NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF STATISTICS & APPLIED PROBABILITY

ST2334 PROBABILITY AND STATISTICS

SEMESTER I, AY 2022/2023

Tutorial 05

This set of questions will be discussed by your tutors during the tutorial in Week 8.

Please work on the questions before attending the tutorial.

1. Let X denote the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day. Let Y denote the number of times a technician is called on an emergency call. Their joint probability distribution is given below.

$f_{X,Y}(x,y)$		x		
		1	2	3
y	1	0.05	0.05	0.1
	2	0.05	0.10	0.35
	3	0	0.2	0.1

   (a) Evaluate the marginal distributions of X and Y.

   (b) Find $P(Y = 3|X = 2)$.

   (c) Find the conditional distribution of Y given $X = 2$.

   (d) Determine whether X and Y are dependent or independent.

2. From a sack of fruit containing 3 oranges, 2 apples, and 3 bananas, a random sample of 4 pieces of fruit is selected. If X is the number of oranges and Y is the number of apples in the sample, find

   (a) the joint probability distribution of X and Y;

   (b) $P(X = 1, Y = 1)$;

   (c) $P(X + Y \leq 2)$;

   (d) $f_X(x)$;

   (e) $f_{Y|X}(y|2)$ and hence $P(Y = 0|X = 2)$.

3. Consider an experiment that consists of two rolls of a balanced die. If X is the number of fours and Y is the number of fives obtained in the two rolls of the die, find

   (a) the joint probability distribution of X and Y;

   (b) $P(2X + Y < 3)$;

   (c) Determine whether X and Y are dependent or independent.

4. Each rear tire on an experimental airplane is supposed to be filled to a pressure of 40 pound per square inch (psi). Let X denote the actual air pressure (in 10 pound per square inch) for the right tire and Y denote the actual air pressure (in 10 pound per square inch) for the left tire. Suppose that X and Y are random variables with the joint density

$$f_{X,Y}(x,y) = \begin{cases} k(x^2+y^2), & 3 \leq x \leq 5; 3 \leq y \leq 5; \\ 0, & \text{elsewhere} \end{cases}$$

(a) Determine k;

(b) Compute $P(3 \le X \le 4 \text{ and } 4 \le Y < 5)$;

(c) Find $f_X(x)$ and hence $P(3.5 < X < 4)$.

5. Two random variables have the joint density

$$f(x_1,x_2) = \begin{cases} x_1x_2, & \text{for } 0 < x_1 < 2, 0 < x_2 < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the probability that both random variables will take on values less than 1.

(b) Find the marginal densities of the two random variables, and check whether the two random variables are independent.

(c) Find the expected value of the random variable whose values are given by $g(x_1,x_2) = x_1 + x_2$.

6. Consider the random variables X and Y that have a joint probability density function given by

$$f(x,y) = x^2e^{-x}, \quad \text{for} \quad x > 0, \quad -1/4 < y < 1/4.$$

(a) Compute the probability $P(X < 1, Y > 0)$.

(b) Find the marginal distributions of X and Y. Are X and Y independent?

Answers

1. (a) $f_X(x) = 0.10, 0.35, 0.55$ for $x = 1,2,3$;
$f_Y(y) = 0.2, 0.5, 0.3$ for $y = 1,2,3$;

(b) $4/7$;

(c) $f_{Y|X}(y|x=2) = 1/7, 2/7$, and $4/7$, for $y = 1,2,3$;

(d) X and Y are dependent.

2. (a) $f(x,y) = \dfrac{\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}}{\binom{8}{4}}, x = 0,1,2,3; y = 0,1,2; 1 \le x+y \le 4$;

(b) 0.2571;

(c) 0.5;

(d) $f_X(x) = \dfrac{\binom{3}{x}\binom{5}{4-x}}{\binom{8}{4}}, \quad x = 0,1,2,3.$

(e) $f_{Y|X}(y|2) = \dfrac{1}{10}\binom{2}{y}\binom{3}{2-y}, \quad y = 0,1,2;$
$P(Y = 0|X = 2) = 0.3.$

3. (a)

$f_{X,Y}(x,y)$		x = 0	1	2	$f_Y(y)$
y	0	4/9	2/9	1/36	25/36
	1	2/9	1/18	0	5/18
	2	1/36	0	0	1/36
$f_X(x)$		25/36	5/18	1/36	1

(b) $11/12$.

(c) dependent.

4. (a) 3/392;

(b) 0.25;

(c) 0.2328.

5. (a) 1/4;

(b) $f_1(x_1) = 0.5x_1; 0 \le x_1 \le 2;$
$f_2(x_2) = 2x_2; 0 \le x_2 \le 1;$

(c) 2.

6. (a) $\dfrac{1}{4}\left(2 - \dfrac{5}{e}\right)$;

(b) $f(x) = 0.5x^2e^{-x}, \quad x > 0;$
$g(y) = 2, \quad -1/4 < y < 1/4;$
X and Y are independent.