

Binomial Expansion

Edward Jex

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Binomial Expansion is a way of easily expanding brackets with two terms to a power. The expansion is given by the general formula:

$$(x + y)^n = \sum_{r=0}^n {}^nC_r \cdot x^{n-r} \cdot y^r$$

where nC_r is equivalent to $\binom{n}{r}$ and defined as:

$${}^nC_r = \frac{n!}{(n-r)! \cdot r!}$$

Note that since pascals triangle is symmetrical it doesn't matter which term way around the powers are.

Example 1:

$$\begin{aligned}(2 + x)^4 &= ({}^4C_0 \cdot 2^4 \cdot x^0) + ({}^4C_1 \cdot 2^3 \cdot x^1) + ({}^4C_2 \cdot 2^2 \cdot x^2) + ({}^4C_3 \cdot 2^1 \cdot x^3) + ({}^4C_4 \cdot 2^0 \cdot x^4) \\ &= (1 \cdot 16 \cdot 1) + (4 \cdot 8 \cdot x) + (6 \cdot 4 \cdot x^2) + (4 \cdot 2 \cdot x^3) + (1 \cdot 1 \cdot x^4) \\ &= x^4 + 8x^3 + 24x^2 + 32x + 16\end{aligned}$$

Example 2:

Find the x^3 coefficient in $(3 + 2x)^5$:

let $n = 5$, $r = 3$

$$\begin{aligned}3^{rd} &= {}^5C_3 \cdot 3^{5-3} \cdot (2x)^3 \\ &= 10 \cdot 3^2 \cdot 8x^3 \\ &= 720x^3\end{aligned}$$

$$\therefore \text{coefficient} = 720$$

Example 3:

Find the constant term in $(x^2 + \frac{1}{x})^9$:

For the two terms to cancel out, the power of $\frac{1}{x}$ must be twice the power of x^2 so that when they are multiplied together they produce $x^0 = 1$, meaning the term is constant or independent \implies We want the term where x^2 is to the power of 3 and $\frac{1}{x}$ is to the power of 6 \implies constant term is:

$$\begin{aligned}t &= {}^9C_3 \cdot (x^2)^3 \cdot \frac{1}{x}^6 \\&= 84 \cdot x^6 \cdot x^{-6} \\&= 84\end{aligned}$$