2. $A = \{A \text{ profitable}\}$

$B = \{B \text{ profitable}\}$

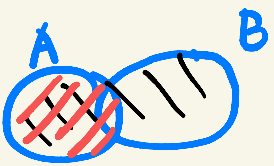
$P(A) = 0.18, \quad P(B) = 0.18$

$P(A \cap B) = 0.05$

(a)
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.05}{0.18} = 0.2778$$

(b)
$$P(A \mid A \overset{\text{or}}{\cup} B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)}$$

$A \cap (A \cup B) = A$



$$= \frac{P(A)}{P(A \cup B)}$$

$$= \frac{P(A)}{P(A) + P(B) - P(A \cap B)}$$

$$= \frac{0.18}{0.18 + 0.18 - 0.05}$$

$$= 0.5806$$

3. $A = \{ \text{TQM Implemented} \}$

$B = \{ \text{sales increased} \}$

(a) $P(A) = \frac{30}{100} = 0.3$

$P(B) = \frac{60}{100} = 0.6$

$P(A \cap B) = \frac{20}{100} = 0.2$ /

$P(A|B) = \frac{20}{60} = \frac{1}{3}$

$P(A \cap B) = P(A|B) \cdot P(B) = \frac{1}{3} \times \frac{3}{5} = \frac{1}{5}$

(b)

$P(A \cap B) = 0.2$

$P(A) \cdot P(B) = 0.3 \times 0.6 = 0.18$

$\therefore P(A \cap B) \neq P(A) \cdot P(B)$

$\therefore$ A and B are dependent

(c) $P(A \cap B) = \frac{18}{100} = 0.18$

$\therefore P(A \cap B) = P(A) \cdot P(B)$

$\therefore$ A and B are independent

5. $P(A_1) = \frac{2}{4} = \frac{1}{2}$

$P(A_2) = \frac{2}{4} = \frac{1}{2}$

$P(A_3) = \frac{2}{4} = \frac{1}{2}$

$P(A_1 \cap A_2) = \frac{1}{4} = P(A_1) \cdot P(A_2)$

$P(A_1 \cap A_3) = \frac{1}{4} = P(A_1) \cdot P(A_3)$

$P(A_2 \cap A_3) = \frac{1}{4} = P(A_2) \cdot P(A_3)$

$\therefore A_1 \perp A_2, \quad A_1 \perp A_3, \quad A_2 \perp A_3$

$\therefore A_1, A_2, A_3$ are pairwise indept

(b) $P(A_1 \cap A_2 \cap A_3) = \frac{1}{4}$

$P(A_1) \cdot P(A_2) \cdot P(A_3) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

$\therefore P(A_1 \cap A_2 \cap A_3) \neq P(A_1) \cdot P(A_2) \cdot P(A_3)$

$\therefore A_1, A_2, A_3$ are not mutually indept

6. (a) $P(A \cap (B \cup C) \cap D)$

$= P(A) \cdot \boxed{P(B \cup C)} \cdot P(D)$

$= 0.95 \times (1 - \boxed{P(B' \cap C')}) \times 0.9$

$= 0.95 \times (1 - P(B') \cdot P(C')) \times 0.9$

$= 0.95 \times (1 - 0.3 \times 0.2) \times 0.9$

$= 0.8037$

(b) $P(C' \mid A \cap (B \cup C) \cap D)$

$$= \frac{P(C' \cap A \cap (B \cup C) \cap D)}{P(A \cap (B \cup C) \cap D)}$$

$P(C' \cap A \cap (B \cup C) \cap D)$

$= P(A \cap D \cap C' \cap (B \cup C))$

$= P(A) \cdot P(D) \cdot P(C' \cap (B \cup C))$

C' is not indept $B \cup C$

$C' \not\perp B \cup C$

$\because C' \not\perp C$

If $C \perp C'$ then $P(C \cap C') = P(C) \cdot P(C')$

$= P(\phi) = 0$

$\therefore P(C) \cdot P(C') = 0$

$P(C) = 0$ or $P(C') = 0$

$P(C) = 0$ or $P(C) = 1$

$P(C) = 0.8 \quad \therefore C \not\perp C'$

☆ $C' \cap (B \cup C) = B \cap C'$

$$= \frac{P(A) \cdot P(D) \cdot P(B) \cdot P(C')}{0.8037}$$

$$= \frac{0.95 \times 0.9 \times 0.7 \times 0.2}{0.8037}$$

$$= 0.1489$$

7. A_1
A_2 = { ith vehicle passes }
A_3 $\quad i = 1, 2, 3$

(b) $P(A_1' \cup A_2' \cup A_3')$

$= 1 - P(A_1 \cap A_2 \cap A_3)$

$= 1 - 0.6^3 = 0.784$

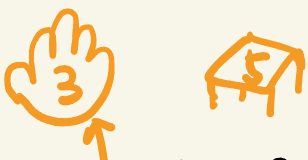
8. E = { enter the house }

U = { unlocked }

K = { get the correct key }

$P(U) = 0.4$

$P(K) = \frac{3}{8}$



$/ \; P(K) = \frac{{}_1C_1 \cdot {}_7C_2}{{}_8C_3} = \frac{1 \times \cancel{7} \times \cancel{6} \times 3 \times \cancel{2}}{\cancel{2} \times 8 \times \cancel{7} \times \cancel{6}}$

$= \frac{3}{8}$

$P(E|U) = 1$,

$$P(E|U') = \underline{P(K)} = \frac{3}{8}$$

$$P(E) = P(E \cap U) + P(E \cap U')$$

$$= P(E|U)\cdot P(U) + P(\underline{E}|U')\cdot P(U')$$

$$= 1 \times 0.4 + \frac{3}{8} \times 0.6$$

$$= 0.625$$