$$-z(p-q) = x-y$$

$$\cancel{-}z(p-q) = \cancel{-}(y-x)$$

$$z(p-q) = (y-x)$$

3) Eliminating f & ϕ from the eqn (or) relation

$$z = f(x+ay) + \phi(x-ay)$$

soln:

Gn, $z = f(x+ay) + \phi(x-ay)$ — ①

D.w.r.t x & y in ①,

$$\Rightarrow \frac{\partial z}{\partial x} = p = f'(x+ay)(1+0) + \phi'(x-ay)(1-0)$$

$$p = f'(x+ay) + \phi'(x-ay) \quad \text{— ②}$$

$$\Rightarrow \frac{\partial z}{\partial y} = q = f'(x+ay)(0+a(1)) + \phi'(x-ay)(0-a(1))$$

$$q = af'(x+ay) - a\phi'(x-ay) \quad \text{— ③}$$

Again D.w.r.t x & y in ② & ③

$$\Rightarrow \frac{\partial^2 z}{\partial x^2} = t = f''(x+ay)(1+0) + \phi''(x-ay)(1+0)$$

$$t = f''(x+ay) + \phi''(x-ay) \quad \text{— ④}$$

$$\Rightarrow \frac{\partial^2 z}{\partial y^2} = r = af''(x+ay)(0+a) - a\phi''(x-ay)(0-a)$$

$$r = a^2 f''(x+ay) + a^2\phi''(x-ay)$$

$$= a^2\left[f''(x+ay) + \phi''(x-ay)\right]$$

$$\boxed{r = a^2 t.}$$

Ans:

Hence the result, $r - a^2t = 0$

where $r = \dfrac{\partial^2 z}{\partial y^2}$ & $t = \dfrac{\partial^2 z}{\partial x^2}$ A.

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integ on both sides.

$$\int \frac{d(x^2+y^2+z^2)}{x^2+y^2+z^2} = \int \frac{dz}{z}$$

$$\log(x^2+y^2+z^2) = \log z + \log b$$

$$\log(x^2+y^2+z^2) - \log z = \log b$$

$$\log\left(\frac{x^2+y^2+z^2}{z}\right) = \log b$$

$$\frac{x^2+y^2+z^2}{z} = b$$

The soln is,

$$\frac{x^2+y^2+z^2}{z} = f(y/z)$$

Ans: $x^2+y^2+z^2 = z\, f(y/z)$.

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taking the Lagrangian multipliers as x, y, z. we get each ratio of (1) equal to

$$\frac{x\,dx + y\,dy + z\,dz}{\Sigma x^2(y^2-z^2)} = \frac{x\,dx + y\,dy + z\,dz}{0}$$

$\therefore \quad x\,dx + y\,dy + z\,dz = 0$

1A

on intg,

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = a$$

$\therefore \boxed{x^2 + y^2 + z^2 = a}$ as one soln.

Taking the Lagrangian multipliers as $1/x, 1/y, 1/z$ we get each ratio of (1) equal to,

$$\frac{\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}}{\Sigma(x^2-y^2)} = \frac{\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}}{0}.$$

$$\therefore \quad \frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = 0$$

on intg,

$$\log x + \log y + \log z = \log b$$

$$\therefore \log(xyz) = \log b$$

$\boxed{xyz = b}$ is another soln.

$\therefore$ the genl soln is $\phi\,(x^2+y^2+z^2,\ xyz) = 0$.

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$C.F = A\cos nx + B\sin nx$ — (3)

The soln of eqn (3),

$y = A\cos nx + B\sin nx$. (P)

$\frac{dy}{dx} = A(-\sin nx)(n) + \cos nx \frac{dA}{dx} + B\cos nx(n) + \sin nx \frac{dB}{dx}$

$\frac{dy}{dx} = -An\sin nx + \cos nx \frac{dA}{dx} + B\cos nx(n) + \sin nx \frac{dB}{dx}$

taking $\cos nx \frac{dA}{dx} + \sin nx \frac{dB}{dx} = 0$

$\frac{dy}{dx} = -An\sin nx + Bn\,\cancel{\sin nx}\cos nx$

$\frac{d^2y}{dx^2} = -An(\cos nx)n - n\sin nx \frac{dA}{dx} + Bn(-\sin nx)(n) + n\cos nx \frac{dB}{dx}$

$\frac{d^2y}{dx^2} = -An^2\cos nx - n\sin nx \frac{dA}{dx} - Bn^2\sin nx + n\cos nx \frac{dB}{dx}$

eqn (1) becomes.

$\frac{d^2y}{dx^2} + n^2y = \sec nx$.

$-An^2\cancel{\cos nx} - n\sin nx \frac{dA}{dx} - Bn^2\cancel{\sin nx} + n\cos nx \frac{dB}{dx} + n^2A\cancel{\cos nx} + nB^2\cancel{\sin nx} = \sec nx$

$-n\sin nx \frac{dA}{dx} + n\cos nx \frac{dB}{dx} = \sec nx$.

($\div$) by n on both sides,

$-\sin nx \frac{dA}{dx} + \cos nx \frac{dB}{dx} = \frac{\sec nx}{n}$

$$\frac{\frac{dA}{dx}}{-\sin nx} = \frac{\frac{dB}{dx}}{\cos nx} = \frac{\sec nx}{n}$$

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The desired integral surface is,

$$x^2+y^2-2z = -2(xyz+1)$$

Ans: $x^2+y^2-2z+2xyz+2=0$

4) Find the integral surface of $x^2p+y^2q+z^2=0$ which passes through the hyperbola $xy=x+y$, $z=1$.

Soln:

Eqn. $x^2p+y^2q+z^2=0$

$$x^2p+y^2q=-z^2$$

The subsidiary eqns are,

$$\frac{dx}{x^2}=\frac{dy}{y^2}=\frac{-dz}{z^2}$$

Taking the terms,

$$\frac{dx}{x^2}=\frac{-dz}{z^2}$$

Integ on both sides, $\Rightarrow \int\frac{dx}{x^2}=-\int\frac{dz}{z^2}$

$$-1/x=-(-1/z)+a$$

$$\boxed{1/x+1/z=a}$$

now taking the terms, $\frac{dy}{y^2}=-\frac{dz}{z^2}$

Intg on both sides, $\Rightarrow \int\frac{dy}{y^2}=-\int\frac{dz}{z^2}$

$$-1/y=-(-1/z)+b$$

$$\boxed{1/y+1/z=b}$$

The genl soln is $\phi(1/x+1/z,\ 1/y+1/z)=0$.

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Taking last two terms,

$$\frac{dy}{2xy} = \frac{dz}{2xz}$$

Intg on both sides,

$$\int \frac{dy}{y} = \int \frac{dz}{z}$$

$$\log y = \log z + \log a$$

$$\log y - \log z = \log a$$

$$\log (y/z) = \log a$$

$$\boxed{y/z = a}$$

Taking first ratio by 2x second ratio by 2y & third ratio by 2z.

$$\frac{2x\,dx + 2y\,dy + 2z\,dz}{2x(x^2-y^2-z^2)+4xy^2+4xz^2} = \frac{dz}{2xz}$$

$$\frac{2x\,dx + 2y\,dy + 2z\,dz}{2x^3-2xy^2-2xz^2+4xy^2+4xz^2} = \frac{dz}{2xz}$$

$$\frac{2x\,dx + 2y\,dy + 2z\,dz}{2x(x^2-y^2-z^2-2y^2+2z^2)} = \frac{dz}{2xz}$$

$$\frac{2x\,dx + 2y\,dy + 2z\,dz}{x^2+y^2+z^2} = dz/z$$

$$\frac{d(x^2)+d(y^2)+d(z^2)}{x^2+y^2+z^2} = dz/z$$

$$\frac{d(x^2+y^2+z^2)}{x^2+y^2+z^2} = dz/z.$$

Working rule to solve the Lag. L.E $Pp + Qq = R$:-

i) Form the aux. eqn $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

ii) solve the aux. eqn and get two independent solns $u = a$ & $v = b$.

iii) The genl soln is $\phi(u, v) = 0$.

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Pblms:

1) Solve $x^2 p + y^2 q = z^2$

soln:

The aux. eqn is $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{z^2}$

Taking $\frac{dx}{x^2} = \frac{dy}{y^2}$

$\Rightarrow -1/x = -1/y + c$

$\therefore 1/x - 1/y = a$

Taking $\frac{dy}{y^2} = \frac{dz}{z^2} \Rightarrow -1/y = -1/z + c$

$\therefore 1/y - 1/z = b$

The genl soln is, $\phi\left(1/x - 1/y, 1/y - 1/z\right) = 0$.

2.) Find the genl soln of,

$x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$

soln:

The aux. eqn is,

$$\frac{dx}{x(y^2 - z^2)} = \frac{dy}{y(z^2 - x^2)} = \frac{dz}{z(x^2 - y^2)} \quad -① $$

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4) Eliminating a & b from $z = ax+by+a$

Soln:

Gn: $z = ax+by+a$

D.w.r.t. x & y,

$\Rightarrow \frac{\partial z}{\partial x} = p = a(1)+0+0 \quad \Rightarrow \frac{\partial z}{\partial x} = p = a$

$\Rightarrow \frac{\partial z}{\partial y} = q = 0+b(1)+0 \quad \Rightarrow \frac{\partial z}{\partial y} = q = b$

$\therefore p = a \; ; \; q = b$

Eliminating a & b,

Ans: $z = px+qy+p$.

5) Eliminating a & b from $\frac{x^2+y^2}{a^2} + \frac{z^2}{b^2} = 1$

Soln:

$\frac{x^2+y^2}{a^2} + \frac{z^2}{b^2} = 1$

$b^2(x^2+y^2) + z^2a^2 = a^2b^2$

$0 \cancel{z} = x^2b^2 + b^2y^2 + z^2a^2 - a^2b^2$ —(1)

D.w.r.t. to x in (1),

$\Rightarrow \frac{\partial z}{\partial x} 0 = 2xb^2 + 0 + 2z\frac{\partial z}{\partial x}a^2 - 0$

$\frac{\partial z}{\partial x} - 2z\frac{\partial z}{\partial x}a^2 = 2xb^2$

$\frac{\partial z}{\partial x}(1-2a^2z) = 2xb^2$

$\frac{\partial z}{\partial x} = p = \frac{2xb^2}{1-2a^2z}$.

D.w.r.t. 'y' in (1),

$\Rightarrow \frac{\partial z}{\partial y} = 0 + b^2(2y) + 2z\frac{\partial z}{\partial y}a^2 - 0$

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Unit - 4 :

Variation of parameters.

Consider $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$ — ①

Let y be a soln of,

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0 \quad — ② \quad \text{so that,}$$

$$\frac{d^2y_1}{dx^2} + P\frac{dy_1}{dx} + Qy_1 = 0 \quad — ③$$

Eliminating Q b/w ② & ③ we get,

$$y_1\frac{d^2y}{dx^2} - y\frac{d^2y_1}{dx^2} + P\left[y_1\frac{dy}{dx} - y\frac{dy_1}{dx}\right] = 0$$

The integral is, $y_1\frac{dy}{dx} - y\frac{dy_1}{dx} = Ae^{-\int P dx}$

Hly integrating, $y = By_1 + Ay_1\int \frac{e^{-\int P dx}}{y_1^2}\,dx$

If y_2 denotes $y_1\int \frac{e^{-\int P dx}}{y_1^2}\,dx_1$

$\therefore$ The above relation, $y = By_1 + Ay_2$.

Problems:

1) Solve $\frac{d^2y}{dx^2} + n^2y = \sec nx$.

Soln:

Gn, $\frac{d^2y}{dx^2} + n^2y = \sec nx$ — ①

$$D^2y + n^2y = \sec nx$$

$$(D^2+n^2)y = \sec nx$$

The aux. eqn is, $m^2+n^2 = 0 \Rightarrow m^2 = -n^2$

$$m = \sqrt{-n^2}$$

$$m \pm in$$

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Since, $\frac{\partial z}{\partial x} = p$; $\frac{\partial z}{\partial y} = q$

$\frac{\partial^2 z}{\partial x^2} = t$; $\frac{\partial^2 z}{\partial y^2} = r$

Eliminating a & b b/w we have a partial D.E of the 1st order of the form,

$$F(x, y, z, p, q) = 0.$$

problms (Eg's)

1) Eliminating a & b from $z = (x+a)(y+b)$

Soln:

Gn, $z = (x+a)(y+b)$ —①

D.w.r.t. x & y in eqn ①,

$\frac{\partial z}{\partial x} = p = (1+0)(y+b)$

$\frac{\partial z}{\partial y} = q = (x+a)(1+0)$

$p = (y+b)$; $q = (x+a)$

Eliminating a & b,

$z = (x+a)(y+b)$

$\boxed{z = pq}$

ii) Elimination of an arbitrary function:

Let u & v be any two functions of x, y, z and be connected by an arbitrary relation,

$\phi(u,v) = 0$ — ①

By eliminating ϕ, we shall form a P.D.E and so that this is linear that is of 1st degree in p & q.

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Diff. first eqn partially w.r.t x & y.

$$\frac{\partial\phi}{\partial u}\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial z}p\right)+\frac{\partial\phi}{\partial v}\left(\frac{\partial v}{\partial x}+\frac{\partial v}{\partial z}p\right)=0 \quad -②$$

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$$\frac{\partial\phi}{\partial u}\left(\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}q\right)+\frac{\partial\phi}{\partial v}\left(\frac{\partial v}{\partial y}+\frac{\partial v}{\partial z}q\right)=0 \quad -③$$

Eliminating $\frac{\partial\phi}{\partial u}$ & $\frac{\partial\phi}{\partial v}$ we have, (dividing ② & ③)

$$(u_x+u_z p)(v_y+v_z q)=(u_y+u_z q)(v_x+v_z p)$$

where, $u_x=\frac{\partial u}{\partial x}$, $u_y=\frac{\partial u}{\partial y}$ etc..

This eqn can be put in the form

$Pp+Qq+R$ where $P=u_y v_z-u_z v_y$,

$Q=u_z v_x-u_x v_z$ and $R=u_x v_y-u_y v_x$.

Eg:)

1) Eliminate the arbitrary function from $z=f(x^2+y^2)$

Soln:

Gn, $z=f(x^2+y^2)$ — ①

Diff partially w.r.t x & y in ①,

$$\frac{\partial z}{\partial x}=p=f'(x^2+y^2)(2x) \quad -②$$

$$\frac{\partial z}{\partial y}=q=f'(x^2+y^2)(2y) \quad -③$$

Eliminating ② & ③

$$\frac{p}{q}=\frac{\cancel{f'(x^2+y^2)}(\cancel{2}x)}{\cancel{f'(x^2+y^2)}(\cancel{2}y)}=\frac{x}{y}$$

$$\boxed{py=qx}$$

$=0, z=1.$

$$\frac{dx/x}{y^2+z} = \frac{dy/y}{-(x^2+z)} = \frac{dz/z}{x^2-y^2} = \frac{dx/x + dy/y}{y^2-x^2}$$

taking the last two terms,

$$\frac{dz/z}{x^2-y^2} = \frac{dx/x + dy/y}{y^2-x^2}$$

$$\frac{dz/z}{-(y^2-x^2)} = \frac{dx/x + dy/y}{y^2-x^2}$$

$$-\frac{dz}{z} = \frac{dx}{x} + \frac{dy}{y}$$

$$\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = 0$$

Intg on both sides,

$$\int \frac{dx}{x} + \int \frac{dy}{y} + \int \frac{dz}{z} = 0$$

$$\log x + \log y + \log z = \log b$$

$$\log(xyz) = \log b$$

$$\boxed{xyz = b}$$

The genl soln is $\phi(x^2+y^2-2z, xyz) = 0$.

Since $x^2+y^2-2z = f(xyz)$

Gvn, $x+y=0$ & $z=1$ $\quad (\because (x+y)^2 = x^2+y^2+2xy$

$(x+y)^2 - 2xy - 2z = f(xyz)$ $\quad (x+y)^2 - 2xy = x^2+y^2+2xy - 2xy$

$0 - 2xy - 2(1) = f(xy)$

$\therefore f(xyz) = -2(xyz+1)$

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is called the aux. eqn of (1). if $u=a$ & $v=b$ are two solns of (2) then $\phi(u,v)=0$ is the genl soln of (1).

generally, the aux. eqn can be solved in (2) ways,

i) Method of Grouping

ii) Method of Multiplication.

Method of Grouping:

In the aux. eqn $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$ if the variables can be separated in any pair of eqns, then we get a soln of the form $u(x,y)=a$ and $v(x,y)=b$.

Method of Multipliers:

Choose any three multipliers l,m,n which may be constants or function of x,y,z. we have

$$\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}=\frac{l\,dx+m\,dy+n\,dz}{lP+mQ+nR}.$$

if it is possible to choose l,m,n such that $lP+mQ+nR=0$ then $l\,dx+m\,dy+n\,dz=0$. if $l\,dx+m\,dy+n\,dz$ is an exact differential then on intg. we get a soln, $u=a$

The multipliers l,m,n are called Lagrangian Multipliers.

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5) Solve $px(y^2+z) - qy(x^2+z) = z(x^2-y^2)$. Find the
(5) surface that contains the straight line $x+y=0, z=1$.

Soln:

Given eqn $px(y^2+z) - qy(x^2+z) = z(x^2-y^2)$

The subsidiary eqn are,

$$\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{z(x^2-y^2)} = \frac{x\,dx + y\,dy}{x^2(y^2+z) - y^2(x^2+z)}$$

$$\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{z(x^2-y^2)} = \frac{x\,dx + y\,dy}{\cancel{x^2y^2} + x^2z - \cancel{x^2y^2} - zy^2}$$

$$'' \qquad '' \qquad = \frac{x\,dx + y\,dy}{z(x^2-y^2)}$$

Taking the last two terms,

$$\frac{dz}{z\cancel{(x^2-y^2)}} = \frac{x\,dx + y\,dy}{z\cancel{(x^2-y^2)}}$$

$$dz = x\,dx + y\,dy$$

on intg,

$$z = \frac{x^2}{2} + \frac{y^2}{2} + a$$

$$2z = x^2 + y^2 + 2a$$

$$x^2 + y^2 = 2z - 2a \qquad [\because -2a = a]$$

$$\boxed{x^2 + y^2 \overset{(-)}{+} 2z = a}$$

The subsidiary eqn can also be written as,

$$\frac{\frac{dx}{x}}{y^2+z} = \frac{dy/y}{-(x^2+z)} = \frac{dz/z}{x^2-y^2} = \frac{\frac{dx}{x} + \frac{dy}{y}}{y^2+z - x^2 - z}$$

$$\frac{\partial z}{\partial y} - 2z\frac{\partial z}{\partial y}a^2 = 2yb^2$$

$$\frac{\partial z}{\partial y}(1-2a^2z) = 2yb^2$$

$$\frac{\partial z}{\partial y} = q = \frac{2yb^2}{1-2a^2z}$$

$$\frac{p}{q} = \frac{2xb^2}{1-2a^2z} \times \frac{1-2a^2z}{2yb^2} = x/y$$

$$\boxed{py = xq}$$

6) Eliminating x & y from $z = e^y f(x+y)$

Soln:

Gn, $z = e^y f(x+y)$ —①

D.wrt x & y in ①

$$\Rightarrow \frac{\partial z}{\partial x} = p = 0 + e^y f'(x+y)\cdot 1$$

$$p = e^y f'(x+y)$$

$$\Rightarrow \frac{\partial z}{\partial y} = f(x+y)e^y + e^y f'(x+y)\cdot 1$$

Ans: $q = z + p$

7) Eliminating x & y from the eqn $(x-h)^2 + (y-k)^2 + z^2 = r^2$

Soln:

Gn, $(x-h)^2 + (y-k)^2 + z^2 = r^2$ —①

D.wrt x & y in ①

$$\Rightarrow 2(x-h) + 0 + 2z\frac{\partial z}{\partial x} = 0$$

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$$\frac{dA/dx}{-\sin nx} = \frac{\sec nx}{n}$$

$$\frac{dA}{dx} = \frac{\sec nx}{n}(-\sin nx)$$

$$dA = -\frac{(\sec nx \sin nx)}{n} \cdot dx$$

$$= -\frac{1}{n}\left(\frac{\sin nx}{\cos nx}\right)dx$$

$$dA = -\frac{1}{n}(\tan nx)\,dx$$

on intg,

$$\int dA = -\frac{1}{n}\int \tan nx\,dx$$

$$A = -\frac{1}{n}\left[-\frac{\log(\cos nx)}{n}\right] + a$$

$$\left[A = \frac{\log(\cos nx)}{n^2} + a\right]$$

Now,

$$\frac{dB/dx}{\cos nx} = \frac{\sec nx}{n}$$

$$\frac{dB}{dx} = \frac{\cos nx \cdot \sec nx}{n} = \frac{1}{n}\left[\frac{\cos nx}{\cos nx}\right]dx$$

$$dB = \frac{1}{n}\,dx$$

on intg,

$$\int dB = \frac{1}{n}\int dx$$

$$B = \frac{1}{n}x + b \Rightarrow \left[\frac{x}{n} + b = B\right]$$

The soln is, $y = A\cos nx + B\sin nx$

$$y = \left[\frac{\log(\cos nx)}{n^2} + a\right]\cos nx + \left[\frac{x}{n} + b\right]\sin nx \quad //.$$

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Since $\frac{1}{x}+\frac{1}{z}=f\left(\frac{1}{y}+\frac{1}{z}\right)$ where f is arbitrary.

If this surface is passes through the hyperbola

$xy=x+y,\ z=1$ we have,

$$\frac{1}{x}+1=f\left(\frac{1}{y}+1\right)$$

from $xy=x+y$

$$1=\frac{x+y}{xy} \Rightarrow \frac{x}{xy}+\frac{y}{xy}=\frac{1}{y}+\frac{1}{x}$$

$$1=\frac{1}{x}+\frac{1}{y}$$

$$\boxed{\frac{1}{x}=1-\frac{1}{y}}$$

$$\Rightarrow \frac{1}{x}+1=1-\frac{1}{y}+1 \Rightarrow 2-\frac{1}{y}$$

$$2+1-1-\frac{1}{y}=3-1-\frac{1}{y}$$

$$=3-\left(1+\frac{1}{y}\right)$$

$$\therefore f\left(\frac{1}{y}+1\right)=3-\left(1+\frac{1}{y}\right)$$

$$f\left(\frac{1}{y}+\frac{1}{z}\right)=3-\left(\frac{1}{z}+\frac{1}{y}\right)$$

Ans:

Hence the req. surface is,

$$\left(\frac{1}{x}+\frac{1}{z}\right)=3-\left(\frac{1}{y}+\frac{1}{z}\right).$$

5.) $(x^2-y^2-z^2)p+2xyq=2xz$

Soln:

Gn, $(x^2-y^2-z^2)p+2xyq=2xz$

The subsidiary eqn are,

$$\frac{dx}{x^2-y^2-z^2}=\frac{dy}{2xy}=\frac{dz}{2xz}$$

$$2(x-h)+2zp=0 \quad -(2)$$

$$\Rightarrow 2(y-k)(1)+0+2z\frac{\partial z}{\partial y}=0$$

$$2(y-k)+2zq=0 \quad -(3)$$

$(2) \Rightarrow \quad 2(x-h) = -2(zp)$

$(3) \Rightarrow \quad 2(y-k) = -2(zq)$

Eliminating (2) & (3).

$$\frac{2(x-h)}{2(y-k)} = \frac{-2(zp)}{-2(zq)}$$

$$\frac{x-h}{y-k} = zp/zq$$

$$x-h=zp \;;\quad y-k=zq$$

Eliminate x & y in (1) eqn,

$$(z^2p^2)+(z^2q^2)+z^2=r^2$$

$$z^2(p^2+q^2+1)=r^2 \;//.$$

Lagrange's Equation:

A partial diff. eqn which is linear in p & q is of the form,

$$Pp+Qq=R \quad -(1)$$

where P, Q, R are functions of x, y, z. this is called Lagrange's linear eqn's.

The system of eqn's, $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R} \quad -(2)$

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partial differential eqn of the first order classification of integral ;

Let the partial D.F be

$F(x, y, z, p, q) = 0$ —①

Let the soln of this be,

$\phi(x, y, z, a, b) = 0$ —②

Where a and b are constants.

The soln of eqn (2) which contains as many constants as there are independent variables is called the comp. Intg of eqn ①.

Singular Integral ;

The elimination of a & b b/w $\phi(x, y, z, a, b) = 0$.

$$\frac{\partial \phi}{\partial a} = 0 \ \& \ \frac{\partial \phi}{\partial b} = 0$$

When it exists is called the sing. Intg.

Deviation of partial D.F ;

i) By elimination of constant ;

Let $\phi(x, y, z, a, b) = 0$ —①

put eqn ① be a relation b/w x, y, z involving two constants a & b.

Diwrt. 'x' & 'y' in ① we get,

$$\frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial z} p = 0 \quad —②$$

$$\frac{\partial \phi}{\partial y} + \frac{\partial \phi}{\partial z} q = 0 \quad —③$$

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2) Eliminate the arbitrary funct. f from

$f(x^2+y^2+z^2,\ z^2-2xy)=0.$

Soln:

Gn, $f(x^2+y^2+z^2,\ z^2-2xy)=0$

Solving, $x^2+y^2+z^2 = F(z^2-2xy)$ — ①

D.w.r.t x & y in eqn ①,

$\Rightarrow 2x+0+2z\frac{\partial z}{\partial x} = F'(z^2-2xy)(2z\frac{\partial z}{\partial x}-2y)$

$2x+2zp = F'(z^2-2xy)(2zp-2y)$ — ②

Now, diff. 'y'

$\Rightarrow 0+2y+2z\frac{\partial z}{\partial y} = F'(z^2-2xy)(2z\frac{\partial z}{\partial x}-2x)$

$2y+2zq = F'(z^2-2xy)(2zq-2x)$ — ③

Eliminating ② & ③,

$$\frac{2(x+zp)}{2(y+zq)} = \frac{F'(z^2-2xy)\,2(zp-y)}{F'(z^2-2xy)\,2(zq-x)}$$

$$\frac{x+zp}{y+zq} = \frac{zp-y}{zq-x}$$

$(x+zp)(zq-x) = (zp-y)(y+zq)$

$xzq-x^2+z^2pq-xzp = yzp-y^2-yzq+z^2pq$

$xz(q-p)-x^2 = yz(p-q)-y^2$

$-xz(p-q) = yz(p-q)-y^2+x^2$

$-xz(p-q)-yz(p-q) = x^2-y^2$

$-z(p-q)(x+y) = (x+y)(x-y)$

$-z(p-q)\cancel{(x+y)} = \cancel{(x+y)}(x-y)$

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$$\frac{dA/dx}{-\sin nx} = \frac{\sec nx}{n}$$

$$\frac{dA}{dx} = \frac{\sec nx}{n}(-\sin nx)$$

$$dA = -\frac{(\sec nx \sin nx)}{n} \cdot dx$$

$$= -\frac{1}{n}\left(\frac{\sin nx}{\cos nx}\right)dx$$

$$dA = -\frac{1}{n}(\tan nx)\,dx$$

on intg,

$$\int dA = -\frac{1}{n}\int \tan nx\,dx$$

$$A = -\frac{1}{n}\left[-\frac{\log(\cos nx)}{n}\right] + a$$

$$\left[A = \frac{\log(\cos nx)}{n^2} + a\right]$$

Now,

$$\frac{dB/dx}{\cos nx} = \frac{\sec nx}{n}$$

$$\frac{dB}{dx} = \frac{\cos nx \cdot \sec nx}{n} = \frac{1}{n}\left[\frac{\cos nx}{\cos nx}\right]dx$$

$$dB = \frac{1}{n}dx$$

on intg,

$$\int dB = \frac{1}{n}\int dx$$

$$B = \frac{1}{n}x + b \implies \left[\frac{x}{n} + b = B\right]$$

The soln is, $y = A\cos nx + B\sin nx$

$$y = \left[\frac{\log(\cos nx)}{n^2} + a\right]\cos nx + \left[\frac{x}{n} + b\right]\sin nx \quad //.$$

partial diff...
of integral
Let the p...
$F(x, y, \ldots$
Let the s...
$\phi(x, y, \ldots$
Where a...
The s...
constants ...
is called ...

Singular Inte...
The ...
When ...

Deviation o...
i) By ...
put ...
two con...
Diw...

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