National University of Singapore

Department of Electrical & Computer Engineering

EE-4502:Electric Drives and Control

Tutorial - 4 (Induction Motor Drives - Solution)

Year 2021-22

1. The synchronous speed is given by

$$N_s = \frac{120 \times 50}{4} = 1500\, rpm$$

The input power is given by

$$P_{in} = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 480\, V \times 60\, A \times 0.85 = 42.4\, kW$$

The air-gap power is given by

$$P_{ag} = P_{in} - P_{SCL} - P_{core} = 42.4\, kW - 2\, kW - 1.8\, kW = 38.6\, kW$$

The slip is given by

$$P_{RCL} = s \times P_{ag} \Rightarrow s = \frac{700\, W}{38.6\, kW} = 0.018$$

The rotor speed is given by

$$N_m = (1 - s) \times N_s = (1 - 0.018) \times 1500\, rpm = 1473\, rpm$$

The rotor frequency, f_r is given by

$$f_r = s \times f_s = 0.018 \times 50\, Hz = 0.9\, Hz$$

The converted output power is given by

$$P_{conv.} = P_{ag} - P_{RCL} = 38.6\, kW - 0.7\, kW = 37.9\, kW$$

The shaft output power is given by

$$P_{out} = P_{conv.} - P_{F\&W} = 37.9\, kW - 0.6\, kW = 37.3\, kW$$

The shaft torque at rated load is given by

$$T_e = \frac{P_{out}}{\omega_m} = \frac{37.3\, kW}{\frac{2\pi}{60} \times 1473\, rpm} = 241.8\, N.m$$

The efficiency is given by

$$\eta = \frac{P_{out}}{P_{in}} = \frac{37.3\, kW}{42.4\, kW} = 88\%$$

2. The parameters given for the IM are:

$$400\,V, 3-phase, f = 50\,Hz, N_{r(rated)} = 1370\,rpm, \triangle - connected$$

$$R_s = 2.0\ \Omega, R_{r'} = 5.0\ \Omega, X_s = 5.0\ \Omega, X_{r'} = 5.0\ \Omega, X_m = 80\ \Omega,$$

The rated-slip is:

$$s_{rated} = \frac{1500\,rpm - 1370\,rpm}{1500\,rpm} = 0.0867$$

The rated rotor current referred to the stator side is:

$$I_{r'} = \frac{400\,V}{\sqrt{\left(2 + \frac{5}{0.0867}\right)^2 + (5+5)^2}} = 6.61\,A$$

The rated torque is given by

$$T_{rated} = \frac{3}{\frac{2\pi}{60} \times 1500} \times (6.61)^2 \times \frac{5}{0.0867} = 48.14\,N.m$$

It is given that load-torque varies in a square manner with the speed. Thus, we have

$$T_l = k{\omega_m}^2 = k \times [\omega_{ms}(1-s)]^2 = k'(1-s)^2$$

At rated and steady-state condition, we have

$$T_{e(rated)} = T_l = 48.14\,N.m = k'(1-0.0867)^2 \Rightarrow k' = 57.7 \Rightarrow T_l = 57.7(1-s)^2$$

The parameter given is $N_r = 1200\,rpm$.

At this speed the slip is

$$s = \frac{1500\,rpm - 1200\,rpm}{1500\,rpm} = 0.2$$

$$T_l = 57.7(1-0.2)^2 = 36.9\,N.m$$

At 1200 rpm with the load torque of 36.9 N.m, we have

$$T_{l@1200rpm} = 36.9\,N.m = \frac{3}{\frac{2\pi}{60} \times 1500} \times \left(\frac{{V_{ph}}^2\,V}{\left(2 + \frac{5}{0.2}\right)^2 + (5+5)^2}\right)^2 \times \frac{5}{0.2} \Rightarrow V_{ph} = 253.2\,V$$

$$I_{r'} = \frac{253.2\,V}{\left(2 + \frac{5}{0.2}\right) + j\,(5+5)^2} = 8.246 - j3.054$$

$$I_m = \frac{V_{ph}}{jX_m} = \frac{253.2}{j80} = -j3.165\,A$$

$$I_s = I_{r'} + I_m = 8.246 - j6.219 = 10.328\angle{-37^0}$$

Line current is

$$I_{sL-L} = \sqrt{3} \times 10.33 = 17.9\,A$$

3. The parameters given for the IM are:

$$440\,V, f = 60\,Hz, P = 4, N_{r(rated)} = 1746\,rpm, T_{rated} = 40\,N.m$$

The synchronous speed is:

$$N_s = \frac{120 \times 60}{4} = 1800\,rpm$$

The slip-speed at rated load is:

$$N_{sl(rated)} = 1800 - 1746 = 54\,rpm$$

(a) The torque-speed characteristics at different frequencies are shown in Fig. 1.

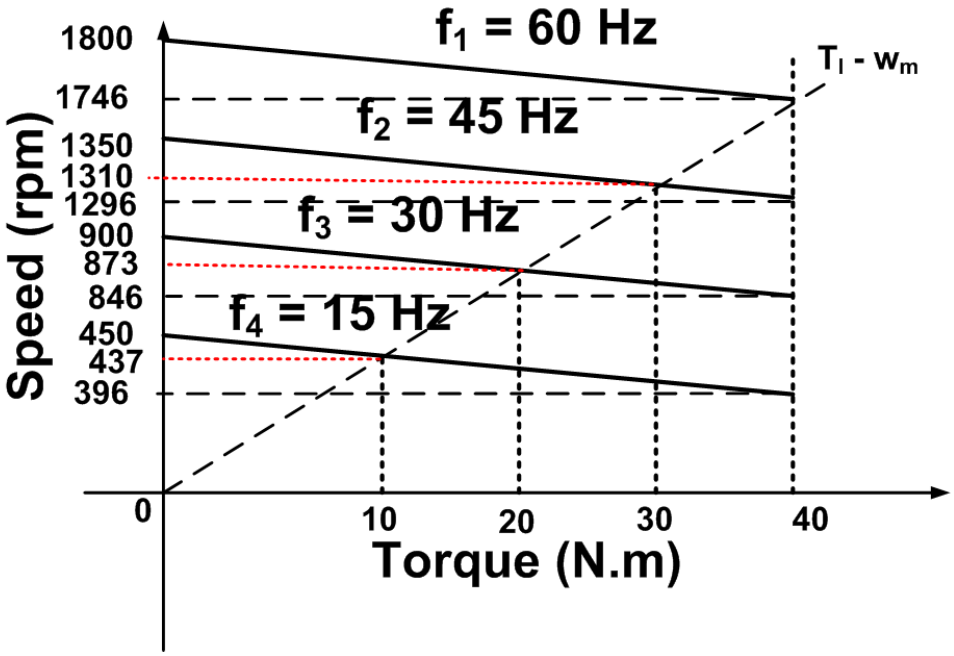

Figure 1:

(b) The load-torque is proportional to speed and is given by:

$$T_l = k \times \omega_m \Rightarrow 40\,N.m = k \times 1746 \Rightarrow k = 0.0229\,N.m/rpm$$

The electromagnetic torque is given by:

$$N_r = m \times T_e + N_s \Rightarrow$$

At rated condition we have

$$1746\,rpm = m \times 40\,N.m + 1800\,rpm \Rightarrow m = -1.35$$

Thus,

$$T_e = \frac{N_s - N_r}{1.35}$$

At steady-state we have

$$T_e = T_l \Rightarrow \frac{N_s - N_r}{1.35} = 0.0229 \times N_r \Rightarrow N_r = \frac{N_s}{1.0309}$$

For

$$f = 60\ Hz, N_s = 1800\ rpm \Rightarrow N_r = \frac{1800}{1.0309} = 1746\ rpm$$
$$\Rightarrow T_l = 0.0229 \times 1746 = 40 N.m$$
$$f = 45\ Hz, N_s = 1350\ rpm \Rightarrow N_r = \frac{1350}{1.0309} = 1309.5\ rpm$$
$$\Rightarrow T_l = 0.0229 \times 1309.5 = 30 N.m$$
$$f = 30\ Hz, N_s = 900\ rpm \Rightarrow N_r = \frac{900}{1.0309} = 873\ rpm$$
$$\Rightarrow T_l = 0.0229 \times 1746 = 20 N.m$$
$$f = 15\ Hz, N_s = 450\ rpm \Rightarrow N_r = \frac{450}{1.0309} = 436.5\ rpm$$
$$\Rightarrow T_l = 0.0229 \times 436.5 = 10 N.m$$

4. The parameters given for the IM are:

$$400\,V, 3 - phase, f = 50\ Hz, N_{r(rated)} = 925\ rpm, \triangle - connected$$

$$R_s = 0.2\ \Omega, R_{r'} = 0.3\ \Omega, X_s = 0.5\ \Omega, X_{r'} = 1.0\ \Omega,$$

(a)

$$T_{max} = \frac{3}{2a\omega_{ms}} \times \frac{{v_{ph}}^2}{R_s + \sqrt{{R_s}^2 + a^2 \left(X_s + X_{r'}\right)^2}}$$

At rated frequency we have, $f = 50\ Hz$, $a = 1$.

$$T_{max(a=1)} = \frac{3}{2 \times 1 \times \frac{2\pi}{60} 1000} \times \frac{400^2}{0.2 + \sqrt{0.2^2 + 1^2 \left(0.5 + 1.0\right)^2}} = 1337.7\ N.m$$

At frequency , $f = 100\ Hz$, we have $a = 2$.

$$T_{max(a=2)} = \frac{3}{2 \times 2 \times \frac{2\pi}{60} 1000} \times \frac{400^2}{0.2 + \sqrt{0.2^2 + 1^2 \left(0.5 + 1.0\right)^2}} = 357.35\ N.m$$

The ratio of maximum torques is:

$$\frac{T_{max(a=2)}}{T_{max(a=1)}} = \frac{357.35}{1337.7} = 0.267$$

(b)

$$I_{r'(rated)} = \frac{400\ V}{\sqrt{\left(0.2 + \frac{0.3}{0.075}\right)^2 + (0.5 + 1.0)^2}} = 89.7\ A$$

$$T_{rated} = \frac{3}{\frac{2\pi}{60} \times 1000} \times (89.7)^2 \times \frac{0.3}{0.075} = 921.8\ N.m$$

$$I_{r'} = 89.7\ A = \frac{400\ V}{\sqrt{\left(0.2 + \frac{0.3}{s}\right)^2 + \left(\frac{75}{50}(0.5 + 1.0)\right)^2}} \Rightarrow s = 0.082$$

$$T_{(a=1.5)} = \frac{3}{(1.5) \times \frac{2\pi}{60} \times 1000} \times (89.7)^2 \times \frac{0.3}{0.082} = 560.9\ N.m$$

The ratio of maximum torques is:

$$\frac{T_{max(a=2)}}{T_{max(a=1)}} = \frac{560.9}{921.8} = 0.608$$

(c)

$$f = 30\ Hz, a = \frac{30}{50} = 0.6, s = \frac{60}{\frac{120 \times 30}{6}} = 0.1$$

$$T_e = \frac{3}{(0.6) \times \frac{2\pi}{60} \times 1000} \times \frac{(0.6 \times 400\ V)^2}{\left(0.2 + \frac{0.3}{0.1}\right)^2 + ((0.6) \times (0.5 + 1.0))^2} \times \frac{0.3}{0.1}$$
$$= 746.66\ N.m$$

5. The parameters given for the IM are:

$$440\ V, 3 - phase, f = 50\ Hz, N_{r(rated)} = 925\ rpm, \triangle - connected$$

(a) For $f = 30\ Hz$ and $T_m = \frac{1}{4} T_{fl}$, we have

$$N_r = \frac{120 \times 30}{6} - \frac{75}{4} = 581.25\ \text{rpm}$$

(b) For $N_r = 500$ rpm and $T_m = 0.6 \times T_{fl}$, we have

$$N_s = 500 + 0.6 \times 75 = 545\ \text{rpm} \Rightarrow f_s = 27.25\ Hz$$

$$I_r^{'} = \frac{\frac{27.25}{50} \times 440\ V}{\sqrt{\left(0.2 + \frac{0.3}{0.082}\right)^2 + \frac{27.25}{50}(1 + 0.5)^2}} = 61.24\ A$$

(c) For $N_r = 750$ rpm and $f_s = 40\ Hz$, we have

$$f_s = 40\ Hz \Rightarrow N_s = 800\ \text{rpm} \Rightarrow N_{sl} = 800 - 750 = 50\ \text{rpm} \Rightarrow T_m = \frac{50}{75} \times T_{fl}$$

$$I_r{}' = \frac{440\,V}{\sqrt{\left(0.2+\frac{0.3}{0.075}\right)^2+(0.5+1.0)^2}} = 98.7\,A \Rightarrow T_{fl} = 1116.31\,N.m \Rightarrow \frac{50}{75}\times T_{fl} = 744.2\,\text{N.m}$$

6. The parameters given for the IM are:

$$440\,V, 3-phase, f = 50\,Hz, P = 4, N_{r(rated)} = 1370\,rpm, Y-connected$$
$$R_s = 1.9\ \Omega, R_{r'} = 2.0\ \Omega, X_s = 3.0\ \Omega, X_{r'} = 3.0\ \Omega$$
$$v_1 = \frac{400}{\sqrt{3}} = 254\,V, v_5 = 100\,V, v_7 = 40\,V,$$

The rated slip is

$$s_{rated} = \frac{1500\,rpm - 1370\,rpm}{1500\,rpm} = 0.087$$

The rated rotor current referred to the stator side is:

$$I_{r'} = \frac{254\,V}{\sqrt{\left(1.9+\frac{2}{0.087}\right)^2+(3+3)^2}} = 9.9\,A$$

The rated torque is given by

$$T_{rated} = \frac{3}{\frac{2\pi}{60}\times 1500}\times(9.9)^2\times\frac{2}{0.087} = 43.1\,N.m$$

The 5th harmonic current is given by

$$I_{s5} = \frac{v_5}{5(X_s+X_{r'})} = \frac{100\,V}{5(3+3)} = 3.33\,A$$

The 7th harmonic current is given by

$$I_{s7} = \frac{v_7}{7(X_s+X_{r'})} = \frac{40\,V}{7(3+3)} = 0.95\,A$$

The rms stator current is given by

$$I_{s(rms)} = \sqrt{{I_1}^2+{I_5}^2+{I_7}^2} = \sqrt{9.9^2+3.33^2+0.95^2} = 10.5\,A$$

The output power is given by

$$P_{out} = T_e\times\omega_m = 43.1\,N.m\times(\frac{2\pi}{60}\times 1370\,rpm) = 6182\,W$$

The copper losses are given by

$$P_{cu,loss} = 3\left[(9.9)^2\times(1.9+2)+(3.33)^2\times(1.9+2)+(0.95)^2\times(1.9+2)\right]$$
$$= 1284.7\,W$$

The efficiency(neglecting iron loss) is given by

$$\eta = \frac{6182}{6182+1284.7} = 82.8\%$$

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