1. Consider the following statements about Peter whom you have not met before.

A: He is not married.

B: He is not married and smokes.

C: He is married.

D: He is married and does not smoke.

(1 mark)

You are to assign probabilities to these statements. Which answer below is consistent with the laws of probability?

You scored 1 / 1 mark

- $\Pr(A) = 0.45, \Pr(B) = 0.5, \Pr(C) = 0.55, \Pr(D) = 0.4.$
- $\Pr(A) = 0.45, \Pr(B) = 0.1, \Pr(C) = 0.6, \Pr(D) = 0.3.$
- $\Pr(A) = 0.45, \Pr(B) = 0.2, \Pr(C) = 0.55, \Pr(D) = 0.5.$
- $\Pr(A) = 0.45, \Pr(B) = 0.4, \Pr(C) = 0.55, \Pr(D) = 0.6.$

2. Let A and B be two events. Which of the following statements is/are true? (1 mark)

You scored 0 / 1 mark

- If $A \neq B$, then $\Pr(A) \neq \Pr(B)$.
- If A and B are independent, then we must have $\Pr(A \cup B) = 1 - (1 - \Pr(A))(1 - \Pr(B))$.
- If $\Pr(A) = 1 - \Pr(B')$, then $\Pr(A) = \Pr(B)$.
- If $(A \cap B') \cup (A' \cap B) = \emptyset$, then $A = B$.

Response Rationale
Please provide a rationale for your answer.

No rationale provided.

3. A worker needs to drive to work from his home daily. There is only one route available, on which there are two speeding cameras working independently. The speeding cameras at each of these locations operates 50% and 75% of the time respectively. Based on the worker's driving habit, he will speed 40% of the time; and whether he will speed at different time points are independent. What is the probability that the worker will not receive a speeding ticket for each day?

Note: whether the camera is working at any time is also independent with whether a driver is speeding when s/he drives through that camera.

(1 mark)

You scored 0 / 1 mark

- 0.56
- 0.48
- 0.36
- 0.72

Response Rationale
Please provide a rationale for your answer.

No rationale provided.

4. Draw 4 balls randomly without replacement from a basket containing 4 blue balls, 4 green balls, and 2 red balls. What is the probability to get 3 blue balls and 1 green ball?

(1 mark)
You scored 1 / 1 mark

- 7/105
- 8/105
- 8/35
- 9/35

Response Rationale
Please provide a rationale for your answer.

No rationale provided.

5. **Fill in the blanks** (1 mark)
You scored 0 / 1 mark

$\Pr(A') = 1/2$, $\Pr(B) = 3/8$ and $\Pr(B'|A) = 3/4$.
Find $\Pr(B \cap A)$.
Answer: 1. {0.125} (Write your answer in the form of 0.xxx, rounded off to three decimal places.)

Enter the correct answer below.

1 0.188

Response Rationale
Please provide a rationale for your answer.

No rationale provided.

6. **Fill in the blanks** (1 mark)
You scored 0 / 1 mark

A new Covid test kit detects the virus in an infected patient 90% of the time. However, it also detects the virus in an uninfected patient 5% of the time. Given that the overall Covid infection rate is 1%, what is the probability of being infected if your test kit detects the virus?
Answer: 1. [0.153, 0.155 (Write your answer in the form of 0.xxx, rounded off to three decimal places.)

Enter the correct answer below.

1 0.010

Response Rationale
Please provide a rationale for your answer.

No rationale provided.

7. **Fill in the blanks** (1 mark)

You scored 1 / 1 mark

How many ways are there to choose an arbitrary number of students (including the possibility of choosing 0 students) from 6 students?

Answer: 1. {64}

Enter the correct answer below.

1 64

Response Rationale
Please provide a rationale for your answer.

No rationale provided.

8. 15 students are present in a class. In how many ways, can they be made to stand in two circles of 8 and 7 students? (1 mark)

You scored 1 / 1 mark

- $_{15}C_7 \times 8! \times 7!$
- $_{15}C_7 \times 7! \times 6!$ (selected)
- $7! \times 6!$
- $7! \times 8!$

Response Rationale
Please provide a rationale for your answer.

No rationale provided.

9. $A \cup (B \cap C) =$ (1 mark)

You scored 1 / 1 mark

- $(A \cup B) \cap C$
- $A \cup B' \cup C'$
- $(A \cap B) \cup (A \cap C)$
- $(A \cup B) \cap (A \cup C)$ (selected)

Response Rationale
Please provide a rationale for your answer.

No rationale provided.

10. Let

$$F(x) = \begin{cases} 0 & x < -2 \\ 0.2 & -2 \leq x < 0 \\ 0.5 & 0 \leq x < 3 \\ 0.65 & 3 \leq x < 4.5 \\ 0.9 & 4.5 \leq x < 7 \\ 1 & x \geq 7 \end{cases}$$

(1 mark)

be the cdf of a random variable X. Which of the following statements is/are correct?

You scored 0 / 1 mark

- [ ] X is a discrete random variable.
- [x] $\Pr(-1 < X < 4) = F(4) - F(-1)$.
- [ ] $X = 7$ has the highest probability.
- [ ] $\Pr(X > 3) = 0.35$.

Response Rationale

Please provide a rationale for your answer.

No rationale provided.

11. **Fill in the blanks** (1 mark)

You scored 0 / 1 mark

Let

$$f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 0.25 & 3 \leq x \leq 5 \\ 0 & \text{elsewhere} \end{cases}$$

be the pdf for a random variable X.

$E(X)$ is 1. [2.33, 2.34 . (Write your answer in form x.xxx, rounded to the third decimal place.)

Enter the correct answer below.

1 1.000

12. Which of the following $f(x)$ can serve as the probability density function of a random variable X: (1 mark)

You scored 0 / 1 mark

- [ ] $f(x) = \begin{cases} \sin(x) & 0 \leq x \leq 3\pi/2 \\ 0 & \text{otherwise} \end{cases}$
- [x] $f(x) = \begin{cases} 0.5\sin(x) & 0 \leq x \leq \pi \\ 0 & \text{otherwise} \end{cases}$
- [x] $f(x) = \begin{cases} \sin(x) & 0 \leq x \leq \pi/2 \\ 0 & \text{otherwise} \end{cases}$
- [x] $f(x) = \begin{cases} 0.5\sin(x) & 0 \leq x \leq \pi/2 \\ 0 & \text{otherwise} \end{cases}$

13. **Fill in the blanks** (1 mark)

You scored 1 / 1 mark

Let

$$f(x) = \begin{cases} x & 0 \le x \le 1 \\ 0.5 & 2 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

be the pdf for a random variable X.

$\Pr(0.5 < X < 2.5)$ is 1. {0.625} . (Write your answer in form x.xxx, rounded to the third decimal place.)

Enter the correct answer below.

1 0.625

14. **Fill in the blanks** (1 mark)

You scored 0 / 1 mark

Consider rolling a dice twice. Let X be the number of times (0, 1 or 2), that the number facing up was larger than 4. The expectation of the reciprocal of $1 + X$, namely $E[(1 + X)^{-1}]$, is 1. [0.7, 0.71 . (Write your answer in form x.xxx, rounded to the third decimal place.)

Enter the correct answer below.

1 4.333

15. The continuous random variable X has the following probability density function

$$f_X(x) = \begin{cases} \frac{1}{4}(1+x), & 0 \le x \le 2; \\ 0, & \text{otherwise.} \end{cases}$$

(1 mark)

The median, m, of a continuous random variable Y satisfies $\Pr(Y < m) = 0.5$. Find the median of X. (Choose the option closest to the answer.)

You scored 1 / 1 mark

- ○ 0.8
- ○ 1.0
- ◉ 1.2
- ○ 1.4
- ○ 1.6

16. Let X be a continuous random variable that only takes on non-negative values. Suppose $E(X) = 4$ and $V(X) = 1$, which of the following statements are necessarily true?

(1 mark)

You scored 0 / 1 mark

- [x] $\Pr(2 < X < 6) \ge \frac{3}{4}$
- [ ] $\Pr(2 \le X \le 6) \ge \frac{3}{4}$
- [ ] $\Pr(X \ge 8) \le \frac{1}{16}$
- [ ] $\Pr(X \ge 10) \le \frac{1}{16}$

17. Let X have probability mass function given by the following table.

x	0	2	5	6
$f(x)$	0.3	0.5	0.1	0.1

(1 mark)

What is $E(X)$?

You scored 1 / 1 mark

- 3.25
- 2.1 (selected)
- 3.5
- 2

18. Let X have cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0; \\ \frac{x^2}{100}, & 0 \leq x \leq 10; \\ 1, & x > 10. \end{cases}$$

(1 mark)

What is $\Pr(X > 3)$?

You scored 1 / 1 mark

- 0.09
- 0.06
- 0.94
- 0.91 (selected)

19. Probability density functions can take on values larger than 1. (1 mark)

You scored 1 / 1 mark

- True (selected)
- False

20. Fill in the blanks (1 mark)

You scored 1 / 1 mark

Let X be a normal random variable, with $E(X) = 10$, $E(X^2) = 200$. Find the value $c > 0$ such that $P(-20 < X < 40) = 1 - 2P(Z > c)$, where Z is a random variable following the standard normal distribution.

Answer: 1. {3}

Enter the correct answer below.

1 3

21. Let $f_{X,Y}(x, y)$ be the joint pdf for the continuous random vector (X, Y). Let $f_X(x)$ and $f_Y(y)$ be the marginal pdf for X and Y, and let $f_{X|Y}(x|y)$ and $f_{Y|X}(y|x)$ be the conditional pdfs. Which of the following statements is wrong?

(1 mark)

You scored 0 / 1 mark

- ○ $f_X(1) = 0$ implies $f_{X,Y}(1, y) = 0$ for all $y \in \mathbb{R}$.
- ○ If $f_{Y|X}(y|x) = f_Y(y)$ for any x such that $f_X(x) > 0$, then X and Y are independent.
- ◉ If X and Y are independent, then $f_{Y|X}(y|x) = f_Y(y)$ for any x such that $f_X(x) > 0$.
- ○ If $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xyf_{X,Y}(x, y)dxdy = \int_{-\infty}^{\infty} f_X(x)dx \int_{-\infty}^{\infty} f_Y(y)dy$, then X and Y are independent.

22. Fill in the blanks (1 mark)

You scored 0 / 1 mark

In a gambling game, a man wins $10 if he gets 5 or 6 facing up in rolling a fair die; he needs to pay $5 otherwise. If the man continues to play the game until he wins **twice**, what is his expected gain?

Answer: 1. {0}

Enter the correct answer below.

1 6

23. Fill in the blanks (1 mark)

You scored 1 / 1 mark

Given that $E(X) = 2, V(X) = 4, E(Y) = 5$ and $V(Y) = 9$.
If $V(2X + 3Y) = 121$, then the correlation between X and Y is 1. [0.32, 0.35 . (Give your answer in a decimal number rounded to 3 decimal places.)

Enter the correct answer below.

1 0.333

24. Fill in the blanks (1 mark)

You scored 0 / 1 mark

Suppose a widget's working lifetime (in years) is an exponential random variable with parameter $\alpha = \frac{1}{2}$. What is the probability that the widget will keep working for another 1 year if it has been working for 3 years?

Answer: 1. [0.6, 0.61 . (Write your answer in the form of 0.xxx, rounded to 3 decimal places.)

Enter the correct answer below.

1 0.135

25. The joint probability function for X and Y is given below. Find $\Pr(X \geq Y)$.

$f_{X,Y}(x,y)$		x = 0	x = 1
y	0	0.24	0.36
	1	0.21	0.19

(1 mark)

You scored 1 / 1 mark

- 0.57
- 0.21
- 0.43
- 0.79
- None of the given options.

26. Ten students have registered for a talk. The number of students who actually turn up is most suitably modeled by a random variable that follows (1 mark)

You scored 1 / 1 mark

- a Bernoulli distribution.
- a Binomial distribution.
- a Poisson distribution.
- a Negative Binomial distribution.

27. Fill in the blanks (1 mark)

You scored 1 / 1 mark

For a Poisson random variable, if the variance of it is 9, then the mean of it is 1. {9} .

Enter the correct answer below.

1 9

28. $f_{X|Y}(x|y) =$ (1 mark)

You scored 1 / 1 mark

- $f_{Y|X}(y|x)$
- $\frac{f_X(x)}{f_Y(y)}$
- $\frac{f_{X,Y}(x,y)}{f_Y(y)}$
- $\frac{f_X(x)}{f_{X,Y}(x,y)}$

29. $Cov(X, Y) = 0$ implies X and Y are independent. (1 mark)

You scored 1 / 1 mark

- True
- False

30. The Gallup Poll has decided to increase the size of its random sample of Canadian voters from about 1500 people to about 4000 people. The effect of this increase is to:

(1 mark)
You scored 0 / 1 mark

- (selected) reduce the bias of the estimator.
- increase the standard error of the estimator.
- reduce the variability of the estimator.
- increase the width of the resultant confidence interval for the parameter.

31. Fill in the blanks (1 mark)
You scored 0 / 1 mark

The mean lifetime of 100 randomly selected pumps made by a particular factory was 200 days. Assuming it is known that the population standard deviation $\sigma = 50$, find a 95% confidence interval for the mean lifetime of pumps made by the factory.

Ans: (1. [189.7, 190.7 , 2. [209.3, 210.3) (Round to 1 decimal place.)

Enter the correct answer below.

1 0.3

2 0.5

32. Fill in the blanks (1 mark)
You scored 0 / 1 mark

We are interested in testing if a coin is fair ($p = 0.5$) against a coin more likely to come up heads ($p > 0.5$). Suppose we toss the coin 6 times and it came up heads 5 times. Compute the p-value of our tossing experiment.

Ans: 1. [0.104, 0.114 (Round to 3 decimal places.)

Enter the correct answer below.

1 0.444

33. In an air-pollution study, ozone measurements were taken in a large California city at 5.00 P.M. The eight readings (in parts per million) are:

$$7.9, 11.3, 6.9, 12.7, 13.2, 8.8, 9.3, 10.6$$

(1 mark)

Assume the population above has mean μ. Which of the following conditions is sufficient in order to construct a 95% C.I. for μ?

You scored 1 / 1 mark

- The observed data consist of a random sample and the population standard deviation is known.
- (selected) The observed data consist of a random sample and the population is normally distributed.
- The observed data consist of a random sample and μ should not be bigger than 13.2 or smaller than 6.9.
- With 95% confidence, $|x - \mu|$ should not be bigger than $s/\sqrt{8}$, where $\bar{x}$ and s are sample mean and sample standard deviation respectively.

34. A manufacturer claims that the average tar content of a certain kind of cigarette is $\mu = 14.0$. In an attempt to show that it {\bf differs} from this value, five measurements are made of the tar content (mg per cigarette):

$$14.5, \quad 14.2, \quad 14.4, \quad 14.3, \quad 14.6.$$

Use test statistic $L = \frac{\bar{X}-14.0}{S/\sqrt{5}}$, which has the computed value $l_{obs} = 5.67$; and the significant level $\alpha = 0.05$, which of the following conclusion is correct?

(1 mark)

You scored 0 / 1 mark

- Since $|l_{obs}| > t_{4,0.025}(= 2.776)$, we reject $\mu = 14.0$ claimed by the manufacturer.
- Since $l_{obs} > t_{4,0.05}(= 2.132)$, we reject $\mu = 14.0$ claimed by the manufacturer.
- Since $l_{obs} > z_{0.05}(= 1.645)$, we reject $\mu = 14.0$ claimed by the manufacturer.
- Since $|l_{obs}| > z_{0.025}(= 1.960)$, we reject $\mu = 14.0$ claimed by the manufacturer.