Unit-1

One words

1) Each performance of random experiment is called a trial.

2) The collection of all possible outcomes of a random exp is called the sample space.

3) sample space is denoted by S.

4) The elements of sample space are called sample points.

5) The outcome of "head turning up" is denoted by H.

6) The outcome of "Tail Turning up" is denoted by T.

7) Any subset A of sample space S is called an event.

8) The event S is called a sure event.

9) The event φ is called an impossible event.

0) $A = \{1, 3, 5\}$ is the event of getting an Odd number.

1) $B = \{2, 4, 6\}$ is the event of getting an even number.

2) $\bar{A}, \bar{B}, \bar{C}$ are called complement events.

3) f/N is called the relative frequency of the event A.

4) $C_1 : P(A) \geq 0$ for all $A \subseteq S$.

5) $C_2 : P(S) = 1$

6) C_3 : If $\{A_n\}$ is any finite (or) infinite seq of disjoint events then $P(\bigcup_i A_i) = \sum_i P(A_i)$.

) P is called the Probability set function.

) The number $P(A)$ is called the Probability of the event A.

) $$P(A) = \frac{\text{No. of cases favourable to A}}{\text{Total no. of cases}}.$$

) If $A_i \cap A_j = \phi \ \forall\ i, j$ with $i \neq j$ then seq. of subsets is said to be mutually disjoint.

If $\bigcup_{n=1}^{\infty} A_n = S$, then seq of events is said to be exhaustive.

22) If seq of events $A_1, A_2 \ldots A_n$ is mutually disjoint & exhaustive, then $P\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} P(A_n) = 1$.

23) $P(A/B) = \frac{P(A \cap B)}{P(B)}$.

24) $P(A/B) \geq 0$

25) $P(A/A) = 1$

26) $P(A_1 \cup A_2 \cup \ldots / B) = P(A_1/B) + P(A_2/B) + \ldots$ provide $A_1, A_2 \ldots$ are mutually disjoint events.

27) $P(A \cap B) = P(B)\, P(A/B)$ is called multiplication therm for probabilities.

28) A is independent of B if $P(A/B) = P(A)$.

29) If A & B are 2 independent events, then $P(A \cap B) = P(A)\, P(B)$

30) A set of events $A_1, A_2 \ldots A_n$ are said to be pairwise independent if $P(A_i \cap A_j) = P(A_i) P(A_j)$

31) The events $A_1, A_2 \ldots A_n$ are independent if $P(A_1 \cap A_2 \cap \ldots \cap A_n) = P(A_1) P(A_2) \ldots P(A_n)$.

32) pairwise independent of n events $\neq$ independence of n events.

3) If A & B are independent events then A & $\bar{B}$ are also independent events.

Unit - 2

1) A fn $x : S \rightarrow R$ which assigns to each elmt $\omega \in S$ one & only one real no's is called a random variable.

2) The space of random variable x is defined to be set of real no's $\xi = \{x(\omega) / \omega \in S\}$.

3) P' defines a probability set fn on ξ.

4) P' is called the induced probability of random variable x.

5) The fn $F : R \rightarrow R$ defined by $F(x) = P(X \leq x)$. where $-\infty < x < \infty$ is called distribution fn of random variable x.

6) If $F(x)$ is distribution fn of random vble x & if $a < b$ then $P(a < x \leq b) = F(b) - F(a)$.

7) The events $a < x \leq b$ & $x \leq a$ are disjoint.

8) $P(X = x_i)$ is also written as $P(x_i)$ or simply P_i.

9) If random vble takes atmost countable no. of values $x_1, x_2 \ldots x_n$ it is called a discrete random vble.

10) $\Sigma P_i = 1$

11) Any subset A of $\mathfrak{Z}$, $P(A) = \sum_{x_i \in A} P(x_i)$.

12) $P(X = x_i) = f(x_i) = p_i$.

13) $F(x) = \sum_{x_i < x} P_i = \sum_{x_i \leq x} f(x_i)$.

14) f is called probability density fn of discrete random vble.

15) A random vble x is said to be continuous random vble if it can take any value in an interval which may be finite or infinite.

16) $P(A) = \int_a^b f(x).dx$.

17) $P(A)$ is also written as $P(A) = P(a < x < b)$

18) $A = \{a\}$.

19) $P(A) = \int_a^a f(x)dx = 0$

20) x is continuous random vble, probability of every set consisting of single pt is zero.

21) $F(x) = \int_{-\infty}^{x} f(t).dt$.

22) $F(x)$ is called distribution fn of continuous random variable x.

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$F'(x) = f(x)$ at each pt where $f(x)$ is continuous.

$F(\infty) = 1$

$F(-\infty) = 0$

$P(a \le x \le b) = F(b) - F(a)$

$F(x)$ is an increasing fn of x.

$P_i = P(x = x_i)i, \; i = 1, 2 \ldots$

29) Mathematical expectation of X, denoted by $E(X)$.

30) $E(X)$ is denoted by $\sum_i P_i x_i$.

31) It is easy to verify the following results if X & Y are random vbls.

32) $E(c) = c$ where c is constant.

33) $E(cX) = c\,E(X)$.

34) $E(aX+b) = a\,E(X) + b$.

35) $E(X+Y) = E(X) + E(Y)$

36) $E(XY) = E(X)E(Y)$. if X & Y are independent random vbls.

37) $E(\psi(x)) = \sum P_i \psi(x_i)$.

①

Statistics - II

Baye's theorm:-

Let $\{A_i\}$ be a sequence of mutually exclusive & exhaustive events in a sample space S s.t $P(A_i) > 0$ for all i. Let B be any event with $P(B) > 0$ then,

$$P(A_i/B) = \frac{P(A_i)\,P(B/A_i)}{\Sigma P(A_i) P(B/A_i)}$$

Pf: By Baye's rule we have,

$$P(A_i/B) = \frac{P(A_i)\,P(B/A_i)}{P(B)} \quad \text{—①}$$

we claim that,

$$P(B) = \sum_i P(A_i)\,P(B|A_i)$$

Since the events $\{A_i\}$ are mutually exclusive & exhaustive, we have,

$\bigcup_i A_i = S$ & A_i's are disjoint.

$\therefore B = B \cap S$

$= B \cap (\bigcup_i A_i)$ ($\because A_i$'s are exhaustive)

$B = \bigcup_i (B \cap A_i)$

$P(B) = P(\bigcup_i (B \cap A_i))$

$P(B) = \sum_i P(B \cap A_i)$ [$\because A_i$'s are mutually disjoint]

$= \sum_i P(A_i) P(B/A_i)$ [$\because P(B/A) = \frac{P(A \cap B)}{P(A)}$]

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Then, $P(B) = \sum_i P(A_i)\, P(B/A_i)$ — ②

sub ② in ①

$$P(A_i/B) = \frac{P(A_i)\, P(B/A_i)}{\sum_i P(A_i)\, P(B/A_i)} \;//$$

2) Boole's inequality:-

If A & B are any 2 events in a sample space then PT i) $P(A\cap B) \ge 1 - P(\bar{A}) - P(\bar{B})$ ii) (Generalised Booles inequality) For the events $A_1, A_2 \dots A_n \dots$ in a sample space $P(\bigcap_i^{\infty} A_i) \ge 1 - \sum_i^{\infty} P(\bar{A_i})$.

Sol: i) For any 2 events A & B we have,

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$P(A\cap B) = [1 - P(\bar{A})] + [1 - P(\bar{B})] - P(A\cup B)$$

$$= 1 - P(\bar{A}) - P(\bar{B}) + [1 - P(A\cup B)]$$

$$\ge 1 - P(\bar{A}) - P(\bar{B}) \quad (\because 0 \le P(A\cup B) \le 1)$$

ii) Let $B_1 = \bigcap_{i=2}^{\infty} A_i$ so that $\bigcap_{i=1}^{\infty} A_i = A_1 \cap B_1$

Now, $P(\bigcap_{i=1}^{\infty} A_i) = P(A_1\cap B_1) \ge 1 - P(\bar{A_1}) - P(\bar{B_1})$

(by (i))

$$= 1 - P(\bar{A_1}) - P\left(\overline{\bigcap_{i=2}^{\infty} A_i}\right)$$

(3)

$$= 1 - P(\bar{A}_1) - P\left(\bigcup_{i=2}^{\infty} \bar{A}_i\right) \geq 1 - P(\bar{A}_1) - \sum_{i=2}^{\infty} P(\bar{A}_i)$$

$$= 1 - \sum_{i=1}^{\infty} P(\bar{A}_i)$$

$$= P\left(\bigcap_{i=1}^{\infty} A_i\right) \geq 1 - \sum_{i=1}^{\infty} P(\bar{A}_i) \,//$$

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One words