Signal Processing Lab 1

1.1 Finding Fourier series coefficients

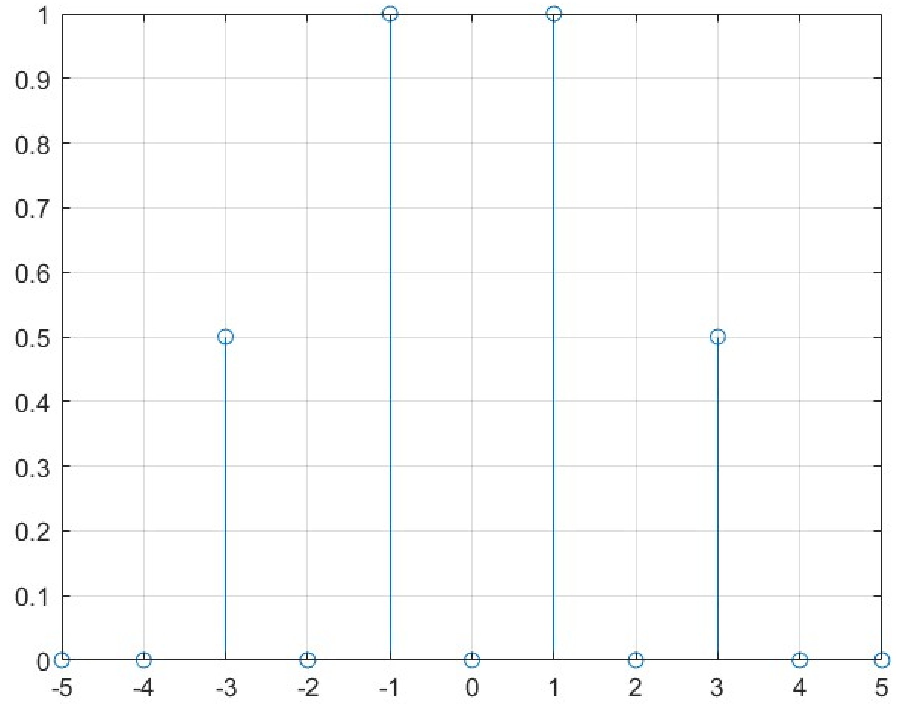
- The Fourier Series coefficients can be determined by the following formula

$$a_k = \frac{1}{T}\int_{\langle T \rangle} x(t) e^{-jk\omega_0 t}\, dt, \qquad k \in \mathbb{Z}$$

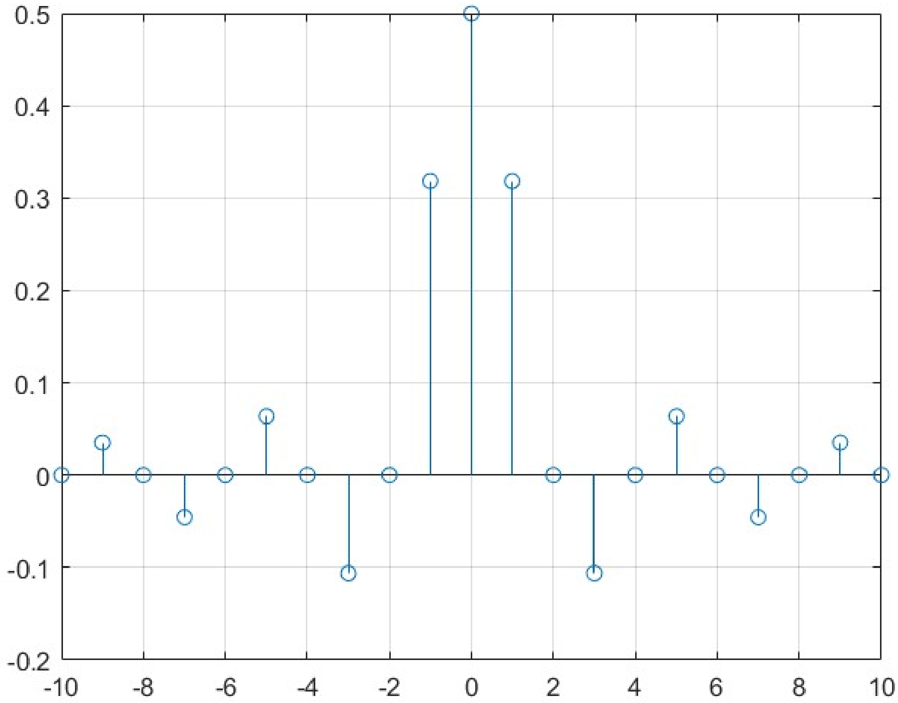
The MATLAB function to find Fourier coefficients is as follows

```
function F = fourierCoeff(t,xt,T,t1,t2,N)
% function to find FS coefficients
% initialize
F = zeros(2*N+1,1);

% for-loop to find coefficents
for nn = 1:2*N+1
  j = 1j;
  expr = xt*exp(-j*(nn - (N + 1))*(2*pi/T)*t);
  F(nn) = (1/T)*int(expr, t, [t1 t2]);
end

end
```

a. The FS coefficients for the function $x(t) = 2\cos(2\pi t) + \cos(6\pi t)$ for T = 1 and N = 5 is as follows



b. And for the periodic square wave with one period, N = 10 and T = 1 is



1.2 FS reconstruction and finite FS approximation error

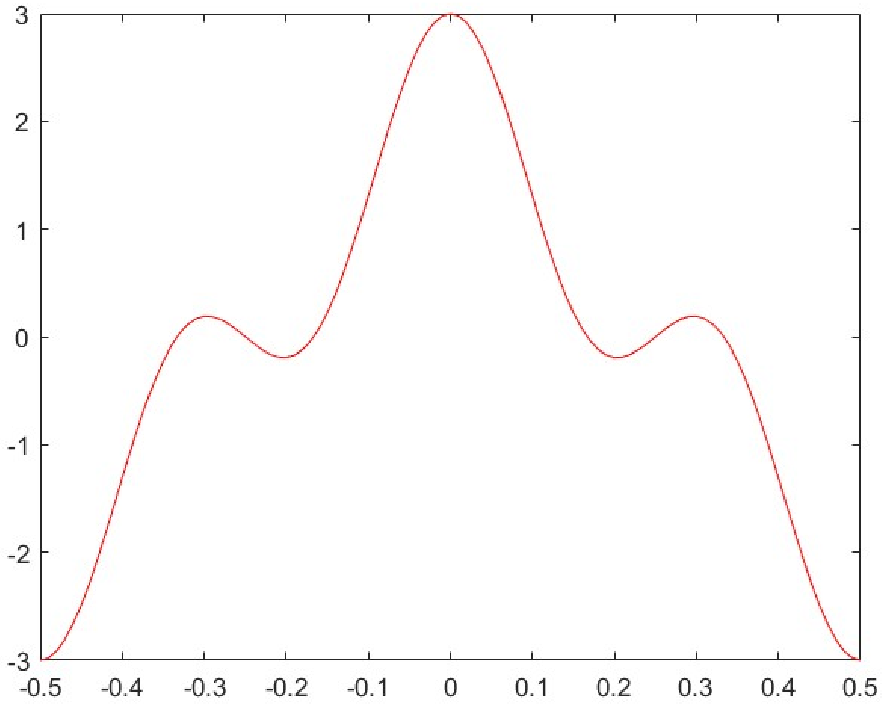
- For a signal the partial Fourier sum is given by

$$\hat{x}(t) = \sum_{k=-N}^{N} a_k \, e^{jk\omega_0 t}$$

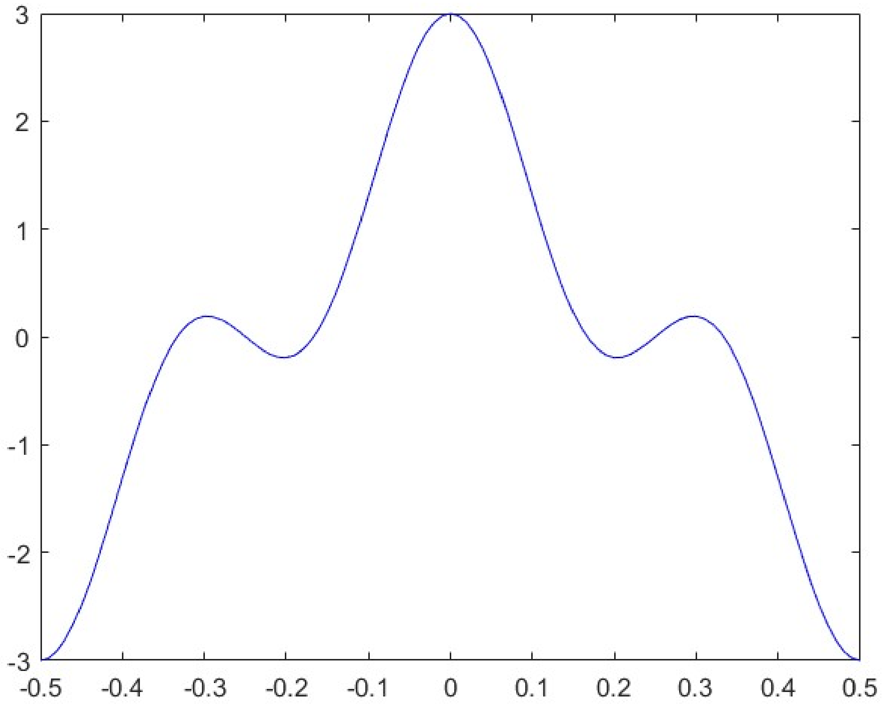
The MATLAB function to find Fourier sum is as follows

```
function y = partialfouriersum (A, T, time_grid)
j=1j;
% Compute N based on the length of A
y = zeros(1,length(time_grid));
si = length(A);
N = (si - 1)/2;

for k = -N:N
    y = y + A(N + k + 1)*exp(1j*k*(2*pi/T)*time_grid);
end

end
```

a. After writing a script to reconstruct the cosine wave of the previous part the output obtained is as follows



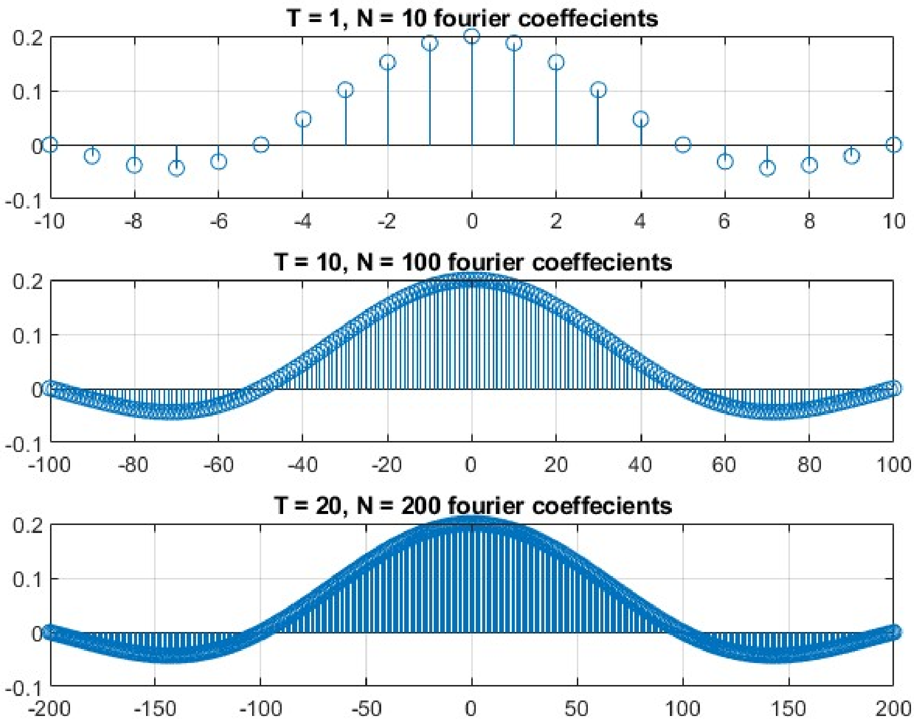
b. After plotting the original and reconstructed signal we almost observed a perfectly reconstructed signal as follows



c. The maximum absolute error for the reconstructed signal was found to be $4.4496e-16$ and the rms error was found to be $9.4800e-17$.

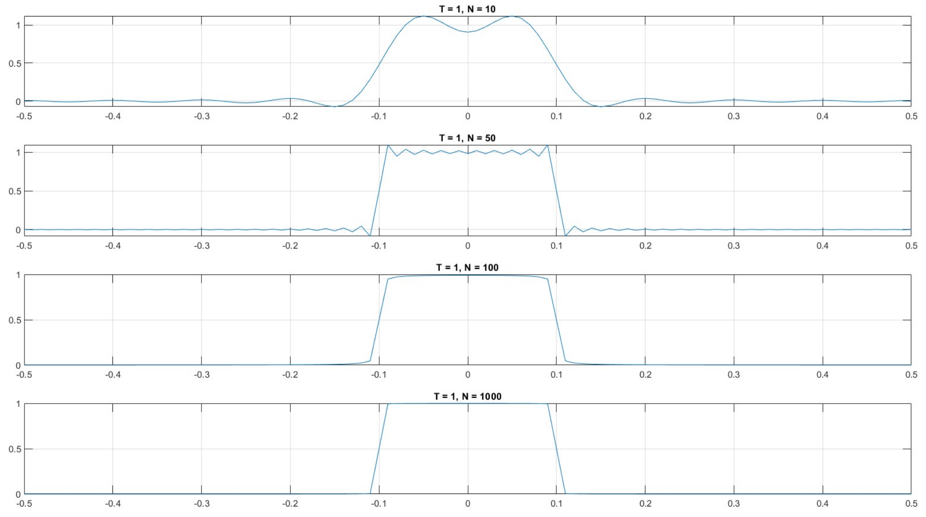
1.3 Gibbs Phenomenon

1. The Fourier series coefficients for a real, periodic square wave with amplitude 1 in $[-0.1, 0.1]$, and T = 1, 10 and 20 is as follows



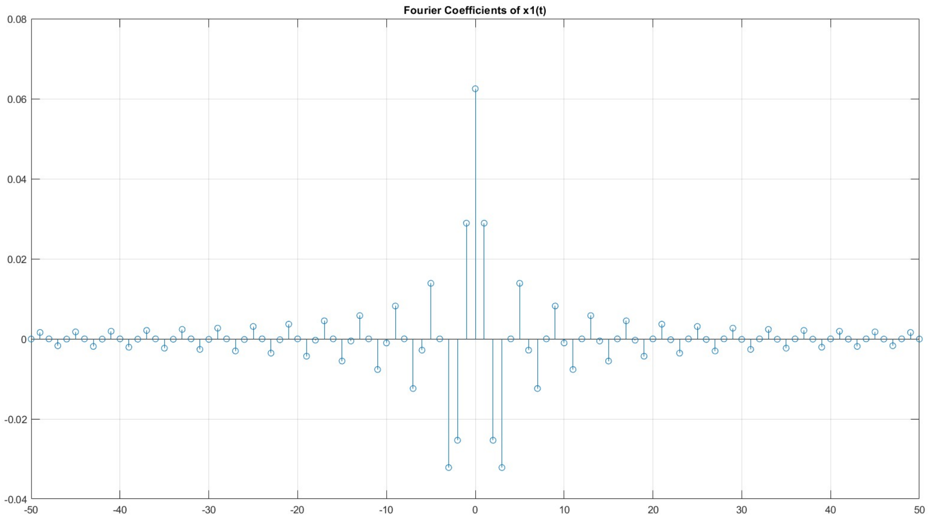
As t goes to infinity the number of FS coefficients increase and it appears as continuous signal.

c. As the number of coeffs keep increasing the better signal we get as compared to that of the original signal. And as N goes to infinity we would be able reconstruct the exact same signal as the original.

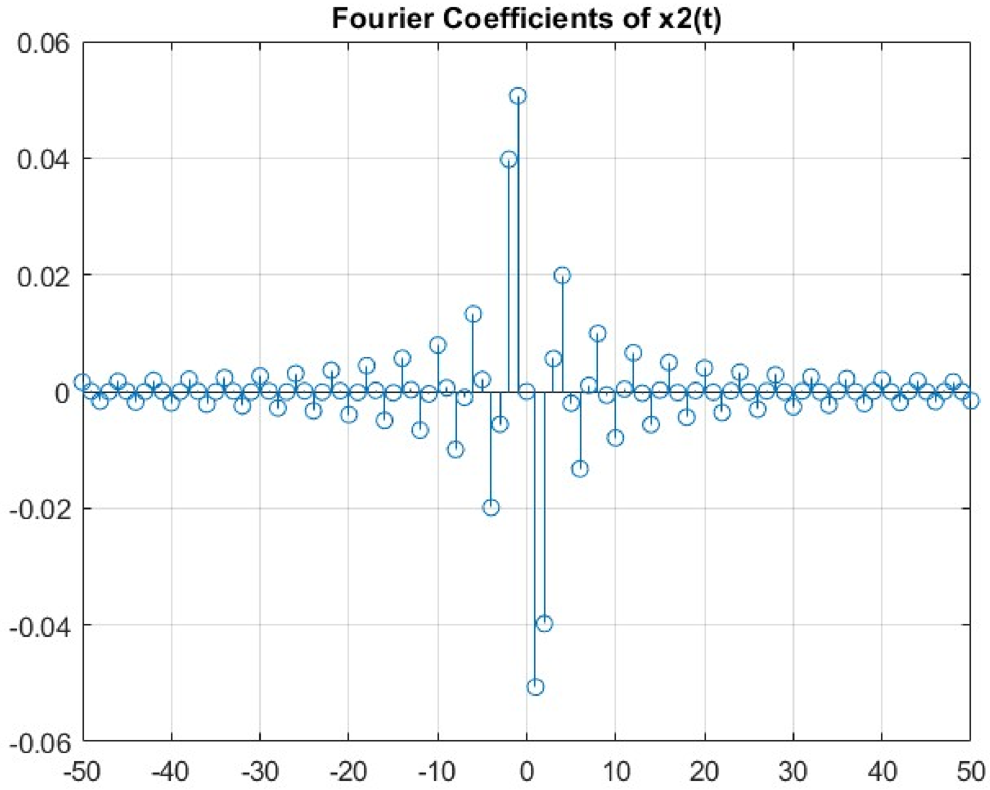


1.4 Fourier series – more examples and symmetry properties

a. FS coefficients of $x1(t)$ are as follows and follow even symmetry.



b. FS coefficients of $x2(t)$ are all zeros in real values because it is an odd signal and the values in imaginary part have non zero values and follow odd symmetry.



c. In the first case the fourier coefficients were non zero because the given signal is real and even. But in case 2, the given signal is odd, so the real fourier coefficients are zeros and the imaginary coeffs are non-zero and have odd symmetry.