Solution to Tutorial 2 Questions

ESP5403 Nanomaterial for Energy Systems

1. A semiconductor has the following parameters:

   $N_C = 1.25 \times 10^{14}$ $(\mathbf{T})^{3/2}$ cm^{-3} $\quad$ $\mu_n = 4200$ $cm^2\, V^{-1} sec^{-1}$

   $N_V = 8.08 \times 10^{13}$ $(\mathbf{T})^{3/2}$ cm^{-3} $\quad$ $\mu_h = 1900$ $cm^2\, V^{-1} sec^{-1}$

   where μ_n & μ_h are assumed to be independent of temperature.

   The minimum resistivity of the intrinsic semiconductor at T = 450 K is supposed to be 6.1×10^5 *ohm-cm*.

   (a) What is the maximum value of the intrinsic carrier concentration at this temperature?

   (b) What is the minimum value of the band gap energy E_g?

Given:

$N_C = 1.25\times10^{14}$ $(\mathrm{T})^{3/2}$ cm^{-3} $\quad$ $\mu_n = 4200\,\mathrm{cm^2\,V^{-1}sec^{-1}}$

$N_V = 8.08\times10^{13}$ $(\mathrm{T})^{3/2}$ cm^{-3} $\quad$ $\mu_h = 1900\,\mathrm{cm^2\,V^{-1}sec^{-1}}$

$\rho_{\min} = 6.1\times10^5\,\Omega m$ $\quad$ $\sigma_{\max} = 0.164\times10^{-5}\,\Omega^{-1}cm^{-1}$

Solution:

$$\sigma_{\max} = q\cdot(\mu_n + \mu_h)\cdot n_i(\max)$$

$$n_i(\max) = \left(\frac{\sigma_{\max}}{q(\mu_n+\mu_h)}\right) = \left(\frac{0.164\times10^{-5}}{1.6\times10^{-19}\times6100}\right)$$

$$= 1.68\times10^9\,cm^{-3}$$

$$n_i^2(\max) = N_C\cdot N_V\cdot\exp\left(\frac{-E_g(\min)}{kT}\right)$$

$$E_g(\min) = KT\,\ell n\left(\frac{N_C\cdot N_V}{n_i^2(\max)}\right)$$

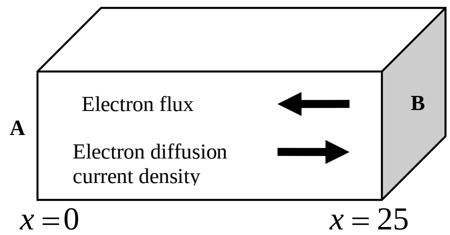
$$E_g(\min) = 8.625\times10^{-5}\times450\times\ell n\left(\frac{1.25\times10^{14}\times8.08\times10^{13}\times450^3}{(1.68\times10^9)^2}\right)$$

$$= 1.56\,eV$$

Results:

$E_g(\min) = 1.56\,eV$

2. Figure shows the directions of electron diffusion flux and current densities in a rectangular bar of Silicon and x is measured in μm. The electron concentration varies linearly from 1×10^{17} to 7×10^{16} cm^{-3} between the two ends at T = 300 K. Calculate the diffusion current density, if the electron mobility is, $\mu_n = 9.62\times10^3\ cm^2\ V^{-1} sec^{-1}$



Solutions:

Given: At x = 25µm, $n_B = 1\times10^{17}$ cm^{-3} and x = 0, $n_A = 7\times10^{16}$ cm^{-3}

Diffusion current density, $J_n\big|_{diff} = eD_n \dfrac{dn}{dx} \cong eD_n \dfrac{\Delta n}{\Delta x}$

$$\Rightarrow J_n\big|_{diff} = 1.6\times10^{-19}\times250\times\left(\frac{1\times10^{17}-7\times10^{16}}{25\times10^{-4}-0}\right)$$

$$= 480\, A/cm^2$$

3. The intrinsic carrier density at 300 K in silicon is 1.5×10^{16} m^{-3}. If the electron and hole motilities are 0.13 and 0.05 $m^2V^{-1}s^{-1}$, respectively, calculate the conductivity of (a) intrinsic silicon and (b) silicon containing 1 donor impurity atom per 10^8 silicon atoms. The atomic weight of Si is 28.09 $g/mole$ and its density = 2.33×10^3 kg/m^3.

Solution:

Given: $n_i = 1.5\times10^{16}$ m^{-3}; $\mu_e = 0.13$ m^{2}V^{-1}s^{-1}; $\mu_h = 0.05$ m^{2}V^{-1}s^{-1}

1 impurity atom/ 10^8 silicon atoms.

σ_i = ? σ_{ext} = ?

(a) $\sigma_i = n_i\, q\, (\mu_e + \mu_h)$

$= 1.5 \times 10^{16} \times 1.6 \times 10^{-19} \times 0.18$

$= 0.432 \times 10^{-3}\ \Omega^{-1}\mathrm{m}^{-1}$

(b) No. of Si atoms per m^3 = $\dfrac{6.023\times10^{23}\times10^{3}\times2.23\times10^{3}}{28.09} = 5\times10^{28}/m^{3}$

The density of the donor impurity, $N_D = \dfrac{5\times10^{28}}{10^{8}} = 5\times10^{20}/m^{3}$

$\sigma_{ext} = \boldsymbol{N_D\, q\, \mu_e} = 5\times10^{20} \times 1.6\times10^{-19} \times 0.13 = 10.4\ \Omega^{-1}\mathrm{m}^{-1}$

Answer: $\sigma_i = 0.432 \times 10^{-3}\ \Omega^{-1}m^{-1}$; $\sigma_{ext} = 10.4\ \Omega^{-1}m^{-1}$

4. An abrupt Si *p*-*n* junction has $N_a = 10^{18}\ cm^{-3}$ on one side and $N_d = 5x10^{15}\ cm^{-3}$ on the other side.

(i) Calculate the Fermi level positions at 300K in the *p* – and *n* – regions

(ii) Draw an equivalent band diagram for the junction and determine the contact potential V_o from the diagram

Solution:

(i)

Fermi level positions at 300 K in the *p* region,

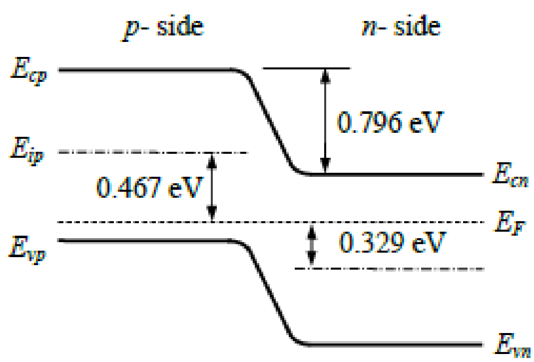
$$E_{ip} - E_F = kT \ln \frac{p_p}{n_i} = 0.0259 \ln \frac{10^{18}}{(1.5\times10^{10})} = \mathbf{0.467\ eV}$$

Fermi level positions at 300 K in the *n* region,

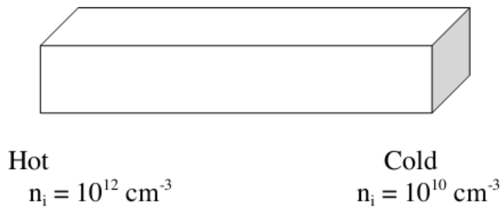
$$E_F - E_{in} = kT \ln \frac{n_n}{n_i} = 0.0259 \ln \frac{5\times10^{15}}{(1.5\times10^{10})} = \mathbf{0.329\ eV}$$

(ii)

$qV_o = 0.467 + 0.329 = \mathbf{0.796\ eV}$



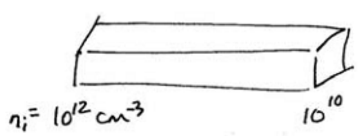
5. A piece of *p*-type Si is shown in Figure below with $N_A = 10^{18}\ cm^{-3}$ and a length of 1 *cm* is heated at one end. This affects the value of n_i as follows:



Figure

Consider only the electrons in the Si, neglecting the motion of the holes. Where do drift and diffusion of the electrons occur? Estimate the electric field at the cold end of the Si.

Solution:



p type: $p = N_A = 10^{18}\text{cm}^{-3}$ for both temperatures. Fully ionized.
Hot End: $n = 10^6\text{cm}^{-3} = n_i^2/N_A$
Cold End: $n = 10^2\text{cm}^{-3}$
So we have diffusion of electrons until the electric field balances the concentra
tion gradient. Neglect hole diff.

Thermal R & G occurs everywhere, but more carriers at hot end.

$$J_n = eD_n\frac{dn}{dx} = en\mu_n\epsilon \text{ at steady state. Diffusion = Drift.}$$

$$\text{Concentration Gradient} = \frac{10^6 - 10^2}{1\text{cm}}\text{cm}^{-3} \approx 10^6\text{cm}^{-4}$$

Substitute:

$$D_n = kT\mu_n/e$$

$$\text{Therefore, } kT\mu_n\frac{dn}{dx} = en\mu_n\epsilon$$

$$\epsilon = \frac{kT}{en}\frac{dn}{dx}$$

At the hot end:

$$n = 10^6 \text{cm}^{-3}, \epsilon = \frac{kT}{e} \cdot \frac{10^6 \text{cm}^{-4}}{10^6 \text{cm}^{-3}} = \frac{kT}{e} = 0.026 \text{V/cm}$$

At the cold end:

$$n = 10^2 \text{cm}^{-3}, \epsilon = 260 \text{V/cm}.$$

6. Consider a silicon *p-n* junction at T=300 K with the following parameters:

$N_d = 10^{16}\ cm^{-3}$ $N_a = 5\times10^{18}\ cm^{-3}$

$D_p = 10\ cm^2/sec$ $D_n = 25\ cm^2/sec$

$\tau_{n0} = 5\times10^{-7}\ sec$ $\tau_{p0} = 10^{-7}\ sec$ & $n_i = 1.5\times10^{10}\ cm^{-3}$

Assume that excess carriers are uniformly generated in the solar cell and for one sun, J_L=15 mA/cm^2. Calculate the open circuit voltage.

Solution:

Given : $N_d = 10^{16}/cm^3$; $N_a = 5 \times 10^{18}/cm^3$; $D_p = 10\ cm^2/sec$; $D_n = 25\ cm^2/sec$;

$\tau_{n0} = 5 \times 10^{-7}$ sec ; $\tau_{p0} = 10^{-7}$ sec ; $n_i = 1.5 \times 10^{10}/cm^3$; $J_L = 15 mA/cm^2$;

kT/q = 0.026 eV (at 300K)

$V_{OC} = (kT/q) \ln (1 + (J_L/J_O))$

$J_O = qn_i^2 [(D_p/ (L_pN_d)) + (D_n/ (L_nN_a))]$

$L_p = (D_p\tau_{p0})^{1/2} = (10 \times 10^{-7})^{1/2} = 10^{-3}$ cm = 10 μm

$L_n = (D_n\tau_{n0})^{1/2} = (25 \times 5 \times 10^{-7})^{1/2} = 3.54 \times 10^{-3}$ cm = 35.4 μm

$J_0 = 1.6 \times 10^{-19} \times 1.5^2 \times 10^{20} [(10/ (10^{-3} \times 10^{16})) + (25 / (3.54 \times 10^{-3} \times 5 \times 10^{18}))]$

$= 3.6 \times 10^{-11}\ A/cm^2$

$V_{OC} = 0.0259 \times \ln [1 + (15 \times 10^{-3}/ 3.6 \times 10^{-11})]$

$= 0.514\ V$

7. An abrupt Si *p-n* junction (A = 10^{-4} cm^2) has the following property at 300 K :

p-side	*n-side*
$N_a = 10^{17} cm^{-3}$ $\tau_n = 0.1\mu s$ $\mu_p = 200 \frac{cm^2}{V-s}$ $\mu_n = 700 \frac{cm^2}{V-s}$	$N_d = 10^{15} cm^{-3}$ $\tau_p = 10\mu s$ $\mu_n = 1300 \frac{cm^2}{V-s}$ $\mu_p = 450 \frac{cm^2}{V-s}$

Assume n_i to be 1.5 x 10^{10} cm^{-3}.

Calculate the following:

(i) The junction is forward biased by 0.5 V. What is the forward current?

(ii) What is the current at reverse bias $-0.5\ V$?

Solution:

$$I = qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)\left(e^{qV/kT} - 1\right) = I_0\left(e^{qV/kT} - 1\right)$$

$$p_n = \frac{n_i^2}{n_n} = \frac{\left(1.5\times10^{10}\right)^2}{10^{15}} = 2.25 \times 10^5 \text{ cm}^{-3}$$

$$n_p = \frac{n_i^2}{p_n} = \frac{\left(1.5\times10^{10}\right)^2}{10^{17}} = 2.25 \times 10^3 \text{ cm}^{-3}$$

For minority carriers,

$$D_p = \frac{kT}{q}\mu_p = 0.0259\times450 = 11.66 \text{ cm}^2\text{/s on the } n \text{ side}$$

$$D_n = \frac{kT}{q}\mu_n = 0.0259\times700 = 18.13 \text{ cm}^2\text{/s on the } p \text{ side}$$

$$L_p = \sqrt{D_p\tau_p} = \sqrt{11.66\times10\times10^{-6}} = 1.08 \times 10^{-2} \text{ cm}$$

$$L_n = \sqrt{D_n\tau_n} = \sqrt{18.13\times0.1\times10^{-6}} = 1.35 \times 10^{-3} \text{ cm}$$

$$I_0 = qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)$$

$$= 1.6\times10^{-19}\times0.0001\left(\frac{11.66}{0.0108}2.25\times10^5 + \frac{18.13}{0.00135}2.25\times10^3\right)$$

$=4.370 \times 10^{-15}$ A

$I = I_0\left(e^{0.5/0.0259} - 1\right) \approx \mathbf{1.058 \times 10^{-6}}$ **A** in forward bias

$I = -I_0 = \mathbf{-4.37 \times 10^{-15}}$ **A** in reverse bias

----