any paynomial is an entire function 1 12 0 Problem -An analytic function in a region with constant modulus is constant. Let \$(2) = u + iv, be analytic in a domain D Given that, 14(2) is constant. ~ ~ ~ ~ C2 (say) =) u2+182 = c Where C is a constant TPI, & is a constant. Now, u2+42=0 Diff partially w to a 2 y we get るいいまするかりまこの。 Duly + 24 Vy = 0 (in) what Vir =0 ->0 Using C-Requation in 0 & 0 una - Vuy =0

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$$u_{x}+v_{x} = (x - y)(2x + 4y) + (x^{2} + 4xy + y^{2})(x)$$

$$= (x - y)(2x + 4y) + (x^{2} + 4xy + y^{2})$$

$$= (x - y)(4x + 2y) + (x^{2} + 4xy + y^{2})$$

$$= (x - y)(4x + 2y) - (x^{2} + 4xy + y^{2})$$

$$= (x - y)(4x + 2y) - (x^{2} + 4xy + y^{2})$$

$$= (x - y)(4x + 2y) - (x^{2} + 4xy + y^{2})$$

$$= (x - y)(x + y) + (x^{2} + 4xy + y^{2})$$
Using these equation in 0 wa get
$$u_{x} - u_{y} = (x - y)(2x + 4y) + (x^{2} + 4xy + y^{2})$$

$$= (x - y)(2x + 4y) + (x^{2} + 4xy + y^{2})$$

$$= (x - y)(2x + 4y) + (x^{2} + 4xy + y^{2})$$

$$= (x - y)(2x + 4y) + (x^{2} + 4xy + y^{2})$$

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$$= (x - y)(2x + 4y) + (x^{2} + 4xy + y^{2})$$

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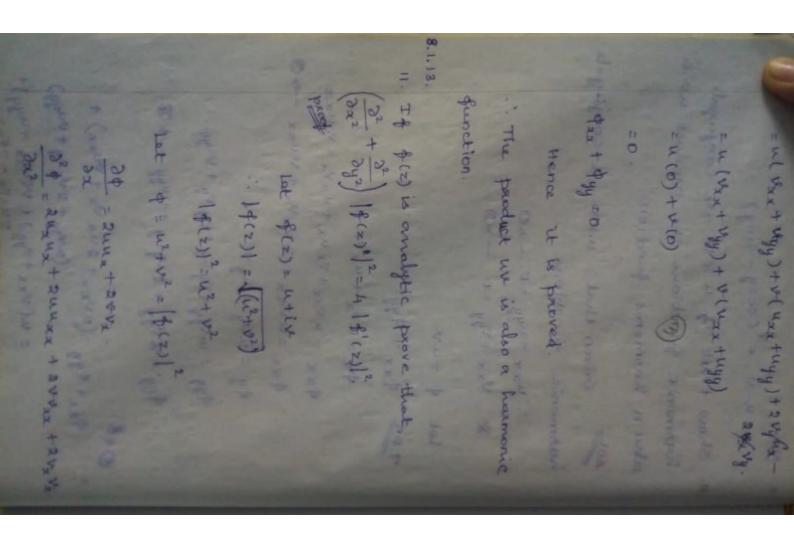
$$= (x - y)(4x + 2y) - (x^{2} + 4xy + y^{2})$$

$$= (x - y)(4x + 2y) + (x^{2} + 4xy + y^{2})$$

$$= (x - y)(4x + 2y) + (x^{2} + 4xy + y^{2})$$

$$= (x - y)(2x + 4xy + 4xy + 4xy + y^{2})$$

$$= (x - y)(2x + 4xy + 4$$

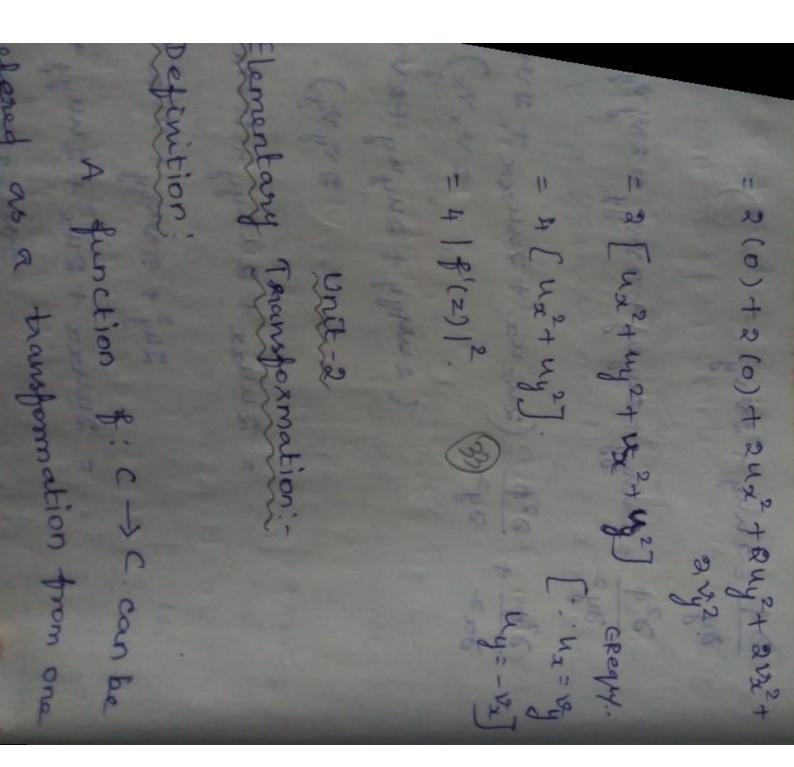


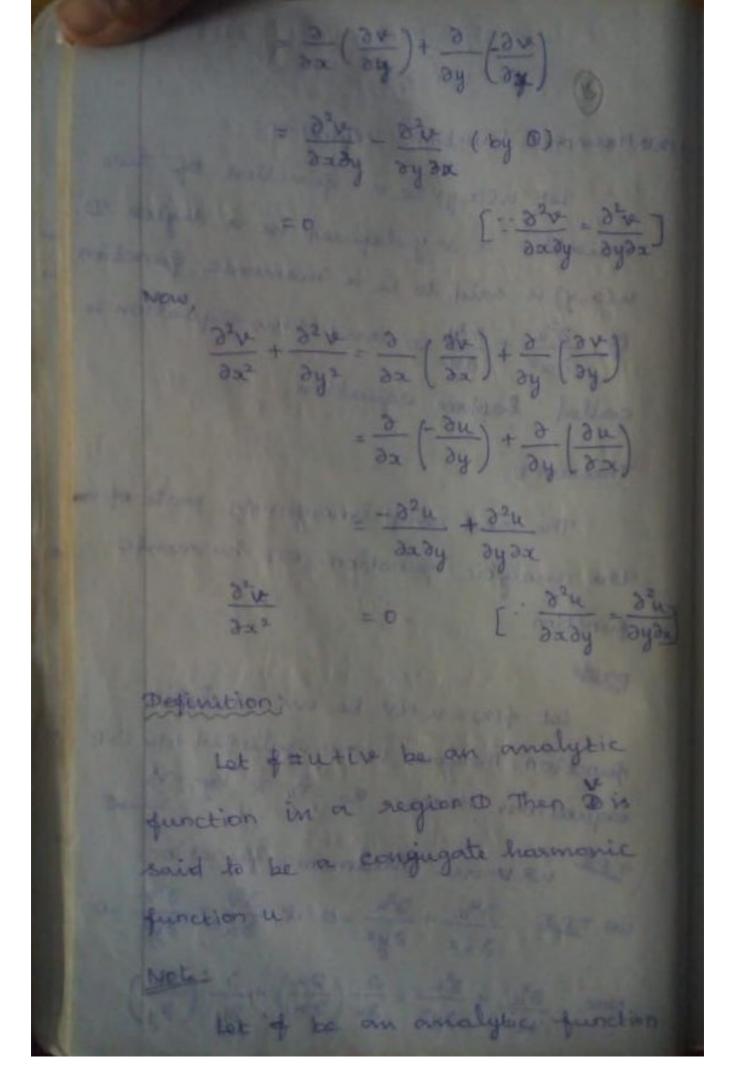
a set u-de excosy - sing). a show that if ux is are conjugate harmonic functions the product us is also a harmonic function. KOL Given that, ux12 are conjugate harmonic functions 2 V2x + 194y = 0 → 10 7.PT Pxx + Hyy = alman all the 64 = we + wx + 1 ウェス = ルレンス+リュルス+ルスレス+レルスス Pax = Litex + a Mx Vxx + Ve uxx -> 3 py = way 4 my x 9yy = u vyy + vy uy + uy vy + vuyy · fyy = a vyy + 2 vy uy + vuyy -> 0. (D+6) Pax + Pyy = (uvxx+ 2 ux vx + vuxx) + 121 -146 1 (asyy + 2 ry my + vagy) = u( vxx + vyy) + v(uxx + uyy)+

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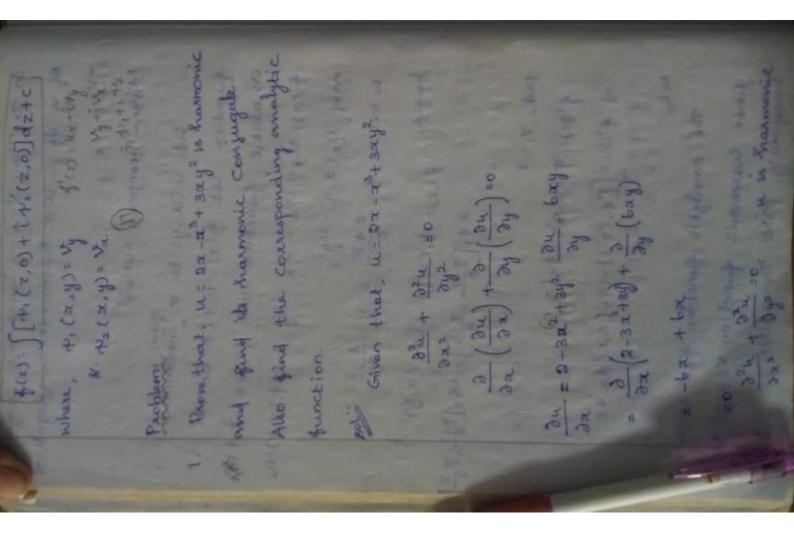
Let v be the harmonic conjugate of w. · +(=) = u + iv is amalytic Lat q (x,y) = ux = cosx coshy - asinx sinhy +22 \$,(z,0) = [cosz - 2sinz(0) + 2z+10)] = 052 + 22 \$ = (x, y) = uy = sinx sinhy + a cosx coshy - 24+4 \$2(2,0) = Sinz(0)+2 cosz(1) - (0)+42 = 2 cosz +4Z ·· \$(z) = [ \$\phi\_1(z,0) + i \phi\_2(z,0) dz + c = [(cosz+22)+i(2cosz+42)dz+c = [ s(cosz+ 2z)dz - is(2 cosz+4z) = [ sinz + 22 - [i(2 sinz + 1422) +c \$(z) = sinz + z2 - aisinz + aiz2 + c . 4(z) = sinz + z2 - 2isinz - aiz2+C

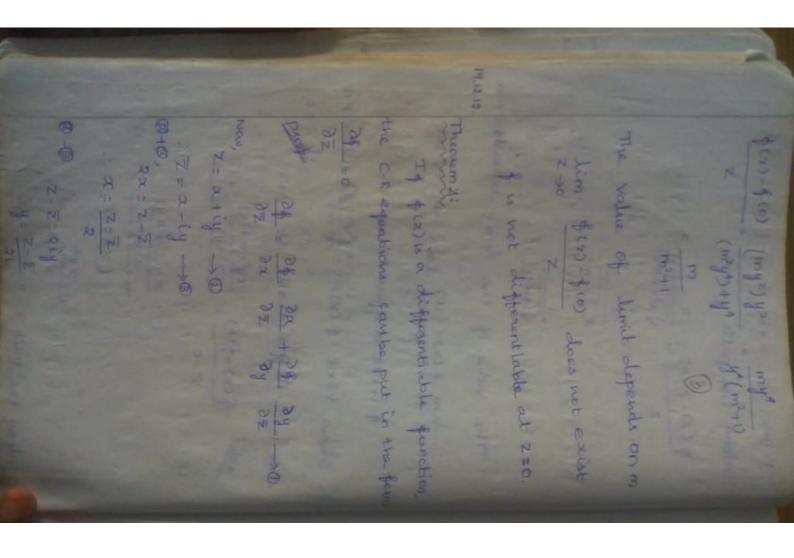
=utiv -. v= 2y-3x3y+ y3 1+4 - 00 p Show that, u(oc,y) = sinoc coshy+ 2 cosesinh + 32-42++24 is harmonic find an amalytic function & (Z) intermes of Z. II ( I I I I I I I u(a,y)= Sinx coshy + 2 cosx sinhy + x2-y2+4xy  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$ du = cosx coshy + a(sinx) sinhy + 2x + 4y. du = sinx sinhy + 2 cosx coshy - 2y+4x or teinx) soony - a cosx sinhy + 2. Du = sinx coshy + 2 cosx sinhy - 2 dy2 of sinx coshy - 2 cosx sin by +2) + (sina coshy + 2 cosx sinhy -2) =0. 1) 1 ( u is harmoning





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ux (2,0) - my (2,0) = -2(1-2522) (1-cosaz) (56)  $\frac{-2}{2 \sin^2 2} = -\cos^2 2$ using C-R equation in @ weget uy tua = -2 sin 2 x sinh = ( sino = 0 ( coshay - cosax) coso = [] ug (z,0) + ux (z,0) = 0 → 3 Adding 3 x 1 24 (2,0) = - cosec22 us (2,6)= 21 cosec2 2 -> 5 1 2 my (2,0) = cosec 2 uy(z,0)= 1 cosec2z -> 0 Now, \$(2) = 4(2,0) + LU(2,0) f(z) = ux(z,0) +ivx(z,0) = ux (2,0)-iuy(2,0) Cosec z - i cosec z ( pr wex 2 (1+1) cosec 2 Integrating 71 (1+1) [cosec 2 dz

$$f(z) = \frac{1+i}{2} \cot z + C$$

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$$f(z) = \int_{-1}^{1} (z, 0) + i + 2 \cdot (z, 0) dz + C$$
where  $f(z) = \frac{1}{2} \cos z + C$ 

$$f(z) = \int_{-1}^{1} \cos z + C \cdot (z, 0) dz + C$$

$$f(z) = \int_{-1}^{1} \cos z + C \cdot (z, 0) dz + C$$

$$= \int_{-1}^{1} (1+i) \int \csc^2 z dz$$

$$= \frac{1}{2} (1+i) \int \csc^2 z dz$$

$$= \frac{1}{2} (1+i) \int \csc^2 z dz$$

$$= \frac{1}{2} (1+i) (-\cot z) + C$$
Note:

i)  $u + v = (x - y) (x^2 + 4xy + y^2)$ 

$$f(z) = u + iv + find + he \cdot analytic + and$$

$$f(z) = u + iv + find + he \cdot analytic + and$$

$$f(z) = u + iv + (x - y) (x^2 + 4xy + y^2)$$

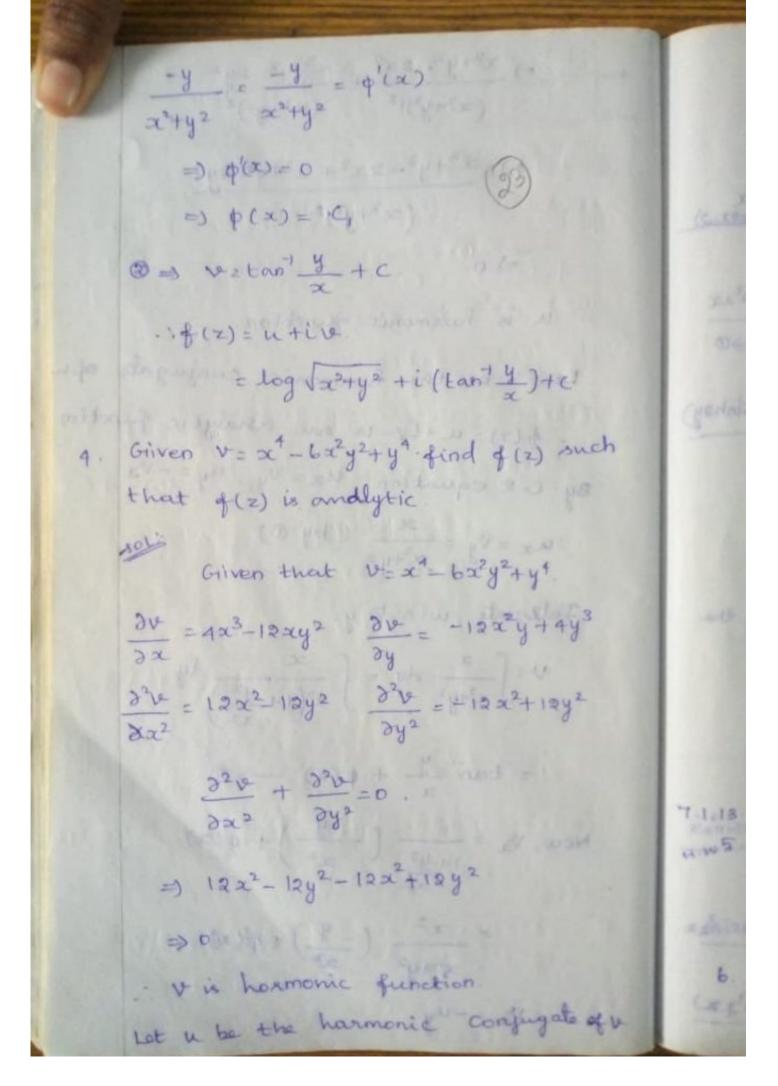
$$u + v = (x - y) (x^2 + 4xy + y^2)$$

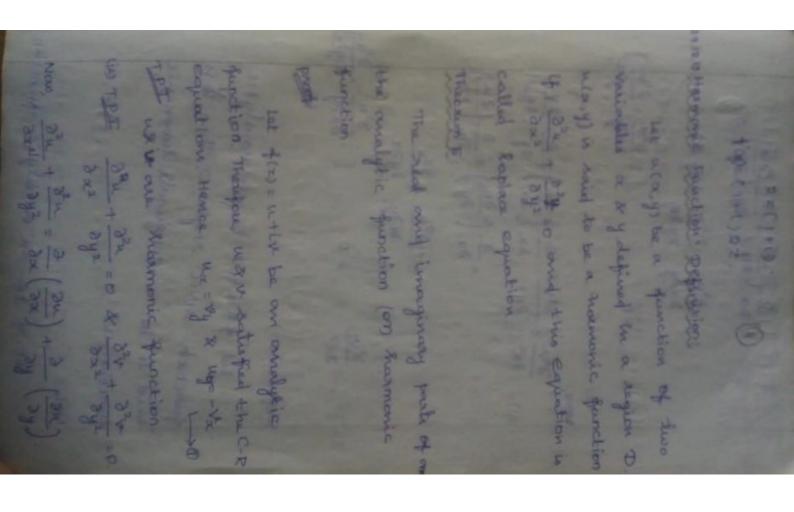
$$u + v = (x - y) (x^2 + 4xy + y^2)$$

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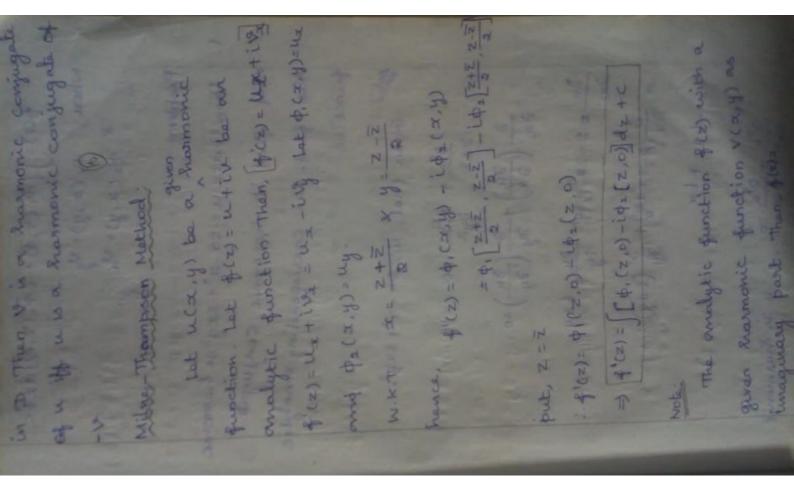
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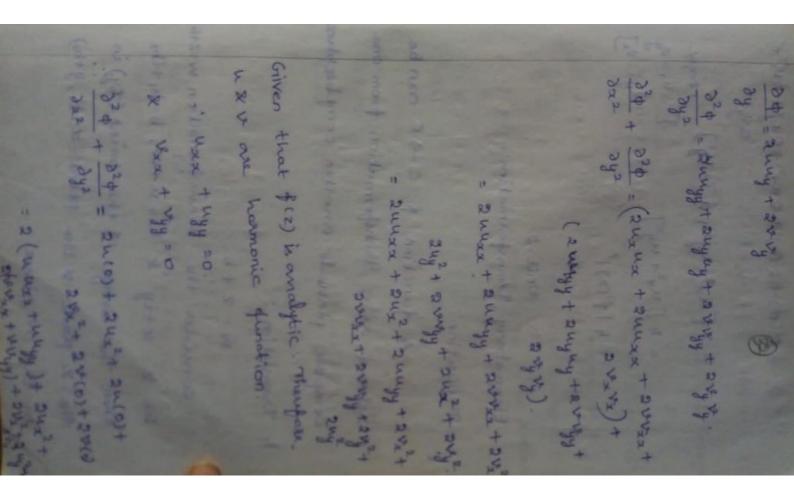


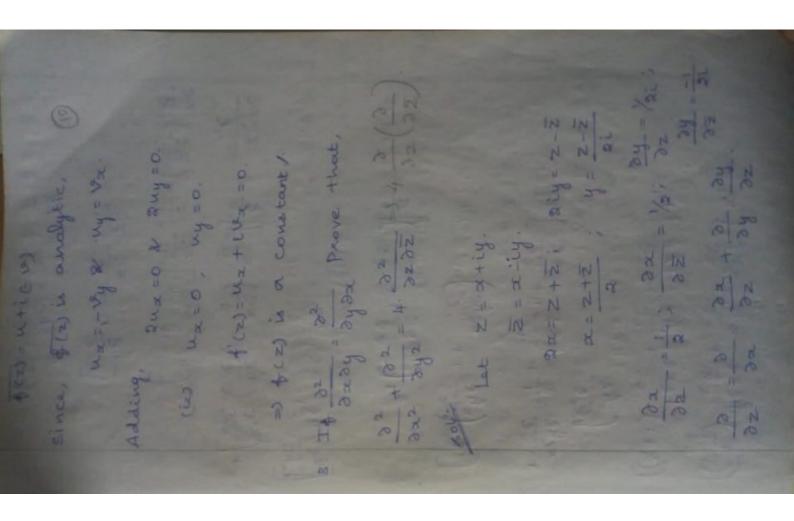
if (2) = notive is analytic +1 (z,0)=0 - (s/4 1) = 4 + 1 V H2(Z,0)= 423 \$(z) = [[+1(z,0)+i+2(z,0)]dz+c = S [0+i423] dz+c = ALSz3dz+C 100 = 41. 21 +0 11 cons ( ( z) = i z + c. 1 1 1 (z) = utive is an amalytic function - find & (z): and u= sin 2x. coshay+ cosax 6. find the analytic function \$(2)=u+10 ch, utu= cosh sy-cos sx

show that u= log \x2+y2 is Examonic and find its conjugates and also find the corresponding analytic function f(=) Given that  $u = \log (\sqrt{x^2 + y^2})$   $= \log (x^2 + y^2)^{\frac{1}{2}}$   $= \frac{1}{2} \log (x^2 + y^2)$ Dx = 1 - 1 x2+y2 x2x 3u = 1/2 x2+y2 x2y 22 = (x2+y2)(1) - x (22) da2 (22+y)2 = x2+y2-2x2 (x3+y2)2 = 7 23+433 3 h (x2+y2)(1)-y (24)



Differentiate partially w. n. to x 2 y 11x + 1x (cosh 2y - cosax) cosax 2 - 51,02x (cashay-cosax)2 (sinax) 4+ 12 = 2 coshay . cos 2x - 2costax - 2 sin2x (cashay-cosox)2 Lyo my + my = (cosh 2y - cos22) (0) - sinax (2 sinh 2y) (coshay - cosas)2 my + my = - 2 sinhay sinax (coshay -cos 2x)2 since &(z) is analytic, utto satisfy the C-R equation ux = vy & uy = -vx Using these equation in @ we get, un-my = 2(coshay - cosaa) cosax -asinaa (cashay - casaa)2 ux (z,0) - uy(z,0) = 2(1-cos2z)cos2z-25in2z = 2 cos 2 z - 2 (cos 2 2 z + sin 2 x) [1- cos 22)2

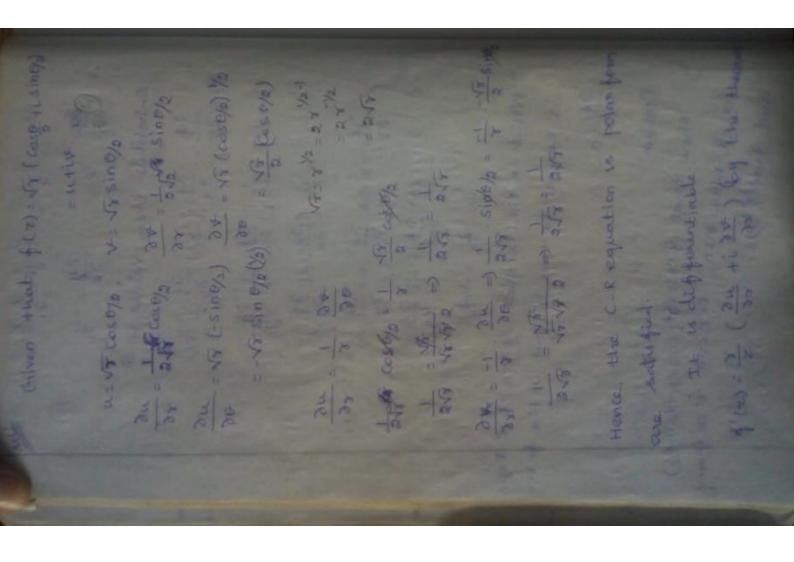


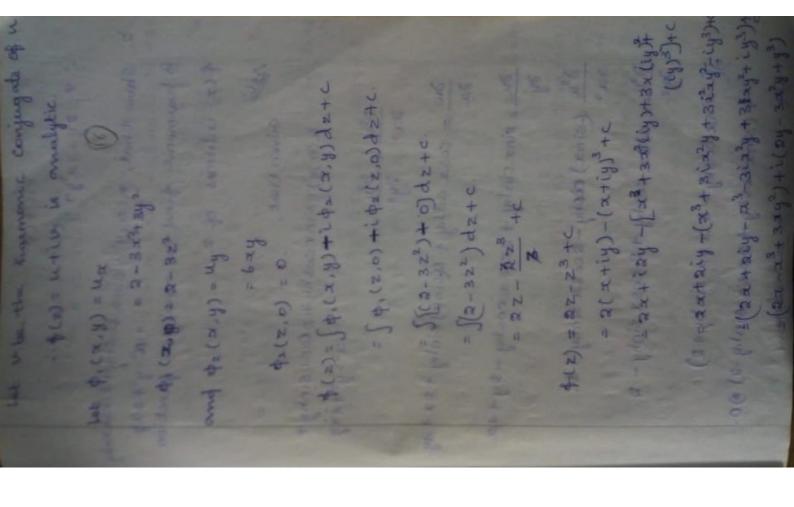


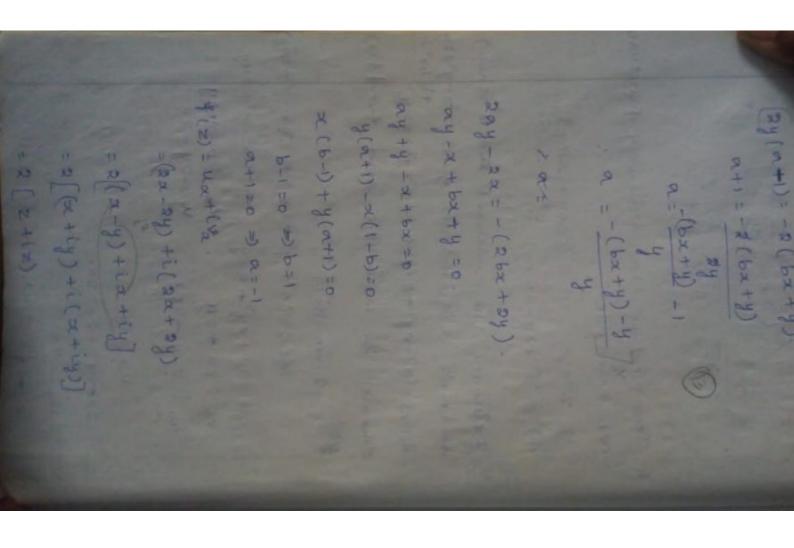
Adding (a) 
$$(z,0) = 6z^2$$
 $(x(z,0) = 3z^2 \rightarrow (x(z,0) + 2)$ 
 $(x(z,0) = 0)$ 
 $(x(z,0) = 0)$ 

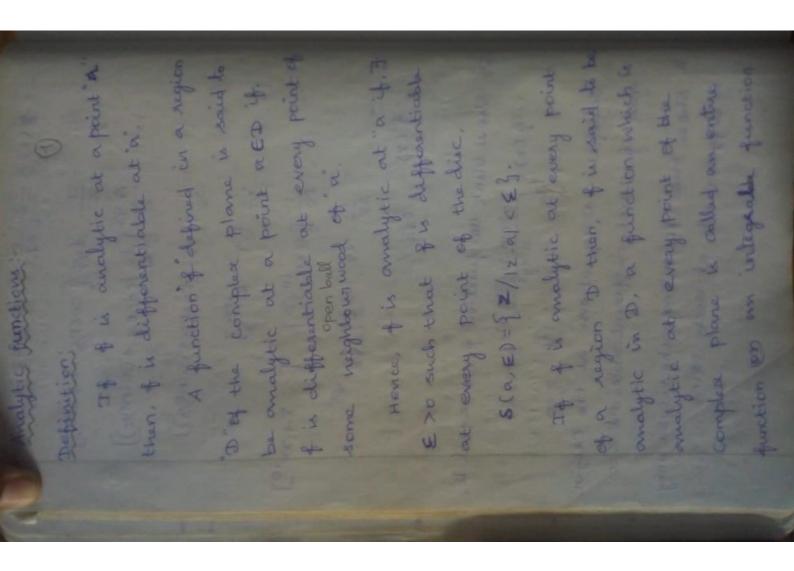
Now,  $(x,0) = 0$ 
 $(x(z,0) = 0)$ 
 $(x(z,0) = 0)$ 

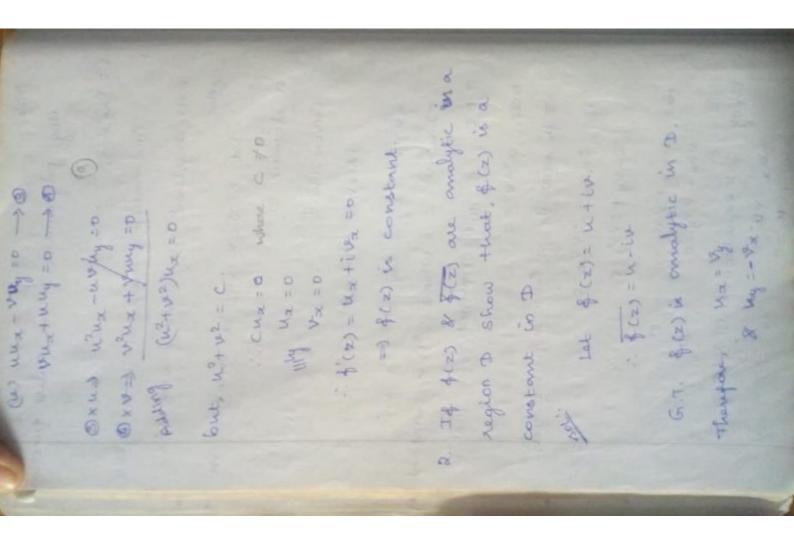
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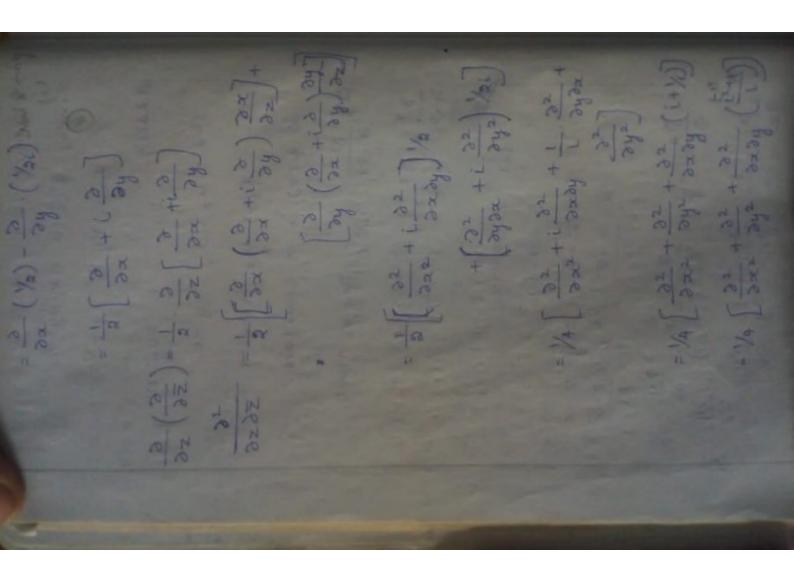


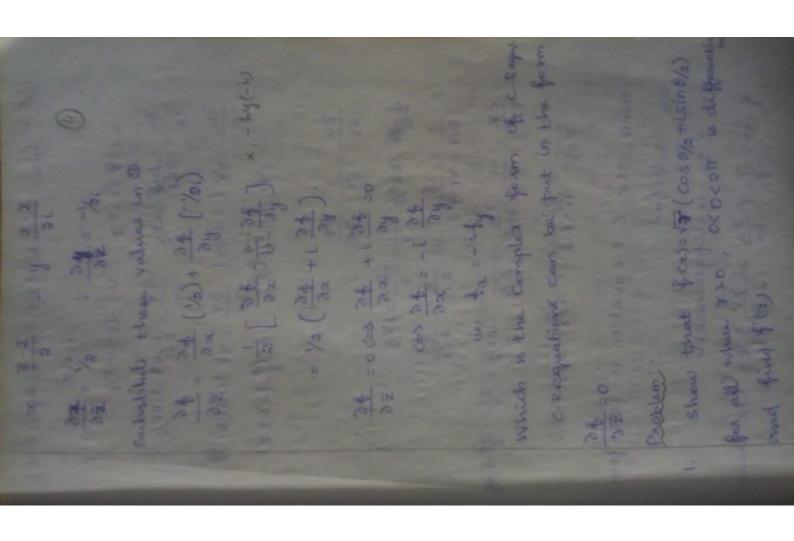


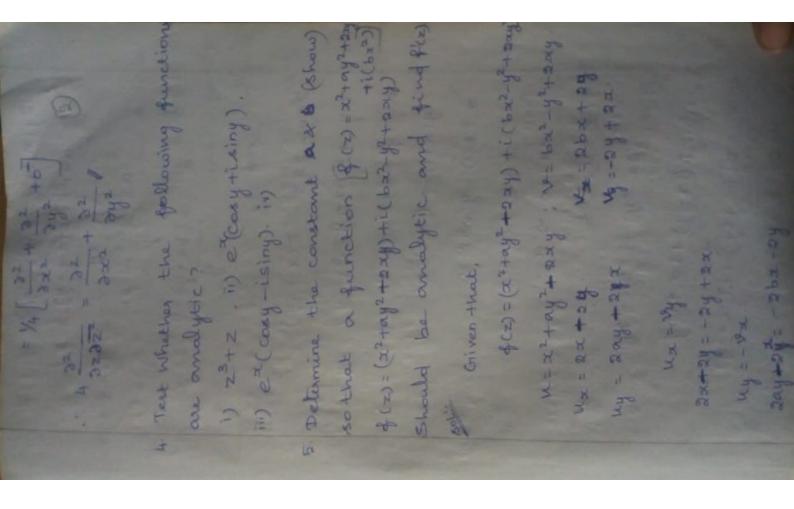


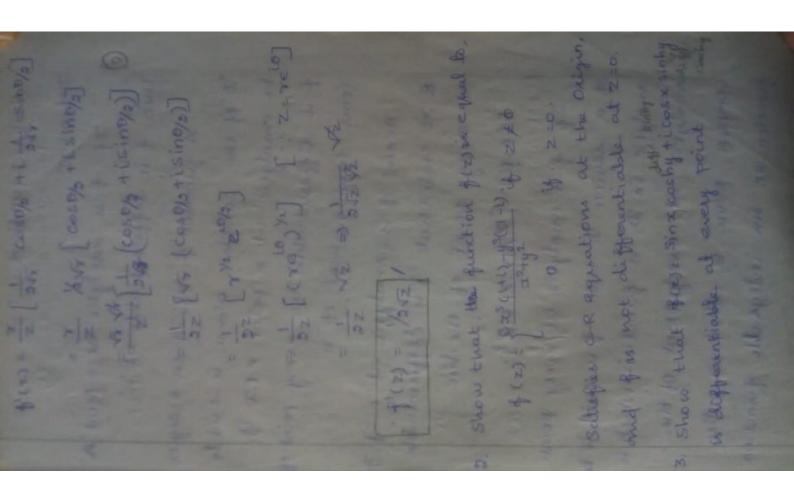












$$x^{2}+y^{2}-2x^{2}+\frac{x^{2}+y^{2}-2y^{2}}{(x^{2}+y^{2})^{2}}$$

$$\Rightarrow x^{2}+y^{2}-2x^{2}+2^{2}+y^{2}-2y^{2}$$

$$= x^{2}+y^{2}-2x^{2}+2^{2}+y^{2}-2y^{2}$$

$$= x^{2}+y^{2}-2x^{2}+2^{2}+y^{2}-2y^{2}$$

$$= x^{2}+y^{2}-2x^{2}+2^{2}+y^{2}-2y^{2}$$

$$= x^{2}+y^{2}-2x^{2}+2^{2}+y^{2}-2y^{2}$$

$$= x^{2}+y^{2}-x^{2}+y^{2}+x$$

$$= x^{2}+x^{2}+x^{2}+x$$

$$= x^{2}+x^{2}+x^{2}+x^{2}+x$$

$$= x^{2}+x^{2}+x^{2}+x^{2}+x$$

$$= x^{2}+$$

