

ST2334 (2020/2021 Semester 1) Solutions to Questions in Tutorial 3Question 1

- (a) $\Pr(A \cap B \cap C) = \Pr(A) \Pr(B|A) \Pr(C|A \cap B) = 0.75(0.9)(0.8) = 0.54$.
- (b) $\Pr(B) = \Pr(A \cap B) + \Pr(A' \cap B) = \Pr(A) \Pr(B|A) + \Pr(A') \Pr(B|A') = (0.75)(0.9) + (0.25)(0.8) = 0.875$.
- (c) $\Pr(A|B) = \Pr(A \cap B) / \Pr(B) = [(0.75)(0.9)]/0.875 = 0.7714$.
- (d) $\Pr(B \cap C) = \Pr(A \cap (B \cap C)) + \Pr(A' \cap (B \cap C))$. But $\Pr(A' \cap (B \cap C)) = \Pr(A') \Pr(B|A') \Pr(C|A' \cap B) = 0.25(0.8)(0.7) = 0.14$. Therefore $\Pr(B \cap C) = 0.54 + 0.14 = 0.68$.
- (e) $\Pr(A|B \cap C) = \Pr(A \cap B \cap C) / \Pr(B \cap C) = 0.54/0.68 = 0.7941$.

Question 2

Let $A = \{\text{product A profitable}\}$, $B = \{\text{product B profitable}\}$ and $C = A \cup B$.

$\Pr(A) = \Pr(B) = 0.18$. $\Pr(A \cap B) = 0.05$. So $\Pr(C) = \Pr(A \cup B) = 0.18 + 0.18 - 0.05 = 0.31$.

- (a) $\Pr(A|B) = \Pr(A \cap B) / \Pr(B) = 0.05/0.18 = 0.2777$.
- (b) $\Pr(A|C) = \Pr(A \cap C) / \Pr(C) = \Pr(A) / \Pr(C) = 0.18/0.31 = 0.5806$.

Question 3

Let $A = \{\text{TQM implemented}\}$ and $B = \{\text{sales increased}\}$.

- (a) $\Pr(A) = 0.3$. $\Pr(B) = 0.6$.
- (b) Since $\Pr(A|B) = 20/60$, therefore, $\Pr(A \cap B) = \Pr(A|B) \Pr(B) = (1/3)0.6 = 0.2$. As $\Pr(A \cap B) \neq \Pr(A) \Pr(B) = 0.18$, therefore A and B are not independent events.
- (c) Since $\Pr(A|B) = 18/60$, therefore, $\Pr(A \cap B) = \Pr(A|B) \Pr(B) = (0.3)0.6 = 0.18$. As $\Pr(A \cap B) = \Pr(A) \Pr(B)$, therefore A and B are independent events.

Question 4

Let B be the event that a component needs rework. Then

$\Pr(B) = \Pr(A_1) \Pr(B|A_1) + \Pr(A_2) \Pr(B|A_2) + \Pr(A_3) \Pr(B|A_3) = (0.5)(0.05) + (0.3)(0.08) + (0.2)(0.1) = 0.069$.

- (a) $\Pr(A_1|B) = [(0.5)(0.05)]/0.069 = 0.3623$.
- (b) $\Pr(A_2|B) = [(0.3)(0.08)]/0.069 = 0.3478$.
- (c) $\Pr(A_3|B) = [(0.2)(0.1)]/0.069 = 0.2899$.

Notice that $\Pr(A_1|B) + \Pr(A_2|B) + \Pr(A_3|B) = 1$.

Question 5

- (a) $\Pr(A_1) = \Pr(\text{draw slip 1 or 4}) = 1/2$. Similarly, $\Pr(A_2) = 1/2$ and $\Pr(A_3) = 1/2$. $\Pr(A_1 \cap A_2) = \Pr(\text{draw slip 4}) = 1/4$, Similarly, $\Pr(A_1 \cap A_3) = 1/4$ and $\Pr(A_2 \cap A_3) = 1/4$. Since $\Pr(A_1 \cap A_2) = \Pr(A_1) \Pr(A_2)$, $\Pr(A_1 \cap A_3) = \Pr(A_1) \Pr(A_3)$ and $\Pr(A_2 \cap A_3) = \Pr(A_2) \Pr(A_3)$, therefore the events A_1 , A_2 and A_3 are pairwise independent.
- (b) $\Pr(A_1 \cap A_2 \cap A_3) = \Pr(\text{draw slip 4}) = 1/4$. But $\Pr(A_1) \Pr(A_2) \Pr(A_3) = 1/8 \neq 1/4$, therefore the events A_1 , A_2 and A_3 are not mutually independent.

Question 6

- (a) Since all the four components work independently, $\Pr(\text{system works}) = \Pr(A \cap (B \cup C) \cap D) = \Pr(A) \Pr(B \cup C) \Pr(D) = (0.95)[0.7 + 0.8 - (0.7)(0.8)](0.9) = 0.8037$.

- (b) $\Pr(C \text{ does not work} \mid \text{system works}) =$
 $\Pr(\text{system works but } C \text{ does not work}) / \Pr(\text{System works}) = \Pr(A \cap B \cap C' \cap D) / \Pr(\text{system works}) = [(0.95)(0.7)(0.2)(0.9)] / 0.8037 = 0.1489.$

Question 7

Let $A_i = \{\text{ith vehicle passes the inspection}\}$. $\Pr(A_1) = \Pr(A_2) = \Pr(A_3) = 0.6$.

- (a) $\Pr(A_1 \cap A_2 \cap A_3) = \Pr(A_1) \Pr(A_2) \Pr(A_3) = (0.6)^3 = 0.216$ since A_i 's are independent.
- (b) $\Pr(\text{At least one failures}) = 1 - \Pr(\text{All pass}) = 1 - 0.216 = 0.784.$
 Or $\Pr(A_1' \cup A_2' \cup A_3') = \Pr((A_1 \cap A_2 \cap A_3)') = 1 - \Pr(A_1 \cap A_2 \cap A_3) = 1 - 0.216 = 0.784.$
- (c) $\Pr(A_1 \cap A_2' \cap A_3') + \Pr(A_1' \cap A_2 \cap A_3') + \Pr(A_1' \cap A_2' \cap A_3) = (0.6)(0.4)(0.4) + (0.4)(0.6)(0.4) + (0.4)(0.4)(0.6) = 0.288.$
- (d) $\Pr(\text{At least one pass}) = 1 - \Pr(\text{All fail}) = 1 - (0.4)^3 = 0.936.$
 $\Pr(\#pass = 3 \mid \#pass \geq 1) = \Pr(\#pass = 3 \cap \#pass \geq 1) / \Pr(\#pass \geq 1) =$
 $\Pr(\#pass = 3) / \Pr(\#pass \geq 1) = 0.216 / 0.936 = 0.2308.$

Question 8

Let $A = \{\text{Get into a house}\}$, $B = \{\text{the house is unlocked}\}$ and $C = \{\text{Agent gets the correct key}\}$

It is given that $\Pr(B) = 0.4$.

$$\Pr(C) = 1/8 + (7/8)(1/7) + (7/8)(6/7)(1/6) = 3/8.$$

$$\text{Alternatively, } \Pr(C) = ({}_1C_1)({}_7C_2)/({}_8C_3) = 3/8.$$

$$\Pr(A) = \Pr(B) \Pr(A|B) + \Pr(B') \Pr(A|B') = 0.4(1) + 0.6(3/8) = 5/8 = 0.625.$$