NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF STATISTICS & APPLIED PROBABILITY

ST2334 PROBABILITY AND STATISTICS

SEMESTER I, AY 2022/2023

Tutorial 01: Solution

This set of questions will be discussed by your tutors during the tutorial in Week 3.

Please work on the questions before attending the tutorial.

1. The NUS library has five copies of a certain text on reserve. Two copies (1 and 2) are first edition, and the other three (3, 4 and 5) are second edition. A student examines these books in random order, stopping only when a second edition has been selected. One possible outcome is 5, and another is 213.

   (a) List the outcomes in the sample space S.

   (b) Let A denote the event that exactly one book must be examined. What outcomes are in A?

   (c) Let B be the event that book 5 is the one selected. What outcomes are in B?

   (d) Let C be the event that book 1 is not examined. What outcomes are in C?

   (e) Perform some event operations based on these events.

   SOLUTION

   (a) $S = \{123, 124, 125, 13, 14, 15, 213, 214, 215, 23, 24, 25, 3, 4, 5\}$;

   (b) $A = \{3, 4, 5\}$;

   (c) $B = \{5, 15, 25, 125, 215\}$;

   (d) $C = \{23, 24, 25, 3, 4, 5\}$;

   (e) $A \cap B = \{5\}$, $A \cup B = \{3, 4, 5, 15, 25, 125, 215\}$, $A \cap B \cap C = \{5\}$; A and B are not mutually exclusive.

2. What can you conclude about the events A and B if

   (a) $A \cup B = A$;

   (b) $A \cap B = A$.

   SOLUTION

   (a) It is always true that for **every** sample point x in Event B, the sample point x is in Event A or Event B, (i.e. $A \cup B$). Since $A \cup B = A$, therefore for **every** sample point x in Event B, the sample point x is in Event A. Hence, $B \subset A$.

   (b) It is always true that for **every** sample point x in Event A and Event B (i.e. $A \cap B$), the sample point x is in Event B. Since $A \cap B = A$, therefore for **every** sample point x in Event A, x is in Event B, Hence, $A \subset B$.

3. How many ways to seat 4 men and 6 women in a row if the 6 women must sit next to each other?

   SOLUTION

   Treat the 6 women as a single unit. Together with the men, there are 5 units. So there are 5! ways to arrange these 5 units.

   $$[6 \text{ women}]\ M_1\ M_2\ M_3\ M_4\ \Longleftarrow 5 \text{ units}$$

   However, there are 6! ways to arrange the women within that unit. So the total number of arrangements required is $5! \times 6! = 86400$.

4. Consider the digits 0, 2, 4, 6, 8 and 9. If each digit can be used only once,

   (a) how many three-digit numbers can be formed?

   (b) how many of these numbers in (a) are odd numbers?

   (c) how many of these odd numbers in (b) are greater than or equal to 620?

   SOLUTION

   (a) Number of choices for the hundreds, tens and ones positions are 5, 5 and 4 respectively. Hence the number of 3-digit numbers formed = $5 \times 5 \times 4 = 100$.

   (b) To ensure it is odd, we place 9 in the ones position. It follows that the number of choices for the ones, hundreds and tens positions are 1, 4 and 4 respectively. Hence, the number of odd 3-digit numbers formed = $4 \times 4 \times 1 = 16$.

   (c) Similarly, number of odd 3-digit numbers greater than 620 with hundreds position greater than $6 = 1 \times 4 \times 1 = 4$; and number of odd 3-digit numbers greter than 620 with hundreds position being 6= $1 \times 3 \times 1 = 3$. Hence the number of 3-digit numbers greater than $620 = 4 + 3 = 7$.

5. An exam paper consists of seven questions. Candidates are asked to answer five questions. Find the number of choices of the five questions if

   (a) no restriction on the choices;

   (b) the first two questions must be answered;

   (c) at least one of the first two questions must be answered and

   (d) exactly two from the first three questions must be answered.

   SOLUTION

   (a) We choose 5 out of 7. Hence, $n = 7$ and $r = 5$. Number of choices is given by $\binom{7}{5} = 7!/(5!2!) = 21$.

   (b) Number of ways to choose 2 questions from the first 2 questions $\binom{2}{2} = 1$. Number of ways to choose three questions from the remaining 5 questions is $\binom{5}{3} = 5!/(3!2!) = 10$. Hence, based on multiplication principle, the number of ways to get the 5 questions for which 2 from the first 2 questions and 3 from the remaining 5 questions is $1 \times 10 = 10$.

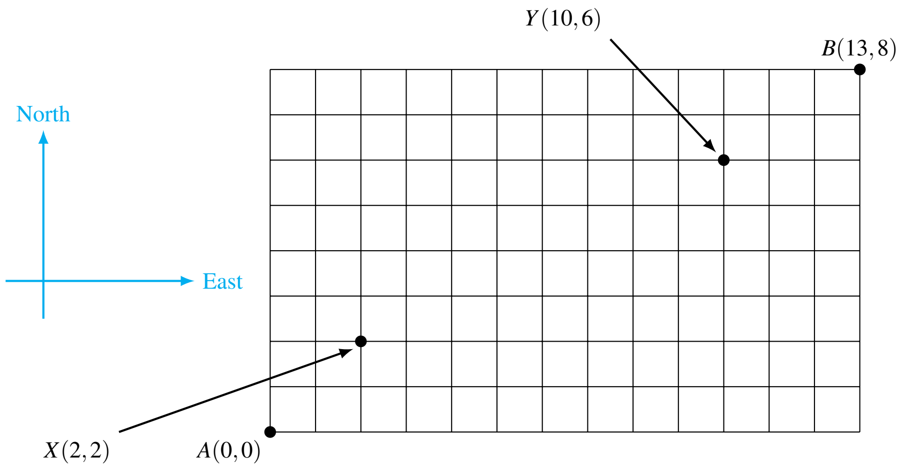
   (c) Similarly, number of choices for selecting exactly 1 question from the first 2 questions and 4 from the remaining 5 questions is $\binom{2}{1} \cdot \binom{5}{4} = 2 \times 5 = 10$.

       Based on Part (b), the number of choices for selecting 2 questions from the first 2 questions and 3 from the remaining 5 questions is 10.

       Based on additional principle, the number of choices if at least one of the first two questions must be answered is $10 + 10 = 20$.

(d) The number of choices for selecting exactly 2 questions from the first 3 questions and 3 from the remaining 4 questions is $\binom{3}{2} \cdot \binom{4}{3} = 12$.

6. Red Riding Hood lives at point $A : (0,0)$ wants to visit her grandmother at point $B : (13,8)$, and Big Bad Wolf lives at $Y : (10,6)$. At each step, she can only go East (Right) or North (Up) along the grid as shown below.



(a) How many ways can she go to visit her grandmother regardless of whether she will pass by Big Bad Wolf?

(b) How many ways can she go to visit her grandmother avoiding the Big Bad Wolf?

(c) Red Riding Hood wants to buy a gift for her grandmother at X (2, 2). How many ways can she go to visit her grandmother stopping by X but avoiding Y?

SOLUTION

(a) Each path from A to B is composed of 21 steps, with 8 steps to the north (N) and 13 steps to the east (E). For example: "ENENNNNEENEEEENEEEENNEE" is one such a path.
Therefore, "the number of ways from A to B" is equivalent to "the number of ways we can choose 8 north moving steps out of 21 steps". That is

$$\binom{21}{8} = \frac{21!}{13!8!} = 203490.$$

(b) Similar in strategy to (a), number of ways from A to Y is $\binom{16}{6} = 16!/(6!10!) = 8008$, i.e., number of ways we can choose 6 north moving steps out of 16.
Number of ways from Y to B is $\binom{5}{2} = 5!/(2!3!) = 10$, i.e., number of ways we can choose 2 north moving steps out of 5.
So, the number of ways from A to B by stopping by Y is $8008(10) = 80080$.
Consequently, the number of ways from A to B without stopping by Y is $203490 - 80080 = 123410$.

(c) Number of ways from A to B stopping by X but not Y:

$$\binom{4}{2} \times \left[\binom{17}{6} - \binom{12}{4} \times \binom{5}{2}\right] = \frac{4!}{2!2!} \times \left(\frac{17}{6!11!} - \frac{12!}{4!8!} \cdot \frac{5!}{2!3!}\right) = 44556.$$