ST2334 (2020/2021 Semester 1) Solutions to Questions in Tutorial 10

Question 1

If $\mu = 20$, then $\Pr(\bar{X} > 24) = \Pr\left(\frac{\bar{X}-20}{4.1/\sqrt{9}} > \frac{24-20}{4.1/\sqrt{9}}\right) = \Pr(T_8 > 2.9268) < 0.01$ since $\Pr(T_8 > 2.9268) < \Pr(T_8 > 2.8965)\ (= 0.01)$. Note $\Pr(T_8 > 2.9268) = 0.00955$ (from Excel: "1-t.dist((24-20)/(4.1/sqrt(9)),8,TRUE)").

We conclude that $\mu > 20$. $\Pr(\bar{X} > 24|\mu = 20)$ being small shows that it is very unlikely to get a mean of 24 if the population mean is really 20.

Question 2

(a) $\Pr(\bar{X}_B - \bar{X}_A \geq 0.2) = \Pr\left(Z > \frac{0.2}{\sqrt{1/36+1/36}}\right) = \Pr(Z > 0.85) = 0.1977$ (from the normal table). Note: $\Pr\left(Z > \frac{0.2}{\sqrt{1/36+1/36}}\right) = 0.1981$ (from Excel: "1-norm.dist(0.2/sqrt(2/36),0,1,TRUE)")

(b) Since the probability in part (a) is not small, therefore it is not unlikely to observe $\bar{X}_B - \bar{X}_A \geq 0.2$ when $\mu_A = \mu_B$. Hence, the conjecture that $\mu_A \neq \mu_B$ is likely not true.

Question 3

(a) $\Pr(S^2 > 9.1) = \Pr\left(\frac{(n-1)S^2}{\sigma^2} > \frac{(24)(9.1)}{6}\right) = \Pr(\chi^2_{24} > 36.4) \approx 0.05$ (from the χ^2-table). Note: $\Pr(\chi^2_{24} > 36.4) = 0.05017$ ((from Excel: "1-chisq.dist(24*9.1/6,24,TRUE)")

(b) $\Pr(3.462 < S^2 < 10.745) = \Pr\left(\frac{(24)(3.462)}{6} < \frac{(n-1)S^2}{\sigma^2} < \frac{(24)(10.745)}{6}\right) = \Pr(13.848 < \chi^2_{24} < 42.98) = 0.95 - 0.01 = 0.94$

Question 4

Since σ_1^2 and σ_2^2 are equal, therefore S_1^2/S_2^2 follows an F distribution with (7, 11) degrees of freedom. Hence $\Pr(S_1^2/S_2^2 < 4.89) = 0.99$ (From F-table). Remark: $\Pr(S_1^2/S_2^2 < 4.89) = 0.99003$ (From Excel: "=f.dist(4.89,7,11,TRUE)")

Question 6

Mine 1:	8260	8130	8350	8070	8340		$S_1^2 = 15750$
Mine 2:	7950	7890	7900	8140	7920	7840	$S_2^2 = 10920$

$$\Pr\left(\frac{S_1^2}{S_2^2} > \frac{15750}{10920}\right) = \Pr\left(\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} > \frac{15750}{10920}\right) = \Pr\big(F_{4,5} > 1.44\big) > 0.05$$

[From F-table, $\Pr\big(F_{4,5} > 5.19\big) = 0.05$. Since $1.44 < 5.19$, therefore $\Pr\big(F_{4,5} > 1.44\big) > \Pr\big(F_{4,5} > 5.19\big) = 0.05$.] Remark: $\Pr\big(F_{4,5} > 1.4423\big) = 0.3436$ [From Excel: "1-f.dist(15750/10920;4;5;TRUE)"].

Since under equal variances assumption, the probability of the ratio of the two sample variances is more extreme than what we observed ($s_1^2/s_2^2 = 1.4423$) is not small, therefore, the equal variances assumption seems to be plausible. Hence, the two variances may be considered equal.

Question 6

(a) $E(U) = E(X)/n = np/n = p$. Since $E(U) = p$, therefore U is an unbiased estimator of p.

(b) $E(V) = \frac{E(X+n/2)}{3n/2} = \frac{np+n/2}{3n/2} = \frac{p+1/2}{3/2} = \frac{2p+1}{3} \neq p$ unless $p = 1$. Since $E(V) \neq p$, therefore V is a biased estimator of p.

<u>Question 7</u>

Y = helium porosity of a coal sample. $Y \sim N(\mu, \sigma^2)$.

(a) It is given that $\sigma = 0.75, n = 20$ and $\bar{y} = 4.85$. Hence a 95% confidence interval for μ is given by $\bar{Y} \pm z_{0.025}\frac{\sigma}{\sqrt{n}} = 4.85 \pm 1.96\frac{0.75}{\sqrt{20}} = 4.85 \pm 0.3287 = (4.5213, 5.7187)$.

(b) The length of a 95% confidence interval is $2\, z_{0.025}\frac{\sigma}{\sqrt{n}}$. Hence the length of 95% CI being 0.4 implies that $2(1.96)\frac{0.75}{\sqrt{n}} = 0.4$. Therefore $n = 54$.

(c) It is given that $S = 0.75,\ n = 20$ and $\bar{y} = 4.85$. Hence a 95% confidence interval for μ is given by $\bar{Y} \pm t_{19;\,0.025}\frac{S}{\sqrt{n}} = 4.85 \pm 2.093\frac{0.75}{\sqrt{20}} = 4.85 \pm 0.351 = (4.499, 5.201)$.

<u>Question 8</u>

(a) 95% confidence interval for μ is given by $\bar{X} \pm z_{0.025}\frac{\sigma}{\sqrt{n}} = 0.310 \pm (1.96)\frac{0.0015}{\sqrt{75}} = 0.310 \pm 0.00034 = (0.3097, 0.3103)$

(b) $n \geq \left(\frac{z_{0.025}\,\sigma}{e}\right)^2 = \left(\frac{1.96\ x\ 0.0015}{0.0005}\right)^2 = (5.88)^2 = 34.6$. Take $n = 35$.

<u>Question 9</u>

A 90% confidence interval for μ is given by $\bar{X} \pm t_{11;0.05}\frac{s}{\sqrt{n}} = 48.50 \pm (1.796)\frac{1.5}{\sqrt{12}} = 48.50 \pm 0.7777 = (47.722, 49.278)$

<u>Question 10</u>

A 94% confidence interval for $\mu_1 - \mu_2$ is given by $(\bar{X}_1 - \bar{X}_2) \pm z_{0.03}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = (80 - 75) \pm (1.88)\sqrt{\frac{5^2}{25} + \frac{3^2}{36}} = 5 \pm 2.102 = (2.898, 7.102)$

<u>Question 11</u>

98% confidence interval for $\mu_1 - \mu_2$ is given by $(\bar{X}_1 - \bar{X}_2) \pm z_{0.01}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} =$

$(12.2 - 9.1) \pm (2.33)\sqrt{\frac{1.1^2}{100} + \frac{0.9^2}{200}} = 3.1 \pm 0.296 = (2.804, 3.396)$

Since the 98% confidence interval does not cover 0 and is in the positive range, the treatment appears to reduce the mean amount of metal removed.

<u>Question 12</u>

Since $E(Z^{2k+1}) = E[(-Z)^{2k+1}] = E[-Z^{2k+1}] = -E[Z^{2k+1}]$, therefore $E(Z^{2k+1}) = 0$.