2.

(a)

			X			
		0	1	2	3	
Y	0	0	$\frac{3}{70}$	$\frac{9}{70}$	$\frac{3}{70}$	
	1	$\frac{2}{70}$	$\frac{18}{70}$	$\frac{18}{70}$	$\frac{2}{70}$	
	2	$\frac{3}{70}$	$\frac{9}{70}$	$\frac{3}{70}$	0	
$f_X(x)$		$\frac{5}{70}$	$\frac{30}{70}$	$\frac{30}{70}$	$\frac{5}{70}$	$\to 1$

$f(0,0) = 0$

$$f(0,1) = \frac{\binom{3}{0}\binom{2}{1}\binom{3}{3}}{\binom{8}{4}} = \frac{2}{70}$$

$$f(0,2) = \frac{\binom{3}{0}\binom{2}{2}\binom{3}{2}}{\binom{8}{4}} = \frac{3}{70}$$

$$\boxed{f(x,y) = \frac{\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}}{\binom{8}{4}}, \quad x = 0,1,2,3, \quad y = 0,1,2, \quad 1 \le x+y \le 4}$$

(b) $P(X=1, Y=1) = f(1,1) = \frac{18}{70} = \frac{9}{35}$

(c) $P(X+Y \le 2) = f(0,1) + f(0,2)$

$+ f(1,0) + f(1,1)$

$$+ f(2,0)$$

$$= \frac{2}{70} + \frac{3}{70} + \frac{18}{70} + \frac{3}{70} + \frac{9}{70}$$

$$= \frac{1}{2}$$

(d)

$$f_X(x) = \frac{\binom{3}{x}\binom{2+3}{4-x}}{\binom{8}{4}}, \quad x = 0, 1, 2, 3$$

(e)

$$f_{Y|X}(y \mid x=2) = \frac{f(2,y)}{\underline{f_X(2)}} = \frac{3}{7}$$

$$= \begin{cases} \frac{9}{70} / \frac{3}{7} = \frac{3}{10}, & y = 0 \\ \frac{18}{70} / \frac{3}{7} = \frac{6}{10}, & y = 1 \\ \frac{3}{70} / \frac{3}{7} = \frac{1}{6}, & y = 2 \end{cases}$$

$$P(Y=0 \mid X=2) = \frac{3}{10}$$

$$f_{Y|X}(y \mid x) = \frac{f(x,y)}{f(x)}$$

$$= \frac{\cancel{\binom{3}{x}}\binom{2}{y}\binom{3}{4-x-y}}{\cancel{\binom{8}{4}}} \Bigg/ \frac{\cancel{\binom{3}{x}}\binom{5}{4-x}}{\cancel{\binom{8}{4}}}$$

$$= \frac{\binom{2}{y}\binom{3}{4-x-y}}{\binom{5}{4-x}}$$

$$f(y \mid x=2) = \frac{\binom{2}{y}\binom{3}{2-y}}{\binom{5}{2}}, \quad y = 0, 1, 2$$

$$P(Y=0 \mid X=2) = \frac{\binom{2}{0}\binom{3}{2}}{\binom{5}{2}} = \frac{3}{10}$$

5. $f(x,y) = 24xy, \quad 0 \le x, y \le 1$

$x + y \le 1$

(a) $f_X(x) = \int_y f(x,y)\,dy$

$0 \le y \le 1$

$x + y \le 1$

$\Rightarrow y \le 1-x$

$\Rightarrow \boxed{0 \le y \le 1-x}$

$$= \int_0^{1-x} 24xy\,dy$$

$$= 12x(1-x)^2, \quad 0 \le x \le 1$$

$f_Y(y) = \int_x f(x,y)\,dx$

$0 \le x \le 1$

$x \le 1-y$

$\Rightarrow 0 \le x \le 1-y$

$$= \int_0^{1-y} 24xy \, dx$$

$$= 12y(1-y)^2 \, , \quad 0 \le y \le 1$$

(b) $f(x) \cdot f(y) = 12x(1-x^2) \cdot 12y(1-y)^2$

$$= 144xy(1-x)^2(1-y)^2$$

$\neq$

$$f(x,y) = 24xy$$

$\therefore$ X and Y are not indept

(c) aim: $P(Y < \frac{1}{8} \mid X > \frac{3}{4})$

$$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)}$$

$$= \frac{24xy}{12x(1-x)^2}$$

$$= \frac{2y}{(1-x)^2} \quad , \quad 0 \le y \le 1-x$$

$$f(y \mid x=\tfrac{3}{4}) = \frac{2y}{(1-\frac{3}{4})^2} = 32y,\quad 0 \le y \le \tfrac{1}{4}$$

$$P(Y < \tfrac{1}{8} \mid X = \tfrac{3}{4}) = \int_0^{\frac{1}{8}} 32y\, dy$$

c.

$$= \frac{1}{4}$$