Function Notation

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What is a function?

$$\begin{aligned} f : A &\to B \\ x &\mapsto f(x) \end{aligned}$$

Where A is the domain (set of the possible inputs to the function), B is the range or co-domain (set of possible outputs from the function) and $x \mapsto f(x)$ is the mapping rule or operation.
The domain can be smaller than it needs to be and likewise the co-domain can be larger than it needs to be.

Example 1:

$$\begin{aligned} f : \mathbb{R} &\to \mathbb{R} \\ x &\mapsto x^2 \end{aligned}$$

$$\begin{aligned} g : \mathbb{Z} &\to \mathbb{Z}^+ \\ x &\mapsto x^2 \end{aligned}$$

$$\begin{aligned} h : \mathbb{R}\backslash\{0\} &\to \mathbb{R} \\ x &\mapsto \frac{1}{x} \end{aligned}$$

$$k : D \to \mathbb{R}$$

D doesn't include the number 0 in the set of reals.

$$x \mapsto 4x^2 + 12x + 73$$

$$D = \{x \in \mathbb{R} \mid \mod x, 2 = 0\}$$

D is the set of even real numbers.

$$\begin{aligned} l : [0, \infty) &\to \mathbb{R} \\ x &\mapsto \sqrt{x} \end{aligned}$$

$[0, \infty)$ represents numbers larger than or equal 0 to smaller than infinity.

Intervals

$$\begin{aligned} [a, b] &= \{x \in \mathbb{R} | a \leqslant x \leqslant b\} \\ (a, b) &= \{x \in \mathbb{R} | a < x < b\} \\ [a, b) &= \{x \in \mathbb{R} | a \leqslant x < b\} \\ (a, b] &= \{x \in \mathbb{R} | a < x \leqslant b\} \end{aligned}$$

Example 2:

$$\begin{aligned} \text{Sine} : &\mathbb{R} \to [-1, 1] \\ &x \mapsto \sin x \\ \text{Cosine} : &\mathbb{R} \to [-1, 1] \\ &x \mapsto \cos x \\ \text{Tangent} : &\{x \in \mathbb{R} | \mod x + \frac{\pi}{2}, \pi \neq 0 \to [-1, 1] \\ &x \mapsto \cos x \end{aligned}$$