Data Distribution

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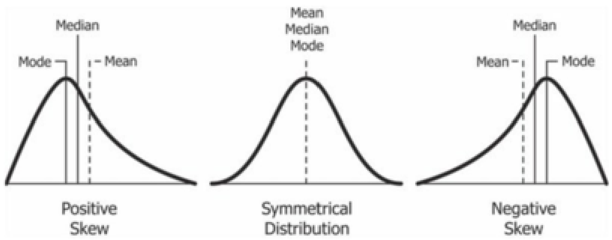
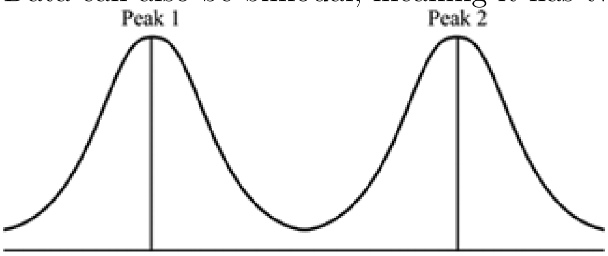
Data can be distributed due to its appearance to help us interpret the spread of data.



Data can also be bimodal, meaning it has two peaks



Measures of Central Tendency

There are four measures of central tendency that we should be aware of:

- Mode - The mode is the value that occurs most frequently. The distribution is uni-modal if there is only one mode. If two non-adjacent values occur mode frequently than the rest, the distribution is said to be bi-modal, even if the frequencies aren't the same.
- Medial - It is the middle value. It is a good measure as it isn't skewed as much by outliers but most representative for symmetrical data.

- Mean - Found by adding all the values together and dividing by the number of values. We use the symbol $\bar{x}$ to denote the mean. It is the best measure for skewed data but also representative for symmetrical data. It is affected by outliers.

- Midrange - This is the average of the highest and lowest values. It is easy to calculate but only useful when the data is symmetrical and contains no outliers.

Measures of Spread

There are three measures of spread we need to know about which help us talk about how varied the data is.

- Range - This is the highest subtract the lowest. Again this is not useful if we have any outliers.

- Interquartile Range - As seen before. We can also find the semi-interquartile range which is half the interquartile range.

- Standard Deviation - This we will learn about separately but is the most widely used measure of spread.

Standard Deviation

Why is it useful?

- Approximately 68% of values lie within 1 standard deviation of the mean.

- 95% lie within 2

If you have a particular values which is more than two standard deviations from the mean it should be investigated as a possible outlier.

$$\begin{aligned}\text{Sum of Squares } &= S_{xx} \\ &= \sum x^2 - n\bar{x}^2 \\ &= \sum x^2 - \frac{(\sum x)^2}{n}\end{aligned}$$

The variance is found by $var = s^2 = \frac{s_{xx}}{n-1}$
The standard deviation is found by the square root of the variance.
$s = \sqrt{\frac{s_{xx}}{n-1}} = \sqrt{var}$

Example 1

Find the standard deviation of 0, 1, 0, 3, 0, 2

$$\begin{aligned} \bar{x} &= \frac{1+2+3}{6} = 1 \\ s_{xx} &= (0^2+1^2+0^2+3^2+0^2+2^2) - 6 \times 1^2 && = 14 - 6 = 8 \\ s &= \sqrt{\frac{s_{xx}}{n-1}} \\ &= \sqrt{\frac{8}{5}} = 1.26 \end{aligned}$$

Frequency tables

If you have a frequency table, we can still work out the standard deviation.
The only thing that changes is our formula for sum of squares:
$s_{xx} = \sum x^2 f - n\bar{x}^2$