problem-07: The following are the weights (in gram) of a randomly selected sample of 11-apples in a shop

70, 85, 92, 90, 95, 79, 80, 85, 90, 85, 95

The weight of apples follows normal distribution with mean μ and variance σ^2. can we conclude that the population variance of apples of the shop is more than 50 gm^2?

solution: The null and alternative hypothesis for this test are

$$H_0: \sigma^2 = 50 \text{ against } H_A: \sigma^2 > 50$$

Under null hypothesis, the test statistic is

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$ which is distributed as χ^2

with $n-1$ df

It is a right tailed test, so, the critical region is given by $\chi^2 \geq \chi^2_{\alpha, n-1}$

Here $\bar{x} = \frac{\sum x_i}{n} = \frac{964}{11} = 86$ and

$$s^2 = \sum \frac{(x_i - \bar{x})^2}{n-1} = \frac{1}{n-1}\left[\sum x_i^2 - \frac{(\sum x_i)^2}{n}\right]$$

$$\frac{1}{10}\left[81930 - \frac{(964)^2}{11}\right] = 57.4$$

so, the computed value of χ^2 ~~is~~ $\frac{(n-1)s^2}{\sigma_0^2}$

$$= \frac{10 \times 57.4}{50} = 11.48$$

Here at 5% level of significance, the critical value is given by $\chi^2_{0.05,10} = 18.307$ which is more than the computed value, so we fail to reject Ho, that means, the variance of the weights of the apples is not more than 50 gm^2.

Source code in R

Ho = NOT same

```
M<- matrix(c(75,105,25,95), ncol=2, byrow=
                                      TRUE)

df = 1
M_lower <- qchisq(0.025, df)
M_upper <- qchisq(0.975, df)
chisq.test(M)
```

Problem-08: A tobacco company claims that there is no relationship between smoking and lung ailments. To investigate the claims a random sample of 300 males in the age group 40-50 are given medical test. The observed sample results are shown below:

Table-01: Number of males according to smoking and lung ailment.

	Found lung ailment	No lung ailment	Total
Smokers	75	105	180
Non-smokers	25	95	120
Total	100	200	300

On the basis of the information, can it be concluded that smoking and lung ailments are independent?

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Solution:

Let us consider the hypothesis that smoking and lung ailments are independent, that means

H_0: Smoking and ailments are indpendent or there is no effect of smoking on lung ailments

Against, H_1: They are not independent, or smoking causes lung ailments.

The test statistic for testing the hypothesis is

$$\chi^2 = \sum \frac{(O_{ij} - E_{ij})^2}{E} \sim \chi^2_{(r-1)(c-1)}$$

It is 2x2 table, so the degree of freedom is 1, we have to calculate expected frequency for a cell independently, and others are to be obtained by adjustment so that the total marginal frequencies remain the same.

Again let $\alpha = 0.05$, then the critical value of χ^2 with 1 df is 3.84 (which is also sometimes written as $\chi^2_{0.05;1} = 3.84$).

Expected frequency for the cell correspon-
ding to first row and first column (E_{11})
is computed as $E_{11} = \frac{R_1 \times C_1}{N} = \frac{180 \times 100}{300}$
$= 60$

So the expected frequencies for the remaining cells are

$E_{12} = R_1 - E_{11} = 180 - 60 = 120$

$E_{22} = C_2 - E_{22} = 200 - 120 = 80,$

$E_{22} = R_2 - E_{21} = 120 - 40 = 80$

$E_{21} = C_1 - E_{11} = 100 - 60 = 40,$

E_{22} can also be computed using R_2 as
Arranging the observed and expected frequencies in the following table, we can easily compute the necessary columns for χ^2

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Table-02: computation of x^2

O	E	$(O-E)^2$	$(O-E)^2/E$
75	60	225	3.75
105	120	225	1.875
25	40	225	5.625
95	80	225	2.8125
		Total	14.0625

Here the observed value of chi-squares is much more than the critical value, therefore, the null hypothesis is rejected at 5% level of significance. That means, it is evident that smoking has significant effect on lung ailments.