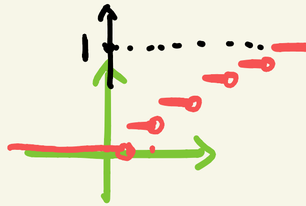
4. (a) $f(y) = ky \quad , \quad y = 1, 2, \cdots, 5$

$$\begin{cases} \sum_{y=1}^{5} ky = 1 \\ ky \geq 0 \end{cases} \Rightarrow k = \frac{1}{15}$$



(d) $F(x) = \sum_{t \leq x} f(t) \qquad y = 1, 2, 3, 4, 5$

$\underline{f(1), f(2), f(3), f(4), f(5)}$

$$\therefore F(y) = \begin{cases} 0 & , \ y < 1 \\ f(1) = \frac{1}{15} & , \ 1 \leq y < 2 \\ f(1) + f(2) = \frac{3}{15} & , \ 2 \leq y < 3 \\ f(1) + f(2) + f(3) = \frac{6}{15} & , \ 3 \leq y < 4 \\ \sum_{t=1}^{4} f(t) = \frac{10}{15} & , \ 4 \leq y < 5 \\ \sum_{t=1}^{5} f(t) = 1 & , \ 5 \leq y \end{cases}$$

$t < 2$

$F(x) = \sum_{t \leq y} f(t) = f(1) + f(2) + f(3)$

6. (a). $f(x) = \begin{cases} k\sqrt{x}, & \boxed{0<x<1} \\ 0, & \end{cases}$

$$\begin{cases} k\sqrt{x} > 0 \\ \int_{-\infty}^{\infty} k\sqrt{x}\,dx = 1 \end{cases}$$

$$\Rightarrow \int_0^1 k\sqrt{x}\,dx = 1$$

$$\Rightarrow \frac{2}{3}kx^{\frac{3}{2}}\Big|_0^1 = 1$$

$$\frac{2}{3}\cdot k = 1$$

$$\Rightarrow k = \frac{3}{2}$$

$F(x) = P(X \le x)$

$F(0.6) = P(X \le 0.6)$

$= P(X < 0.6)$

(b) $P(0.3 < x < 0.6)$

$= F(0.6) - F(0.3)$

$= 0.6^{\frac{3}{2}} - 0.3^{\frac{3}{2}} = 0.3004$

(b) $F(x) = \int_{-\infty}^{x} f(t)\,dt$

For $x \le 0$, $F(x) = \int_{-\infty}^{x} \underset{=0}{f(t)}\,dt = \int_{-\infty}^{0} 0\,dt = 0$

For $0<x<1$, $F(x) = \int_{-\infty}^{x} f(t)\,dt$

$$= \int_{-\infty}^{0} f(t)\,dt + \int_0^x f(t)\,dt$$

$$= \int_{-\infty}^{0} 0\,dt + \int_0^x \frac{3}{2}\sqrt{t}\,dt$$

$$= 0 + t^{\frac{3}{2}}\Big|_0^x$$

$$= x^{\frac{3}{2}}$$

For $x \geq 1$, $F(x) = \int_{-\infty}^{x} f(t)\,dt$

$$= \int_{-\infty}^{0} \overset{0}{\overset{\parallel}{f(t)}}\,dt + \overset{k\sqrt{x}}{\overset{\parallel}{\underset{\underset{1}{\parallel}}{\boxed{\int_{0}^{1} f(t)\,dt}}}}$$

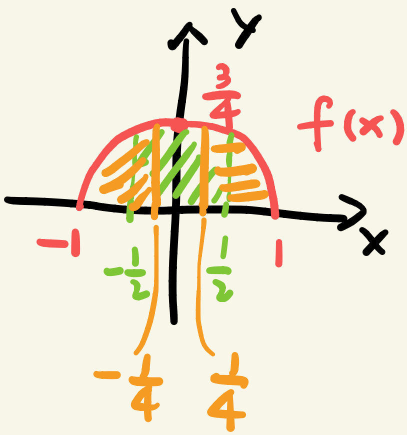
$$+ \int_{1}^{x} \overset{0}{\overset{\parallel}{f(t)}}\,dt$$

$$= 0 + 1 + 0$$

$$= 1$$

$$\therefore F(x) = \begin{cases} 0 & , \quad x \leq 0 \\ x^{\frac{3}{2}} & , \quad 0 < x < 1 \\ 1 & , \quad 1 \leq x \end{cases}$$

7. $f(x) = \begin{cases} \frac{3}{4}(1-x^2) & , \ -1 \leq x \leq 1 \\ 0 & \end{cases}$



(a) $P\left(-\frac{1}{2} < X < \frac{1}{2}\right)$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} f(x)\,dx$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{3}{4}(1-x^2)\,dx$$

$$= \frac{3}{4}\left(x - \frac{x^3}{3}\right)\Big|_{-\frac{1}{2}}^{\frac{1}{2}}$$

$$= \frac{11}{16}$$

(b) $P\left(X < -\frac{1}{4} \text{ or } X > \frac{1}{4}\right)$

$$= \int_{-1}^{-\frac{1}{4}} f(x)\,dx + \int_{\frac{1}{4}}^{1} f(x)\,dx$$

$$= \int_{-1}^{\frac{1}{4}} \frac{3}{4}(1-x^2)\,dx + \int_{\frac{1}{4}}^{1} \frac{3}{4}(1-x^2)\,dx$$

$$= \frac{81}{256} + \frac{81}{256}$$

$$= \frac{81}{128}$$

(c) For $x < -1$, $F(x) = \int_{-\infty}^{x} 0\,dt = 0$

For $-1 \le x \le 1$, $F(x) = \int_{-\infty}^{-1} 0\,dt + \int_{-1}^{x} f(t)\,dt$

$$= 0 + \int_{-1}^{x} \frac{3}{4}(1-t^2)\,dt$$

$$= \frac{3}{4}\left(t - \frac{t^3}{3}\right)\Big|_{-1}^{x}$$

$$= \frac{3}{4}\left(x - \frac{x^3}{3}\right) + \frac{1}{2}$$

For $1<x$, $F(x) = \boxed{\int_{-\infty}^{-1} 0\,dt}$ ^{0} $+ \int_{-1}^{1} f(t)\,dt$

$+ \int_{1}^{x} 0\,dt$ ^{0}

$\int_{-\infty}^{\infty} f(x)\,dx = 1$

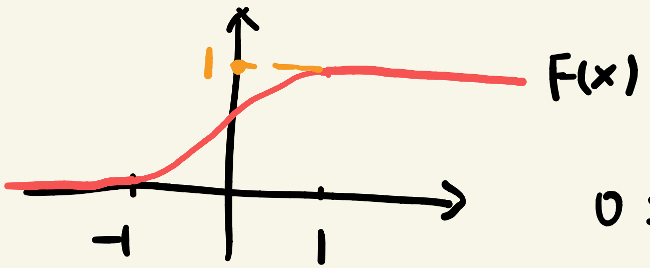
$\Downarrow$

$\int_{-1}^{1} f(x)\,dx = 1$

$= 0 + 1 + 0$

$= 1$

$$\therefore F(x) = \begin{cases} 0, & x < -1 \\ \frac{3}{4}\left(x - \frac{x^3}{3}\right) + \frac{1}{2}, & -1 \le x \le 1 \\ 1, & 1 \le x \end{cases}$$



$0 \le F(x) \le 1$

8. (a) $12 \text{ min} = \frac{1}{5} h$

$$P\left(X < \tfrac{1}{5}\right) = P\left(X \le \tfrac{1}{5}\right)$$

$$= F\left(\tfrac{1}{5}\right)$$

$$= 1 - e^{-\frac{8}{5}}$$

$$= 0.7981$$

$\frac{1}{5}$

$\downarrow$

def $F(x) = P(X \le x)$

(b) $F(x) = \begin{cases} 0, & \boxed{x \le 0} \\ 1 - e^{-8x}, & \boxed{x > 0} \end{cases}$

$f(x) = \dfrac{dF(x)}{dx} = \begin{cases} 0, & x \le 0 \\ \dfrac{d(1-e^{-8x})}{dx} = 8e^{-8x}, & x > 0 \end{cases}$