Solving Equations

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Equations with Indices

If we can write an equation in terms of the same bases, we can equate the powers to help us solve them.

Example 1

$$\begin{cases} 2^{x-y} = 64^2 \\ 3^{x+y} = 1 \end{cases}$$

Find x, y

$$\begin{cases} 2^{x-y} = 2^1 2 \Rightarrow x - y = 12 \\ 3^{x+y} = 3^0 \Rightarrow x + y = 0 \end{cases}$$

$$2x = 12$$

$$\begin{cases} x = 6 \\ y = -6 \end{cases}$$

Quadratics

Factorising

$(x - \alpha)$ is a factor $\iff x = \alpha$
Given an equation of the form $x^2 + bx + c$, the roots should add to b and multiply to c.

Difference of two squares $x^2 - y^2 = (x + y)(x - y)$

Completing the square Given an equation x^2+bx+c, complete square form is $(x+\frac{b}{2})-\frac{b^2}{4}+c$.
This can easily be rearranged to get x as the subject.
Given the equation ax^2+bx+c, the completed square form can be rearranged to produce the quadratic formula.

0.0.1 The Quadratic Formula

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ for an equation $ax^2+bx+c=0$.

The Discriminant

Looking at the quadratic formula, you can see that there would be no real solutions if $b^2-4ac<0$.
$>0 \iff$ 2 real roots
$=0 \iff$ 1 real root (repeated)

Hidden Quadratics

Polynomials of higher power can be solved like quadratics if they fit the form $ax^{2d}+bx^d+c$.

Example 2

For which values of k, does the following equation have repeated roots?

$$\begin{aligned} x^2+kx+2k&=0\\ k^2-8k&=0\\ k(k-8)&=0\\ k=0 \qquad & \qquad k=-8 \end{aligned}$$

Example 3

$$\begin{aligned} x^4-3x^2-4&=0\\ (x^2-4)(x^2+1)&=0\\ x&=\pm 2 \end{aligned}$$