Assignment 1Ø

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1a. $\mathcal{L}\{\cosh at \cos at\} = \frac{1}{2}\mathcal{L}\{(e^{at}+e^{-at})\cos at\}$

$= \frac{1}{2}\mathcal{L}\{e^{at}\cos at\} + \frac{1}{2}\mathcal{L}\{e^{-at}\cos at\}$

$= \frac{1}{2}\left[\frac{s-a}{(s-a)^2+a^2}\right] + \frac{1}{2}\left[\frac{s+a}{(s+a)^2+a^2}\right]$

b. $\mathcal{L}\{\sinh at \sin at\} = \frac{1}{2}\mathcal{L}\{(e^{at}-e^{-at})\sin at\}$

$= \frac{1}{2}\mathcal{L}\{e^{at}\sin at\} - \frac{1}{2}\mathcal{L}\{e^{-at}\sin at\}$

$= \frac{1}{2}\left[\frac{a}{(s-a)^2+a^2}\right] - \frac{1}{2}\left[\frac{a}{(s+a)^2+a^2}\right]$

2c. Using partial fraction expansion:

let $\frac{k^2}{s(s^2+k^2)} = \frac{A}{s} + \frac{Bs+C}{s^2+k^2}$

$k^2 = A(s^2+k^2) + Bs^2 + Cs$

comparing coefficients of s,

$C = 0$

comparing coefficients of s^2,

$A + B = 0$

$A = -B \quad —(1)$

Comparing constant k,

$k^2 = Ak^2$

$A = 1 \quad —(2)$

Sub (2) into (1)

$B = -1$

$\Rightarrow A = 1,\ B = -1,\ C = 0$

$\Rightarrow \mathcal{L}^{-1}\left\{\frac{k^2}{s(s^2+k^2)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{s}{(s^2+k^2)}\right\}$

$= 1 - \cos(kt)$

$d\theta(t) = \int \frac{2}{}$

$\int dt = \int$

hen $t=0$,

$\Rightarrow \int_0^{\frac{\pi}{4}} dt$

$\frac{\pi}{4}$:

$1dt$:

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Using convolution theorem:

$$\mathcal{L}^{-1}\left\{\frac{k^2}{s(s^2+k^2)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} * \mathcal{L}^{-1}\left\{\frac{k^2}{s^2+k^2}\right\}$$

$$= 1 * \mathcal{L}^{-1}\left\{k\left(\frac{k}{s^2+k^2}\right)\right\}$$

$$= 1 * \left[\mathcal{L}^{-1}\{k\} * \mathcal{L}^{-1}\left\{\frac{k}{s^2+k^2}\right\}\right]$$

$$= 1 * \left[\mathcal{L}^{-1}\{k\} * \sin(kt)\right]$$

Since $\mathcal{L}\{\delta(t)\} = 1 \Rightarrow \mathcal{L}^{-1}\{k\} = k\delta(t)$

$$\Rightarrow \mathcal{L}^{-1}\left\{\frac{k^2}{s(s^2+k^2)}\right\} = 1 * \left[k\delta(t) * \sin(kt)\right]$$

$$= 1 * k\sin(kt)$$

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b. Using partial fraction expansion,

Let $\dfrac{s}{(s^2+a^2)(s^2+b^2)} = \dfrac{s}{[s-ia][s+ia][s-ib][s+ib]}$

$\Rightarrow \dfrac{s}{(s^2+a^2)(s^2+b^2)} = \dfrac{A}{s-ia} + \dfrac{B}{s+ia} + \dfrac{C}{s-ib} + \dfrac{D}{s+ib}$ — (1)

Multiply (1) by $(s-ia)$ on both sides, limits $\Rightarrow s \to ia$

$\Rightarrow A = \lim_{s\to ia}\left\{\dfrac{s}{(s+ia)(s^2+b^2)}\right\} = \dfrac{ia}{2ia(-a^2+b^2)} = \dfrac{1}{2(b^2-a^2)}$

Multiply (1) by $(s+ia)$ on both sides, limits $\Rightarrow s \to -ia$

$\Rightarrow B = \lim_{s\to -ia}\left\{\dfrac{s}{(s-ia)(s^2+b^2)}\right\} = \dfrac{-ia}{-2ia(-a^2+b^2)} = \dfrac{1}{2(b^2-a^2)}$

Multiply (1) by $(s-ib)$ on both sides, limits $\Rightarrow s \to ib$

$\Rightarrow C = \lim_{s\to ib}\left\{\dfrac{s}{(s^2+a^2)(s+ib)}\right\} = \dfrac{ib}{(-b^2+a^2)(2ib)} = \dfrac{1}{2(a^2-b^2)}$

Multiply (1) by $(s+ib)$ on both sides, limits $\Rightarrow s \to -ib$

$\Rightarrow D = \lim_{s\to -ib}\left\{\dfrac{s}{(s^2+a^2)(s-ib)}\right\} = \dfrac{-ib}{(-b^2+a^2)(-2b)} = \dfrac{1}{2(a^2-b^2)}$

$$\Rightarrow \frac{s}{(s^2+a^2)(s^2+b^2)} = \frac{1}{2(b^2-a^2)}\left[\frac{1}{s-ia} + \frac{1}{s+ia}\right] + \frac{1}{2(a^2-b^2)}\left[\frac{1}{s-ib} + \frac{1}{s+ib}\right]$$

Since $\mathcal{L}\{e^{\pm i\omega t}\} = \dfrac{1}{s \mp i\omega}$,

$$\Rightarrow \mathcal{L}^{-1}\left\{\frac{s}{(s^2+a^2)(s^2+b^2)}\right\} = \frac{1}{2(b^2-a^2)}\left[e^{iat} + e^{-iat}\right] + \frac{1}{2(a^2-b^2)}\left[e^{ibt} + e^{-ibt}\right]$$

$$= \frac{1}{b^2-a^2}\cos(at) + \frac{1}{a^2-b^2}\cos(bt)$$

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Using convolution theorem:

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+a^2)(s^2+b^2)}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s^2+a^2)}\right\} * \mathcal{L}^{-1}\left\{\frac{1}{(s^2+b^2)}\right\}$$

$$= \cos(at) * \frac{1}{b}\sin(bt)$$

$$= \int_0^t \cos(au)\frac{1}{b}\sin[b(t-u)]\,du$$

$$= \frac{1}{b}\int_0^t \cos(au)\sin[bt-bu]\,du$$

$$= \frac{1}{b}\left[\sin(bt-bu)\frac{\sin(au)}{a}\right]_0^t - \frac{1}{b}\int_0^t -b\cos(bt-bu)\left[\frac{\sin(au)}{a}\right]du$$

$$= \frac{1}{b}(0) + \frac{1}{b}\int_0^t \frac{b}{a}\cos(bt-bu)\sin(au)\,du$$

$$= \frac{1}{a}\left[\cos(bt-bu)\left(-\frac{\cos(au)}{a}\right)\right]_0^t + \int_0^t \frac{b\cos(au)}{a}\sin(bt-bu)\,du$$

$$= \frac{1}{a}\left[-\frac{\cos(at)}{a} + \frac{1}{a}\cos(bt)\right] + \frac{b}{a}\int_0^t \cos(au)\sin(bt-bu)\,du$$

$$\Rightarrow \frac{1}{b}\int_0^t \cos(au)\sin[bt-bu]\,du = \frac{1}{a}\left[\frac{1}{a}\cos(bt) - \frac{\cos(at)}{a}\right] + \frac{b}{a}\int_0^t \cos(au)\sin(bt-bu)\,du$$

$$\Rightarrow \int_0^t \cos(au)\sin(bt-bu)\,du = \frac{1}{a^2}[\cos(bt)-\cos(at)]\left(\frac{1}{b}-\frac{b}{a^2}\right)^{-1}$$

$$= \left(\frac{b^2}{a^2-b^2}\right)[\cos(bt)-\cos(at)]$$

$$\Rightarrow \mathcal{L}^{-1}\left\{\frac{s}{(s^2+a^2)(s^2+b^2)}\right\} = \frac{1}{b}\left(\frac{b^2}{a^2-b^2}\right)[\cos(bt)-\cos(at)]$$

$$= \frac{b}{a^2-b^2}[\cos(bt)-\cos(at)]$$