

NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF STATISTICS & APPLIED PROBABILITY
ST2334 PROBABILITY AND STATISTICS
SEMESTER I, AY 2022/2023

Tutorial 07

This set of questions will be discussed by your tutors during the tutorial in Week 10.

Please work on the questions before attending the tutorial.

1. A box contains 2 red marbles and 98 blue ones. Draws are made at random with replacement. In n draws from the box, there is better than a 50% chance for a red marble to appear at least once. What is the smallest possible value for n ?
2. Suppose that, on average, 1 person in 1000 makes a numerical error in preparing his or her income tax return. 10,000 forms are selected at random and examined.
 - (a) Find the probability that 6, 7, or 8 of the forms contain an error.
 - (b) Find the mean and variance of the number of persons among 10,000 who make an error in preparing their tax returns.
3. A couple decides they will continue to have children until they have two males. Assuming that $P(\text{male}) = 0.5$.
 - (a) What is the probability that their second male is their seventh child?
 - (b) What is the expected number of children for the couple?
4. Three people toss a fair coin and the odd man pays for coffee. If the coins all turn up the same, they are tossed again.
 - (a) Find the probability that fewer than 4 tosses are needed.
 - (b) Provide a general formula for the probability of at most x tosses are needed.
5. Hospital administrators in large cities anguish about problems with traffic in emergency rooms in hospitals. For a particular hospital in a large city, the staff on hand cannot accommodate the patient traffic if there are more than 10 emergency cases in a given hour. It is assumed that patient arrival follows a Poisson process and historical data suggest that, on the average, 5 emergencies arrive per hour. Find the probability that
 - (a) In a given hour, there is no emergency.
 - (b) In a given hour the staff can no longer accommodate the traffic?
 - (c) More than 20 emergencies arrive during a 3-hour shift of personnel?
6. A notice is sent to all owners of a certain type of automobile, asking them to bring their cars to a dealer to check for the presence of a particular type of defect. Suppose that only 0.05% of the cars have the defect. Consider a random sample of 10,000 cars.
 - (a) What are the expected value and variance of the number of cars in the sample that have the defect?
 - (b) What is the (approximate) probability that at least 10 sampled cars have the defect?
 - (c) What is the (approximate) probability that no sampled cars have the defect?

7. A company rents time on a computer for periods of t hours, for which it receives \$600 an hour. The number of times the computer breaks down during t hours is a random variable having the Poisson distribution with $\lambda = 0.8t$, and if the computer breaks down x times during t hours, it costs $50x^2$ dollars to fix it. How should the company select t in order to maximize its expected profit?
8. Compute the following:
- (a) $\sum_{x=1}^{\infty} \frac{x}{2^x}$;
- (b) $\sum_{x=1}^{\infty} \frac{x^2}{2^x}$.
9. You arrive at the bus stop at 10 a.m., knowing that the bus will arrive at some time uniformly distributed between 10 a.m. and 10:30 a.m.
- (i) What is the probability that you will have to wait longer than 10 minutes?
- (ii) If the bus has not yet arrived at 10:15 a.m., what is the probability that you will have to wait at least an additional 10 minutes?

Answers

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| 1. 35 | 6. (a) 5,5; |
| 2. (a) 0.2657; | (b) 0.0318; |
| (b) 10; 9.99. | (c) 0.0067. |
| 3. (a) 0.0469; | 7. 8.75 |
| (b) 4. | 8. (a) 2; |
| 4. (a) 63/64; | (b) 6. |
| (b) $1 - (1/4)^x$. | 9. $\frac{2}{3}, \frac{1}{3}$ |
| 5. (a) 0.00673; | 9. (a) 2/3; |
| (b) 0.0137; | (b) 1/3. |
| (c) 0.0830. | |