

ST2234 (2020/2021 Semester 2) Solutions to Questions in Tutorial 11Question 1 $X = \text{lifetime. } X \sim N(\mu, 40^2)$

- (a) Test
- $H_0: \mu = 800$
- against
- $H_1: \mu \neq 800$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{788 - 800}{40/\sqrt{30}} = -1.64$$

Since $|z_{obs}| = 1.64 < z_{0.025} (= 1.96)$, therefore we do not reject H_0 .Alternatively, $p\text{-value} = 2 \min\{\Pr(Z < -1.64), \Pr(Z > -1.64)\} = 2(0.0505) = 0.1010$. Since $p\text{-value} > \alpha (= 0.05)$, we do not reject H_0 .

- (b) 95% confidence interval for
- $\mu: \bar{X} \pm Z \frac{\sigma}{\sqrt{n}} = 788 \pm 1.96 \frac{40}{\sqrt{30}} = (773.69, 802.31)$
- .

Yes, 800 is plausible.

- (c) Under
- H_0
- ,
- H_0
- is not rejected if
- $-1.96 < Z < 1.96$
- or
- $\mu - 1.96 \frac{\sigma}{\sqrt{n}} < \bar{X} < \mu + 1.96 \frac{\sigma}{\sqrt{n}}$
- or
- $785.79 < \bar{X} < 814.31$
- .

When $\mu = 790$ (i.e. H_0 is false), $\bar{X} \sim N\left(790, \frac{40^2}{30}\right)$. $\Pr(\text{Do not reject } H_0 | \mu = 790) = \Pr(785.79 < \bar{X} < 814.31 | \mu = 790) =$

$$\Pr\left(\frac{785.79 - 790}{40/\sqrt{30}} < \frac{\bar{X} - 790}{40/\sqrt{30}} < \frac{814.31 - 790}{40/\sqrt{30}}\right) = \Pr(-0.591 < Z < 3.329) = 1 - 0.9999 - 0.2774 = 0.7225.$$

- (d) When
- $\mu = 790$
- , Power =
- $1 - \Pr(\text{Type II error} | \mu = 790) = 1 - 0.7225 = 0.2775$
- .

Question 2 $X = \text{content of lubricant. } X \sim N(\mu, \sigma^2)$

- (a)
- $H_0: \mu = 10$
- against
- $H_1: \mu \neq 10$

$$\text{From the data, } \bar{x} = 10.06, s = 0.24585. \text{ Hence, } t_{obs} = \frac{\bar{x} - 10}{s/\sqrt{10}} = \frac{10.06 - 10}{0.246/\sqrt{10}} = 0.772.$$

Since $|t_{obs}| = 0.772 < t_{9,0.005} (= 3.25)$, therefore we do not reject H_0 Alternatively, $p\text{-value} = 2 \min\{\Pr(T < 0.772), \Pr(T > 0.772)\} > 2(0.10)$ (From t-table). $p\text{-value} = 0.4599$ (from statistical software). Since $p\text{-value} > \alpha (= 0.01)$, therefore we do not reject H_0 .

- (b)
- $H_0: \sigma^2 = 0.03$
- against
- $H_1: \sigma^2 \neq 0.03$

$$\chi_{obs}^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(9)(0.246)^2}{0.03} = 18.13 \text{ which falls between } \chi_{9,0.975}^2 (= 2.70) \text{ and } \chi_{9,0.025}^2 (= 19.023).$$

Hence, we do not reject H_0 .Alternatively, $\Pr(\chi_9^2 > 18.13)$ is between 0.025 and 0.05 since $\Pr(\chi_9^2 > 16.92) = 0.05$ and $\Pr(\chi_9^2 > 19.02) = 0.025$. Hence $0.05 < p\text{-value} < 0.10$.Since the $p\text{-value} > 0.05$. We do not reject H_0 .(Remark: $p\text{-value } \Pr(\chi_9^2 > 18.13) = 0.0673$ from statistical software)

- (c) 99% confidence interval for
- $\sigma^2 = \left(\frac{(n-1)s^2}{\chi_{9,0.005}^2}, \frac{(n-1)s^2}{\chi_{9,0.995}^2}\right) = \left(\frac{9(0.246)^2}{23.589}, \frac{9(0.246)^2}{1.735}\right) = (0.023, 0.314)$
- . Note:
- $\chi_{9,0.005}^2$
- satisfies
- $\Pr(W > \chi_{9,0.005}^2) = 0.005$
- with
- $W \sim \chi^2(9)$
- .

Question 3 $X = \text{amount of soft drink dispensed. } X \sim N(\mu, \sigma^2)$

$$\text{From the data, we have } n = 25, s^2 = 2.03. \text{ Hence } \chi_{obs}^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(24)(2.03)}{1.15} = 42.37$$

Since the observed test statistic $> \chi_{24,0.05}^2 (= 36.415)$, we reject H_0 at 5% significance level.Alternatively, $p\text{-value}$ is between 0.01 and 0.025 as $\Pr(\chi_{24}^2 > 39.364) = 0.025$ and $\Pr(\chi_{24}^2 > 42.98) = 0.01$ [Exact $p\text{-value} = 0.0117$]

Question 4 X_A = tensile strength of thread A $\sim N(\mu_A, 6.28^2)$ X_B = tensile strength of thread B $\sim N(\mu_B, 5.61^2)$

- (a) Test
- $H_0: \mu_A - \mu_B = 12$
- against
- $H_1: \mu_A - \mu_B > 12$

From the data, we have $n_A = 50, \bar{x}_A = 86.7, n_B = 50, \bar{x}_B = 77.8$. Hence

$$z = \frac{(86.7 - 77.8) - (12)}{\sqrt{\frac{6.28^2}{50} + \frac{5.61^2}{50}}} = -2.60$$

Since $z_{obs} < z_{0.05} (= 1.645)$, we do not reject H_0 .Alternatively, $p\text{-value} = \Pr(Z > -2.60) = 1 - 0.0047 = 0.9953$.Since $p\text{-value} > \alpha (= 0.05)$. We do not reject H_0 .

- (b) We committed an error if our decision of not rejecting
- H_0
- is wrong. Hence it is Type II error. (Type I error is committed if our decision of rejecting
- H_0
- is wrong.)

Question 5 X_A = grades of students in the 3-semester-hour course $\sim N(\mu_A, \sigma^2)$ X_B = grades of students in the 4-semester-hour course $\sim N(\mu_B, \sigma^2)$ From the data, $n_A = 18, \bar{x}_A = 77, s_A = 6; n_B = 12, \bar{x}_B = 84, s_B = 4$. Hence,

$$s_p = \sqrt{\frac{(n_A - 1)s_A^2 + (n_B - 1)s_B^2}{n_A + n_B - 2}} = 5.3050$$

- (a) 99% confidence interval for
- $\mu_B - \mu_A = (\bar{X}_B - \bar{X}_A) \pm t_{28, 0.005} s_p \sqrt{\frac{1}{n_B} + \frac{1}{n_A}} =$

$$(84 - 77) \pm (2.763)(5.304) \sqrt{\frac{1}{12} + \frac{1}{18}} = (1.537, 12.463).$$

Or 99% confidence interval for $\mu_A - \mu_B = (\bar{X}_A - \bar{X}_B) \pm t_{28, 0.005} s_p \sqrt{\frac{1}{n_B} + \frac{1}{n_A}} =$

$$(77 - 84) \pm (2.763)(5.304) \sqrt{\frac{1}{12} + \frac{1}{18}} = (-12.463, -1.537).$$

- (b)
- $H_0: \mu_A - \mu_B = 0$
- against
- $H_1: \mu_A - \mu_B > 0$

$$t_{obs} = \frac{\bar{x}_A - \bar{x}_B}{s_p \sqrt{\frac{1}{n_B} + \frac{1}{n_A}}} = \frac{77 - 84}{(5.304) \sqrt{\frac{1}{12} + \frac{1}{18}}} = -3.541$$

Since $t_{obs} = -3.541 < t_{28; 0.05} (= 1.701)$, therefore, we do not reject H_0 .[Note: Exact p-value = $\Pr(T > -3.541) = 0.9993$ (from statistical software)]Question 6 X_R = gasoline consumption by radial tires X_B = gasoline consumption by belted tires $d = X_R - X_B, d \sim N(\mu_d, \sigma_d^2)$ From the data, $n_d = 12, \bar{x}_d = 0.1417, s_d = 0.1975$

- (a) 95% confidence interval for
- $\mu_d = \bar{x}_d \pm t_{11, 0.025} \frac{s_d}{\sqrt{n_d}} = 0.1417 \pm 2.201 \frac{0.1975}{\sqrt{12}} = (0.0162, 0.2672)$

- (b)
- $H_0: \mu_d = 0$
- against
- $H_1: \mu_d > 0$

$$t_{obs} = \frac{\bar{x}_d}{s_d / \sqrt{n}} = \frac{0.14167}{0.1975 / \sqrt{12}} = 2.485 > t_{11; 0.05} (= 1.796). \text{ Reject } H_0$$

Alternatively, $0.01 < p\text{-value} < 0.025$ since $\Pr(T > 2.306) = 0.025$ and $\Pr(T > 2.8965) = 0.01$ (Refer to the t-table). Reject H_0 .[Note: Exact p-value = $\Pr(T > 2.485) = 0.01515$ (from statistical software)]

Question 7

X_M = the length of time taken to assemble a product by men $\sim N(\mu_M, \sigma_M^2)$

X_W = the length of time taken to assemble a product by women $\sim N(\mu_W, \sigma_W^2)$

$H_0: \sigma_M^2 = \sigma_W^2$ against $H_1: \sigma_M^2 > \sigma_W^2$

From the data, $n_M = 11$, $s_M = 6.1$, $n_W = 14$, $s_W = 5.3$

Hence, $F_{obs} = \frac{s_M^2}{s_W^2} = \frac{6.1^2}{5.3^2} = 1.325$

Since $F_{obs} = 1.325 < F_{10,13;0.05} (= 2.67)$, therefore, we do not reject H_0 .

[Note: Exact p-value = $\Pr(F > 1.325) = 0.3117$ (from statistical software)]

At $\alpha = 0.05$, we do not have enough evidence to conclude that the variance of the times for women is less than that for men.

Question 8

X_1 = the running times of film produced by company I $\sim N(\mu_1, \sigma_1^2)$

X_2 = the running times of film produced by company I $\sim N(\mu_2, \sigma_2^2)$

(a) $H_0: \sigma_1^2 = \sigma_2^2$ against $H_1: \sigma_1^2 \neq \sigma_2^2$

From the data, $n_1 = 5$, $s_1^2 = 78.8$, $n_2 = 7$, $s_2^2 = 913.3333$

Hence, $F_{obs} = \frac{s_1^2}{s_2^2} = \frac{78.8}{913.3333} = 0.0863 < F_{4,6;0.975} (= 1/F_{6,4;0.025} = 1/9.20 =$

$0.1087)$. Reject H_0 .

Alternatively, p -value = $2 \min\{\Pr(F < 0.086), \Pr(F > 0.086)\} = 2 \min\{0.01639, 0.98361\} = 2(0.01639) = 0.0328 < 0.05$. Reject H_0 .

(b) 95% confidence interval for $\frac{\sigma_1^2}{\sigma_2^2} = \left(\frac{s_1^2}{s_2^2} \frac{1}{F_{4,6;0.025}}, \frac{s_1^2}{s_2^2} F_{6,4;0.025} \right) =$

$\left(\frac{78.8}{913.33} \frac{1}{6.23}, \frac{78.8}{913.33} (9.20) \right) = (0.01385, 0.79375)$

(c) 95% confidence interval for $\frac{\sigma_1}{\sigma_2} = (\sqrt{0.01385}, \sqrt{0.79375}) = (0.1177, 0.8909)$

Question 9

We have $E(W) = E(a_1X_1 + \dots + a_nX_n) = a_1E(X_1) + \dots + a_nE(X_n) = a_1\mu_1 + \dots + a_n\mu_n$.

Also recall variance of sum of independent random variables is the sum of their variances. Therefore, $V(W) = V(a_1X_1 + \dots + a_nX_n) = V(a_1X_1) + \dots + V(a_nX_n) = a_1^2\sigma_1^2 + \dots + a_n^2\sigma_n^2$.