

NATIONAL UNIVERSITY OF SINGAPORE
Department of Statistics and Applied Probability

(2020/21) Semester 2

ST2334 Probability and Statistics

Tutorial 4

1. From a box containing 4 one-cent coins and 2 five-cent coins, 3 coins are selected at random *without replacement*. Let X denote the total amount of the selected coins.
 - (a) What are the possible values for X ?
 - (b) Find the probability function of X .
2. Let W be a random variable giving the number of heads minus the number of tails in three tosses of a coin.
 - (a) What are the possible values for W ?
 - (b) Find the probability function of W assuming the coin is fair.
 - (c) Find the probability function of W assuming the coin is biased so that a head is twice as likely to occur as a tail.
3. Determine the value c so that the following function can serve as a probability function of a discrete random variable X .

$$f_X(x) = \begin{cases} c(x^2 + 4), & x = 0, 1, 2, 3; \\ 0, & \text{otherwise.} \end{cases}$$

4. A contractor is required by the city planning department to submit 1, 2, 3, 4, or 5 forms (depending on the nature of the project) in applying for a building permit. Let Y = the number of forms required of the next application. The probability that y forms are required is known to be proportional to y — that is, $f_Y(y) = ky$ for $y = 1, 2, \dots, 5$.
 - (a) What is the value of k ?
 - (b) What is the probability that at most three forms are required?
 - (c) What is the probability that between two and four forms (inclusive) are required?
 - (d) Find the cumulative distribution function (c.d.f.) of Y .
5. An insurance company offers its policyholders a number of different premium payment options. For a randomly selected policyholder, let X = the number of months between successive payments. The c.d.f. of X is as follow.

$$F_X(x) = \begin{cases} 0, & x < 1, \\ 0.30, & 1 \leq x < 3, \\ 0.40, & 3 \leq x < 4, \\ 0.45, & 4 \leq x < 6, \\ 0.60, & 6 \leq x < 12, \\ 1, & 12 \leq x. \end{cases}$$

- (a) What is the probability function of X ?
- (b) Using the c.d.f., compute $\Pr(3 \leq X \leq 6)$ and $\Pr(4 \leq X)$.

6. Consider the probability density function

$$f_X(x) = \begin{cases} k\sqrt{x}, & 0 < x < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Evaluate the constant k .
 - (b) Find the cumulative distribution function $F_X(x)$ and use it to evaluate $\Pr(0.3 < X < 0.6)$.
7. Suppose the distance X between a point target and a shot aimed at the point in a coin-operated target game is a continuous random variable with p.d.f.

$$f_X(x) = \begin{cases} \frac{3}{4}(1 - x^2), & -1 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Compute $\Pr\left(-\frac{1}{2} < X < \frac{1}{2}\right)$.
 - (b) Compute $\Pr\left(X < -\frac{1}{4} \text{ or } X > \frac{1}{4}\right)$.
 - (c) Find the c.d.f. of X .
8. The waiting time, in hours, between successive speeders spotted by a radar unit is a continuous random variable with cumulative distribution

$$F_X(x) = \begin{cases} 0, & x \leq 0, \\ 1 - e^{-8x}, & x > 0. \end{cases}$$

- (a) Find the probability of waiting less than 12 minutes between successive speeders.
- (b) Find the probability density function of X .

Answers to selected problems

1. (a) 3, 7 and 11.

(b)

x	3	7	11
$f_X(x)$	1/5	3/5	1/5

2. (a) 3, -1, 1, 3.

(b)

w	3	1	-1	-3
$f_W(w)$	1/8	3/8	3/8	1/8

(c)

w	3	1	-1	-3
$f_W(w)$	8/27	12/27	6/27	1/27

3. $c = 1/30$.

4. (a) $k = 1/15$.

(b) 0.4.

(c) 0.6.

(d)

$$F_Y(y) = \begin{cases} 0, & y < 1; \\ 1/15, & 1 \leq y < 2; \\ 3/15, & 2 \leq y < 3; \\ 6/15, & 3 \leq y < 4; \\ 10/15, & 4 \leq y < 5; \\ 1, & 5 \leq y. \end{cases}$$

5. (a)

x	1	3	4	6	12
$f_X(x)$	0.3	0.1	0.05	0.15	0.4

(b) 0.3, 0.6.

6. (a) $3/2$.

(b)

$$F_X(x) = \begin{cases} 0, & x < 0; \\ x^{3/2}, & 0 \leq x \leq 1; \\ 1, & x \geq 1. \end{cases}$$

$F_X(x) = 0$ for $x < 0$; $F_X(x) = x^{3/2}$ for $0 \leq x < 1$; $F_X(x) = 1$ for $x \geq 1$.

(c) 0.3004.

7. (a) 0.6875.

(b) 0.6328.

(c)

$$F_X(x) = \begin{cases} 0, & x < -1; \\ (2 + 3x - x^3)/4, & -1 \leq x \leq 1; \\ 1, & x > 1. \end{cases}$$

8. (a) 0.7981.

(b)

$$f_X(x) = \begin{cases} 8e^{-8x}, & x \geq 0; \\ 0, & x < 0. \end{cases}$$