AC Drives

- AC machines are the economical workhorse of modern industries.
- They are rugged, reliable and less expensive.
- However, the machine dynamics and structure are much more complex as compared to that of the DC machines.
- They need external power converters capable of providing variable voltage and/or variable frequency for VSD applications.
- Typically AC drives are of two types:
  - Synchronous motors
  - Asynchronous (induction) motors
    - Squirrel-cage type (most widely used)
    - Slip-ring or wound rotor type (rarely used these days)

Induction Motor Equivalent Circuit

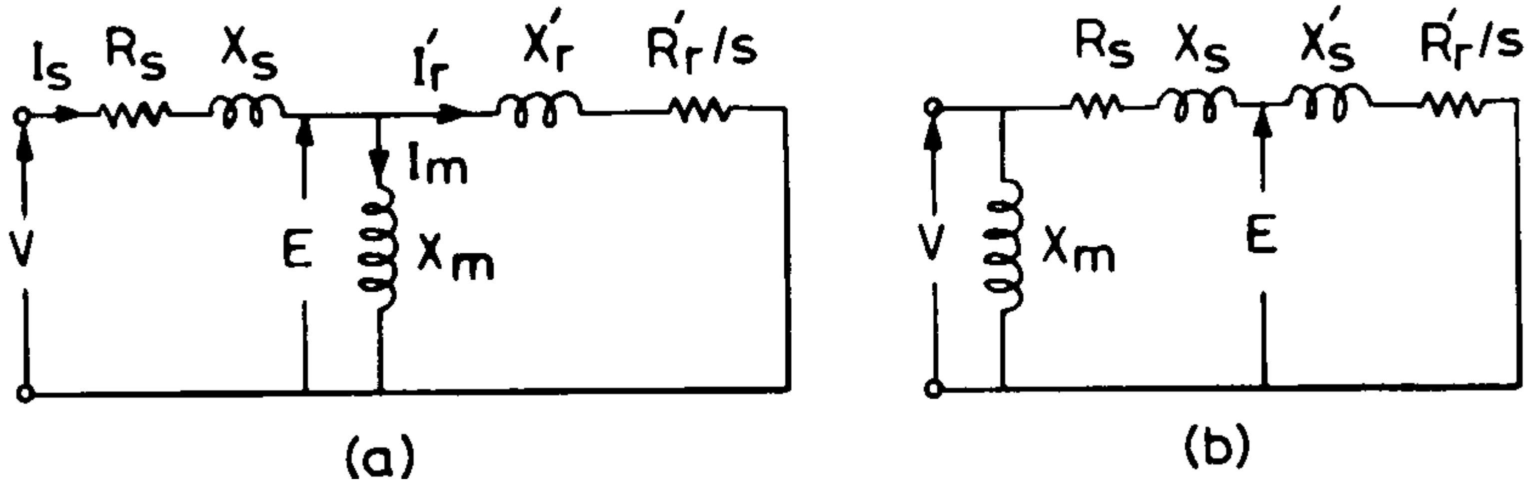
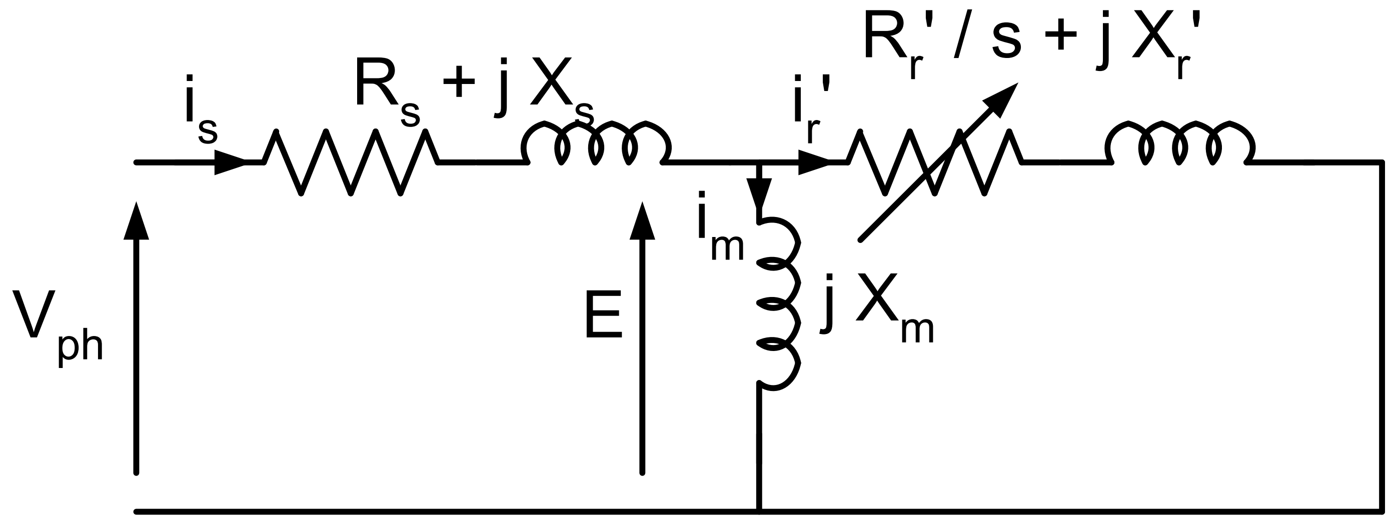


Fig.4.1: Per-phase stator referred equivalent circuit of IM.

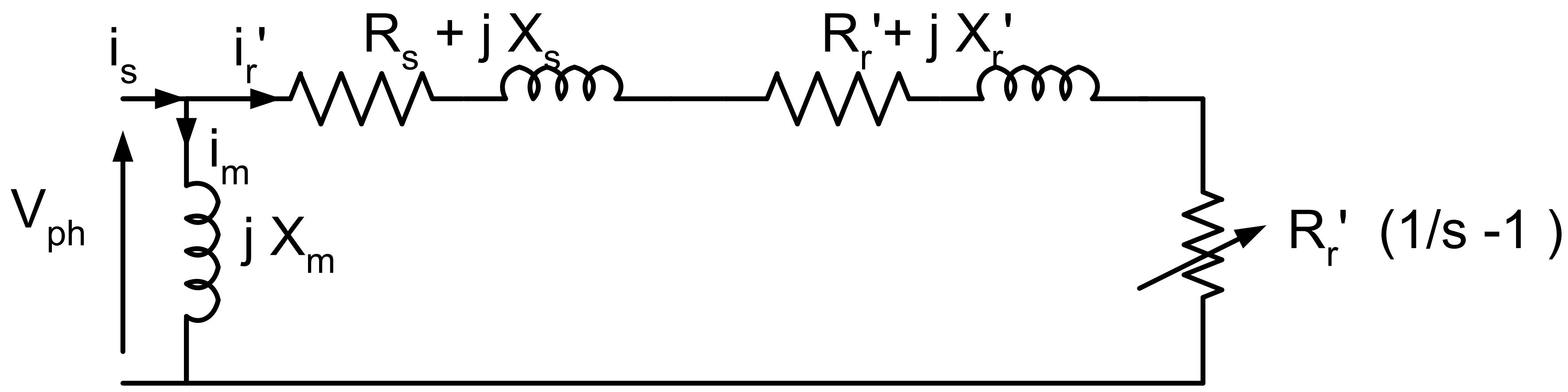
$$Slip, s = \frac{\omega_{ms} - \omega_m}{\omega_{ms}} \quad (4.1)$$

$$\omega_{ms} = \frac{120 \times f_s}{P} \times \frac{2 \times \pi}{60} (rad/s) \; (4.2)$$

ω_m - motor speed and ω_{ms} - synchronous speed



(a) per-phase **exact** equivalent circuit



(b) per-phase **approximate** equivalent circuit

$$\omega_m = (1 - s) \times \omega_{ms} \quad (4.3)$$

From the equivalent circuit, we have

$$i_r' = \frac{V_{ph}}{\sqrt{\left(R_S + \frac{R_r'}{s}\right)^2 + (X_S + X_r')^2}} \quad (4.4)$$

$$\text{Air} - \text{gap power}, P_{ag} = 3 \times (i_r')^2 \times \frac{R_r'}{s} \quad (4.5)$$

$$\text{Rotor copper}-\text{loss}, P_{cu} = 3 \times (i_r')^2 \times R_r' = s \times P_{ag} \quad (4.6)$$

$$\text{Converted power}, P_{conv.} = \left(P_{ag} - P_{cu}\right) = 3 \times (i_r')^2 \times R_r' \left(\frac{1}{s} - 1\right) \quad (4.7)$$

$$\text{Torque, } T_e = \left(\frac{P_{conv.}}{\omega_m}\right) = \frac{3}{\omega_{ms}} \times (i_r')^2 \times \frac{R_r'}{s}$$

$$= \frac{3}{\omega_{ms}} \times \left(\frac{{V_{ph}}^2}{\left(R_S + \frac{R_r'}{s}\right)^2 + (X_S + X_r')^2}\right) \times \frac{R_r'}{s} \quad (4.8)$$

$$\text{Slip at maximum torque, } S_{max} = \pm \frac{R_r'}{\sqrt{R_S^2 + (X_S + X_r')^2}} \quad (4.9)$$

$$T_{max} = \frac{3}{2\omega_{ms}} \times \left(\frac{{V_{ph}}^2}{R_S \pm \sqrt{R_S^2 + (X_S + X_r')^2}}\right) \quad (4.10)$$

Losses and the Power Flow Diagram

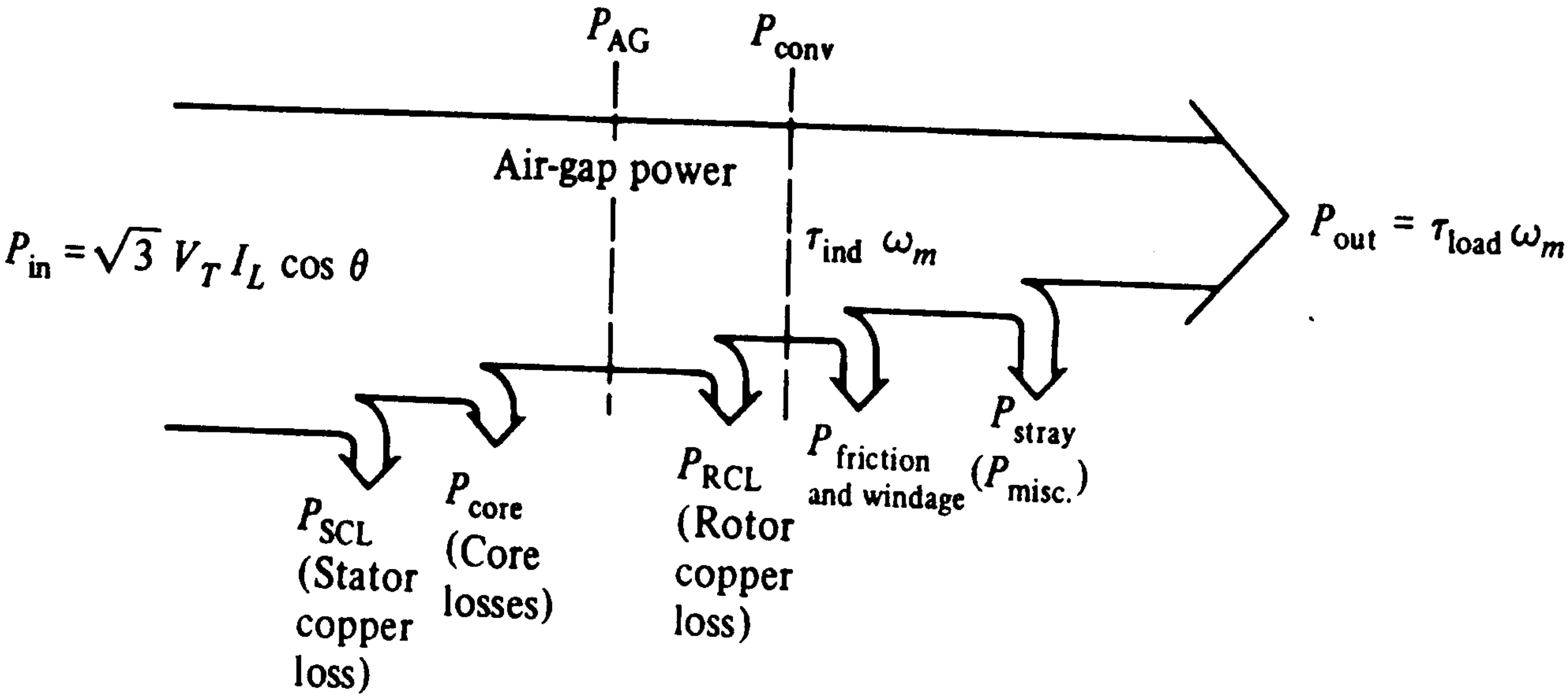
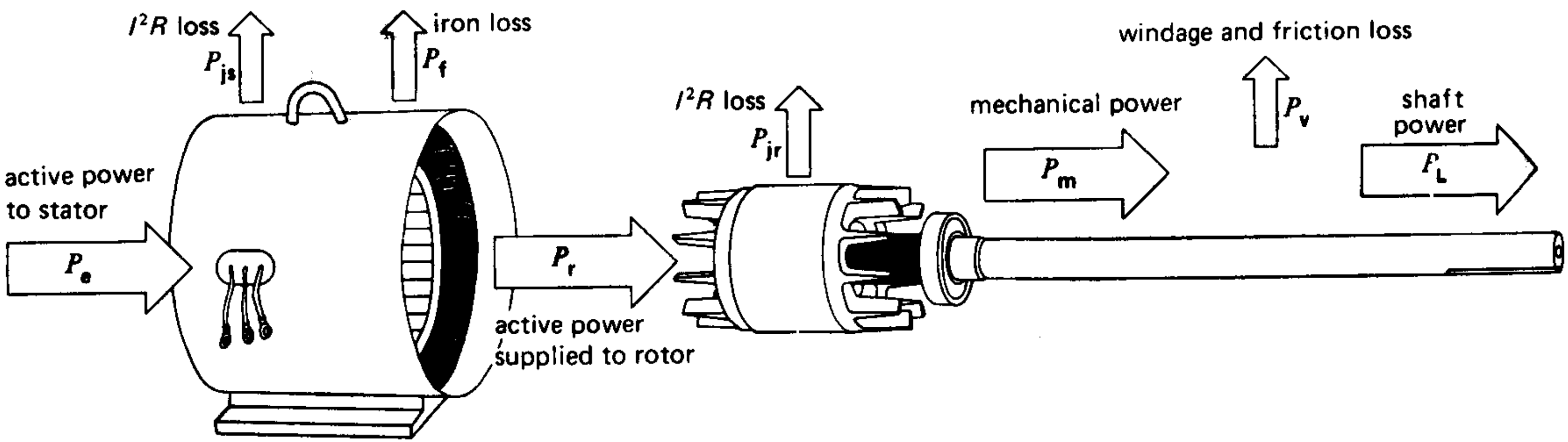


Figure : The power-flow diagram of an induction motor.



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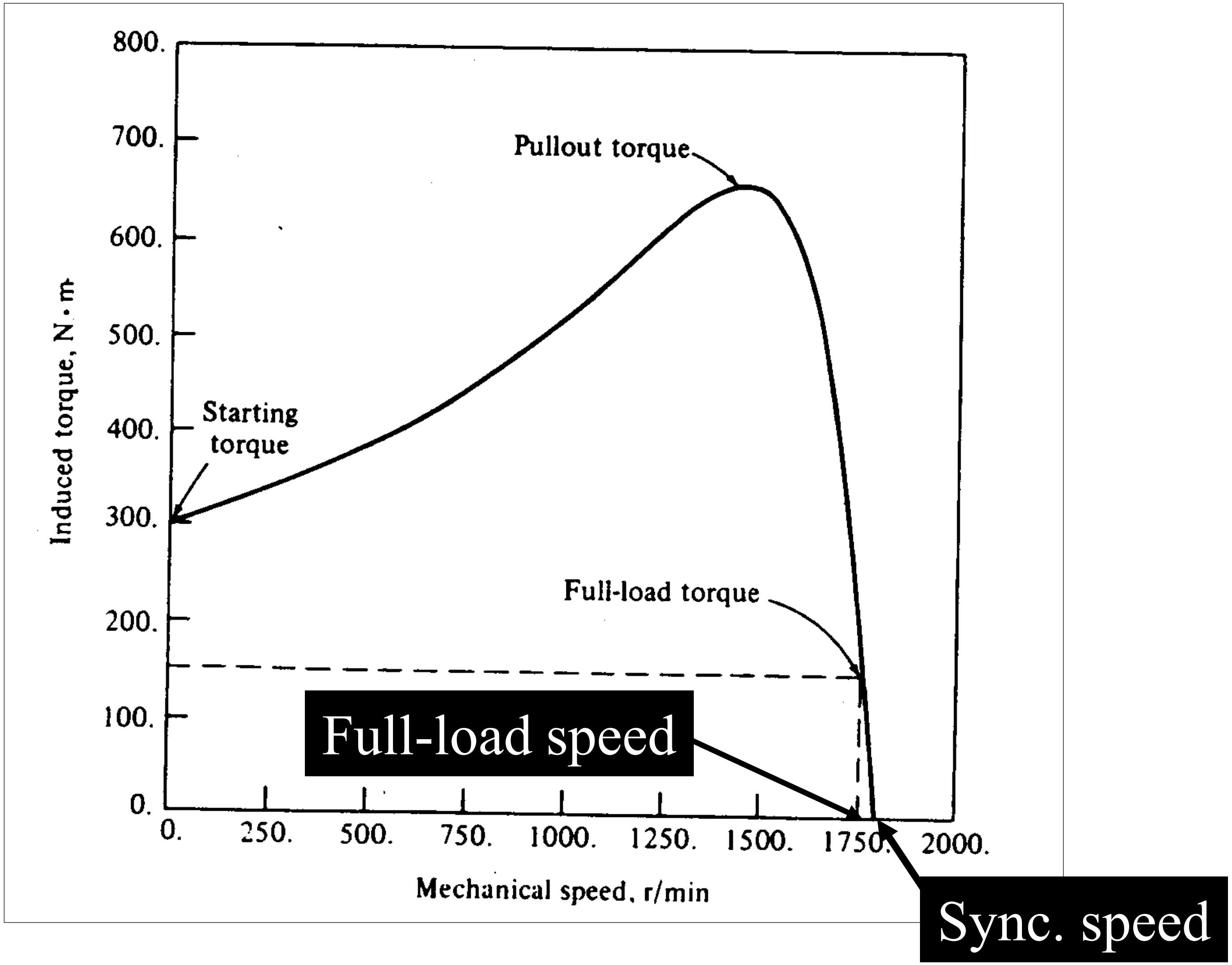
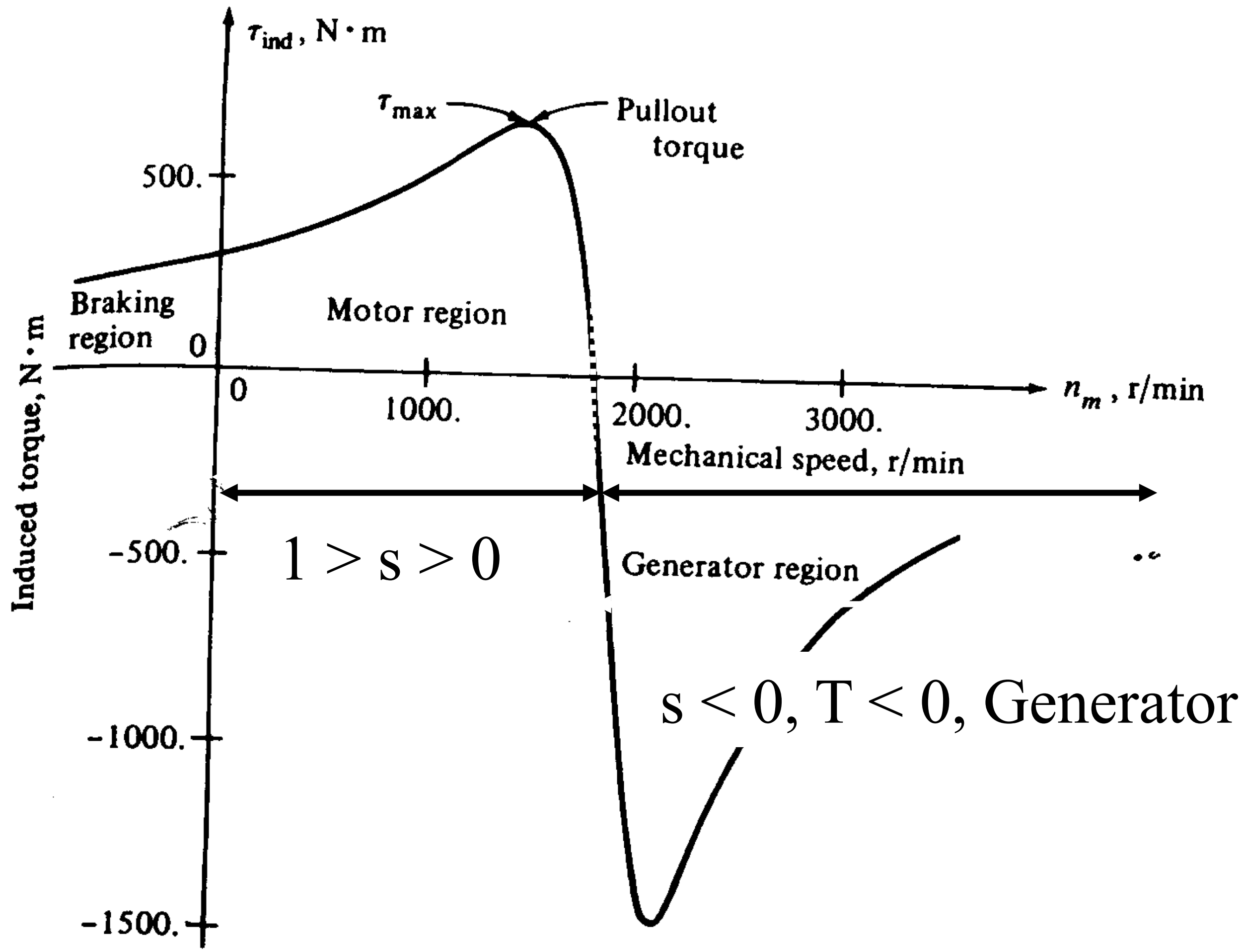


Figure : Induction motor torque-speed curve.



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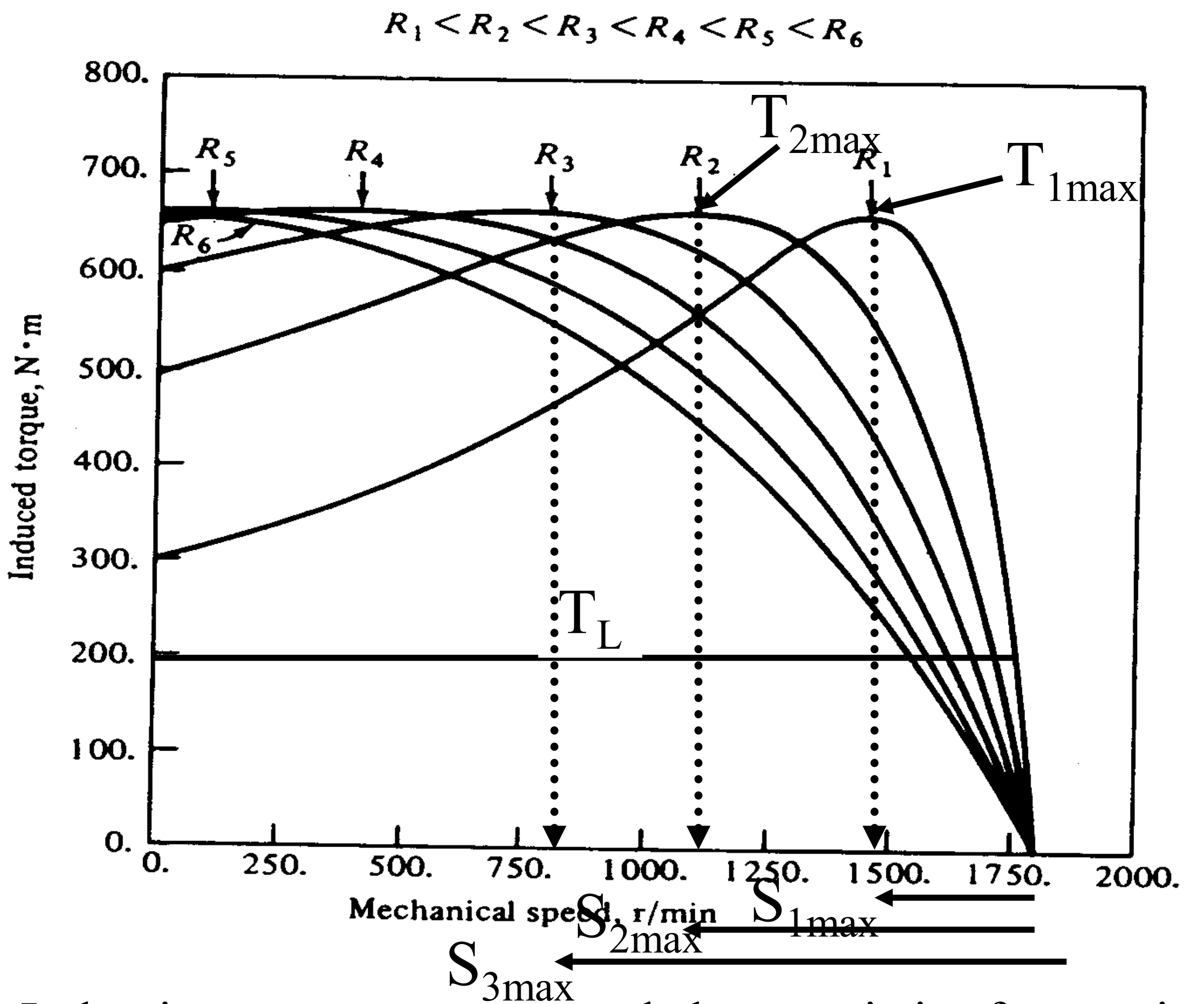


Figure: Induction motor torque-speed characteristics for varying R_2

Stator Voltage Control of IM

- Torque developed by an IM at a given slip is proportional to the square of the applied stator voltage.
- Slip at which the IM operates is dictated by the load torque-speed characteristic and the applied stator voltage.
- Thus, by varying the stator voltage magnitude in a step-less manner speed control of IM can be obtained without changing the supply frequency.
- As the stator voltage magnitude is reduced the rotor speed decreases for a given load torque. Also the maximum torque developed by the motor is also reduced with a reduction in stator voltage.
- For constant load torque the range of speed control is limited.

- Alternatively, if a high rotor-resistance IM is chosen then speed control can be achieved over a wider range.

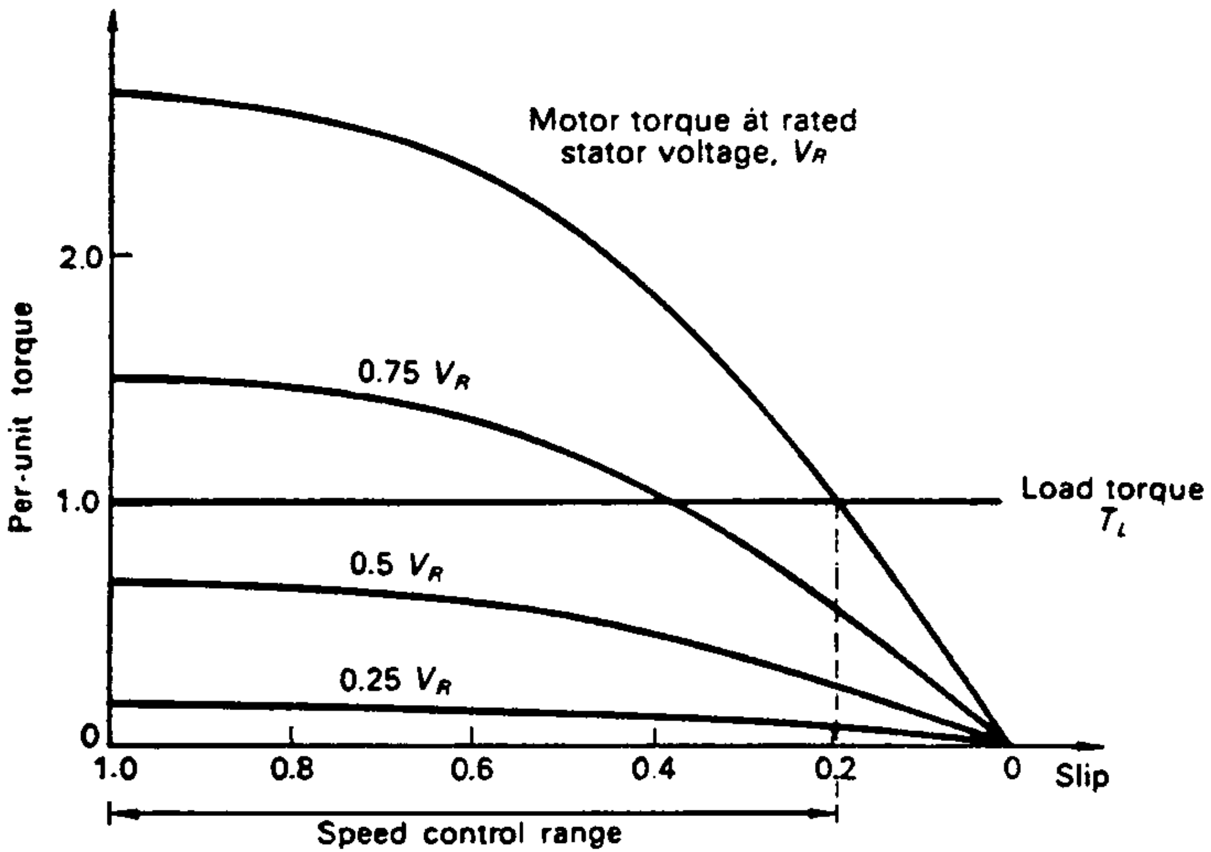


Fig.4.2: Speed control of a constant-torque load by stator voltage control of an IM.

- From eqn. (4.7) we have

$$P_{R,cu} = s \times P_{ag},\ P_{conv.} = (1 - s) \times P_{ag}$$

- From the above two eqns. it is clear that with increase in slip the rotor copper loss increases and at the same time $P_{conv.}$ decreases – making the drive system highly inefficient at low speeds.
- Low-speed performance is poor because at a given slip the stator current is proportional to stator applied voltage whereas torque is proportional to the square of the applied voltage
  - resulting a lower torque per ampere at reduced speeds
  - large current is required to produce a certain torque.

- For fan and pump type of loads the load torque varies as square of the speed and therefore load torques at starting as well as low speeds are low which can be met with stator voltage control without causing overheating.

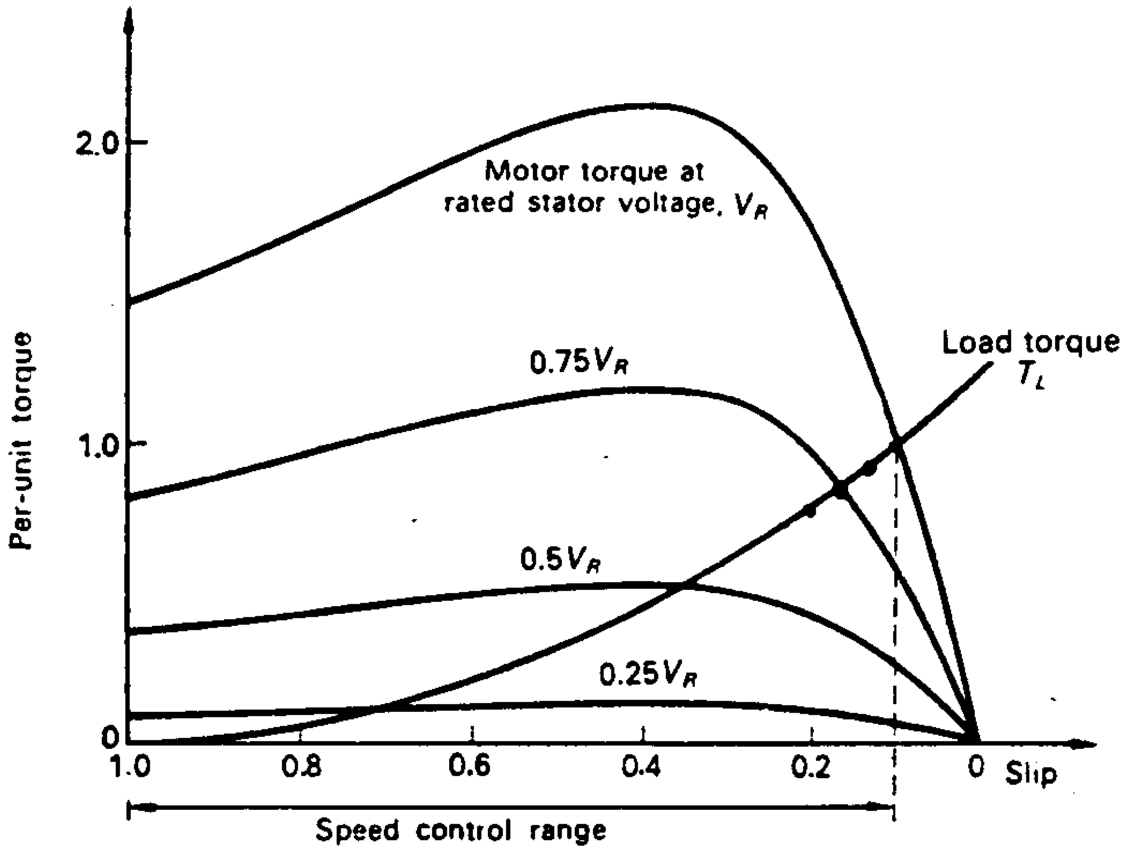
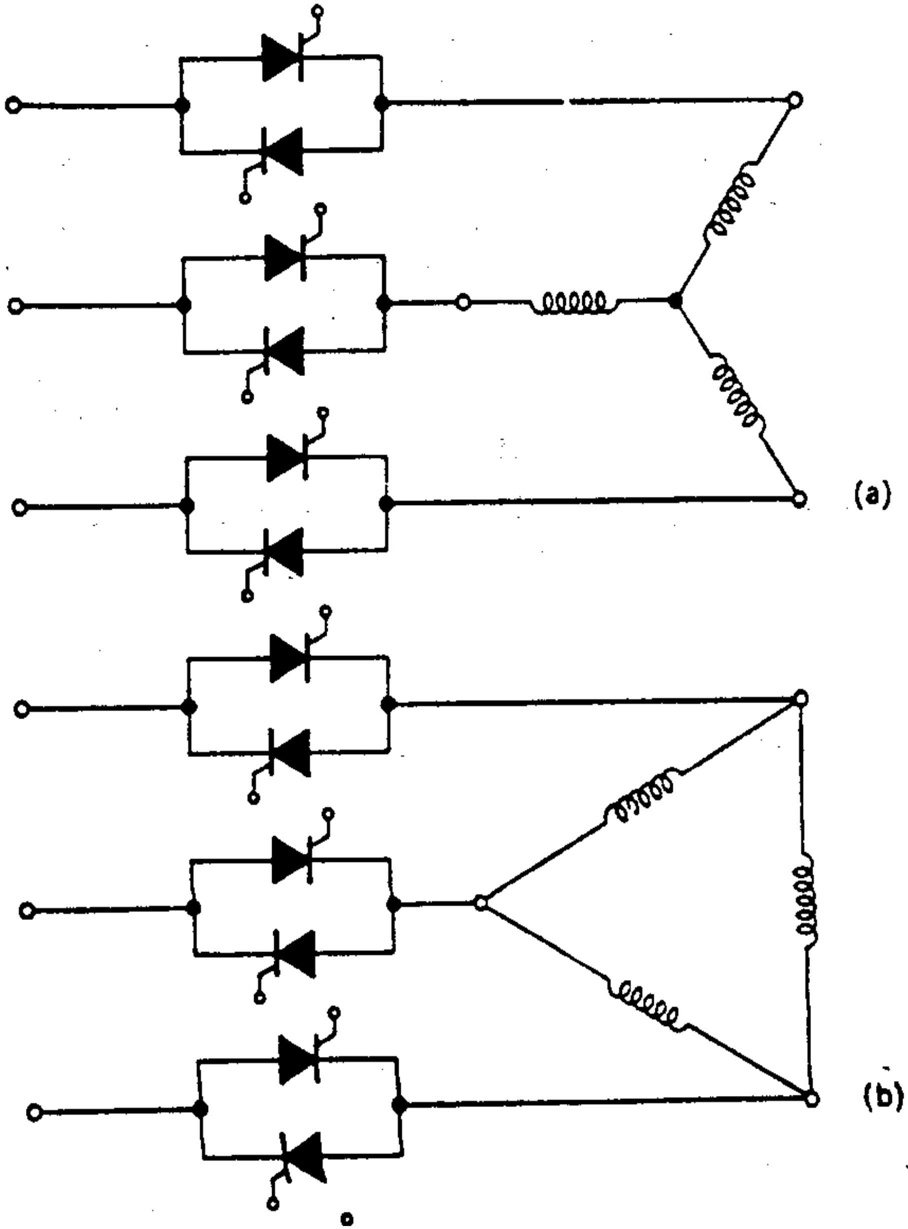


Fig.4.3: Speed control of a constant-torque load by stator voltage control of an IM for fan and pump type of loads.

- Stator voltage control can be achieved by connecting anti-parallel thyristors in each line as shown in Fig. 4.4. By varying the conduction angle the effective rms voltage can be varied from zero to full supply voltage.
- The motor is subjected to chopped sine-wave voltage and stator currents are rich in harmonics.
- Satisfactory operations can be achieved with small and medium size IMs up to 100 hp.
- Stator voltage control eliminates the complex circuitry of variable-frequency scheme and consequently cheaper to install.
- However, operating efficiency is poor, motor de-rating is necessary at low speeds.



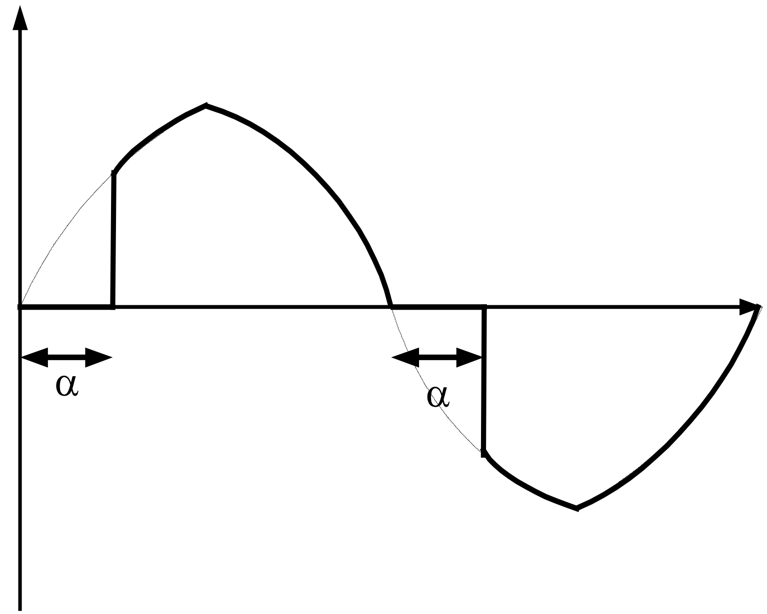


Fig.4.4: Connections for stator voltage control of a three-phase IM.

- At light loads it is not necessary to apply rated flux – reduced stator voltage can be applied reducing the flux and also reducing the core losses which results in savings in power consumption and improves power factor.

AC Voltage Controller

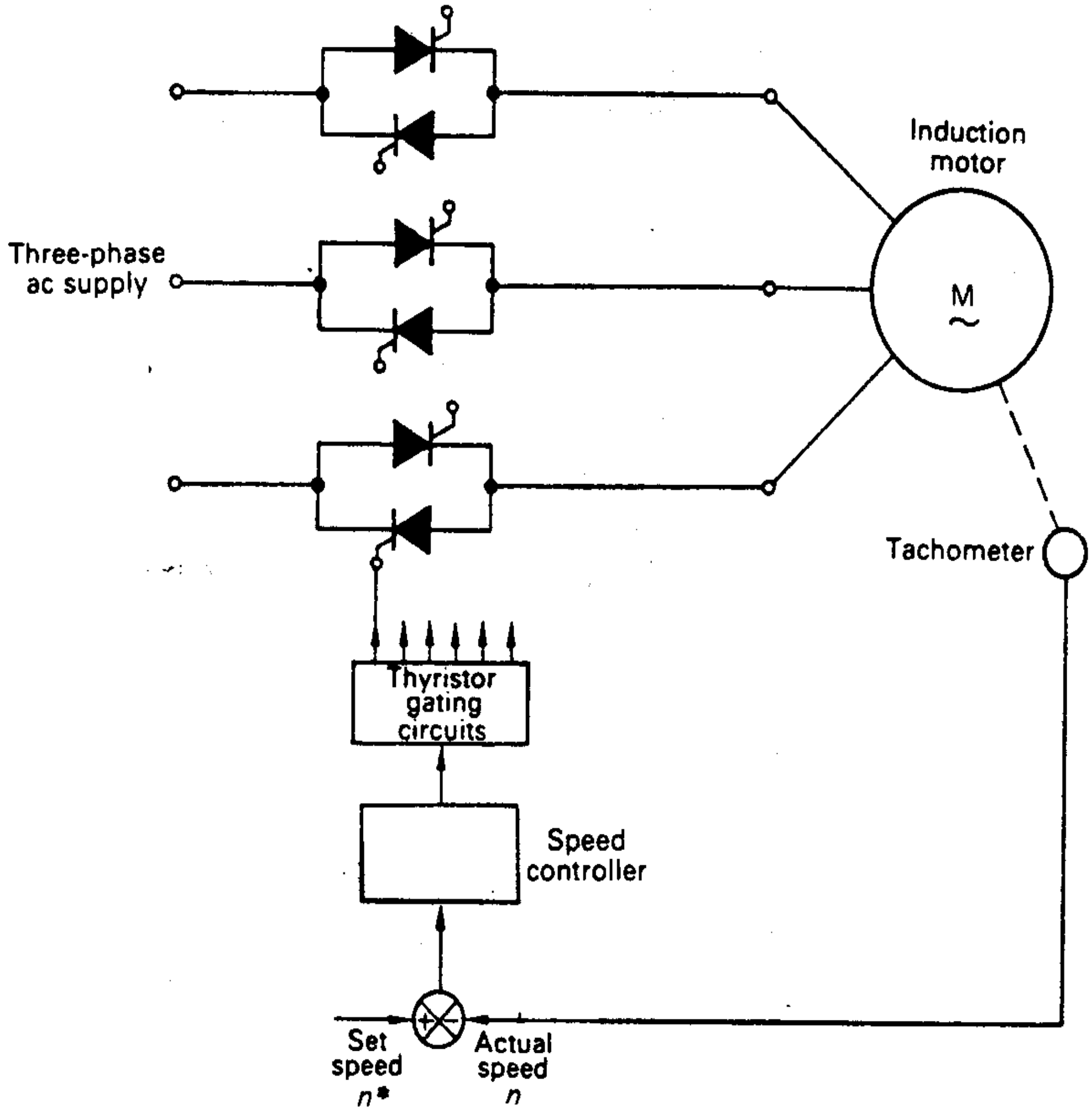


Fig.4.5: Closed-loop system for IM speed control by stator voltage control.

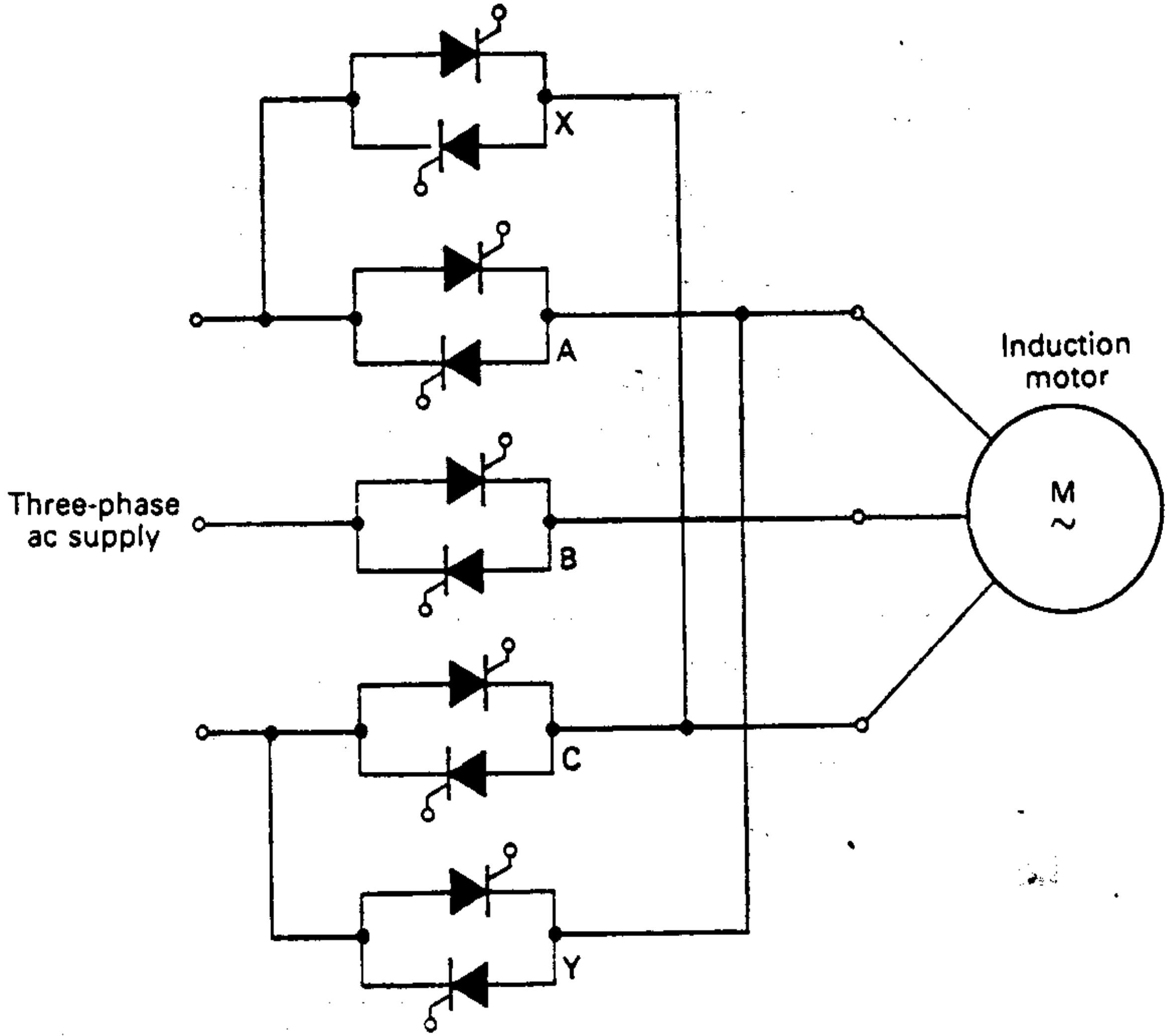


Fig.4.6: Thyristor based power converter configuration for a reversible IM drive using ac voltage controller

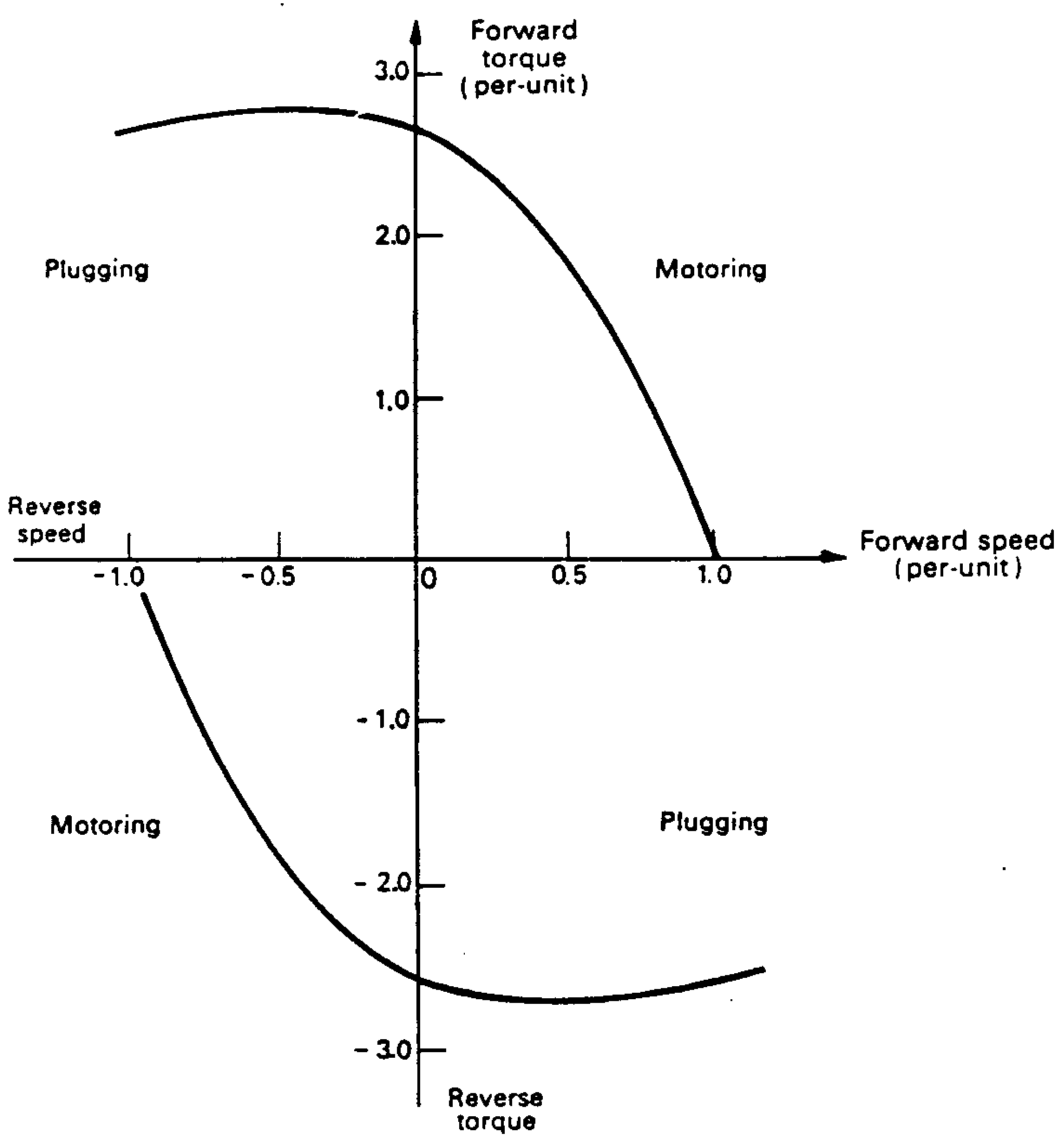


Fig.4.7: Torque-speed characteristics of a reversible induction motor drive.

AC Voltage controller for IM Drive

- Fan and pump drives requiring no braking single-quadrant IM drive can be used.

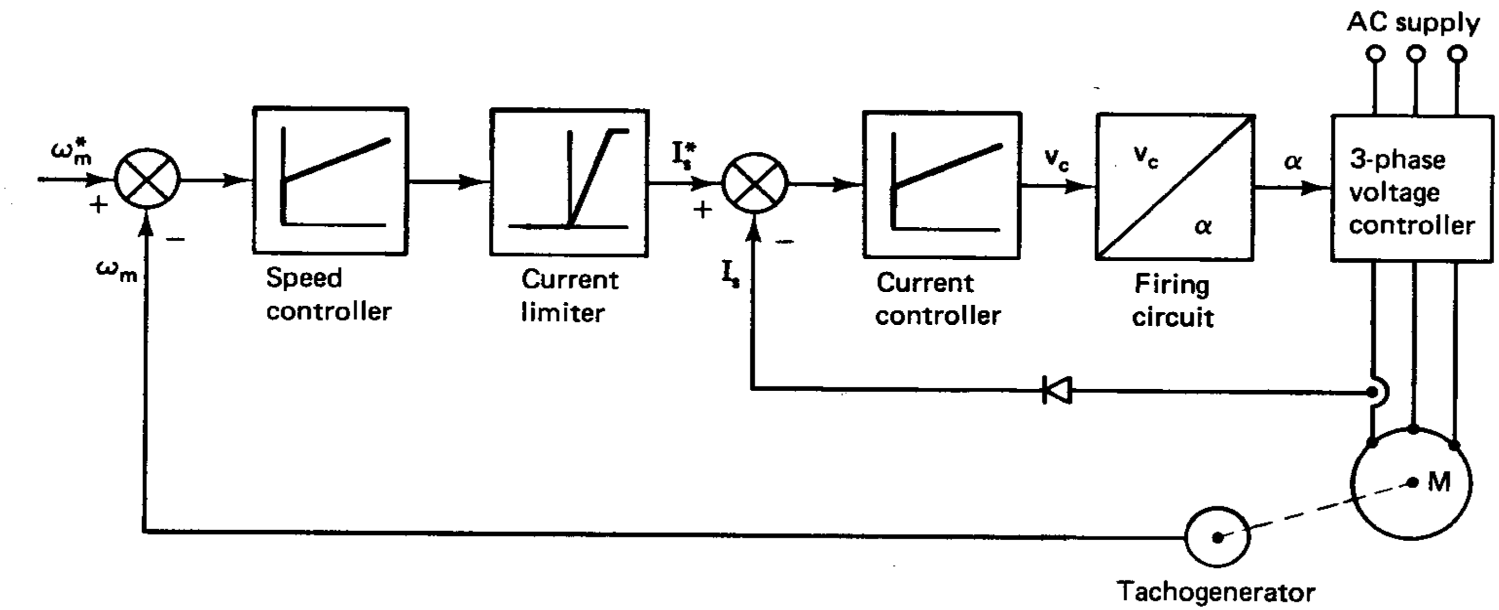


Fig.4.8: Single-quadrant closed-loop speed control of IM drive using 3-phase AC voltage controller.

Four-quadrant AC Voltage Controller for IM Drive

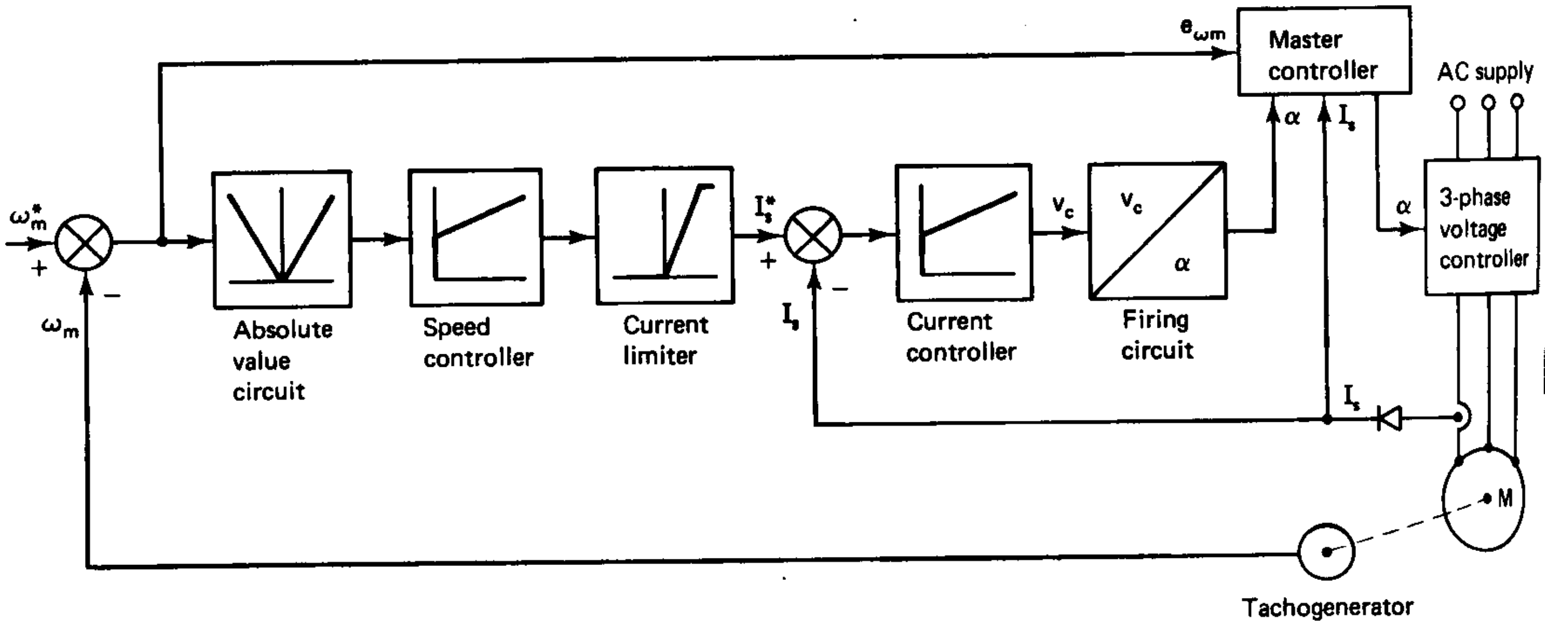


Fig.4.9: Four-quadrant closed-loop speed control of IM drive using 3-phase AC voltage controller.

- Speed error, $e_{\omega m}$ is used to detect speed reversal or reduction and accordingly directs the master controller to give gating signals to the correct thyristor groups.

Example-1: A 2.8 kW, 400 V, 50 Hz, 4-pole, 1370 rpm, delta connected squirrel-cage induction motor has the following parameters referred to the stator:

$$R_s = 2\ \Omega,\ \ R_{r'} = 5\ \Omega,\ \ X_s = X_{r'} = 5\ \Omega,\ \ X_m = 80\ \Omega$$

Motor speed is controlled by stator voltage control. When driving a fan load it runs at rated speed at rated applied voltage.

Determine

(i) Motor terminal voltage, current and torque at 1200 rpm.

(ii) Motor speed, current, and torque for the terminal voltage of 300 V.

Solution:

$$N_s = \frac{120 \times 50}{4} = 1500rpm = 157.1rad/s$$

$$S_{rated} = \frac{1500 - 1370}{1500} = 0.087$$

$$\text{At full−load } T_e = \frac{3}{157.1} \frac{400^2}{\left(2 + \frac{5}{0.087}\right)^2 + (5+5)^2} \times \frac{5}{0.087} = 48.12\text{N.m}$$

$$\text{For fan load, } T_l = k(1-s)^2$$

$$\text{At full−load steady−state, } T_e = T_l \Rightarrow 48.12\ N.m = k(1-0.087)^2$$

$$\Rightarrow k = 57.7$$

$$\text{So, at any speed, } T_l = 57.7 \times (1-s)^2$$

$$(i)\ \text{At N} = 1200\ \text{rpm},\ s = \frac{1500 - 1200}{1500} = 0.2$$

$$T_l = 57.7 \times (1 - 0.2)^2 = 36.93N.m$$

$$T_l = 36.93N.m = T_e = \frac{3}{157.1} \frac{{V_{ph}}^2 \times \frac{5}{0.2}}{\left(2 + \frac{5}{0.2}\right)^2 + (5+5)^2}$$

$$\Rightarrow V_{ph} = 253.2V = V_{L-L}$$

$$\bar{i}'_r = \frac{V_{ph}}{\left(R_S + \frac{R'_r}{S}\right) \quad + j(X_S + X'_r)} = \frac{253.2}{\left(2 + \frac{5}{0.2}\right) \quad + j(5+5)}$$

$$= 8.79\angle -20.32^0$$

$$\bar{i}_m = \frac{V_{ph}}{jX_m} = \frac{253.2}{j80} = 3.165\angle -90^0$$

$$\bar{i}_s = \bar{i}_{r\prime} + \bar{i}_m = 8.79\angle -20.32^0 + 3.165\angle -90^0 = 10.33\angle -37^0$$

$$\text{At } V_{l-l} = V_{ph} = 300V$$

$$T_e = \frac{3}{157.1} \frac{(300)^2}{\left(2+\frac{5}{s}\right)^2 + (5+5)^2} \times \frac{5}{s} = \frac{8593.25s}{104s^2 + 20s + 25}$$

$$\text{At steady-state } \; T_e = T_l$$

$$\frac{8593.25s}{104s^2 + 20s + 25} = 57.7(1-s)^2$$

$$104s^4 - 188s^3 + 89s^2 - 179s + 25 = 0 \Rightarrow s \approx 0.147 (\text{trial \& error})$$

$$T_e = T_l = 57.7(1-0.147)^2 = 41.98 \text{ N.m}$$

$$N_r = (1-s)N_s = (1-0.147) \times 1500 = 1279.5 \text{ rpm}$$

$$\bar{i_s} = \bar{i_{r'}} + \bar{i_m} = \frac{V_{ph}}{\left(R_S + \frac{R_r'}{s}\right) \quad + j(X_S + X_r')} + \frac{V_{ph}}{jX_m}$$

$$= \frac{300}{\left(2 + \frac{5}{0.147}\right) + j(10)} + \frac{300}{j80} = 9.75\angle - 37.3^0$$

$$i_{s(l-l)} = \sqrt{3} \times 9.75 = 16.88\text{A}$$

Variable Frequency Control of AC Drive

- If the stator winding impedance voltage drop is neglected then we have

$$V \approx E = 4.44 \times f_s \times \varphi \times N \times k_w \; (4.17)$$

- If the supply frequency, f_s is reduced keeping the terminal voltage, V constant then flux, ϕ will increase.
- Increase in flux would cause saturation in the core, leading to line current distortions and higher magnetising current. This results in higher stator copper as well as core losses.
- Alternatively, if the supply frequency, f_s is increased keeping the terminal voltage, V constant then flux, ϕ will decrease reducing the torque producing capability of the motor.

- Therefore, it is necessary to vary the terminal voltage, *v* with the supply frequency, f_s so that the **ratio (*v/f*) is constant,** which would maintain **constant flux** in the machine.

- Above rated frequency the **terminal voltage is held constant** due to the supply side limitation and hence flux, ϕ decreases with increase in frequency.

- We can define the variable "*a*" as:

$$a = \frac{f}{f_{rated}} \quad (4.18)$$

- For variable frequency operation, we would use the variable, *a*.

Operations below rated frequency

- If flux, ϕ is constant, it implies that the magnetising current, I_m is also constant. At rated motor operation, we have

$$|I_{m(rated)}| = \frac{E_{rated}}{X_{m(rated)}} = \frac{E_{rated}}{f_{rated}} \frac{1}{2\pi L_m} \quad (4.19)$$

- At any operating frequency *f*, the magnetising current and back-emf be I_m and E respectively.

$$I_m = \frac{E}{a \times X_{m(rated)}} = \frac{E}{a} \frac{1}{\left(2\pi\, f_{rated}\, L_m\right)} \qquad (4.20)$$

- For I_m to remain constant and equal to $I_{m(rated)}$ from eqns. 4.19 and 4.20 we have

$$E = a \times E_{rated} \ (4.21)$$

- Eqn. 4.21 tells us that flux, ϕ will remain constant if the back-emf changes in the same ratio as the frequency i.e. when (E/f) remains constant.

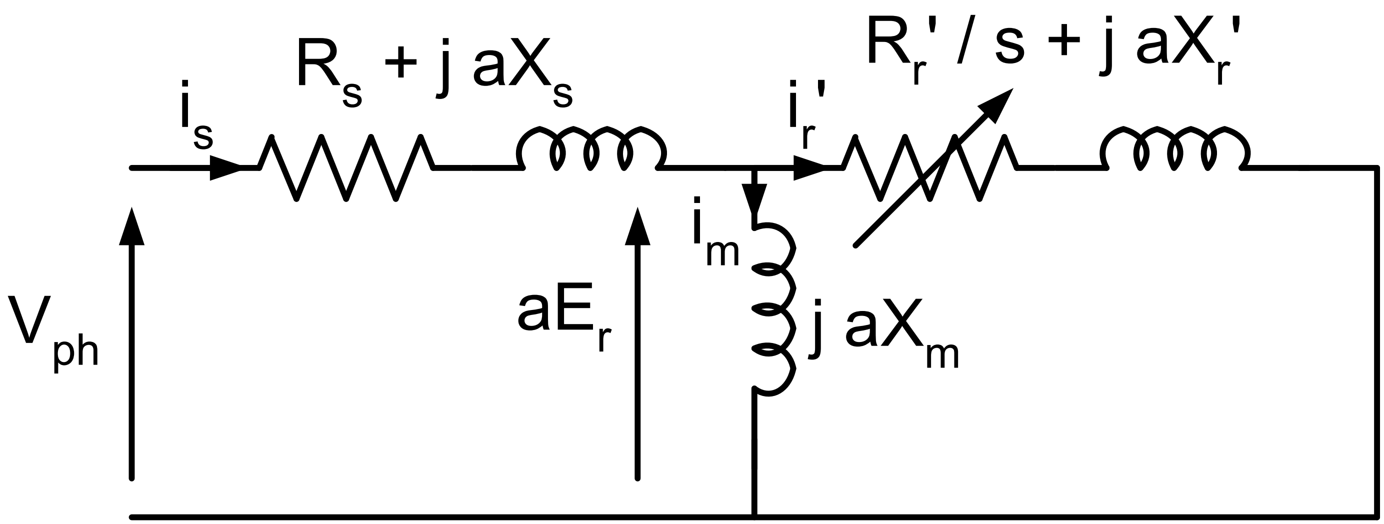


Fig.4.10: Equivalent circuit of IM at rated voltage and rated frequency.

$$I_{r'} = \frac{aE_{rated}}{\sqrt{\left(\frac{R_{r'}}{s}\right)^2 + (aX_{r'(rated)})^2}} = \frac{E_{rated}}{\sqrt{\left(\frac{R_{r'}}{as}\right)^2 + (X_{r'(rated)})^2}} \quad (4.22)$$

$$T = \frac{3}{\omega_{ms(rated)}} \left(\frac{E_{rated}^2}{\left(\frac{R_{r'}}{as}\right)^2 + X_{r'(rated)}^2} \right) \left(\frac{R_{r'}}{as} \right) \quad (4.23)$$

- For maximum torque the necessary condition (eqn. 4.9) is:

$$s_{\max} = \pm \frac{R_r'}{\sqrt{R_S^2 + \left[a \times \left(X_{S(rated)} + X_{r'(rated)}\right)\right]^2}} \approx \pm \frac{R_{r'}}{a \times X_{r'(rated)}} \quad (4.24)$$

- Approximation is that stator impedance is negligible.

- Substituting eqn. 4.24 in eqn. 4.23 we have:

$$T_{\max} = \pm \frac{3}{2\omega_{ms(rated)}} \frac{E_{rated}{}^{2}}{X_{r'(rated)}} \quad (4.25)$$

- From eqn. 4.25 it can be stated that for variable frequency control the maximum torque T_{max} remains constant for all frequencies during both motoring as well as braking, provided the flux(E/f) is maintained constant.

- If "$a \times s$" is maintained constant then according to eqn. 4.22 and 4.23 both $I_{r'}$ and T are also constants.

- The slip, s at any frequency, f can be defined as:

$$s = \frac{a\omega_{ms(rated)} - \omega_m}{a\omega_{ms(rated)}} \quad (4.26)$$

- From eqn. 4.26 we can define the slip speed, ω_{sl} as the difference between the synchronous speed at any frequency f and the rotor speed ω_m and is given as follows:`

$$\omega_{sl} = a \times \omega_{ms(rated)} - \omega_m \quad (4.27)$$

- It can be noted that the slip speed, ω_{sl} is the drop in the motor speed from its synchronous speed, when the machine is loaded and is the same for all frequencies as can be seen in Fig 4.11.
- For operations in the region $s < s_{max}$, we have $\frac{R'r}{as} \rangle\rangle Xr'$

and substituting this condition into the torque eqn. 4.23 we have:

$$T_{em} = \frac{3}{\omega_{ms(rated)}}\left(\frac{E_{rated}^2}{\left(\frac{R_{r\prime}}{as}\right)}\right) = \left[\frac{3}{\omega_{ms(rated)}}\left(\frac{E_{rated}^2}{R_{r\prime}}\right)\right] \times (as) = const. \times \omega_{sl}$$

$$= const.\,(a\omega_{ms} - \omega_m)$$

$$\omega_m = -\frac{1}{const.}T_{em} + a\omega_{ms}(y = mx + c) \quad (4.28)$$

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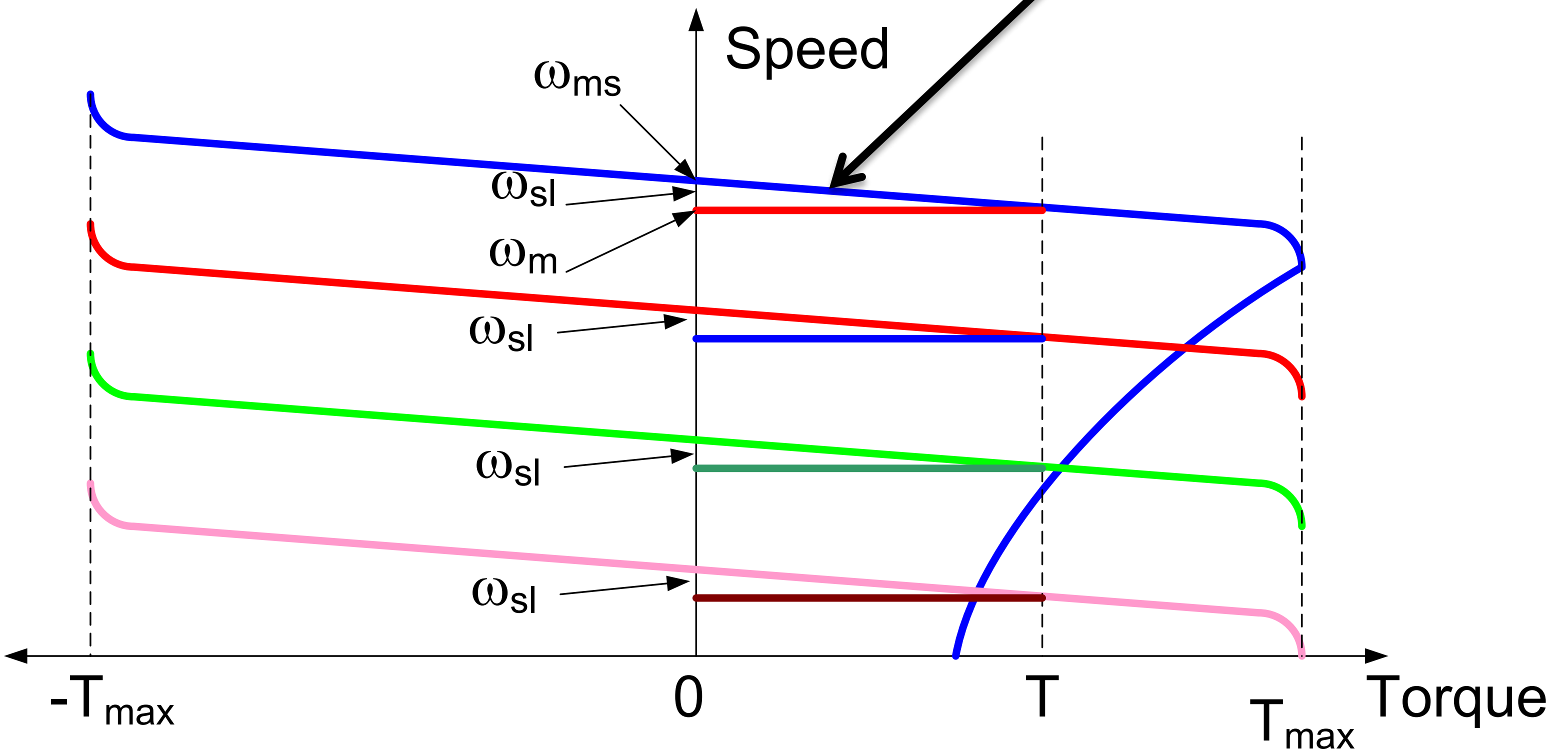


Fig.4.11: IM torque-speed characteristics with variable frequency control at constant flux .

- Eqn. 4.28 indicates that torque as a function of speed is a straight line and eqn. 4.27 indicates that torque–speed characteristics are parallel to each (torque remaining the same the slip-speed is the same at different frequencies) other as shown in Fig. 4.11.

- For operations at constant flux requires closed-loop control and measurement of flux. Direct measurement of flux is difficult and measurement of *E* requires more computational effort, hence flux is controlled indirectly by operating the machine at constant *(V/f)* ratio instead of constant *(E/f)*.

- At any operating frequency, *f* the synchronous speed, voltage and the reactance get modified to:

$$\omega_{ms} \rightarrow (a \times \omega_{ms(rated)}) V \rightarrow (a \times V_{rated}) X \rightarrow (a \times X_{rated})$$

Substituting into the torque eqn. 4.8 we have:

$$T = \frac{3}{\omega_{ms(rated)}} \times \frac{V_{rated}^2}{\left(\frac{R_s}{a} + \frac{R_{r\prime}}{as}\right)^2 + \left(X_{s(rated)} + X_{r\prime(rated)}\right)^2} \times \left(\frac{R_{r\prime}}{as}\right) \quad (4.29)$$

- The maximum torque (eqn. 4.10) is given by:

$$T_{max} = \frac{3}{2\omega_{ms(rated)}} \times \frac{V_{rated}^2}{\left(\frac{R_s}{a}\right) \pm \sqrt{\left(\frac{R_s}{a}\right)^2 + \left(X_{s(rated)} + X_{r\prime(rated)}\right)^2}} \quad (4.30)$$

- For larger *f* it follows that $(R_s/a) << (X_s + X_{r'})$ and we can have the following conclusion: $T_{max,}$ is constant both in motoring and generating mode.

$$T_{max} = \frac{3}{2\omega_{ms(rated)}} \times \frac{V_{rated}^2}{\pm\left(X_{s(rated)} + X_{r\prime(rated)}\right)} \quad (4.30(a))$$

- However, at low frequency the effect of R_s cannot be neglected and hence from eqn. 4.30 we can see that the motoring torque decreases where as generating torque increase as shown in Fig. 4.12 and more clearly in Fig. 4.13.
- With (*v/f*) control the terminal voltage and reactances are reduced by a factor of "*a*" whereas the stator resistance, R_s remains fixed.
- Thus, at low frequency the voltage drop across R_s reduces the back-emf, *E* and hence the flux (*E/f*). This reduction in flux reduces the torque producing capability.

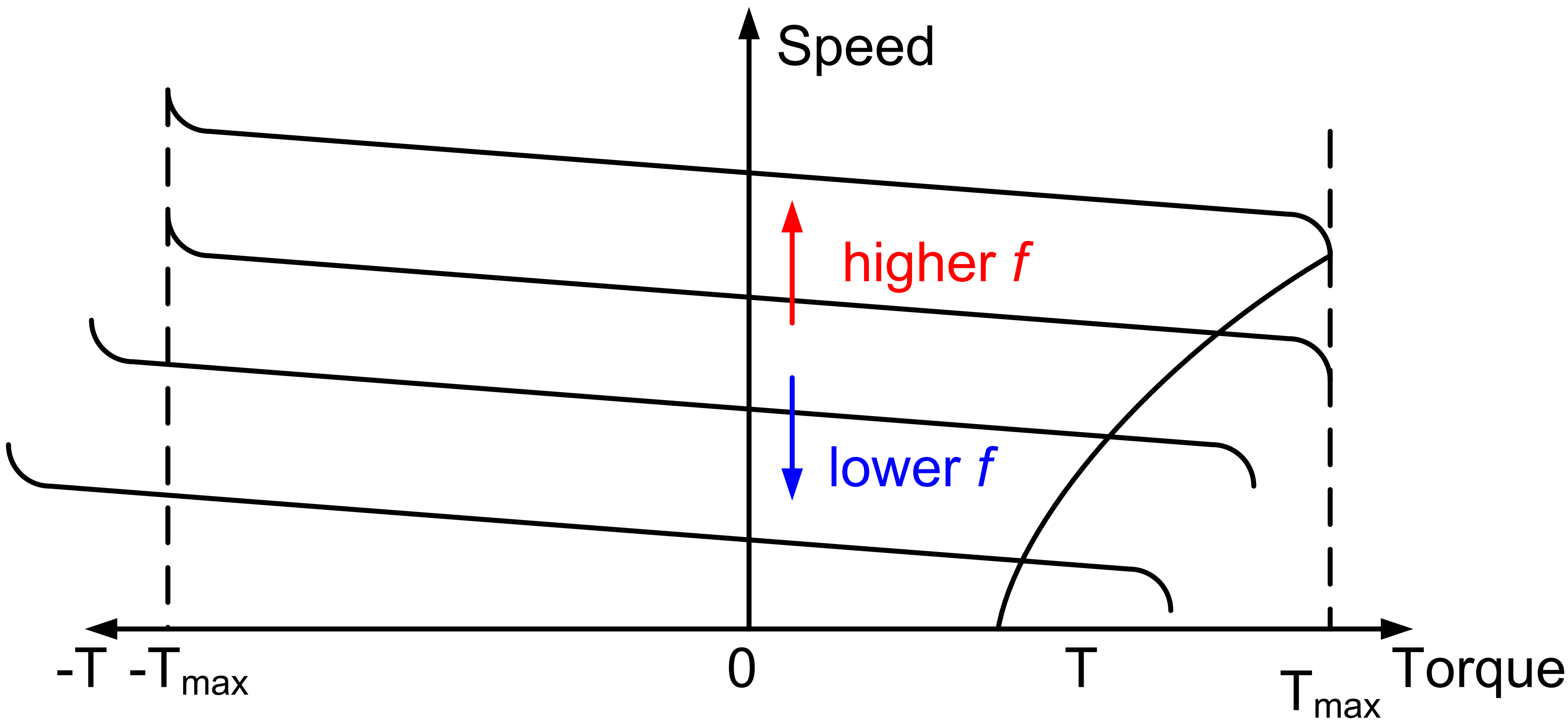


Fig.4.12: IM torque-speed characteristics with variable frequency control at constant (v/f) ratio .

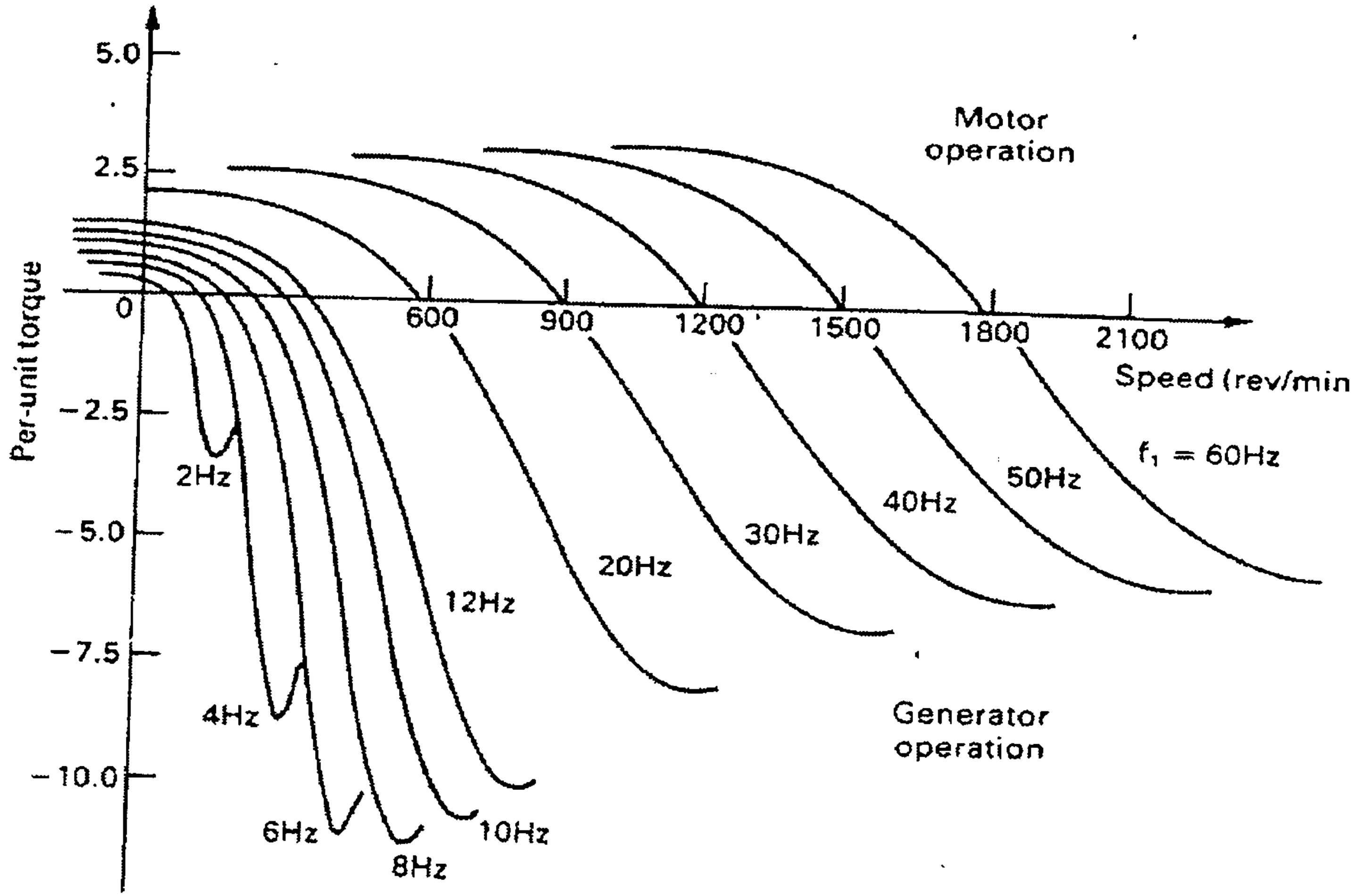


Fig.4.13: IM torque-speed characteristics with variable frequency control at constant (*v/f*) ratio.

- In order to make use of the full torque capability the (*v/f*) ratio is increased at low frequencies to compensate for the resistance drop as shown in Fig. 4.14.
- This voltage boosting maintains constant maximum torque at all frequencies during motoring region.

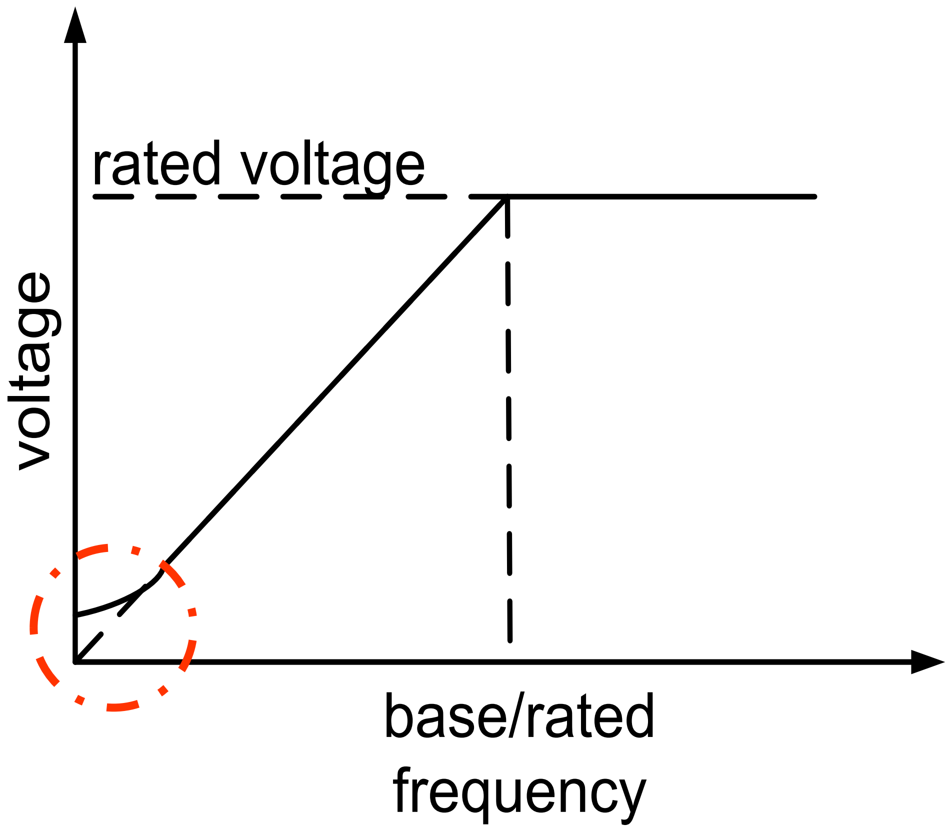


Fig.4.14:Voltage-frequency relationship in IM .

Operations above the rated frequency

- Operations above the rated frequency is carried out at constant voltage i.e. rated voltage. As the voltage is held constant flux decreases with increase in frequency and the machine operates in the **field weakening mode**.

- The maximum torque(eqn.4.10) is given by:

$$T_{max} = \frac{3}{2\omega_{ms(rated)}a}\left[\frac{V_{rated}^2}{(R_s) \pm \sqrt{(R_s)^2 + a^2\left(X_{s(rated)} + X_{r\prime(rated)}\right)^2}}\right] \quad (4.31)$$

- Since *a* > 1 from eqn. 4.31 it is clear that the maximum toque decreases with increase in frequency beyond the rated value as shown in Fig. 4.15.

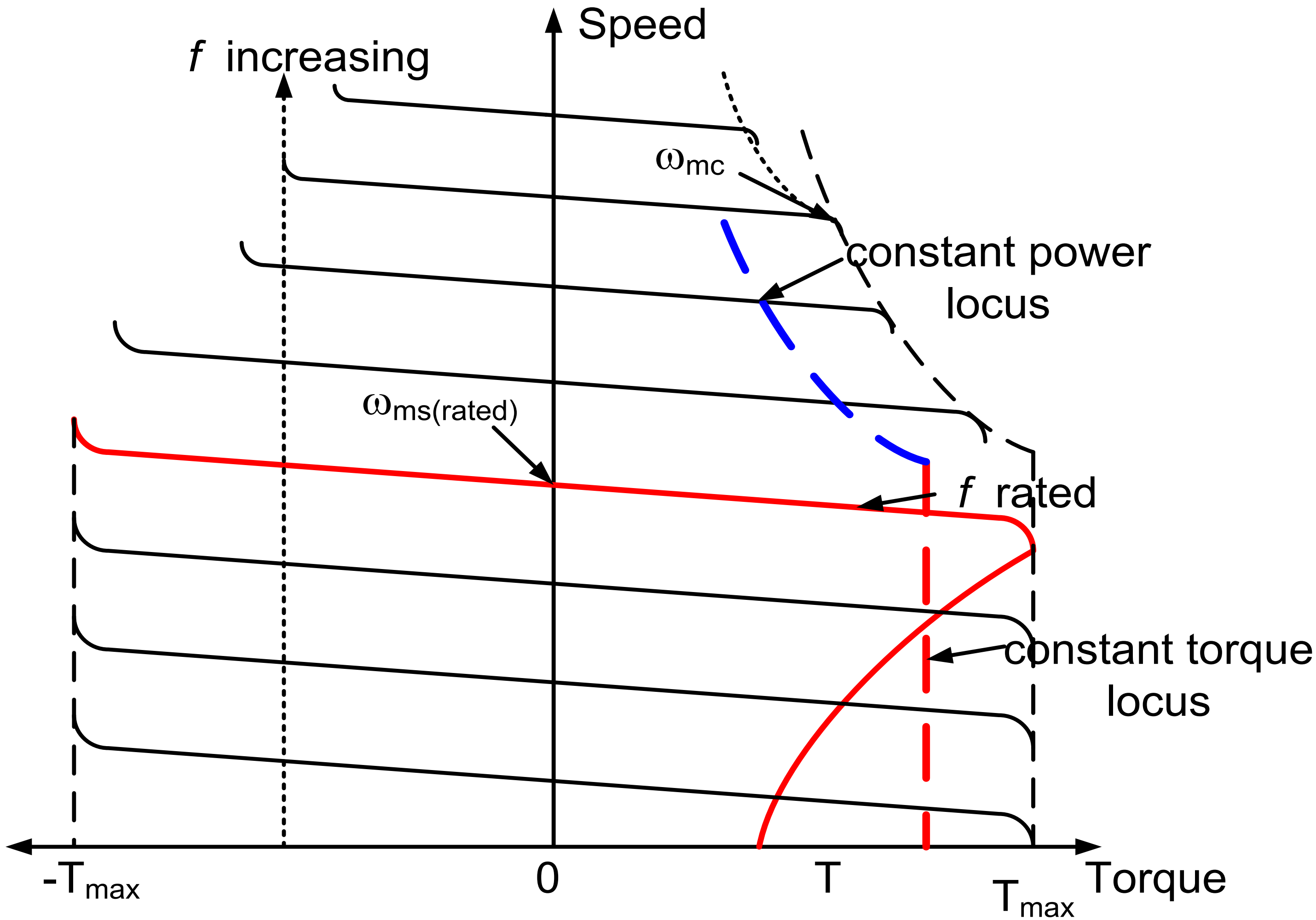


Fig. 4.15:Torque speed characteristics of IM with variable frequency control.

- For a > 1 we have:

$$I_{r'} = \frac{V_{rated}}{\sqrt{\left(R_s + \frac{R_{r'}}{s}\right)^2 + a^2\left(X_{s(rated)} + X_{r'(rated)}\right)^2}} \quad (4.32)$$

- If slip, s is small, $\frac{R'_r}{s} >> [R_s + a(X_s + X_{r'})]$

$$I_{r'} = \frac{V_{rated}}{R_{r'}/s} = \frac{V_{rated}}{R_{r'}/\left(\frac{a\omega_{ms(rated)} - \omega_m}{a\omega_{ms(rated)}}\right)} = \frac{V_{rated}}{R_{r'}/\left(\frac{\omega_{sl}}{a\omega_{ms(rated)}}\right)} = \frac{V_{rated} \times \omega_{sl}}{R_{r'} \times a\omega_{ms(rated)}} \quad (4.33)$$

- Thus, the slip-speed, ω_{sl} can be expressed as:

$$\omega_{sl} = \frac{R_{r'}\omega_{ms(rated)}}{V_{rated}}(aI_{r'}) \quad (4.34)$$

- Thus, for a constant $I_{r'}$ and hence I_s, ω_{sl} increase linearly with "*a*" as shown in Fig. 4.16.

- For smaller slip, E and $I_{r'}$ can be considered to be in phase (ref. Fig. 4.10). If we neglect the rotor copper loss then:

$$P_{mot\_conv.} = 3EI_{r'} \quad (4.35)$$

- If we further assume that the stator drop is neglected $E \approx V$ and hence $P_{mot\_conv.} = 3V_{rated}I_{r'}$ (4.36)

- For a given $I_{r'}$ and hence I_s, P_m is constant for ($2 > a > 1$). The variation of various variables such as torque, stator current, slip-speed, voltage as a function of *a* are shown in Fig. 4.16.

- Thus, variable frequency control provides a highly efficient variable speed AC drive with excellent steady-state and transient performance.

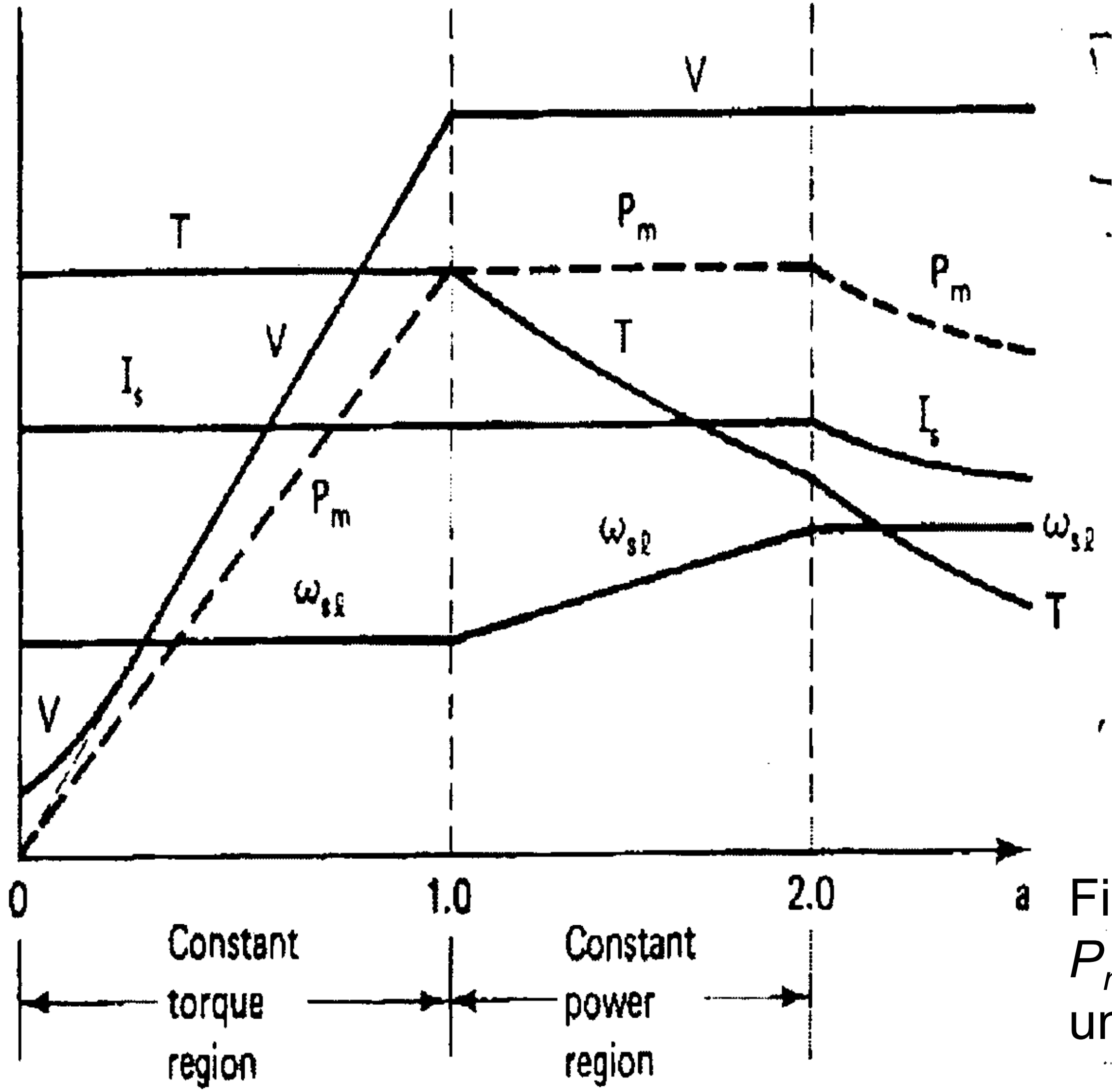


Fig.4.16: V, I_S, T, P_m, and ω_{sl} vrs per-unit frequency a.

Example 2: A star-connected squirrel-cage induction motor has the following ratings and parameters:

$$400V, 50Hz, 4-pole, 1370rpm, Rs = 2\Omega, Rr' = 3\Omega,$$
$$Xs = Xr' = 3.5\Omega$$

Motor is controlled by a voltage source inverter at constant (*v/f*) ratio. Inverter allows frequency variations from 10 Hz to 50 Hz.

a) Obtain a plot between the breakdown (maximum) torque and frequency.

b) Calculate starting torque and current of this drive as a ratio of their values when motor is started at rated voltage and frequency.

Solution:

Y-connected , 400V, f = 50 Hz, P = 4, N_r = 1370 rpm, R_s = 2Ω, $R_{r'}$ =3Ω, X_s = $X_{r'}$ = 3.5, 10 < f < 50 Hz

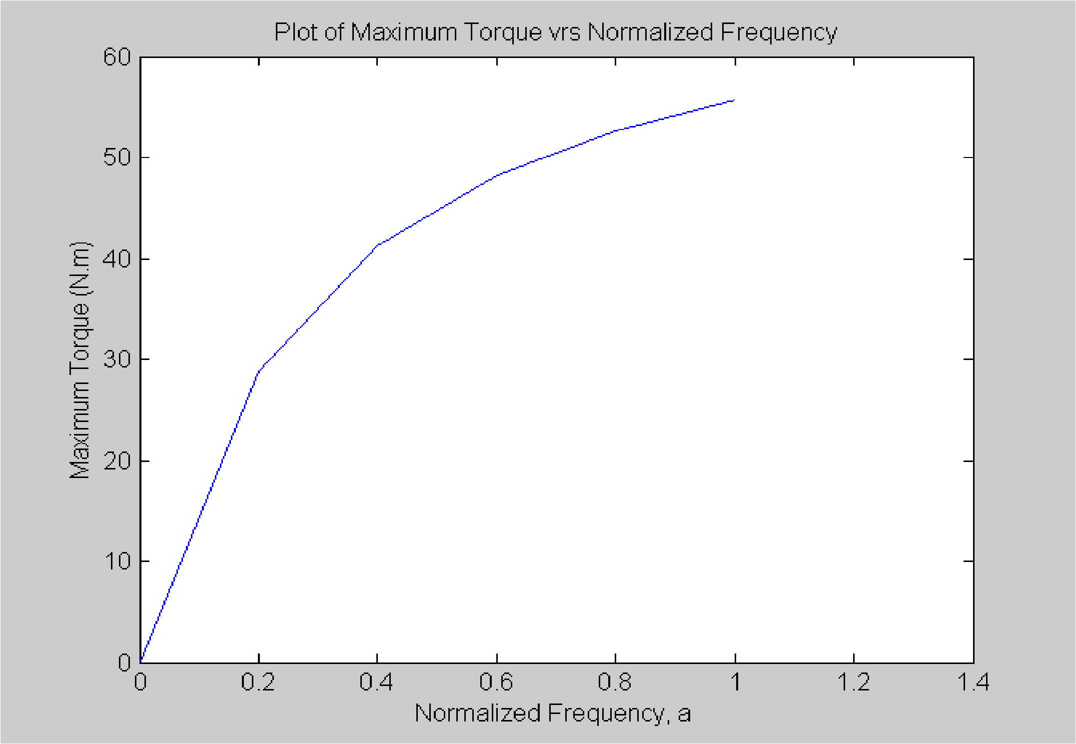
$$T_{max} = \frac{3}{2\omega_{ms}} \frac{{V_{rated}}^2}{\left[\frac{R_s}{a} \pm \sqrt{\left(\frac{R_s}{a}\right)^2 + (X_s + X_{r\prime})^2}\right]}$$

$$\omega_{ms,rated} = \frac{2\pi}{60} \times \frac{120 \times 50}{4} = 157 rad/s \qquad V_{rated} = \frac{400}{\sqrt{3}} = 230.94V$$

- Substituting in above eqn. For T_{max}, we have

$$T_{max} = \frac{509.55}{\frac{2}{a} \pm \sqrt{\left(\frac{2}{a}\right)^2 + 49}}$$

a	1	0.8	0.6	0.4	0.2
f (Hz)	50	40	30	20	10
T_{max}(motoring)	54.9	51.29	45.96	37.46	22.95



Note that as f reduces T_{max} also reduces.

(b) At starting s = 1.0.

$$T_{st} = \frac{3}{\omega_{ms}} \frac{{V_{rated}}^2}{\left(\frac{R_s + R_{r'}}{a}\right)^2 + (X_s + X_{r'})^2} \times \frac{R_{r'}}{a}$$

$At\ \ f = 50Hz, \qquad a = 1$

$$T_{st} = \frac{3}{157} \frac{(230.94)^2}{\left(\frac{2+3}{1}\right)^2 + (3.5+3.5)^2} \times \frac{3}{1}$$
$$= 41.31N.m$$

$$I_{st} = \frac{230.94}{\sqrt{(2+3)^2 + (3.5+3.5)^2}}$$
$$= 26.84A$$

- At f = 10 Hz, a = 0.2

$$T_{st} = \frac{3}{157} \frac{(230.94)^2}{\left(\frac{2+3}{0.2}\right)^2 + 7^2} \frac{3}{0.2}$$
$$= 22.68N.m$$

$$I_{st} = \frac{230.94}{\sqrt{\left(\frac{2+3}{0.2}\right)^2 + 7^2}}$$
$$= 8.89A$$

$$\frac{T_{st}(a=0.2)}{T_{st}(a=1)} = \frac{22.68}{41.31} = 0.549 \quad \frac{I_{st}(a=0.2)}{I_{st}(a=1)} = \frac{8.89}{26.84} = 0.331$$

$$\frac{T_{st}(a=1.0)}{I_{st}(a=1.0)} = \frac{41.31}{26.84} = 1.54 \quad \frac{T_{st}(a=0.2)}{I_{st}(a=0.2)} = \frac{22.68}{8.89} = 2.55$$

- **Note that the starting (torque/current ratio) 2.55 (for a = 0.2) is higher than that at rated frequency (for a = 1.0).**
- Thus, by reducing the frequency we are increasing the torque/current ratio.

Example 3: The (v/f) ratio of the variable frequency drive of Example 2 is controlled to get a <u>constant breakdown torque at all speeds</u>. Lowest frequency of the inverter is extended to 5 Hz. Calculate and plot *v* against *f* and compare with that used in Example 2.

Solution:

$$400V, f = 50Hz, P = 4, N_r = 1370rpm,$$
$$R_s = 2\Omega, R_{r\prime} = 3\Omega, X_s = X_{r\prime} = 3.5\Omega$$

$$T_{\max} = \frac{3}{2\omega_{ms}} \frac{V_{rated}^2}{\frac{R_S}{a} + \sqrt{\left(\frac{R_S}{a}\right)^2 + (X_S + X_{r\prime})^2}}$$

$$= \frac{3}{2a\omega_{ms}} \frac{a^2 V_{rated}^2}{R_S + \sqrt{R_S^2 + a^2 (X_S + X_{r\prime})^2}}$$

$$= \frac{3}{2a\omega_{ms}} \frac{Vph^2}{R_S + \sqrt{R_S^2 + a^2(X_S + X_{r\prime})^2}}$$

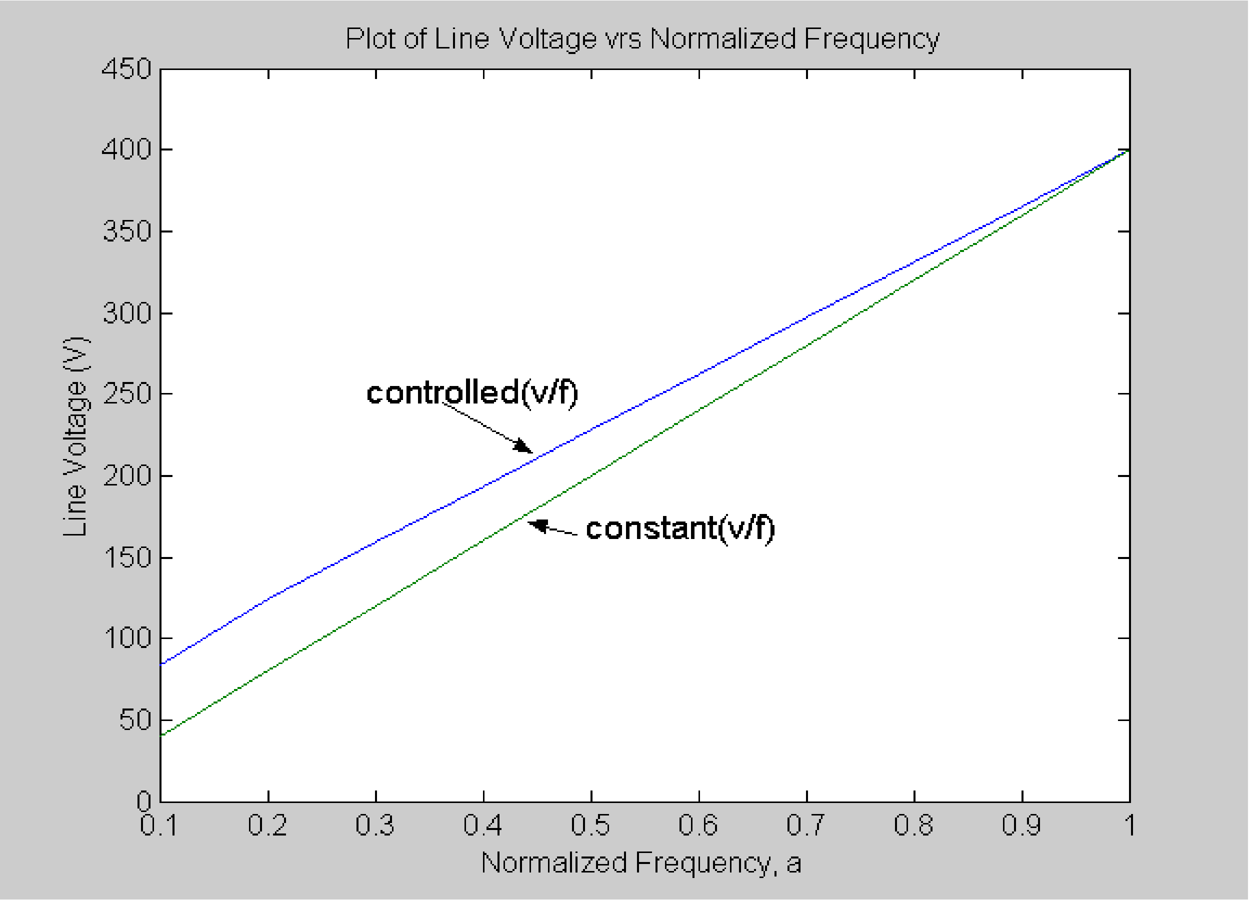
$$3V_{ph}^2 = (2a\omega_{ms}) \times T_{\max} \quad \times \left[R_s + \sqrt{R_s^2 + a^2(X_s + X_{r\prime})^2}\right]$$

$$V_L^2 = (2a\omega_{ms}) \times T_{\max} \quad \times \left[R_s + \sqrt{R_s^2 + a^2(X_s + X_{r\prime})^2}\right]$$

$$V_L = \sqrt{(2a \times 157.1) \times (54.9) \times \left[2 + \sqrt{2^2 + a^2(3.5 + 3.5)^2}\right]}$$

T_{max} = 54.9 N.m @ a = 1 i.e. f = 50 Hz

a	1	0.8	0.6	0.4	0.2	0.1
f (Hz)	50	40	30	20	10	5
V_L (Volts)	399.9	331	262.3	193.7	123.7	84.3



- Note that in order to maintain constant T_{max} at all frequency it is necessary to control (v/f) ratio rather than maintaining it constant.

Example 4: Calculate **approximate values** of the following for inverter-fed induction motor drive of Example 2.

a) Speed for a frequency of 30 Hz and 80% of full-load torque.

b) Frequency for a speed of 1000 rpm and full-load torque.

c) Torque for a frequency of 40 Hz and speed of 1100 rpm.

At rated condition: $N_s = \frac{120 \times 50}{4} = 1500rpm, N_{r(rated)} = 1370rpm$

$$N_{sl(rated)} = N_s - N_r = 1500 - 1370 = 130rpm$$

(i) For $f_s = 30Hz, N_s = \frac{120 \times 30}{4} = 900rpm, N_r = ??$

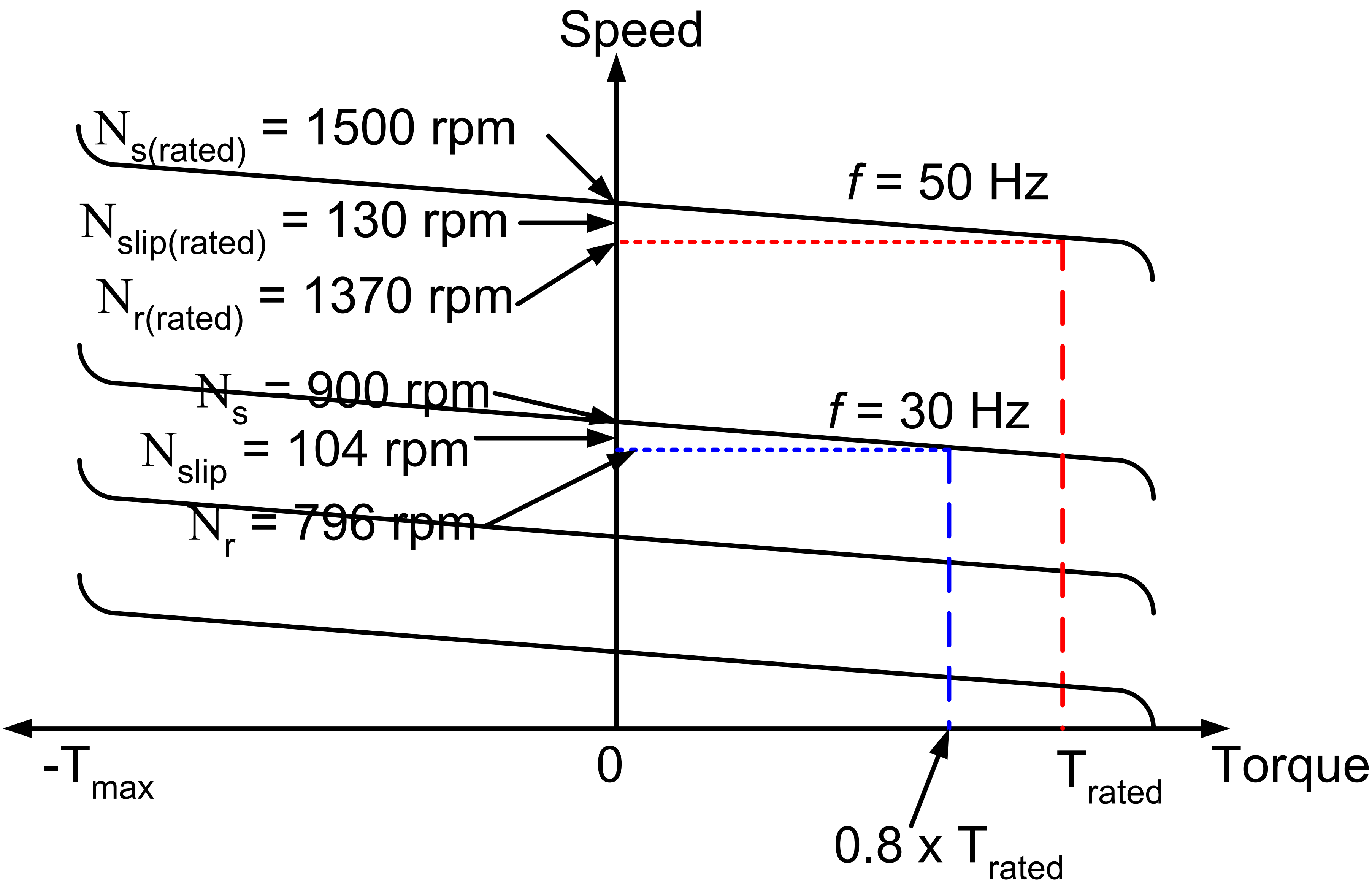
For $T = 0.8 \times T_{fl}, N_{sl} = 0.8 \times 130rpm = 104rpm$

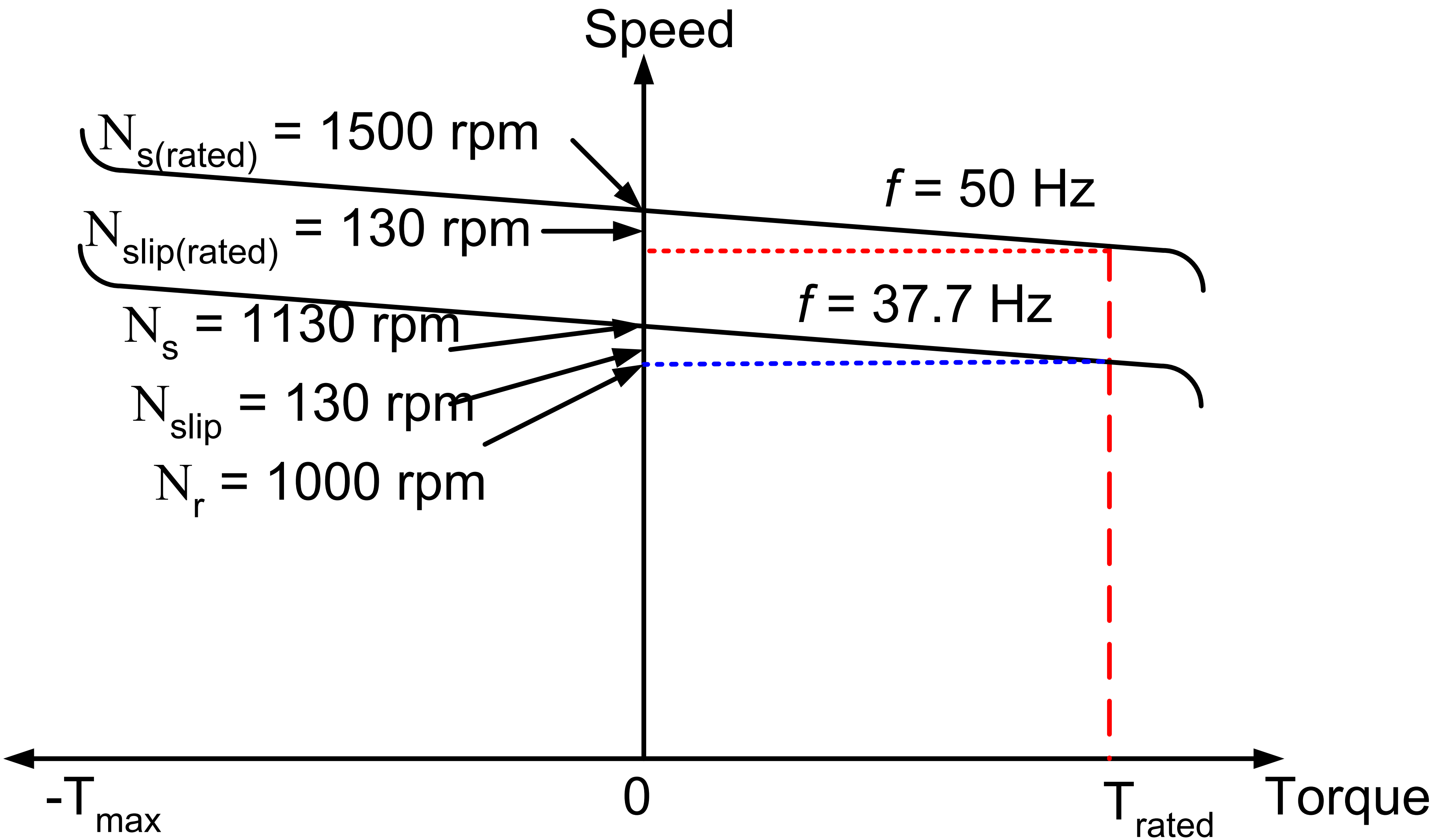
$N_r = 900 - 104 = 796rpm$

(ii) N_r = 1000 rpm, $T = T_{fl}$, slip-speed = 130 rpm, f_s =??

$$N_s = N_r + slip - speed = 1000 + 130 = 1130rpm$$

$$N_s = 1130 = \frac{120 \times f_s}{4} \Rightarrow f_s = 37.67Hz$$


Speed
Ns(rated) = 1500 rpm
Nslip(rated) = 130 rpm
Nr(rated) = 1370 rpm
f = 50 Hz
Ns = 900 rpm
Nslip = 104 rpm
Nr = 796 rpm
f = 30 Hz
-Tmax
0
Trated
Torque
0.8 x Trated



Speed
$N_{s(rated)}$ = 1500 rpm
$N_{slip(rated)}$ = 130 rpm
f = 50 Hz
f = 37.7 Hz
N_s = 1130 rpm
N_{slip} = 130 rpm
N_r = 1000 rpm
$-T_{max}$
0
T_{rated}
Torque

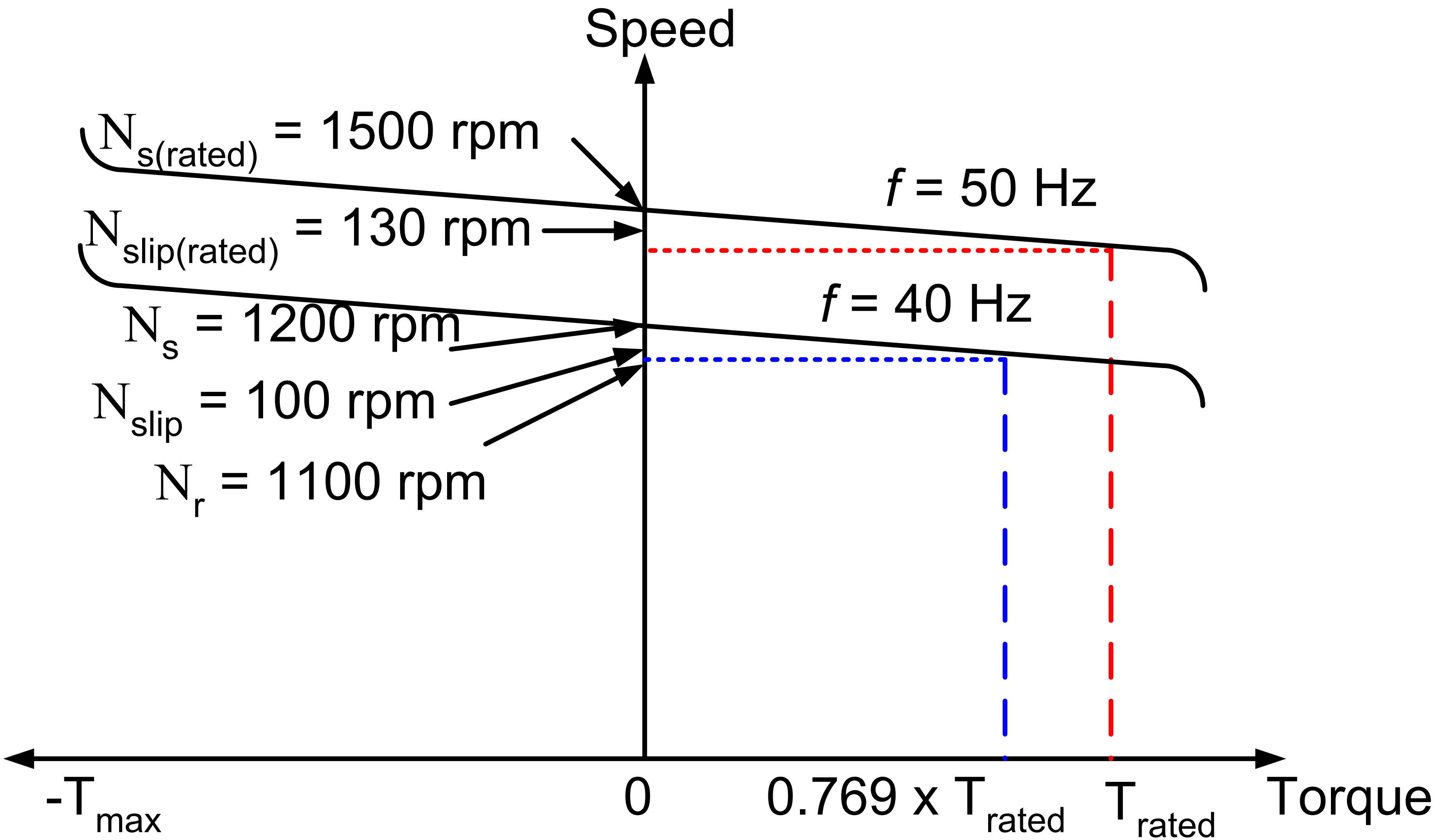
$$\text{(iii)}\ fs = 40Hz, Nr = 1100rpm, T = ??$$

$$Ns = \frac{120 \times 40}{4} = 1200rpm \Rightarrow slip - speed = 1200 - 1100 = 100rpm$$

$$s_{fl} = \frac{130rpm}{1500rpm} = 0.0867$$

$$T_{fl} = \frac{3}{157} \frac{(230.94)^2 \times \left(\frac{3}{0.0867}\right)}{\left(2 + \frac{3}{0.0867}\right)^2 + 7^2} = 25.4N.m$$

$$T = \frac{100}{130} \times T_{fl} = 0.769 \times 25.4. N.m = 19.5N.m$$


Speed
$N_{s(rated)}$ = 1500 rpm
$N_{slip(rated)}$ = 130 rpm
f = 50 Hz
f = 40 Hz
N_s = 1200 rpm
N_{slip} = 100 rpm
N_r = 1100 rpm
$-T_{max}$
0
0.769 x T_{rated}
T_{rated}
Torque

Example 5: For regenerative braking of inverter-fed induction motor drive of Example 2. Determine approximate values of the following:

i. Speed for a frequency of 30 Hz and 80% of full-load torque

ii. Frequency for a speed of 1000 rpm and full-load torque

iii. Torque for a frequency of 40 Hz and speed of 1300 rpm

iv. What will be the answers to (i) and (iii) above. If the drive works under DYNAMIC BRAKING?

$$(i) fs = 30Hz, T = 0.8Tfl, Nr = ? \quad Ns = \frac{120 \times 30}{4} = 900rpm$$

$$N_{sl} = 130rpm\ @ \quad T_{fl}$$

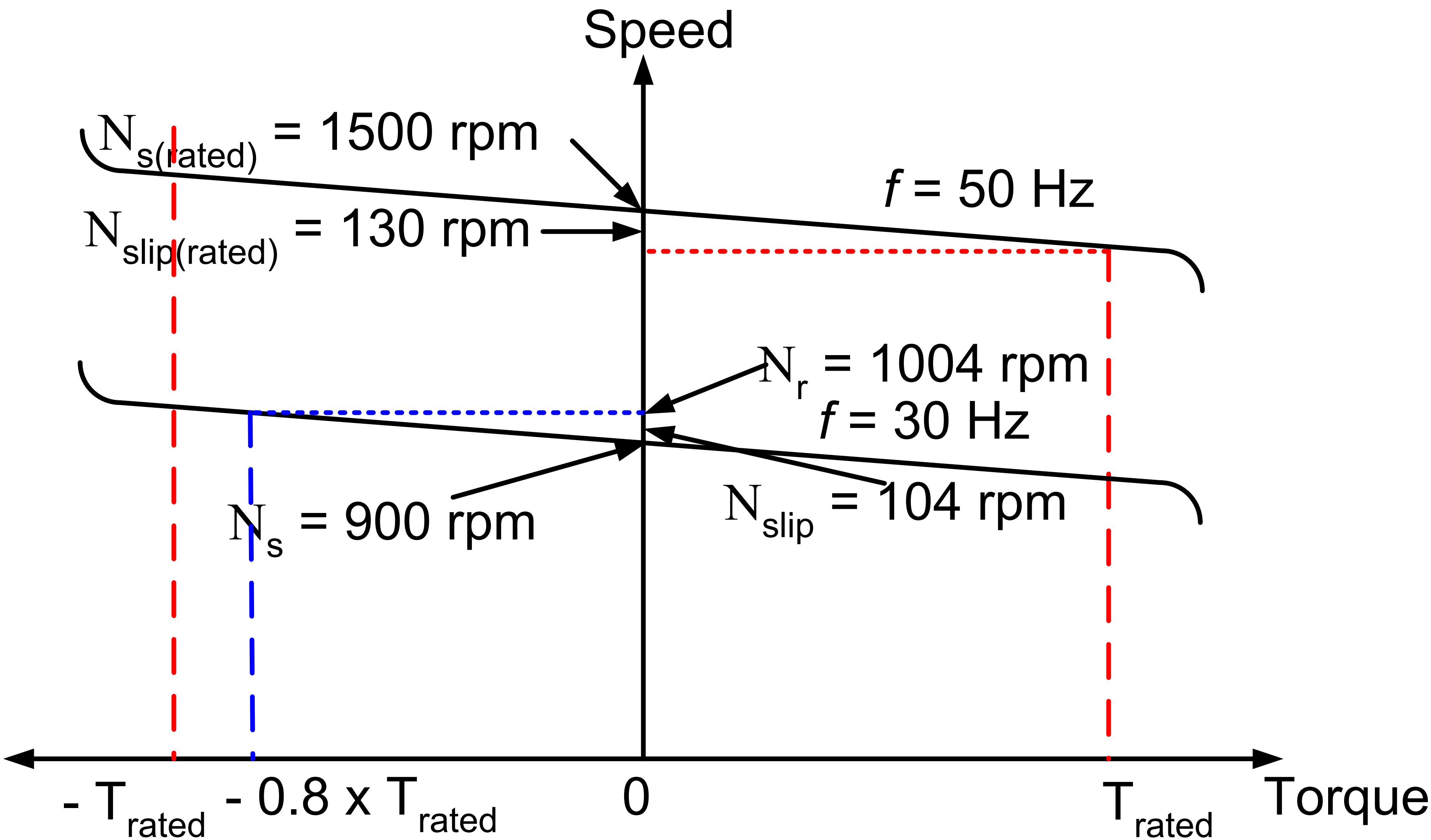
$$N_r = N_s + N_{sl} = 900 + 130 \times 0.8 = 1004rpm$$

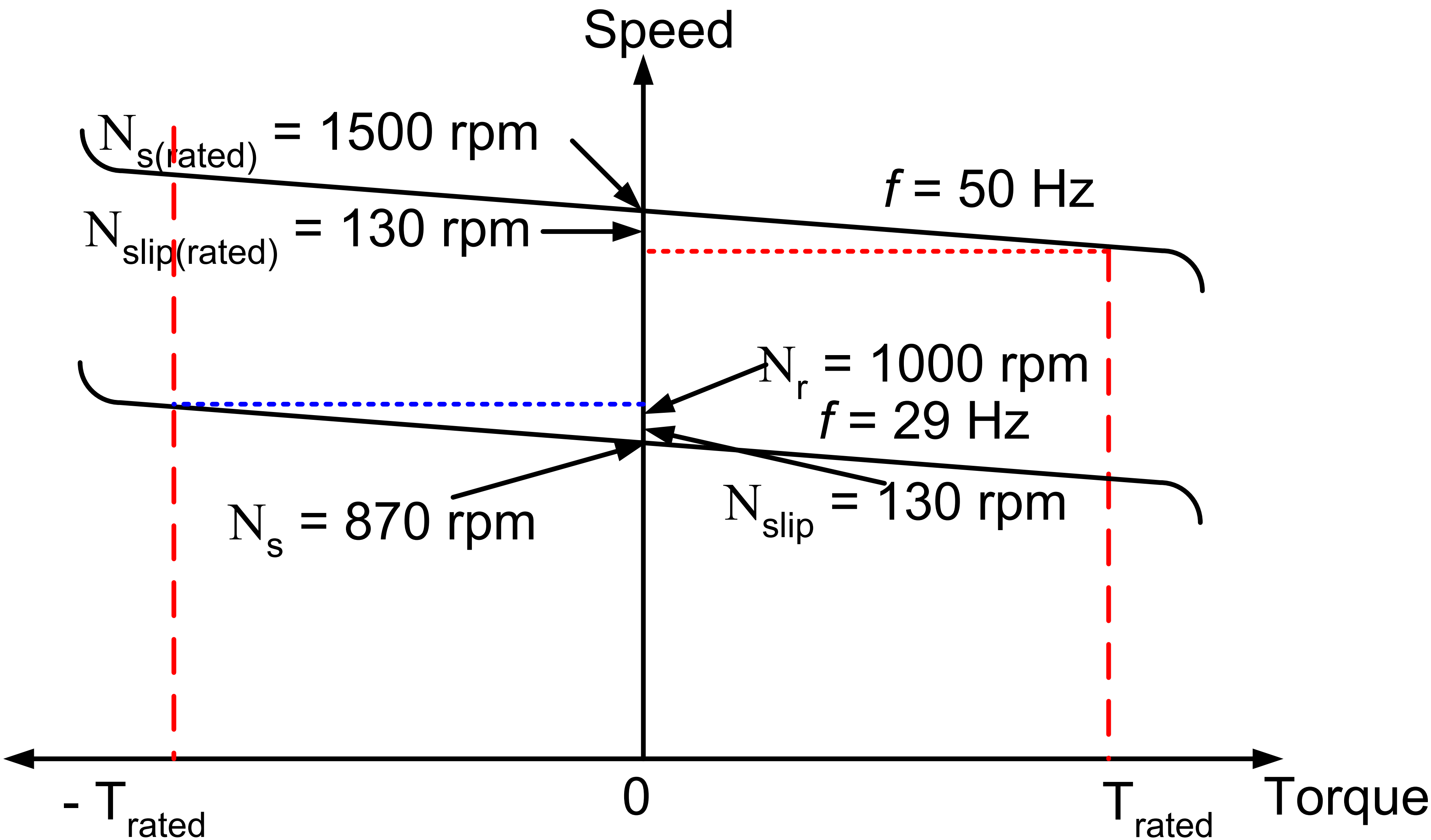
$$(ii) f_s = ??, \quad Nr = 1000rpm, T = Tfl, Nsl = 130rpm$$

$$Ns = Nr - Nsl = 1000 - 130 = 870rpm$$
$$\Rightarrow fs = (870 \times 4)/120 = 29Hz$$

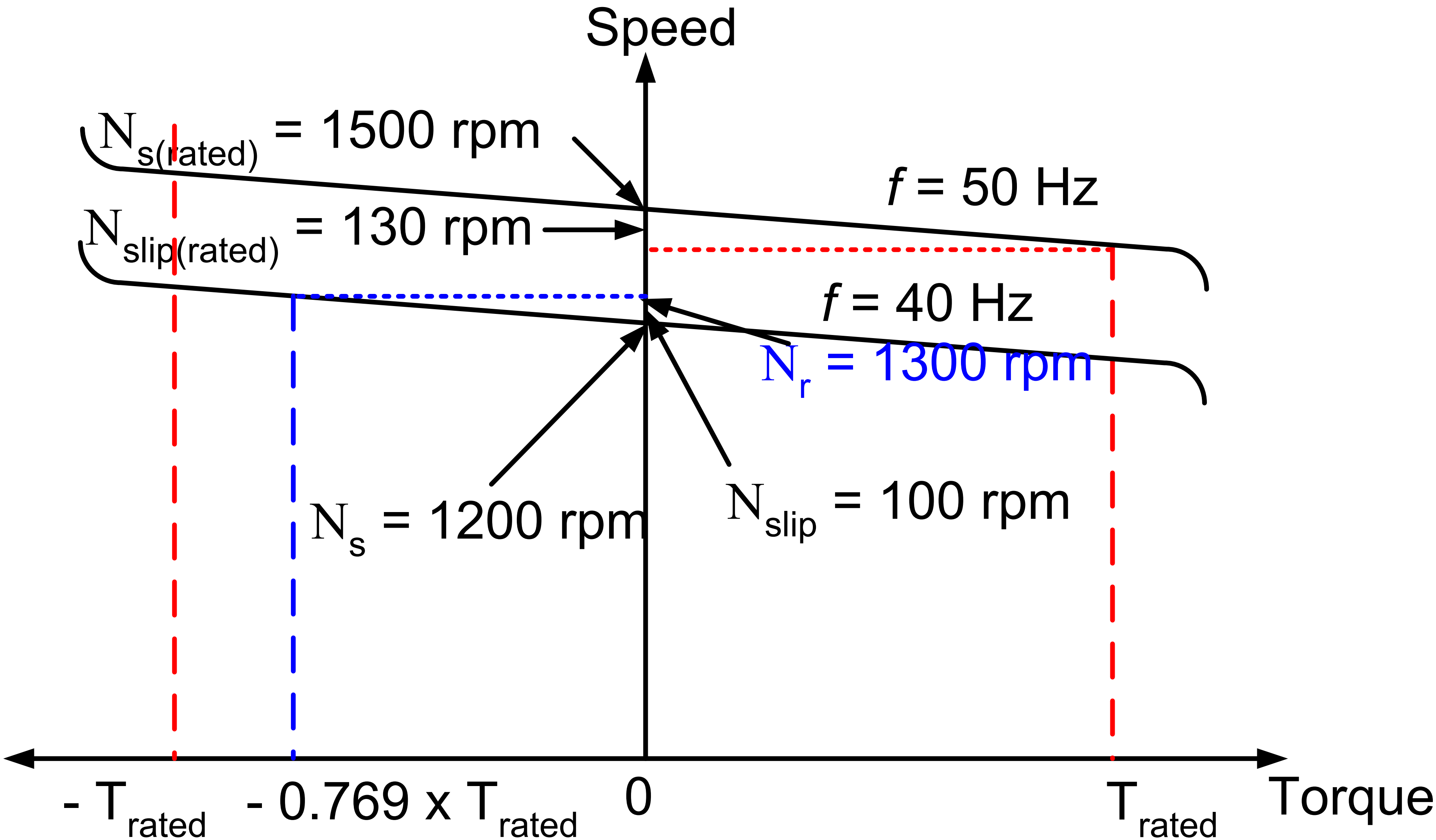
$$(iii) f_s = 40Hz, N_r = 1300rpm, N_s = \frac{120 \times 40}{4} = 1200rpm$$

$$slip - speed = -100rpm, T = -\frac{100}{130} \times Tfl = -\frac{100}{130} \times 25.4 = -19.53N.m$$


Speed
Ns(rated) = 1500 rpm
f = 50 Hz
Nslip(rated) = 130 rpm
Nr = 1004 rpm
f = 30 Hz
Ns = 900 rpm
Nslip = 104 rpm
- Trated
- 0.8 x Trated
0
Trated
Torque



Speed
$N_{s(rated)}$ = 1500 rpm
f = 50 Hz
$N_{slip(rated)}$ = 130 rpm
N_r = 1000 rpm
f = 29 Hz
N_s = 870 rpm
N_{slip} = 130 rpm
- T_{rated}
0
T_{rated}
Torque



Speed
$N_{s(rated)}$ = 1500 rpm
f = 50 Hz
$N_{slip(rated)}$ = 130 rpm
f = 40 Hz
N_r = 1300 rpm
N_s = 1200 rpm
N_{slip} = 100 rpm
- T_{rated}
- 0.769 x T_{rated}
0
T_{rated}
Torque

Example 6: Calculate the motor breakdown torque for inverted-fed motor drive of Example2 for a frequency of 60Hz as a ratio of its value at 50Hz.

$$400V, f = 50Hz, P = 4, N_r = 1370rpm$$

$$R_s = 2\Omega, R_{r'} = 3\Omega, X_s = X_{r'} = 3.5\Omega$$

$$a = \frac{f}{f_{rated}} = \frac{60}{50} = 1.2\rangle 1, V_{rated} = \frac{400}{\sqrt{3}} = 230.94V$$

$$T_{max} = \frac{3}{2a\omega_{ms}} \frac{V^2{}_{ph(rated)}}{R_s + \sqrt{R_s{}^2 + a^2(X_s + X_{r'})^2}}, \omega_{ms} = \frac{120 \times 50}{4} \times \frac{2\pi}{60} = 157rad/s$$

$$T_{max} = \frac{3}{2 \times 157 \times 1.2} \frac{(230/94)^2}{2 + \sqrt{2^2 + (1.2)^2(3.5 + 3.5)^2}} = 32.92N.m$$

$$T_{max}(a = 1) = 54.88N.m > T_{max}(a = 1.2) = 32.92N.m$$

Inverters for AC motor control

- The inverters for ac motor control can broadly be classified into two categories: (a) voltage source inverter and (b) current-source inverter.
- Voltage-source inverters(VSI) are those which are powered form a stiff or low impedance voltage source. Due to the low internal impedance of the inverter, the terminal voltage remains constant irrespective of the load variation.
- Current-source inverters(CSI) are supplied with a controlled current source with high internal impedance. Because of the large impedance, the terminal voltage varies widely with changes in the load, however, load current remains constant.
- VSIs are more suitable for multi-motor drive as compared to CSIs. We will focus our study only on VSIs.

Three-phase Voltage Source Inverter

- The power circuit and the corresponding voltage and current waveforms are shown in Fig. 4.17.
- Each thyristors conduct for a period of 180º in one cycle and the firing signals to each thyristors are delayed by an angle of 60º in the order of their serial number. At any given instant only three power switches out of six are conducting.
- The expressions for the line and phase voltage are given as

$$V_{AB} = \frac{2\sqrt{3}}{\pi} Vd \left[\sin\left(\omega t + \frac{\pi}{6}\right) + \frac{1}{5}\sin\left(5\omega t - \frac{\pi}{6}\right) + \dots \right] \quad (4.37)$$

$$V_{AN} = \frac{2}{\pi} V_d \left[\sin \omega\, t + \frac{1}{5} \sin 5\, \omega t + \dots \right] \quad (4.38)$$

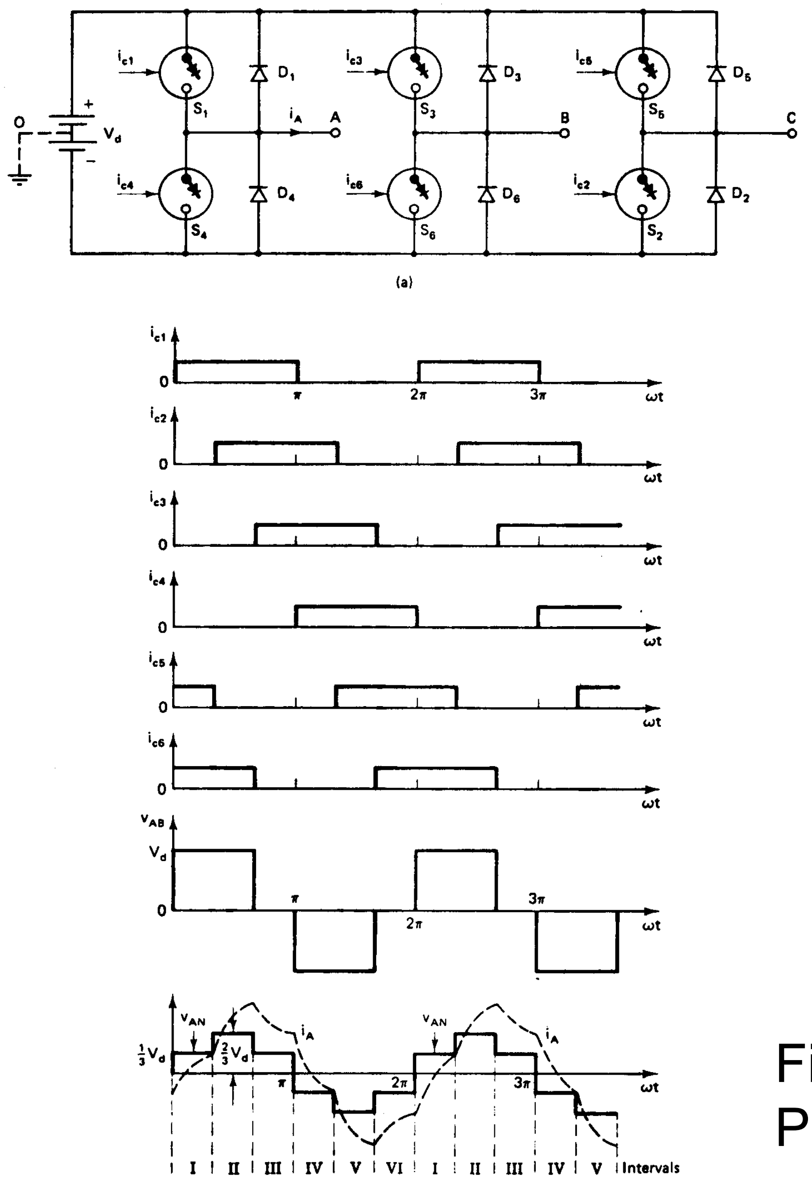
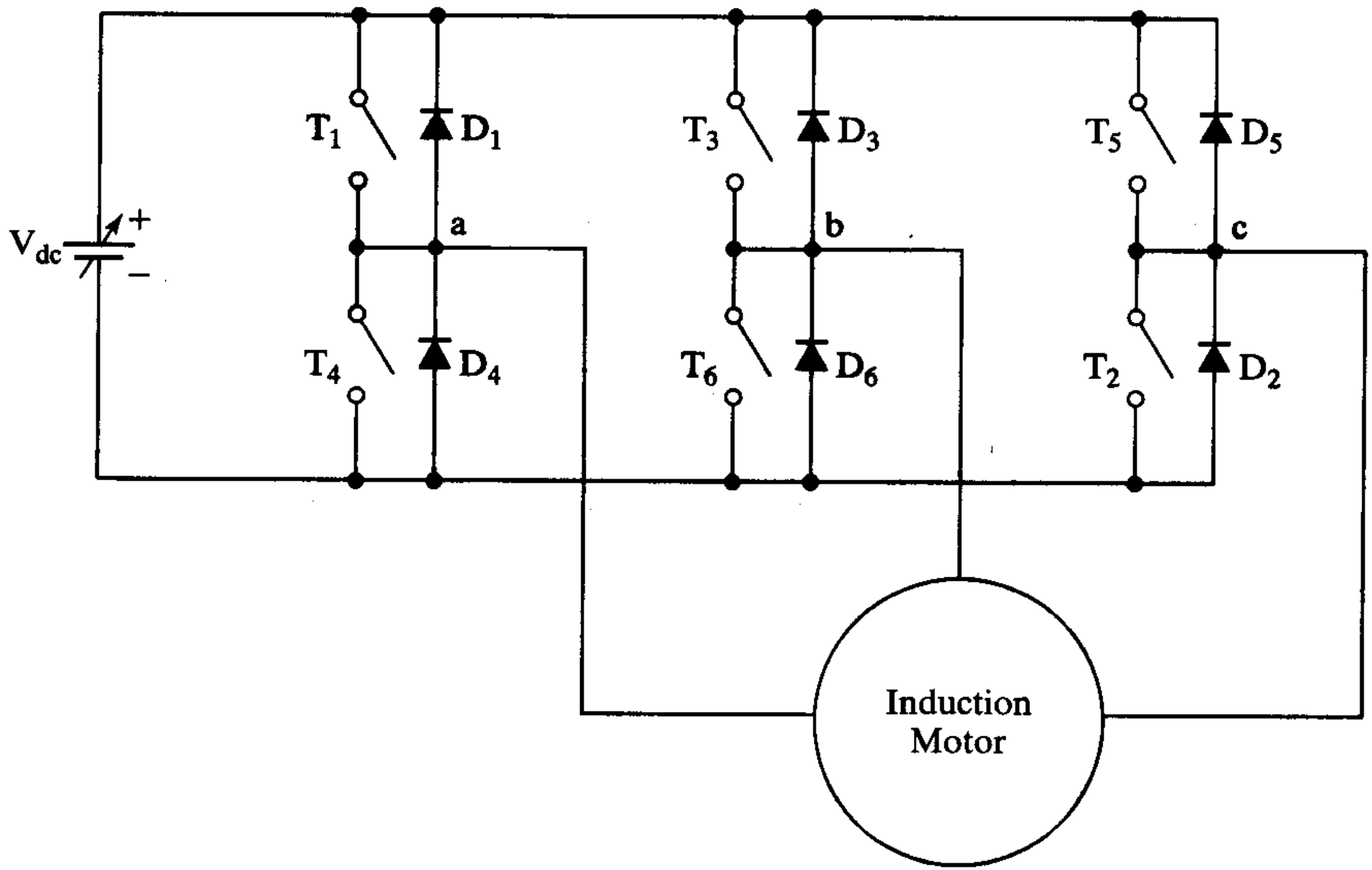
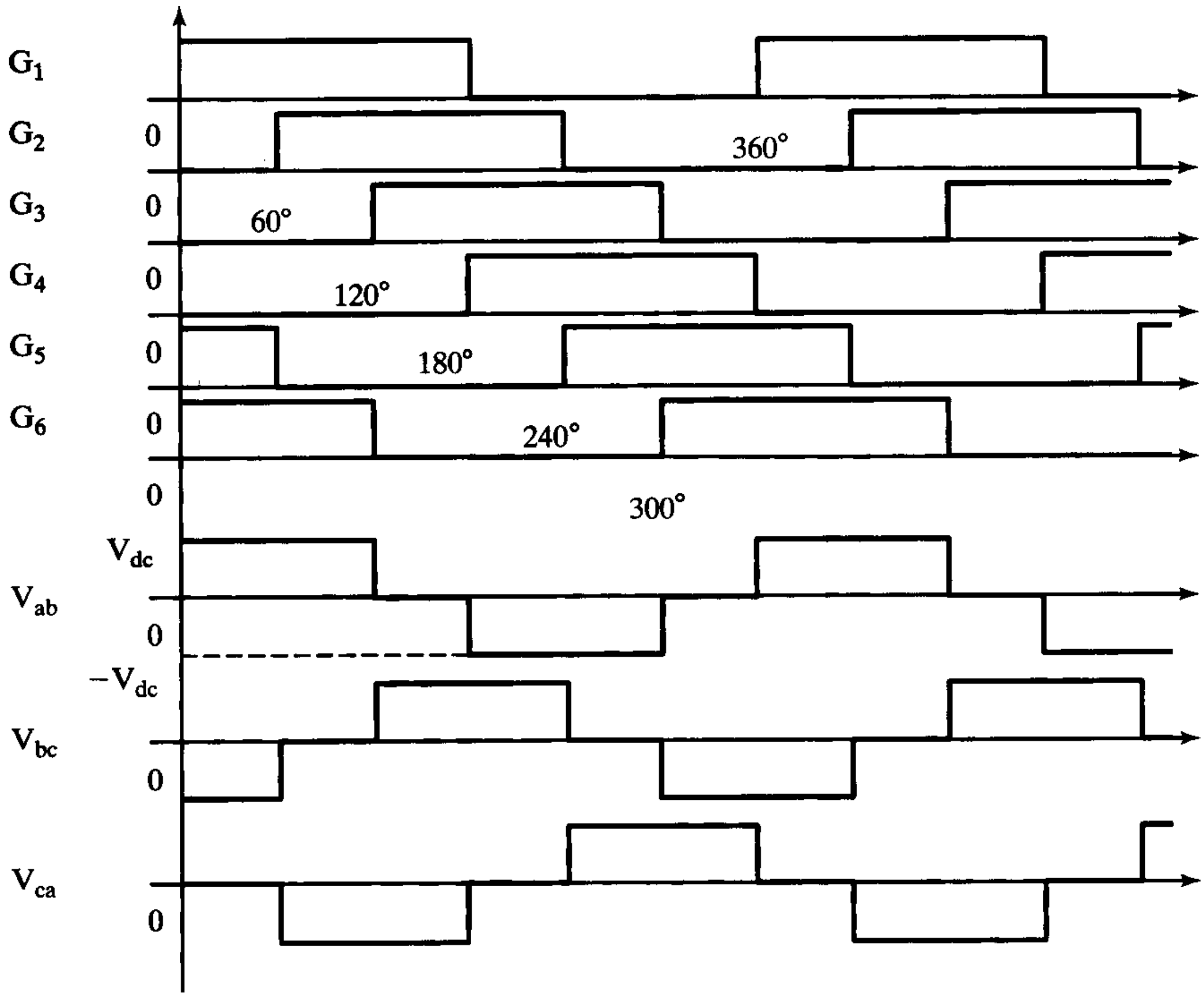
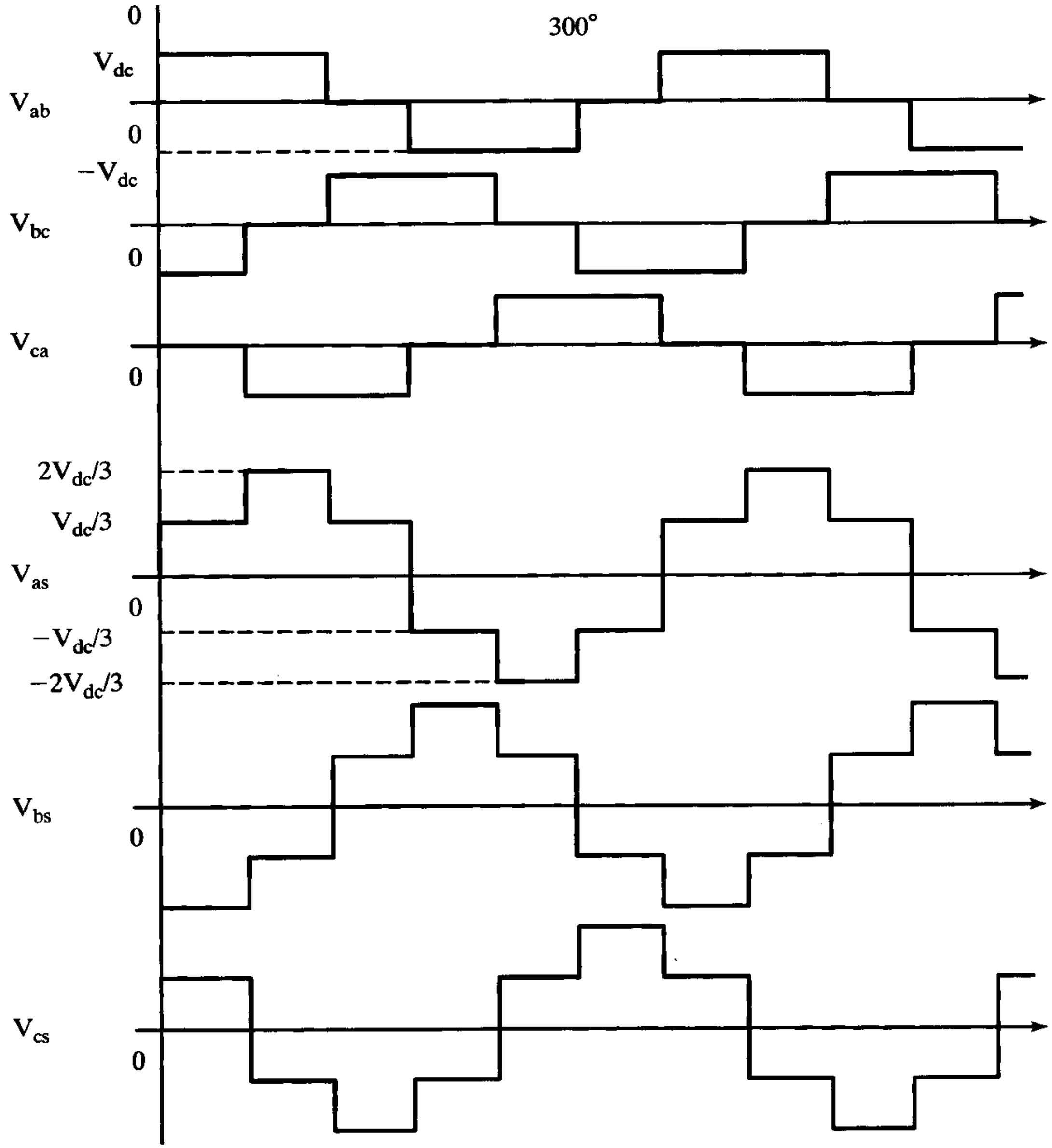


Fig. 4.17: Three Phase VSI


V_dc
+
–
T1
D1
T3
D3
T5
D5
a
b
c
T4
D4
T6
D6
T2
D2
Induction
Motor


G_1
G_2
G_3
G_4
G_5
G_6
0
360°
60°
120°
180°
240°
300°
V_{dc}
V_{ab}
$-V_{dc}$
V_{bc}
V_{ca}


0
300°
V_{dc}
V_{ab}
0
$-V_{dc}$
V_{bc}
0
V_{ca}
0
$2V_{dc}/3$
$V_{dc}/3$
V_{as}
0
$-V_{dc}/3$
$-2V_{dc}/3$
V_{bs}
0
V_{cs}
0

$$V_{AB} = V_{AN} - V_{BN}, V_{BC} = V_{BN} - V_{CN}, V_{CA} = V_{CN} - V_{AN},$$
$$V_{AB} - V_{CA} = (V_{AN} - V_{BN}) - (V_{CN} - V_{AN}) = 2V_{AN} - (V_{BN} + V_{CN})$$
For a three−phase system we have,
$$V_{AN} + V_{BN} + V_{CN} = 0$$
$$\therefore V_{AB} - V_{CA} = 3V_{AN} \Rightarrow V_{AN} = \frac{(V_{AB} - V_{CA})}{3}$$
$$\text{Similarly, } V_{BN} = \frac{(V_{BC} - V_{AB})}{3} \text{ and } V_{CN} = \frac{(V_{CA} - V_{BC})}{3}$$

- The phase voltages are shifted from the line voltages by 30^0.

- RMS value of the fundamental phase voltage is

$$V_1 = \frac{\sqrt{2}}{\pi} V_d \quad (4.39)$$

- For ac motor speed control, it is required to control the magnitude and frequency of the supply voltage.
- From eqn. 4.38 it is clear that the frequency of the fundamental voltage can be varied by varying the time period of one cycle, however, the magnitude remains fixed so long as the dc supply voltage V_d is fixed.
- The dc link voltage can be controlled by using any of the following methods:
  – If the supply voltage is dc then dc chopper can be used to control the dc voltage at the input of the inverter as shown in Fig. 4.18(b).
  – If the supply voltage is ac then the phase controlled rectifier can be sued as shown in Fig. 4.18(a).

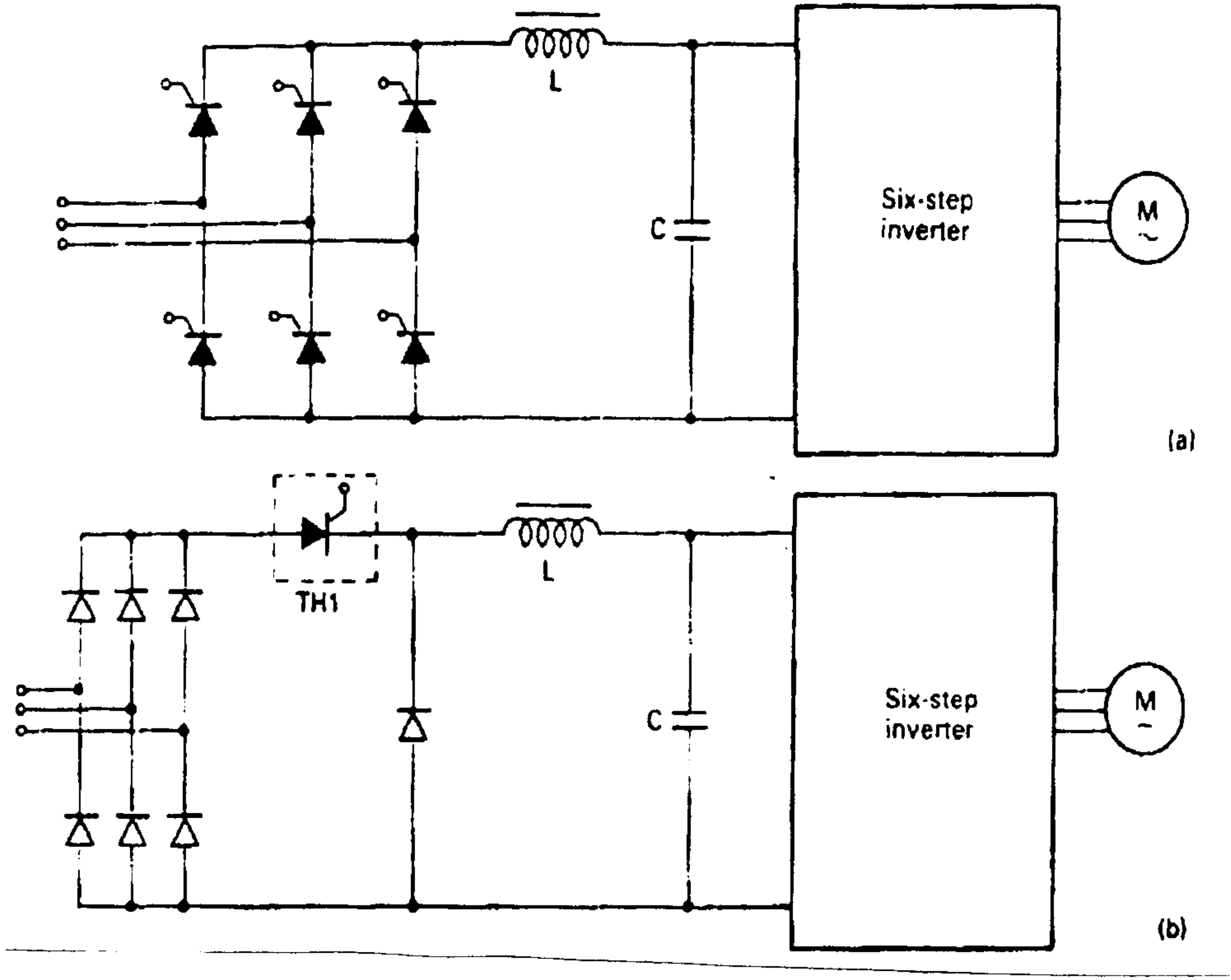


Fig. 4.18: Voltage control in six-step VSI.

- However, the disadvantage of using this scheme is the high magnitude of low-frequency harmonics.
- This problem can be overcome by using an un-controlled rectifier together with a chopper to control the inverter input voltage as shown in Fig. 4.18(b). This scheme is however more expensive.
- The disadvantages of the six-step inverters are :
  - Due to the presence of low frequency harmonics, the losses are increased at all speed causing de-rating - more critical at low speeds.
  - Torque pulsations are present at low speeds due to 5, 7, 11 and 13^{th} harmonics.
  - If phase controlled rectifier is used it deteriorates the line side power factor (p.f. = k cos α).

PWM Inverter

- The PWM inverters have the provision to control the output voltage within itself and hence can be supplied from a fixed dc bus.
- When the supply is ac the dc voltage is obtained by having a diode rectifier bridge giving unity fundamental power factor (p.f. = (I_{s1}/I_s) DPF = 0.9 cos α = 0.9).
- Moreover, due to low harmonic content at the diode bridge output, the filter components are smaller in size providing faster response.
- The low harmonic contents in the inverter output voltage provides smoother operation, free from torque pulsation and cogging.

- In the sinusoidal PWM inverter the reference sine modulating wave is compared with a common triangular wave, and the natural points of intersections provide the switching instants of the power semiconductor devices.
- Such a scheme consisting of 12 cycles of carrier wave is shown in Fig. 4.19(a). The phase and line voltage in the case of PWM inverter are as shown in Fig. 4.19(b).
- The sine modulating wave has a variable amplitude A and the carrier triangular wave has a fixed amplitude A_m as shown in Fig. 4.19(b). The ratio of the amplitudes of the reference wave to the carrier wave is called as the modulation index, m.

$$m = \frac{A}{Am} \quad (4.40)$$

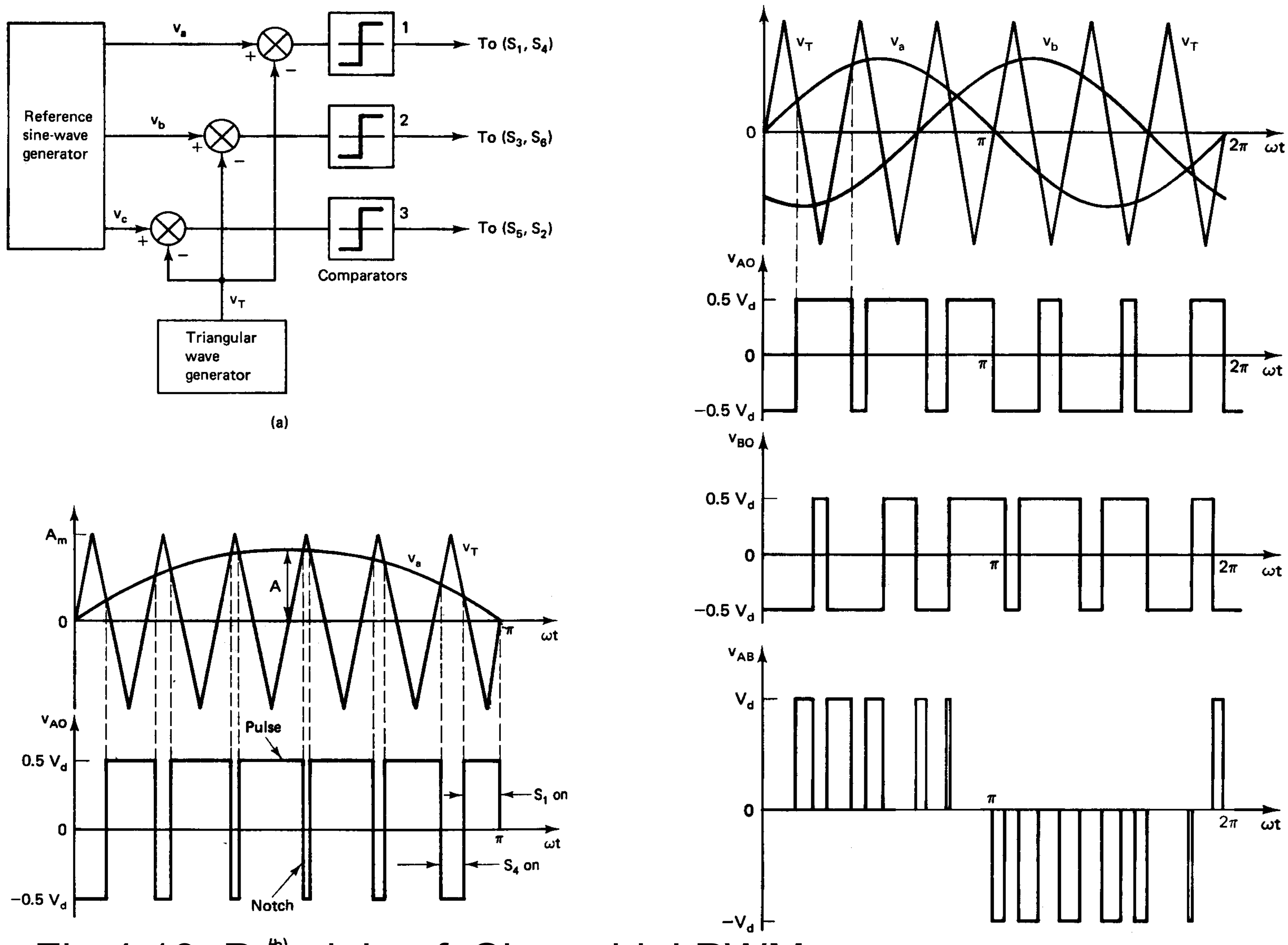


Fig.4.19: Principle of Sinusoidal PWM.

- The fundamental rms component of the waveform V_{A0} is given by:

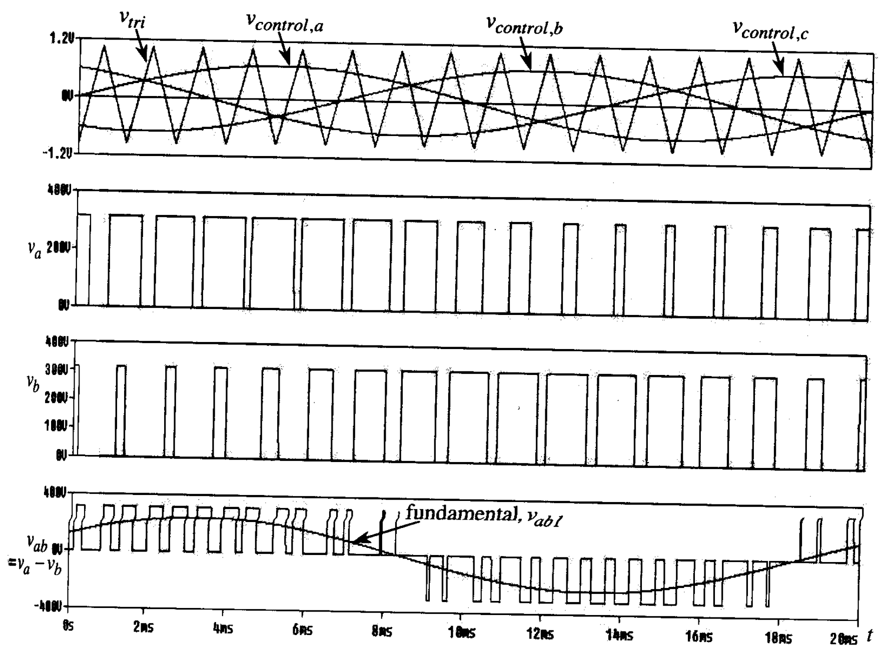
$$V_1 = \frac{mV_d}{2\sqrt{2}} \quad (4.41)$$

- From eqn. 4.41 it can be seen that the fundamental voltage increases linearly with *m*, until $m = 1$. As $m \rightarrow 1$, the notch width at the centre approaches zero causing problem for the power semiconductor devices in terms of switching.

- The waveform V_{A0} consists of harmonics which are odd multiples of the carrier frequency f_c. The waveform also contains side bands centred around multiples of f_c and given by:

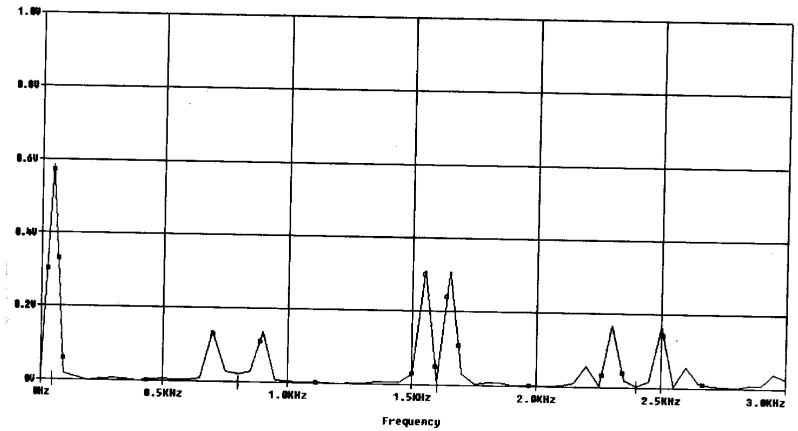
$$f_h = Kf_c \pm kf (4.42)$$

where *K* and *k* are integers and $(K + k)$ is an odd number.



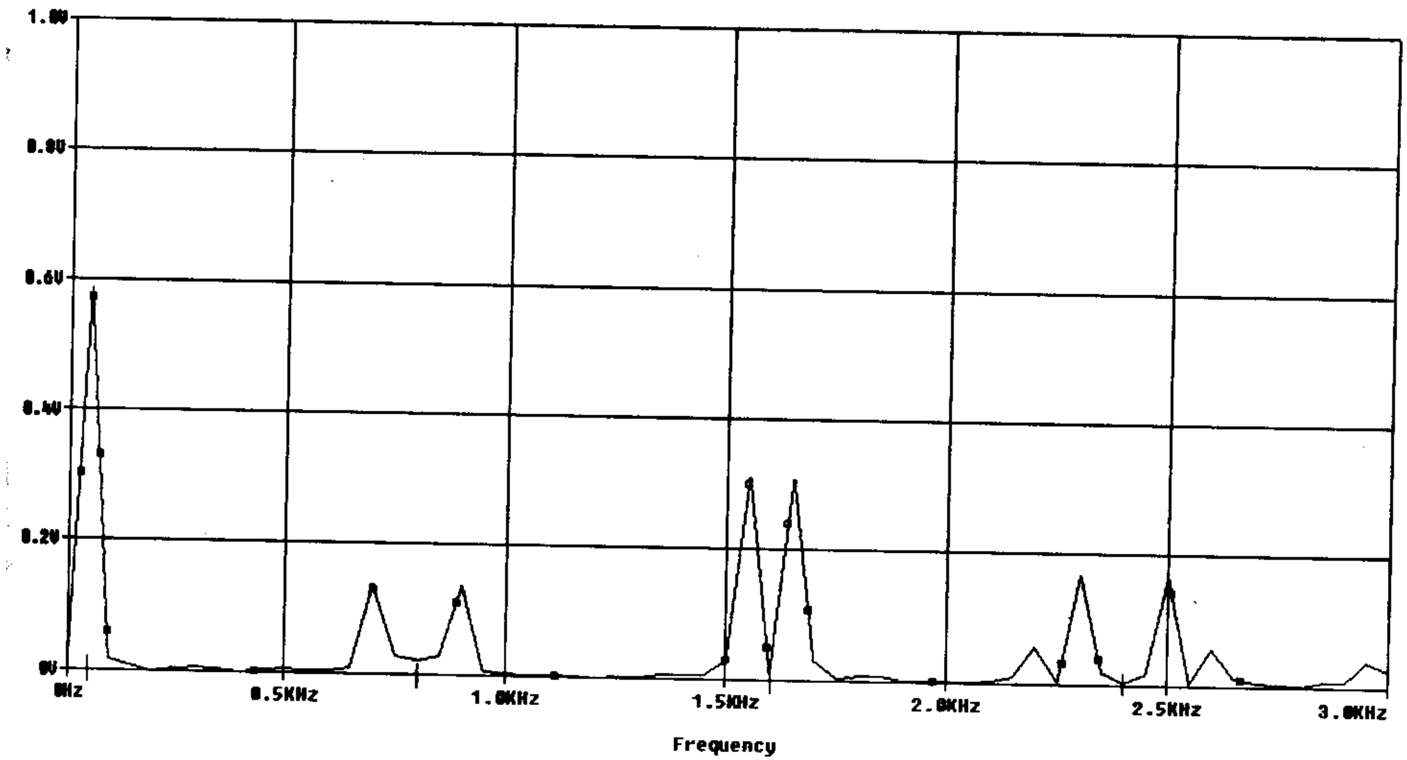
v_{tri}
$v_{control,a}$
$v_{control,b}$
$v_{control,c}$
1.2V
0V
-1.2V
v_a
400V
200V
0V
v_b
400V
300V
200V
100V
0V
fundamental, v_{ab1}
v_{ab}
$= v_a - v_b$
400V
0V
-400V
0s
2ms
4ms
6ms
8ms
10ms
12ms
14ms
16ms
18ms
20ms
t

(a)



1.0V
0.8V
0.6V
0.4V
0.2V
0V
0Hz
0.5KHz
1.0KHz
1.5KHz
2.0KHz
2.5KHz
3.0KHz
Frequency

(b)



(b)

$f = 50$ Hz, $f_c = 16 \times 50 = 800$ Hz,

$f_h = (K = 0) \times (f_c = 800) \pm (k = 1) \times (f = 50 \text{ Hz}) = 50$ Hz

$f_h = (K = 1) \times (f_c = 800) \pm (k = 2) \times (f = 50 \text{ Hz}) = 700$ Hz and 900 Hz

$f_h = (K = 2) \times (f_c = 800) \pm (k = 1) \times (f = 50 \text{ Hz}) = 1550$ Hz and 1650 Hz

- The magnitude of the band frequency harmonics decrease rapidly with the increasing distance from the band centre.
- The carrier ratio, *p* can be defied as:

$$p = \frac{f_c}{f} \quad (4.43)$$

- When *p is large the frequency of the harmonics will be large and hence can be easily filtered out by the machine leakage inductance.* This results in a near sinusoidal current waveform.

- The modulation is called synchronous when *p* is an integer and is a multiple of 3. When the above mentioned condition is not satisfied the modulation is called as asynchronous or free-running.

- The fundamental output voltage of the PWM inverter can be increased by increasing the modulation index, *m* to unity.
- The corresponding amplitude of the line-to-neutral voltage for a star connected load is $V_d/2$. For a six-step inverter the corresponding amplitude of the line-to-neutral voltage is $2V_d/\pi$.
- Thus for a given link voltage, V_d the PWM inverter has a fundamental voltage capability which is only 78% of that of a six-step inverter.
- In order to improve the dc link voltage utilisation it is necessary to increase the modulation index, *m* over 1 and is termed as over-modulation as shown in Fig. 4.20.
- For large *m* the only intersection takes place at the zero crossover points and hence the output becomes a un-modulated square-wave.

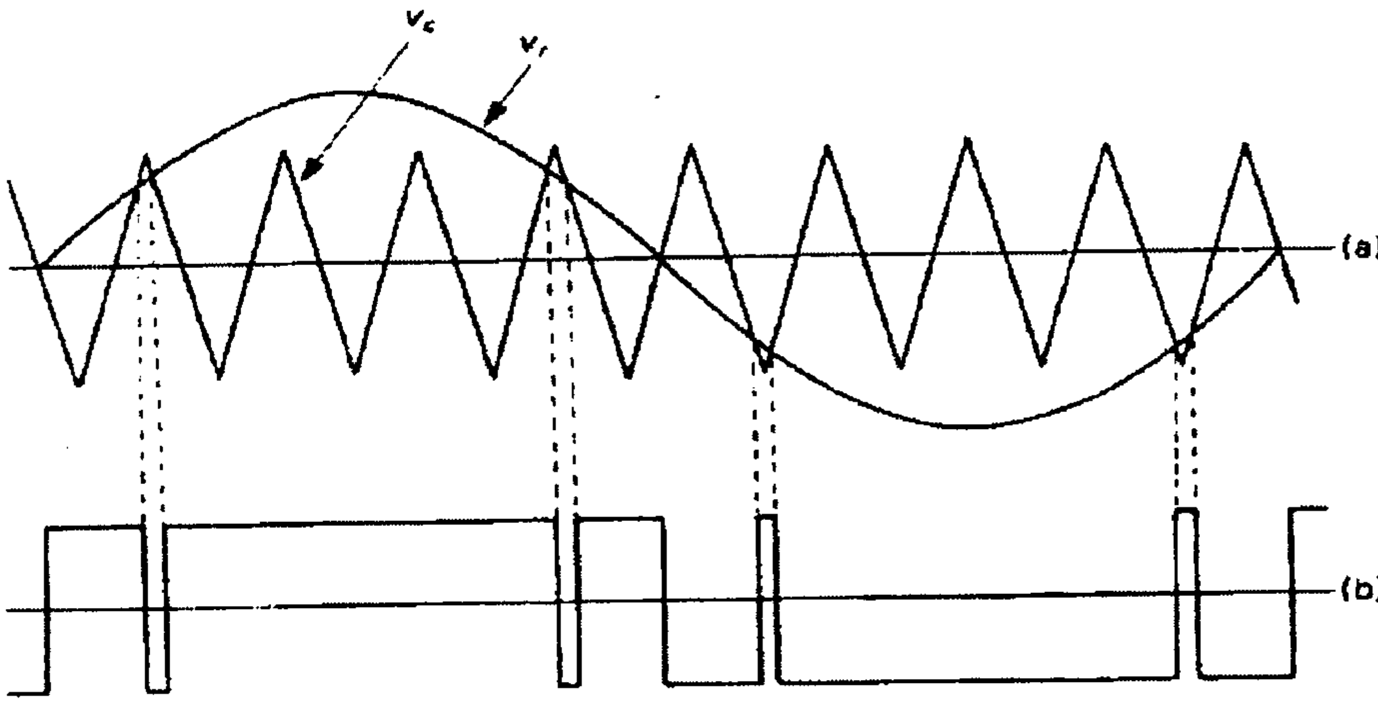


Fig. 4.20: Over-modulation in a sinusoidal PWM.

• Constant volts/Hertz operation is carried out by varying the modulation index linearly with frequency of the reference wave to provide a fundamental voltage which is proportional to the frequency.

- At low fundamental frequency very high carrier ratios are feasible resulting near sinusoidal output current waveforms. As the fundamental output frequency is reduced, the carrier ratio, *p* increases in integer multiples at certain preset values. This is to limit the inverter switching frequency in between certain minimum and maximum limits.
- As the reference frequency increases, the carrier ratio, *p* reduces and operation in unsynchronised mode takes place. This introduces sub-harmonics of the reference frequency and cause low frequency torque and speed pulsations known as frequency beats.

- As the operating frequency is increased further, the carrier ratio, *p* reduces to six and possibly to three to ease the transition to six-step mode of operation.
- Advantages of PWM inverters:
  - It provides smooth low speed operations as compared to the six-step inverter.
  - It has better transient response, as voltage and frequency control both are carried out within the inverter.
  - Due to the diode bridge the fundamental power factor presented to the ac utility supply is always high.
  - The constant dc-link voltage allows several inverters can be connected to the same dc-link resulting significant savings.

Operation with Non-sinusoidal Supplies

- The output voltages and currents from a power-semiconductor converter are **non-sinusoidal** in nature.
- Any non-sinusoidal waveform can be resolved into its fundamental component as well as harmonic components using Fourier Series Analysis.
- Let the fundamental component of the three-phase voltages be:

$$V_{AN} = V_1\, sin(\omega t), V_{BN} = V_1\, sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$V_{CN} = V_1 \sin\left(\omega t - \frac{4\pi}{3}\right) \;(4.44)$$

- The third harmonic voltages are given by:

$$V_{AN} = V_3\, sin3(\omega t), V_{BN} = V_3\, sin3\left(\omega t - \frac{2\pi}{3}\right) = V_3\, sin\,3\,(\omega t)$$

$$V_{CN} = V_3 sin3\left(\omega t - \frac{4\pi}{3}\right) = V_3 sin3(\omega t)\ (4.45)$$

- Thus, the 3rd harmonic voltages do not produce any rotating magnetic field and are known as zero sequence harmonics.
- The 5th harmonic voltages are given by:

$$V_{AN} = V_5 sin5(\omega t)$$

$$V_{BN} = V_5 sin5\left(\omega t - \frac{2\pi}{3}\right) = V_5\, sin\left(5\omega t - \frac{4\pi}{3}\right)$$

$$V_{CN} = V_5 sin5\left(\omega t - \frac{4\pi}{3}\right) = V_5\, sin\left(5\omega t - \frac{2\pi}{3}\right)\ (4.46)$$

- The phase sequence is *A-C-B*, which is opposite to that of the fundamental i.e. A-B-C. In general, for $k = 6n - 1$, where n is an integer, the phase sequence would be opposite to that of the fundamental (e.g. for $n = 1$, $k = 5$). These are known as negative sequence harmonics.
- The 7th harmonic phase voltages are given by:

$$V_{AN} = V_7 sin7(\omega t), V_{BN} = V_7 sin7\left(\omega t - \frac{2\pi}{3}\right) = V_7\, sin\left(7\omega t - \frac{2\pi}{3}\right)$$

$$V_{CN} = V_7 sin7\left(\omega t - \frac{4\pi}{3}\right) = V_7 sin\left(7\omega t - \frac{4\pi}{3}\right) \quad (4.47)$$

- The phase sequence is *A-B-C*, which is the same as the fundamental. In general, for $k = 6n + 1$, where n is an integer, the phase sequence would be the same as that of the fundamental (e.g. for $n = 1$, $k = 7$). These are known as positive-sequence harmonics.

Harmonic Equivalent Circuit

- The equivalent-circuit valid for any harmonic voltages and currents are as shown in Fig. 4.21, k – harmonic order.

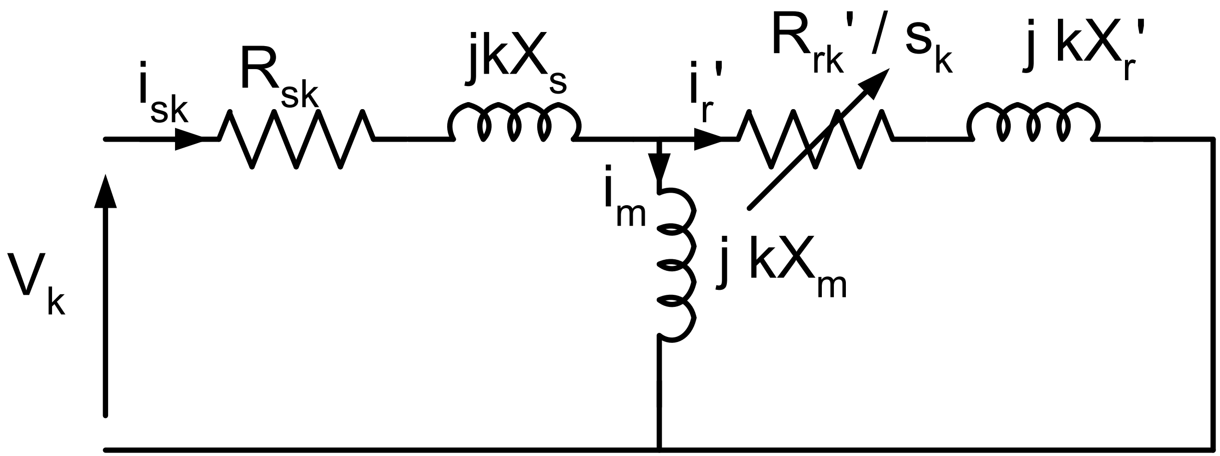


Fig. 4.21: Equivalent circuit of IM.

- The harmonic slip-, $s_{k,}$ can be defined as:

$$s_k = \frac{k\omega_{ms} \pm \omega_m}{k\omega_{ms}} = 1 \pm \frac{(1-s)}{k} \quad (4.48)$$

- The negative sign is for positive sequence harmonics and positive sign is for negative-sequence harmonics.

- If the fundamental slip varies in the range 0 < s < 1, then

$$s_5 = 1 + \frac{1-s}{5} \Rightarrow 1.0 < s_5 < 1.2 \quad s_7 = 1 - \frac{1-s}{7} \Rightarrow 0.857 < s_7 < 1.0$$

$$s_{11} = 1 + \frac{1-s}{11} \Rightarrow 1.0 < s_{11} < 1.09 \quad s_{13} = 1 - \frac{1-s}{13} \Rightarrow 0.92 < s_{13} < 1.0$$

- For higher values of harmonic order, *k* we can assume that $s_k \approx 1.0$.
- Except at low frequency, the magnetising reactance jkX_m is much higher than the rotor impedance in parallel, therefore jkX_m can be treated as open-circuit as shown in Fig.4.22.
- Moreover, at higher harmonic order *k*, the reactance $k(X_s+X_{r'}) >> R_{sk} + (R_{r'k}/s_k)$.

- Thus, the equivalent circuit gets modified as shown in Fig. 4.22.

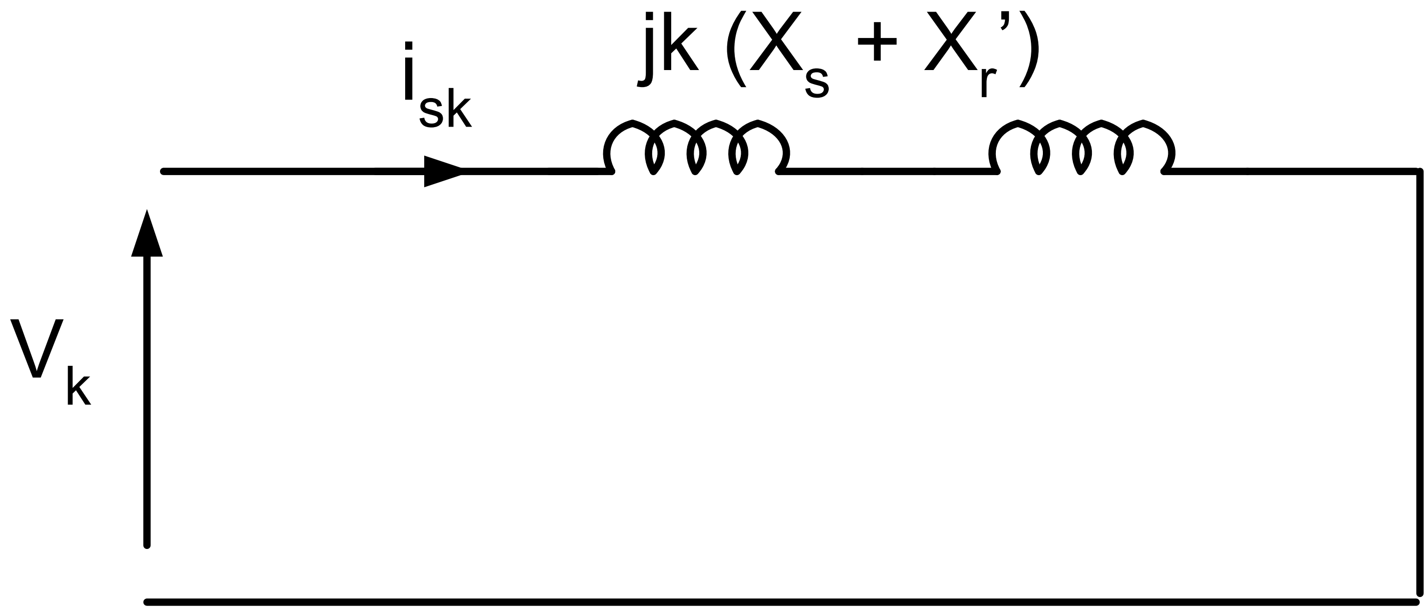


Fig. 4.22: Harmonic equivalent circuit of IM.

Harmonic Torques

- In an induction motor the rotor and stator MMFs interact with each other to produce the steady-state torque as given by $T_{ind} = k(F_s \times F_r)$.
- It can be shown mathematically that the contribution due to the harmonic torques is negligible in comparison with the fundamental torque.
- From the equivalent circuit of Fig. 4.22, the expression for the k^{th} harmonic torque is given by

$$T_k = \pm \frac{3}{k\omega_{ms}} \left(i_{sk}^2\right) \frac{R_{r'k}}{s_k} \quad (4.49)$$

- For $k = 1$, the fundamental torque is given by

$$T_1 = \frac{3}{\omega_{ms}}(i_{r\prime}^2)\frac{R_{r\prime}}{s} \quad (4.50)$$

  - For small s,

$$s_k = 1 \pm \frac{(1-s)}{k} \approx 1 \pm \frac{1}{k} \quad (4.51)$$

- The ratio of the k^{th} harmonic torque to the fundamental torque is given by

$$\frac{T_k}{T_1} = \frac{\left(\frac{3}{k\omega_{ms}}(i_{sk}^2)\frac{R_{r\prime k}}{s_k}\right)}{\left(\frac{3}{\omega_{ms}}(i_{r\prime}^2)\frac{R_{r\prime}}{s}\right)} = \left(\frac{i_{sk}}{i_{r\prime}}\right)^2\left(\frac{R_{r\prime k}}{R_{r\prime}}\right)\left(\frac{s}{k \pm 1}\right) \quad (4.52)$$

- For an example, let us assume that the starting current be five times the rated rotor current referred to the stator side.
- At starting *s = 1*,

$$R_s + \frac{R_{r\prime}}{s(=1)} << (X_s + X_{r\prime})$$

$$\therefore I_{s1} = \frac{V_1}{(X_s + X_{r\prime})} = 5 \qquad I_{r\prime} \Rightarrow I_{r\prime} = \frac{V_1}{5(X_s + X_{r\prime})}$$

$$\text{For a six}-\text{step VSI,} \qquad V_k = \frac{V_1}{k} \text{(ref. Eqn. 4.38)}$$

$$I_{s5} = \frac{V_5\left(=\frac{V_1}{5}\right)}{5(X_s + X_{r\prime})} = \frac{V_1}{25(X_s + X_{r\prime})}, \frac{I_{s5}}{I_{r\prime}} = \frac{\frac{V_1}{25(X_s + X_{r\prime})}}{\frac{V_1}{5(X_s + X_{r\prime})}} = 0.2$$

- Let the rotor resistance of the 5th harmonics be three times that of the fundamental rotor resistance due to skin effect and the full-load slip be 0.04.

$$\frac{I_{s5}}{I_{r\prime}} = 0.2, \frac{R_{r\prime 5}}{R_{r\prime}} = 3, s = 0.04$$

$$\frac{T_k}{T_1} = \pm\left(\frac{i_{sk}}{i_{r\prime}}\right)^2 \left(\frac{R_{r\prime k}}{R_{r\prime}}\right)\left(\frac{s}{k \pm 1}\right)$$

$$\Rightarrow \frac{T_5}{T_1} = -(0.2)^2(3)\left(\frac{0.04}{5+1}\right) = -0.0008\,T_5$$

$$= -0.08 \times T_1\%$$

- Thus, the contribution of the 5th harmonic torque in comparison with the fundamental is negligible.
- The negative sign is due to the negative sequence of the 5th harmonic component.

- The harmonic currents instead produce copper losses as given by

$$P_h = \sum_{k=5,7,11,13,\ldots\ldots} I_{sk}^2 (R_{sk} + R_{r\prime k}) \quad (4.52)$$

- This reduces the efficiency of the system and hence increases thermal loading of the machine.
- Thus, it may be necessary to de-rate the motor when operated with non-sinusoidal supply.
- The rotor MMF and stator MMF produced by harmonic currents are not necessarily stationary w.r.t. each other. They produce pulsating torques with zero average value.
- For example, the 5th and 7th harmonic rotor MMFs interact with fundamental stator MMF to produce pulsating torque which pulsates at six times the fundamental frequency.

- Pulsating torque causes jitter in the machine speed and is highly noticeable at low speeds.
- However, if the system has high rotor-load inertia then the effect of torque ripples on speed would be smoothened out.

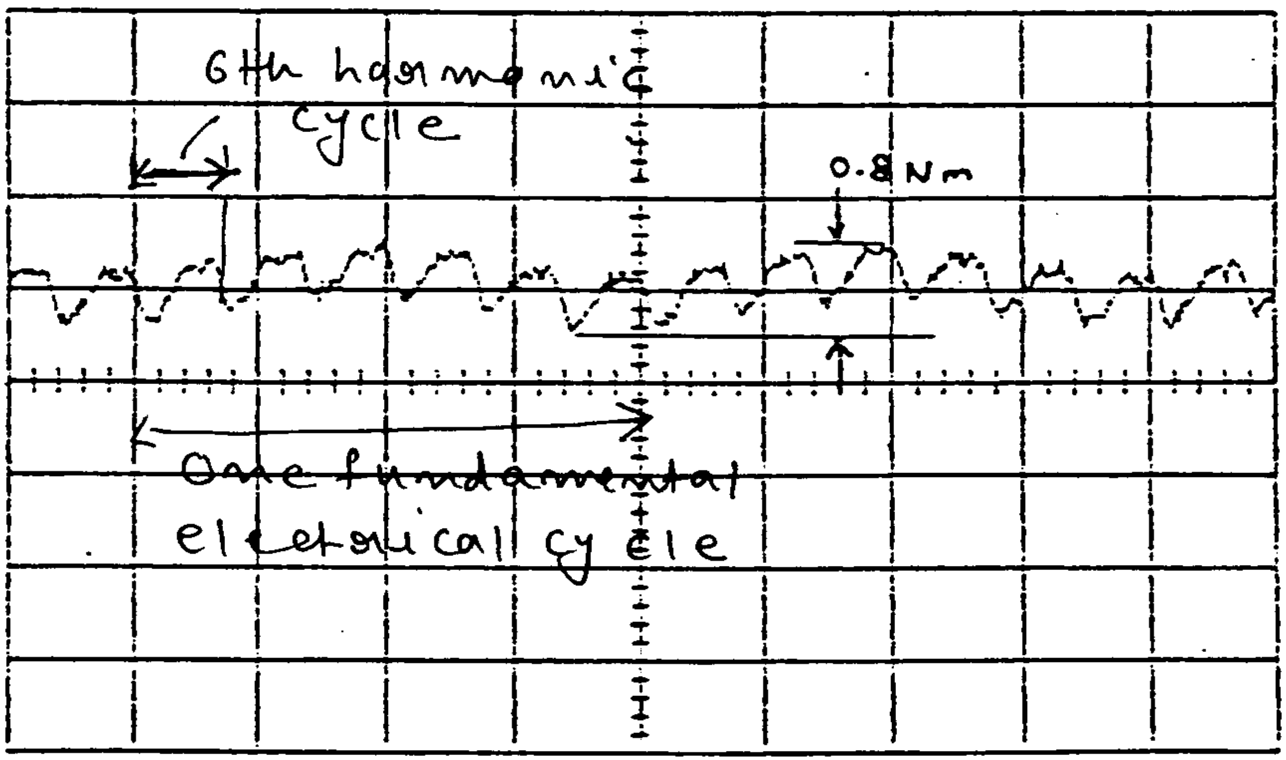


Fig. 4.23: Pulsating motor torque.

Example 7: A 440 V, 50 Hz, 6-pole, 960 rpm, star connected induction motor has the following parameters per-phase referred to the stator:

$R_s = 0.6\ \Omega$, $R_{r'} = 0.3\ \Omega$, $X_s = X_{r'} = 1.0\ \Omega$, $X_m = \infty$

The motor is fed from a non-sinusoidal voltage source. The fundamental component of the source voltage is 440 V. 5^{th} and 7^{th} harmonic components are 20% and 14% of the fundamental respectively.

Higher harmonics can be ignored. Skin-effect causes the rotor resistance to increase three times for the for the 5^{th} harmonic and four times for the 7^{th} harmonic components.

Calculate the de-rating of the motor due to the non-sinusoidal supply. Also calculate the rated motor torque with non-sinusoidal supply.

Neglect friction and windage and core losses.

$$V_{1(l-l)} = 440V, V_{1(ph.)} = \frac{440V}{\sqrt{3}} = 254V$$

$$V_{5(ph.)} = \frac{V_{1(ph.)}}{5} = 0.2 \times 254V = 50.8V$$

$$V_{7(ph.)} = \frac{V_{1(ph.)}}{7} = 0.14 \times 254V = 35.56V$$

$$R_{r'5} = 3 \times R_{r'}, \ R_{r'7} = 4 \times R_{r'}$$

At rated condition:

$$s_{fl} = \frac{N_s - N_r}{N_s} = \frac{1000 - 960}{1000} = 0.04$$

$$I_{r'} = \frac{V_{1(ph.)}(= 254V)}{\sqrt{\left(0.6 + \frac{0.3}{0.04}\right)^2 + (1+1)^2}} = 30.45A$$

$$P_{out} = P_{mech.} = P_{ag} \times (1 - s) = 3 \times (30.45)^2 \times \frac{0.3}{0.04} \times (1 - 0.04)$$
$$= 20.03kW$$

Motor copper loss at rated condition:

$$P_{cu1} = 3 \times (30.45)^2 \times (0.6 + 0.3) = 2.5kW$$

$$I_{s5} = \frac{V_5}{5(X_s + X_{r\prime})} = \frac{50.8V}{5(1+1)\Omega} = 5.08A$$

$$I_{s7} = \frac{V_7}{7(X_s + X_{r\prime})} = \frac{35.56V}{7(1+1)\Omega} = 2.54A$$

$$P_{cu5} = 3 \times (5.08)^2 \times (0.6 + 3 \times 0.3) = 0.116kW$$

$$P_{cu7} = 3 \times (2.54)^2 \times (0.6 + 4 \times 0.3) = 0.035kW$$

$$P_{cuh} = P_{cu5} + P_{cu7} = 0.116 + 0.035kW = 0.1508kW$$

Copper losses allowed by fundamental component is:

$$P_{cu1} = P_{cu} - P_{cuh} = 2.5kW - 0.1508kW = 2.349kW$$

Let the fundamental current be I_{s1}

$$P_{cu1} = 2.349kW = 3 \times (I_{s1})^2 \times (0.6 + 0.3) \Rightarrow I_{s1} = 29.5A$$

$$I_{s1} = 29.5A = \frac{254V}{\sqrt{\left(0.6 + \frac{0.3}{s}\right)^2 + (1+1)^2}} \Rightarrow s = 0.0386$$

$$P_{mech.} = 3 \times (I_{r'})^2 \times R_{r'} \times \left(\frac{1}{s} - 1\right)$$

$$= 3 \times (29.5)^2 \times 0.3 \times \left(\frac{1}{0.038} - 1\right) = 19.5kW$$

$P_{mech.} = 20kW,$ when fed from a sinusoidal supply.

$$\% \text{ derating} = \frac{20kW - 19.5kW}{20kW} = 2.5\%$$

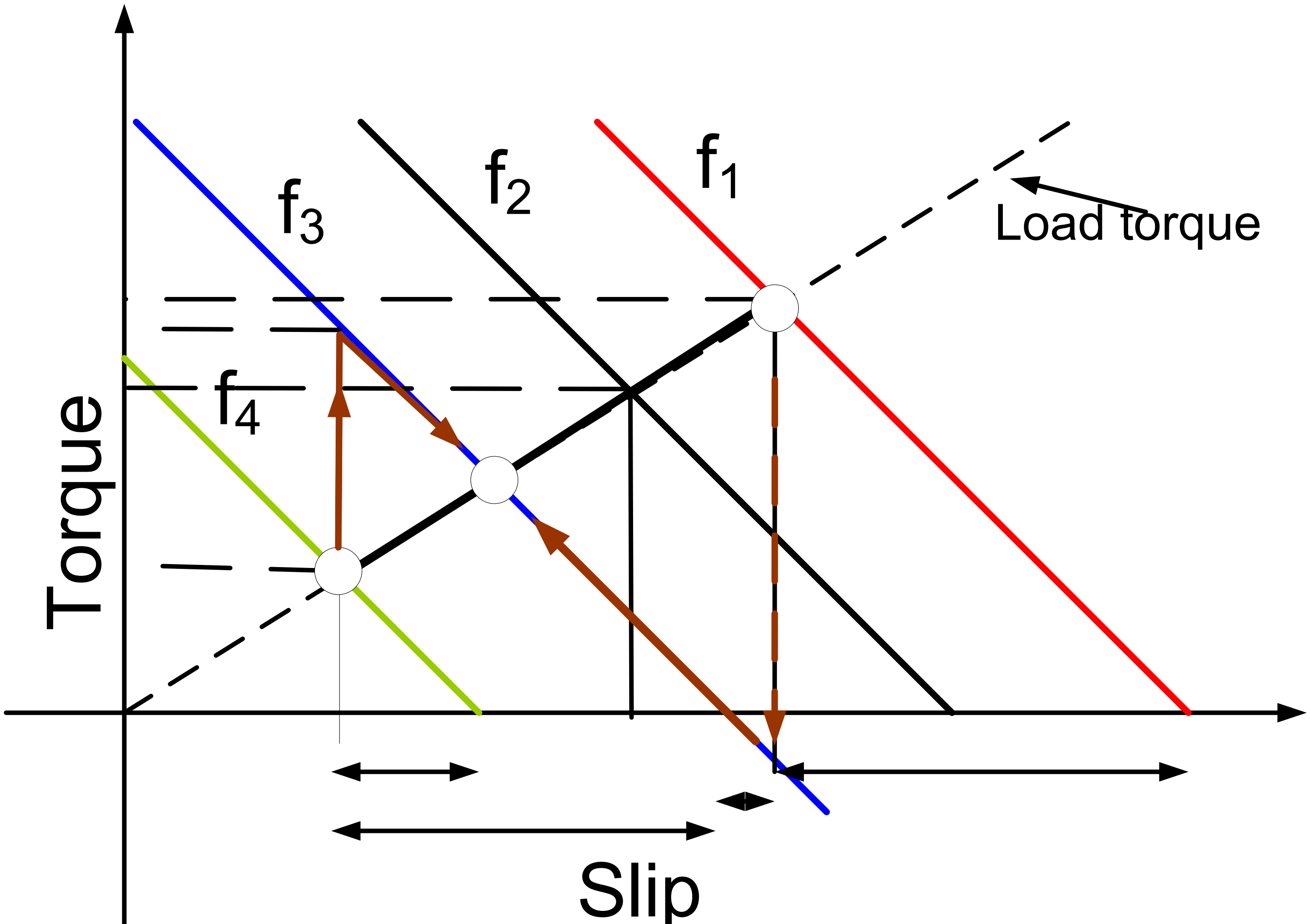
$$T_{ind} = \frac{3}{104.72} \times (29.5)^2 \times \frac{0.3}{0.0386} = 193.8N.m$$

When fed from a pure sinusoidal supply

$$T_{ind} = \frac{3}{104.72} \times (30.45)^2 \times \frac{0.3}{0.04} = 199.22N.m > 193.8N.m$$

Scalar Control of IM Drive

- In scalar(*v/f*) control of IM drive, both torque and flux are functions of stator voltage (*v*) and its frequency (*f*).
- Thus, there is a coupling effect between flux and torque i.e. if torque is to be increased then we need to increase the stator frequency (slip) and when we change the stator frequency the corresponding stator voltage needs to be changed so as to maintain constant flux by making (*v/f*) ratio constant.
- During transient-state it is not possible to maintain the vectorial relationship between the flux- and torque-producing components of the stator current and therefore degrades the drive performance.



Torque
f_3
f_2
f_1
f_4
Load torque
Slip

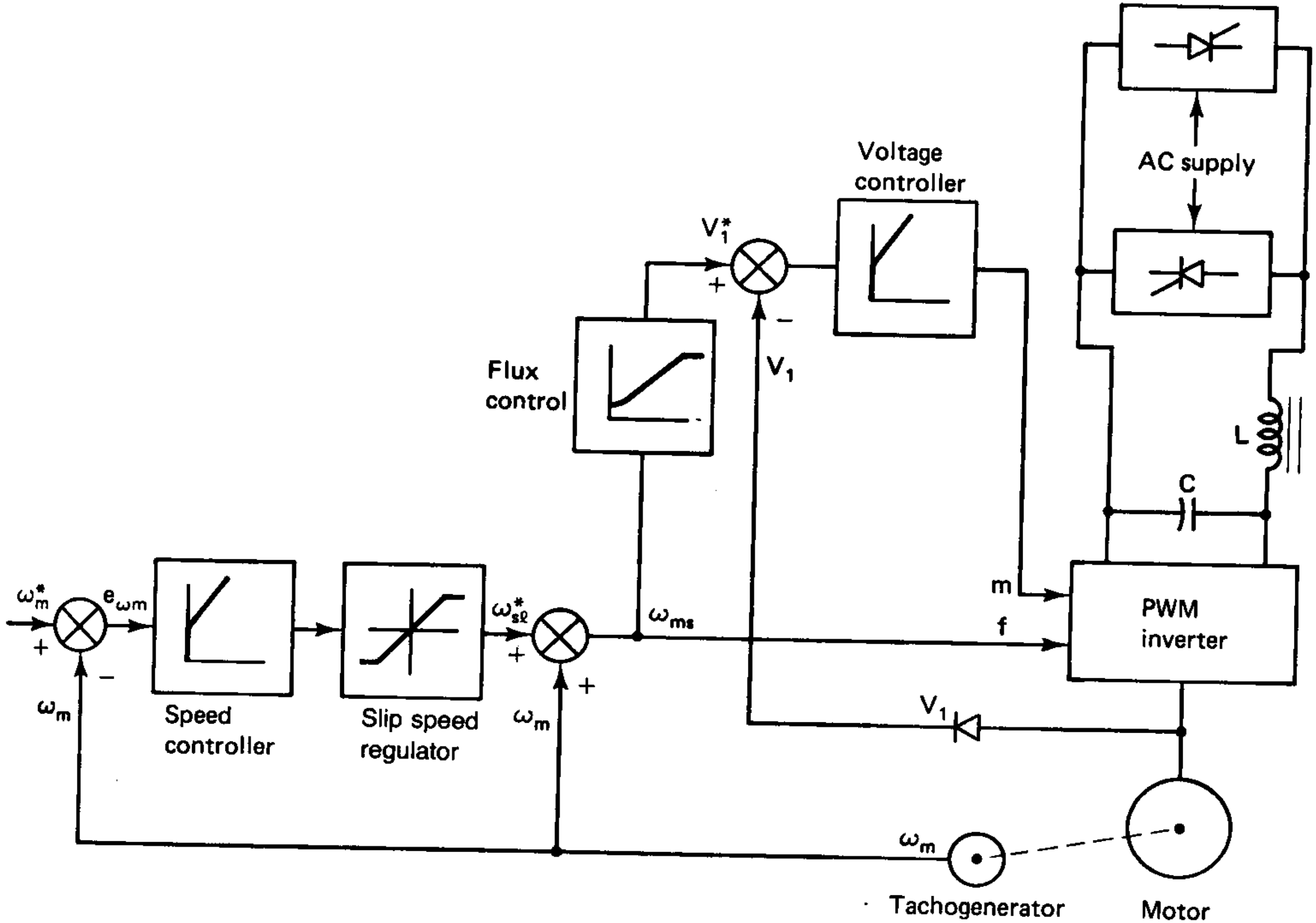


Fig. 4.24: Closed-loop slip-speed controlled PWM inverter drive with regenerative braking.

Field-oriented Control of IM Drive

- The drawback of scalar control of IM drive can be overcome by using vector or field oriented control (FOC).
- In FOC of IM drive, the complex and coupled structure of induction machine is transformed into a rotating reference-frame in which the machine can be considered just as a decoupled separately-excited DC machine.
- Just like in a separately-excited dc motor the armature and field currents are decoupled and therefore can be controlled independently (which allows it to maintain constant flux below the base speed).
- Similarly in ac machines, we split the stator current, i_s into two decoupled orthogonal currents namely, the direct-axis current, i_{ds} and quadrature-axis current, i_{qs} as in Fig. 4.25.

- The q-axis current, i_{qs}, is analogous to armature current, i_a and the d-axis current, i_{ds} is analogous to the field current, i_f of a dc machine respectively.
- The electromagnetic torque is given by

$$T_e = k_t \phi_{rd} i_{qs} \quad (4.53)$$

where φ_{rd} is the rotor flux which is function of i_{ds} and is maintained constant.

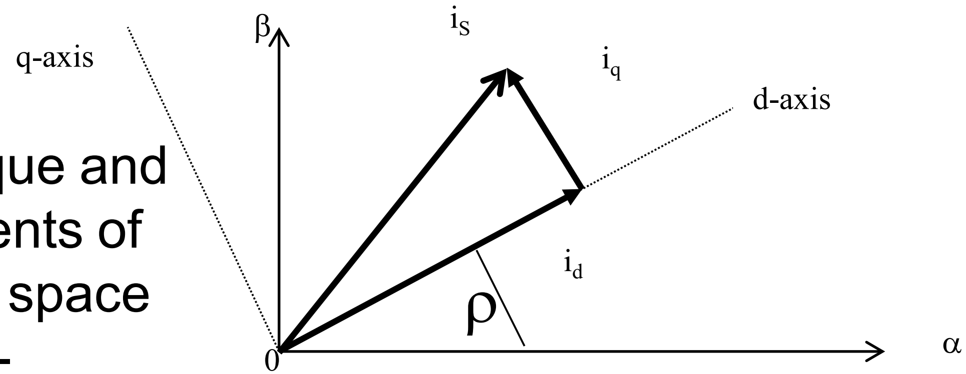


Fig. 4.25:Torque and field components of stator current space vector in field-coordinates

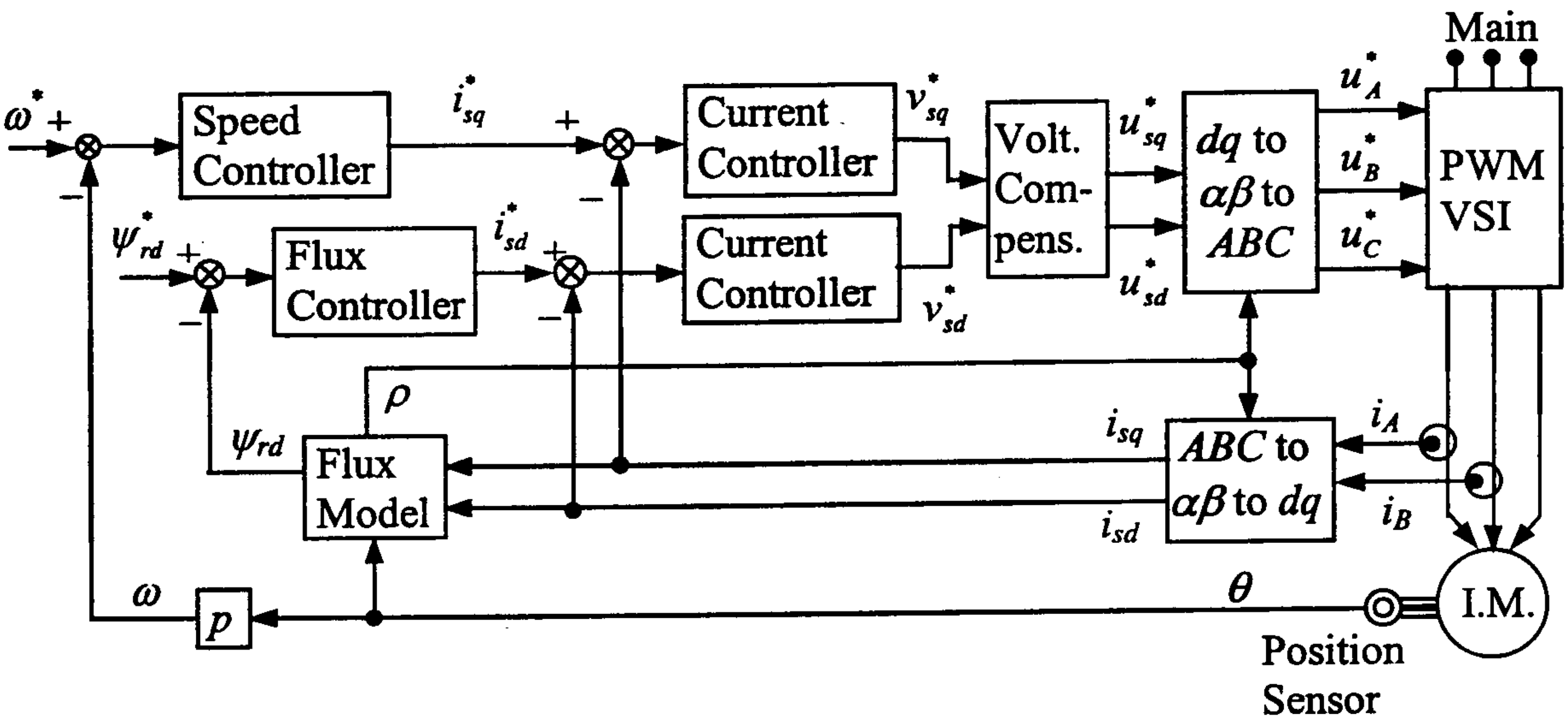


Fig.4.26: Field-oriented control of induction motor drive

- The drawbacks of field-oriented control are:
  - The flux model which is used to estimate the angle ρ is parameter dependent rotor resistance and it varies with temperature which affects the estimation of flux angle.
  - Rotor position encoder is an essential requirement.
  - The transformations of currents from stator reference frame (a-b-c) to rotor reference frame (d-q) and vice-versa take time and therefore the sampling time is large and the drive transient response may be slower.

Direct Torque Control of IM Drive

- DTC was developed by Takahashi (1984/85) and Depenbrock (1985) independently.
- In DTC the electromagnetic torque and flux are controlled directly by selecting optimum inverter voltage vectors.
- The choice of the optimum voltage vector is made to restrict the torque and flux within the hysteresis bands and obtain fastest torque and flux responses without using any PWM operation.

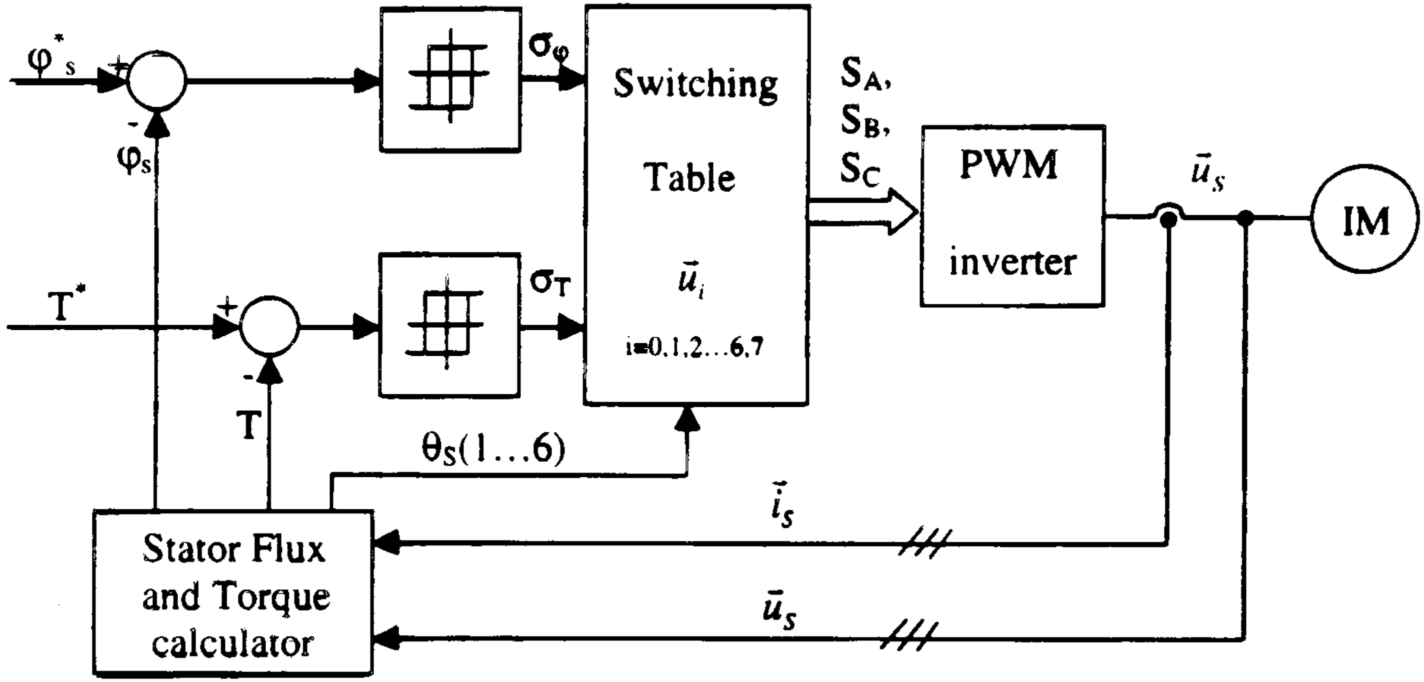


Fig. 4.27: Direct Torque Control of IM Drive

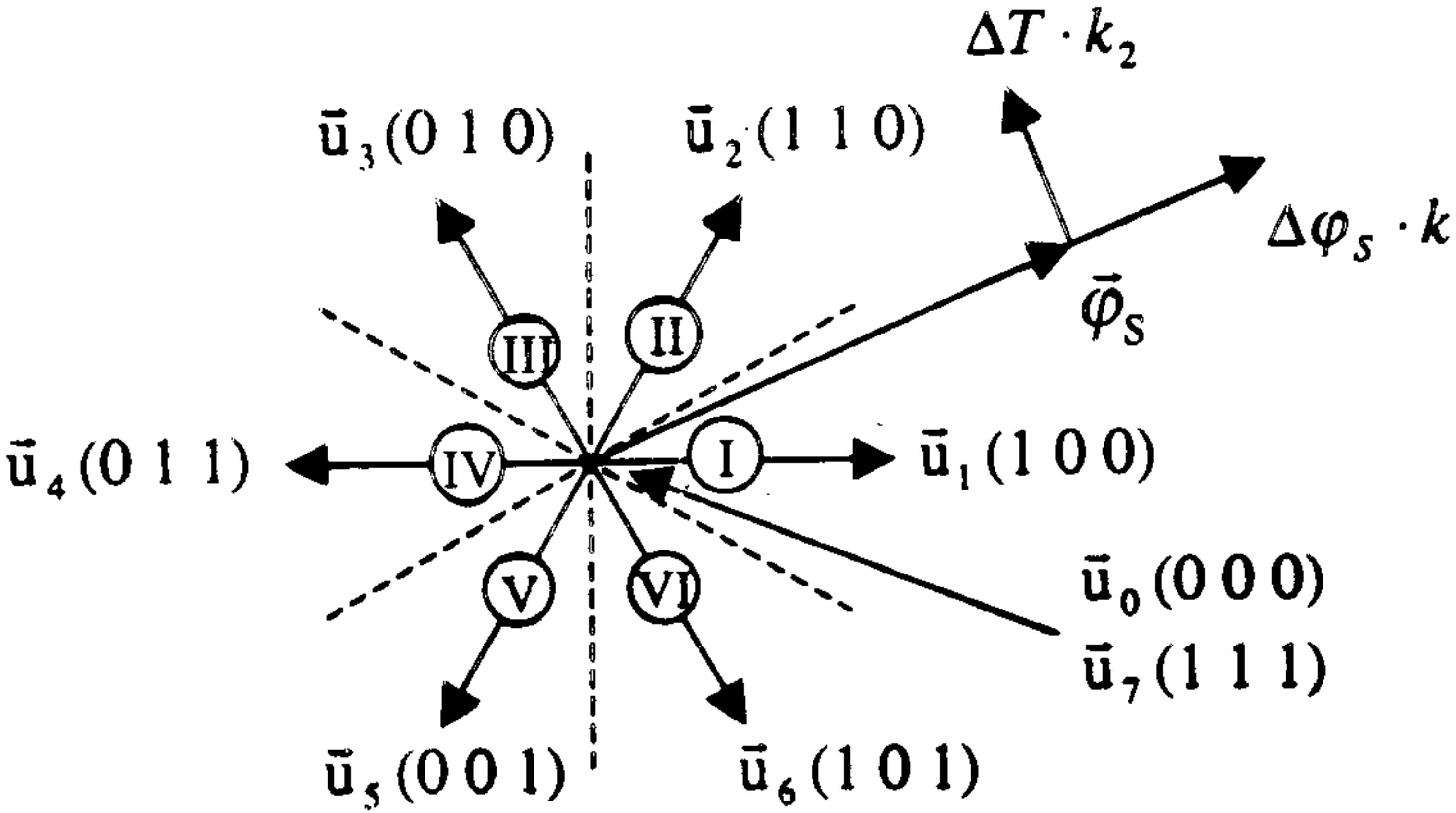


Fig.4.28: Stator Voltage Vectors and their components on flux and torque axes.

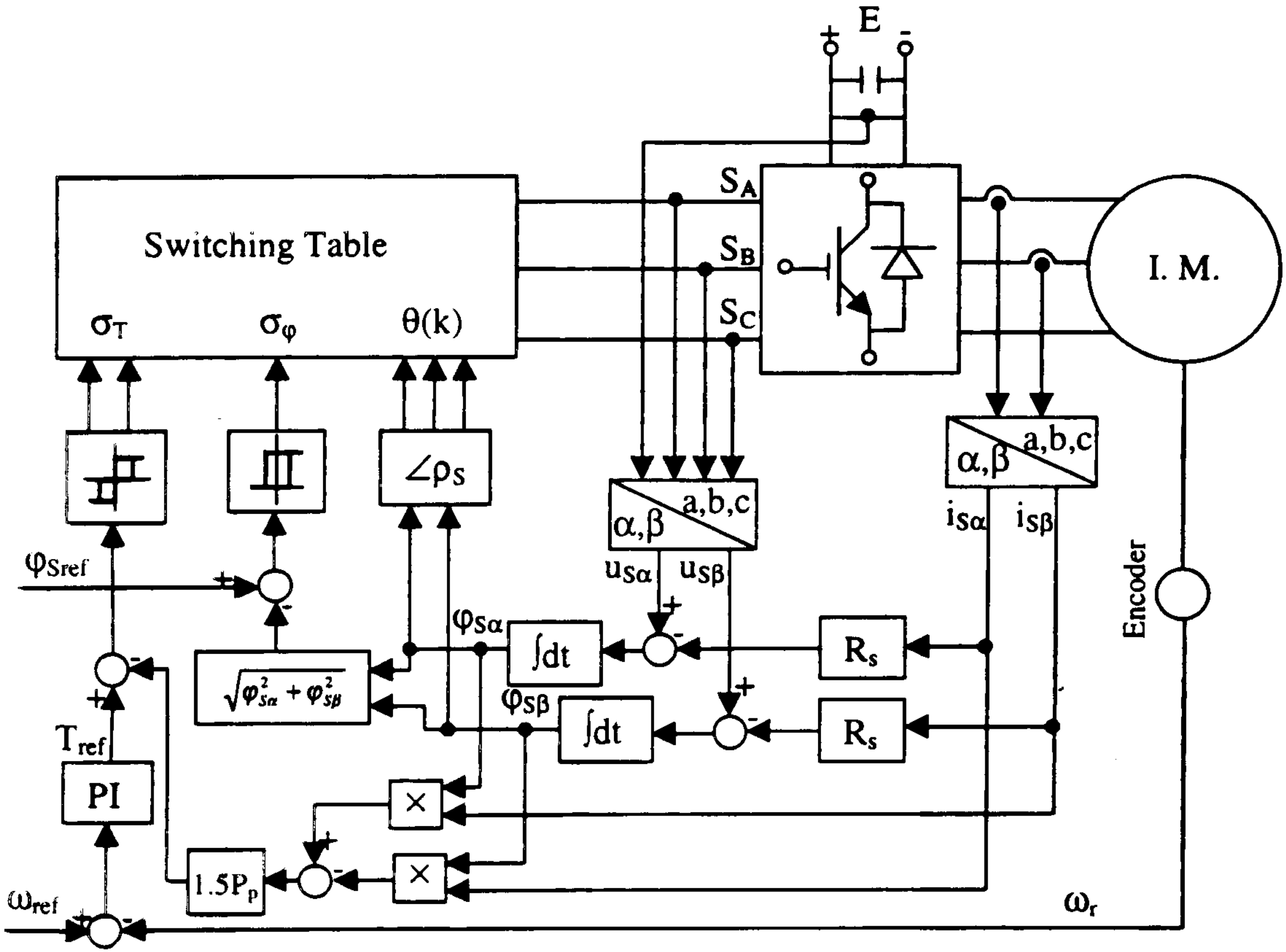


Fig.4.29: Speed control of IM Drive based on DTC.

- Advantages of DTC
  - Fastest torque and flux responses.
  - No encoder is required (except for speed control feedback info.).
  - No inner current control loops.
  - No PWM block so saves in computation time and therefore shorter sampling time can be used.
  - Robust as compared to FOC as flux and torque estimations is dependent on stator resistance R_s only.
- Disadvantage
  - Due to hysteresis control the inverter operates with a variable switching frequency.

Summary

- AC Drives are gradually replacing DC Drives.
- AC Drives: (a) Synchronous Motor Drives (b) Asynchronous motor Drives.
- Asynchronous Motor Drives: (1) Squirrel-cage IM and (2) slip-ring IMs.
- Squirrel-cage IMs are the modern industry work-horses.
- IM speed can be controlled by: (a) stator voltage control and (b) frequency control.
- Frequency control is preferred over stator voltage control due to high-efficiency.

- In variable-frequency control (v/f) ratio is maintained constant for speed control below the base speed and above base-speed flux decreases.
- With (v/f) control the motor torque-speed characteristics are straight-lines and parallel to each other.
- With constant-flux operation the maximum-torque remains constant for varying frequency (below base frequency).
- Inverters are used to produce variable-voltage and variable-frequency AC supply from DC supply.

- Inverters can be (a) VSI and (b) CSI.
- VSIs are more widely used and can be (a) six-step square-wave type or (b) PWM type.
- Six-step square-wave type VSIs are used only for very high-power drives and produce considerable amount of low-frequency(5^{th}, 7^{th},11^{th} and 13^{th}) harmonics.
- PWM VSIs are more popular due to their superior performance.
- Harmonics produced by VSIs do not contribute significantly to average torque but do produce torque-pulsations as well as losses in the machine.

- Scalar-control does provide satisfactory steady-state performance but dynamic performance is not as good as separately-excited DC drives.
- The drawback of scalar-control is overcome by using vector-control which provides de-coupled flux and torque control.
- The alternative to vector-control is DTC which provides fastest torque-response.

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