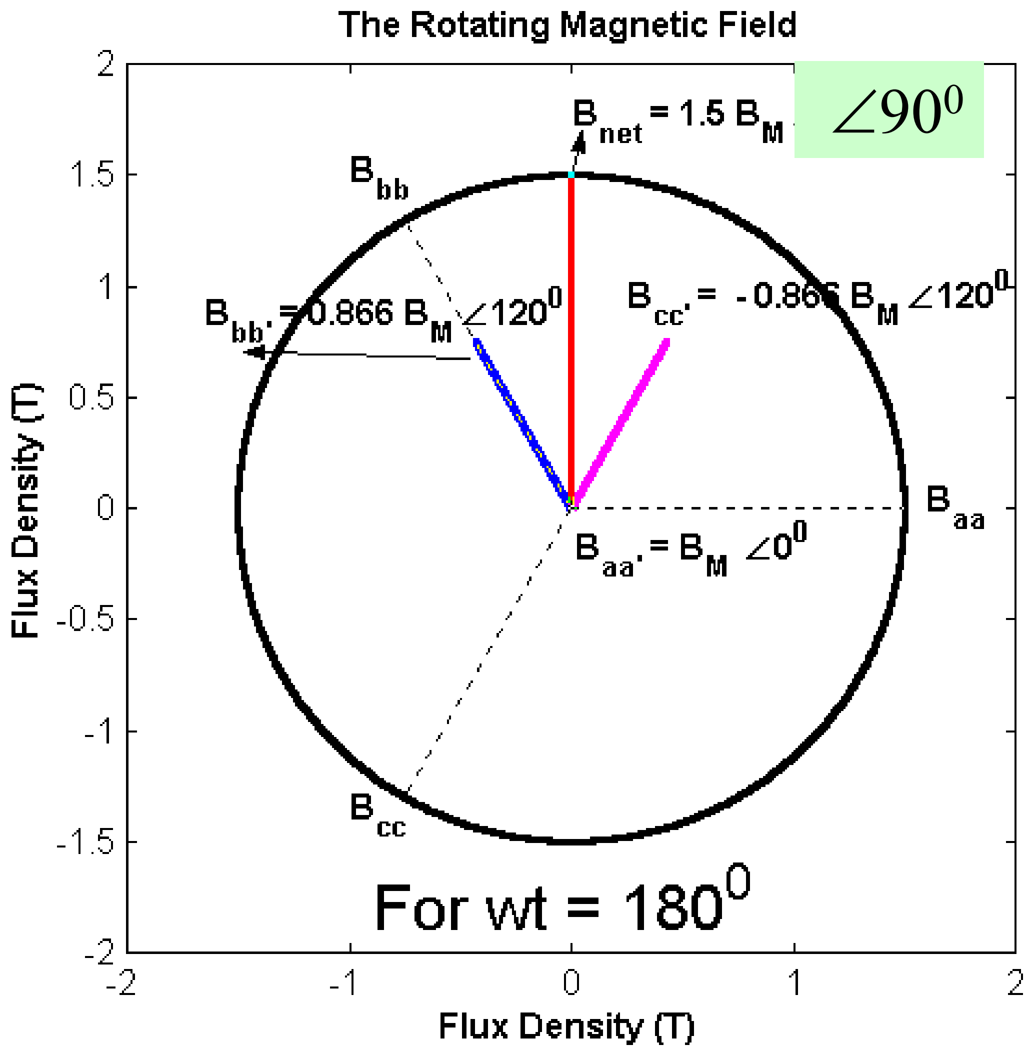


Figure 5.14(c): The net magnetic field vector for $\omega t = 180^\circ$

- For example for $\omega t = 270^0$, we have

$$i_{aa'}(t) = I_M \sin(270^o), \Rightarrow B_{aa'(t)} = B_M \sin(270°)\angle 0° = -B_M \angle 0°$$

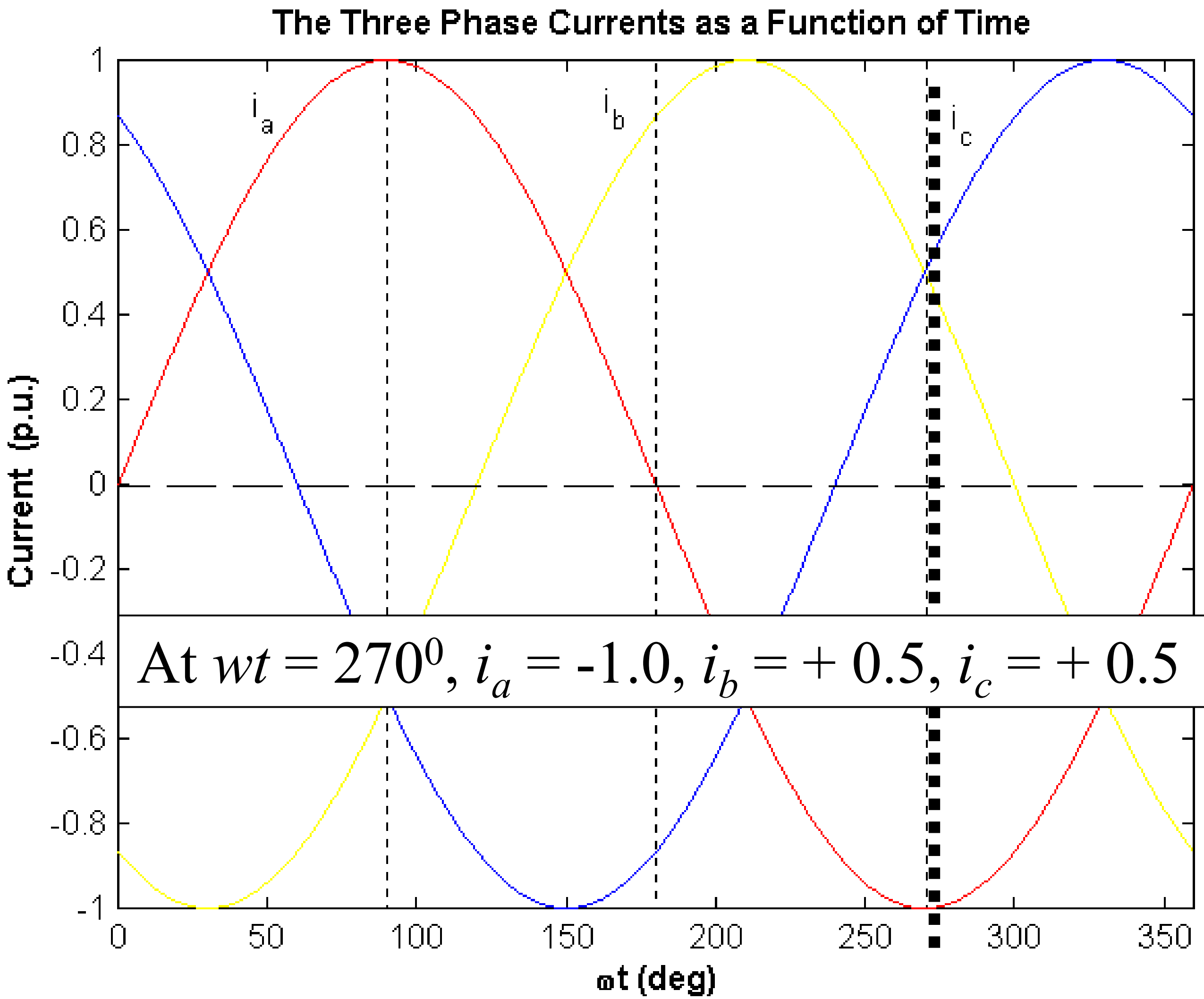
$$i_{bb'}(t) = I_M \sin(270^o - 120^o), \Rightarrow B_{bb'(t)} = B_M \sin(150°)\angle 120° = 0.5B_M \angle 120°$$

$$i_{cc'}(t) = I_M \sin(270^o - 240°) \Rightarrow B_{cc'(t)} = B_M \sin(30°)\angle 240° = 0.5B_M \angle 240°$$

- The resulting magnetic field from all three coils will be

$$B_{net} = B_{aa'} + B_{bb'} + B_{cc'}$$
$$= (-B_M)\angle 0° + 0.5B_M \angle 120° + 0.5B_M \angle 240°$$
$$= 1.5B_M \angle 180°$$

- The resulting net magnetic field is shown in Fig. 5.14(d).



At $wt = 270^0$, $i_a = -1.0$, $i_b = +\ 0.5$, $i_c = +\ 0.5$

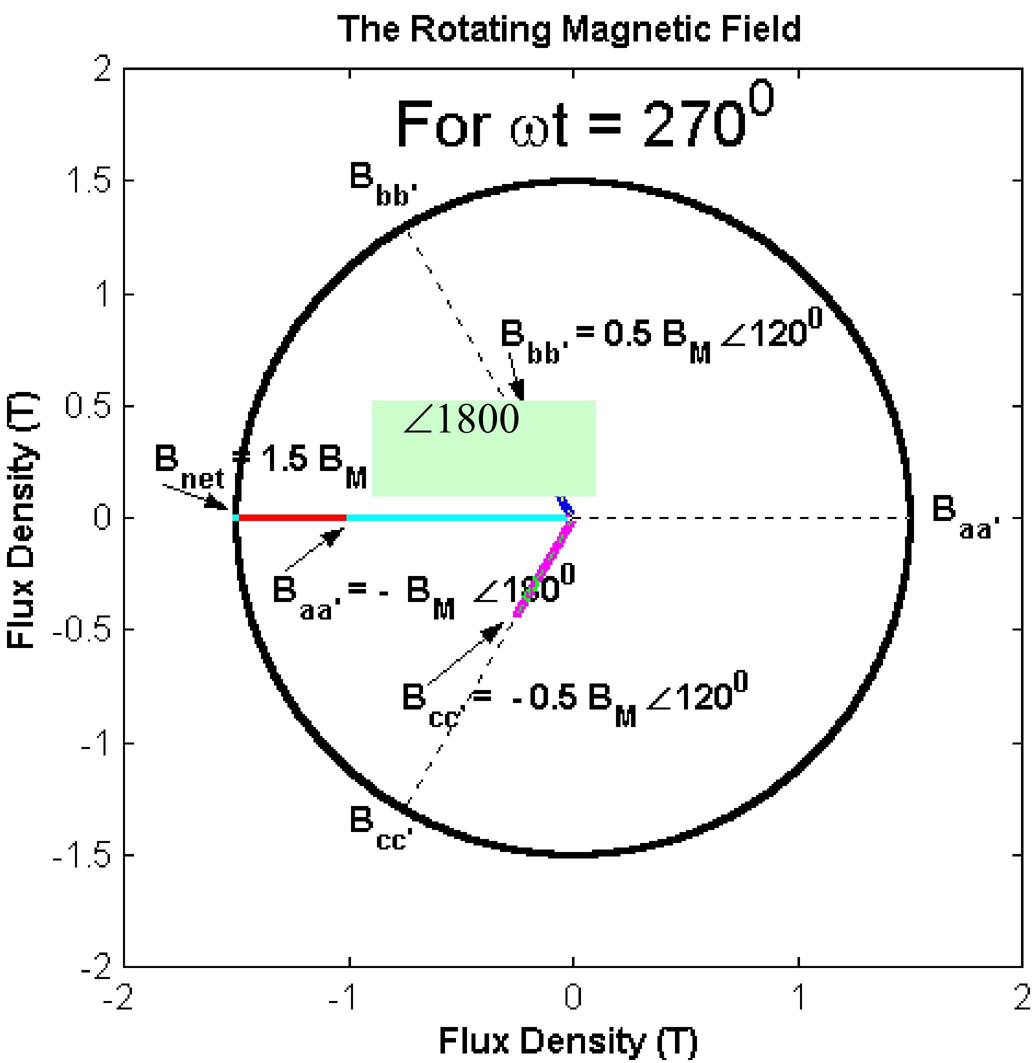


Figure 5.14(d): The net magnetic field vector for $\omega t = 270^\circ$

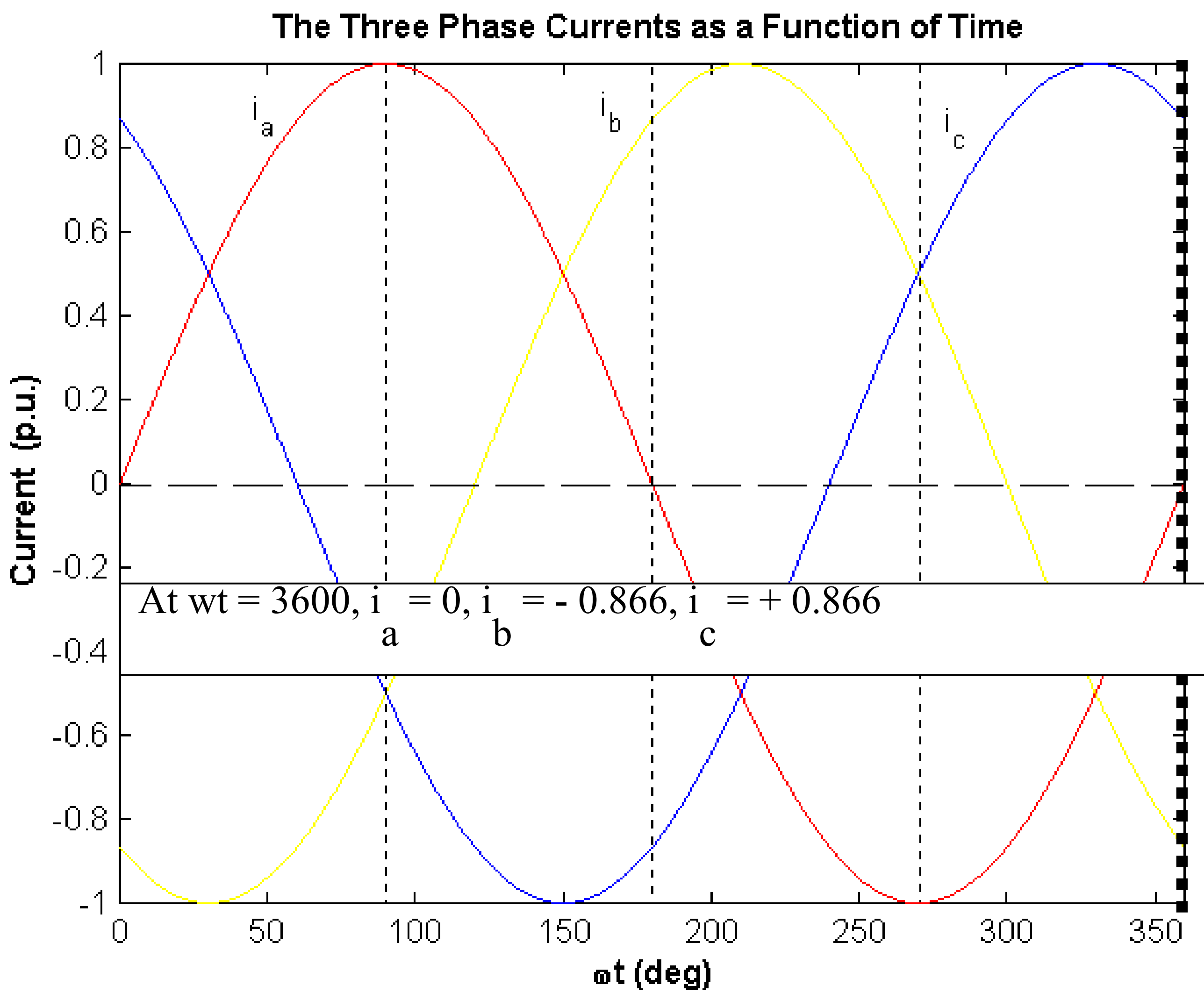
- For example for ωt = 360^0 = 0^0, we have

$$i_{aa'}(t) = I_M \sin(360^o), \Rightarrow B_{aa'(t)} = B_M \sin(360°)\angle 0° = 0\angle 0°$$
$$i_{bb'}(t) = I_M \sin(360^o - 120^o), \Rightarrow B_{bb'(t)} = B_M \sin(240°)\angle 120° = (-0.866)B_M\angle 120°$$
$$i_{cc'}(t) = I_M \sin(360^o - 240°) \Rightarrow B_{cc'(t)} = B_M \sin(120°)\angle 240° = 0.866B_M\angle 240°$$

- The resulting magnetic field from all three coils will be

$$B_{net} = B_{aa'} + B_{bb'} + B_{cc'}$$
$$= 0\angle 0° + (-0.866)B_M\angle 120° + 0.866B_M\angle 240°$$
$$= 1.5B_M\angle 270°$$

- The resulting net magnetic field is shown in Fig. 5.14(e).



At wt = 3600, i_a = 0, i_b = - 0.866, i_c = + 0.866

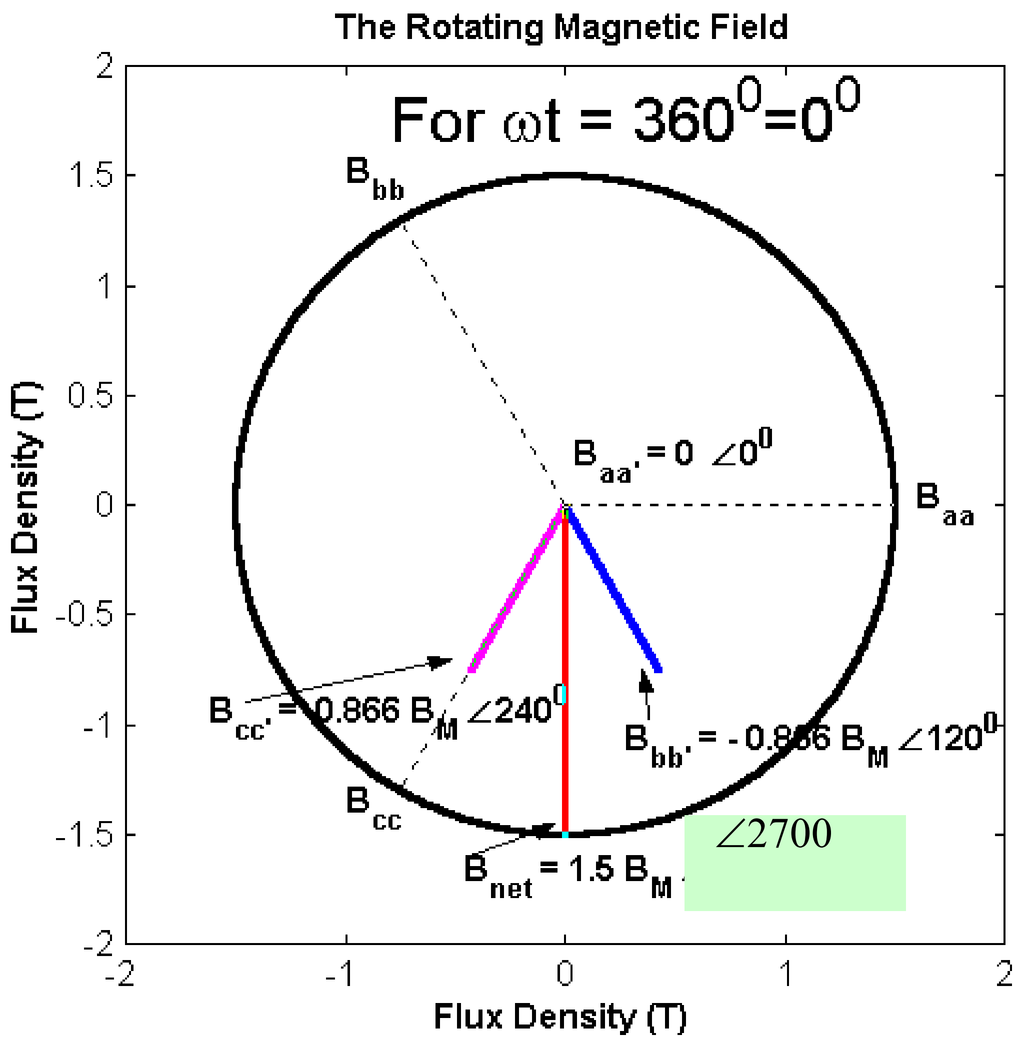


Figure 5.14(e): The net magnetic field vector for ωt = 360°

- Notice that although the direction of the net magnetic field B_{net} has changed (from $\angle -90° \Rightarrow \angle 0° \Rightarrow \angle 90° \Rightarrow \angle 180° \Rightarrow \angle 270° \Rightarrow \angle 0°$), its amplitude is always constant ($1.5B_M$).

- Mtalab demonstration of Rotating Magnetic Field.

- Therefore we can conclude that,

  "the resultant magnetic field has a constant magnitude of the 1.5 B_M and it rotates at an angular velocity of ω rad/s".

- The rotating magnetic field can be considered as moving magnet as shown in Fig. 5.15.

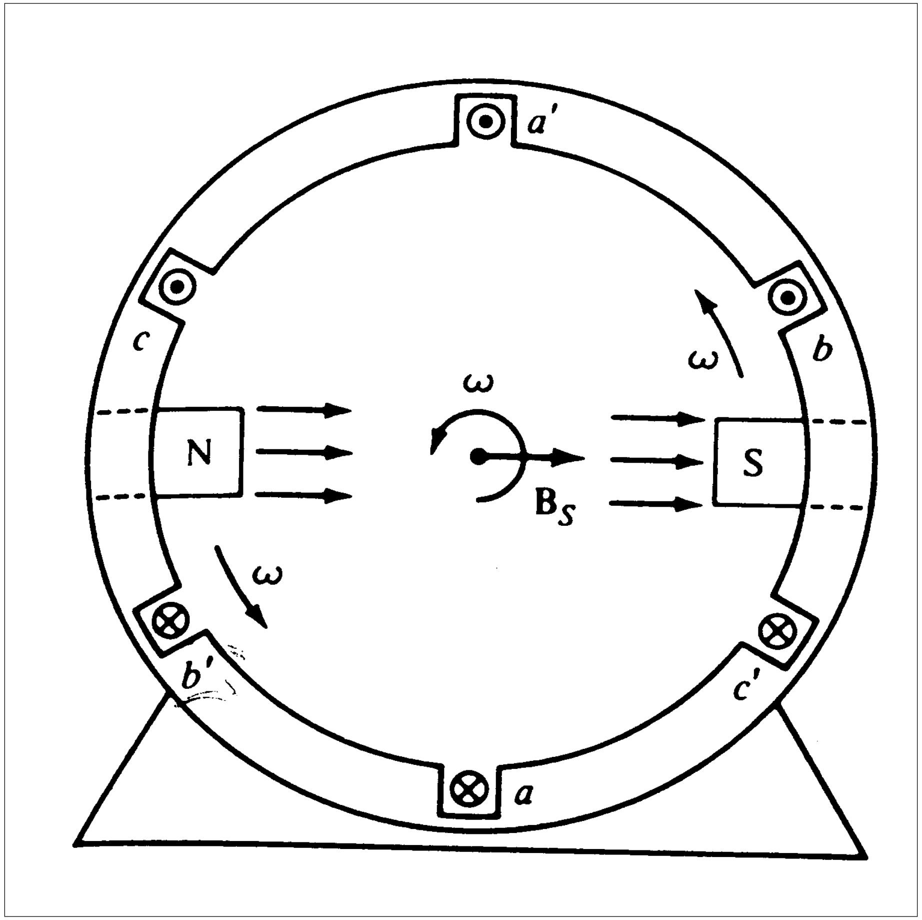


Figure 5.15: The rotating magnetic field in a stator represented as moving north and south poles.

Reversing the Direction of Magnetic Field Rotation

- *If the currents in any two of the three coils are swapped, the direction of the magnetic field's rotation can be reversed.*
- This means that the direction of rotation of an AC motor can be achieved by switching the connections on any two of the three coils i.e. if the normal sequence is *A-B-C* and we switch B and C connections then the sequence would be *A-C-B*.

The Relationship between Electrical Frequency and the Speed of Magnetic Field Rotation

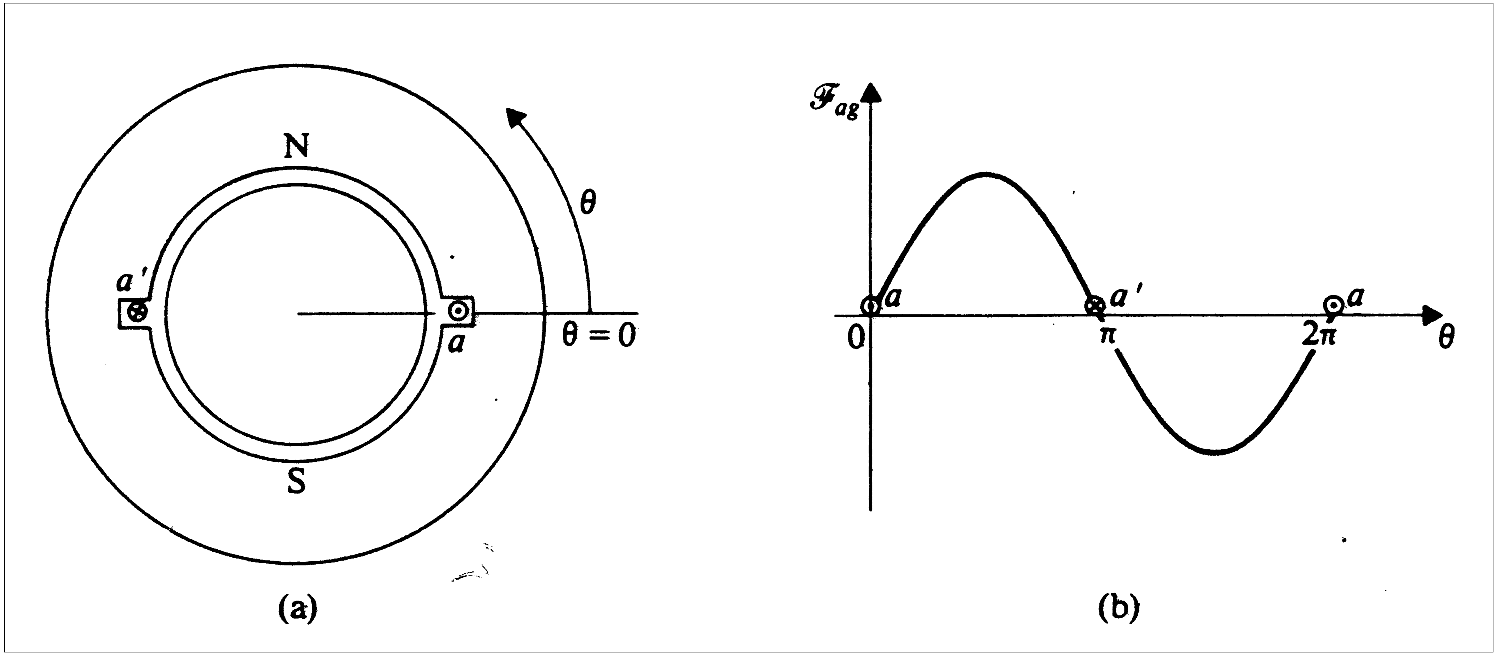


Figure 5.16: Two-pole stator MMF distribution

- These magnetic poles complete one mechanical rotation around the stator surface for each electrical cycle of the applied stator current as can be seen from Fig. 5.16.

- Thus, the mechanical speed of rotation of the rotating magnetic field in revolutions per second is equal to the electrical frequency in Hz:

$$f_e = f_m \text{ and } \quad \omega_e = \omega_m \quad \text{for two pole machine} \qquad (5.10)$$

- Now, *if we assume that this pattern of the windings is repeated twice within the same stator space then the stator with windings will look like Fig. 5.17(a).*

- When a set of three-phase stator currents is applied to the stator windings, it will produce two north and two south poles as shown in Fig. 5.17(b).

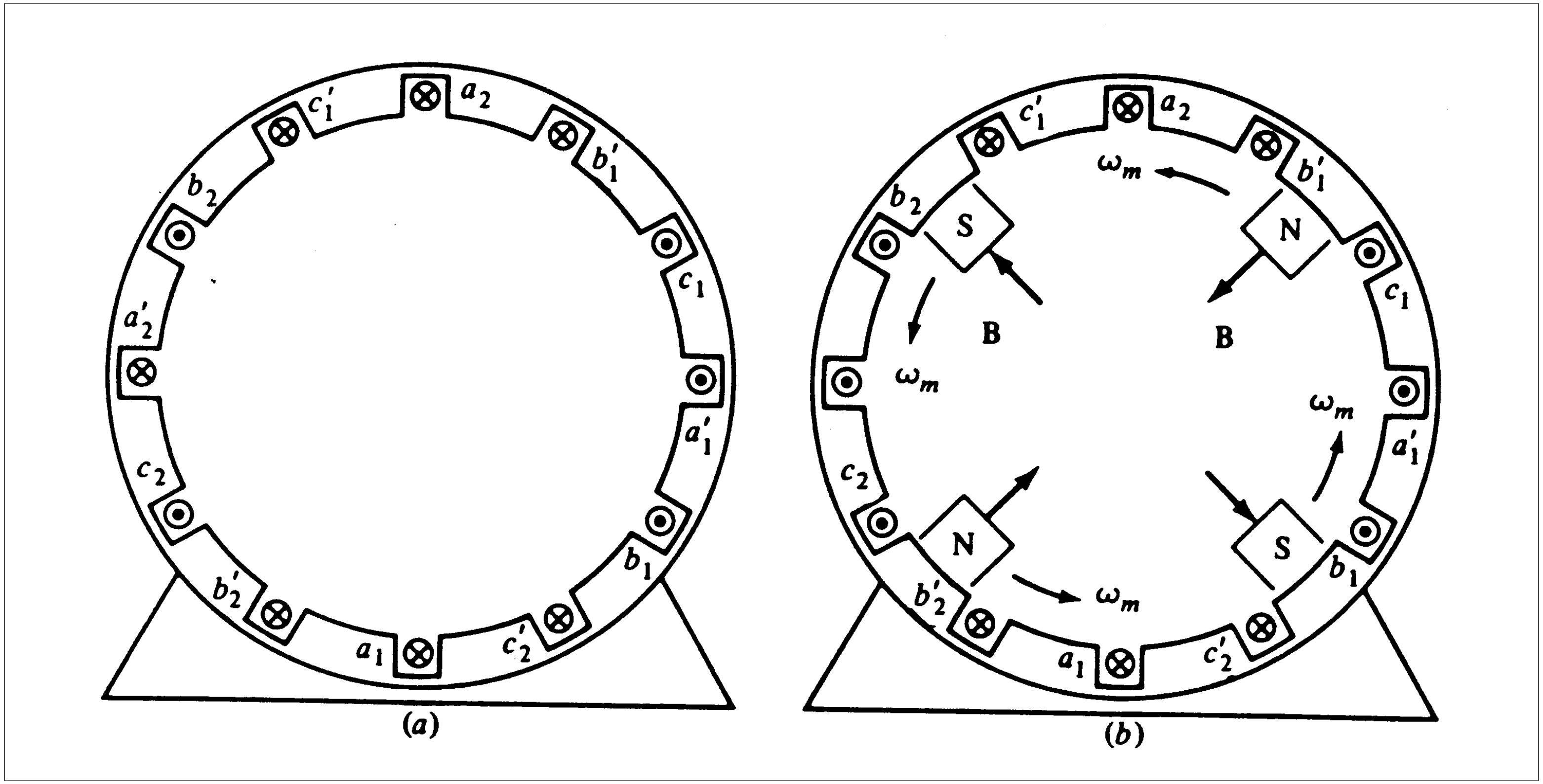


Figure 5.17: (a) A simple four-pole stator winding. (b) The resulting stator magnetic poles.

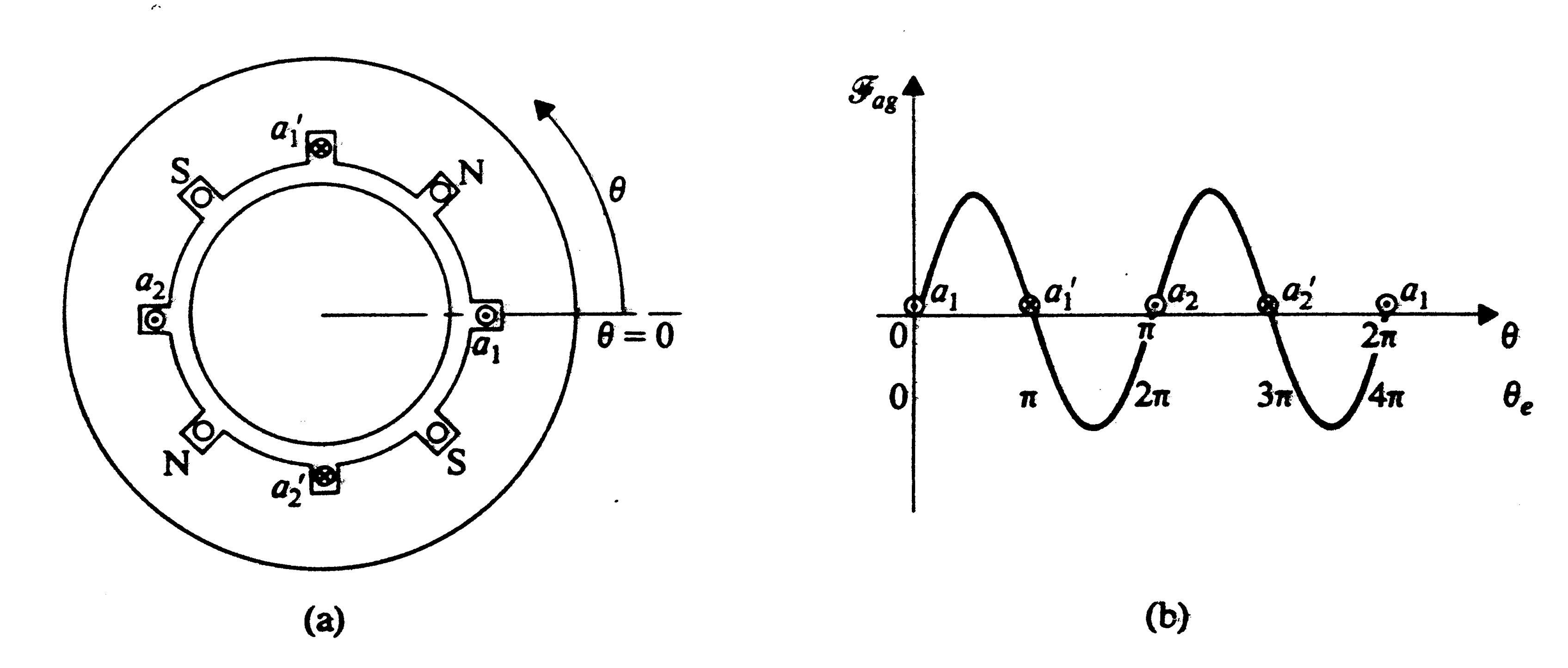


Figure 5.18: Four-pole stator MMF distribution

- In such four-pole windings, a pole moves only halfway around the stator surface in one electrical cycle. In other words, the MMF distribution will move two cycle for every one cycle of the mechanical motion as shown in Fig. 6.18.
- Notice that $f_m = n_m /60$, we have relationship between electrical frequency, f_e *in Hz* to the mechanical speed of the rotating magnetic field as:

$$f_e = \frac{P}{2} \times f_m = \frac{P}{2} \times \frac{n_m}{60} = \frac{n_m P}{120} \quad or \quad n_m = \frac{120 f_e}{P} \qquad (5.13)$$

where n_m is the speed in rpm.

- We refer this mechanical speed of the rotating magnetic field as the synchronous speed , n_s of the ac machine

$$n_s = \frac{120 \times f_e}{P} \qquad (5.14)$$

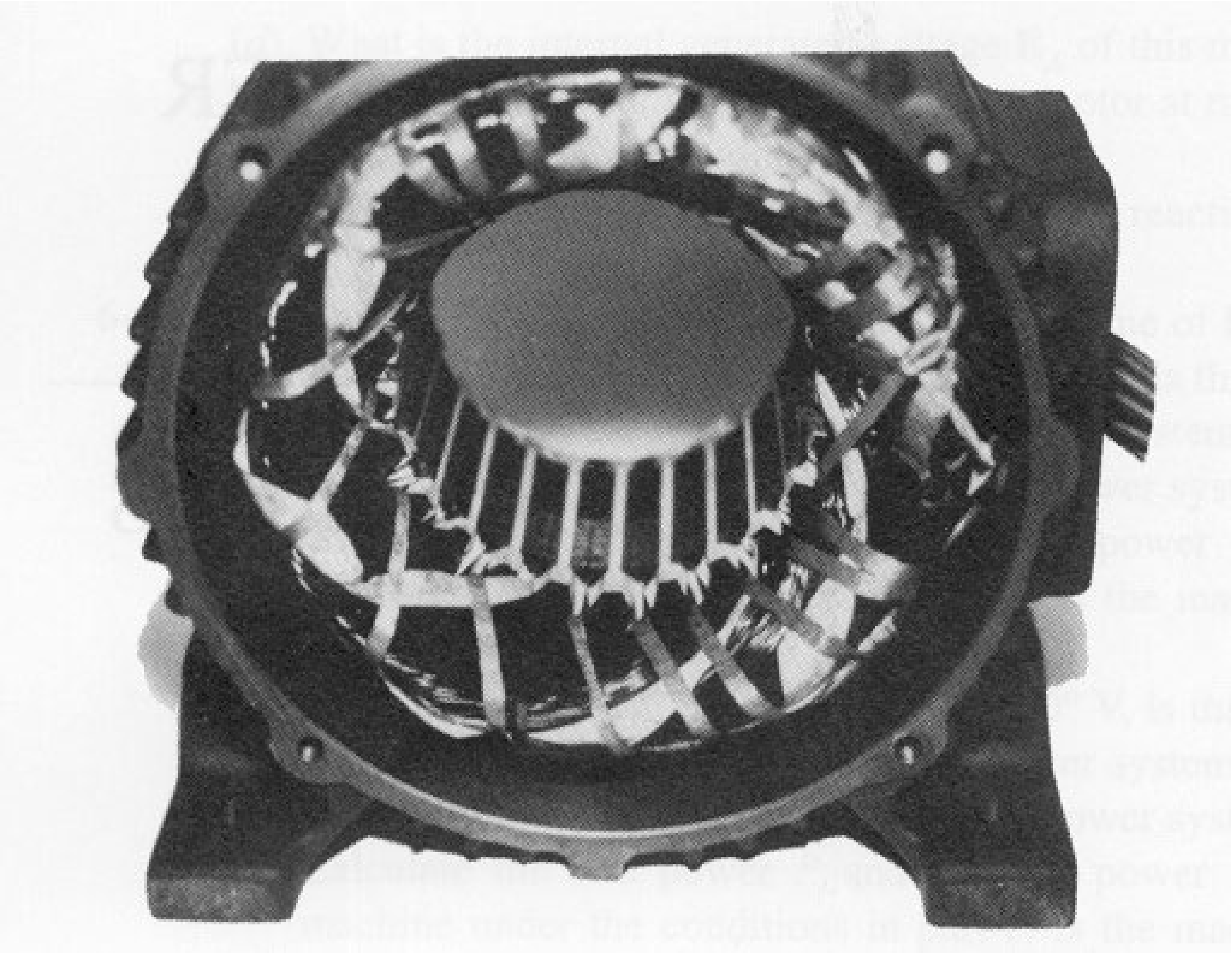
Induction Motor

- In induction machines rotor voltage (which produces the rotor currents and hence the rotor magnetic field, $\boldsymbol{B_R}$) is **induced** in the rotor winding (by induction or transformer action) thus the name *induction motor.*

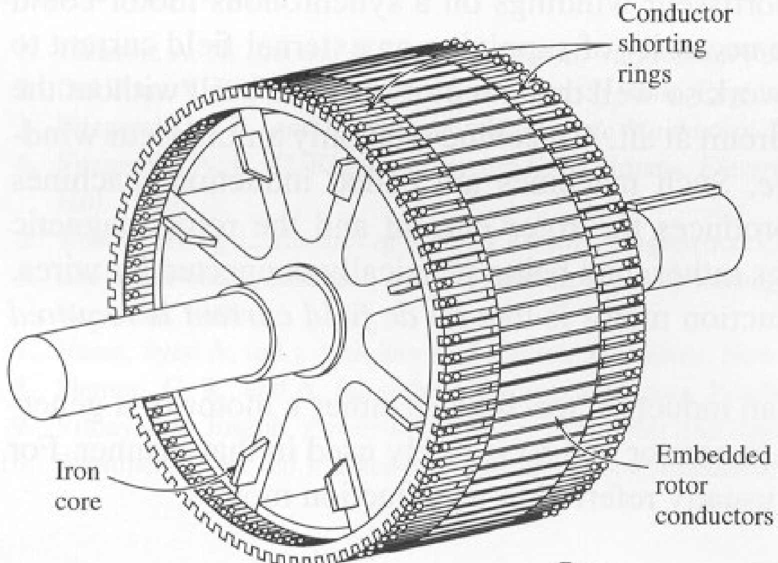
- Induction machines are usually operated as *motors* rather than as generators.

Induction Machine Construction

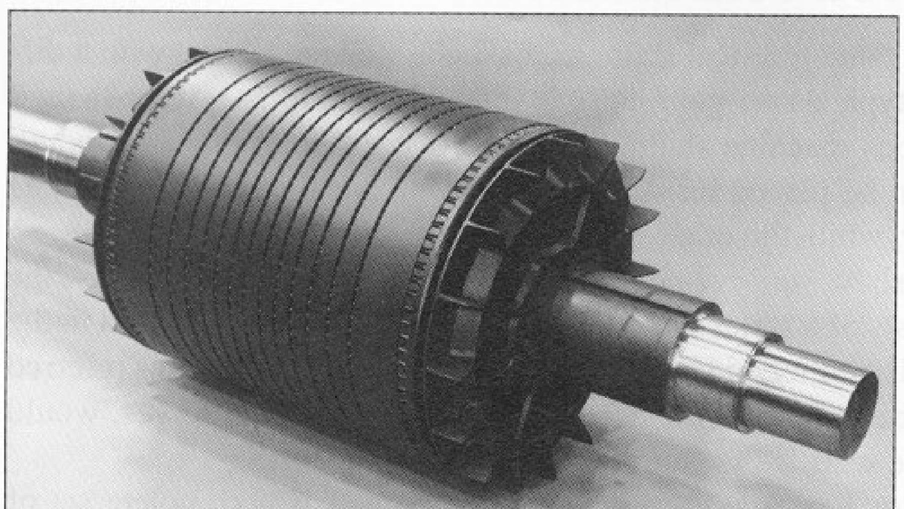
- Stator consists of a set of three-phase windings.
- Rotors are two types – squirrel-cage rotor and the other is called wound rotor.
- A squirrel-cage rotor consists of conducting bars and are shorted by end rings at either end – no external access to the rotor windings possible.
- A wound rotor consists of three-phase windings and the end points brought out to the slip-rings and are shorted through brushes – external access to the rotor windings possible.
- Squirrel cage induction motors are simpler, more economical and more rugged than the wound rotor induction motors, therefore preferred in industry.



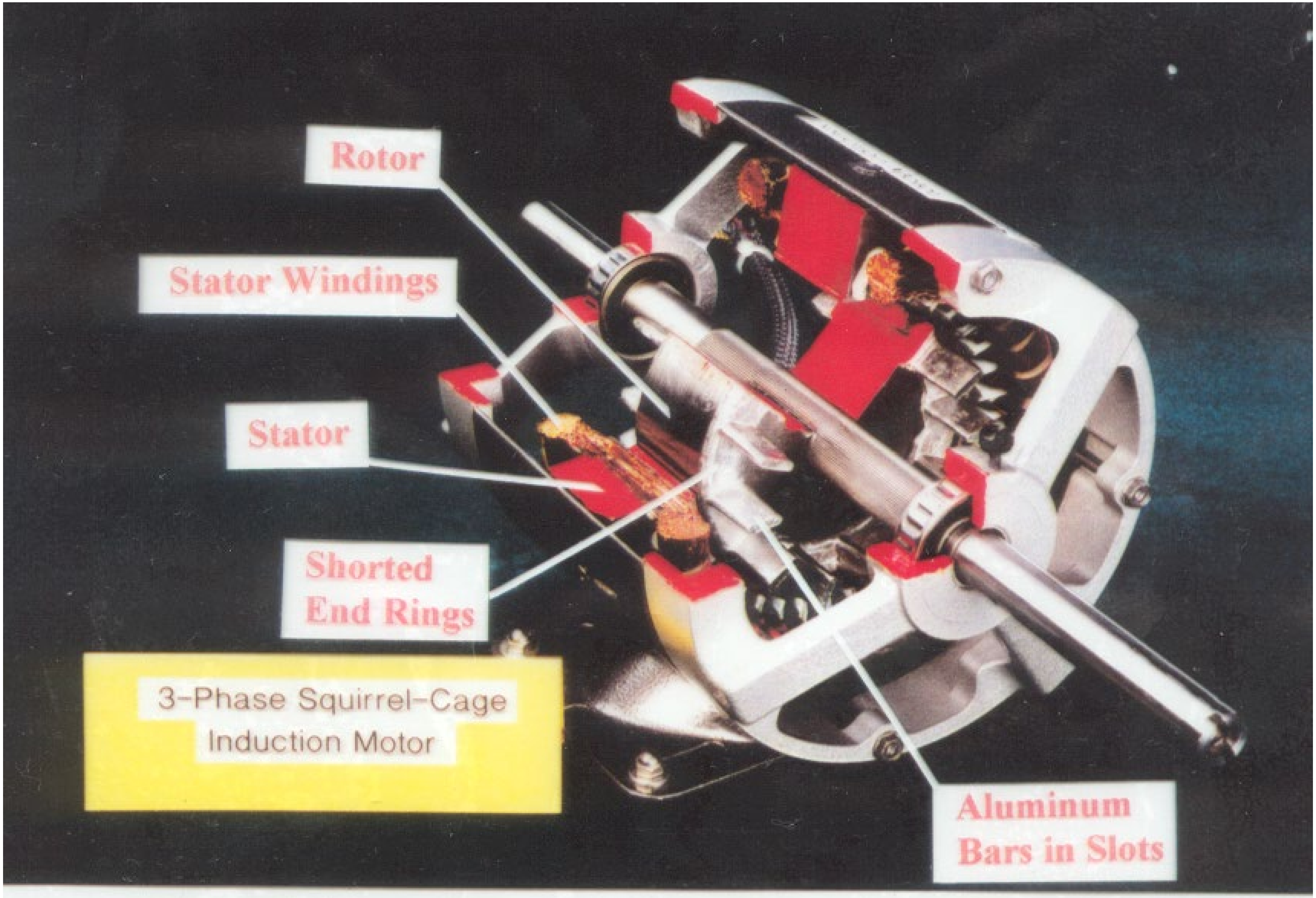
Three phase stator windings.

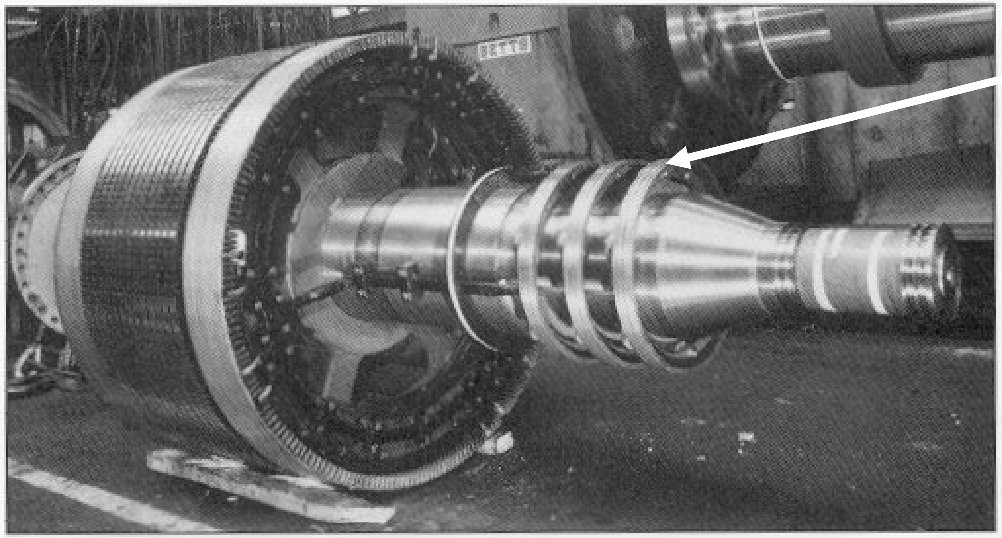


Rotor

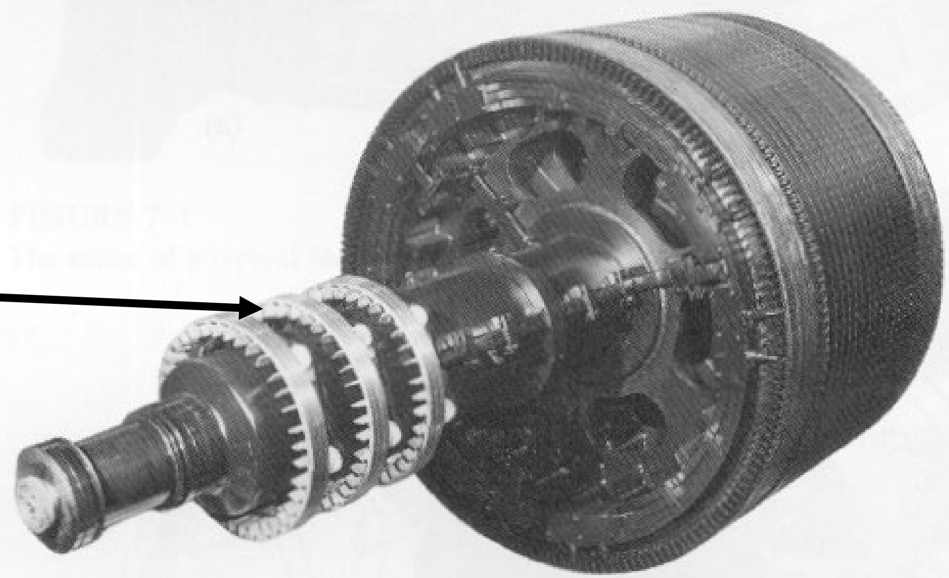


Squirrel-cage rotor


Rotor
Stator Windings
Stator
Shorted End Rings
3-Phase Squirrel-Cage Induction Motor
Aluminum Bars in Slots

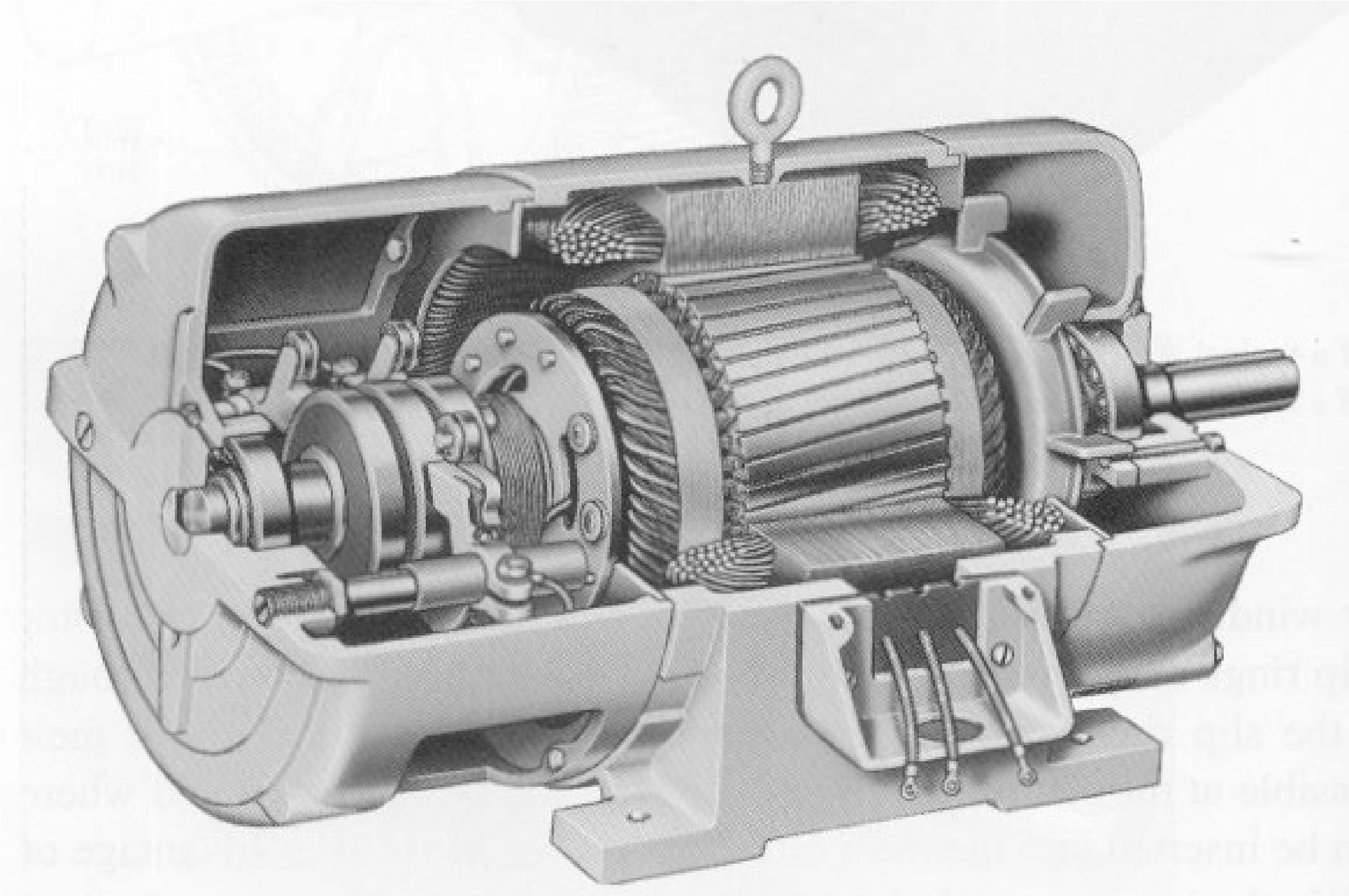


Slip rings

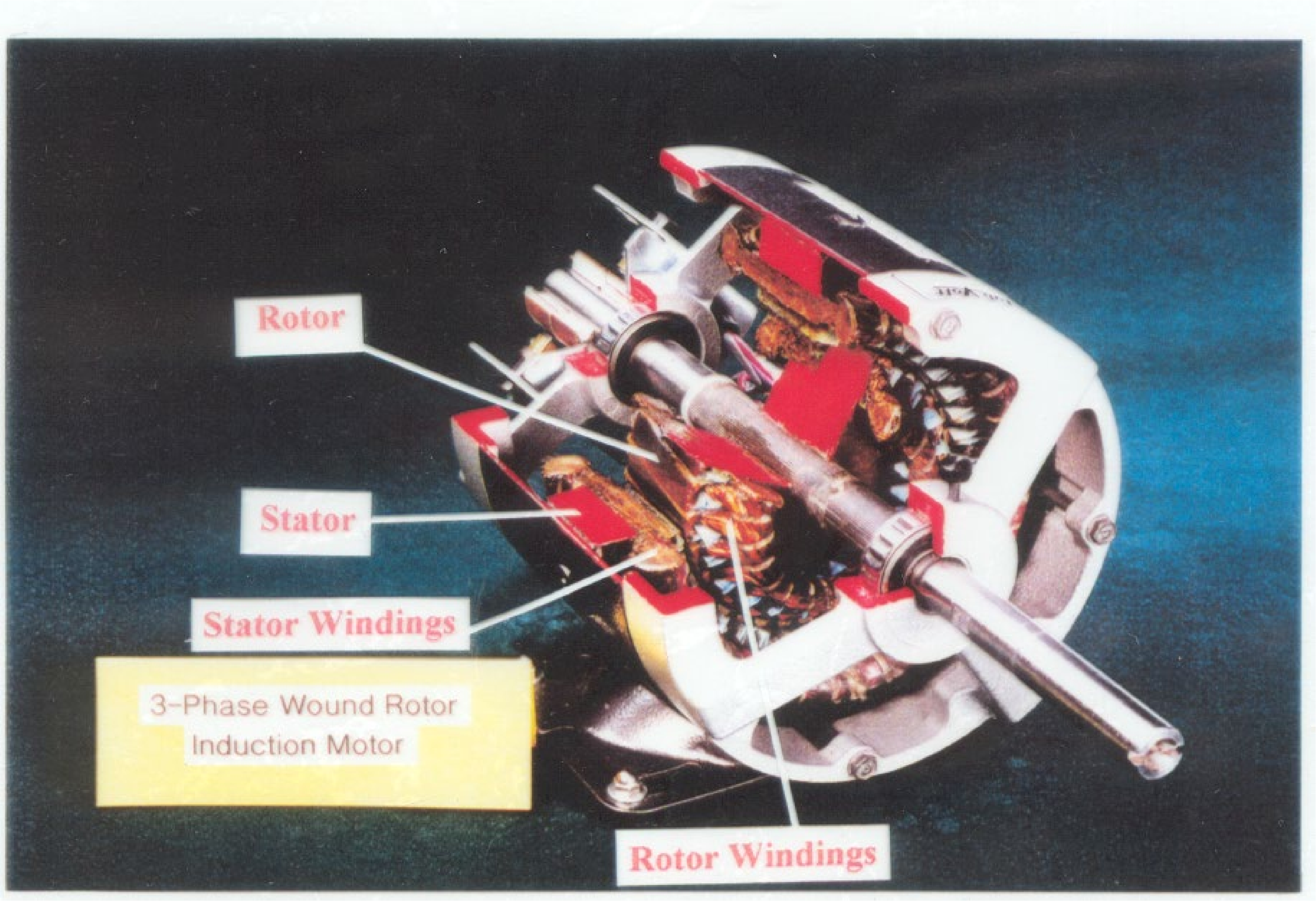


Slip rings

Wound-rotor or slip-ring rotor.



Wound-rotor induction motor.



Rotor
Stator
Stator Windings
3-Phase Wound Rotor
Induction Motor
Rotor Windings

Basic Concept of Induction Motor Operation

- The three-phase stator currents produce a rotating magnetic field represented by flux-density, $\boldsymbol{B_S}$ whose speed of rotation is:

$$n_s = \frac{120 f_e}{P} \quad rpm \qquad (6.1)$$

- When the flux-density, $\boldsymbol{B_s}$ passes over the rotor conductors it induces a voltage in it according to:

$$e_{ind} = (v \times B) \cdot l \qquad (6.2)$$

- It is the relative velocity, v between the rotor bars and the stator magnetic flux-density, $\boldsymbol{B_S}$ which produces e_{ind} on the rotor.
- This induced voltages cause the three-phase rotor currents – which in turn produce the rotating rotor magnetic flux-density, $\boldsymbol{B_R}$ – which interacts with $\boldsymbol{B_S}$ to produce the torque as given by eqn. $T_{ind} = k\,(\boldsymbol{B_s} \times \boldsymbol{B_R})$.
- Therefore, an induction motor can run and speed up to near synchronous speed, n_s but will never be able to operate at the synchronous speed.

Concept of Rotor Slip

- Two terms are commonly used to define the relative motion of rotor: (1) slip-speed and (2) slip.
- One is slip-speed: $$n_{slip} = n_s - n_m \quad (6.3)$$
- The other is slip: $$s = \frac{n_s - n_m}{n_s}(\times 100\%) = \frac{\omega_s - \omega_m}{\omega_s}(\times 100\%) \quad (6.4)$$

• Note that when the rotor rotates at synchronous speed, $n_{slip} = 0$ and $s = 0$.
• Alternatively, when the rotor is stationary $n_{slip} = n_s$ and $s = 1$.
• For normal motor speeds $0 < n_m < n_s$, we have $0 < s < 1$.
• The mechanical speed of the rotor shaft n_m can be expressed in terms of the synchronous speed, n_s as

$$n_m = (1 - s)n_s \quad or \quad \omega_m = (1 - s)\omega_s \quad (6.5)$$

The Electrical Frequency of the Rotor

- The induction motor is sometimes referred to as rotating transformer.
- Just like a transformer, the primary (stator) winding induces voltage in the secondary (rotor) windings, however, unlike a transformer, the secondary side electrical signal frequency may not be the same as the primary.
- If the rotor is blocked i.e. $n_m = 0$, the slip, $s = 1$ and $f_r = f_e$ (just like in a transformer).
- Alternately, if the rotor runs at synchronous speed i.e. for $n_m = n_s$, the slip, $s = 0$ and $f_r = 0$.

- If you assume that you were sitting on the rotor, then you would find that the rotor would be slipping behind the rotating stator magnetic field by the slip speed $(n_s - n_m)$.
- Rotor frequency will correspond to this slip speed. Therefore, from eqn. 7.1 we have

$$f_r = \frac{(n_s - n_m)P}{120} = \frac{sn_sP}{120} = s\left(\frac{n_sP}{120}\right) = sf_e \qquad (6.6)$$

- Just like in a transformer, the voltage induced in the rotor circuit of an IM is given by

$$E_R = 4.44 f_r N_{Ph} \phi\, k_{wR} = 4.44 s f_e N_{Ph} \phi\, k_{wR}$$
$$= sE_{R0} \qquad [E_{R0} = 4.44 f_e N_{Ph} \phi\, k_{wR}] \qquad (6.7)$$

where E_{R0} – induced voltage on the rotor circuit at standstill.

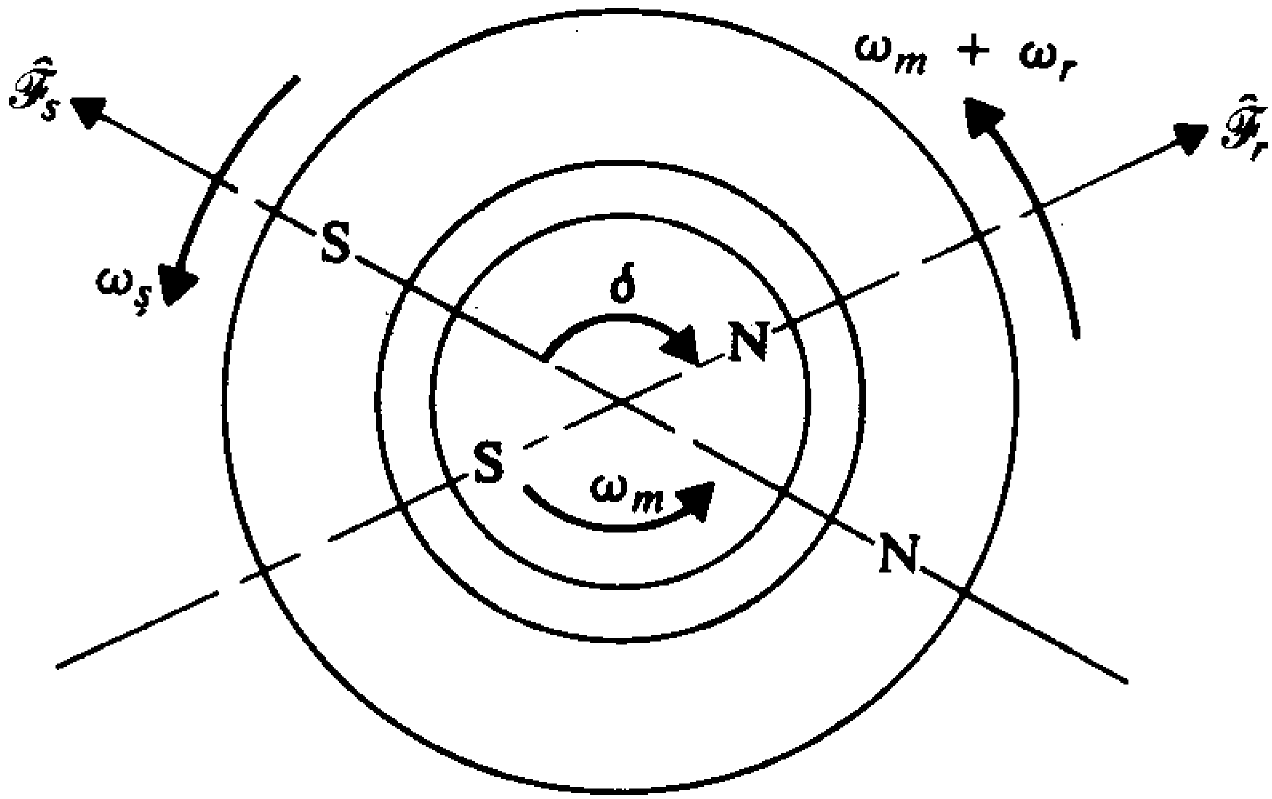
- The three-phase induced currents in the rotor would produce a rotating magnetic field represented by flux-density, $\boldsymbol{B_R}$ of its own whose speed of rotation w.r.t. rotor is given by

$$n_r = \frac{120 f_r}{P} = \frac{120 \times s f_e}{P} = s n_s \qquad (6.8)$$

- Since the rotor itself rotates at a speed of n_m w.r.t. the stationary stator, thus, the rotor magnetic field, $\boldsymbol{B_R}$ rotates at a speed of n_s w.r.t. the stationary stator.

$$(\mathrm{n_m} + \mathrm{n_r}) = (1 - \mathrm{s})\mathrm{n_s} + \mathrm{s\, n_s} = \mathrm{n_s} \qquad (6.8a)$$

- Therefore, the stator as well as the rotor magnetic fields both rotate at the same synchronous speed, n_s w.r.t stator.
- The two magnetic fields interact with each other to produce the torque. As the two magnetic fields tend to align, it can be visualized that the stator magnetic field is dragging the rotor magnetic field as the motor rotates.



$\hat{\mathscr{F}}_s$
ω_s
S
N
δ
S
ω_m
N
$\omega_m + \omega_r$
$\hat{\mathscr{F}}_r$

Example1: A 208 V, 10 hp, four-pole, 50 Hz, star(Y) – connected induction motor has a full (rated) – load slip of 5 %.

a) What is the synchronous speed of the motor?

b) What is the rotor speed of this motor at rated load?

c) What is the rotor frequency of this motor at rated load?

d) What is the speed of the rotating magnetic field?

e) What is the slip speed in rpm at rated load?

f) What is the speed of the rotor field relative to the (i) rotor structure, (ii) stator structure and (iii) stator rotating field?

g) What is the shaft torque of this motor at rated load?

Solutions :

$$(a) \qquad n_s = \frac{(120)f_e}{P} = \frac{(120)(50)}{4} = 1500rpm$$

$$(b) \qquad n_m = (1-s)n_s = (1-0.05)(1500) = 1425rpm$$

$$(c) \qquad f_r = sf_e = (0.05)(50) = 2.5Hz$$

$$(d)\ Rotating\ magnetic\ field\ always\ rotates\ at\ synchronous\ speed,\ n_s = 1500\ rpm.$$

$$(e)\ Slip\ speed = sn_s = (0.05)(1500) = 75rpm$$

(f) (i) Rotor field rotates at slip-speed of sn_s = 75 rpm w.r.t. the rotor structure itself.

(ii) Rotor field rotates at the same speed as that of the rotating magnetic field w.r.t the stator and therefore the relative speed between the two is zero.

(iii) Rotor field, B_r rotates at synchronous speed, n_s w.r.t the stator and the stator field also rotates at synchronous speed w.r.t. the stator and therefore the relative speed between the two is zero .

$$(g) \qquad T_{shaft} = \frac{P_{out}}{\omega_m} = \frac{(10)(746)}{(2\pi/60)(1425)} = 50N.m$$

References

1. Electromechanical Energy Devices and Power Systems – Yamee & Bala – Chapter 8

2. Electric Machinery Fundamentals – Chapman – 3rd Edition – Chapter 7.

3. Principles of Electric Machines and Power Electronics – P C Sen – 2nd Edition – Chapter 5.