ST2334 (2020/2021 Semester 1) Solutions to Questions in Tutorial 2

Question 1

Let A = {The factory will be set up in City A}, B = {The factory will be set up in City B}.

It is given that $\Pr(A) = 0.7, \Pr(B) = 0.4$ and $\Pr(A \cup B) = 0.8$.

(a) $\Pr(A \cap B) = \Pr(A) + \Pr(B) - \Pr(A \cup B) = 0.7 + 0.4 - 0.8 = 0.3$.

(b) $\Pr(A' \cap B') = \Pr((A \cup B)') = 1 - \Pr(A \cup B) = 1 - 0.8 = 0.2$.

Question 2

(a) Number of ways to choose 5 out of 30 qualified applicants = ${}_{30}C_5$ = 30!/(5!25!) = 142506.

(b) Number of ways to choose 5 out of 30 qualified applicants such that none of the minority is hired = ${}_{7}C_0 \times {}_{23}C_5$ = 1 × 23!/(5!18!) = 33649. Therefore the desired probability is 33649/142506 = 0.2361.

(c) Number of ways to choose 5 out of 30 qualified applicants such that one minority is hired = ${}_{7}C_1 \times {}_{23}C_4$ = 7× [23!/(4!19!)] = 61985.

Let A_0 and A_1 denote the events that no minority and one minority is hired respectively. Hence $\Pr(A_1)$ = 61985/142506 = 0.4350.

From part (b), $\Pr(A_0)$ = 0.2361.

Therefore Pr(at most one minority is hired) = $\Pr(A_0) + \Pr(A_1)$ = 0.6711.

Question 3

Number of possible hands of 5 cards is ${}_{52}C_5$ = 52(51)(50)(49)(48)/5! = 2598960.

(a) Number of spade flush hands is ${}_{13}C_5 \times {}_{13}C_0 \times {}_{13}C_0 \times {}_{13}C_0$ = 1287.

Similarly, the number of heart flush hands is ${}_{13}C_0 \times {}_{13}C_5 \times {}_{13}C_0 \times {}_{13}C_0$ = 1287 and so on.

Pr(a flush hand) = 4(1287)/2598960 = 5148/2598960 = 0.001981.

(b) Number of straight hands with 1 as the smallest card is $({}_{4}C_1)^5 \times ({}_{4}C_0)^8 - 4$ = 1020.

Similarly, the number of straight hands with 2 as the smallest card is ${}_{4}C_0 \times ({}_{4}C_1)^5 \times ({}_{4}C_0)^8 - 4$ = 1020 and so on. The smallest card can be any one from 1 to 10.

Pr(a straight hand) = 10(1020)/2598960 = 10200/2598960 = 0.003925.

Question 4

Let A_i, i = 1, 2 denote the event that the motorist stops at light i.

We have $\Pr(A_1) = 0.4$, $\Pr(A_2) = 0.5$ and $\Pr(A_1 \cup A_2) = 0.6$.

(a) $\Pr(A_1 \cap A_2) = \Pr(A_1) + \Pr(A_2) - \Pr(A_1 \cup A_2) = 0.4 + 0.5 - 0.6 = 0.3$.

(b) Stops at exactly one light = $(A_1 \cap A_2') \cup (A_1' \cap A_2)$

But $\Pr(A_1 \cap A_2') = \Pr(A_1) - \Pr(A_1 \cap A_2) = 0.4 - 0.3 = 0.1$ and

$\Pr(A_1' \cap A_2) = \Pr(A_2) - \Pr(A_1 \cap A_2) = 0.5 - 0.3 = 0.2$.

Hence Pr(Stops at exactly one light) = 0.1 + 0.2 = 0.3.

(c) $\Pr(A_1' \cap A_2') = \Pr((A_1 \cup A_2)') = 1 - \Pr(A_1 \cup A_2) = 1 - 0.6 = 0.4$.

(d) $\Pr(A_2 \mid A_1) = \Pr(A_1 \cap A_2) / \Pr(A_1) = 0.3/0.4 = 0.75$.

Question 5

Number of possible 9-digit numbers with no restriction = ${}_9C_1 \times ({}_{10}C_1)^8 = 9(10)^8$

(a) There are 9 ways to choose the first digit, and also 9 ways (in order not repeat the number chosen for the first/previous digit) to choose the second digit, and so on until the ninth digit. Hence, the number of 9-digit numbers with no two consecutive digits are the same = $({}_9C_1)^9 = 387420489$.
The probability that no two consecutive digits are the same in a randomly selected 9-digit number = $({}_9C_1)^9/[{}_9C_1 \times ({}_{10}C_1)^8] = 387420489/[9(10)^8] = 0.4305$

(b) In a 9-digit number, there are 8 places (except the first digit) where we can place the three zeros, number of ways of doing so = ${}_8C_3 = 56$. For the other places, there are 6 of them, we have 9 choices (except the choice of zero), number of ways doing so = $({}_9C_1)^6 = 9^6$. So the number of 9-digit numbers with 0 appears as a digit for a total of 3 times = ${}_8C_3 \times ({}_9C_1)^6 = 29760696$.
The probability that a 9-digit number with 0 appears as a digit for a total of 3 being selected = ${}_8C_3 \times ({}_9C_1)^6/[{}_9C_1 \times ({}_{10}C_1)^8] = 29760696/[9(10)^8] = 0.03307$.

Question 6

Let A = {Player A wins the game} and B = {Player B enters the game}.
It is given that $\Pr(A|B) = 1/6$, $\Pr(A|B') = 3/4$ and $\Pr(B) = 1/3$.
Hence $\Pr(B') = 1 – \Pr(B) = 2/3$.
Applying the total probability law, $\Pr(A) = \Pr(A|B)\Pr(B) + \Pr(A|B')\Pr(B') = (1/6)(1/3) + (3/4)(2/3) = 5/9$.

Question 7

Let M_1 = {the selected bottle was filled on machine I}, M_2 = {the selected bottle was filled on machine II} and N = {a nonconforming bottle was selected}
It is given that $\Pr(N \cap M_1) = 0.01$, $\Pr(N \cap M_2) = 0.025$. $\Pr(M_1) = \Pr(M_2) = 0.5$

(a) $\Pr(N) = \Pr((N \cap M_1) \cup (N \cap M_2)) = 0.01 + 0.025 = 0.035$
(b) $\Pr(M_2) = 0.5$
(c) $\Pr(M_2 \cap N') = \Pr(M_2) - \Pr(M_2 \cap N) = 0.5 – 0.025 = 0.475$.
(d) $\Pr(M_1 \cup N') = \Pr(M_1) + \Pr(N') - \Pr(M_1 \cap N')$.
But $\Pr(N') = 1 – \Pr(N) = 1 – 0.035 = 0.965$, $\Pr(M_1 \cap N') = \Pr(M_1) – \Pr(M_1 \cap N) = 0.5 – 0.01 = 0.49$, therefore $\Pr(M_1 \cup N') = 0.5 + 0.965 – 0.49 = 0.975$.
(e) $\Pr(N \mid M_1) = \Pr(N \cap M_1) / \Pr(M_1) = 0.01/0.5 = 0.02$.
(f) $\Pr(M_1 \mid N) = \Pr(N \cap M_1) / \Pr(N) = 0.01/0.035 = 0.2857$
(g) The events are different and the conditions are different. The answer in part (e) is the probability of having a nonconforming item given the condition that the item was from machine I (i.e $\Pr(N|M_1)$). The answer in part (f) is the probability of having an item from machine I given that it was a nonconforming item (i.e. $\Pr(M_1|N)$).

Question 8

Let P = {the women is pregnant} and T = {test result is positive}. Hence, P^C = {the woman is not pregnant} and T^C = {test result is negative}.
We have $\Pr(P) = 0.75$, $\Pr(T \mid P) = 0.99$, $\Pr(T \mid P^C) = 0.02$.
Hence $\Pr(T) = \Pr(P)\Pr(T \mid P) + \Pr(P')\Pr(T \mid P^C) = 0.75(0.99) + 0.25(0.02) = 0.7475$.
(a) $\Pr(P \mid T) = \Pr(P \cap T)/\Pr(T) = 0.75(0.99)/0.7475 = 0.9933$.
(b) $\Pr(P^C \mid T^C) = \Pr(P^C \cap T^C)/\Pr(T^C) = (1 – 0.02)(0.25)/(1 – 0.7475) = 0.9703$.