Vectors

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Idea

- (physics) Linear displacement of objects.
- (maths) Do geometric algebra.
- Two vectors are the same if they have the same direction and magnitude.
- Vectors represent directed line segments.

Representation

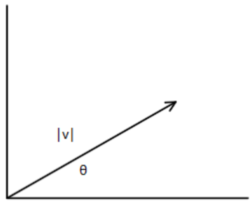
Component representation:

$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

Basis representation:

$\vec{v} = v_1\vec{e_1} + v_2\vec{e_2} + \cdots + v_n\vec{e_n}$
In text book $\vec{e_1}, \vec{e_2}, \vec{e_3}$ etc. may be written as $\vec{i}, \vec{j}, \vec{k}$
$\vec{e_n}$ represents a unit vector along one of the axes. E.g. in 2 dimensions, $\vec{e_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Polar representation:



Where $\vec{v} = |\vec{v}| \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix}$

Operations

Addition

Vector addition is commutative (order doesn't matter) and associative (bracketing doesn't matter).

Example 1:

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} p_1 + q_1 \\ p_2 + q_2 \end{bmatrix}$$

$$\begin{aligned} \vec{u} + \vec{w} + \vec{v} &= \vec{u} + (\vec{w} + \vec{v}) \\ &= \vec{u} + (\vec{v} + \vec{w}) \end{aligned}$$

Scalar Multiplication

$\lambda\vec{v} \qquad \lambda \in \mathbb{R}$

λ Scales the vector but keeps the direction if positive, reverses if negative.

Example 2:

$$\lambda \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} \lambda v_1 \\ \lambda v_2 \\ \vdots \\ \lambda v_n \end{bmatrix}$$

$$|\lambda \vec{v}| = \lambda |\vec{v}|$$

$$|\vec{v}| = \sqrt{v_1^2 + v_2^2 \cdots + v_n^2}$$

What happens when $\lambda = 0$? We need to define a zero vector:

$$\vec{o} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Zero vector is the additive identity

Vectors and coordinates

Vector space is any type that the same algebraic rule as a vector (commutativity, associativity and distributivity).

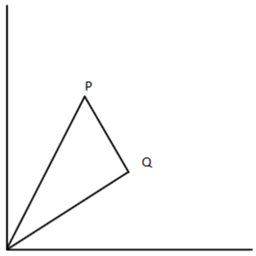
How can we link vectors back to coordinates?

A vector can represent the displacement from the origin of a point. $\vec{v} = \overrightarrow{OP}$ Represents the displacement of point P relative to the origin as we always need a frame of reference. This is called a position vector.

Applications of vectors

We can use vectors to represent shapes with position vectors to vertices. From there you can then do calculations with the vectors.

Example 3:



$$\overrightarrow{PQ} = \overrightarrow{OP} - \overrightarrow{OP}$$

You can also use vectors to represent the equation of a line:

$$\begin{aligned} v &\in \mathbb{R}^n \\ \vec{x} : \mathbb{R} &\to \mathbb{R}^n \\ t &\mapsto \vec{v}t + \vec{x_0} \end{aligned}$$