NATIONAL UNIVERSITY OF SINGAPORE
Department of Statistics and Applied Probability

(2020/21) Semester 2 ST2334 Probability and Statistics Tutorial 10

1. A normal population with unknown variance has a mean of 20. Is one likely to obtain a random sample of size 9 from this population with a standard deviation being 4.1 and a mean being larger than or equal to 24? If not, what conclusion would you draw?

2. Two different box-filing machines are used to fill cereal boxes on the assembly line. The critical measurement influenced by these machines is the weight of the product in the machines. Engineers are quite certain that the variance of the weight of product is $\sigma^2 =$ 1 gram. Experiments are conducted using both machines with sample sizes of 36 each. The sample averages for machine A and B are $\bar{x}_A = 4.5$ grams $\bar{x}_B = 4.7$ grams. Engineers seemed surprised that the two sample averages for the filling machines were so different.
   (a) Use the central limit theorem to determine $\Pr(\bar{X}_B - \bar{X}_A \geq 0.2)$ under the condition that $\mu_A = \mu_B$.
   (b) Do the aforementioned experiments seem to, in any way, strongly support a conjecture that the two population means for the two machines are different? Explain using your answer in (a).

3. Find the probability that a random sample of 25 observations, from a normal population with variance $\sigma^2 = 6$, will have a variance S^2
   (a) Greater than 9.1
   (b) Between 3.462 and 10.745.

4. If S_1^2 and S_2^2 represent the variances of independent random samples of size $n_1 = 8$ and $n_2 = 12$, taken from normal populations with equal variances, find $\Pr(S_1^2/S_2^2 < 4.89)$.

5. Consider the following measurements of the heat producing capacity of the coal produced by two mines (in millions of calories per ton):
   Mine 1: 8260 8130 8350 8070 8340
   Mine 2: 7950 7890 7900 8140 7920 7840
   Assume that the populations are normal, can it be considered that the two population variances are equal?

6. Let X be a binomial random variable with parameters n and p.
   (a) Let $U = X/n$. Show that U is an unbiased estimator of p.
   (b) Let $V = \frac{X+n/2}{3n/2}$. Show that V is a biased estimator of p.

7. Assume that the helium porosity (in percentage), Y, of coal samples taken from any seam is normally distributed.
   (a) If the true standard deviation of Y is 0.75, compute a 95% confidence interval for the average porosity of a certain seam if the average porosity for 20 specimens from the seam was 4.85.
   (b) How large a sample size is necessary if the length of the 95% interval is to be 0.40?

(c) If the variance of Y is unknown, and the sample standard deviation for the sample in (a) is 0.75, compute a 95% confidence interval for the average porosity of a certain seam.

8. Many cardiac patients wear implanted pacemakers to control their heartbeat. A plastic connector module mounts on the top of the pacemaker. Assume that the dimension of the connector modules to be approximate normally distributed with a standard deviation of 0.0015.
   (a) A random sample of 75 modules has an average of 0.310 inch. Find a 95% confidence interval for the mean of all connector modules made by a certain manufacturing company.
   (b) How large a sample is needed if we wish to be 95% confident that the sample mean will be within 0.0005 inch of the true mean?

9. A random sample of 12 shearing pins is taken in a study of the Rockwell hardness of the head on the pin. Measurements on the Rockwell hardness were made for each of the 12, yielding an average value of 48.50 with a sample standard deviation of 1.5. Assuming the measurements to be normally distributed, construct a 90% confidence interval for the mean Rockwell hardness.

10. A random sample of size $n_1 = 25$ taken from a normal population with a standard deviation $\sigma_1 = 5$ has a mean $\bar{x}_1 = 80$. A second random sample of size $n_2 = 36$ taken from a different normal population with a standard deviation $\sigma_2 = 3$ has a mean $\bar{x}_2 = 75$. Find a 94% confidence interval for $\mu_1 - \mu_2$.

11. A study was conducted to determine if a certain metal treatment has any effect on the amount of metal removed in a pickling operation. A random sample of 100 pieces was immersed in a bath for 24 hours without the treatment, yielding an average of 12.2 millimeters of metal removed and a sample standard deviation of 1.1 millimeters. A second sample of 200 pieces was exposed to the treatment, followed by the 24-hour immersion in the bath, resulting in an average removal of 9.1 millimeters of metal with a sample standard deviation of 0.9 millimeters. Compute a 98% confidence interval estimate for the difference between the population means. Does the treatment appear to reduce the mean amount of metal removed?

12. Recall a property of standard normal distribution: if $Z \sim N(0,1)$, then $-Z \sim N(0,1)$. Make use of this property to show that $E(Z) = E(Z^3) = E(Z^5) = \cdots = 0$. That is, all odd moments of a standard normal are 0.

Answers to selected problems

1. 0.00955 (from statistical software); Between 0.005 and 0.01 (from t-table). No, it is unlikely
2. (a) 0.1981 (from statistical software); 0.1977 (From z-table)
   (b) No, it does not support the conjecture that that $\mu_A \neq \mu_B$
3. (a) ≈ 0.05
   (b) 0.94
4. 0.99003 (from statistical software); 0.99 (From χ^2-table)
5. $\Pr(F_{4,5} > 1.4423) = 0.3436$ (from statistical software); $\Pr(F_{4,5} > 1.4423) > 0.05$ (From the F-table). Variances are considered equal
6. (a) $E(U) = p$
   (b) $E(V) = \frac{1}{3}(2p + 1) \neq p$
7. (a) $(4.521, 5.719)$
   (b) $n = 54$
   (c) $(4.499, 5.201)$
8. (a) $(0.3097, 0.3103)$
   (b) $n = 35$
9. (47.722, 49.278) (From t-table, $t_{11;0.05} = 1.796$)
10. (2.898, 7.102) (From z-table, $z_{0.03} = 1.88$)
11. (2.804, 3.396) (From z-table, $z_{0.01} = 2.33$). Yes, it is likely.