Conditional Probability

DEFINITION 1 (CONDITIONAL PROBABILITY)
For any two events A and B with $P(A) > 0$, the conditional probability of B given that A has occurred is defined by

$$P(B|A) = \frac{P(A \cap B)}{P(A)}.$$

The Multiplication Rule

$$P(A \cap B) = P(A)P(B|A)$$

or

$$P(A \cap B) = P(B)P(A|B)$$

Independence

DEFINITION 1 (INDEPENDENCE)
Two events A and B are independent if and only if

$$P(A \cap B) = P(A)P(B).$$

Two events A and B that are not independent are said to be ***dependent****.*

CHECK FOR INDEPENDENCE
The properties of independence, unlike the mutually exclusive property, cannot be shown on a Venn diagram. This means you can't trust your intuition.

In general, the only way to check for independence for events A and B is by checking if

$$P(AB) = P(A)P(B).$$

Rule of Total Probability

THEOREM 2 (RULE OF TOTAL PROBABILITY OR BAYES FORMULA 1)
If $B_1, \ldots, B_n$ is a partition of S, then for any A,

$$P(A) = \sum_{i=1}^{n} P(B_i A) = \sum_{i=1}^{n} P(B_i) P(A|B_i)$$
$$= P(B_1)P(A|B_1) + \cdots + P(B_n)P(A|B_n).$$

Bayes' Theorem

Theorem 8 (Bayes' Theorem)
Let $B_1, \ldots, B_n$ be a partition of S. For any event A, and any $k \in 1, \ldots, n$

$$P(B_k|A) = \frac{P(B_k)P(A|B_k)}{P(B_1)P(A|B_1) + \cdots + P(B_n)P(A|B_n)} = \frac{P(B_k)P(A|B_k)}{\sum_{i=1}^{n} P(B_i)P(A|B_i)}.$$