Polynomials

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February 21, 2020

The order / degree of a polynomial is the highest power of the variable it contains. For example, a polynomial is order 3.

Division

Long division is the best method when it is not known if there is a remainder or not. Otherwise it can be done by inspection.

Example 1

Divide $2x^3 - 3x^2 + x - 6$ by $x - 2$

$$
\begin{array}{r}
2x^2 + x + 3 \\
x - 2 \overline{)\,2x^3 - 3x^2 + x - 6} \\
-(2x^3 - 4x^2) \\
\hline
x^2 \\
-(x^2 - 2x) \\
\hline
3x - 6 \\
-(3x - 6) \\
\hline
0
\end{array}
$$

$\Rightarrow 2x^3 - 3x^2 + x - 6 = (2x^2 + x + 3)(x - 2)$

The Factor Theorem

We can use the factor theorem to help us to solve algebraic equations of order greater than 2.

The factor theorem is as follows:
If $(x-\alpha)$ is a factor of $f(x)$ then $f(\alpha)=0$ and α is the root of the equation of $f(x)=0$.

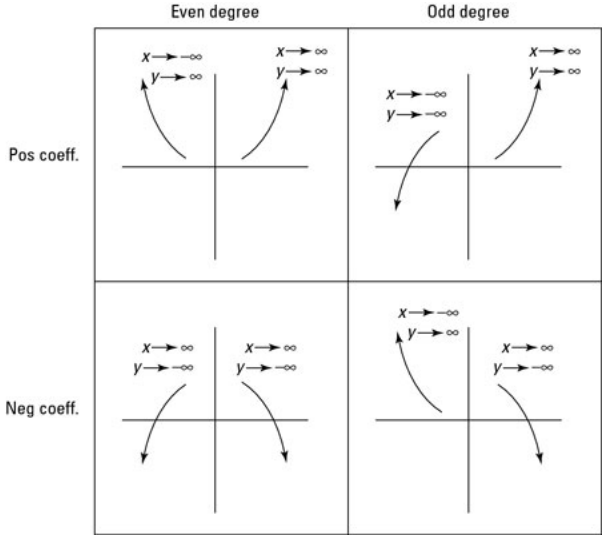
Example 2

Show that $(x-1)$ is a linear factor of $2x^3-5x^2-6x+9$

$$\begin{aligned} f(1)=0 &\Rightarrow (x-1)\text{is a factor by the factor theorum} \\ 2x^3-5x^2-6x+9 &= (x-1)(2x^2-3x-9) \\ &= (x-1)(2x+3)(x-3) \\ &\Rightarrow x=1,-\frac{3}{2},3 \end{aligned}$$

Sketching Polynomials

To sketch polynomials, we must find where it crosses the x-axis and y-axis. We also need to know what order the polynomial is so that we know the general shape.



Turning Points

A point where the gradient is 0. If a polynomial is of order n, it can at most n-1 turning points.