3(a). $E(X) = \boxed{\sum_{k=1}^{\infty} P(X \geq k)}$

$$
\begin{aligned}
P(X \geq 1) &= P(X=1) + P(X=2) + P(X=3) + \cdots \\
P(X \geq 2) &= \qquad\quad\;\; P(X=2) + P(X=3) + P(X=4) + \cdots \\
P(X \geq 3) &= \qquad\qquad\qquad\qquad\;\; P(X=3) + P(X=4) + \cdots \\
P(X \geq 4) &= \qquad\qquad\qquad\qquad\qquad\qquad\;\; P(X=4) + \cdots \\
5 \\
6 \\
\vdots
\end{aligned}
$$

$+$

$$\sum_{k=1}^{\infty} P(X \geq k) = 1 \cdot P(X=1) + 2 \cdot P(X=2) + 3P(X=3) + 4P(X=4) + \cdots + n \cdot P(X=n) + \cdots$$

Dis.

$f(x) = P(X=x)$

$$= \sum_{x=1}^{\infty} x \cdot P(X=x)$$

$$= \sum_{x=1}^{\infty} x \cdot f(x)$$

$$= E(X)$$

Tail Sum Formula

X_1, X_2, X_3

(b) $P(M \geq k) = P(X_1 \geq k, X_2 \geq k, X_3 \geq k)$

$P(X \geq k) = \frac{7-k}{6}$ $\quad = P(X_1 \geq k) \cdot P(X_2 \geq k) \cdot P(X_3 \geq k)$

$P(X \geq 1) = \frac{6}{6}$ $\quad = \left(\frac{7-k}{6}\right)^3$

$P(X \geq 2) = \frac{5}{6}$ $\quad P(M \geq 1) = \left(\frac{6}{6}\right)^3 = 1$

$P(X \geq 3) = \frac{4}{6}$ $\quad P(M \geq 2) = \left(\frac{5}{6}\right)^3$

$P(X \geq 4) = \frac{3}{6}$ $\quad P(M \geq 3) = \left(\frac{4}{6}\right)^3$

$P(X \geq 5) = \frac{2}{6}$ $\quad \vdots$

$P(X \geq 6) = \frac{1}{6}$ $\quad P(M \geq 6) = \left(\frac{1}{6}\right)^3$

$$E(M) = \sum_{k=1}^{\infty} P(M \geq k)$$

$P(M \geq 7) = 0$

$$= \sum_{k=1}^{6} P(M \geq k)$$

$$= \sum_{k=1}^{6} \left(\frac{7-k}{6}\right)^3$$

$$= \frac{6^3 + 5^3 + \cdots + 1^3}{6^3} = 2.0417$$

1.(b) (i) $V(x) = \sum (x-\underset{E(x)}{\underset{\|}{\mu}})^2 f(x)$

$E(x) = \sum x f(x)$

$= 2\times 0.01 + 3\times 0.25 + 4\times 0.4$

$+ 5\times 0.3 + 6\times 0.04$

$= 4.11$

$V(X) = \sum (x-4.11)^2 \cdot f(x)$

$= 2.11^2\times 0.01 + 1.11^2\times 0.25$

$+ 0.11^2\times 0.4 + 0.89^2\times 0.3$

$+ 1.89^2\times 0.04$

$= 0.7379$

(ii) $V(X) = \boxed{E(X^2)} - [E(x)]^2 = 17.63 - 4.11^2$

$= 0.7379$

$E(X^2) = \sum x^2 f(x) = 17.63$

6.

$$E[(X-1)^2] = E(X^2 - 2X + 1)$$

① $$= \boxed{E(X^2) - 2E(X) + 1 = 10}$$

$$E[(X-2)^2] = E(X^2 - 4X + 4)$$

② $$= \boxed{E(X^2) - 4E(X) + 4 = 6}$$

From ① and ② $\Rightarrow$ $\begin{cases} \boxed{E(X) = 3.5} \\ E(X^2) = 16 \end{cases}$

$$V(X) = E(X^2) - [E(X)]^2 = 16 - 3.5^2 = 3.75$$

7. $\mu = 10$, $\sigma^2 = 4 \Rightarrow \sigma = 2$

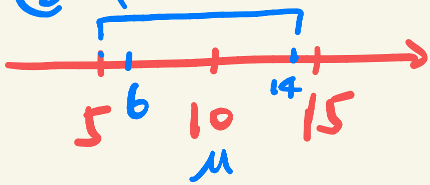
7. (a) $P(5 < X < 15)$

$= P(|X-10| < 5)$ $\quad k\sigma = 5 \Rightarrow k = \frac{5}{2}$

$\geq 1 - \frac{1}{2.5^2} = 0.84$

① $(5 < X < 15)$

② $(6 < X < 14)$



(b) $P(5 < X < 14)$ ×

$\leq P(5 < X < 15)$

≥ 0.84 ×

$a < b \geq c$

★ $P(5 < X < 14)$ ✓

$> P(6 < X < 14)$

$= P(|X-10| < 4)$ $\quad k\sigma = 4 \Rightarrow k = 2$

$\geq 1 - \frac{1}{2^2} = 0.75$

$a > b \geq c$

$\Rightarrow a > c$

(c) $P(|X-10| < 3)$ $\quad \Rightarrow k\sigma = 3 \Rightarrow k = \frac{3}{2}$

$\geq 1 - \frac{1}{1.5^2} = \frac{5}{9}$

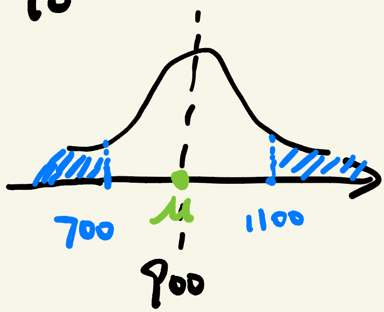
(d) $P(|X-10| \geq 3) \leq \frac{1}{1.5^2} = \frac{4}{9}$

(e) $P(|X-10| \geq c) \leq 0.04$

$$\frac{1}{k^2} = 0.04 \Rightarrow k = 5$$

$$c = k\sigma = 5 \times 2 = 10$$

9. $\mu = 900, \quad \sigma = 50$



$$P(X < 700) = \frac{1}{2} P(\underline{X < 700 \text{ or } X > 1100})$$

$k\sigma = 200$

$k = 4$

$$= \frac{1}{2} P(|X - 900| > 200)$$

$$\leq \frac{1}{2} \times \frac{1}{16} = \frac{1}{32}$$