Wind Power

① Power in the wind, $P_w = \frac{1}{2}\rho A v^3$

- ↳ $P_w \propto v^3$ ⇒ ↑height, ↑v
  - ⇒ ↓tower obstruction, ↑v
  - ⇒ ↓frictional surface, ↑v
- ↳ $P_w \propto A$ ⇒ for HAWT: $A = \frac{\pi}{4}D^2$ ⇒ $P_w \propto D^2$
  - ⇒ for VAWT: $A = \frac{2}{3}\pi D H$ ⇒ $P_w \propto D$
- ↳ At 15°C & 1atm: $\rho = 1.225\ kg/m^3$ $\left(\rho = \frac{m}{V}\right)$
- ↳ $P_w \propto \rho$ ⇒ ↓altitude, ↑ρ
  - ⇒ winter, ↑ρ
  - ⇒ $\rho = 1.225 K_T K_A$

TABLE 6.1 Density of Dry Air at a Pressure of 1 Atmosphere[a]

Temperature (°C)	Temperature (°F)	Density (kg/m^3)	Density Ratio (K_T)
−15	5.0	1.368	1.12
−10	14.0	1.342	1.10
−5	23.0	1.317	1.07
0	32.0	1.293	1.05
5	41.0	1.269	1.04
10	50.0	1.247	1.02
15	**59.0**	**1.225**	1.00
20	68.0	1.204	0.98
25	77.0	1.184	0.97
30	86.0	1.165	0.95
35	95.0	1.146	0.94
40	104.0	1.127	0.92

TABLE 6.2 Air Pressure at 15°C as a Function of Altitude

Altitude (meters)	Altitude (feet)	Pressure (atm)	Pressure Ratio (K_A)
0	0	1	1
200	656	0.977	0.977
400	1312	0.954	0.954
600	1968	0.931	0.931
800	2625	0.910	0.910
1000	3281	0.888	0.888
1200	3937	0.868	0.868
1400	4593	0.847	0.847
1600	5249	0.827	0.827
1800	5905	0.808	0.808
2000	6562	0.789	0.789
2200	7218	0.771	0.771

② Friction Coefficients:

- ↳ $\left(\frac{P}{P_0}\right) = \left(\frac{v}{v_0}\right)^3 = \left(\frac{H}{H_0}\right)^{3\alpha}$
- ↳ $\left(\frac{v}{v_0}\right) = \left(\frac{H}{H_0}\right)^{\alpha}$
- ↳ open terrain: $\alpha = \frac{1}{7}$
- ↳

Terrain Characteristics	Friction Coefficient α
Smooth hard ground, calm water	0.10
Tall grass on level ground	0.15
High crops, hedges and shrubs	0.20
Wooded countryside, many trees	0.25
Small town with trees and shrubs	0.30
Large city with tall buildings	0.40

Extracted

③ Power delivered by rotor, $P_b = \frac{1}{2}\rho A v^3 \cdot C_p$

- ↳ Rotor efficiency, $C_p = \frac{1}{2}(1+\lambda)(1-\lambda^2)$, where $\lambda = \frac{v_d}{v}$ — downwind
- ↳ max. blade efficiency: $\lambda = \frac{v_d}{v} = \frac{1}{3}$
  - ⇒ max. $C_p = 59.3\%$ (Betz efficiency)

④ $TSR = \frac{\text{Rotor tip speed}}{\text{Wind speed}, v} = \frac{rpm \times \pi D}{60 \cdot v}$

- ↳ rotor too slow: too much wind pass ⇒ ↓η
- ↳ rotor too fast: turbulence ⇒ ↓η
- ↳ gear ratio = $\frac{\text{generator rpm}}{\text{rotor rpm}}$
- ↳ overall $\eta = \frac{P_{out}}{P_{in}}$

⑤ Average wind speed, $(v^3)_{avg}$:

- ↳ average of v^3
- ↳ find v^3, then average

⑥ Rayleigh Distribution: $f(v) = \frac{2v}{c^2}\exp\left[-\left(\frac{v}{c}\right)^2\right]$, $(k=2)$

- ↳ ↑c, ↑v
- ↳ $f(v) = \frac{\pi v}{2\bar{v}^2}\exp\left[-\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^2\right]$, where $\bar{v}$ = average v

⑦ Average Power by Rayleigh, $\bar{P} = \frac{6}{\pi}\cdot\frac{1}{2}\rho A \bar{v}^3$

- ↳ $\bar{P} = 1.91 \cdot P_{@\bar{v}}$
- ↳ $(v^3)_{avg} = \frac{6}{\pi}\bar{v}^3 = 1.91\bar{v}^3$

⑧ Finding Annual Energy using Rayleigh:

↳

Step 1: Find the probability of each wind speed. How?

Step 2: Find the energy produced at each wind speed.

Step 3: Annual energy generated = summation of energy produced at each wind speed

↳ $f(v) = \frac{\pi v}{2\bar{v}^2}\exp\left[-\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^2\right]$, where $\bar{v}$ = average v

⑨ Capacity Factor, $CF = \frac{\text{Actual energy delivered}}{\text{Rated power} \times 8760}$

- ↳ Annual energy (kWh/year) = P_R (kW) × 8760 (h/yr) × CF
- ↳ $CF = 0.087\bar{v} - \frac{P_R}{D^2}$

1 year = 8760 h

Hydro Energy

① $P = g \times \eta \times \overset{m^3/s}{Q} \times \overset{m}{H}$, where $g \approx 10$

↳ Power ∝ (Head × Flow)

↳ Net head = Gross head − Head loss due to pipe losses

② $E = P \times t$

$1\,l = 1000\,m^3$

③ Efficiency of Hydropower Plants:

$$\eta = \frac{\text{Elect. power delivered to "busbar"}}{\text{Potential energy of head water}}$$

↳ ~75% to 95%

↳ VERY efficient! ⇒ most efficient renewable energy

1 year = 8760 h

$$1\,m^3/s = \frac{1}{(15.85 \times 10^3)}\,gpm$$

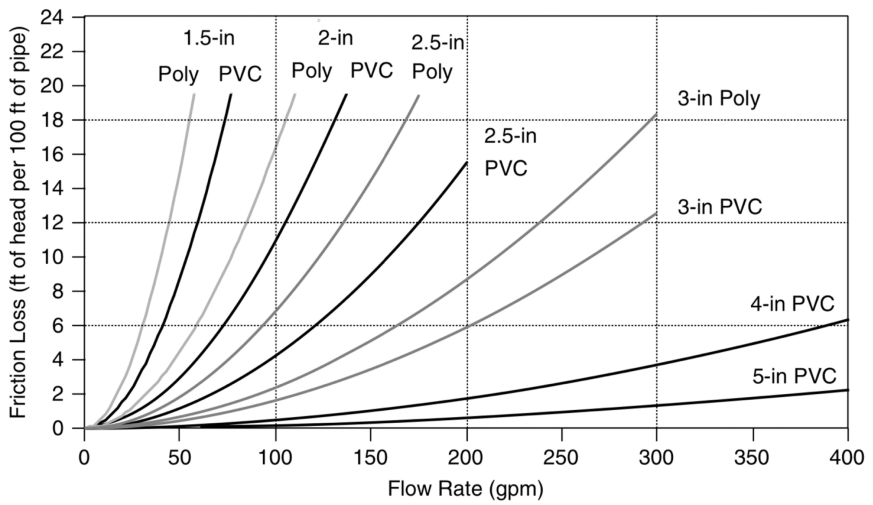

Figure 4.19 Friction head loss, in feet of head per 100 ft of pipe, for 160 psi PVC piping and for polyethylene, SDR pressure-rated pipe.

Ocean Energy

① Power Output of Tidal Turbines:

↳ $P = \frac{1}{2} \rho A v^3$

↳ $\rho_{sea} = 1025\,kg/m^3$ (for underwater)

↳ similar to wind turbines

↳ can produce more power than wind turbines

↳ needs strong foundation in presence of corrosion etc.

Biofuel Energy

① Biomass = Organic molecules of H_2 + Atoms of O_2 + N_2
+ Alkali, Alkaline earth + Heavy metals