T1, Q11, (a) (b) $A \subset B$ ✗

$A \subseteq B$ ($A = B$ or $A \subset B$)

T2:

3. (a). A: flush

$$_{13}C_5 \times 4 = 5148$$

\# total 5-card : $_{52}C_5 = 2598960$

$$P(A) = \frac{5148}{2598960} = 0.001981$$

(b) B: straight

number   #s   suits

\#s

A 2 3 4 5 #s

2 3 4 5 6 #s

3 4 5 6 7 #s

⋮

10 J Q K A #s

$10 \times \#s$

$10 \times (4^5 - 4) = 10200$

A 2 3 4 5

↓ ↓ ↓ ↓ ↓

$4\ 4\ 4\ 4\ 4 = 4^5 - \underbrace{4}_{\text{straight flush}} = \#s$

$$P(B) = \frac{10200}{2598960} = 0.003925$$

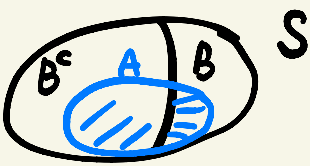
5. $\underset{\substack{\uparrow \\ 0 \\ \times}}{\overset{a}{\_}}\ \underset{\substack{\uparrow \\ a \\ \times}}{\overset{b\,0}{\_}}\ \underset{\substack{\uparrow \\ b \\ \times}}{\_}\ \overset{0}{\_}\ \_\ \_\ \_\ \overset{0}{\_}\ \_$ 0, 1, ..., 9 10 $a\ \times$

(a) $9 \times 9 \times 9 \times \cdots \times 9 = 9^9$

$$P = \frac{9^9}{9 \times 10^8} = 0.4305$$

(b) ${}_8C_3 \times 9^6 = 56 \times 9^6$

$$P = \frac{56 \times 9^6}{9 \times 10^8} = 0.03307$$

6. 

$A = \{A \text{ wins}\}$

$B = \{B \text{ enters}\}$

$$P(A) = P(A \cap B) + P(A \cap B^c)$$
$$= P(B) \cdot P(A|B) + P(B^c) \cdot P(A|B^c)$$
$$= \frac{1}{3} \times \frac{1}{6} + \frac{2}{3} \times \frac{3}{4}$$
$$= \frac{1}{18} + \frac{1}{2} = \frac{5}{9}$$

7. $M_1 = \{\text{filled by machine I}\}$ M_1

$M_2 = \{\text{filled by machine II}\}$ M_1^c

$N = \{\text{Nonconforming}\}$

conditional

"and" = "$\cap$" "or" = "$\cup$" "given/if" = "$|$"

$P(M_1 \cap N) = 0.01$, $P(M_2 \cap N) = 0.025$

$P(M_1) = P(M_2) = 0.5$

(a) $P(N) = P(N \cap M_1) + P(N \cap M_2)$

$= 0.01 + 0.025$

$= 0.035$

(b) $P(M_2) = 0.5$

(c) $P(M_2 \cap N^c) = P(M_2) - P(M_2 \cap N)$

$= 0.5 - 0.025$

$= 0.475$

(d) $P(M_1 \cup N^c) = P(M_1) + P(N^c) - P(M_1 \cap N^c)$

★ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$\begin{aligned} P(M_1 \cap N^c) &= P(M_1) - P(M_1 \cap N) \\ &= 0.5 - 0.01 \\ &= 0.499 \end{aligned}$$

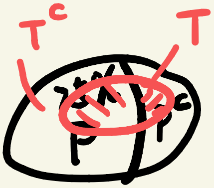
$$\begin{aligned} P(M_1 \cup N^c) &= P(M_1) + P(N^c) - P(M_1 \cap N^c) \\ &= 0.5 + (1 - 0.035) - 0.499 \\ &= 0.975 \end{aligned}$$

(e) $P(\underset{\text{event}}{N} \mid \underset{\text{condition}}{M_1}) = \dfrac{P(N \cap M_1)}{P(M_1)} = \dfrac{0.01}{0.5} = 0.02$

(f) $P(M_1 \mid N) = \dfrac{P(N \cap M_1)}{P(N)} = \dfrac{0.01}{0.035} = 0.2857$

(g) They are different cond. prob.

events & conditions are different.

8. 

S = do the test

$P = \{ \text{Pregnant} \}$ $\quad Pr(P) = 0.75$

$T = \{ \text{Test positive} \}$

event

$$\begin{cases} P(T|P^c) = 0.02 & \text{false positive} \\ P(T^c|P^c) = 1 - 0.02 = 0.98 & ✓ \end{cases}$$

$$\begin{cases} P(T|P) = 0.99 & \text{valid positive} \\ P(T^c|P) = 1 - 0.99 = 0.01 \end{cases}$$

$$\boxed{\begin{array}{l} P(A) + P(A^c) = 1 \\ \text{event} \\ P(T|P^c) + P(T^c|P^c) = 1 \end{array}}$$

(a) $P(P|T) = \dfrac{P(P \cap T)}{P(T)}$

$$P(P \cap T) = P(P) \cdot P(T|P) \quad ✓$$

$$= P(T) \cdot P(P|T)$$

$$= \frac{P(P) \cdot P(T|P)}{P(T \cap P) + P(T \cap P^c)}$$

$$= \frac{P(P)\cdot P(T|P)}{P(P)\cdot P(T|P) + P(P^c)\cdot P(T|P^c)}$$

$$= \frac{0.75 \times 0.99}{0.75 \times 0.99 + 0.25 \times 0.02}$$

$$= \frac{0.7425}{0.7475} \quad = P(T)$$

$$= 0.9933$$

(b) $$P(P^c|T^c) = \frac{P(P^c \cap T^c)}{P(T^c)}$$

$$= \frac{P(P^c)\cdot P(T^c|P^c)}{1 - P(T)}$$

$$= \frac{0.25 \times 0.98}{1 - 0.7475}$$

$$= 0.9703$$