NATIONAL UNIVERSITY OF SINGAPORE
Department of Statistics and Applied Probability

(2020/21) Semester 1 ST2334 Probability and Statistics Tutorial 6

1. Let X denote the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day. Let Y denote the number of times a technician is called on an emergency call. Their joint probability distribution is given below.

$f_{X,Y}(x,y)$		x		
		1	2	3
y	1	0.05	0.05	0.1
	2	0.05	0.10	0.35
	3	0	0.2	0.1

(a) Evaluate the marginal distribution of X.
(b) Evaluate the marginal distribution of Y.
(c) Find $\Pr(Y = 3|X = 2)$
(d) Find the conditional distribution of Y given $X = 2$.
(e) Determine whether X and Y are dependent or independent.

2. From a sack of fruit containing 3 oranges, 2 apples, and 3 bananas, a random sample of 4 pieces of fruit is selected. If X is the number of oranges and Y is the number of apples in the sample, find

(a) the joint probability distribution of X and Y;
(b) $\Pr(X = 1, Y = 1)$;
(c) $\Pr(X + Y \leq 2)$;
(d) $f_X(x)$;
(e) $f_{Y|X}(y|2)$ and hence $\Pr(Y = 0|X = 2)$.

3. Consider an experiment that consists of two rolls of a balanced die. If X is the number of fours and Y is the number of fives obtained in the two rolls of the die, find

(a) the joint probability distribution of X and Y;
(b) $\Pr(2X + Y < 3)$.
(c) Determine whether X and Y are dependent or independent.

4. Each rear tire on an experimental airplane is supposed to be filled to a pressure of 40 pound per square inch (psi). Let X denote the actual air pressure (in 10 pound per square inch) for the right tire and Y denote the actual air pressure (in 10 pound per square inch) for the left tire. Suppose that X and Y are random variables with the joint density

$$f_{X,Y}(x,y) = \begin{cases} k(x^2 + y^2), & 3 \leq x \leq 5; \quad 3 \leq y \leq 5; \\ 0, & \text{otherwise} \end{cases}$$

(a) Determine k.
(b) Compute $\Pr(3 \leq X \leq 4 \text{ and } 4 \leq Y < 5)$.
(c) Find $f_X(x)$ and hence $\Pr(3.5 < X < 4)$.

5. A candy company distributes boxes of chocolates with a mixture of creams, toffees, and cordials. Suppose that the weight of each box is 1 kilogram, but the individual weights of the creams, toffees, and cordials vary from box to box. For a randomly selected box, let X and Y represent the weights of the creams and the toffees, respectively, and suppose that the joint density function of these variables is

$$f_{X,Y}(x,y) = \begin{cases} 24xy, & 0 \le x \le 1, \quad 0 \le y \le 1, \quad x + y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the marginal density for the weight of the creams and the marginal density for the weight of the toffees.
(b) Determine if X and Y are independent.
(c) Find the probability that the weight of the toffees in a box is less than $1/8$ of a kilogram if it is known that creams constitute $3/4$ of the weight.

Answers to selected problems

1. (a)

x	1	2	3
$f_X(x)$	0.10	0.35	0.55

(b)

y	1	2	3
$f_Y(y)$	0.20	0.50	0.30

(c) 4/7

(d)

y	1	2	3
$f_{Y\|X}(y\|x=2)$	1/7 = 0.14286	2/7 = 0.28571	4/7 = 0.57143

(e) X and Y are dependent

2. (a) $f_{X,Y}(x,y) = \begin{cases} \dfrac{\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}}{\binom{8}{4}}, & x = 0,1,2,3;\ \ y = 0,1,2;\ \ 1 \le x + y \le 4 \\ 0, & \text{otherwise.} \end{cases}$

(b) 0.2571 (or 9/35)

(c) 0.5

(d) $f_X(x) = \begin{cases} \dfrac{\binom{3}{x}\binom{5}{4-x}}{\binom{8}{4}}, & x = 0,1,2,3; \\ 0, & \text{otherwise.} \end{cases}$

(e) $f_{Y|X}(y|x) = \begin{cases} \frac{1}{10}\binom{2}{y}\binom{3}{2-y}, & y = 0,1,2; \\ 0, & \text{otherwise.} \end{cases}$, 0.3

3. (a)

$f_{X,Y}(x,y)$		x		
		0	1	2
y	0	4/9	2/9	1/36
	1	2/9	1/18	0
	2	1/36	0	0

(b) 11/12
(c) X and Y are dependent

4. (a) 3/392

(b) 0.25

(c) $f_X(x) = (3x^2 + 49)/196$ for $3 \le x \le 5$, 0.2328

5. (a) $f_X(x) = \begin{cases} 12x(1-x)^2, 0 \le x \le 1; \\ 0, \quad \text{otherwise.} \end{cases}$ $f_Y(y) = \begin{cases} 12y(1-y)^2, 0 \le y \le 1; \\ 0, \quad \text{otherwise.} \end{cases}$

(b) X and Y are not independent

(c) $f_{Y|X}(y|x = 3/4) = 32y$ for $0 \le y \le 1/4$, 0.25