Assignment 3 Fasignment 8 1a. E = m v(t). v(t) + Q(t) Dince total energy is comeaved dE = 0 ⇒ d m v(t) · v(t) + d (p(r) = 0 In [dv(t) . v(t) + v(t) . dv(t)] = - dy (v(7) m [dow v(x) + v(x) dow)] = - 70(7) di * - - - sine of = T(t), m [dv(v) + v(v) - dv(v)] = - 29(7). v(v) => : m dv(+) = - v((7) (shown)

b.
$$-\nabla \mathcal{Q}(\vec{r}) = -\frac{GMm}{R^3} (x\hat{e}_x + y\hat{e}_y - 2z\hat{e}_z)$$
 $\Rightarrow \frac{\partial \mathcal{Q}(\vec{r})}{\partial x} \hat{e}_x + \frac{\partial \mathcal{Q}(\vec{r})}{\partial y} \hat{e}_y + \frac{\partial \mathcal{Q}(\vec{r})}{\partial z} \hat{e}_z = \frac{GMm}{R^3} (x\hat{e}_x + y\hat{e}_y - z\hat{e}_z)$

Comparing \hat{e}_x component,

 $\frac{\partial \mathcal{Q}(\vec{r})}{\partial x} = \frac{GMm}{R^3} \frac{x^2}{2} + f_1(y,z)$

Comparing \hat{e}_y component,

 $\frac{\partial \mathcal{Q}(\vec{r})}{\partial y} = \frac{GMm}{R^3} \frac{y}{2}$
 $\frac{\partial f_1(y,z)}{\partial y} = \frac{GMm}{R^3} \frac{y}{2} + f_3(z)$

Comparing \hat{e}_z component,

 $\frac{\partial \mathcal{Q}(\vec{r})}{\partial y} = \frac{GMm}{R^3} \frac{y^2}{2} + f_3(z)$

Comparing \hat{e}_z component,

 $\frac{\partial \mathcal{Q}(\vec{r})}{\partial z} = \frac{GMm}{R^3} \frac{y^2}{2} + f_3(z)$
 $\frac{\partial f_1(y,z)}{\partial z} = \frac{GMm}{R^3} \frac{y^2}{2} + f_3(z)$

2a. ACR) = Mo Sino e-Ar êo 7 = rsinOcos & ex + rsinOsind ey + rcosO ez h = | sind cospex + sind sind by + coso by = \(\left(\sin\text{O} \cos\text{\$\pi} \right)^2 + \left(\cos\text{\$\pi} \right)^2 + \left(\cos\text{\$\pi} \right)^2 = | sin 20 (cos 2 d + sin 20) + cos 20 = 1

> h= () = | r coso cos p ex + r coso sinp ey - r sin o ez 1 = /(rcos0 cosp) + (rcos0 sinp) + (rsin 0)2 = Nr2 cos20 (cos20 + sin20) + (251200

ho = de = | -rsindsind ex traindsind ey 1 = N(rsinOsino)2 + (rsinOsino)2 = [r'sin'0 (sin'0 + cos'0)

= rsin0

 $\nabla X \vec{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \cdot \hat{\ell}_r & h_2 \cdot \hat{\ell}_0 & h_3 \cdot \hat{\ell}_0 \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \theta} \\ h_1 A_r & h_2 A_0 & h_3 A_0 \end{vmatrix}$ = +2 sino (30 h3 A4 - 30 12 A0) & + (3 1, Ar - 3 1, Ax) + 20 + (3-KA - 30 KA,) rsno &] = 1 [() de rsin 0 to to sin 0 e-r) er - (2 rono ho Asind e-Ar) rêo] = rising [(Mo Ao e- Ar 2 sin20) er - (2 rsin 0 fr + sin8 e- Ar) real = 1 [1.A. e > 2 0 sin 20] = - [41 sin 20 d e - 1) & = rising [40 e 2 2 sin Ocard êr + 40 sin o 2 rent ea] = Moto 2 cost e- dr dr + Nond e- dr do]

Assignment 3 8-0 $\nabla X (\nabla X \vec{A}) = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \hat{e}_c & h_3 \hat{e}_d \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \Phi} \end{vmatrix}$ h, (VXA), h, (VXA), h, (VXA) = = 1 (3 h, (\(\fix A)_0 - \fix \(\fix A)_0 \) \(\hat{e}_2 \) + () h (VXA), - d h (VXA) rêo + (3 h, (VXA) - 30 h, (VXA),) rsino 20 = 1 to Ao (- 3 to Sino e- Ar) êr + (2 2000 e-7) r 60 + (de r Asino e-Ar - do 2coso e-Ar) rsino do = 1 MoAo [- 2sinde - 2r + 2sinde - 2r] rsindê. = MoAosino e-2r [2 - 2] ê => VX[VXAG)] = No. Ao sind e >r (= - 1) ê = No. J(1) == J(r) = A sind e-2 (= - 1) ex

priments
$$2b \ \exists x \ \exists = re_{r} \times \vec{J} = \underbrace{e_{r}}_{c} \underbrace{e_{o}}_{o} \underbrace{e_{o}}_{o}$$

$$= \underbrace{\left[-r \frac{A_{o} \sin \theta}{4\pi} e^{-\lambda r} \left(\frac{2}{r^{3}} - \frac{\lambda^{2}}{r^{2}}\right)\right]}_{c} \underbrace{e_{o}}_{c}$$

$$= \underbrace{\frac{A_{o} \sin \theta}{4\pi}}_{c} \underbrace{e^{-\lambda r} \left(\lambda^{2} - \frac{2}{r^{2}}\right)}_{c} \underbrace{e_{o}}_{c}$$

$$m = \underbrace{\frac{1}{2} \iint_{c} x \ \vec{J}(x) \ dV}_{c}$$

$$= \underbrace{\frac{1}{2} \underbrace{A_{o}^{2} \iint_{c} \sin \theta}_{c} \underbrace{e^{-\lambda r} \left(\lambda^{2} - \frac{2}{r^{2}}\right)}_{c} \underbrace{e_{o}}_{c} \ dV}_{c}$$

$$dV = \underbrace{h_{c} h_{2} h_{3} \ dr \ dO \ d\varphi}_{c}$$

$$= +2 \sin \theta \ dr \ dO \ d\varphi}_{c}$$

$$= \underbrace{e_{o} \quad e^{-\lambda r} \left(\lambda^{2} - \frac{2}{r^{2}}\right)}_{c} \underbrace{e_{o}}_{c} \ dV$$

$$\underbrace{e_{o} = \frac{1}{2} \underbrace{e_{o}}_{c} = \frac{1}{2} \left(r \cos \theta \cos \varphi \ \hat{e}_{x} + r \cos \theta \sin \theta \ \hat{e}_{y} - r \sin \theta \ \hat{e}_{z}\right)}_{c}$$

since to is independent of r,

Assignment of Evaluate J (12 x2-2) e-r dr, $\int_{r=0}^{\infty} \lambda^2 r^2 - 2 e^{-\lambda r} dr = \lambda^2 \left[-\frac{r^2}{\lambda} e^{-\lambda r} \right]_0^{\infty} - \lambda^2 \int_0^{\infty} -\frac{2r}{\lambda} e^{-\lambda r} dr$ $+ \left[\frac{2}{\lambda} e^{-\lambda r}\right]_{0}^{\infty}$ $= \left[-\lambda_r^2 e^{-\lambda r} + 2\lambda e^{-\lambda r} \right]_0^\infty + 2\lambda \left[-\frac{r}{\lambda} e^{-\lambda r} \right]_0^\infty$ - Jo te-dr $= [-\lambda r^{2}e^{-\lambda r} + 2\lambda e^{-\lambda r} - 2re^{-\lambda r} - \frac{1}{2}e^{-\lambda r}]^{2}$ · · m =0

Sa. C = a cosh & cosncos & ex + a cosh & cosn sine Ey + a sinh & single DE = asinh & councos dex + a sinh & cos n sind ey + a cosh & sing ez To = - a cosh & sinh cos & êx - a cosh & sinh sin & ey ta sinh & cosh & z de = -a cosh & cosn sno ex + a cosh & cosn coso ey h= | de | = |(asinh & cospcoso)2 + (asinh & cosp sin d)2+ (acosh & sin p)2 = /a2 sinh28 cos2n(cos20 + sin20) +a2 cosh28 sin2n = a sinh 28 (1-sin2n) + a2 cosh 28 sin2n =alsinh28-sinh28sin2n tosh28sin2n = a sinh 2 & + sin 2 n (cosh 2 & - sin h 2 &) = ansinh28 tsin2n h, = | = N (-acosh&sin(coso) + (-acosh&sin(sino) + (asinh&coso))2 = a cosh 8 sin 1(cosptin) + sinh 8cos n = a cosh Esin 1 + sinh E(1-sin) = a cosh & sin'n + sinh & - sinh & sin'n =qsin2 (cosh&-sinh28) + sinh & = a sinh 8 t sin ??

AUTONIE .

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$$\hat{e}_{\varepsilon} \cdot \hat{e}_{\gamma} = \frac{1}{\lambda_1 \lambda_2} \frac{\partial \varepsilon}{\partial 8} \cdot \frac{\partial \varepsilon}{\partial \gamma}$$

$$\hat{e}_{\varepsilon} \cdot \hat{e}_{\varphi} = \frac{1}{h_{i}h_{i}} \frac{\partial f}{\partial \theta} \cdot \frac{\partial c}{\partial \varphi}$$

$$\frac{1}{2} = 0$$

$$\frac{\partial}{\partial x} = \frac{1}{h_1 h_2} = \frac{\partial}{\partial x} = \frac{\partial}{\partial x}$$

$$\left\{ \begin{array}{ll} \hat{e}_{8} & \hat{e}_{n} = 0 \\ \hat{e}_{8} & \hat{e}_{0} = 0 \end{array} \right\} \Rightarrow \text{ orthogonal}$$

$$\left\{ \begin{array}{ll} \hat{e}_{8} & \hat{e}_{0} = 0 \\ \hat{e}_{n} & \hat{e}_{0} = 0 \end{array} \right\} \Rightarrow \text{ orthogonal}$$

4229 Assignment 3 ent 2 Assignment 9 (g,0)d EE = 1, 25 = avsinh8+sin'n (asinh8concost & +asinh8cosq sintly + cosh Esing (2) = Jsix 8+sin2n (sinh & cosposab ex + sinh & cospandey + cosh & suple) ên = 1 de - Tsinhistsinin (-cosh & singross dex - cosh & singrindey + sinh Ecosp êz) ex = 1 de = cosh 8cosn (-cosh 8 cosn sind êx + cosh 8 cosn cosp ey) =-sind êx + cost êy $\hat{e}_{n} \times \hat{e}_{\varepsilon} = \begin{vmatrix} \hat{e}_{x} & \hat{e}_{y} & \hat{e}_{z} \\ (\hat{e}_{n})_{x} & (\hat{e}_{n})_{y} & (\hat{e}_{n})_{z} \end{vmatrix}$ (lêx) (lex) lexz = sinh28 + sin2n [(-cosh28 xn2nsnø - sinh28cos2nsnø) ex + (sinh2 & cos2 n cost + cosh2 &sm2 n cost) ey + (-sinh&cosh&sing cosq cosq sind) et] = onhi & +sinin [-sind (cashi &sinin + sinhi &cosin) Ex + cosp (sinh 28 cos 2 + cosh 28 sin 2 n) êy] = sinh2 & +sin2 {-sind [(Itsinh28)sin2 + sinh28(1-sin2)] ex + cos \$ [(Hsinh28)sin2n + sinh28(1-sin2n)] &y}

Assignment 3 $=-\sin\phi \, \hat{e}_x + \cos\phi \, \hat{e}_y = \hat{e}_\phi$ êaxên= Johans (sinh&cosncosd)êx + (sonbacosn sing)êy +(osh & sing sin2 + cosh & sing(052 \$) eq = Isinh & + sing [(sinh & cosn cos d) ex + (sinh & cosn snd) ey +(cosh&san) êz] = ê8 Exx & = Jank & tsing [-cosh & sing cost & - cosh & sing sing by 三月 + (sinh & cosnces2 & + sinh & cosn sin2) êz] = Jsinh28+sin2n [-cosh&singcosd êx-cosh&singsindey + sinh & cosn êz] = ên êi xêj = Eleijkêk ; i,j=1,2,3 2) right-handed

