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Lecture on Kalman Filter
State Observer or State Estimator

ESP 3201
Dr Ng Gee Wah

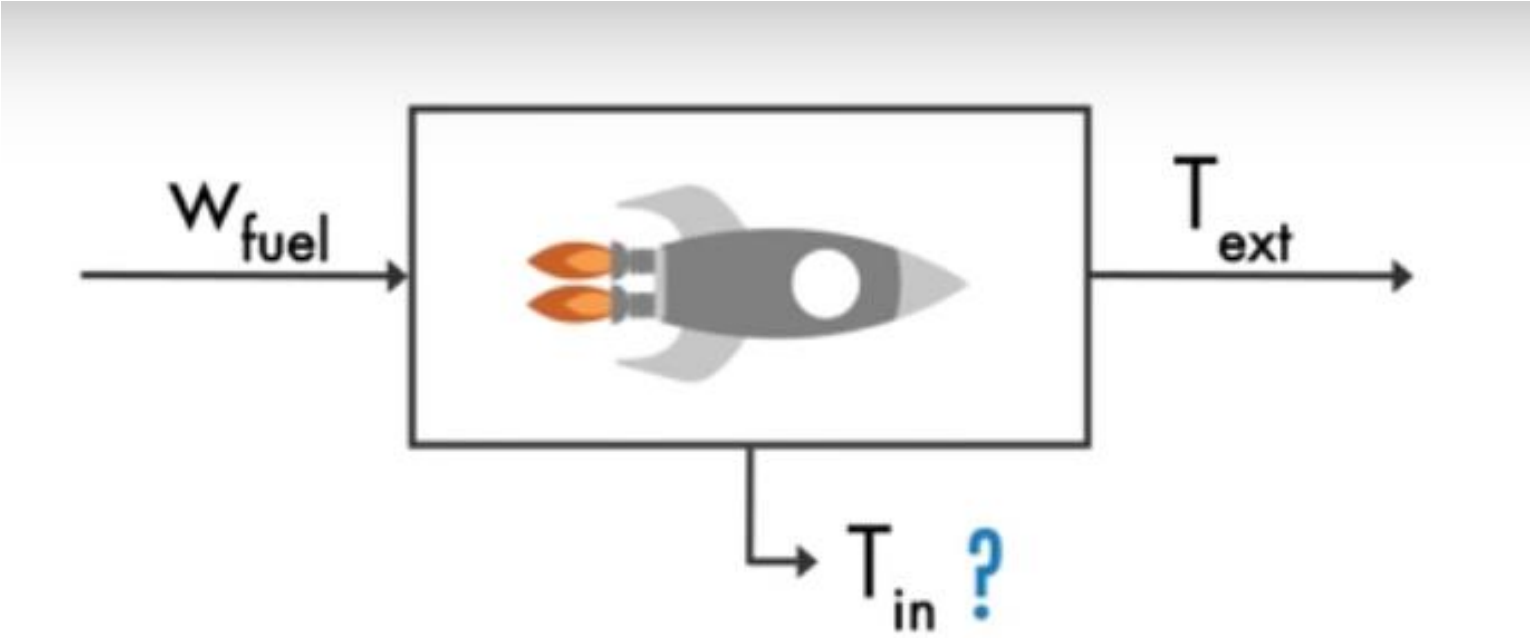
State Observer or State Estimator

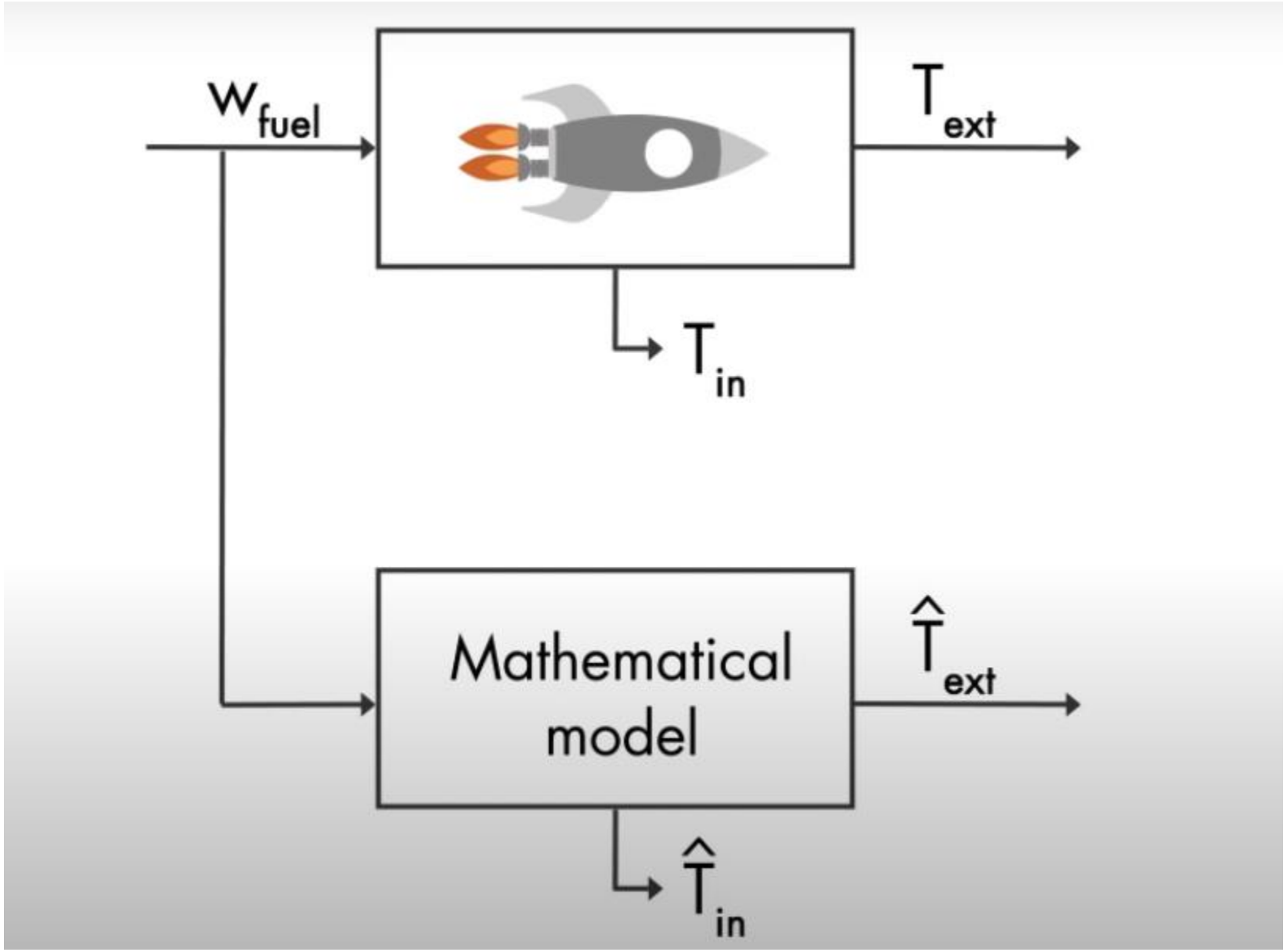
- An example when you play guessing game such as guess what is in the cage or guess what is his mood?
  - Give an input such as a cookie and observe his facial expression; this indirectly measure his mood.

- In control theory, a state observer or state estimator is a system that provides an estimate of the internal state of a given real system, from measurements of the input and output of the real system.

Example of State Observer or State Estimator

- An spacecraft example of estimated state of the internal temperature of its engine $\hat{T}$





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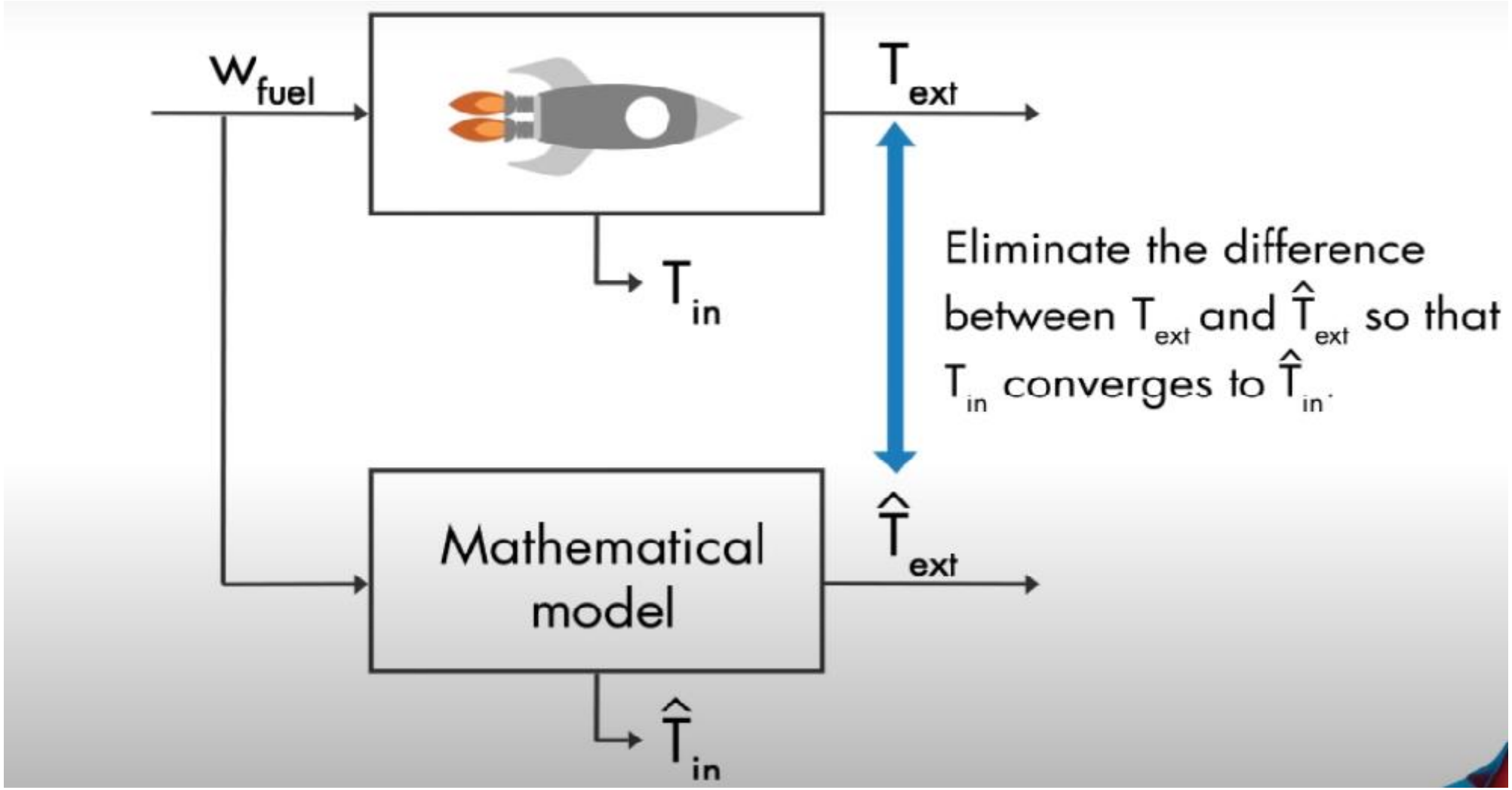
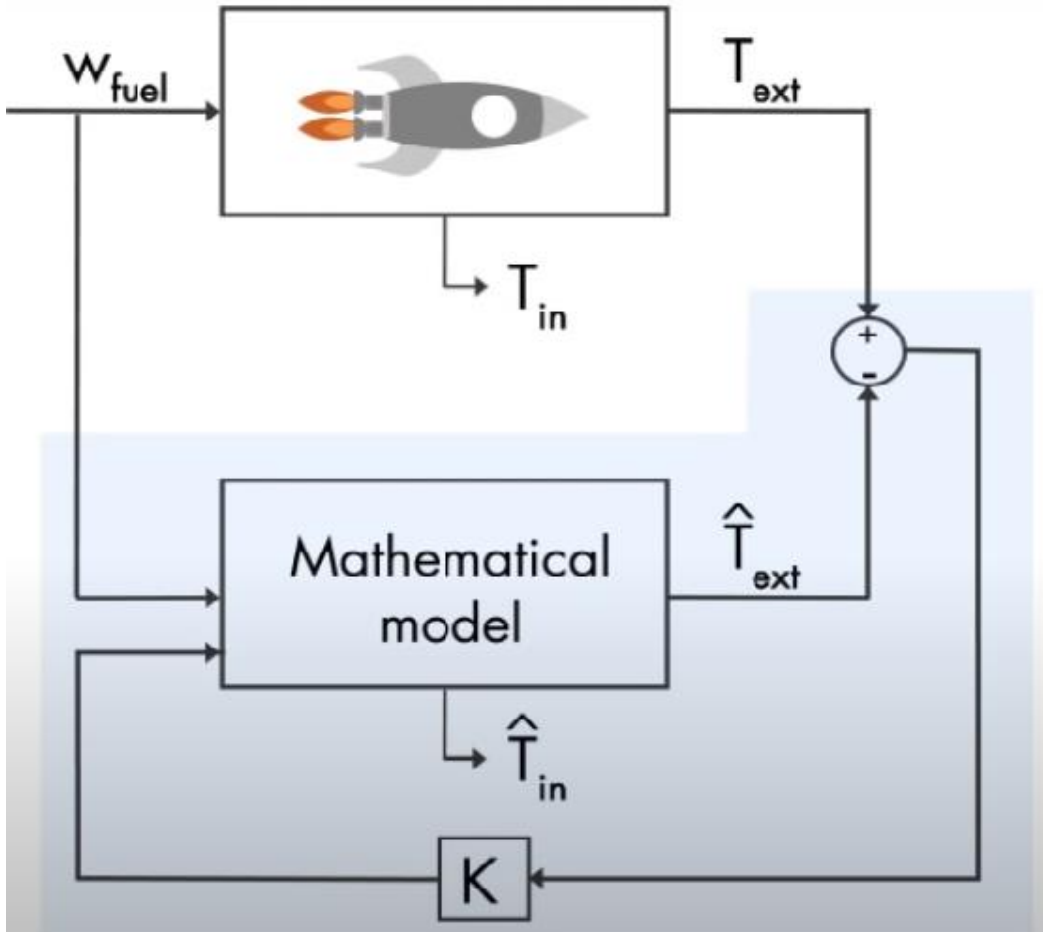
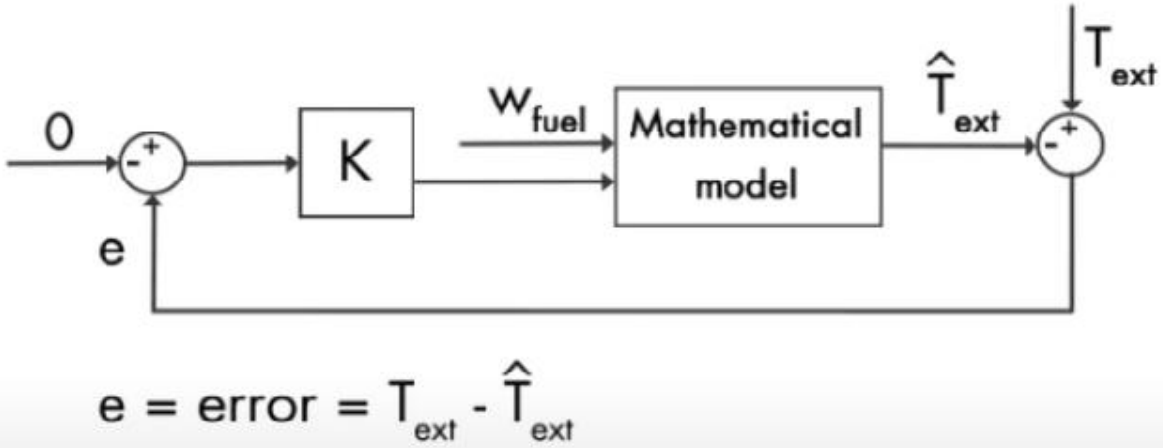


State Observer or State Estimator

- Estimated state $\hat{T}$





$e = error = T_{ext} - \hat{T}_{ext}$

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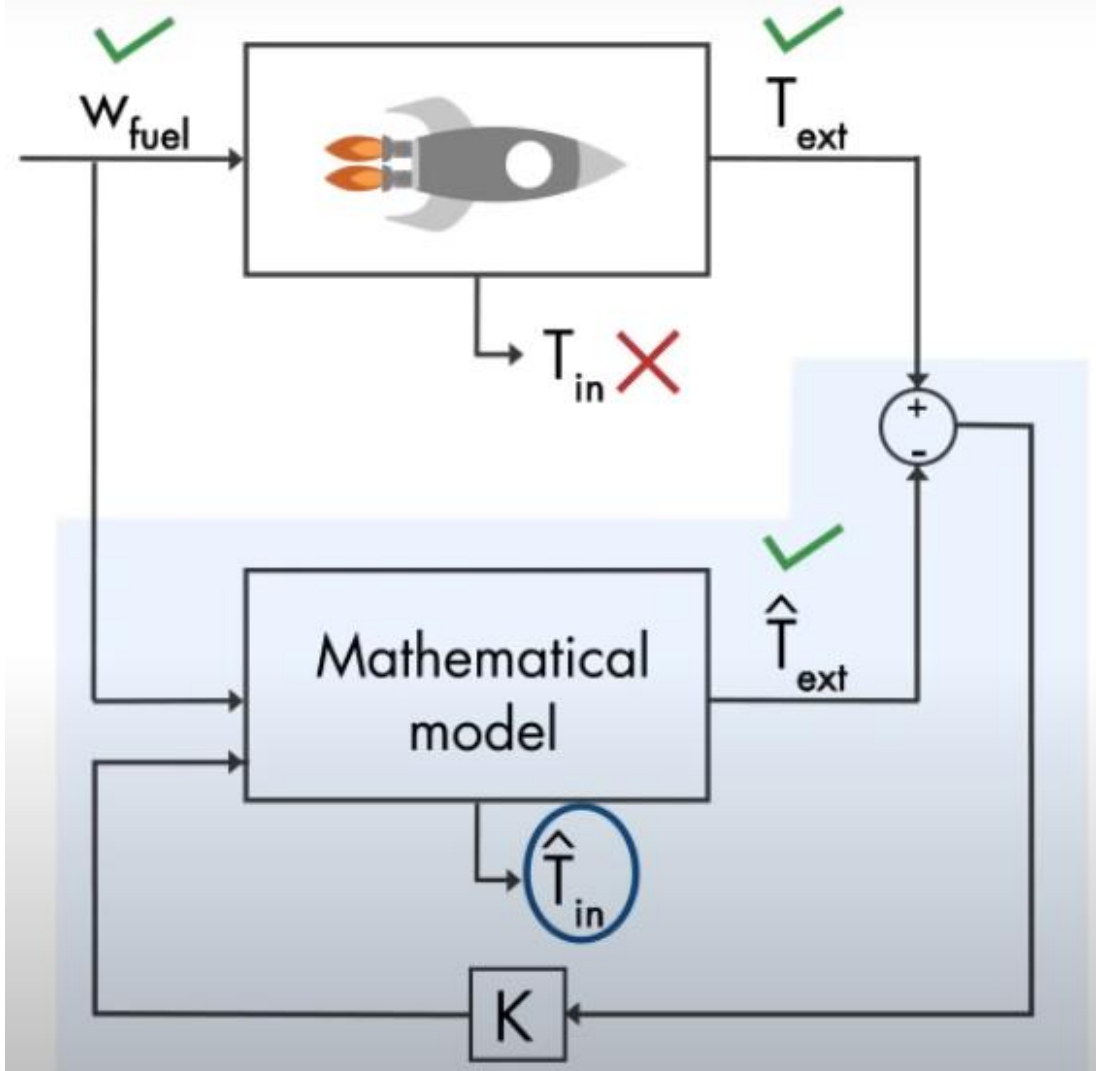

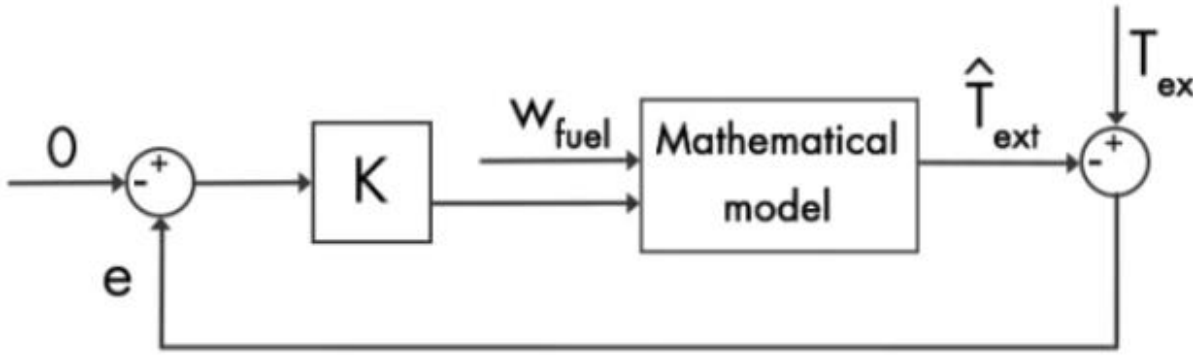

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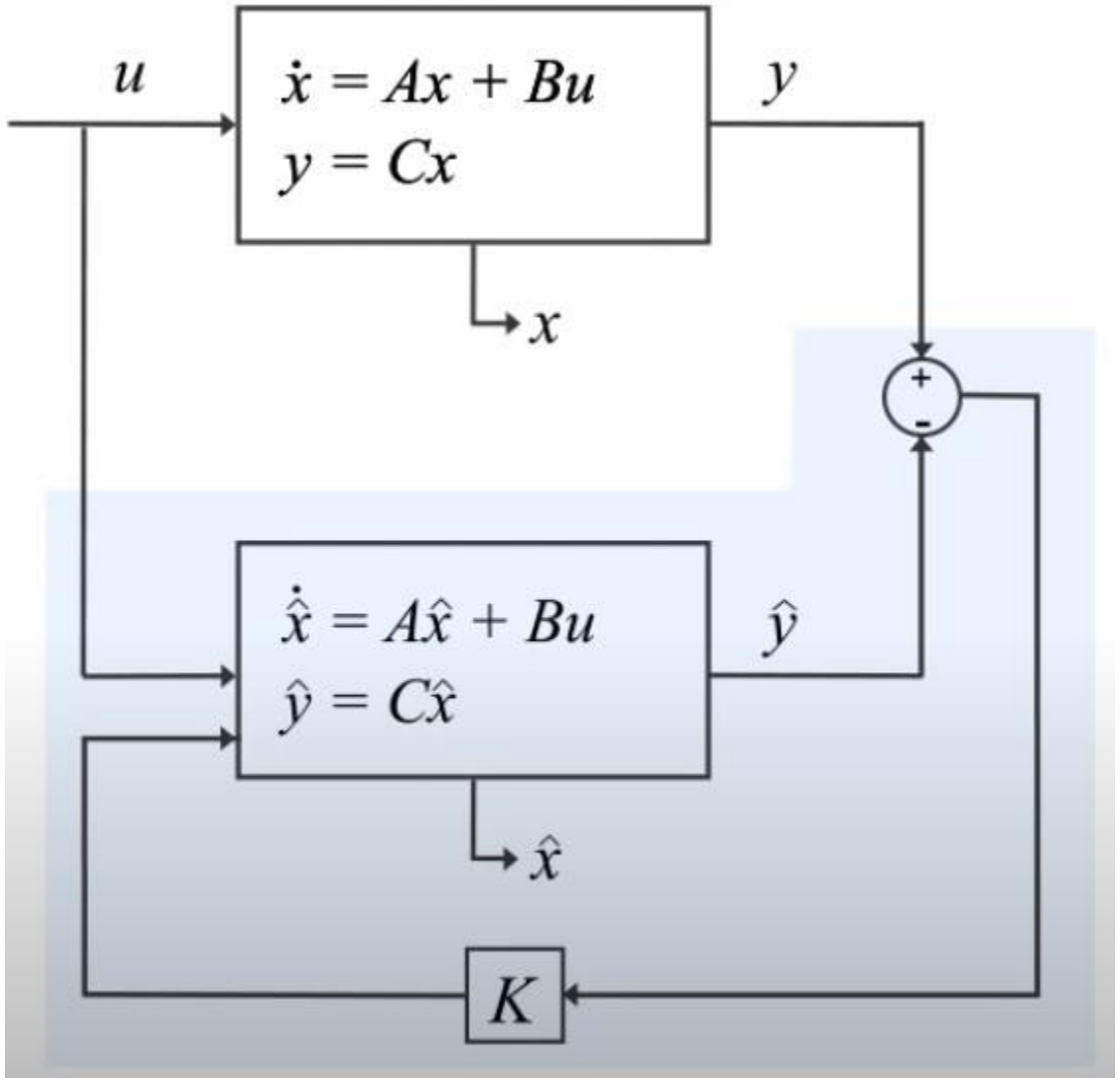

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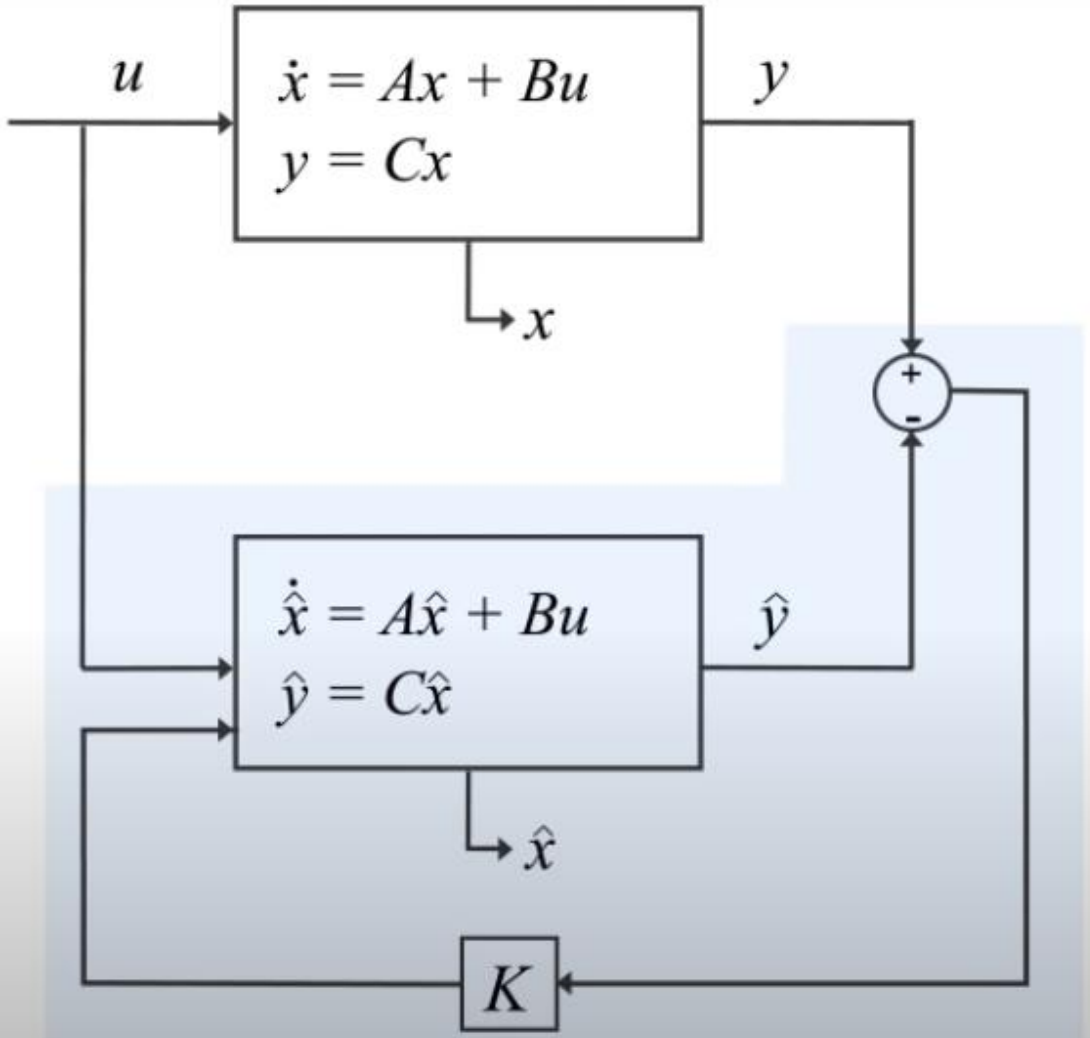


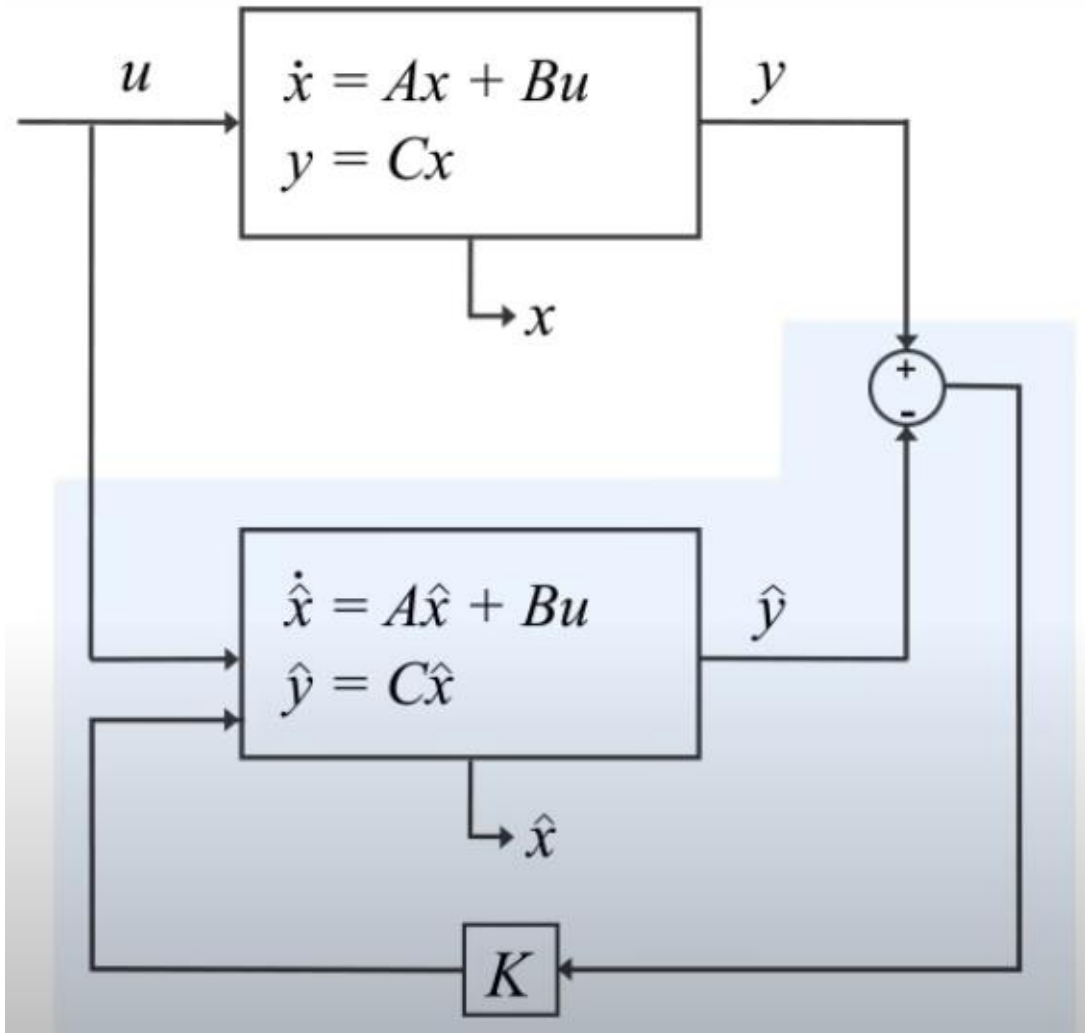
$$e_{obs} = x - \hat{x}$$

$$\dot{x} = Ax + Bu \qquad y = Cx$$

$$\dot{\hat{x}} = A\hat{x} + Bu + K(y - \hat{y}) \qquad \hat{y} = C\hat{x}$$

$$\underbrace{\dot{x} - \dot{\hat{x}}}_{\dot{e}_{obs}} = \underbrace{Ax - A\hat{x}}_{Ae_{obs}} + \underbrace{Bu - Bu}_{0} + \underbrace{K(y - \hat{y})}_{K(C\underbrace{(x - \hat{x})}_{e_{obs}})} \qquad y - \hat{y} = C\underbrace{(x - \hat{x})}_{e_{obs}}$$

State Observer or State Estimator



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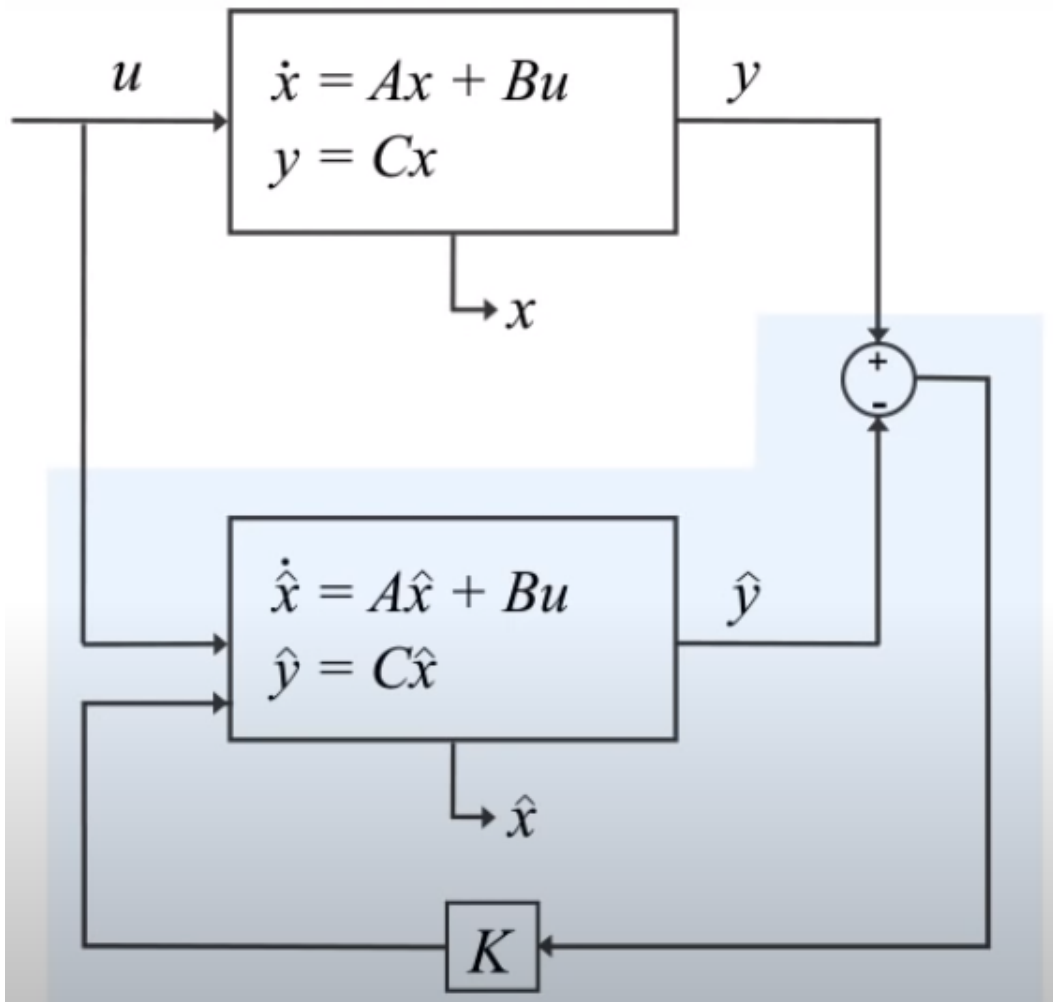
State Observer or State Estimator



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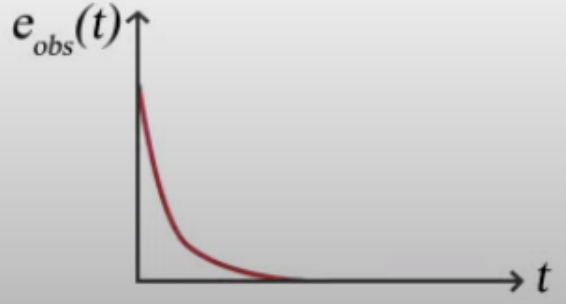
$$\dot{e}_{obs} = (A - KC)e_{obs} \qquad y - \hat{y} = Ce_{obs}$$

$$\hookrightarrow e_{obs}(t) = e^{(A - KC)t}e_{obs}(0)$$

If $(A\text{-}KC) < 0$, then $e_{obs} \rightarrow 0$ as $t \rightarrow \infty$. So, $\hat{x} \rightarrow x$.



Thank you for your attention