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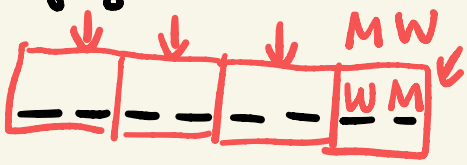
4(a). _ _ _ _ _ _ _ _

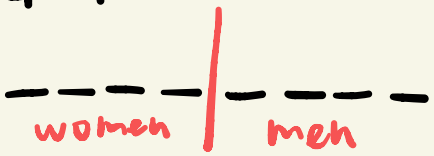
$_8P_8 = 8! = 40320$

(b)



$_4P_4 \times 2^4 = 4! \times 16 = 384$

(c)



$_4P_4 \times {}_4P_4 = 4! \times 4! = 576$

(0,0) (13,8)

6. (a) A→B

13→ 8↑

13+8 = 21 steps

$_{21}C_{13} \times {}_8C_8 = \frac{21!}{13!\,8!} \times 1 = 203490$

$_{21}C_8 \times {}_{13}C_{13} = \frac{21!}{8!\,13!} \times 1$

(b) $A \to B$: { $A \to Y \to B$ encounter BBW

$A \to B$ Avoid BBW

total − #A -> Y -> B

$A \to Y \to B$:

(0,0) (10,6)

$A \to Y$: $10 \to 6\uparrow$ 16 steps ${}_{16}C_{10}$

(10,6) (13,8)

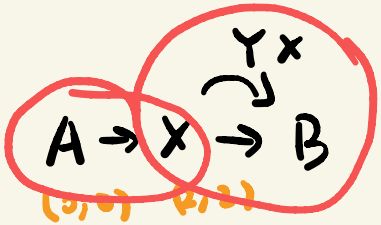
$Y \to B$: $3 \to 2\uparrow$ 5 steps ${}_{5}C_{3}$ ${}_{5}C_{2}$

$A \to Y \to B$: ${}_{16}C_{10} \times {}_{5}C_{3} = 80080$

Avoid BBW : 203490 − 80080

= 123410

(c)



7426

(3,0) (7,2)

$A \to X$: $2 \to 2\uparrow$ 4 steps ${}_{4}C_{2} = 6$

$X \to B$: { encounter BBW : $X \to Y \to B$ 4950
Avoid BBW : #total − $\#X \to Y \to B$

12376 − 4950 = 7426

(2,2)
$X \to Y \to B$:

(2,2) $X \to Y$ (10,6) : 8 → 4↑ 12 steps $_{12}C_8$

$$_{12}C_8 = \frac{12!}{8!4!} = {}_{12}C_4$$

$$_{12}C_4 = \frac{12!}{4!8!}$$

(10,6) (13,8)
$Y \to B$: 3 → 2↑ 5 steps $_5C_3$

$X \to Y \to B$: $_{12}C_8 \times {}_5C_3 = 4950$

(2,2) (13,8)
$X \to B$: 11 → 6↑ 17 steps

$$_{17}C_{11} = 12376$$

$$6 \times 7426 = 44556$$

7(b). A: 9 × B: 27 × C: 15

${}_9C_1 \times {}_{27}C_1 \times {}_{15}C_1 = 9 \times 27 \times 15 = 3645$

$3645 \div 365 \simeq 10$ years

8. White consonants : w h t

vowels : i e

(c) _ _ _ _ _ C V C V C ${}_3P_3 \times {}_2P_2 = 3! \times 2! = 12$

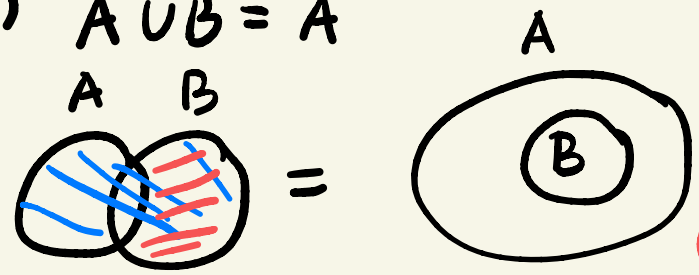
9. □□□□□□ ← / □□□ ← ${}_9C_6 \times {}_6P_6 \times {}_3P_3 = 362880$

${}_9P_6 \times {}_3P_3 = 362880$

${}_9P_6 = \frac{9!}{3!} \times 3!$

${}_9C_6 \times {}_6P_6 \times {}_3P_3 = \frac{9!}{6!3!} \times 6! \times 3!$

11. (a) $A \cup B = A$



$B \subseteq A$

every $e \in B$, $e \in A$

Proof: ① If $B = \emptyset$ ✓, $A \cup B = A \cup \emptyset = A$, holds

② If $B \neq \emptyset$, then there is at least 1 element in B.

For every element $e \in B$,

$e \in A \cup B$ ($B \subseteq A \cup B$)

$\because A \cup B = A$

$\therefore e \in A$

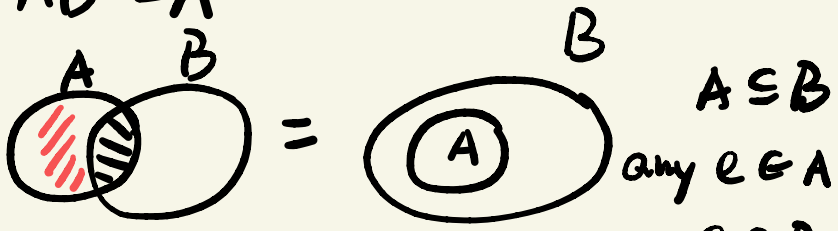
$\therefore B \subseteq A$

Conclude ① and ②, $B \subseteq A$. (Since $\emptyset \subseteq A$)

$A \cup B = A$, $B \subseteq \underbrace{A \cup B}_{A}$

$\Rightarrow B \subseteq A$

(b) $A \cap B = A$



$A \subseteq B$

any $e \in A$

$e \in B$

Proof: ① If $A \cap B = \emptyset$

$\because A \cap B = A$

$\therefore A = \emptyset$ ✓

② If $A \cap B \neq \emptyset$, there is at least one element in $A \cap B$.

For any element $e \in A \cap B$, (A)

$e \in B$ ($A \cap B \subseteq B$)

$\because A = A \cap B$

For any element $e \in A$, $e \in B$

$A \subseteq B$

Conclude ① and ②, $A \subseteq B$ (Since $\emptyset \subseteq B$)

$A \cap B = A$, $A \subseteq B$ ($A \cap B$)

$\Rightarrow \quad A \subseteq B$

"$\subseteq$" "$\subset$" "$\subset$": $\subset / =$

$A \subseteq B$

$A \subset B$

$A = B$