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Assignment 3

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1a. $\varepsilon = \frac{m}{2}\vec{v}(t)\cdot\vec{v}(t) + \varphi(\vec{r})$

Since total energy is conserved

$$\frac{d\varepsilon}{dt} = 0 \Rightarrow \frac{d}{dt}\frac{m}{2}\vec{v}(t)\cdot\vec{v}(t) + \frac{d}{dt}\varphi(\vec{r}) = 0$$

$$\frac{m}{2}\left[\frac{d\vec{v}(t)}{dt}\cdot\vec{v}(t) + \vec{v}(t)\cdot\frac{d\vec{v}(t)}{dt}\right] = -\frac{d}{dt}\varphi(\vec{r})$$

$$\frac{m}{2}\left[\frac{d\vec{v}(t)}{dt}\cdot\vec{v}(t) + \vec{v}(t)\cdot\frac{d\vec{v}(t)}{dt}\right] = -\nabla\varphi(\vec{r})\cdot\frac{d\vec{r}}{dt}$$

since $\frac{d\vec{r}}{dt} = \vec{v}(t)$,

$$\frac{m}{2}\left[\frac{d\vec{v}(t)}{dt}\cdot\vec{v}(t) + \vec{v}(t)\cdot\frac{d\vec{v}(t)}{dt}\right] = -\nabla\varphi(\vec{r})\cdot\vec{v}(t)$$

$$\Rightarrow \therefore m\frac{d\vec{v}(t)}{dt} = -\nabla\varphi(\vec{r}) \text{ (shown)}$$

b. $-\nabla\varphi(\vec{r}) = -\frac{GMm}{R^3}(x\hat{e}_x + y\hat{e}_y - 2z\hat{e}_z)$

$$\Rightarrow \frac{\partial\varphi(\vec{r})}{\partial x}\hat{e}_x + \frac{\partial\varphi(\vec{r})}{\partial y}\hat{e}_y + \frac{\partial\varphi(\vec{r})}{\partial z}\hat{e}_z = \frac{GMm}{R^2}(x\hat{e}_x + y\hat{e}_y - z\hat{e}_z)$$

Comparing $\hat{e}_x$ component,

$$\frac{\partial\varphi(\vec{r})}{\partial x} = \frac{GMm}{R^3}x$$

$$\varphi(\vec{r}) = \frac{GMm}{R^3}\frac{x^2}{2} + f_1(y,z)$$

Comparing $\hat{e}_y$ component,

$$\frac{\partial\varphi(\vec{r})}{\partial y} = \frac{GMm}{R^3}y$$

$$\frac{\partial f_1(y,z)}{\partial y} = \frac{GMm}{R^3}y$$

$$f_1(y,z) = \frac{GMm}{R^3}\frac{y^2}{2} + f_2(z)$$

Comparing $\hat{e}_z$ component,

$$\frac{\partial\varphi(\vec{r})}{\partial z} = \frac{GMm}{R^3}(-2z)$$

$$\frac{\partial f_2(z)}{\partial z} = -2z\frac{GMm}{R^3}$$

$$f_2(z) = -\frac{GMm}{R^3}z^2 + c'$$

$$\Rightarrow \varphi(\vec{r}) = \frac{GMm}{R^3}\frac{x^2}{2} + f_1(y,z) = \frac{GMm}{R^3}\frac{x^2}{2} + \frac{GMm}{R^3}\frac{y^2}{2} + f_2(z)$$

$$= \frac{GMm}{R^3}\frac{x^2}{2} + \frac{GMm}{R^3}\frac{y^2}{2} - \frac{GMm}{R^3}z^2 + c'$$

$$= \frac{GMm}{R^3}\left(\frac{x^2}{2} + \frac{y^2}{2} - z^2\right) + c$$

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2a. $\vec{A}(\underline{r}) = \frac{\mu_0}{4\pi} \frac{A_0 \sin\theta}{r} e^{-\lambda r} \hat{e}_\phi$

$\vec{r} = r\sin\theta\cos\phi\, \hat{e}_x + r\sin\theta\sin\phi\, \hat{e}_y + r\cos\theta\, \hat{e}_z$

$h_1 = \left|\frac{\partial \underline{r}}{\partial r}\right| = |\sin\theta\cos\phi\, \hat{e}_x + \sin\theta\sin\phi\, \hat{e}_y + \cos\theta\, \hat{e}_z|$

$= \sqrt{(\sin\theta\cos\phi)^2 + (\sin\theta\sin\phi)^2 + (\cos\theta)^2}$

$= \sqrt{\sin^2\theta(\cos^2\phi + \sin^2\phi) + \cos^2\theta} = 1$

$h_2 = \left|\frac{\partial \underline{r}}{\partial \theta}\right| = |r\cos\theta\cos\phi\, \hat{e}_x + r\cos\theta\sin\phi\, \hat{e}_y - r\sin\theta\, \hat{e}_z|$

$= \sqrt{(r\cos\theta\cos\phi)^2 + (r\cos\theta\sin\phi)^2 + (r\sin\theta)^2}$

$= \sqrt{r^2\cos^2\theta(\cos^2\phi + \sin^2\phi) + r^2\sin^2\theta}$

$= r$

$h_3 = \frac{\partial \underline{r}}{\partial \phi} = |-r\sin\theta\sin\phi\, \hat{e}_x + r\sin\theta\sin\phi\, \hat{e}_y|$

$= \sqrt{(r\sin\theta\sin\phi)^2 + (r\sin\theta\sin\phi)^2}$

$= \sqrt{r^2\sin^2\theta(\sin^2\phi + \cos^2\phi)}$

$= r\sin\theta$

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$$\nabla\times\vec{A}=\frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\hat{e}_r & h_2\hat{e}_\theta & h_3\hat{e}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ h_1A_r & h_2A_\theta & h_3A_\phi \end{vmatrix}$$

$$=\frac{1}{r^2\sin\theta}\Big[\left(\frac{\partial}{\partial\theta}h_3A_\phi-\cancelto{0}{\frac{\partial}{\partial\phi}h_2A_\theta}\right)\hat{e}_r$$

$$+\left(\cancelto{0}{\frac{\partial}{\partial\phi}h_1A_r}-\frac{\partial}{\partial r}h_3A_\phi\right)r\hat{e}_\theta$$

$$+\left(\cancelto{0}{\frac{\partial}{\partial r}h_2A_\theta}-\cancelto{0}{\frac{\partial}{\partial\theta}h_1A_r}\right)r\sin\theta\,\hat{e}_\phi\Big]$$

$$=\frac{1}{r^2\sin\theta}\Big[\left(\frac{\partial}{\partial\theta}r\sin\theta\frac{\mu_0}{4\pi}\frac{A_0\sin\theta}{r}e^{-\lambda r}\right)\hat{e}_r$$

$$-\left(\frac{\partial}{\partial r}r\sin\theta\frac{\mu_0}{4\pi}\frac{A_0\sin\theta}{r}e^{-\lambda r}\right)r\hat{e}_\theta\Big]$$

$$=\frac{1}{r^2\sin\theta}\Big[\left(\frac{\mu_0A_0}{4\pi}e^{-\lambda r}\frac{\partial}{\partial\theta}\sin^2\theta\right)\hat{e}_r-\left(\frac{\partial}{\partial r}r\sin\theta\frac{\mu_0}{4\pi}\frac{A_0\sin\theta}{r}e^{-\lambda r}\right)r\hat{e}_\theta\Big]$$

$$=\frac{1}{r^2\sin\theta}\Big[\left(\frac{\mu_0A_0}{4\pi}e^{-\lambda r}\frac{\partial}{\partial\theta}\sin^2\theta\right)\hat{e}_r-\left(\frac{\mu_0A_0}{4\pi}\sin^2\theta\frac{\partial}{\partial r}e^{-\lambda r}\right)\hat{e}_\theta\Big]$$

$$=\frac{1}{r^2\sin\theta}\Big[\frac{\mu_0A_0}{4\pi}e^{-\lambda r}\cdot2\sin\theta\cos\theta\,\hat{e}_r+\frac{\mu_0A_0}{4\pi}\sin^2\theta\cdot\lambda re^{-\lambda r}\hat{e}_\theta\Big]$$

$$=\frac{\mu_0A_0}{4\pi}\Big[\frac{2\cos\theta}{r^2}e^{-\lambda r}\hat{e}_r+\frac{\lambda\sin\theta}{r}e^{-\lambda r}\hat{e}_\theta\Big]$$

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$$\nabla\times(\nabla\times\vec{A}) = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\hat{e}_r & h_2\hat{e}_\theta & h_3\hat{e}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ h_1(\nabla\times\vec{A})_r & h_2(\nabla\times\vec{A})_\theta & h_3(\nabla\times\vec{A})_\phi \end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta}\Big[\Big(\cancelto{0}{\frac{\partial}{\partial\theta}h_3(\nabla\times\vec{A})_\phi} - \frac{\partial}{\partial\phi}h_2(\nabla\times\vec{A})_\theta\Big)\hat{e}_r$$

$$+\Big(\frac{\partial}{\partial\phi}h_1(\nabla\times\vec{A})_r - \cancelto{0}{\frac{\partial}{\partial r}h_3(\nabla\times\vec{A})_\phi}\Big)r\hat{e}_\theta$$

$$+\Big(\frac{\partial}{\partial r}h_2(\nabla\times\vec{A})_\theta - \frac{\partial}{\partial\theta}h_1(\nabla\times\vec{A})_r\Big)r\sin\theta\,\hat{e}_\phi$$

$$= \frac{1}{r^2\sin\theta}\frac{\mu_0A_0}{4\pi}\Big[\Big(\cancelto{0}{-\frac{\partial}{\partial\phi}r\frac{\lambda\sin\theta}{r}}e^{-\lambda r}\Big)\hat{e}_r$$

$$+\Big(\cancelto{0}{\frac{\partial}{\partial\phi}\frac{2\cos\theta}{r^2}}e^{-\lambda r}\Big)r\hat{e}_\theta$$

$$+\Big(\frac{\partial}{\partial r}r\frac{\lambda\sin\theta}{r}e^{-\lambda r} - \frac{\partial}{\partial\theta}\frac{2\cos\theta}{r^2}e^{-\lambda r}\Big)r\sin\theta\,\hat{e}_\phi\Big]$$

$$= \frac{1}{r^2\sin\theta}\frac{\mu_0A_0}{4\pi}\Big[-\lambda^2\sin\theta\,e^{-\lambda r} + \frac{2\sin\theta}{r^2}e^{-\lambda r}\Big]r\sin\theta\,\hat{e}_\phi$$

$$= \frac{\mu_0A_0\sin\theta}{4\pi}e^{-\lambda r}\Big[\frac{2}{r^3} - \frac{\lambda^2}{r}\Big]\hat{e}_\phi$$

$$\Rightarrow \nabla\times[\nabla\times\vec{A}(\vec{r})] = \frac{\mu_0A_0\sin\theta}{4\pi}e^{-\lambda r}\Big(\frac{2}{r^3}-\frac{\lambda^2}{r}\Big)\hat{e}_\phi = \mu_0\vec{J}(\vec{r})$$

$$\therefore \vec{J}(\vec{r}) = \frac{A_0\sin\theta}{4\pi}e^{-\lambda r}\Big(\frac{2}{r^3}-\frac{\lambda^2}{r}\Big)\hat{e}_\phi$$

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2b $\underset{\sim}{r} \times \vec{J} = r\hat{e}_r \times \vec{J} = \begin{vmatrix} \hat{e}_r & \hat{e}_\theta & \hat{e}_\phi \\ r & 0 & 0 \\ 0 & 0 & J_\phi \end{vmatrix}$

$$= \left[-r \frac{A_0 \sin\theta}{4\pi} e^{-\lambda r}\left(\frac{2}{r^3} - \frac{\lambda^2}{r}\right)\right]\hat{e}_\theta$$

$$= \frac{A_0 \sin\theta}{4\pi} e^{-\lambda r}\left(\lambda^2 - \frac{2}{r^2}\right) \hat{e}_\theta$$

$$\underset{\sim}{m} = \frac{1}{2}\iiint \underset{\sim}{r} \times \vec{J}(\underset{\sim}{r})\, dV$$

$$= \frac{1}{2}\frac{A_0}{4\pi}\iiint \sin\theta\, e^{-\lambda r}\left(\lambda^2 - \frac{2}{r^2}\right) \hat{e}_\theta\, dV$$

$$dV = h_1 h_2 h_3\, dr\, d\theta\, d\phi$$

$$= r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$\hat{e}_\theta = \frac{1}{h_2}\frac{\partial \underset{\sim}{r}}{\partial \theta} = \frac{1}{r}\left(r\cos\theta\cos\phi\, \hat{e}_x + r\cos\theta\sin\phi\, \hat{e}_y - r\sin\theta\, \hat{e}_z\right)$$

$$= \cos\theta\cos\phi\, \hat{e}_x + \cos\theta\sin\phi\, \hat{e}_y - \sin\theta\, \hat{e}_z$$

$$\Rightarrow \underset{\sim}{m} = \frac{A_0}{8\pi}\iiint e^{-\lambda r}\sin\theta\left(\lambda^2 - \frac{2}{r^2}\right)\hat{e}_\theta\, r^2 \sin\theta\, dr\, d\theta\, d\phi$$

since $\hat{e}_\theta$ is independent of r,

$$\Rightarrow \underset{\sim}{m} = \frac{A_0}{8\pi}\iint \sin^2\theta\, \hat{e}_\theta \int \left(\lambda^2 - \frac{2}{r^2}\right)e^{-\lambda r} r^2\, dr\, d\theta\, d\phi$$

Evaluate $\int (\lambda^2 r^2 - 2) e^{-\lambda r} dr$,

$$\int_{r=0}^{\infty} (\lambda^2 r^2 - 2) e^{-\lambda r} dr = \lambda^2 \left[-\frac{r^2}{\lambda} e^{-\lambda r} \right]_0^{\infty} - \lambda^2 \int_0^{\infty} -\frac{2r}{\lambda} e^{-\lambda r} dr$$

$$+ \left[\frac{2}{\lambda} e^{-\lambda r} \right]_0^{\infty}$$

$$= \left[-\lambda r^2 e^{-\lambda r} + 2\lambda e^{-\lambda r} \right]_0^{\infty} + 2\lambda \left[-\frac{r}{\lambda} e^{-\lambda r} \right]_0^{\infty}$$

$$- \int_0^{\infty} -\frac{1}{\lambda} e^{-\lambda r} dr$$

$$= \left[-\lambda r^2 e^{-\lambda r} + 2\lambda e^{-\lambda r} - 2r e^{-\lambda r} - \frac{1}{\lambda^2} e^{-\lambda r} \right]_0^{\infty}$$

$$= 0$$

$\therefore \vec{m} = 0$

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3a. $\underset{\sim}{r} = a\cosh\xi\cos\eta\cos\phi\,\hat{e}_x + a\cosh\xi\cos\eta\sin\phi\,\hat{e}_y + a\sinh\xi\sin\eta\,\hat{e}_z$

$$\frac{\partial \underset{\sim}{r}}{\partial \xi} = a\sinh\xi\cos\eta\cos\phi\,\hat{e}_x + a\sinh\xi\cos\eta\sin\phi\,\hat{e}_y + a\cosh\xi\sin\eta\,\hat{e}_z$$

$$\frac{\partial \underset{\sim}{r}}{\partial \eta} = -a\cosh\xi\sin\eta\cos\phi\,\hat{e}_x - a\cosh\xi\sin\eta\sin\phi\,\hat{e}_y + a\sinh\xi\cos\eta\,\hat{e}_z$$

$$\frac{\partial \underset{\sim}{r}}{\partial \phi} = -a\cosh\xi\cos\eta\sin\phi\,\hat{e}_x + a\cosh\xi\cos\eta\cos\phi\,\hat{e}_y$$

$$h_1 = \left|\frac{\partial \underset{\sim}{r}}{\partial \xi}\right| = \sqrt{(a\sinh\xi\cos\eta\cos\phi)^2 + (a\sinh\xi\cos\eta\sin\phi)^2 + (a\cosh\xi\sin\eta)^2}$$

$$= \sqrt{a^2\sinh^2\xi\cos^2\eta(\cos^2\phi + \sin^2\phi) + a^2\cosh^2\xi\sin^2\eta}$$

$$= a\sqrt{\sinh^2\xi(1-\sin^2\eta) + a^2\cosh^2\xi\sin^2\eta}$$

$$= a\sqrt{\sinh^2\xi - \sinh^2\xi\sin^2\eta + \cosh^2\xi\sin^2\eta}$$

$$= a\sqrt{\sinh^2\xi + \sin^2\eta(\cosh^2\xi - \sinh^2\xi)}$$

$$= a\sqrt{\sinh^2\xi + \sin^2\eta}$$

$$h_2 = \left|\frac{\partial \underset{\sim}{r}}{\partial \eta}\right| = \sqrt{(-a\cosh\xi\sin\eta\cos\phi)^2 + (-a\cosh\xi\sin\eta\sin\phi)^2 + (a\sinh\xi\cos\eta)^2}$$

$$= a\sqrt{\cosh^2\xi\sin^2\eta(\cos^2\phi + \sin^2\phi) + \sinh^2\xi\cos^2\eta}$$

$$= a\sqrt{\cosh^2\xi\sin^2\eta + \sinh^2\xi(1-\sin^2\eta)}$$

$$= a\sqrt{\cosh^2\xi\sin^2\eta + \sinh^2\xi - \sinh^2\xi\sin^2\eta}$$

$$= a\sqrt{\sin^2\eta(\cosh^2\xi - \sinh^2\xi) + \sinh^2\xi}$$

$$= a\sqrt{\sinh^2\xi + \sin^2\eta}$$

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$$h_3 = \left|\frac{\partial r}{\partial \phi}\right| = \sqrt{(-a\cosh\xi\cos\eta\sin\phi)^2 + (a\cosh\xi\cos\eta\cos\phi)^2}$$

$$= a\sqrt{\cosh^2\xi\cos^2\eta\sin^2\phi + \cosh^2\xi\cos^2\eta\cos^2\phi}$$

$$= a\sqrt{\cosh^2\xi\cos^2\eta\,(\sin^2\phi + \cos^2\phi)}$$

$$= a\cosh\xi\cos\eta$$

$$\hat{e}_\xi \cdot \hat{e}_\eta = \frac{1}{h_1 h_2}\frac{\partial r}{\partial \xi}\cdot\frac{\partial r}{\partial \eta}$$

$$= \frac{1}{h_1 h_2}\Big[-a^2\sinh\xi\cosh\xi\sin\eta\cos\eta\cos^2\phi - a^2\sinh\xi\cosh\xi\sin\eta\cos\eta\sin^2\phi$$
$$+ a^2\sinh\xi\cosh\xi\sin\eta\cos\eta\Big]$$

$$= \frac{1}{h_1 h_2}\Big[-a^2\sinh\xi\cosh\xi\sin\eta\cos\eta(\cos^2\phi + \sin^2\phi)$$
$$+ a^2\sinh\xi\cosh\xi\sin\eta\cos\eta\Big]$$

$$= 0$$

$$\hat{e}_\xi \cdot \hat{e}_\phi = \frac{1}{h_1 h_3}\frac{\partial r}{\partial \xi}\cdot\frac{\partial r}{\partial \phi}$$

$$= \frac{1}{h_1 h_3}\Big[-a^2\sinh\xi\cosh\xi\cos^2\eta\sin\phi\cos\phi + a^2\sinh\xi\cosh\xi\cos^2\eta\sin\phi\cos\phi\Big]$$

$$= 0$$

$$\hat{e}_\eta \cdot \hat{e}_\phi = \frac{1}{h_2 h_3}\frac{\partial r}{\partial \eta}\cdot\frac{\partial r}{\partial \phi}$$

$$= \frac{1}{h_2 h_3}\Big[a^2\cosh^2\xi\sin\eta\cos\eta\sin\phi\cos\phi - a^2\cosh^2\xi\sin\eta\cos\eta\sin\phi\cos\phi\Big]$$

$$= 0$$

$$\left.\begin{array}{l} \hat{e}_\xi \cdot \hat{e}_\eta = 0 \\ \hat{e}_\xi \cdot \hat{e}_\phi = 0 \\ \hat{e}_\eta \cdot \hat{e}_\phi = 0 \end{array}\right\} \Rightarrow \text{orthogonal}$$

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$$\hat{e}_\xi = \frac{1}{h_1}\frac{\partial r}{\partial \xi} = \frac{1}{a\sqrt{\sinh^2\xi + \sin^2\eta}}\left(a\sinh\xi\cos\eta\cos\phi\,\hat{e}_x + a\sinh\xi\cos\eta\sin\phi\,\hat{e}_y + \cosh\xi\sin\eta\,\hat{e}_z\right)$$

$$= \frac{1}{\sqrt{\sinh^2\xi + \sin^2\eta}}\left(\sinh\xi\cos\eta\cos\phi\,\hat{e}_x + \sinh\xi\cos\eta\sin\phi\,\hat{e}_y + \cosh\xi\sin\eta\,\hat{e}_z\right)$$

$$\hat{e}_\eta = \frac{1}{h_2}\frac{\partial r}{\partial \eta} = \frac{1}{\sqrt{\sinh^2\xi + \sin^2\eta}}\left(-\cosh\xi\sin\eta\cos\phi\,\hat{e}_x - \cosh\xi\sin\eta\sin\phi\,\hat{e}_y + \sinh\xi\cos\eta\,\hat{e}_z\right)$$

$$\hat{e}_\phi = \frac{1}{h_3}\frac{\partial r}{\partial \phi} = \frac{1}{\cosh\xi\cos\eta}\left(-\cosh\xi\cos\eta\sin\phi\,\hat{e}_x + \cosh\xi\cos\eta\cos\phi\,\hat{e}_y\right)$$

$$= -\sin\phi\,\hat{e}_x + \cos\phi\,\hat{e}_y$$

$$\hat{e}_\eta \times \hat{e}_\xi = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ (\hat{e}_\eta)_x & (\hat{e}_\eta)_y & (\hat{e}_\eta)_z \\ (\hat{e}_\xi)_x & (\hat{e}_\xi)_y & (\hat{e}_\xi)_z \end{vmatrix}$$

$$= \frac{1}{\sinh^2\xi + \sin^2\eta}\Big[(-\cosh^2\xi\sin^2\eta\sin\phi - \sinh^2\xi\cos^2\eta\sin\phi)\,\hat{e}_x + (\sinh^2\xi\cos^2\eta\cos\phi + \cosh^2\xi\sin^2\eta\cos\phi)\,\hat{e}_y + \begin{pmatrix} -\sinh\xi\cosh\xi\sin\eta\cos\eta\cos\phi\sin\phi \\ +\sinh\xi\cosh\xi\sin\eta\cos\eta\cos\phi\sin\phi \end{pmatrix}\hat{e}_z\Big]$$

$$= \frac{1}{\sinh^2\xi + \sin^2\eta}\Big[-\sin\phi(\cosh^2\xi\sin^2\eta + \sinh^2\xi\cos^2\eta)\,\hat{e}_x + \cos\phi(\sinh^2\xi\cos^2\eta + \cosh^2\xi\sin^2\eta)\,\hat{e}_y\Big]$$

$$= \frac{1}{\sinh^2\xi + \sin^2\eta}\Big\{-\sin\phi\left[(1+\sinh^2\xi)\sin^2\eta + \sinh^2\xi(1-\sin^2\eta)\right]\hat{e}_x + \cos\phi\left[(1+\sinh^2\xi)\sin^2\eta + \sinh^2\xi(1-\sin^2\eta)\right]\hat{e}_y\Big\}$$

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$$= -\sin\phi\,\hat{e}_x + \cos\phi\,\hat{e}_y = \hat{e}_\phi$$

$$\hat{e}_\phi \times \hat{e}_\eta = \frac{1}{\sqrt{\sinh^2\xi + \sin^2\eta}}\Big[(\sinh\xi\cos\eta\cos\phi)\,\hat{e}_x + (\sinh\xi\cos\eta\sin\phi)\,\hat{e}_y$$
$$+ (\cosh\xi\sin\eta\sin^2\phi + \cosh\xi\sin\eta\cos^2\phi)\,\hat{e}_z$$

$$= \frac{1}{\sqrt{\sinh^2\xi + \sin^2\eta}}\Big[(\sinh\xi\cos\eta\cos\phi)\,\hat{e}_x + (\sinh\xi\cos\eta\sin\phi)\,\hat{e}_y$$
$$+ (\cosh\xi\sin\eta)\,\hat{e}_z\Big] = \hat{e}_\xi$$

$$\hat{e}_\xi \times \hat{e}_\phi = \frac{1}{\sqrt{\sinh^2\xi + \sin^2\eta}}\Big[-\cosh\xi\sin\eta\cos\phi\,\hat{e}_x - \cosh\xi\sin\eta\sin\phi\,\hat{e}_y$$
$$+ (\sinh\xi\cos\eta\cos^2\phi + \sinh\xi\cos\eta\sin^2\phi)\,\hat{e}_z\Big]$$

$$= \frac{1}{\sqrt{\sinh^2\xi + \sin^2\eta}}\Big[-\cosh\xi\sin\eta\cos\phi\,\hat{e}_x - \cosh\xi\sin\eta\sin\phi\,\hat{e}_y$$
$$+ \sinh\xi\cos\eta\,\hat{e}_z\Big] = \hat{e}_\eta$$

$$\hat{e}_i \times \hat{e}_j = \sum_{k=1}^{3} \epsilon_{ijk}\,\hat{e}_k \;; \quad i,j = 1,2,3$$

$\Rightarrow$ right-handed

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a.

b. From (a),

$$h_1 = \left|\frac{\partial r}{\partial \xi}\right|, \quad h_2 = \left|\frac{\partial r}{\partial \eta}\right|, \quad h_3 = \left|\frac{\partial r}{\partial \phi}\right| \xrightarrow{\text{rearranging such that system is right-handed}} \begin{aligned} h_1 &= \left|\frac{\partial r}{\partial \eta}\right| = a\sqrt{\sinh^2\xi + \sin^2\eta} \\ h_2 &= \left|\frac{\partial r}{\partial \xi}\right| = a\sqrt{\sinh^2\xi + \sin^2\eta} \\ h_3 &= \left|\frac{\partial r}{\partial \phi}\right| = a\cosh\xi\cos\eta \end{aligned}$$

$$\nabla \times \vec{A} = \frac{1}{h_1 h_2 h_3}\begin{vmatrix} h_1 \hat{e}_\eta & h_2 \hat{e}_\xi & h_3 \hat{e}_\phi \\ \frac{\partial}{\partial \eta} & \frac{\partial}{\partial \xi} & \frac{\partial}{\partial \phi} \\ h_1 A_\eta & h_2 A_\xi & h_3 A_\phi \end{vmatrix}$$

$$(\nabla \times \vec{A})_\eta = \frac{1}{a^3(\sinh^2\xi + \sin^2\eta)\cosh\xi\cos\eta}\left[\frac{\partial}{\partial \xi} a\cosh\xi\cos\eta A_\phi - \frac{\partial}{\partial \phi} a\sqrt{\sinh^2\xi + \sin^2\eta}\, A_\xi\right] a\sqrt{\sinh^2\xi + \sin^2\eta}\;\hat{e}_\eta$$

$$= \frac{1}{a^2\sqrt{\sinh^2\xi + \sin^2\eta}\,(\cosh\xi\cos\eta)}\left[a\cos\eta\cosh\xi \frac{\partial}{\partial \xi} A_\phi + a\cos\eta A_\phi \frac{\partial}{\partial \xi}\cosh\xi - a\sqrt{\sinh^2\xi + \sin^2\eta}\,\frac{\partial A_\xi}{\partial \phi} - aA_\xi \frac{\partial}{\partial \xi}\sqrt{\sinh^2\xi + \sin^2\eta}\right]\hat{e}_\eta$$

$$= \frac{1}{a(\cosh\xi\cos\eta)\sqrt{\sinh^2\xi + \sin^2\eta}}\left[\cos\eta\cosh\xi \frac{\partial}{\partial \xi} A_\phi + \cos\eta A_\phi \sinh\xi - \sqrt{\sinh^2\xi + \sin^2\eta}\,\frac{\partial A_\xi}{\partial \phi} - A_\xi \frac{1}{2}(\sinh^2\xi + \sin^2\eta)^{-\frac{1}{2}}(2)\sinh\xi\cosh\xi\right]\hat{e}_\eta$$

$$= \frac{1}{a}\left[\frac{1}{\sqrt{\sinh^2\xi + \sin^2\eta}}\frac{\partial}{\partial \xi} A_\phi + \frac{\tanh\xi}{\sqrt{\sinh^2\xi + \sin^2\eta}} A_\phi - \frac{1}{\cosh\xi\cos\eta}\frac{\partial}{\partial \phi} A_\xi - \frac{\tanh\xi\cos\eta}{(\sinh^2\xi + \sin^2\eta)^{\frac{1}{2}}} A_\xi\right]\hat{e}_\eta$$