

From this table x_1 is the entering variable & s_2 is the leaving variable & 1 is the pivotal element

First Iteration Method

Basis		cost c_j	3	2	0	0	Ratio
C_B	B	soln x_B	x_1	x_2	s_1	s_2	
0	s_1	2	0	2	1	-1	1
3	x_1	2	1	-1	0	1	-2
Optimality is not obtained		$z_j = 6$	3	-3	0	3	
		$z_j - c_j$	0	-5	0	3	

From this table x_2 is the entering variable & s_1 is the leaving variable & 2 is the pivotal element

Second Iteration Method

Basis		cost c_j	3	2	0	0	Ratio
C_B	B	soln x_B	x_1	x_2	s_1	s_2	
2	x_2	1	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	
3	x_1	3	1	0	$\frac{1}{2}$	$\frac{1}{2}$	
Optimality is obtained		$z_j = 11$	3	2	$\frac{5}{2}$	$\frac{1}{2}$	
		$z_j - c_j$	0	0	$\frac{5}{2}$	$\frac{1}{2}$	

Hence $z_j - c_j \geq 0$

Hence the optimality is attained

Hence the optimal soln is $x_1 = 3$ $x_2 = 1$

The max value of the obj fn is $z = 11$

operation Research

Simplex Method

1. $\text{Max } z = 4x_1 + 10x_2$

subject to constraints is,

$$2x_1 + x_2 \leq 50$$

$$2x_1 + 5x_2 \leq 100$$

$$2x_1 + 3x_2 \leq 90$$

$$x_1, x_2 \geq 0$$

Sol

G.T. $\text{Max } z = 4x_1 + 10x_2$

Subject to constraint is,

$$2x_1 + x_2 \leq 50$$

$$2x_1 + 5x_2 \leq 100$$

$$2x_1 + 3x_2 \leq 90$$

$$x_1, x_2 \geq 0$$

The std form of LPP is,

$$\text{Max } z = 4x_1 + 10x_2 + 0s_1 + 0s_2 + 0s_3$$

Subject to constraint is,

$$2x_1 + x_2 + s_1 = 50$$

$$2x_1 + 5x_2 + s_2 = 100$$

$$2x_1 + 3x_2 + s_3 = 90$$

$$x_1, x_2, s_1, s_2, s_3 \geq 0$$

The initial basic feasible solution is got by putting $x_1 = 0$

$$x_2 = 0.$$

$$s_1 = 50 \quad s_2 = 100 \quad s_3 = 90.$$

2. Max $Z = 3x_1 + 2x_2$

Subject to constraint is,

$$x_1 + x_2 \leq 4$$

$$x_1 - x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

sol:-

Gr.T

$$\text{Max } Z = 3x_1 + 2x_2$$

subject to constraint is,

$$x_1 + x_2 \leq 4$$

$$x_1 - x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

The std form of LPP is,

$$\text{Max } Z = 3x_1 + 2x_2 + 0s_1 + 0s_2$$

subject to constraint.

$$x_1 + x_2 + s_1 = 4$$

$$x_1 - x_2 + s_2 = 0$$

$$x_1, x_2, s_1, s_2 \geq 0$$

The initial basic feasible solution is got by putting,

$$x_1 = 0, x_2 = 0, s_1 = 4, s_2 = 0.$$

Initial simplex Table :-

Basis		cost c_j	3	2	0	0	Ratio
c_B	B	soln x_B	x_1	x_2	s_1	s_2	
0	s_1	4	1	1	1	0	4
0	s_2	2	1	-1	0	1	2 ←
Optimality is not obtained		$Z_j = 0$ $Z_j - c_j$	0 -3 ↑	0 -2	0 0	0 0	

First Iteration Method

Basis		cost c_j	3	2	0	0	Ratio
CB	B	soln x_B	x_1	x_2	s_1	s_2	
0	s_1	3	0	1/2	1	-1/2	6 ←
3	x_1	3	1	1/2	0	1/2	6
optimality is not attained		$z_j = 9$	3	3/2	0	3/2	
		$z_j - c_j$	0	-1/2	0	3/2	

From this table x_2 is entering variable & s_1 is the leaving variable & 1/2 is pivotal element

2nd Iteration Method :

Basis		cost c_j	3	2	0	0	Ratio
CB	B	soln x_B	x_1	x_2	s_1	s_2	
2	x_2	6	0	1	2	-1	
3	x_1	0	1	0	-1	1	
Optimality is attained		$z_j = 12$	3	2	3	1	
		$z_j - c_j$	0	0	3	1	

Hence $z_j - c_j \geq 0$

Hence the optimality is attained

Hence the optimal soln is $x_1 = 0$ $x_2 = 6$

The max value of the obj fn is $\text{Max } z = 12$

Graphical Method

4. $\text{Max } Z = 2x_1 + 4x_2$
s.t. $x_1 + 2x_2 \leq 5$
 $x_1 + x_2 \leq 4$
where $x_1, x_2 \geq 0$

sol. $\text{G.T. Max } Z = 2x_1 + 4x_2$
s.t. $x_1 + 2x_2 \leq 5$
 $x_1 + x_2 \leq 4$
 $x_1, x_2 \geq 0$

The problem is std form taken $x_1 \times x_2$ axis - first constraint

Now consider $x_1 + 2x_2 = 5$

when $x_1 = 0$ $x_2 = 0$

$2x_2 = 5$ $x_1 = 5$
 $x_2 = 2.5$

pt A (0, 2.5) B (5, 0)

Now consider $x_1 + x_2 = 4$

when $x_1 = 0$ $x_2 = 0$

$x_2 = 4$ $x_1 = 4$

pt C (0, 4) D (4, 0)

solving, $x_1 + 2x_2 = 5$ — (1)

$x_1 + x_2 = 4$ — (2)

$x_1 + 2x_2 = 5$

$x_1 + x_2 = 4$

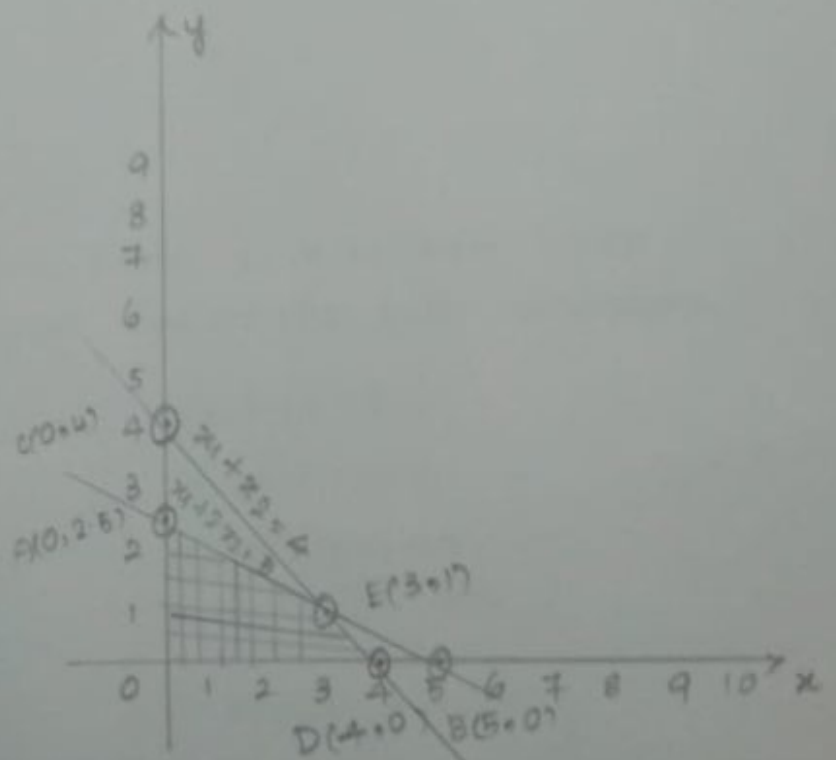
(1) (2) (1)

$x_2 = 1$

sub x_2 value in (2) eqn,

$x_1 + 1 = 4$

$x_1 = 3$



$$\text{3) Max } z = 3x_1 + 2x_2$$

st. to

$$x_1 + x_2 \leq 6$$

$$2x_1 + x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

sol

$$\text{G.T. Max } z = 3x_1 + 2x_2$$

st. to

$$x_1 + x_2 \leq 6$$

$$2x_1 + x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

The std form of lpp is

$$\text{Max } z = 3x_1 + 2x_2 + 0s_1 + 0s_2$$

$$\text{st. to } x_1 + x_2 + s_1 = 6$$

$$2x_1 + x_2 + s_2 = 6$$

$$x_1, x_2, s_1, s_2 \geq 0$$

The initial basic feasible soln is got by putting $x_1 = 0, x_2 = 0$
 $s_1 = 6, s_2 = 6$

Initial simplex Table:-

Basis		cost c_j	3	2	0	0	Ratio
C_B	B	soln x_B	x_1	x_2	s_1	s_2	
0	s_1	6	1	1	1	0	6
0	s_2	6	2	1	0	1	3 ←
Optimality not attained		$Z_j = 0$	0	0	0	0	
		$Z_j - c_j$	-3 ↑	-2	0	0	

From this table x_1 is entering variable & s_2 is leaving variable & 2 is the pivotal element

The pt of intersection E is (3,1)

Hence the region of feasibility is the closed polygon of ODEA

The extreme pt of feasibility region is gn by O (0,0) A (0,2.5) D (4,0) E (3,1)

$$Z \text{ at } O (0,0) = 2x_1 + 4x_2 = 2(0) + 4(0) = 0$$

$$Z \text{ at } D (4,0) = 2(4) + 4(0) = 8$$

$$Z \text{ at } E (3,1) = 2(3) + 4(1) = 10$$

$$Z \text{ at } A (0,2.5) = 2(0) + 4(2.5) = 10$$

Now Max Z is obtained at the pt E (3,1)

The optimal solu is $x_1 = 3$ $x_2 = 1$

The max value is 10.

5. Max $Z = 2x_1 + x_2$

$$\text{s.t. } 10x_1 + x_2 \leq 1$$

$$-3x_1 + x_2 \geq 3$$

$$x_1 + x_2 \geq 0$$

sol

Ort Max $Z = x_1 + x_2$

$$\text{s.t. } 10x_1 + x_2 \leq 1$$

$$-3x_1 + x_2 \geq 3$$

$$x_1 + x_2 \geq 0$$

This problem is std form taken x_1 & x_2 axis since $x_1 + x_2 \geq 0$ the feasibility region lies in the first constraint

Now consider $x_1 + x_2 = 1$

$$x_1 = 0 \quad x_2 = 0$$

$$x_2 = 1 \quad x_1 = 1$$

pt A (0,1) B (1,0)

$$-3x_1 + x_2 = 3$$

$$x_1 = 0 \quad x_2 = 0$$

$$x_2 = 3 \quad -3x_1 = 3$$

$$x_1 = 3/-3$$

$$x_1 = -1$$

$$C (0,3) D (-1,0)$$

Initial Simplex Table :-

Basis		cost c_j	4	10	0	0	0	Ratio
C_B	B	soln x_B	x_1	x_2	s_1	s_2	s_3	
0	s_1	50	2	1	1	0	0	50
0	s_2	100	2	5	0	1	0	20 ←
0	s_3	90	2	3	0	0	1	30
optimality is not obtained		$z_j = 0$	0	0	0	0	0	
		$z_j - c_j$	-4	-10 ↑	0	0	0	

From this table x_2 is entering variable, s_2 is the leaving variable & 5 is the pivotal element.

First Iteration Table

Basis		cost c_j	4	10	0	0	0	Ratio
C_B	B	soln x_B	x_1	s_2	s_1	s_2	s_3	
0	s_1	30	8/5	0	1	-1/5	0	
10	x_2	20	2/5	1	0	1/5	0	
0	s_3	30	4/5	0	0	-3/5	1	
optimality not obtained		$z_j = 200$	4	10	0	2	0	
		$z_j - c_j$	0	0	0	2	0	

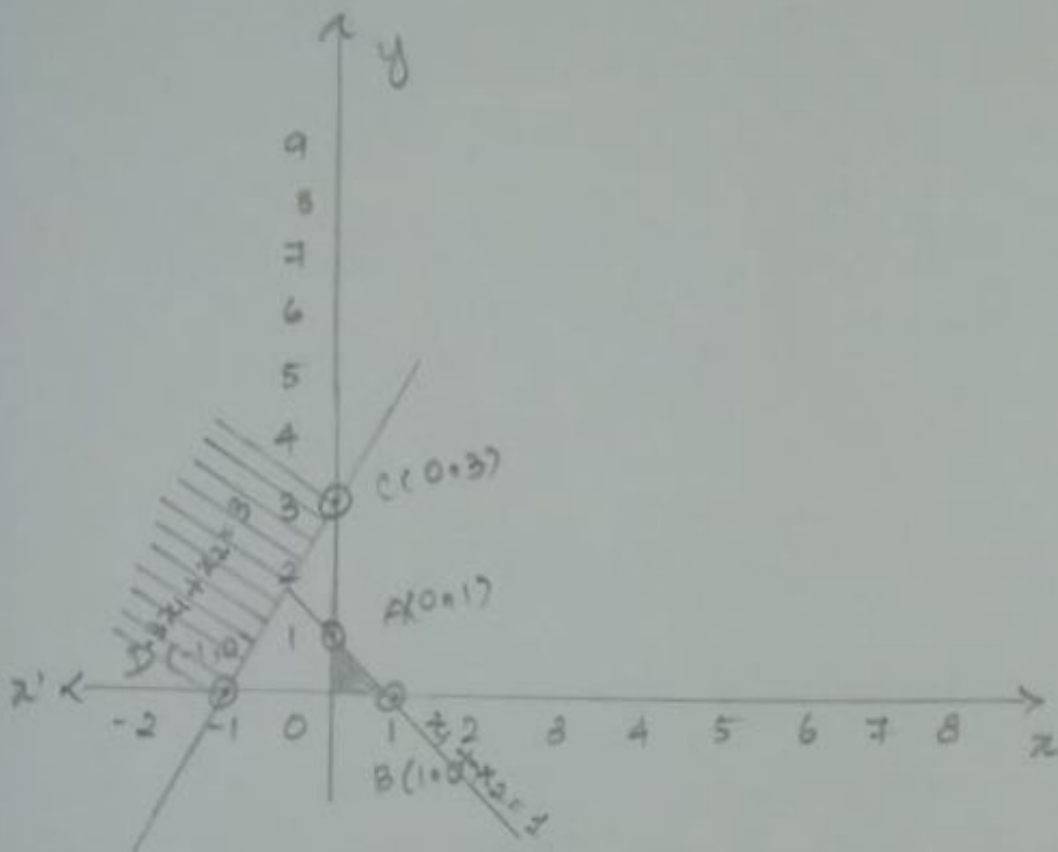
Hence $z_j - c_j \geq 0$

Hence the optimality is attained

Hence the optimal soln is $x_1 = 0$ $x_2 = 20$

The max value of the obj fn is 200.

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The half plane determined by the constraints $x_1 + x_2 \leq 1$ & the half plane determined by the constraint $-3x_1 + x_2 \geq 3$

\therefore Hence the feasible region is empty

Hence the LPP has infeasible solution.