National University of Singapore

Department of Electrical & Computer Engineering

EE4502: Electric Drives & Control

TUTORIAL - 1: Fundamentals of Electrical Drives (Solution)

Year 2022-2023

1. For operation in quadrant-I, we have at steady-state

   $$T_m\,(= 200 - 0.3N_A \text{ N.m}) = T_l\,(= 80 \text{ N.m}) \Rightarrow N_A = 400 \text{ rpm and } T_A = 80 \text{ N.m}$$

   For operation in quadrant-II, we have at steady-state

   $$T_m\,(= 200 - 0.3N_B \text{ N.m}) = T_l\,(= -100 \text{ N.m}) \Rightarrow N_B = 1000 \text{ rpm and } T_B = -100 \text{ N.m}$$

   For operation in quadrant-III, we have at steady-state

   $$T_m\,(= -200 - 0.3N_C \text{ N.m}) = T_l\,(= -100 \text{ N.m}) \Rightarrow N_C = -333.33 \text{ rpm and } T_C = -100 \text{ N.m}$$

   For operation in quadrant-IV, we have at steady-state

   $$T_m\,(= -200 - 0.3N_D \text{ N.m}) = T_l\,(= 80 \text{ N.m}) \Rightarrow N_D = -933.33 \text{ rpm and } T_D = 80 \text{ N.m}$$

   For all the equilibrium points we have,

   $$\frac{dT_l}{dN} = 0$$

   and

   $$\frac{dT_m}{dN} = -0.3$$

   Thus, the necessary condition for stability i.e.

   $$\frac{dT_l}{dN}[= 0] > \frac{dT_m}{dN} = [-0.3]$$

   is satisfied and therefore the equilibrium points are stable.

2. The load power requirement is given by

   $$P_{load} = F \times v = 1000N \times 25m/s = 25kW$$

   The load-power is supplied by a rotational motor through a speed reduction gear-box which has an efficiency of 87%. Thus, the motor output power is

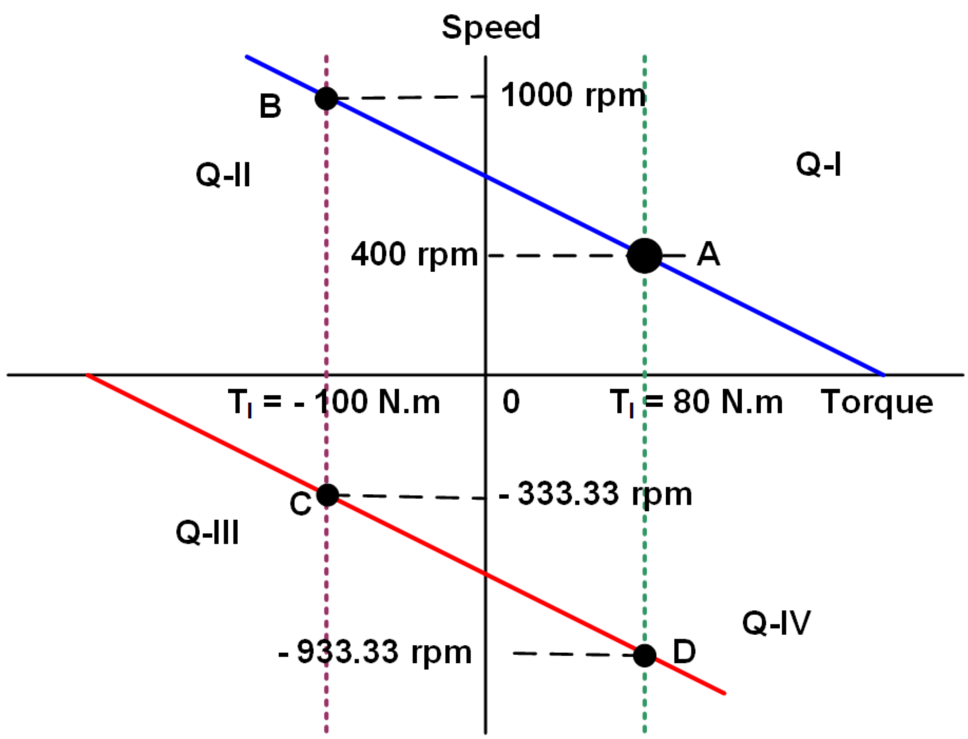

Figure 1:

$$P_{out} = \frac{P_{load}}{\eta} = \frac{25\,kW}{0.87} = 28.73\,kW$$

The linear velocity of the strip is related to the angular speed of the motor by the relationship $v = r \times \omega$. Thus, for the two different diameters of the mandrel the corresponding two speeds are:

$$\omega_{l,1} = \frac{v}{r_1} = \frac{25}{0.075\,m} = 333.33\,rad/s \Rightarrow \omega_{m1} = \frac{\omega_{l,1}}{a = 0.5} = 666.67\,rad/s$$

The corresponding motor torque at this speed is

$$T_{em,1} = \frac{P_out}{\omega_{m1}} = \frac{28.73\,kW}{666.67\,rad/s} = 43.1\,N.m$$

$$\omega_{l,2} = \frac{v}{r_2} = \frac{25}{0.125\,m} = 200.0\,rad/s \Rightarrow \omega_{m2} = \frac{\omega_{l,2}}{a = 0.5} = 400.0\,rad/s$$

(Please take note as to why we choose $a = 0.5$ rather than $a = 2$.)

The corresponding motor torque at this speed is

$$T_{em,2} = \frac{P_{out}}{\omega_{m1}} = \frac{28.73\,kW}{400.0\,rad/s} = 71.8\,N.m$$

3. The motor and winch inertias and speed are:

$$J_m = 0.5\ \ kg.m^2 \text{ and } J_w = 0.3\ \ kg.m^2, \omega_m = 104.72 \text{ rad/s}$$

The equivalent inertia referred to the motor shaft is:

$$\begin{aligned} J_{eq} &= J_m + J_w + M\left(\frac{v}{\omega_m}\right)^2 \\ &= 0.5 + 0.3 + 500\left(\frac{1.5}{104.72}\right)^2 \\ &= 0.9026 \;\; kg.m^2 \end{aligned}$$

Similarly, the equivalent torque referred to the motor shaft is:

$$\begin{aligned} T_{eq} &= T_{lo} + (F_1)\left(\frac{v_1}{\omega_m}\right) \\ &= 100 N.m + (500 \times 9.81)\left(\frac{1.5}{104.72}\right) \\ &= 170.26 \;\; \text{N.m} \end{aligned}$$

The dynamic torque required to accelerate the drive is

$$T_{dyn} = J_{eq} \times \frac{d\omega_m}{dt} = 0.9026 \times \frac{\frac{2\pi}{60} \times 1000\, rpm}{12} = 7.9\, N.m$$

Thus, the total motor torque required is

$$T_{em} = 170.26 + 7.9 = 178.1\, N.m$$

4. The parameters given are:

$R_a = 0.35\,\Omega, k\phi = 0.5\, V/(rad/s), J_m = 0.02\, kg.m^2, J_l = 0.04\, kg.m^2, T_L = 2\, N.m, I_{a-max} = 15\, A,$

$$\omega_m = 300 \text{ (rad/s)}$$

$$T_{m,max} = k\phi I_{a,max} = 0.5 \times 15 = 7.5\, N.m$$

$$J_{eq} = J_m + J_l = 0.06\, kg.m^2$$

Applying eqn. of motion we have

$$7.5\, N.m = 2.0\, N.m + 0.06\, kg.m^2 \;\times \frac{d\omega_m}{dt} \Rightarrow \frac{d\omega_m}{dt} = 91.67\, rad/s^2$$

From acceleration we have

$$\omega_m(t) = 91.67 \times t + c \Rightarrow c = 0 \Rightarrow \omega_m(t) = 91.67 \times t$$

The armature voltage is given by

$$v_a(t) = e_a(t) + i_a(t) \times R_a = 0.5 \times 91.67t + 15A \times 0.35\,\Omega = 45.84t + 5.25\, V$$

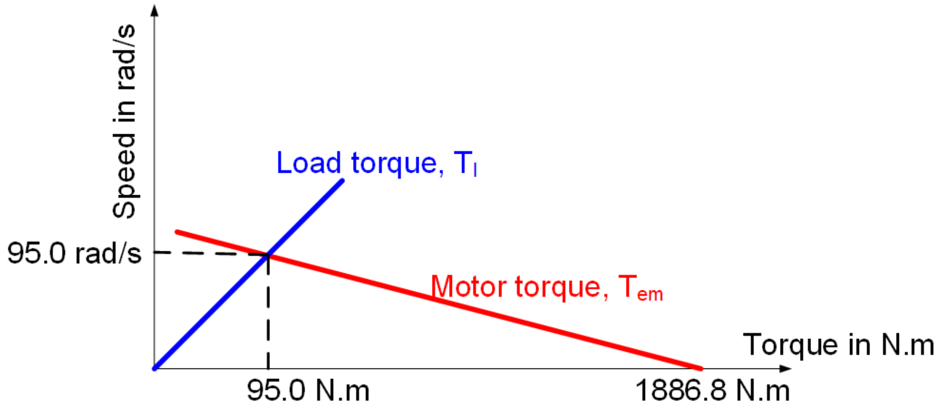

Figure 2:

5. We have at steady-state

   $T_{em}\,(= 1886.8 - 18.86 \times \omega_m\ \text{N.m}) = T_l\,(= \omega_m\ \text{N.m}) \Rightarrow \omega_m = 94.97\ \text{rad/s} = 907\ \text{rpm}$ and

   $$T_{em}\ \text{at}\ \ (\omega_m = 94.97\ \text{rad/s}) = 95.0\ \text{N.m}$$

   For steady-state equilibrium point to be stable the necessary condition is:

   $$\frac{dT_l}{d\omega_m} > \frac{dT_{em}}{d\omega_m}$$

   In our case

   $$\frac{dT_l}{d\omega_m} = 1.0 \quad \text{and} \quad \frac{dT_{em}}{d\omega_m} = -18.86$$

   For the equilibrium operating point ($\omega_m = 94.97\ rad/s$), we have

   $$\left[\frac{dT_l}{d\omega_m} = 1.0\right] > \left[\frac{dT_{em}}{d\omega_m} = -18.86\right]$$

   thus, the operating point is stable.

6. The parameters given are:

   $$v = 12\,m/s, F = 5800\,N, d_1 = 0.65\,m, d_2 = 1.24\,m, a = 7.1, \eta_{gear-box} = 0.98$$

   (a) Knowing the relationship between linear velocity of the strip and the angular velocity of the rotational motor, we have the angular velocity of the motor when the mandrel is empty as:,

   $$\omega_{l1} = \frac{v}{r_1} = \frac{12\,m/s}{0.65/2} = 36.9\,rad/s = 352.6\,rpm$$

   When the mandrel is full, we have

   $$\omega_{l2} = \frac{v}{r_2} = \frac{12\,m/s}{1.24/2} = 19.35\,rad/s = 184.8\,rpm$$

The load power is given by

$$P_{load} = F \times v = 5800\,N \times 12\,m/s = 69.6\,kW$$

Thus, the corresponding load torques when the mandrel is empty is given by

$$T_{l1} = \frac{P_{load}}{\omega_{l1}} = \frac{69.6\,kW}{36.9\,rad/s} = 1886.2\,N.m$$

Similarly, the corresponding load torque when the mandrel is full is given by

$$T_{l2} = \frac{P_{load}}{\omega_{l2}} = \frac{69.6\,kW}{19.35\,rad/s} = 3597\,N.m$$

The nature of the load is constant power type.

(b) Taking the gear-box ratio into account we have,

$$\omega_{m1} = \omega_{l1} \times a = 352.6\,rad/s \times 7.1 = 262\,rad/s = 2503.5\,rpm$$

When the mandrel is full, we have

$$\omega_{m2} = \omega_{l2} \times a = 184.8\,rad/s \times 7.1 = 137\,rad/s = 1312\,rpm$$

Thus, the corresponding motor torque when the mandrel is empty is given by

$$T_{m1} = \frac{T_{l1}}{\eta_{gear-box} \times a} = \frac{1886.2\,N.m}{0.98 \times 7.1} = 271.1\,N.m$$

Similarly, the corresponding motor torque when the mandrel is full is given by

$$T_{m2} = \frac{T_{l2}}{\eta_{gear-box} \times a} = \frac{3597\,N.m}{0.98 \times 7.1} = 517\,N.m$$

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