ODL

A Python framework for rapid prototyping in inverse problems

Jevgenija Rudzusika[1] Jonas Adler[2] Holger Kohr[3] Ozan Öktem[1]

[1] Department of Mathematics
KTH - Royal Institute of Technology, Stockholm

[2] Google DeepMind, London

[3] Thermo Fisher Scientific, Eindhoven

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Conclusion: Need a *common software framework* to exchange implementations of concepts and methods.

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Initial situation: No existing framework fit our purpose.

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- **Tomography module**
  Acquisition *geometries* and *forward operators* for tomographic applications.

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Main components:

- Library of atomic mathematical components
  - Deformation operators
  - Function transforms: wavelet, Fourier, shearlet, …
  - Differential operators: partial derivative, gradient, Laplacian, …
  - Discretization-related: (re-)sampling, interpolation, domain extension, …

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- Utility functions
  - Visualization: Slice viewer, real time plotting, …
  - Phantoms: Shepp-Logan, FORBILD, Defrise, …
  - Data I/O: MRC2014, Mayo Clinic, …

Operator Discretization Library

Main components:

- User-contributed modules
  "Fast track" for experimental or slightly exotic code
  – Figures of Merit (FOMs) for image quality assessment
  – Handlers for specific data formats or geometries
  – Functionality to download and import public datasets
  – Wrappers for Deep Learning frameworks: Tensorflow, Theano, Pytorch, ...

Design principle: modularity

Consider a TV minimization problem

$$\min_{f \in X} \left[\|\mathcal{T}(f) - g\|_Y^2 + \lambda \, \mathrm{TV}(f) \right]$$

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$\rightsquigarrow$ (almost) freely exchangeable “modules” in the mathematical formulation

$\rightsquigarrow$ ODL maps them to software objects as closely as possible

Design principle: abstraction

Landweber's method: Determine f from given data $g = \mathcal{T}(f)$ and initial guess f_0 by

$$f_{k+1} = f_k + \omega[\partial\mathcal{T}(f_k)]^*\big(g - \mathcal{T}(f_k)\big), \quad k = 0, 1, \ldots, K-1$$

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- Uses abstract properties of operators in the iteration:
  $\rightsquigarrow$ `T(f)` $\longleftrightarrow \mathcal{T}(f)$ (operator evaluation)
  $\rightsquigarrow$ `T.derivative(f)` $\longleftrightarrow \partial\mathcal{T}(f)$ (derivative *operator* at f)
  $\rightsquigarrow$ `T.derivative(f).adjoint` $\longleftrightarrow [\partial\mathcal{T}(f)]^*$ (adjoint of the derivative at f)

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- Lots of tools to build complex operators from simple ones: operator arithmetic `T + S`, composition `T * S`, product space operators etc.
- There are *many* readily implement operators in ODL, all implementing the above interface

Design principle: compartmentalization

- Separates the “what” (abstract interface) of an object class from the “how” (concrete implementation)
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- Allows building generic APIs with the possibility for a new implementation in the future (*extensibility*)
  Example: `L1Norm` as a concrete realization of the abstract `Functional`
- Documentation is bundled with the object and immediately visible to the user

Example: Tomography

Inverse Problem: Determine attenuation coefficient $\mu\colon \Omega \to \mathbb{R}$ from its ray transform $\mathcal{P}\colon L^2(\Omega) \to Y$ defined as

$$\mathcal{P}(\mu)(\ell) := \int_\ell \mu(x)\mathrm{d}x$$

for all lines ℓ.

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Regularization: Conjugate gradient (CGLS) with early termination

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Implementation steps:

- Set up uniformly *discretized* image space $L^2(\Omega)$ with a rectangular domain Ω and $n_x \times n_y$ pixels

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- Solve inverse problem using CGLS
- Display the results

Example: Tomography

```
# Create reconstruction space and ray transform
space = odl.uniform_discr([-20, -20], [20, 20], shape=(256, 256))
geometry = odl.tomo.parallel_beam_geometry(space, num_angles=1000)
ray_transform = odl.tomo.RayTransform(space, geometry)

# Create artificial data with around 5 % noise (data max = 10)
phantom = odl.phantom.shepp_logan(space, modified=True)
g = ray_transform(phantom)
g_noisy = g + 0.5 * odl.phantom.white_noise(ray_transform.range)

# Solve inverse problem
x = space.zero()
odl.solvers.conjugate_gradient_normal(ray_transform, x, g_noisy, niter=20)

# Display results
phantom.show('Phantom')
g_noisy.show('Noisy data')
x.show('CGLS after 20 iterations')
```

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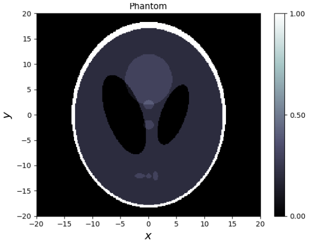

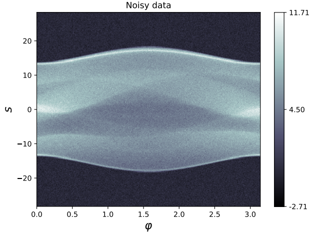

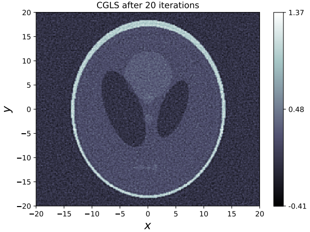

Conclusions and Outlook

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- Anyone is welcome to use and/or contribute!

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