National University of Singapore

Department of Electrical & Computer Engineering

EE4502: Electric Drives & Control

TUTORIAL - 1: Fundamentals of Electrical Drives (Solution)

Year 2021-2022

1. For operation in quadrant-I, we have at steady-state

   $T_m\,(= 200 - 0.3N_A \text{ N.m}) = T_l\,(= 120 \text{ N.m}) \Rightarrow N_{Q-I} = 266.7 \text{ rpm} \text{and} T_{m-Q-I} = 120 \text{ N.m}$

   For operation in quadrant-II, we have at steady-state

   $T_m\,(= 200 - 0.3N_B \text{ N.m}) = T_l\,(= -100 \text{ N.m}) \Rightarrow N_{Q-II} = 1000 \text{ rpm and } T_{m-Q-II} = -100 \text{ N.m}$

   For operation in quadrant-III, we have at steady-state

   $T_m\,(= -200 - 0.3N_C \text{ N.m}) = T_l\,(= -100 \text{ N.m}) \Rightarrow N_{Q-III} = -333.33 \text{ rpm and } T_{m-Q-III} = -100 \text{ N.m}$

   For operation in quadrant-IV, we have at steady-state

   $T_m\,(= -200 - 0.3N_D \text{ N.m}) = T_l\,(= 120 \text{ N.m}) \Rightarrow N_D = -1066.7 \text{ rpm and } T_{m-Q-IV} = 120 \text{ N.m}$

   For all the equilibrium points we have,

   $$\frac{dT_l}{dN} = 0$$

   and

   $$\frac{dT_m}{dN} = -0.3$$

   Thus, the necessary condition for stability i.e.

   $$\frac{dT_l}{dN}[= 0] > \frac{dT_m}{dN} = [-0.3]$$

   is satisfied and therefore the equilibrium points are stable.

2. The load power requirement is given by

   $$P_{load} = F \times v = 1000N \times 25m/s = 25kW$$

   The load-power is supplied by a rotational motor through a speed reduction gear-box which has an efficiency of 87%. Thus, the motor output power is

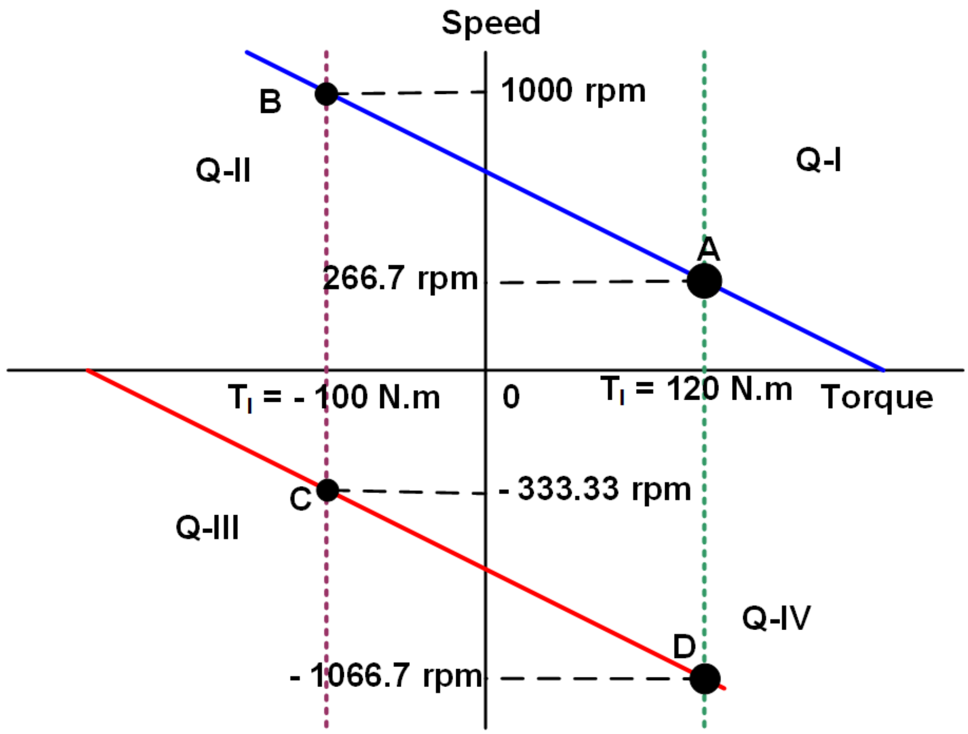

Figure 1:

$$P_{out} = \frac{P_{load}}{\eta} = \frac{25\,kW}{0.87} = 28.73\,kW$$

The linear velocity of the strip is related to the angular speed of the motor by the relationship $v = r \times \omega$. Thus, for the two different diameters of the mandrel the corresponding two speeds are:

$$\omega_{l,1} = \frac{v}{r_1} = \frac{25}{0.075\,m} = 333.33\,rad/s \Rightarrow \omega_{m1} = \frac{\omega_{l,1}}{a = 0.5} = 666.67\,rad/s$$

The corresponding motor torque at this speed is

$$T_{em,1} = \frac{P_{out}}{\omega_{m1}} = \frac{28.73\,kW}{666.67\,rad/s} = 43.1\,N.m$$

$$\omega_{l,2} = \frac{v}{r_2} = \frac{25}{0.125\,m} = 200.0\,rad/s \Rightarrow \omega_{m2} = \frac{\omega_{l,2}}{a = 0.5} = 400.0\,rad/s$$

(Please take note as to why we choose $a = 0.5$ rather than $a = 2$.)

The corresponding motor torque at this speed is

$$T_{em,2} = \frac{P_{out}}{\omega_{m1}} = \frac{28.73\,kW}{400.0\,rad/s} = 71.8\,N.m$$

3. The motor and winch inertias and speed are:

$$J_m = 0.5\ kg.m^2 \text{ and } J_w = 0.3\ kg.m^2, \omega_m = 104.72 \text{ rad/s}$$

The equivalent inertia referred to the motor shaft is:

$$\begin{aligned} J_{eq} &= J_m + J_w + M\left(\frac{v}{\omega_m}\right)^2 \\ &= 0.5 + 0.3 + 500\left(\frac{1.5}{104.72}\right)^2 \\ &= 0.9026 \;\; kg.m^2 \end{aligned}$$

Similarly, the equivalent torque referred to the motor shaft is:

$$\begin{aligned} T_{eq} &= T_{lo} + (F_1)\left(\frac{v_1}{\omega_m}\right) \\ &= 100N.m + (500 \times 9.81)\left(\frac{1.5}{104.72}\right) \\ &= 170.26 \;\; \text{N.m} \end{aligned}$$

The dynamic torque required to accelerate the drive is

$$T_{dyn} = J_{eq} \times \frac{d\omega_m}{dt} = 0.9026 \times \frac{\frac{2\pi}{60} \times (1000 - 0)}{12} = 7.9 \; N.m$$

Thus, the total motor torque required is

$$T_{em} = 170.26 + 7.9 = 178.1 \; N.m$$ = (motor torque) + (dynamic torque)

4. For $V = 100V, N = 1000$ rpm, $R_a = 10.0\,\Omega$, $J = 0.05kg.m^2$ and $P_{in} = 250W$, we have,

$$P_{in} = V_a \times I_a \Rightarrow 250\;W = 100\;V \times I_a \Rightarrow I_a = 2.5\;A$$

$$E_a = V_a - I_a \times R_a = 100\;V - 2.5\;A \times 10\;\Omega = 75\;V = k\phi \times \frac{2\pi}{60} \times 1000\;\text{rpm} \Rightarrow k\phi = 0.72\;\text{V/(rad/s)}$$

$$T_l = T_{em} = 0.72 \times 2.5\;A = 1.8\;\text{N.m}$$

During speed reversal we have the armature voltage reversed from 100 V to -100 V.

Thus, the equation of motion when the motor is in quadrant-II is given by

$$\begin{aligned} T_{em} &= T_l + J_{eq}\frac{d\omega_m}{dt} \\ 0.72 \times \left[\frac{-100V - 0.72 \times \omega_m}{10\;\Omega}\right] &= 1.8\;N.m + 0.05\;kg.m^2 \times \frac{d\omega_m}{dt} \\ 0.964\frac{d\omega_m}{dt} + \omega_m &= -173.6 \\ \omega_m(t) &= -173.6\,(1 - exp(-t/0.964)) + 104.72exp(-t/0.964) \end{aligned}$$

At time t, $\omega_m(t) = 0$ rad/s (speed is reducing from a higher steady-state speed of 104.72 rad/s to lower value of 0 rad/s) and substituting in the above expression for speed we can solve for time, t.

$$\omega_m(t) = 0 = -173.6\,(1 - exp(-t/0.964)) + 104.72exp(-t/0.964) \Rightarrow t = 0.45\,\text{sec}$$

Now, when the motor drive enters into the third-quadrant the load torque is expected to change its polarity and therefore it becomes -1.8 N.m.

Thus, the new equation of motion becomes

II I
III IV

$(Kg)\left(\frac{Tem - Kg\,wm}{Ra}\right]$

$$\begin{aligned}
T_{em} &= T_l + J_{eq}\frac{d\omega_m}{dt} \\
0.72 \times \left[\frac{-100V - 0.72 \times \omega_m}{10\,\Omega}\right] &= -1.8\,N.m + 0.05\,kg.m^2 \times \frac{d\omega_m}{dt} \\
0.964\frac{d\omega_m}{dt} + \omega_m &= -103.8 \\
\omega_m(t) &= -103.84\,(1 - exp(-t/0.964))
\end{aligned}$$

At time t, $\omega_m(t) = -0.98 \times 103.84$ rad/s (speed is increasing from 0 rad/s in the reverse direction to higher value of -103.84 rad/s) and substituting in the above expression for speed we can solve for time, t.

$$\omega_m(t) = 0.98 \times (0 - 103.84)\text{rad/s} = -103.84\,(1 - exp(-t/0.964)) \Rightarrow t = 3.77\,\text{sec}$$

Thus, the total time taken by the drive from 1000 rpm in quadrant-I to -103.84 rad/s in quadrant-3 is $t = 0.45 + 3.8 = 4.25$ sec.

5. We have at steady-state

$T_{em}\,(= 1886.8 - 18.86 \times \omega_m \ \text{N.m}) = T_l\,(= \omega_m \ \text{N.m}) \Rightarrow \omega_m = 94.97 \ \text{rad/s} = 907 \ \text{rpm}$ and

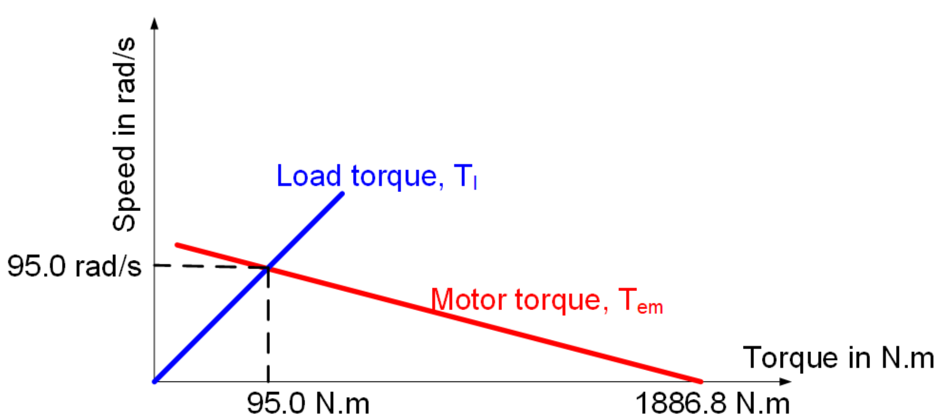

Figure 2:

$$T_{em} \ \text{at} \ (\omega_m = 94.97 \ \text{rad/s}) = 95.0 \ \text{N.m}$$

For steady-state equilibrium point to be stable the necessary condition is:

$$\frac{dT_l}{d\omega_m} > \frac{dT_{em}}{d\omega_m}$$

In our case

$$\frac{dT_l}{d\omega_m} = 1.0 \quad \text{and} \quad \frac{dT_{em}}{d\omega_m} = -18.86$$

For the equilibrium operating point ($\omega_m = 94.97\ rad/s$), we have

$$\left[\frac{dT_l}{d\omega_m} = 1.0\right] > \left[\frac{dT_{em}}{d\omega_m} = -18.86\right]$$

thus, the operating point is <u>stable</u>.

6. The parameters given are:

$R_a = 0.35\,\Omega, k\phi = 0.5\,V/(rad/s), J_m = 0.02\,kg.m^2, J_l = 0.04\,kg.m^2, T_L = 2\,N.m, I_{a-max} = 15\,A,$

$$\omega_m = 300\ \text{(rad/s)}$$

$$T_{m,max} = k\phi I_{a,max} = 0.5 \times 15 = 7.5\,N.m$$

$$J_{eq} = J_m + J_l = 0.06\,kg.m^2$$

Applying eqn. of motion we have

$$7.5\,N.m = 2.0\,N.m + 0.06\,kg.m^2\ \times \frac{d\omega_m}{dt} \Rightarrow \frac{d\omega_m}{dt} = 91.67\,rad/s^2$$

The time taken to reach the steady-state speed of 300 rad/s is

$$dt = \frac{d\omega_m}{91.67\,rad/s} = 3.27\,sec$$

From acceleration we have

$$\omega_m(t) = 91.67 \times t + c \Rightarrow c = 0 \Rightarrow \omega_m(t) = 91.67 \times t$$

The armature voltage is given by

$$v_a(t) = e_a(t) + i_a(t) \times R_a = 0.5 \times 91.67t + 15A \times 0.35\,\Omega = 45.84t + 5.25\,V$$

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