Tutorial 10

This set of questions will be discussed by your tutors during the tutorial in Week 13.

Please work on the questions before attending the tutorial.

1. In 64 randomly selected hours of production, the mean and the standard deviation of the number of acceptable pieces produced by a automatic stamping machine are $\bar{x} = 1,038$ and $s = 146$. At the 0.05 level of significance, does this enable us to reject the null hypothesis $\mu = 1,000$ against the alternative hypothesis $\mu > 1,000$?

2. A manufacturer claims that the average tar content of a certain kind of cigarette is $\mu = 14.0$. In an attempt to show that it differs from this value, five measurements are made:

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   14.5 14.2 14.4 14.3 14.6
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   Show that the difference between the mean of this sample, $\bar{x} = 14.4$, and the average tar claimed by the manufacturer, $\mu = 14.0$, is significant at $\alpha = 0.05$. Assume normality.

3. Suppose that in the last question, the first measurement is recorded incorrectly as 16.0 instead of 14.5. Show that, even though the mean of the sample increases to $\bar{x} = 14.7$, the null hypothesis $H_0 : \mu = 14.0$ is not rejected at level $\alpha = 0.05$.

   Explain the apparent paradox that even though the difference between observed $\bar{x}$ and μ has increased, the null hypothesis is no longer rejected.

4. A study based on a sample size of 36 reported a mean of 87 with a margin of error of 10 for 95% confidence.

   (a) Give the 95% confidence interval for the population mean μ.

   (b) You desire a margin of error of 2.5 with the same confidence level. What is the sample size that will give you that kind of accuracy? Assume that we know the population variance.

   (c) You are asked to test the hypothesis that $\mu = 80$ against a two sided alternative at $\alpha = 0.05$. What is your conclusion?

5. In a study, the mean CAP (cumulative average point) of a random sample of 49 final year students is calculated to be 4.5. The standard deviation for this sample is given as 0.75.

   (a) Find a 95% confidence interval for the mean CAP of the entire final year class.

   (b) The university administration claims that the mean CAP for the entire final year class is 4.3. Does our study offer evidence against this claim? Explain.

6. Suppose we wish to test the hypothesis

$$H_0 : \mu = 2 \text{ vs } H_1 : \mu \neq 2$$

   and found a two-sided p-value of 0.03. Separately, a 95% confidence interval for μ is computed to be $(1.5, 4.0)$. Are these two results compatible? Why or why not?

7. The dynamic modulus of concrete is obtained for two different concrete mixes. For the first mix, $n_1 = 33$, $\bar{x} = 115.1$, and $s_1 = 0.47$ psi. For the second mix, $n_2 = 31$, $\bar{y} = 114.6$, and $s_2 = 0.38$ psi. Test, with $\alpha = 0.05$, the null hypothesis of equality of mean dynamic modulus versus the two-sided alternative.

8. Obtain a 95% confidence interval for the difference in mean dynamic modulus in Question 7.

9. Two procedures for etching integrated circuits are to be compared. Given 10 units, five are prepared using etching procedure A and five are prepared using etching procedure B.

   (a) The response is the percent of area on the integrated circuit where the etching was inadequate. Suppose the results are

Prodedure A	Prodedure B
5	1
2	3
9	4
6	0
3	2

   Find a 95% confidence interval for the difference in means.
   (Summary statistics are given as $\bar{x} = 5$, $s_1 = 2.73$ and $\bar{y} = 2$, $s_2 = 1.58$.)

   (b) What assumptions did you make for your answer to part (b)?

10. A manager is considering instituting an additional 15-minute coffee break if it can be shown to decrease the number of errors that employees commit. The manager divided a sample of 20 employees into two groups of 10 each. Members of one group followed the same work schedule as before, but the members of the other group were given a 15-minute coffee break in the middle of the day. The following data give the total number of errors committed by each of the 20 workers over the next 20 working days.

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    Coffee break group:    8, 7,  5,  8, 10, 9, 7, 8,  4, 5
    No-break group:        7, 6, 14, 12, 13, 8, 9, 6, 10, 9
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    (Summary statistics are given as $\bar{x} = 7.1$, $s_1 = 1.91$, $\bar{y} = 9.4$ and $s_2 = 2.84$.)

    (i) Using a suitable test, at the 5 percent level of significance, test the hypothesis that instituting a coffee break reduces the mean number of errors. What is your conclusion?

    (ii) What assumptions have you made?

    (iii) What is the p-value of your test?

11. A taxi company manager is trying to decide whether the use of radial tires instead of regular belted tires improves fuel economy. Twelve cars were equipped with radial tires and driven over a prescribed test course. Without changing drivers, the same cars were then equipped with regular belted tires and driven once again over the test course. The gasoline consumption, in kilometers per liter, was recorded as follows:

Car	1	2	3	4	5	6	7	8	9	10	11	12
Radial Tires	4.2	4.7	6.6	7	6.7	4.5	5.7	6	7.4	4.9	6.1	5.2
Belted Tires	4.1	4.9	6.2	6.9	6.8	4.4	5.7	5.8	6.9	4.7	6	4.9

    (a) Find a 95% confidence interval for the difference of the true mean gasoline consumption between cars equipped with radial tires and cars equipped with belted tires.

(b) Can we conclude that cars equipped with radial tires give better fuel economy than those equipped with belted tires? Assume the populations to be normally distributed. Use a p-value in your conclusion.

Answers

1. Reject H_0.

2. Reject H_0;
   p-value: 0.0048.

3. Do not reject H_0;
   p-value: $2P(t_4 > 2.11) = 0.1025$.

4. (a) $(77, 97)$;

   (b) 576;

   (c) Do not reject H_0.

5. (a) $(4.29, 4.70)$;

   (b) No.

6. No.

7. Reject H_0;
   p-value: approx. 0.00000.

8. $(0.29, 0.71)$

9. (a) $(-0.26, 6.26)$;

   (b) Normality and independence.

10. (a) Reject;

    (b) Normality;

    (c) p-value: 0.02378.

11. (a) $(0.0162, 0.2672)$;

    (b) 0.01515.