NATIONAL UNIVERSITY OF SINGAPORE
Department of Statistics and Applied Probability

(2020/21) Semester 1 ST2334 Probability and Statistics Tutorial 11

1. An electrical firm manufactures light bulbs that have a lifetime that is approximately normally distributed with a mean of 800 hours and a standard deviation of 40 hours. A random sample of 30 bulbs has an average life of 788 hours.
   (a) Is there any evidence that the average lifetime of the light bulbs from the manufacturer is different from 800 hours? Use $\alpha = 0.05$.
   (b) Construct a 95% confidence interval for μ. Is 800 hours a plausible value for μ?
   (c) Find the probability of committing a Type II error if the true mean lifetime is in fact 790 hours.
   (d) What is the power of the test if the true mean lifetime is in fact 790 hours?

2. The contents of a random sample of 10 containers of a particular lubricant are 10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3, and 9.8 liters. Assume that the distribution of contents is normal.
   (a) Use a 0.01 level of significant to test the hypothesis that the average content of containers of lubricant is different from 10 liters.
   (b) If the variance of the content is suspected to be different from 0.03, test the hypothesis that $\sigma^2 = 0.03$ against $\sigma^2 \neq 0.03$.
   (c) Construct a 99% confidence interval for σ^2.

3. A soft-drink dispensing machine is said to be out of control if the variance of the contents **exceeds** 1.15 deciliters. If a random sample of 25 drinks from this machine has a variance of 2.03, does this sample indicate at the 0.05 level of significance that the machine is out of control? Assume that the contents are approximately normally distributed.

4. A manufacturer claims that the average tensile strength of thread A **exceeds** the average tensile strength of thread B by **at least 12 kilograms**. To test his claim, 50 pieces of each type of thread are tested under similar conditions. Type A thread had an average tensile strength of 86.7 kilograms with known standard deviation of $\sigma_A = 6.28$ kilograms, while type B thread had an average tensile strength of 77.8 kilograms with known standard deviation of $\sigma_B = 5.61$ kilograms.
   (a) Test the manufacturer's claim at $\alpha = 0.05$.
   (b) Which type of error has possibly been committed in the conclusion in part (a)?

5. Students may choose between a 3-semester-hour course in physics without labs and a 4-semester-hour course with labs. The final written examination is the same for each section. If 12 students in the section with labs made an average examination grade of 84 with a standard deviation of 4, and 18 students in the section without labs made an average grade of 77 with a standard deviation of 6. Assume the populations to be approximately normally distributed with equal variances.
   (a) Find a 99% confidence interval for the difference between the average grades for the two courses.
   (b) Is there any evidence to conclude that the 3-semester-hour course has a **higher** average grades? Use a 0.05 level of significance.

6. A taxi company manager is trying to decide whether the use of radial tires instead of regular belted tires improves fuel economy. Twelve cars were equipped with radial tires and driven over a prescribed test course. Without changing drivers, the same cars were then equipped with regular belted tires and driven once again over the test course. The gasoline consumption, in kilometers per liter, was recorded as follows:

Car	1	2	3	4	5	6	7	8	9	10	11	12
Radial Tires	4.2	4.7	6.6	7	6.7	4.5	5.7	6	7.4	4.9	6.1	5.2
Belted Tires	4.1	4.9	6.2	6.9	6.8	4.4	5.7	5.8	6.9	4.7	6	4.9

(a) Find a 95% confidence interval for the difference of the true mean gasoline consumption between cars equipped with radial tires and cars equipped with belted tires.

(b) Can we conclude that cars equipped with radial tires give **better** fuel economy than those equipped with belted tires? Assume the populations to be normally distributed. Use a *p*-value in your conclusion.

7. A study is conducted to compare the length of time between men and women to assemble a certain product. Past experience indicates that the distribution of times for both men and women is approximately normal but the variance of the times for women is **less** than that for men. A random sample of times for 11 men and 14 women produced the following data:

Men	Women
$n_1 = 11$	$n_2 = 14$
$s_1 = 6.1$	$s_2 = 5.3$

At 0.05 level significance, test the hypothesis that $\sigma_1^2 = \sigma_2^2$ against the alternative that $\sigma_1^2 > \sigma_2^2$. State your conclusion.

8. The following data represent the running times of films produced by 2-motion-picture companies:

Company	Time (minutes)						
1	102	86	98	109	92		
2	81	165	97	134	92	87	114

(a) Test the hypothesis at the 0.05 level of significance that $\sigma_1^2 = \sigma_2^2$ against the alternative that $\sigma_1^2 \neq \sigma_2^2$, where$\sigma_1^2$ and σ_2^2 are the variances for the running times of films produced by company 1 and company 2, respectively. Report the *p*-value (using software).

(b) Find a 95% confidence interval for σ_1^2/σ_2^2.

(c) Find a 95% confidence interval for σ_1/σ_2 .`

9. You can assume this property of normal distribution: Suppose that $X_k \sim N(\mu_k, \sigma_k^2)$ for $k = 1, 2, \ldots, n$, then $W = a_1 X_1 + \cdots + a_n X_n$ is also normal distributed. [That is, linear combination of normally distributed random variables is normally distributed.]
Compute the mean and the variance of W if the random variables X_k's are assumed to be independent.

Answers to selected problems

1. (a) $z_{obs} = -1.64$. H_0 is not rejected since $|z_{obs}| < z_{0.025}(= 1.96)$ (or p-value = 0.1010 > 0.05).
   (b) (773.7, 802.3) Yes, 800 is plausible.
   (c) $\Pr(785.79 < \bar{X} < 814.31 \mid \mu = 790) = \Pr(-0.591 < Z < 3.329) = 0.7225$
   (d) 0.2775
2. (a) $t_{obs} = 0.772 < t_{0.005;9}$ $(= 3.25)$, p-value > 0.20; do not reject H_0.
   (b) $\chi^2_{obs} = 18.13 < \chi^2_{0.025;9}$ $(= 19.023)$. 0.05 < p-value < 0.10; do not reject H_0.
   (c) (0.0230, 0.3140)
3. $\chi^2_{obs} = 42.37 > \chi^2_{24;0.05}$ $(= 36.415)$. Reject H_0 at 5% significance level
   0.01 < p-value < 0.025. (Exact p-value$= 0.0117$)
4. (a) $z_{obs} = -2.60 < z_{0.05}(= 1.645)$. Do not reject H_0. p-value = 0.9953; do not reject H_0.
   (b) Type II error.
5. (a) $s_p = 5.305$. (1.537, 12.463) for $\mu_B - \mu_A$ or (–12.463, –1.537) for $\mu_A - \mu_B$.
   (b) $t_{obs} = -3.54 < t_{11;0.05} = 1.701$; Do not reject H_0. (Exact p-value$= 0.9993$)
6. (a) $\bar{x}_d = 0.1417$, $s_d = 0.1975$, (0.0162, 0.2672)
   (b) $t_{obs} = 2.485 > t_{11;0.05}(= 1.796)$ or 0.01 < p-value < 0.025 [since $\Pr(T > 2.201) = 0.025$ and $\Pr(T > 2.718) = 0.01$]. Reject H_0 at 5% significance level. (Exact p-value $= 0.01515$).
7. $F_{obs} = 1.325 < F_{10,13;0.05}(= 2.67)$. Do not reject H_0. [Note: Exact p-value = $\Pr(F > 1.325) = 0.3117$ (from statistical software)]
8. (a) $F_{obs} = 0.0863 < F_{4,6;0.975}$ $(= 0.1087)$. Reject H_0 at 5% significance level.
   (b) (0.01385, 0.79375)
   (c) (0.1177, 0.8909)
9. $E(W) = a_1\mu_1 + \cdots + a_n\,\mu_n$
   $V(W) = a_1^2\sigma_1^2 + \cdots + a_n^2\sigma_n^2$