Trigonometry

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The Radian

π radians = 180 degrees

Arc length

$$\begin{aligned} l &= \frac{\theta 2\pi r}{2\pi} \\ &= r\theta \end{aligned}$$

Sector area

$$\begin{aligned} A &= \frac{\theta}{2\pi}\pi r^2 \\ &= \frac{r^2\theta}{2} \end{aligned}$$

Trigonometry

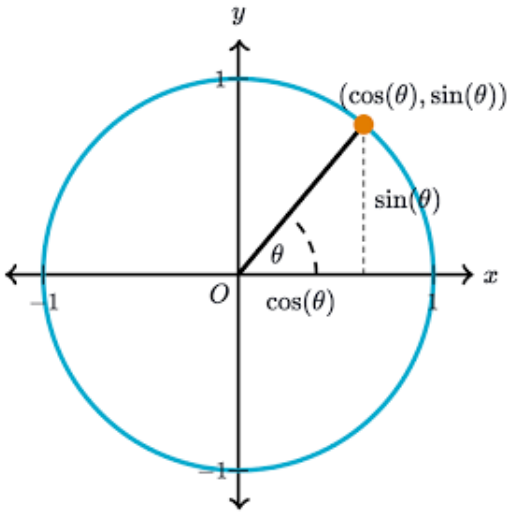
Values of sin, cos and tan can be worked out by using triangles.

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	NaN

The unit circle

Identities

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$
$$\sin^2\theta + \cos^2\theta \equiv 1$$



sin and cos graphs have a period of 2π, tan has a period of π.
sin and tan have a rotational symmetry about the origin.
cos has a line of symmetry on the y axis

$$\cos -\theta = \cos\theta$$
$$\sin -\theta = -\sin\theta$$
$$\tan -\theta = -\tan\theta$$

Solving Equations

Be careful not to divide by an expression that may be 0 as you may lose solutions. Also note that there may be many solutions in a given range. Drawing a CAST diagram or graph sketch may be useful.

Example 1

$$\begin{aligned}
\text{Solve } \sin\theta - 2\cos\theta = 0 \qquad & \text{for } 0 \leqslant \theta < 2\pi \\
\sin\theta &= 2\cos\theta \\
\frac{\sin\theta}{\cos\theta} &= 2 \\
\tan\theta &= 2 \\
\theta &= \arctan 2 \\
\theta &= 1.107, 4.249
\end{aligned}$$

Note, two solutions.

Example 2

$$\begin{aligned}
\text{Solve } 2\cos\theta\sin\theta = \cos\theta \qquad & \text{for } 0 \leqslant \theta < 2\pi \\
2\cos\theta\sin\theta - \cos\theta &= 0 \\
\cos\theta(2\sin\theta - 1) &= 0 \\
\cos\theta = 0 \qquad & \sin\theta = \frac{1}{2} \\
\theta &= \frac{\pi}{6}, \frac{\pi}{4}, \frac{5\pi}{6}, \frac{3\pi}{2}
\end{aligned}$$

Example 3

$$\begin{aligned}
\text{Solve } \sin^2\theta + \sin\theta = \cos^2\theta \qquad & \text{for } 0 \leqslant \theta < 2\pi \\
\sin^2\theta + \sin\theta &= 1 - \sin^2\theta \\
2\sin^2\theta + \sin\theta &= 1 \\
2\sin^2\theta + sin\theta - 1 &= 0 \\
(\sin\theta + 1)(2\sin\theta - 1) &= 0 \\
\sin\theta = -1 \qquad & \sin\theta = \frac{1}{2} \\
\theta &= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}
\end{aligned}$$

Sine and Cosine rules

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$

Area of a triangle $= \frac{1}{2}ab\sin C$