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Date No.

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$C = 98, D = 6, M = 99, K = 7$

1. By Newton's second law of motion,

$$\Sigma F = ma$$

$$f(t) - Ky(t) = M\frac{d^2y(t)}{dt^2}$$

$$99\frac{d^2y(t)}{dt^2} + 7y(t) = f(t)$$

2. From differential eqn obtained in qn 1,

$$99\frac{d^2y(t)}{dt^2} + 7y(t) = f(t)$$

Applying Laplace transform,

$$\Rightarrow 99[s^2Y(s) - sy(0) - y'(0)] + 7Y(s) = F(s)$$

Assuming $y(0) = 0$ and $y'(0) = 0$,

$$\Rightarrow 99s^2Y(s) + 7Y(s) = F(s)$$

$$\frac{Y(s)}{F(s)} = \frac{1}{99s^2+7}$$

$$\therefore G(s) = \frac{Y(s)}{F(s)} = \frac{1}{99s^2+7}$$

poles $\Rightarrow 99s^2 + 7 = 0$

$$s = \pm j\sqrt{\frac{K}{M}}$$

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3. $\mathcal{F}(s) = \int_{0^-}^{\infty} (A \sin \omega_f t) e^{-st} \, dt$

$= A \left(\dfrac{\omega_f}{s^2 + \omega_f^2} \right) = \dfrac{10}{s^2 + 100}$

$Y(s) = G(s) \cdot \mathcal{F}(s)$

$= \left(\dfrac{1}{99s^2 + 7} \right) \cdot \left(\dfrac{10}{s^2 + 100} \right)$

$= \dfrac{10}{(99s^2 + 7)(s^2 + 100)}$

$= \dfrac{1}{9893(s^2 + \frac{7}{99})} - \dfrac{1}{9893(s^2 + 100)}$

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4. From $G(s)$ calculated in qn 2,

$$G(s) = \frac{1}{99s^2 + 7}$$

$$= \frac{\frac{1}{99}}{s^2 + \frac{7}{99}} \quad \text{——} \ (1)$$

For second order system,

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \text{——} \ (2)$$

comparing (1) & (2),

$$\omega_n^2 = \frac{7}{99}$$

$$\omega_n = \sqrt{\frac{7}{99}}$$

$$\omega_f = \sqrt{\frac{K}{M}} = \sqrt{\frac{7}{99}} = \omega_n$$

Since $\omega_f = \omega_n$, the cantilever experiences resonance.

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5. $$M_1 \frac{d^2 x(t)}{dt} = K_1 (y(t) - x(t))$$

Assuming $x(0)$ and $x'(0) = 0$,

$$M_1 s^2 X(s) = K_1 Y(s) - K_1 X(s)$$

$$X(s) = \frac{K_1 Y(s)}{M_1 s^2 + K_1} \quad \text{—— (1)}$$

$$M \frac{d^2 y(t)}{dt^2} = f(t) - Ky(t) - K_1 (y(t) - x(t))$$

Assuming $y(0)$ and $y'(0) = 0$,

$$M s^2 Y(s) = F(s) - KY(s) - K_1 Y(s) + K_1 X(s)$$

sub in (1),

$$\Rightarrow M s^2 Y(s) = F(s) - KY(s) - K_1 Y(s) + \frac{K_1^2 Y(s)}{M_1 s^2 + K_1}$$

$$MM_1 s^4 Y(s) + K_1 M s^2 Y(s) = F(s) M_1 s^2 + K_1 F(s) - M_1 s^2 K Y(s) - K_1 K Y(s) - K_1 Y(s) M_1 s^2 - K_1^2 Y(s) + K_1^2 Y(s)$$

$$(MM_1 s^4 + K_1 MM_1 s^2 + M_1 s^2 K + K_1 K + K_1 M_1 s^2) Y(s) = (M_1 s^2 + K_1) F(s)$$

$$\frac{Y(s)}{F(s)} = \frac{M_1 s^2 + K_1}{MM_1 s^4 + K_1 M s^2 + M_1 K s^2 + K_1 M_1 s^2 + K_1 K}$$

$$G(s) = \frac{M_1 s^2 + K_1}{99 M_1 s^4 + (99 K_1 + 7 M_1 + K_1 M_1) s^2 + 7 K_1}$$

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6. $$G_2(s) = \frac{\frac{1}{99M_1}(M_1 s^2 + K_1)}{s^4 + \left(\frac{K_1}{M_1} + \frac{K_1+7}{99}\right)s^2 + \frac{7K_1}{99M_1}}$$

$$= \frac{\frac{1}{99M_1}(M_1 s^2 + K_1)}{\left(s^2 + \frac{K_1}{M_1}\right)\left(s^2 + \frac{K_1+7}{99}\right) - \frac{K_1^2}{99M_1}}$$

Comparing with given form of $\frac{C(M_1 s^2 + K_1)}{(s^2+\omega_{n1}^2)(s^2+\omega_{n2}^2)}$,

$$C = \frac{1}{99M_1} \Rightarrow M_1 = \frac{1}{99C}$$

$$\frac{K_1^2}{99M_1} = 0 \Rightarrow K_1 = 0$$

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7. Numerator of $G(s) \Rightarrow \frac{1}{99M_1}(M_1 s^2 + K_1)$

Zeroes $\Rightarrow s^2 - \frac{K_1}{M_1}$

$s^2 \Rightarrow \omega^2 = \frac{K}{M}$

$\therefore \sqrt{\frac{K}{M}} = \sqrt{\frac{K_1}{M_1}}$

$K_1 = 0.0707 M_1$ (3.1.6.)

8.