ST2334 (2020/2021 Semester 2) Solutions to Questions in Tutorial 1

Question 1

(a) $S = \{123, 124, 125, 13, 14, 15, 213, 214, 215, 23, 24, 25, 3, 4, 5\}$
(b) $A = \{3, 4, 5\}$
(c) $B = \{5, 15, 25, 125, 215\}$
(d) $C = \{23, 24, 25, 3, 4, 5\}$
(e) $A \cap B = \{5\} \neq \emptyset$. Hence A and B are not mutually exclusive events.

Question 2

We have $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 3, 5, 7, 9\}$, $C = \{2, 3, 4, 5\}$ and $D = \{1, 6, 7\}$.
(a) A or $C = A \cup C = \{2, 3, 4, 5, 6, 8, 10\}$.
(b) $A\ and\ B = A \cap B = \emptyset$.
(c) The complement of $C = C' = \{1, 6, 7, 8, 9, 10\}$.
(d) Since $A \cap C = \{2, 4\}$ and $D' = \{2, 3, 4, 5, 8, 9, 10\}$, hence $A \cap C \cap D' = \{2, 4\}$.

Question 3

(a) Number of choices for the hundreds, tens and ones positions are 5, 5 and 4 respectively. Hence the number of 3-digit numbers formed $= 5 \times 5 \times 4 = 100$.
(b) To ensure it is odd, we place 9 in the ones position. It follows that the number of choices for the ones, hundreds and tens positions are 1, 4 and 4 respectively. Hence, the number of odd 3-digit numbers formed $= 4 \times 4 \times 1 = 16$.
(c) Similarly, number of odd 3-digit numbers > 620 with hundreds position $> 6 = 1 \times 4 \times 1 = 4$; and number of odd 3-digit numbers > 620 with hundreds position being $6 = 1 \times 3 \times 1 = 3$. Hence the number of 3-digit numbers $>620 = 4 + 3 = 7$.

Question 4

(a) ${}_8P_8 = 8! = 40320$.
(b) Let A, B, C and D represent the four couples. Number of ways to permute these four couples $= {}_4P_4 = 4! = 24$.
For each of these 24 permutations, we can permute the husband and wife in each couple, hence the number of ways to permute $= 2!\,2!\,2!\,2! = 16$.
Therefore, the number of ways that they can be seated if each couple is to sit together $= 4! \times (2!\,2!\,2!\,2!) = 384$.
(c) Number of ways to permute 4 husbands $= 4!$ and number of ways to permute 4 wives $= 4!$. Hence the number of ways that they can be seated together if all the men sit together to the right of all the women $= 4! \times 4! = 576$.

Question 5

(a) We choose 5 out of 7. Hence, $n = 7$ and $r = 5$. Number of choices is given by ${}_7C_5 = 7!/(5!\,2!) = 21$.
(b) Number of ways to choose 2 questions from the first 2 questions $= {}_2C_2 = 1$. Number of ways to choose three questions from the remaining 5 questions $= {}_5C_3 = 5!/(3!\,2!) = 10$.
Hence, number of ways to get the 5 questions for which 2 from the first 2 questions and 3 from the remaining 5 questions $= {}_2C_2 \times {}_5C_3 = 1(10) = 10$.
(c) Similarly, number of choices for selecting 1 question from the first 2 questions and 4 from the remaining 5 questions $= {}_2C_1 \times {}_5C_4 = (2)(5) = 10$.
Number of choices for selecting 2 question from the first 2 questions and 3 from the remaining 5 questions $= {}_2C_2 \times {}_5C_3 = (1)(10) = 10$.

Therefore, the number of choices if at least one of the first two questions must be answered $= 10 + 10 = 20$.

(d) Number of choices for selecting exactly 2 questions from the first 3 questions and 3 from the remaining 4 questions $= {}_3C_2 \times {}_4C_3 = (3)(4) = 12$.

Question 6

(a) Each path from A to B can be represented by a permutation of 8 U(N)'s and 13 R(E)'s (or choose 8 steps (or numbers) out of 21 steps (or numbers) for the U's).
For example, 8 numbers (2, 4, 5, 6, 9, 13, 18, 19) represent the path
RURUUURRURRRURRRRUURR
Number of ways from A to B is the same as the number of ways to choose 8 numbers out of 21 numbers $= {}_{21}C_8 = 21!/(13!\,8!) = 203490$.

(b) Number of ways from A to $Y = {}_{16}C_6 = 16!/(6!\,10!) = 8008$. (Choose 6 U's out of 16 steps.)
Number of ways from Y to $B = {}_5C_2 = 5!/(2!\,3!) = 10$. (Choose 2 U's out of 5 steps.)
Hence the number of ways from A to B stopping at $Y = 8008(10) = 80080$.
Therefore, the number of ways from A to B without stopping at $Y = 203490 - 80080 = 123410$.

(c) Number of ways from A to B stopping at X but not $Y = {}_4C_2 \times ({}_{17}C_6 - {}_{12}C_4 \times {}_5C_2) =$
$[4!/(2!\,2!)] \times [17!/(6!\,11!)] - [12!/4!\,8!] \times [5!/(2!\,3!)] = 44556$.

Question 7

(a) ${}_9C_1 \times {}_{27}C_1 = 9(27) = 243$.

(b) ${}_9C_1 \times {}_{27}C_1 \times {}_{15}C_1 = 9(27)(15) = 3645 \approx 10$ (years).

Question 8

(a) The number of permutations begin with a consonant $= {}_3P_1 \times {}_4P_4 = 3(4!) = 72$.

(b) The number of permutations end with a vowel $= {}_2P_1 \times {}_4P_4 = 2(4!) = 48$.

(c) The number of permutations have the consonants and vowels alternating $=$
$3(2)(2)(1)(1) = 3!\,2! = 12$.
Alternatively, as there is only one pattern CVCVC to meet the specification, we may consider to permute the 3 consonants and to permute the 2 vowels. Hence the number of permutations $= {}_3P_3 \times {}_2P_2 = (3!)(2!) = 12$.

Question 9

Number of ways to select 6 houses to be on 1 side of the street $= {}_9C_6 = 9!/(6!\,3!) = 84$.
For each of these selections, the number of ways to arrange the houses $= {}_6P_6 \times {}_3P_3 = 6!\,3! = 4320$.
Therefore the number of ways to place these houses $= {}_9C_6 \times {}_6P_6 \times {}_3P_3 = 362880$.

Question 10

Number of ways to arrange 3 oaks, 4 pines and 2 maples $= 9!/(3!\,4!\,2!) = 1260$.

Question 11

(a) It is always true that for every sample point x in Event B, the sample point x is in Event A or Event B, (i.e. $A \cup B$). Since $A \cup B = A$, therefore for every sample point x in Event B, the sample point x is in Event A. Hence, $B \subset A$.

(b) It is always true that for every sample point x in Event A and Event B (i.e.$A \cap B$), the sample point x is in Event B. Since $A \cap B = A$, therefore for every sample point x in Event A, x is in Event B. Hence, $A \subset B$.