National University of Singapore

Department of Electrical & Computer Engineering

EE-4502:Electric Drives and Control

Tutorial - 3 (DC Motor Drives - Solution)

Year 2021-22

1. If we neglect armature copper loss $\Rightarrow \; (R_a \approx 0)$. Thus, we have

$$V_a = E_a + I_a \times R_a \simeq E_a \, (= k\phi\omega_m) \Rightarrow \omega_m = \frac{V_a}{k\phi}$$

$$T_{em} = k\phi I_a \Rightarrow I_a = \frac{T_{em}}{k\phi}$$

$$I_{a,rated} = 500\, A \text{and} N_{rated} = 800\, rpm$$

   (a) Load torque is maintained constant i.e. $T_{load} = const.$

$$\omega_m = \frac{V_a}{k\phi} = \frac{0.5\, pu}{0.8\, pu} \times 800\, rpm = 500\,\text{rpm}$$

$$T_{em} = T_{load} = \text{const.} \rightarrow I_a = \frac{1}{k\phi} \times 500\text{ A} = 625\text{ A}$$

   (b) Load torque is maintained constant i.e. $T_{load} \propto {\omega_m}^2$

$$\omega_m = \frac{V_a}{k\phi} = \frac{0.5\, pu}{0.8\, pu} \times 800\, rpm = 500\,\text{rpm}$$

$$T_{em} = T_{load} = \propto {\omega_m}^2 \rightarrow I_a = \frac{1}{0.8} \times 0.625^2 \times 500\text{ A} = 244.14\text{ A}$$

2. The parameters given are:

$$V_{a,rated} = 250\, V, N_{rated} = 500\, rpm, R_a = 0.13\,\Omega, I_{a,rated} = 60\, A$$

   At rated operating conditions, the back-emf constant is:

$$k_E = \frac{V_a - I_a \times R_a}{\omega_m} = \frac{250\, V - 60\, A \times 0.13\,\Omega}{(\frac{2\pi}{60}) \times 500\, rpm} = 4.62\, V/(rad/s)$$

   (a) At rated braking torque, we have

$$V_a = E_a + (-I_a) \times R_a \Rightarrow 250\, V = E_a - 60\, A \times 0.13\,\Omega \Rightarrow E_a = 257.8\, V$$

$$N = \frac{E_a}{k_E} = \frac{257.8}{4.62} \times \frac{60}{2\pi} = 532.85\,rpm$$

(b) Under dynamic braking, we have $V_a = 0\,V$ and external resistance, R_{ext} is connected in series with the armature to limit the armature current to twice its rated value.

$$V_a = E_a + (-I_a) \times (R_a + R_{ext}) \Rightarrow 0\,V = 242.2\,V - (60 \times 2\,A) \times (0.13 + R_{ext})\,\Omega$$

$$R_{ext} = 1.9\,\Omega$$

At rated braking torque we have,

$$V_a = E_a + (-I_a) \times (R_a + R_{ext}) \Rightarrow 0\,V = E_a - 60\,A \times (0.13 + 1.9)\,\Omega$$

$$E_a = 121.2\,V \Rightarrow N = \frac{E_a}{k_E} = \frac{121.2}{4.62} \times \frac{60}{2\pi} = 250.5\,rpm$$

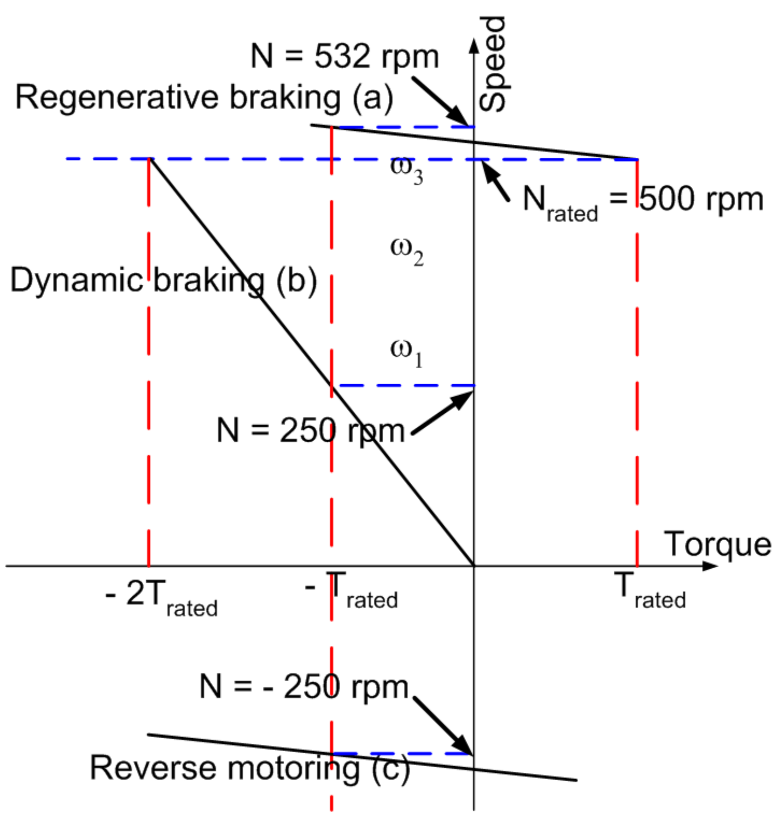

Figure 1:

(c)

$$V_a = E_a + (I_a) \times R_a = \left(-4.62 \times \frac{2\pi}{60} \times \frac{500}{2}\right) + (-60\ A) \times 0.13 = -128.9\ V$$

3. The parameters given are:

$$V_{dc} = 500\ V, I_{a,rated} = 20\ A, R_a = 0.5\ \Omega,$$

$$L_a = 20\ mH, \text{and } f_s = 1000\ Hz$$

At rated operating conditions, the back-emf constant is:

$$k_E = \frac{V_a - I_a \times R_a}{\omega_m} = \frac{500\ V - 20\ A \times 0.5\ \Omega}{(\frac{2\pi}{60}) \times 1170\ rpm} = 4.00\ V/(rad/s)$$

(a) The back-emf at 800 rpm is:

$$E_a = 4 \times \left[\left(\frac{2\pi}{60}\right) \times 800\right] = 335.1\ V$$

The corresponding armature voltage is

$$V_a = 335.1\ V + 20\ A \times 0.5\ \Omega = 345.1\ V$$

Thus, the duty-cycle of the chopper is

$$D = \frac{345.1\ V}{500\ V} = 0.69$$

(b) The corresponding armature current is as shown in Fig. 2.

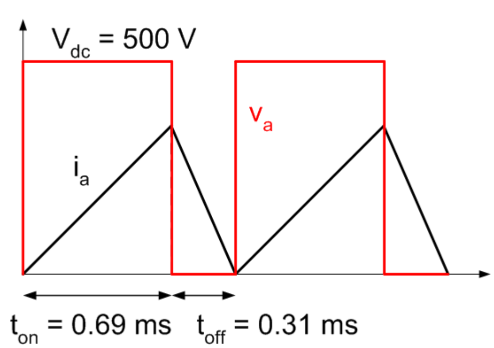

Figure 2:

The corresponding voltage eqn. during the current rise period is

$$v_a = i_a \times R_a + L_a \times (\frac{di_a}{dt}) + e_a$$

$$\Rightarrow 500\,V = i_a \times 0.5\,\Omega + 20.0 \times 10^{-3} \times \left(\frac{di_a}{dt}\right) + 4 \times \left[\left(\frac{2\pi}{60}\right) \times 800\right]$$

Assuming the speed remains constant and solving the differential equation we get,

$$i_a(t) = 329.8\,(1 - exp(-t/0.04))\;\; A$$

At the end of the ON period ($t = 0.69ms$), we have,

$$i_a(t = 0.69ms) = 329.8\,(1 - exp(-0.00069/0.04))\;\; A = 5.64\; A$$

Assuming that the rise of the current is linear, the average armature current for this operating condition is

$$I_a = \frac{1}{2} \times 5.64\; A = 2.82\; A$$

This gives rise to a torque of

$$T_{em} = k_T \times I_a = 4.0 \times 2.82\; A = 11.3\; N.m$$

4. The parameters given are:

$$V_{rated} = 230\;V, I_{a,rated} = 90\;A, N_{rated} = 500\;rpm, R_a = 0.115\;\Omega,$$

$$L_a = 11\;mH, V_s = 230\;V\;DC, \text{and } f_s = 400\;Hz$$

Under rated condition, we have

$$E_{a,rated} = 230\;V - 90.0\;A \times 0.115\;\Omega = 219.65\;V, k\phi_{rated} = \frac{219.65\;V}{500} = 0.439\;V/rpm$$

(a) For $\delta = 0.5$ and $I_a = 90\;A$, we have

$$E_a = 0.5 \times 230\;V - 90\;A \times 0.115\;\Omega = 104.7\;V \Rightarrow N = \frac{104.7\;V}{219.65\;V} \times 500 = 238.2\;rpm$$

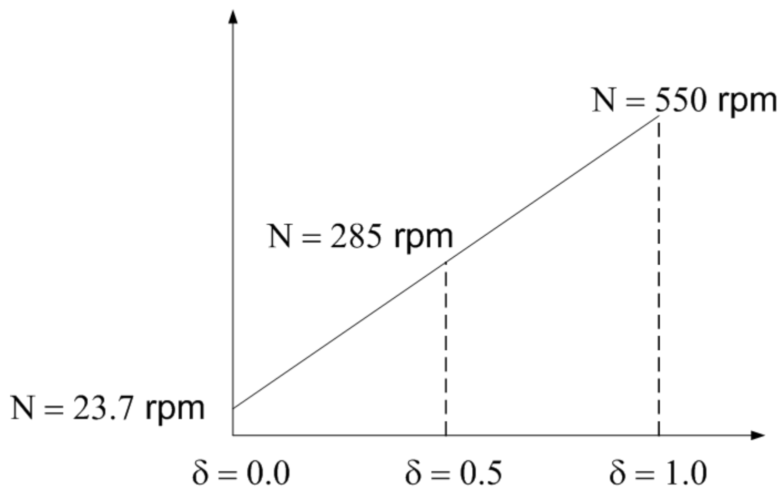

Figure 3:

(b) For braking at rated torque $I_a = -90\ A$. We have

$$\begin{aligned}\delta \times V_s &= E_a + I_a \times R_a \\ \delta \times 230 &= 0.439 \times N - 90 \times 0.115 \\ \delta &= 0.0019 \times N - 0.045\end{aligned}$$

For $\delta = 0$, we have

$$0 = 0.0019 \times N_{min} - 0.045 \Rightarrow N_{min} = 23.7\ rpm$$

5. The parameters given are:

$$R_a = 0.4\,\Omega, L_a = 1.5\,mH, k_E = k_T = 0.5, J_m = 0.02kg.m^2,$$

$$T_{rated} = 4\ N.m, V_s = 200\ V, f_s = 25\ kHz$$

At $N = 1500\,rpm$, we have

$$V_a = \frac{2\pi}{60} \times 1500 \times 0.5 + \frac{3}{0.5} \times 0.4 = 80.95\ V$$

Duty-cycle

$$D = \frac{V_a}{V_s} = \frac{80.95}{200} = 0.4$$

Turn-on time

$$t_{on} = D \times T_s = 0.4 \times \frac{1}{25\ kHz} = 16\ \mu s$$

Turn-off time

$$t_{off} = T_s - t_{on} = 40\,\mu s - 16\,\mu s = 24\,\mu s$$

The average armature current is

$$I_a = \frac{T}{k_T} = \frac{3\,N.m}{0.5\,N.m/A} = 6\,A$$

During the on-period we have

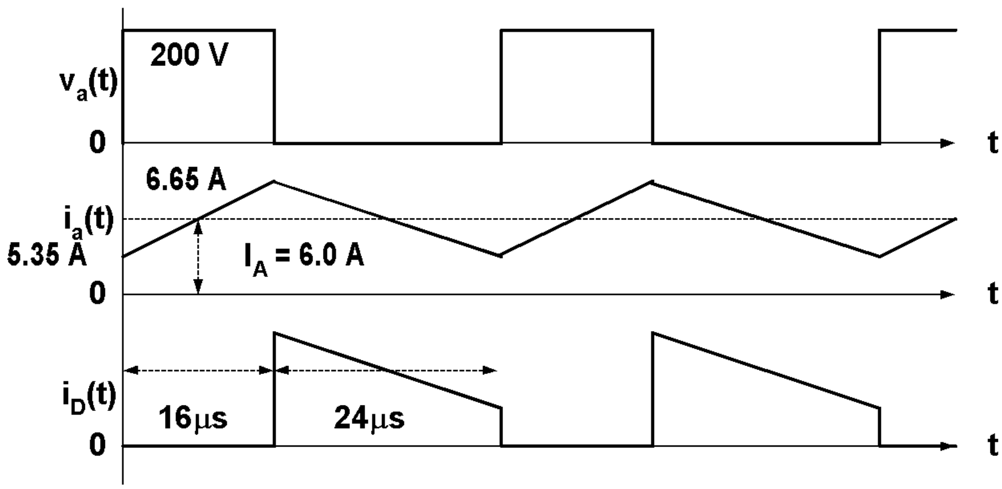

Figure 4:

$$\begin{aligned}
v_a(t) &= e_a(t) + i_a(t) \times R_a + L_a \frac{di_a}{dt} \\
&\approx e_a(t) + L_a \frac{di_a}{dt} \\
200\,V - 157.1 \times 0.5 &= 1.5 \times 10^{-3} \frac{di_a}{16\,\mu s} \\
di_a &= 1.3\,A
\end{aligned}$$

Thus, we have

$$i_{a,min} = 6\,A - \frac{1.3\,A}{2} = 5.35\,A, i_{a,max} = 6\,A + \frac{1.3\,A}{2} = 6.65\,A$$

(b) At 1200 rpm, we have

$$V_a = \frac{2\pi}{60} \times 1200 \times 0.5 + (-10\,A) \times 0.5 = 57.83\,V$$

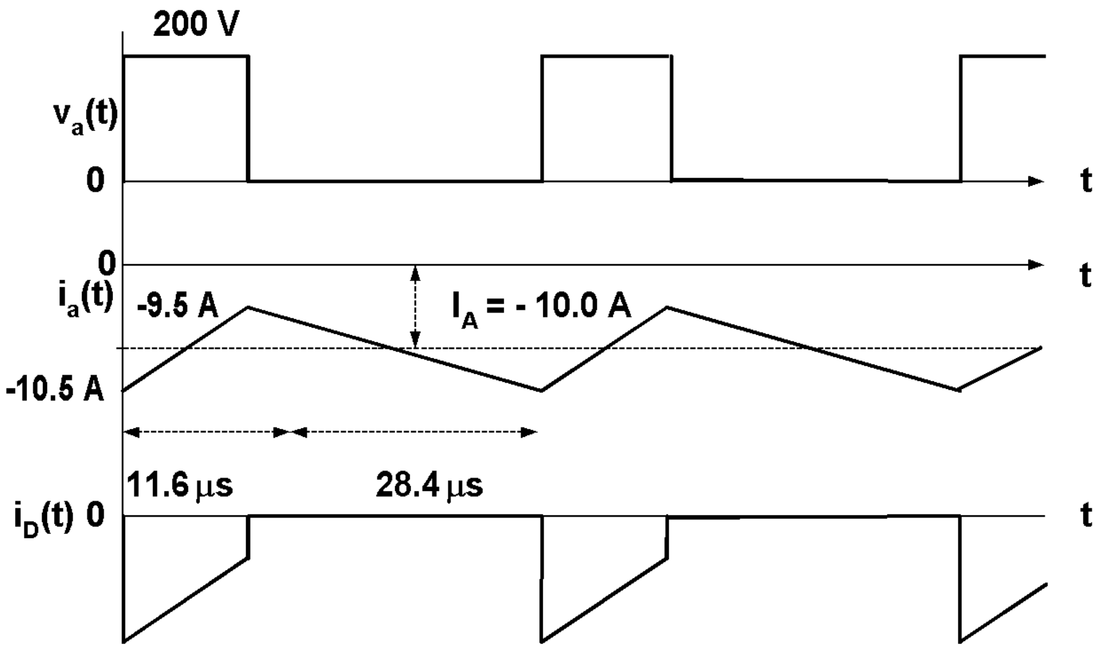

Figure 5:

Duty-cycle

$$D = \frac{V_a}{V_s} = \frac{57.83}{200} = 0.289$$

Turn-on time

$$t_{on} = D \times T_s = 0.289 \times \frac{1}{25\,kHz} = 11.6\,\mu s$$

Turn-off time

$$t_{off} = T_s - t_{on} = 40\,\mu s - 16\,\mu s = 28.4\,\mu s$$

During the period when the diode is ON, we have

$$\begin{aligned} v_a(t) &= e_a(t) + i_a(t) \times R_a + L_a \frac{di_a}{dt} \\ &\approx e_a(t) + L_a \frac{di_a}{dt} \\ 200\,V - 125.7 \times 0.5 &= 1.5 \times 10^{-3} \frac{di_a}{11.6\,\mu s} \\ di_a &= 1.06\,A \end{aligned}$$

Thus, we have

$$i_{a,min} = -10.0\,A - \frac{1.06\,A}{2} = -10.53\,A, i_{a,max} = -10.0\,A + \frac{1.06\,A}{2} = -9.47\,A$$

6. The parameters given are:

$$V_{dc} = 200\,V, T_l = c{\omega_m}^2, R_a = 0.5\,\Omega, k\phi = 0.6\,N.m/A$$

For $T_{l1} = 9\,N.m$, we have

$$I_{a1} = \frac{9}{0.6} = 15\,A$$

For $N_2 = 500\,rpm$

$$T_{l2} = 9 \times \left(\frac{500}{1000}\right)^2 = 2.25\,N.m, I_{a2} = \frac{2.25}{0.6} = 3.75\,A$$

At 500 rpm,

$$E_{a2} = k\phi\omega_{m2} = 0.6 \times \frac{2\pi}{60} \times 500 = 31.4\,V$$

Thus,

$$V_{a2} = 31.4\,V + 3.75\,A \times 0.5\,\Omega = 33.3\,V \Rightarrow \delta_2 = \frac{33.3}{200} = 0.17$$

For $N_3 = 200\,rpm$

$$T_{l3} = 9 \times \left(\frac{200}{1000}\right)^2 = 0.36\,N.m, I_{a3} = \frac{0.36}{0.6} = 0.6\,A$$

At 200 rpm,

$$E_{a3} = k\phi\omega_{m3} = 0.6 \times \frac{2\pi}{60} \times 200 = 12.57\,V$$

Thus,

$$V_{a3} = 12.57\,V + 0.6\,A \times 0.5\,\Omega = 12.87\,V \Rightarrow \delta_3 = \frac{12.9}{200} = 0.064$$

Thus, the range over which δ varies is $\boxed{0.064 < \delta < 0.17}$.

7. The parameters given are

$$V_{a,rated} = 220\,V, N_{rated} = 1750\,rpm, N = 1500\,rpm, R_a = 0.067\,\Omega,$$

$$V_{dc} = 240\,V, f_s = 400\,Hz, k\phi = 1.28\,N.m/A, i_{a,max} = 290\,A$$

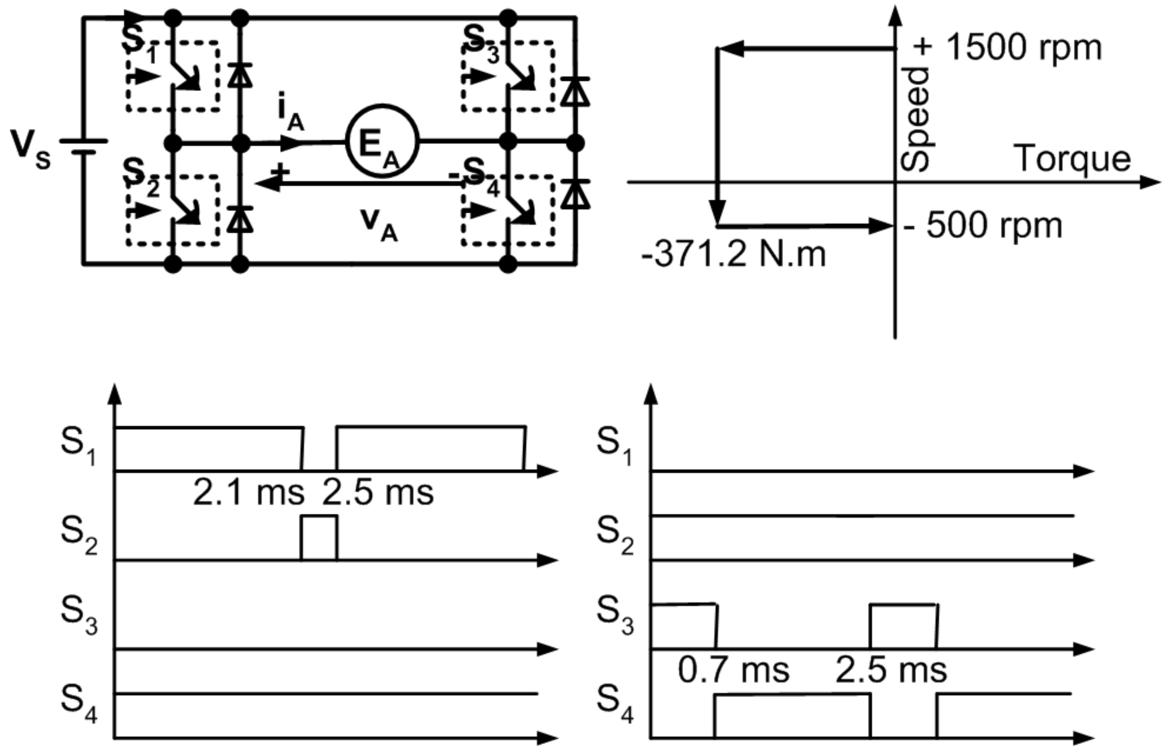

Figure 6: Chopper Circuit

At $N = 1500\,rpm$ we have

$$E_a = 1.28 \times \frac{2\pi}{60} \times 1500 = 201.1\,V$$

$$\Rightarrow V_a = 201.1 + 0 \times 0.067\,\Omega = 201.1\,V$$

$$\Rightarrow t_{ON} = \left(\frac{201.1}{240}\right) \times \left(\frac{1}{400}\right) = 2.1\,ms$$

Since the operation is in quadrant-I, switch S_4 is permanently ON while switch S_3 is completely OFF. The corresponding gating signals for all the switches are as shown in Fig. 6.

For operations at 1500 rpm in quadrant-II just after deceleration begins,at $N = 1500\,rpm$ we have

$$E_a = 1.28 \times \frac{2\pi}{60} \times (1500) = 201.1\,V$$

$$\Rightarrow V_a = 201.1 - 290\,A \times 0.067\,\Omega = 181.67\,V$$

$$\Rightarrow t_{ON} = \left(\frac{181.67}{240}\right) \times \left(\frac{1}{400}\right) = 1.9\,ms$$

For operations at -500 rpm in quadrant-III, we have At $N = -500\,rpm$ we have

$$E_a = 1.28 \times \frac{2\pi}{60} \times (-500) = -67.03\,V$$
$$\Rightarrow V_a = -67.03 + 0 \times 0.067\,\Omega = -67.03\,V$$
$$\Rightarrow t_{ON} = \left(\frac{67.03}{240}\right) \times (\frac{1}{400}) = 0.7\,ms$$

Since the operation is in quadrant-III, switch S_2 is permanently ON while switch S_1 is completely OFF. The corresponding gating signals for all the switches are as shown in Fig. 6.

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