NATIONAL UNIVERSITY OF SINGAPORE
Department of Statistics and Applied Probability

(2020/21) Semester 2 ST2334 Probability and Statistics Tutorial 8

1. According to *Chemical Engineering Progress (Nov, 1990)*, approximately 30% of all pipework failures in chemical plants are caused by operator error.
   (a) What is the probability that out of the next 20 pipework failures at least 10 are due to operator error?
   (b) What is the probability that no more than 4 out of 20 such failures are due to operator error?
   (c) Suppose, for a particular plant, that out of the random sample of 20 such failures, exactly 5 are operational errors. Do you feel that the 30% figure stated above applies to this plant? Explain in not more than two short sentences.

2. In testing a certain kind of truck tire over a rugged terrain, it is found that 25% of the trucks fail to complete the test without a blowout. Of the next 15 trucks tested, find
   (a) The probability of zero blowouts.
   (b) The probability of at least 8 blowouts.
   (c) Expected number of blowouts.
   (d) According to Chebyshev's theorem, what interval does the number of trucks having blowouts in the next 15 trucks fall with at least ¾ chance?

3. Suppose that, on average, 1 person in 1000 makes a numerical error in preparing his or her income tax return. 10,000 forms are selected at random and examined.
   (a) Find the probability that 6, 7, or 8 of the forms contain an error.
   (b) Find the mean and variance of the number of persons among 10,000 who make an error in preparing their tax returns.
   (c) According to Chebyshev's theorem, what interval does the number of persons who make errors in preparing their income tax returns among 10,000 persons fall with at least 8/9 chance?

4. The probability that a person, living in a certain city, owns a dog is estimated to be 0.3. Find the probability that the tenth person randomly interviewed in that city is the fifth one to own a dog.

5. A couple decides they will continue to have children until they have two males. Assuming that $\Pr(\text{male}) = 0.5$.
   (a) What is the probability that their second male is their seventh child?
   (b) What is the expected number of children for the couple?

6. Three people toss a fair coin and the odd man pays for coffee. If the coins all turn up the same, they are tossed again.
   (a) Find the probability that fewer than 4 tosses are needed.
   (b) Provide a general formula for the probability of at most *x* tosses are needed.

7. A secretary makes 2 errors per page, on average.
   (a) Find the variance of the number of errors per page.
   (b) What is the probability that on the next page he or she will make 4 or more errors? No errors?

8. Hospital administrators in large cities anguish about problems with traffic in emergency rooms in hospitals. For a particular hospital in a large city, the staff on hand cannot accommodate the patient traffic if there are more than 10 emergency cases in a given hour. It is assumed that patient arrival follows a Poisson process and historical data suggest that, on the average, 5 emergencies arrive per hour. Find the probability that
   (a) In a given hour, there is no emergency.
   (b) In a given hour the staff can no longer accommodate the traffic?
   (c) More than 20 emergencies arrive during a 3-hour shift of personnel?

9. A notice is sent to all owners of a certain type of automobile, asking them to bring their cars to a dealer to check for the presence of a particular type of defect. Suppose that only 0.05% of the cars have the defect. Consider a random sample of 10000 cars.
   (a) What are the expected value and standard deviation of the number of cars in the sample that have the defect?
   (b) What is the (approximate) probability that at least 10 sampled cars have the defect?
   (c) What is the (approximate) probability that no sampled cars have the defect?

10. Suppose that a large conference room for a certain company can be reserved for no more than 4 hours. The use of the conference room is such that both long and short conferences occur quite often. Assume that length X of a conference has a uniform distribution on the interval [0,4].
   (a) What is the probability density function of X?
   (b) What is the probability that any given conference lasts at least 3 hours?
   (c) Find $E(X)$ and $V(X)$.

11. The length of time for one individual to be served at a cafeteria is a random variable having an exponential distribution with a mean of 4 minutes. Find the probability that
   (a) A person is served in more than 3 minutes.
   (b) A person is served in less than 3 minutes.
   (c) A person is served in less than 3 minutes on at least 4 of the next 6 days?

Answers to selected problems

1.	(a) 0.0480	(b) 0.2375		
	(c) 0.1789 is not very small so it is not a rare event. Thus $p = 0.30$ is reasonable.			
2.	(a) 0.0134	(b) 0.0173	(c) 3.75	(d) $1 \le X \le 7$
3.	(a) 0.2657	(b) 10; 9.99	(c) $1 \le X \le 19$	
4.	0.0515			
5.	(a) 0.0469	(b) 4		
6.	(a) 0.9844	(b) $1 - \left(\frac{1}{4}\right)^x$		
7.	(a) 2	(b) 0.1429; 0.1353		
8.	(a) 0.0067	(b) 0.0137	(c) 0.0830	
9.	(a) $\mu = 5, \sigma = 2.2355$	(b) $\Pr(X \ge 10) \approx 0.0318$		(c) $\Pr(X = 0) \approx 0.0067$
10.	(a) $f_X(x) = \begin{cases} \frac{1}{4}, & 0 \le x \le 4, \\ 0, & \text{otherwise} \end{cases}$		(b) 0.25	(c) 2; 1.3333
11.	(a) 0.4724	(b) 0.5276	(c) 0.3968	