NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF STATISTICS & APPLIED PROBABILITY

# ST2334 PROBABILITY AND STATISTICS

SEMESTER I, AY 2022/2023

## Tutorial 02: Solution

**This set of questions will be discussed by your tutors during the tutorial in Week 4.**

**Please work on the questions before attending the tutorial.**

1. The probability that a Singapore company will set up a factory in City A is 0.7. The probability that it will set up a factory in City B is 0.4 and the probability that it will set up in either City A or City B or both is 0.8. What is the probability that the company will set up a factory

   (a) in both cities?

   (b) in neither city?

   SOLUTION
   Let

$$\begin{aligned} A &= \{\text{The factory will be set up in City A}\} \\ B &= \{\text{The factory will be set up in City B}\}. \end{aligned}$$

   It is given that $P(A) = 0.7$, $P(B) = 0.4$ and $P(A \cup B) = 0.8$.

   (a) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.7 + 0.4 - 0.8 = 0.3$.

   (b) $P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - 0.8 = 0.2$.

2. Suppose there are 500 applicants for five equivalent positions at a factory and the company is able to narrow the field to 30 equally qualified applicants. Seven of the finalists are minority candidates. Assume that the five who are chosen are selected at random from this final group of thirty.

   (a) In how many ways can the selection be made?

   (b) What is the probability that none of the minority candidates is hired?

   (c) What is the probability that no more than one minority candidates are hired?

   SOLUTION

   (a) The number of ways to choose 5 out of 30 qualified applicants is $\binom{30}{5} = 142506$.

   (b) The number of ways to choose 5 out of 30 qualified applicants such that none of the minority is hired is $\binom{23}{5} = 33649$. Therefore the desired probability is $33649/142506 = 0.2361$.

   (c) The number of ways to choose 5 out of 30 qualified applicants such that one minority is hired is $\binom{7}{1} \times \binom{23}{4} = 61985$.
   Let $A_0$ and $A_1$ denote the events that no minority and one minority is hired respectively. Hence $P(A_1) = 61985/142506 = 0.4350$.
   From Part (b), $P(A_0) = 0.2361$.
   Therefore $P(\text{at most one minority is hired}) = P(A_0) + P(A_1) = 0.6711$.

3. Consider 5-card poker hands dealt from a standard 52 card deck. Two important events are

   $A = \{$You draw a flush$\}$; a flush means: 5 cards from the same suit;

   $B = \{$You draw a straight$\}$; straight means: values of the 5 cards are in sequence, e.g., 9 of diamonds, 10 of hearts, jack of hearts, queen of spades and king of spades), assuming that aces can be high or low.

   **Note**: we consider a straight flush, i.e., 5 consecutive cards of the same suit, is not a straight.

   If you are dealt a 5-card hand, find the following probability:

   (a) $P(A)$

   (b) $P(B)$

   SOLUTION

   Number of possible hands of 5 cards is

$$\binom{52}{5} = \frac{52(51)(50)(49)(48)}{5!} = 2598960.$$

   (a) Number of spade flush hands is $\binom{13}{5} = 1287$.

   Similarly, the number of heart flush hands is also $\binom{13}{5} = 1287$, and so on.

$$P(A) = P(\text{a flush hand}) = \frac{4(1287)}{2598960} = 0.001981.$$

   (b) Number of straight hands with 1 as the smallest card is $\binom{4}{1}^5 - 4 = 1020$. Similarly, the number of straight hands with 2 as the smallest card is $\binom{4}{1}^5 - 4 = 1020$ and so on. The smallest card can be any one from 1 to 10.

$$P(\text{a straight hand}) = \frac{10(1020)}{2598960} = 0.003925.$$

4. There are two intersections with traffic lights along the route taken a motorist in driving to work. The probability that he must stop at the first light is 0.4, the probability that he must stop at the second light is 0.5, and the probability that he must stop at least one of the two lights is 0.6. What is the probability that he must stop

   (a) at both lights?

   (b) at exactly one light?

   (c) at neither light?

   (d) at the second light given that he has stopped at the first light?

   (e) Is the event stopping at first traffic light independent of the event stopping at the second traffic light?

   SOLUTION

   Let $A_i, i = 1,2$ denote the event that the motorist stops at light $i$. We have $P(A_1) = 0.4$, $P(A_2) = 0.5$, and $P(A_1 \cup A_2) = 0.6$.

(a) $P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = 0.4 + 0.5 - 0.6 = 0.3.$

(b) $\{$stop at exactly one light$\} = (A_1 \cap A_2') \cup (A_1' \cap A_2).$
On the other hand $P(A_1 \cap A_2') = P(A_1) - P(A_1 \cap A_2) = 0.4 - 0.3 = 0.1$ and
$P(A_1' \cap A_2) = P(A_2) - P(A_1 \cap A_2) = 0.5 - 0.3 = 0.2.$
Hence $P(\text{stop at exactly one light}) = 0.1 + 0.2 = 0.3.$

(c) $P(A_1' \cap A_2') = P((A_1 \cup A_2)') = 1 - P(A_1 \cup A_2) = 1 - 0.6 = 0.4.$

(d) $P(A_2|A_1) = \dfrac{P(A_1 \cap A_2)}{P(A_1)} = \dfrac{0.3}{0.4} = 0.75.$

(e) $P(A_2|A_1) = 0.75 \neq 0.5 = P(A_2)$. So $A_1$ and $A_2$ are not independent.

5. Consider 9-digit numbers where each digit is one of the 10 integers $0, 1, 2, \ldots, 9$.

(a) What is the probability that no two consecutive digits are the same in a randomly selected 9-digit number?

(b) What is the probability that 0 appears as a digit for a total of 3 times in a randomly selected 9-digit number?

SOLUTION

Number of possible 9-digit numbers with no restriction is $9 \times 10^8$.

(a) There are 9 ways to choose the first digit, and also 9 ways (in order not to repeat the number chosen for the first/previous digit) to choose the second digit, and so on until the ninth digit. Hence, the number of 9-digit numbers with no two consecutive digits the same is $9(9)(9)\ldots(9) = 9^9$.
The probability that no two consecutive digits are the same in a randomly selected 9-digit number is

$$\frac{9^9}{9 \times 10^8} = 0.4305.$$

(b) In a 9-digit number, there are 8 places (except the first digit) where we can place the three zeros, number of ways of doing so is $\binom{8}{3} = 56$. For the other places, there are 6 of them, we have 9 choices (except the choice of zero), number of ways doing so is $9^6$. So the number of 9-digit numbers with 0 appears as a digit for a total of 3 times is $56 \times 9^6$.
The probability that a 9-digit number with 0 appears as a digit for a total of 3 being selected is

$$\frac{56 \times 9^6}{9 \times 10^8} = 0.03307.$$

6. A soft-drink bottling company maintains records concerning the number of unacceptable bottles of soft drink obtained from the filling and capping machines. Based on the past data, the probability that a bottle came from machine I and was nonconforming is 0.01, and the probability that a bottle came from machine II and was nonconforming is 0.025. Half the bottles are filled on machine I and the other half are filled on machine II. If a filled bottle of soft drink is selected at random, what is the probability that

(a) it is a nonconforming bottle?

(b) it was filled on machine II?

(c) it was filled on machine II and is a conforming bottle?

(d) It was filled on machine I or is a conforming bottle?

(e) Suppose you know that the bottle was produced on machine I. What is the probability that it is nonconforming?

(f) Suppose you know that the bottle is nonconforming. What is the probability that it was produced on machine I?

(g) Explain the difference in the answers to (e) and (f).

SOLUTION

Let

$$M_1 = \{\text{the selected bottle was filled on machine I}\},$$

$$M_2 = \{\text{the selected bottle was filled on machine II}\},$$

$$N = \{\text{a nonconforming bottle was selected}\}.$$

It is given that $P(N \cap M_1) = 0.01$, $P(N \cap M_2) = 0.025$, and $P(M_1) = P(M_2) = 0.5$.

(a) $P(N) = P((N \cap M_1) \cup (N \cap M_2)) = 0.01 + 0.025 = 0.035$.

(b) $P(M_2) = 0.5$;

(c) $P(M_2 \cap N') = P(M_2) - P(M_2 \cap N) = 0.5 - 0.025 = 0.475$.

(d) $P(M_1 \cup N') = P(M_1) + P(N') - P(M_1 \cap N')$.
On the other hand $P(N') = 1 - P(N) = 1 - 0.035 = 0.965$; $P(M_1 \cap N') = P(M_1) - P(M_1 \cap N) = 0.5 - 0.01 = 0.49$.
Therefore
$$P(M_1 \cup N') = 0.5 + 0.965 - 0.49 = 0.975.$$

(e) $P(N|M_1) = P(N \cap M_1)/P(M_1) = 0.01/0.5 = 0.02$.

(f) $P(M_1|N) = P(N \cap M_1)/P(N) = 0.01/0.035 = 0.2857$.

(g) The events are different and the conditions are different. The answer in part (e) is the probability of having a nonconforming item given the condition that the item was from machine I (i.e $P(N|M_1)$). The answer in part (f) is the probability of having an item from machine I given that it was a nonconforming item (i.e. $P(M_1|N)$).