Formula Sheet for EE4502

1. Chapter1:Introduction to Electric Drives

$$T_{em} \text{ or } T_m = T_l + J\frac{d\omega_m}{dt} \quad (1.1) \qquad T_{fric.} = T_S + T_V + T_C \approx T_V\left(= B\omega_m\right) \quad (1.2)$$

$$\left[\frac{dT_l}{d\omega_m} - \frac{dT_{em}}{d\omega_m}\right] > 0 \quad (1.12) \qquad \frac{JR_a}{(k_e)^2}\frac{d\omega_m}{dt} + \omega_m = \left(\frac{V}{k_e} - \frac{R_a T_l}{(k_e)^2}\right)(= \omega_{ss})$$

$$\omega_m(t) = \omega_{ss}\left(1 - e^{-t/\tau_m}\right) + \omega_{mo}e^{-t/\tau_m} \quad (1.13)$$

$$J_{eq} = \left(J_m + J_{l1}\right) + a^2 J_{l2} \quad (1.15)$$

$$T_l = T_{l1} + \frac{aT_{l2}}{\eta_1} \quad (1.16)$$

$$J_{eq} = \left(J_{l1} + J_m\right) + \left(\frac{v}{\omega_m}\right)^2 M_2 \quad (1.17) \qquad T_l = T_{l1} + \left(\frac{F_2}{\eta_2}\right) \times \left(\frac{v}{\omega_m}\right) \quad (1.18)$$

2. Chapter2: Sizing of Electric Motors for VSD

$$\tau\frac{d\theta}{dt} + \theta = \theta_{ss} \quad (2.3) \qquad \theta(t) = \theta_{ss}\left(1 - e^{-t/\tau}\right) + \theta_1 e^{-t/\tau} \quad (2.4)$$

$$\frac{C}{D'}\frac{d\theta}{dt} + \theta = 0 \quad (2.5), \quad \theta(t) = \theta_2 e^{-t/\tau'} \quad (2.6)$$

$$\frac{C}{D'}\frac{d\theta}{dt} + \theta = \frac{p_1'}{D'} \quad (2.7), \quad \theta(t) = \theta'_{ss}(1 - e^{-t/\tau'}) + \theta_2 e^{-t/\tau'} \quad (2.8)$$

$$i_{eq} = \sqrt{\frac{I_{1,rms}^2 \times t_1 + I_{2,rms}^2 \times t_2 + I_{3,rms}^2 \times t_3 + \ldots + I_{n,rms}^2 \times t_n}{t_1 + t_2 + t_3 + \ldots + t_n}} \quad (2.9)$$

$$\text{Motor overload factor for short-term duty}: k = \sqrt{\frac{1+\alpha}{1 - e^{-t_r/\tau}} - \alpha} \quad (2.15)$$

$$\text{Overloading factor for Intermittent Periodic-duty } k = \sqrt{(\alpha+1)\frac{1 - e^{-(t_r/\tau_r + t_s/\tau_s)}}{1 - e^{-(t_r/\tau_r)}} - \alpha} \quad (2.19)$$

3. Chapter3: DC Drives

$$E_a = k_e\phi\omega_m = k_E\omega_m \quad (3.1),$$

$$V_a = E_a + I_a R_a \quad (3.2),$$

$$T_{em} = k_e\phi I_a = k_T I_a \quad (k_E = k_T) \quad (3.3)$$

$$e_a = k_e\phi\omega_m = k_E\omega_m \quad (3.4)$$

$$v_a = e_a + i_a R_a + L_a\frac{di_a}{dt} \quad (3.5)$$

$$T_{em} = k_e\phi i_a = k_T i_a \quad (3.6)$$

$$\frac{d\omega_m}{dt} = \frac{1}{J_{eq}}\left(T_{em} - T_{mech} - B\omega_m\right) \quad (3.7)$$

$$\omega_m = \frac{V_a}{k_e\phi} - \frac{R_a}{(k_e\phi)^2}T_{em} \quad (3.8)$$

$$\omega_m = \frac{\frac{2V_m}{\pi}\cos\alpha}{k_e\phi} - \frac{R_a}{(k_e\phi)^2}T_{em} \quad (3.10)$$

$$V_A = \frac{1}{T}\int_0^T v_a\,dt = \delta V_s \quad (3.11)$$

$$V_A = \frac{1}{T}\int_0^T v_a dt = \frac{1}{T}\int_{\delta T}^T V_s dt = \delta' V_s, \ (3.22) V_s = \frac{V_A}{\delta'} = \frac{V_A}{1-\delta} \Rightarrow (V_A \le V_s \le \infty).$$

Chapter 4: Induction Motor Drives

Induction motor torque under rated operating condition	$T_{e,rated} = \left[\dfrac{3}{\omega_{ms(rated)}}\right] \dfrac{V_{ph}^2}{\left(R_s + \dfrac{R_r'}{s_{rated}}\right)^2 + \left(X_s + X_r'\right)^2} \left(\dfrac{R_r'}{s_{rated}}\right)$
Induction motor maximum torque with variable frequency operating condition below rated frequency	$T_{e,\max} = \left[\dfrac{3}{2\omega_{ms(rated)}}\right] \dfrac{V_{rated}^2}{\dfrac{R_s}{a} \pm \sqrt{\left(\dfrac{R_s}{a}\right)^2 + \left(X_s + X_r'\right)^2}}$
Induction motor maximum torque with variable frequency operating condition above rated frequency	$T_{\max} = \dfrac{3}{2\omega_{ms(rated)}a}\left[\dfrac{V_{rated}^2}{(R_s) \pm \sqrt{(R_s)^2 + a^2\left(X_{s(rated)} + X_{r'(rated)}\right)^2}}\right]$
Rotor copper losses	$P_{RCL} = sP_{AG} = 3I_2^2 R_2$
Converted/mechanical power output	$P_{conv} = (1-s)P_{AG}$
Induced voltage	$V \approx E = 4.44 \times f_s \times \phi \times N \times k_w$
Break down torque	$T_{ind,\max} = \dfrac{3}{2\omega_s} \dfrac{V_{ph}^2}{\left[R_s + \sqrt{R_s^2 + (X_s + X_r')^2}\right]}$
Slip at breakdown torque	$s_{\max} = \dfrac{R_r'}{\sqrt{R_s^2 + (X_s + X_r')^2}}$
Starting torque for an induction motor	$T_{ind,start} = \dfrac{3}{\omega_s} \dfrac{V_{ph}^2}{(R_s + R_r')^2 + (X_s + X_r')^2} \dfrac{R_r'}{1}$
Starting current for an induction motor	$I_{2,start} = \dfrac{V_{ph}}{\sqrt{(R_s + R_r')^2 + (X_s + X_r')^2}}$
Induced torque	$T_{ind} = \dfrac{P_{conv}}{\omega_m} = \dfrac{(1-s)P_{AG}}{(1-s)\omega_s} = \dfrac{P_{AG}}{\omega_s}$
Rotor frequency	$f_r = \dfrac{(n_s - n_m)P}{120} = \dfrac{sn_sP}{120} = s\left(\dfrac{n_sP}{120}\right) = sf_e$
Load torque for a fan load	$T_l = k(1-s)^2$

Three-phase AC Supply

$$\textit{For star-connected load}: V_L = \sqrt{3}V_{ph}, \qquad I_L = I_{ph}$$

$$\textit{For delta-connected load}: V_L = V_{ph}, \qquad I_L = \sqrt{3}I_{ph}$$