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SOLUTION FOR

P.T.A MODEL QUESTIONS

10

MATHEMATICS

PREPARED BY

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STUDENTS

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Model Paper - I

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PART I

1) (a) (8,6)
2) (a) 1
3) (d) do not intersect
4) (c) $BD \cdot CD = AD^2$
5) (b) Parallel to y axis
6) (b) 7
7) (c) $\frac{8\pi h^2}{a}$
8) (a) $P(A) > 1$
9) (d) Both A.P and G.P
10) (c) $(x-a)^2(x^2+ax+a^2)$
11) (d) 2^{pq}
12) (b) 2
13) (c) zero
14) (b) kaleidoscope

PART-2

15) $a = 532$; $b = 21$

$a = bq + r$

$532 = 21q + r$

$532 = 21(25) + 7$

completed row = 25
remaining row = 7

16) $x^2 = a$; $x^4 - 13x^2 + 42 = 0$

$(x^2)^2 - 13x^2 + 42 = 0$

$a^2 - 13a + 42 = 0$

$(a-7)(a-6) = 0$ $\quad \therefore a = x^2$

$x^2 = 7$ $\qquad x^2 = 6$

$x = \pm 7$ $\qquad x = \pm 6$

17) order of $A = p \times q$

order of $B = q \times r$

order of $AB = p \times r$

order of BA = not

18) $f(x) = x^2 - 2$; $x \in \{-2, -1, 0, 3\}$

$f(-2) = (-2)^2 - 2$
$= 4 - 2$
$= 2$

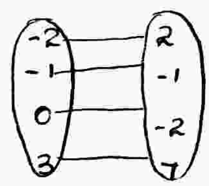
$f(-1) = (-1)^2 - 2$
$= 1 - 2$
$= -1$

$f(0) = 0 - 2$
$= -2$

$f(3) = 3^2 - 2$
$= 9 - 2$
$= 7$

(i) element of f = 2, -1, -2, 7.

(ii)



$\therefore$ All having unique image, so it is a function.

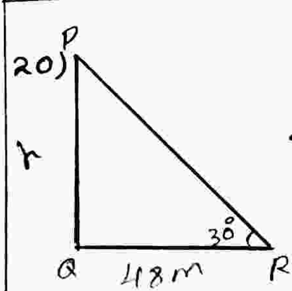
19) $\frac{PS}{PQ} = \frac{PR}{PR}$

$\frac{2}{2+1} = \frac{4}{4+2} = \frac{\cancel{4}}{\cancel{6}} = \frac{2}{3}$

$\frac{2}{3} = \frac{2}{3}$

$\angle P$ is common $\therefore \Delta PST \sim \Delta PQR$

20)



$\tan\theta = \frac{PQ}{QR}$

$\tan 30° = \frac{h}{48}$

$\frac{1}{\sqrt{3}} \times \frac{h}{48} \Rightarrow h\sqrt{3} = 48$

$h = \frac{\cancel{48}^{16}}{\cancel{3} \times 3} = h = 16\sqrt{3}$

$\therefore h = 16\sqrt{3}$

21) $h = 24$ cm ; volume of cone $= 11088$ sq cm

volume of cone $= 11088$

$\frac{1}{3}\pi r^2 h = 11088$

$\frac{1}{3} \times \frac{22}{7} \times r^2 \times \overset{8}{\cancel{24}} = 11088$

$r^2 = \cancel{11088} \times \frac{7}{\cancel{22}} \times \frac{1}{8}$

$r^2 = 63 \times 7$

$= 441$ $\quad \therefore r = 21$ cm

22) $P(A) = \frac{2}{3}$; $P(B) = \frac{2}{5}$; $P(A \cup B) = \frac{1}{3}$

$P(A \cap B) = ?$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

$\frac{1}{3} = \frac{2}{3} + \frac{2}{5} - P(A \cap B)$

$P(A \cap B) = \frac{2}{3} + \frac{2}{5} - \frac{1}{3} = \frac{10+6-5}{15}$

$= \frac{11}{15}$

23) $A(m,n)$; $B = \phi$

(i) $A \times B = (m \times n) \times \phi$

$= \phi$

(ii) $A \times A = \{m, n) \times (m, n]$

$= \{(m,m), (m,n), (n,m), (n,n)\}$

24) $n = \left(\frac{l-a}{d}\right) + 1$

$= \left(\frac{183-9}{6}\right) + 1 = \frac{\overset{29}{\cancel{174}}}{\cancel{6}} + 1$

$= 29 + 1$

$\therefore n$ is even.

$\frac{n}{2}, \frac{n}{2}+1 \Rightarrow \frac{30}{2}, \frac{30}{2}+1$

$\Rightarrow 15, 16$

$t_{15} = a + 14d$ $\qquad t_{16} = a + 15d$

$= 9 + (14)(6)$ $\qquad = 9 + 15(6)$

$= 9 + 84$ $\qquad = 9 + 90$

$= 93$ $\qquad = 99$

$\therefore t_{15} = 93$; $t_{16} = 99$

25) Kumaran's age $= x$

2 years ago $= (x-2)$

after 4 years $= (x+4)$

$(x-2)(x+4) = 1 + 2x$

$x^2 + 2x - 8 = 1 + 2x$

$x^2 + 2x - 8 - 1 - 2x = 0$

$x^2 + 0x - 9$

$(x-3)(x+3) = 0$

$x = 3$ $\therefore$ Kumaran age $= 3$

26) $(4, -3)$; $m = \frac{-7}{5}$

$y - y_1 = m(x - x_1)$

$y + 3 = -\frac{7}{5}(x-4)$

$5(y+3) = -\frac{7}{5}(x-4)$

$5y + 15 = -7x + 28$

$7x + 5y + 15 - 28 = 0$

$7x + 5y - 13 = 0$

27) The standard deviation of 20 observation $= \sqrt{6}$

If multiple of 3, The standard deviation $= 3\sqrt{6}$

$$=(3\sqrt{6})^2$$
$$= 9 \times 6$$
$$= 54$$

28) $n = 25$

$r = \frac{t_2}{t_1} = \frac{3}{1} = 3$

$1 + 3 + 9 + \ldots\ldots n$

$S_n = \frac{a(r^n - 1)}{r-1}$

$= \frac{1 \times 3^{25} - 1}{3-1} = \frac{3^{25} - 1}{2}$

29) $t(c) = f;\ F = \frac{9}{5}c + 32.$

(i) $t(0) = \frac{9}{5}(0) + 32.$

$= 32$

(ii) $t(28) = \frac{9}{5}(28) + 32$

$= \frac{\cancel{252}^{50.4}}{\cancel{5}} + 32 = 82.4$

(iii) $t(c) = 212$

$\frac{9}{5}c + 32 = 212 \Rightarrow \frac{9}{5}c = 212 - 32$

$= \frac{9}{5}c = 180$

$c = \frac{180 \times 5}{9}$

$c = 100°$

(iv) $c = F;\ c = \frac{9}{5}c + 32$

$c - \frac{9}{5}c = 32$

$\frac{5c - 9c}{5} = 32. \Rightarrow \frac{-4c}{5} = 32$

$= -4c = 32 \times 5$

$c = \frac{\cancel{160}^{40}}{\cancel{-4}}$ $c = -40$

$\therefore c = -40$

30) $10^2 + 11^2 + 12^2 + \ldots\ldots\ldots + 24^2$

$= 1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + 24^2)$

$- (1^2 + 2^2 + \ldots\ldots + 9^2)$

$S_n = \frac{n(n+1)(2n+1)}{6}$

$S_{24} - S_9 = \frac{24(24+1)(2(24)+1)}{6} - \frac{9(9+1)(2(9)+1)}{6}$

$= \frac{24 \times 25 \times 49}{6} - \frac{9 \times 10 \times 19}{6}$

$= 4900 - 285$

$= 4615$ sq. cm.

31) $A(B+C) = AB + AC$

$B + C = \begin{bmatrix} 1 & 2 \\ -4 & 2 \end{bmatrix} + \begin{bmatrix} -7 & 6 \\ 3 & 2 \end{bmatrix}$

$= \begin{bmatrix} -6 & 8 \\ -1 & 4 \end{bmatrix}$

$A(B+C) = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix} \times \begin{bmatrix} -6 & 8 \\ -1 & 4 \end{bmatrix}$

$= \begin{bmatrix} -6-1 & 8+4 \\ 6-3 & -8+12 \end{bmatrix}$

$= \begin{bmatrix} -7 & 12 \\ 3 & 4 \end{bmatrix} \to (1)$

$AB + AC$

$AB = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ -4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1-4 & 2+2 \\ -1-12 & -2+6 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ -13 & 4 \end{bmatrix}$

$AC = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix} \times \begin{bmatrix} -7 & 6 \\ 3 & 2 \end{bmatrix}$

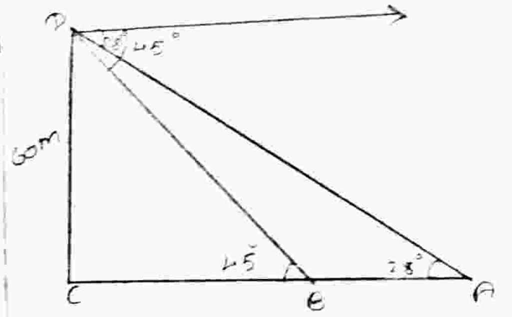
$= \begin{bmatrix} -7+3 & 6+2 \\ 7+9 & -6+6 \end{bmatrix} = \begin{bmatrix} -4 & 8 \\ 16 & 0 \end{bmatrix}$

$AB + AC = \begin{bmatrix} -3 & 4 \\ -13 & 4 \end{bmatrix} + \begin{bmatrix} -4 & 8 \\ 16 & 0 \end{bmatrix}$

$= \begin{bmatrix} -7 & 12 \\ 3 & 4 \end{bmatrix} \Rightarrow (2)$

$\therefore$ Hence Proved.

33)



$\tan 45^\circ = \frac{DC}{BC} = \frac{60}{BC}$

$BC = 60$

$\tan 28^\circ = \frac{DC}{AC}$

$0.5317 = \frac{60}{AC} \Rightarrow AC = \frac{60}{0.5317}$

$= 112.85$

$AB = AC - BC$

$= 52.85m.$

34) cylinder

$h = 10\,cm$

$d = 4.5\,cm;$

$r = 2.25\,cm$

circle

$d = 1.5\,cm$

$r = 0.75\,cm$

thick $h = 2mm$

$= 0.2cm$

$= \frac{\text{volume of cylinder}}{\text{volume of sphere}}$

$= \frac{2.25 \times 2.25 \times 10}{0.75 \times 0.75 \times 0.2}$

$= \frac{\cancel{22.5}^{4.5} \times 22.5 \times \cancel{10}^{5} \times 10^2}{\cancel{75}_{15} \times \cancel{75}_{15} \times \cancel{2}}$

$= 225 \times 2$

$= 450$ coin

35) $A = 8$

x	f	$d = x - A$	fd	fd^2
4	7	-4	-28	112
6	3	-2	-6	12
8	5	0	0	0
10	9	2	18	36
12	5	4	20	80
			$\Sigma fd = 4$	$\Sigma fd^2 = 240$

$\sigma = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left[\frac{\Sigma fd}{\Sigma f}\right]^2}$

$= \sqrt{\frac{240}{29} - \left(\frac{4}{29}\right)^2}$

$= \sqrt{\frac{240 \times 29 - 16}{29 \times 29}}$

$= \sqrt{\frac{6944}{29 \times 29}}$

$\sigma \simeq 2.87$

36) $A = \{1, 2, 3, 4, 5, 6, 7\}$

$B = \{2, 3, 5, 7\}$

$C = \{2\}$

LHS

$A(B-C)$: $B - C = \{2, 3, 5, 7\} - \{2\}$

$= \{3, 5, 7\}$

$A(B-C) = \{1, 2, 3, 4, 5, 6, 7\} \times \{3, 5, 7\}$

$= \{(1,3)\ (2,3)\ (3,3)\ (4,3)$
$(5,3)\ (6,3)\ (7,3))\ (1,5)\ (2,5)\ (3,5)$
$(4,5)\ (6,5)\ (7,5)\ (1,7)\ (2,7)$
$(3,7)\ (4,7)\ (5,7)\ (6,7)\ (7,7)$

$\hookrightarrow$ 1

RHS

$A\times B = \{1,2,3,4,5,6,7\} \times \{2,3,5,7\}$
$= \{(1,2)(2,2)(3,2)(4,2)(5,2)$
$(6,2)(7,2)(1,3)(2,3)(3,3)(4,3)$
$(5,3)\ (6,3)\ (7,3)(1,5)(2,5)(3,5)$
$(4,5)\ (5,5)\ (6,5)\ (7,5)\ \ (7,7)$
$(1,7)\ (2,7)(3,7)\ (4,7)\ (5,7)\ (6,7)$

$A\times C = \{1,2,3,4,5,6,7\} \times \{2\}$
$= \{(1,2)(2,2)(3,2)(4,2)(5,2)(6,2)$
$(7,2)\}$

$(A\times B)-(A\times C) = \{(1,3)\ (2,3)\ (4,3)$
$(5,3)\ (6,3)\ (7,3)\ (1,5)\ (2,5)$
$(3,5)\ (4,5)\ (6,5)\ (7,5)\ (1,7)\ (2,7)$
$(3,7)\ (4,7)\ (5,7)\ (6,7)\ (7,7) \rightarrow$ 2

LHS = RHS

) squaring on both side $(+x-y)$

$(x-y)S_n = (x-y)(x+y) + (x-y)(x^2+xy+y^2) + (x-y)(x^3+x^2y+xy^2-y^3) + n$ terms

$= (x^2-y^2)+(x^3-y^3)+(x^4-y^4)+$ n terms

$= (x^2+x^3+x^4+\dots\dots + n \text{ term}) - (y^2+y^3+y^4+\dots\dots + n \text{ term})$

$(x-y)S_n = \dfrac{x(x^n-1)}{x-1} - \dfrac{y(y^n-1)}{y-1}$

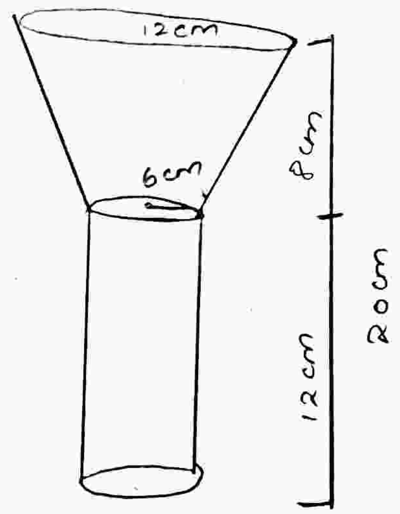
$S_n = \dfrac{a(r^n-1)}{r-1}$

$a = x^2$

$r = \dfrac{x^3}{x^2} = x$

3a) $R = 12cm;\ r = 6cm;\ h_2 = 12cm$

$h_1 = 20-12$
$= 8cm$



$l = \sqrt{(R-r)^2 + h_1^2}$
$= \sqrt{(6)^2 + 8^2}$
$= \sqrt{36+64} = \sqrt{100}$
$l = 10cm$

other surface area $= 2\pi rh + \pi(R+r)l$
$= \pi(2\pi h_2 + (R+r)l)$
$= \frac{22}{7}[(2\times 6\times 12)+(18\times 10)$
$= \pi[144\times 180]$
$= \frac{22}{7}\times 324.$
$= 1018.28$

38) $\frac{1}{x} = p$; $\frac{1}{y} = q$; $\frac{1}{z} = r$

$$\frac{p}{2} + \frac{q}{4} - \frac{r}{3} = \frac{1}{4}$$

x by 12 $\Rightarrow 6p + 3q - 4r = 3 \rightarrow$ (1)

$$p = \frac{q}{3} \rightarrow (2)$$

$$p - \frac{q}{5} + 4r = \frac{32}{15}$$

$$15p - 3q + 60r = 32 \rightarrow (3)$$

By simplify

$$6p + 3q - 4r = 3 \rightarrow (1)$$

$$3p = q \rightarrow (2)$$

$$15p - 3q + 60r = 32 \rightarrow (3)$$

Substitute (2) in (1)

$$15p - 4r = 3 \rightarrow (4)$$

$\div$ by 2 in (3)

$$15p - 3q + 60r = 32$$

$$15p - 3(3p) + 60r = 32$$

$$6p + 60r = 32.$$

$\div$ by 2

$$3p + 30r = 16 \Rightarrow (5)$$

solve (4) + (5)

$$15p - 4r = 3$$

$$15p + 150r = 80$$

$$(-)\quad (-)\quad (-)$$

$$-154r = -77$$

$r = \frac{1}{2}$ sub in (5)

$3p + 30\cdot\frac{1}{2} = 16 \Rightarrow p = \frac{1}{3}$

$p = \frac{1}{3}$ in (2) $\quad q = 1 \quad \therefore x = 3$

$y = 1$

$r = \frac{1}{2}$; $p = \frac{1}{3}$; $q = 3/p \quad z = 2$

40) $n(A) = 28$; $n(B) = 30$;

$n(A \cap B) = 18$

$$P(A) = \frac{n(A)}{n(S)} = \frac{28}{50}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{30}{50}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{18}{50}$$

i) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$$= \frac{28}{50} - \frac{18}{50} = \frac{10}{50}$$

$$= \frac{1}{5}$$

ii) $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$$= \frac{30}{50} - \frac{18}{50} = \frac{6}{25}$$

iii) $P(A \cap \bar{B}) \cup (\bar{A} \cap B)$

$$= P(A \cap \bar{B}) + P(\bar{A} \cap B)$$

$$= \frac{1}{5} + \frac{6}{25} = \frac{11}{25}$$

41) let the base = b; let the height = h

Area $= \frac{1}{2} \times b \times h = 48$ sq.cm

base $b = h + 4$ (gn)

$$\frac{1}{2} \times (h+4) h = 48$$

$$h^2 + 4h - 96 = 0$$

$$(h+12)(h-8) = 0$$

$$h = 8$$

base $= 8 + 4$

$= 12$cm

Area of a triangle = 5 sq. unit

$$\frac{1}{2}\left[(x_1y_2+x_2y_3+x_3y_1)-(x_2y_1+x_3y_2+x_1y_3)\right]=5$$

$$\frac{1}{2}\left[(-4+3y+x)-(3-2x+2y)\right]=5$$

$$-4+3y+x-3+2x-2y=10$$

$$3x+y-7=10$$

$$3x+y=17 \rightarrow (1)$$

Gn: $y=x+3$

$y=x+3$ in (1)

$$3x+x+3=17$$

$$4x=17-3$$

$$4x=14$$

$$x=\frac{14}{4}=\frac{7}{2}$$

sub x in $y=x+3$

$$y=\frac{7}{2}+3 \Rightarrow y=\frac{7+6}{2}=\frac{13}{2}$$

Third vertex is $\left[\frac{7}{2}, \frac{13}{2}\right]$

PART-IV

43) (b)

$$2x+y+4z=15 \rightarrow (1)$$

$$x-2y+3z=13 \rightarrow (2)$$

$$3x+y-z=2 \rightarrow (3)$$

$$(1) \Rightarrow 2x+y+\cancel{4z}=15$$

$$(3)\times 4 \quad 12x+4y-\cancel{4z}=8$$

$$14x+5y=23 \rightarrow (4)$$

$$(3)\times 3 \Rightarrow 9x+3y-\cancel{3z}=6$$

$$10x+y=19 \rightarrow (5)$$

$$(4) \rightarrow 14x+5y=23$$

$$(5)\times 5 \quad \underset{(-)}{50x}+\underset{(-)}{\cancel{5y}}=\underset{(-)}{95}$$

$$-36x=-72$$

$$x=2$$

sub $x=2$ in (5)

$$10(2)+y=19$$

$$20+y=19$$

$$y=19-20 \quad \therefore y=1$$

sub x and y in (1)

$$2(2)-1+4z=15$$

$$3+4z=15$$

$$4z=15-3$$

$$4z=12$$

$$z=3$$

$x=2;\ y=1;\ z=3$

44) b) By menelaus theorem

$$\frac{AE}{EB}\times\frac{BF}{FC}\times\frac{CD}{DA}=1$$

$AE=EB=2;\ DA=1$

$$FC=FB+BC$$

$$=BF+3$$

By Pyth.

$$AC^2=AB^2+BC^2$$

$$25+16+9$$

$$AC=5$$

$$CD=AC-AD$$

$= 5 - 1$

$= 4$

Sub FC, AE, EB, BA, CD in

$$\frac{2}{2} \times \frac{BF}{BF+3} \times \frac{4}{1} = 1$$

$$4BF = BF + 3$$

$$4BF - BF = 3$$

$$BF = 1$$

43)(a) graph.

$y = x^2 + x - 2$

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
x	-3	-2	-1	0	1	2	3
-2	-2	-2	-2	-2	-2	-2	-2
y	4	0	-2	-2	0	4	10

Pts are (-3,4) (-2,0) (-1,-2) (0,-2) (1,0) (2,4) (3,10)

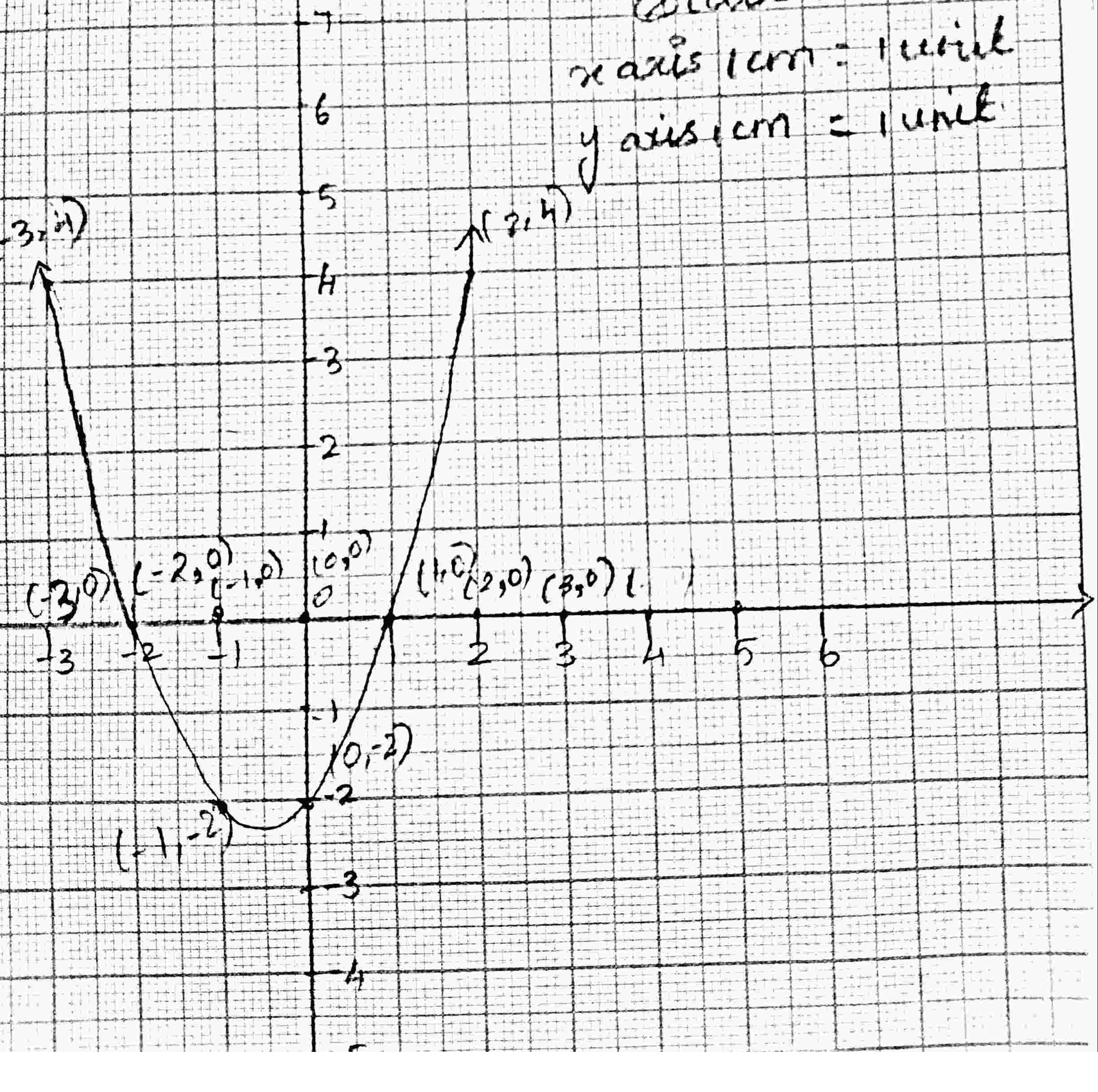
$y = x^2 + x - 2$

$0 = x^2 + \underset{0}{\underline{x - 2}}$

x	-3	-2	-1	0	1	2	3	4
y	0	0	0	0	0	0	0	0

Pts are (-3,0) (-2,0) (-1,0) (0,0) (1,0) (2,0) (3,0) a-9

Scale
x axis 1cm = 1 unit
y axis 1cm = 1 unit



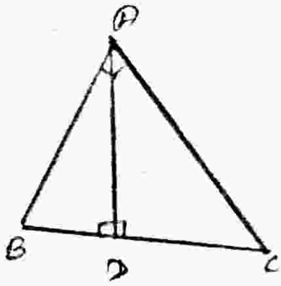
32)

Pythagoras theorem:

Statement:

In a right angled triangle, the square on the hypotenus is equal to the sum of the square on the other sides

Proof:



Given : In $\triangle ABC$, $\angle B = 90^\circ$

To Prove : $AB^2 + AC^2 = BC^2$

construction : Draw $AD \perp BC$

Statement	Reason.
Compare $\triangle ABC$ and $\triangle ABD$ $\angle B$ is common $\angle BAC = \angle BDA = 90^\circ$ Therefore, $\triangle ABC \sim \triangle ABD$ $\frac{AB}{BD} = \frac{BC}{AB}$ $AB^2 = BC \times BD \rightarrow (1)$	Given $\angle BAC = 90^\circ$ and by construction $\angle BDA = 90^\circ$ By AA similarity
Compare $\triangle ABC$ and $\triangle ADC$ $\angle C$ is common $\angle BAC = \angle ADC = 90^\circ$ Therefore, $\triangle ABC \sim \triangle ADC$ $\frac{BC}{AC} = \frac{AC}{DC}$ $AC^2 = BC \times DC \rightarrow (2)$	Given $\angle BAC = 90^\circ$ and by construction $\angle CDA = 90^\circ$ By AA similarity.

Adding (1) + (2)

$$AB^2 + AC^2 = BC \times BD + BC \times DC$$

$$= BC(BD + DC) = BC \times BC$$

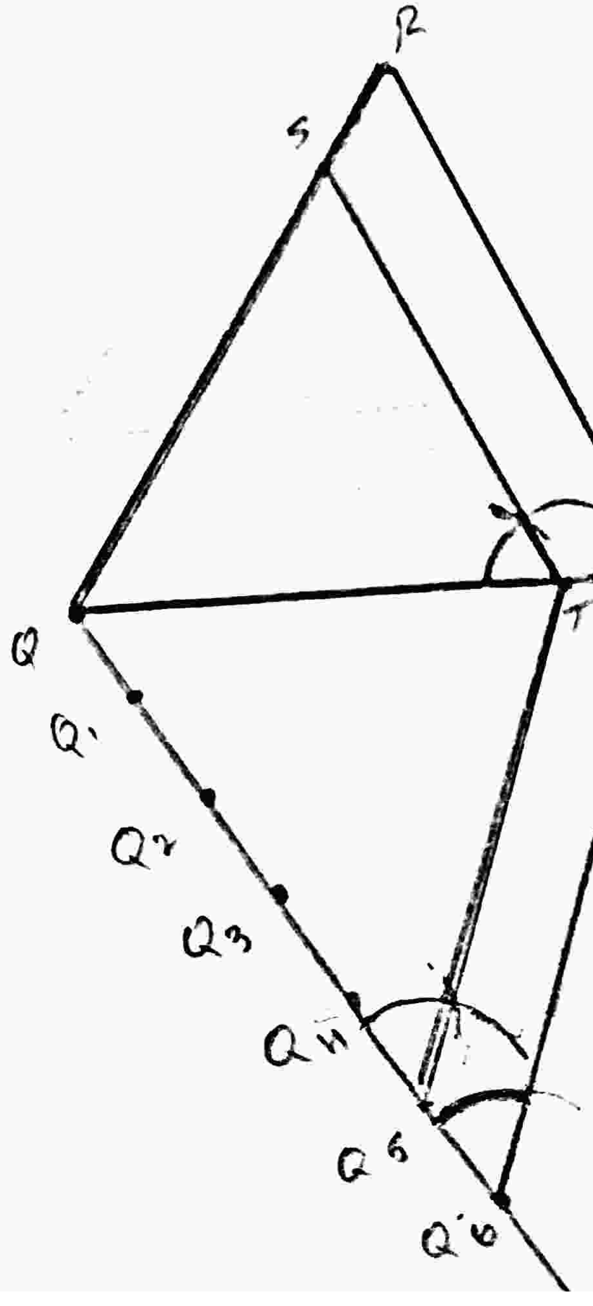
$$AB^2 + AC^2 = BC^2$$

$\therefore$ Hence Proved.

a=11

44) geometry:

a)



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PTA MODEL - 5

PART - I

1) d) quadratic
2) a) 0, 1, 8
3) c) 31 m
4) a) $\frac{94}{7}$
5) ~~a)~~ ~~2×3~~ real and unequal roots
6) a) 2×3
7) d) 15 cm.
8) c) 9
9) b) 7
10) b) $b^2 - a^2$
11) d) 3:1:2
12) a) TSA of solid sphere
13) c) 32·25
14) a) $P(A) > 1$

PART - II

15)
i) The range of f.

$f(x) = x^2$

$f(1) = 1^2 = 1$

$f(2) = 2^2 = 4$

$f(3) = 3^2 = 9$

$f(4) = 4^2 = 16$

Range of f is $\{1, 4, 9, 16\}$

ii) It is one - one function

16) $t_2 - t_1 = t_3 - t_2$

$18 - k - (3 + k) = 5k + 1 - (18 - k)$

$18 - k - 3 - k = 5k + 1 - 18 + k$

$15 - 2k = 6k - 17$

$32 = 8k$

$\therefore k = \frac{32}{8}$

$k = 4$

17) $a = -7$, $r = 6$

$G \cdot P = a, ar, ar^2 \ldots\ldots$

$ar = -7 \times 6 = -42$

$ar^2 = -7 \times 6^2 = -252$

The required $G \cdot P$ is

$-7, -42, -252 \ldots\ldots$

18) $$\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}} = \sqrt{\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}}$$

$$= \frac{4}{3}\left|\frac{a^4b^6c^8}{f^6g^2h^7}\right|$$

19) $21, 18, 15, \ldots\ldots$

$a = 21$, $d = 18 - 21$, $d = -3$

$t_n = -81$

$t_n = a + (n-1)d$

$-81 = 21 + (n-1)(-3)$

$-81 - 21 = (n-1)(-3)$

$\frac{-102}{-3} = n - 1$

$n - 1 = 34$ $\quad n = 35$

$t_n = 0$

$0 = 21 + (n-1)(-3)$

$-21 = (n-1)(-3)$

$\frac{-21}{-3} = n - 1$

$n - 1 = 7$ $\quad n = 8$

$t_8 = 0$

It is a term of A.P

20) $y = x + 3$

$x \in \{0, 1, 2, 3, 4, 5\}$

$y = 0 + 3 = 3$

$y = 1 + 3 = 4$

$y = 2 + 3 = 5$

$y = 3 + 3 = 6$

$y = 4 + 3 = 7$

$y = 5 + 3 = 8$.

i) Domain $= \{0, 1, 2, 3, 4, 5\}$

ii) Range $= \{3, 4, 5, 6, 7, 8\}$

21) $3A - 9B =$

$$= 3\begin{bmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{bmatrix} - 9\begin{bmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 12 & 27 \\ 24 & 9 & 21 \end{bmatrix} - \begin{bmatrix} 63 & 27 & 72 \\ 9 & 36 & 81 \end{bmatrix}$$

$$= \begin{bmatrix} -63 & -15 & -45 \\ 15 & -27 & -60 \end{bmatrix}$$

22)

By Angle Bisector Theorem

$$\frac{AB}{BD} = \frac{AC}{DC} \Rightarrow \frac{6}{4} = \frac{x}{3}$$

$\Rightarrow 18 = 4x$

$\Rightarrow \frac{18}{4} = x$

$\Rightarrow x = 4.5\text{cm}$

23) Slope of $x - 2y + 3 = 0$.

Slope $= \frac{-\text{coefficient of } x}{\text{coefficient of } y}$

$m_1 = \frac{-1}{-2} = \frac{1}{2}$.

Slope of $6x + 3y + 8 = 0$.

$m_2 = \frac{-6}{3} = 2$.

$m_1 \times m_2 = \frac{1}{2} \times (-2) = -1$

Hence perpendicular

24. $\dfrac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}=\left(\dfrac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}\right)\left(\dfrac{\sec\theta-\tan\theta}{\sec\theta-\tan\theta}\right)$

$=\left(\dfrac{\sec\theta-\tan\theta}{\sec^2\theta-\tan^2\theta}\right)$

$\dfrac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}=\dfrac{(\sec\theta-\tan\theta)^2}{1}$

$\sqrt{\dfrac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}}=\sec\theta-\tan\theta$

$=\dfrac{1}{\cos\theta}-\dfrac{\sin\theta}{\cos\theta}$

$=\dfrac{1-\sin\theta}{\cos\theta}$

25) $h=45\,cm$, $R=28\,cm$, $r=7\,cm$

Volume $=\frac{1}{3}\pi(R^2+Rr+r^2)h$ cu·units

$=\frac{1}{3}\times\frac{22}{7}\times\left[28^2+(28\times7)+7^2\right]\times45$

$=\frac{1}{3}\times\frac{22}{7}\times1029\times45$

$=48510\ cm^3$

26) $L=28$

$S=18$

$R=L-S$

$R=28-18$

$R=10$ years.

27) i) Atleast one tail

$A\rightarrow$ Atleast one tail

$A=\{HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(A)=7$

$P(A)=\dfrac{n(A)}{n(S)}=\dfrac{7}{8}$.

ii) Atmost one head

$B\rightarrow$ Atmost one head

$B=\{HTT, THT, TTH, TTT\}$

$n(B)=4$.

$P(B)=\dfrac{n(B)}{n(S)}=\dfrac{4}{8}=\dfrac{1}{2}$.

28. $px^2+(\sqrt{3}-\sqrt{3})x-1=0$

$x=\frac{1}{\sqrt{3}}$

$P\left(\frac{1}{\sqrt{3}}\right)^2+(\sqrt{3}-\sqrt{2})\left(\frac{1}{\sqrt{3}}\right)-1=0$

$P\left(\frac{1}{3}\right)+\frac{\sqrt{3}}{\sqrt{3}}-\frac{\sqrt{2}}{\sqrt{3}}-1=0$

$\frac{P}{3}+1-\frac{\sqrt{2}}{3}-1=0$

$\frac{P}{3}=\frac{\sqrt{2}}{\sqrt{3}}$

$P=\frac{3\sqrt{2}}{\sqrt{3}}$

$P=\frac{\sqrt{3}\sqrt{3}\sqrt{2}}{\sqrt{3}}$

$P=\sqrt{6}$

29) Gn:

$A = \{0,1\} \quad B = \{2,3,4\}$

$C = \{3,5\}$.

To find:

$A\times(B\cap C) = (A\times B)\cap(A\times C)$

LHS.

$A\times(B\cap C)$

$B\cap C = \{2,3,4\}\cap\{3,5\}$.

$= \{3\}$

$A\times(B\cap C) = \{0,1\}\times\{3\}$

$= \{(0,3),(1,3)\} \rightarrow 1$

$A\times B = \{0,1\}\times\{2,3,4\}$

$= \{(0,2),(0,3),(0,4),$
$(1,2),(1,3),(1,4)\}$

$A\times C = \{0,1\}\times\{3,5\}$.

$= \{(0,3),(0,5),(1,3),(1,5)\}$

$(A\times B)\cap(A\times C) = \{(0,3),(1,3)\}$

$\rightarrow 2$

From 1 & 2.

Hence proved

$A\times(B\cap C) = (A\times B)\cap(A\times C)$

30. $f(x) = 2x+3$

$g(x) = 1-2x$

$h(x) = 3x$

$f(g\circ h)$

$g\circ h = g(h(x))$

$= g(3x)$

$= 1-2(3x)$

$= 1-6x$

$f = [2(1-6x)+3]$

$= [2-12x+3]$

$= 5-12x \rightarrow 1$

$(f\circ g)\circ h$

$f\circ g = f(g(x))$

$= f(1-2x)$

$= 2(1-2x)+3$

$= 2-4x+3$

$= 5-4x$.

$= h(5-4x)$.

$= h(5-4(3x))$

$= 5-12x \rightarrow 2$

From 1 & 2

$f(g\circ h) = (f\circ g)\circ h$

Hence proved

31. $400+700+1000+\dots+n = 6500$

$a = 400 \quad d = t_2 - t_1$

$= 700 - 400 = 300$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$65,000 = \frac{n}{2}[2\times400+(n-1)(300)]$

$65,000\times2 = n(800+300n-300)$

$130000 = n(500+300n)$

$130000 = 100n(5+3n)$

$1300 = n(5+3n)$

$1300 = 5n+3n^2$

$3n^2+5n-1300 = 0$.

$\left(n+\frac{65}{3}\right)(n-20) = 0$

$n = -\frac{65}{3}$ Negative not possible

$n = 20$.

32. $(1^3+2^3+\cdots 20^3) - (1^3+2^3+\cdots 9^3)$

$$= \left(\frac{n(n+1)}{2}\right)^2 - \left(\frac{n(n+1)}{2}\right)^2$$

$n = 20 \qquad n = 9$

$$= \left(\frac{20\times 21}{2}\right)^2 - \left(\frac{9\times 10}{2}\right)^2$$

$= (210)^2 - (45)^2$

$= 44100 - 2025$

$= 42075$

---

33. $x+y+z=5 \rightarrow (1)$

$2x - y + z = 9 \rightarrow (2)$

$x - 2y + 3z = 16 \rightarrow (3)$

$(1)\times 2 \Rightarrow 2x+2y+2z=10$

$(2)\times 1 \Rightarrow 2x - y + z = 9$

$3y + z = 1 \Rightarrow (4)$

$(2)\times 1 \Rightarrow 2x - y + z = 9$

$(3)\times 2 \Rightarrow 2x - 4y + 6z = 32$

$3y - 5z = -23 \Rightarrow (5)$

$(4) \Rightarrow 3y + 4 = 1$

$3y = 1 - 4$

$3y = -3$

$y = \frac{-3}{3} = -1$

$y = -1$

$(1) \Rightarrow x+y+z=5$

$x - 1 + 4 = 5$

$x + 3 = 5$

$x = 5 - 3$

$x = 2$

Therefore

$x = 2 \quad y = -1$

$z = 4$

---

34)

$$\begin{array}{r|l} & 3x^2+2x+4 \\ \hline 3x^2 & 9x^4+12x^3+28x^2+ax+b \\ & 9x^4 \\ \hline & 12x^3+28x^2 \\ & 12x^3+8x^2 \\ \hline & 24x^2+ax+b \\ & 24x^2+16x+16 \\ \hline & 0 \end{array}$$

$a - 16 = 0 \qquad b - 16 = 0$

$a = 16 \qquad b = 16.$

---

35)

$$A^2 = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & -1-3 \\ 2+6 & -2+9 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix}$$

$$4A = 4\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -4 \\ 8 & 12 \end{bmatrix}$$

$$5I_2 = 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$A^2 - 4A + 5I_2 =$

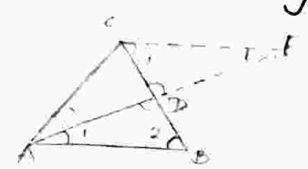
$$= \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ 8 & 12 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1-4+5 & -4+4+0 \\ 8-8+0 & 7-12+5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

36. STATEMENT

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of corresponding side containing the angle.



PROOF

Given: In ΔABC, AD is the internal bisector

To prove: $\frac{AB}{AC} = \frac{BD}{CD}$

Construction: Draw a line through C parallel to AB. Extend AD to meet line through C at E.

	Statement	Reason
1	$\angle AEC = \angle BAE = \angle 1$	Two parallel lines cut by a transversal make alternate angles equal.
2	ΔACE $AC = CE \rightarrow (1)$	In ΔACE: $\angle CAE = \angle CEA$
3	$\Delta ABD \sim \Delta ECD$ $\frac{AB}{CE} = \frac{BD}{CD}$	By AA similarity
4	$\frac{AB}{AC} = \frac{BD}{CD}$	From (1) $AC = CE$ Hence proved

37) Area = 28

$$\frac{1}{2}\left\{\begin{matrix}(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - \\ (x_2y_1 + (x_3y_2 + x_4y_3 + x_1y_4)\end{matrix}\right\}$$

$$\frac{1}{2}\left\{\begin{matrix} -4 & -3 & 3 & 2 & -4 \\ -2 & k & 2 & 3 & -2 \end{matrix}\right\} = 28$$

$$\frac{1}{2}\left\{\begin{matrix}(-4k + 6 + 9 - 4) - \\ (6 + 3k - 4 - 12)\end{matrix}\right\} = 28$$

$$(-4k + 11) - (3k + 10) = 28 \times 2$$

$$-4k + 11 - 3k + 10 = 56$$

$$-7k + 21 = 56$$

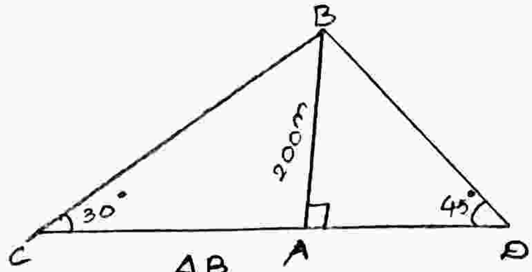
$$-7k = 56 - 21$$

$$-7k = 35$$

$$k = \frac{-35}{7}$$

$$k = -5$$

38:



$$\tan\theta = \frac{AB}{AC}.$$

$$\tan 30^\circ = \frac{AB}{AC} = \frac{200}{AC}.$$

$$\frac{1}{\sqrt{3}} = \frac{200}{AC}$$

$$AC = 200\sqrt{3} \rightarrow (1)$$

In ΔBAD

$$\tan\theta = \frac{AB}{AD}$$

$$\tan 45^\circ = \frac{200}{AD}$$

$$1 = \frac{200}{AD}$$

$$AD = 200 \rightarrow (2)$$

$$CD = AC + AD$$

$$= 200\sqrt{3} + 200$$

$$CD = 200(\sqrt{3} + 1)$$

$$= 200 \times 2.732$$

$$= 546.4$$

39. Let h and r be the height and radius of the cylinder respectively.

$h = 15cm \quad r = 6cm$.

Volume of the container $= \pi r^2 h$ cu. units

$= \frac{22}{7} \times 6 \times 6 \times 15$.

Let $r_1 = 3cm \quad h_1 = 9 cm$ of cone

Volume of one ice cream cone =

(Volume of the cone + Volume of the hemisphere)

$= \frac{1}{3}\pi r_1^2 h_1 + \frac{2}{3}\pi r^3$

$= \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 9 + \frac{2}{3} \times \frac{22}{7} \times 3 \times 3 \times 3$

$= \frac{22}{7} \times 9(3+2) = \frac{22}{7} \times 45$

No. of cones $= \frac{\text{Volume of the cylinder}}{\text{Volume of 1 ice cream cone}}$

No. of ice cream cone =

$$\frac{\frac{22}{7} \times 6 \times 6 \times 15}{\frac{22}{7} \times 45} = 12$$

No. of ice cream = 12.

40.

Cydrical well	Hollow Cydrical embankment
$D = 3m$	$r = \frac{3}{2}$
$r = \frac{3}{2}m$	$R = \frac{3}{2} + 4$
	$= \frac{11}{2}m$
$h = 14m$	$h = 2$

Volume of Cylindrical well = Volume of Hollow Cylindrical Embankment

$\pi r^2 h = \pi(R^2 + r^2)h$

$\frac{3}{2} \times \frac{3}{2} \times 14 = \left(\frac{121}{4} - \frac{9}{4}\right)h$

$\frac{3 \times 3 \times 7}{2} = \frac{112}{4} \times h$

$\frac{33 \times 7}{2} \times \frac{4}{112} = h$

$h = \frac{9}{8} m$

$h = 1.12 m$.

41\. Assumed mean = 1

Time Taken	No. of students	mid Value	$d = x - 11$	$f_i d_i$	$f_i d_i^2$
8.5-9.5	6	9	-2	-12	24
9.5-10.5	8	10	-1	-8	8
10.5-11.5	17	11	0	0	0
11.5-12.5	10	12	1	10	10
12.5-13.5	9	13	2	18	36
	$\Sigma f = 50$			$\Sigma fd = 8$	$\Sigma fd^2 = 78$

$N = 50$, $\Sigma f_i d_i = 8$ $\Sigma f_i d_i^2 = 78$

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$= \sqrt{\frac{78}{50} - \left(\frac{8}{50}\right)^2}$$

$$= \sqrt{1.56 - (0.16)^2}$$

$$= \sqrt{1.56 - 0.03}$$

$$= \sqrt{1.53} = 1.236 \Rightarrow 1.24$$

42. Total no. of cards = 52

$n(S) = 52$

$A \rightarrow$ getting Queen

$n(A) = 4$

$P(A) = \frac{n(A)}{n(S)}$

$= \frac{4}{52}$

$B \rightarrow$ getting Diamond

$n(B) = 13$

$P(B) = \frac{n(B)}{n(S)}$

$= \frac{13}{52}$

$C \rightarrow$ getting black cards

$n(C) = 26$

$P(C) = \frac{n(C)}{n(S)}$

$= \frac{26}{52}$

$P(A \cap B) = \frac{1}{52}$

$P(B \cap C) = 0$

$P(C \cap A) = \frac{2}{52}$

$A \cap B \cap C = 0$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - 0 - \frac{2}{52} + 0$

$= \frac{4 + 13 + 26 - 1 - 0 - 2 + 0}{52}$

$= \frac{40}{52} = \frac{10}{13}$

a) $y = x^2+3x+2$

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$3x$	-12	-9	-6	-3	0	3	6	9	12
2	2	2	2	2	2	2	2	2	2
y	6	2	0	0	2	6	12	20	30

Points : $(-4,6), (-3,2), (-2,0), (-1,0), (0,2), (1,6),$
$(2,12), (3,20), (4,30)$.

$$y = x^2+3x+2$$
$$0 = x^2+2x+1$$
$$\overline{y = x+1.}$$

x	-4	-2	-1	0	2	4
y	-3	-1	0	1	3	5

b) Let speed be x

Let time be y

Distance = speed x time.

Let the distance be $= xy$

From condition I:

$(x+10)(y-2) = xy$

$\cancel{xy} - 2x + 10y - 20 = \cancel{xy}$

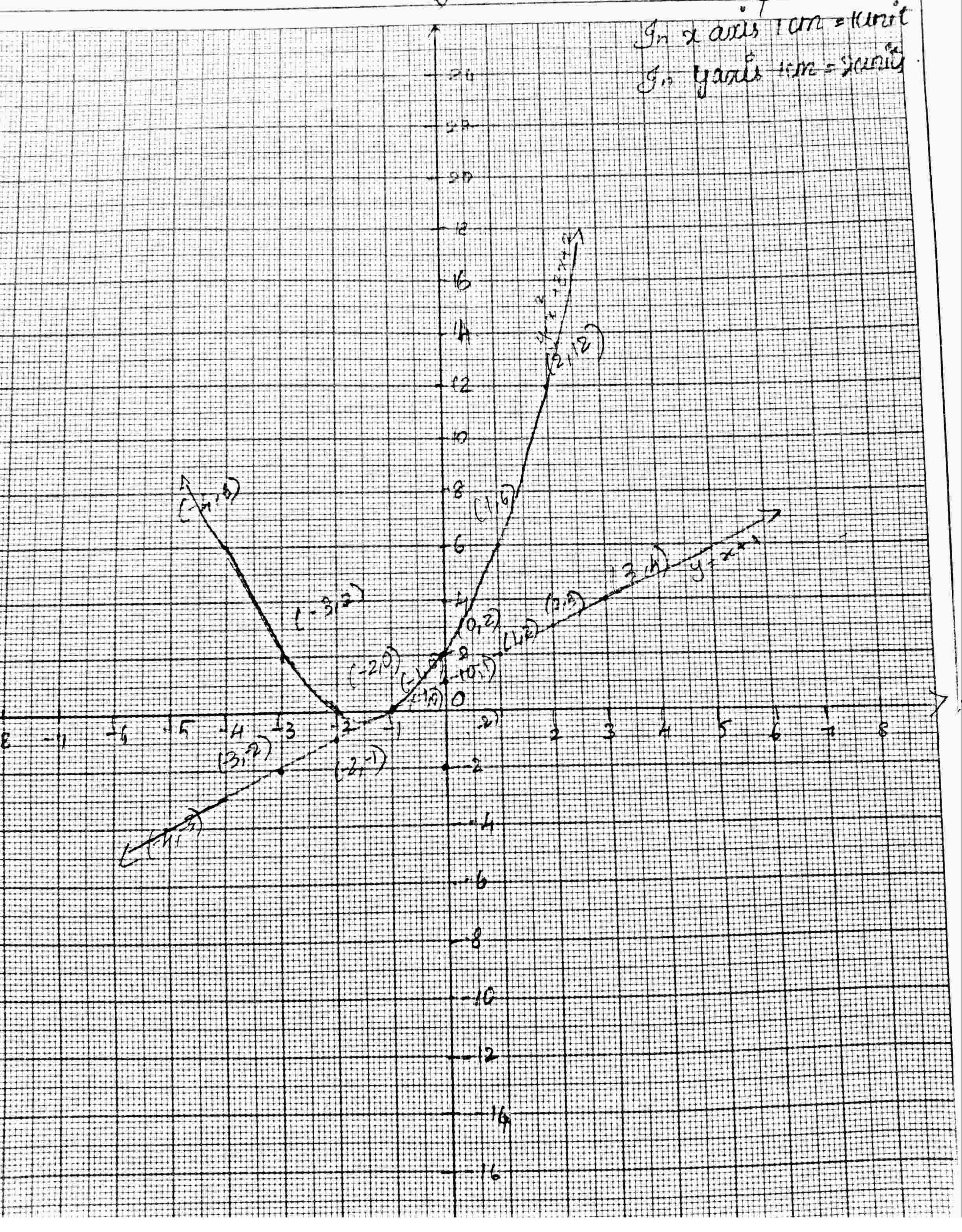
$-2x + 10y = 20$

$x - 5y = -10 \rightarrow (1)$

Scale

In x axis 1cm = 1unit

In y axis 1cm = 2units



From Condition (2)

$$(x-10)(y+3) = xy$$

$$xy + 3x - 10y - 30 = xy$$

$$3x - 10y = 30 \rightarrow (2)$$

$$(2) \Rightarrow 3x - 10y = 30$$

$$(1) \times 3 \Rightarrow 3x - 15y = -30$$

$$5y = 60$$

$$y = 12$$

$$x - 5(12) = -10$$

$$x + 60 = -10$$

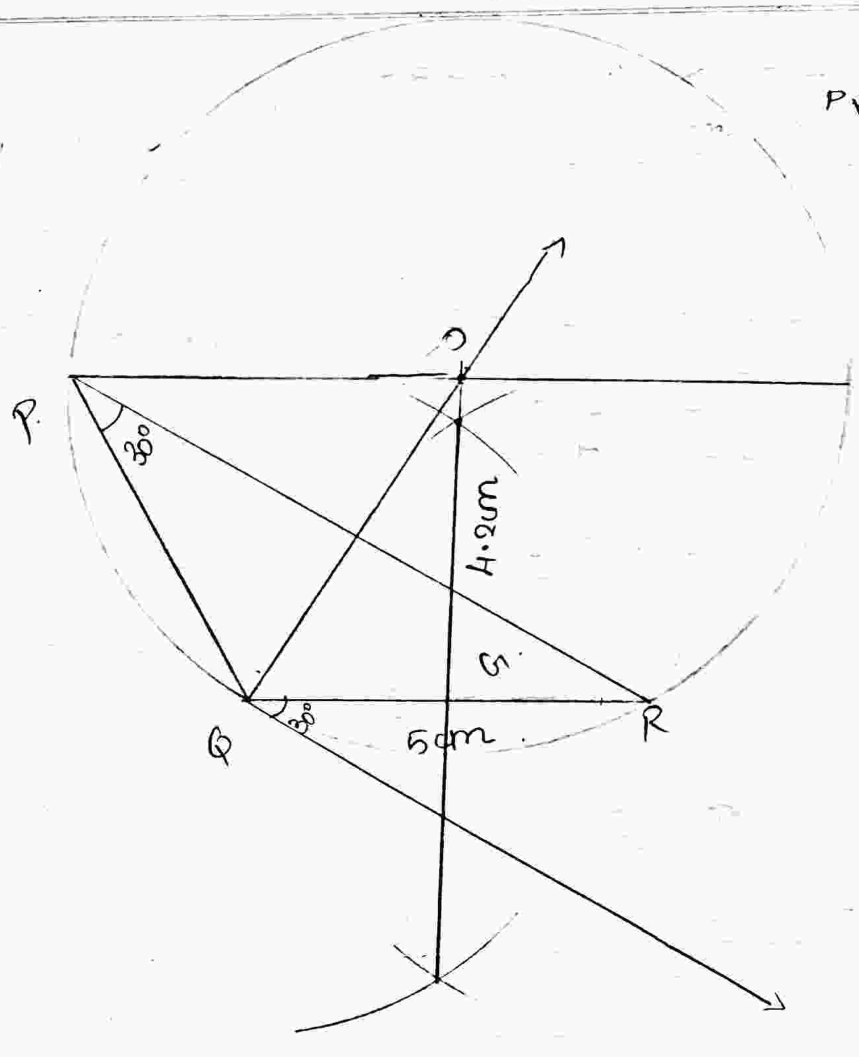
$$x = 50 \text{ km/hr}$$

$$\text{Distance} = xy$$

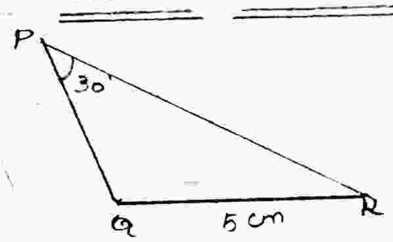
$$= 50 \times 12$$

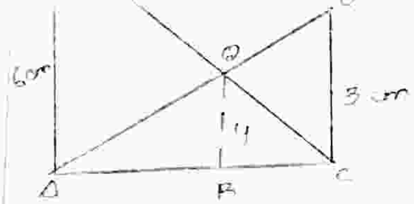
$$= 600 \text{ km}.$$

44) a)



ROUGH DIAGRAM





In the ΔPAC and ΔBQC

$\angle PAC = \angle QBC = 90^\circ$

$\angle C$ is common

$\Delta PAC \sim \Delta QBC$

$$\frac{AP}{BQ} = \frac{AC}{BC}$$

$$\frac{6}{y} = \frac{AC}{BC}$$

$$\frac{BC}{AC} = \frac{y}{6} \rightarrow ①$$

In the ΔACR and ΔQBC

$\angle ACR = \angle QBC = 90^\circ$

$\angle A$ is common

$\Delta ACR \sim \Delta ABQ$

$$\frac{RC}{QB} = \frac{AC}{AB}$$

$$\frac{3}{y} = \frac{AC}{AB}$$

$$\frac{AB}{AC} = \frac{y}{3} \rightarrow ②$$

By adding ① & ②

$$\frac{BC}{AC} + \frac{AB}{AC} = \frac{y}{6} + \frac{y}{3}$$

$$\frac{BC+AB}{AC} = \frac{y}{6} + \frac{y}{3}$$

$$\frac{AC}{AC} = \frac{y}{6} + \frac{y}{6}$$

$$1 = \frac{3y+6y}{18}$$

$$9y = 18$$

$$\frac{18}{9} = y$$

$$y = 2$$

J.Y. Sharu Nivetha
RDM 100%
Tution.

PTA Model no: 6.

PART-I

1) (b) $(2,-1)$.

2) (d) not a function

3) (d) A is larger than B by 1

4) (d) 1

5) (b) $\left(y+\frac{1}{y}\right)^2$

6) (b) $\begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$

7) (a) $(x-5)(x-3)$

8) (b) 4cm.

9) (d) 90°

10) (b) $-\sqrt{3}$

11) (d) 60°

12) (a) 2:1

13) (b) $5/6$

14) c) arithmetic mean

PART-II

15) $f(x) = 3x-5$ $f = \{(x, 3x-5) \mid x \in R\}$

(a,4) means the image of a is 4

$f(a) = 4$

(1,b) means the image of 1 is b

$f(1) = b \Rightarrow b = -2$

$3(1)-5 = b$

$b = -2$.

16) $(x,-2)$ $(-5,y)$

$x = -2$

$y = -5$

17) $t_{18} - t_{14} = 32$

$(a+17d) - (a+13d) = 32$

$4d = 32$

$d = 8$

18) $3x \equiv 1 \pmod{15}$

$3x-1 = 15k$

$3x = 15k+1$

$x = \frac{15k+1}{3}$

$x = 5k+\frac{1}{3}$.

$5k+\frac{1}{3}$ cannot be an integer.

(50)

19) $1+3+5 \dots +55$

$n = \frac{l-a}{d} + 1$

$= \frac{55-1}{2} + 1 = 28.$

$Sn = 28^2$

$Sn = 784.$

20) $2x^2 - 2\sqrt{6}x + 3 = 0.$

$2x^2 - \sqrt{6}x - \sqrt{6}x + 3$

$\sqrt{2}x(\sqrt{2}x - \sqrt{3}) - \sqrt{3}(\sqrt{2}x - \sqrt{3})$

$= (\sqrt{2}x - \sqrt{3})(\sqrt{2}x - \sqrt{3})$

$(\sqrt{2}x - \sqrt{3})^2 = 0.$

$x = \frac{\sqrt{3}}{\sqrt{2}}$

21) a number $= x$

reciprocal number $= \frac{1}{x}$

$x - \frac{1}{x} = \frac{24}{5}$

$\frac{x^2 - 1}{x} = \frac{24}{5}$

$5x^2 - 5 = 24x.$

$5x^2 - 24x - 5 = 0.$

$(x-5)(5x+1) = 0.$

$x = 5, -1/5$

22) $a = 7, b = a, c = 2$

$\alpha + \beta = b/a = -a/7$

$\alpha\beta = c/a = 2/7$

$\beta - \alpha = -13/7$

$\alpha - \beta = 13/7$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$\left(\frac{13}{7}\right)^2 = \left(\frac{-a}{7}\right)^2 - 4\left(\frac{2}{7}\right)$

$\frac{169 + 56}{49} = \frac{a^2}{49}$

$225 = a^2$

$a = \pm 15$

23) $(-2, 6)$ $(4, 8)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{8-6}{4+2} = \frac{2}{6} = \frac{1}{3}$

$(8, 12)$ $(x, 24)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{24 - 12}{x - 8}.$

$m_1 \times m_2 = -1$

$\frac{1}{3} \times \frac{12}{x-8} = -1$

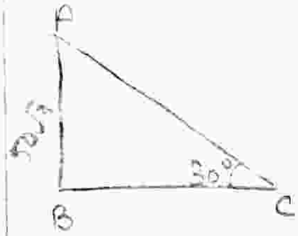
$\frac{4}{x-8} = -1$

$4 = -x + 8$

$x = 4$

(F3)

24)



$AB = 50\sqrt{3}$

$BC = x$

$\theta = 30^\circ$

$$\tan 30^\circ = \frac{50\sqrt{3}}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{x}$$

$$x = 50 \times 3$$

$$x = 150\ m$$

26.

$n = 21$

$$\sigma = \sqrt{\frac{n^2-1}{12}}$$

$$= \sqrt{\frac{21^2-1}{12}}$$

$$= \sqrt{\frac{440}{12}}$$

$$= \sqrt{36.66}$$

$$\sigma = 6.05$$

25) TSA of sphere = TSA of hemi sphere

$$4\cancel{\pi}r^2 = 3\cancel{\pi}r^2$$

$$r^2 = \frac{3r^2}{4}$$

$$r_1 = \frac{\sqrt{3}r}{2}$$

$$\frac{V_1}{V_2} = \frac{4/3\,\pi r^3}{2/3\,\pi r^3}$$

$$= \frac{2r_1^3}{r_2^3}$$

$$= 2\left(\frac{\sqrt{3}r_2}{2}\right)^3$$

$$= \frac{2 \times 3\sqrt{3}\, r_2^3}{r_2^3}$$

$$\frac{V_1}{V_2} = \frac{3\sqrt{3}}{4}$$

$$V_1 : V_2 = 3\sqrt{3} : 4$$

27) $P(A) = 0.5$

$P(A \cap B) = 0.3$

$P(A \cup B) \leq 1$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

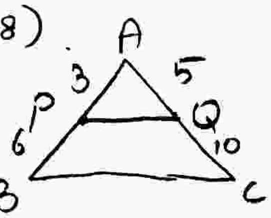
$= 0.5 + P(B) - 0.3 \leq 1$

$P(B) = 0.3 - 0.5 \leq 1$

$P(B) = \leq -0.2$

$P(B) = \leq 0.8.$

28)



Thales Theorem,

$$\frac{AP}{PB} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{AQ}{QC} = \frac{5}{10} = \frac{1}{2}$$

$PQ \parallel BC$

$$\frac{AP}{PQ} = \frac{AB}{BC}$$

$$\frac{3}{PQ} = \frac{9}{BC}$$

$$BC = 3PQ$$

PART - III

29)

i) $\frac{2x+1}{x-9}$

$x - 9 = 0$

$x = 9.$

$R = \{9\}$

ii) $\sqrt{x-2}$

Domain $= \{2, 3, 4, 5 \ldots \alpha\}$

$= \{2, \alpha\}.$

30) $f(x) = x^5$

$f(x) = f(y)$

$x^5 = y^5$

$x = y.$

f is one - one function.

$g(x) = x^4$

$g(x) = 8(y)$

$x^4 = y^4$

$x = \pm y$

g is not one - one function.

$fog = f(g(x))$

$= f(x^4)$

$= (x^4)^5$

$fog = x^{20}.$

$fog(x) = fog(y).$

$x^{20} = y^{20}$

$x = \pm y$

fog is not one - one function.

31) $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_p = \frac{p}{2}[2a + (p-1)d]$

$ap^2 + bp = \frac{p}{2}[2a + (p-1)d]$

$2ap^2 + 2bp - 2ap = p(p-1)d$

$\frac{2p(ap+b-a)}{p(p-1)} = d$

$d = \frac{2(ap+b-a)}{(p-1)}.$

32)

$P = 60{,}000 \quad r = 5\%.$

$n = 5$

$A = P\left[1 + \frac{r}{100}\right]^n$

(F5)

$$= 60000\left[1+\frac{5}{100}\right]^5$$

$$= 60000\left[\frac{105}{100}\right]^5$$

$$= 60000\left[\frac{21}{20}\right]^5$$

$$= 60000 \times \frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}$$

$$= 76576.8$$

33)

$$a = c^2 - ab$$
$$b = -2(a^2 - bc)$$
$$c = b^2 - ac$$
$$\Delta = 0$$
$$b^2 - 4ac = 0$$
$$[-2(a^2-bc)]^2 - 4(c^2-ab)(b^2-ac) = 0$$
$$4[a^4 + b^2c^2 - 2a^2bc - b^2c^2 + ac^3 + ab^3 - a^2bc) = 0$$
$$4a[a^3+b^3+c^3 - 3abc] = 0$$
$$4a = 0 \qquad a^3+b^3+c^3-3abc = 0$$
$$a = 0\ ;\ a^3+b^3+c^3 = 3abc$$

34)

$$f(x) = a^2+4a-12$$
$$(a+b)(a-2)$$
$$g(x) = a^2-5a+6$$
$$= (a-2)(a-3)$$
$$GCD = (a-2)$$
$$f(x)\times g(x) = GCD \times LCM$$
$$LCM = \frac{f(x)\times g(x)}{GCD}$$
$$= \frac{(a+6)\cancel{(a-2)}(a-2)(a-3)}{\cancel{(a-2)}}$$
$$LCM = (a+6)(a-2)(a-3).$$

35)

$$BC = \begin{bmatrix}0 & 3\\ -1 & 5\end{bmatrix}\times\begin{bmatrix}-1 & 5\\ 1 & 3\end{bmatrix}$$
$$= \begin{bmatrix}3 & 9\\ 6 & 10\end{bmatrix}$$
$$A(BC) = \begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\times\begin{bmatrix}3 & 9\\ 6 & 10\end{bmatrix}$$
$$= \begin{bmatrix}3+12 & 9+20\\ 9+24 & 27+40\end{bmatrix}$$
$$= \begin{bmatrix}15 & 29\\ 33 & 67\end{bmatrix} \rightarrow (1)$$
$$A\times B = \begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\times\begin{bmatrix}0 & 3\\ -1 & 5\end{bmatrix}$$
$$= \begin{bmatrix}-2 & 13\\ -4 & 29\end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -2 & 13 \\ -4 & 29 \end{bmatrix} \times \begin{bmatrix} -1 & 5 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2+13 & -10+39 \\ 4+29 & -20+87 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 29 \\ 33 & 67 \end{bmatrix} \rightarrow (2).$$

$(1) = (2)$.

LHS = RHS

---

36. $QS = 3SR$

$QR = QS + SR$

$= 3SR + SR = 4SR$.

$SR = \frac{1}{4} QR \rightarrow (1)$

$QS = 3SR$

$SR = \frac{QS}{3} \rightarrow (2)$

From (1) and (2)

$\frac{1}{4} QR = \frac{QS}{3}$

$QS = \frac{3}{4} QR$

In the right ΔPQS

$PQ^2 = PS^2 + QS^2 \rightarrow (4)$

In ΔPSR

$PR^2 = PS^2 + SR^2$

Sub (4) & (5)

$PQ^2 - PR^2 = PS^2 + QS^2 - PS^2 - SR^2$

$= QS^2 - SR^2$

$PQ^2 - PR^2 = \left[\frac{3}{4} QR\right]^2 - \left[\frac{QR}{4}\right]^2$

From (3) and (1)

$= \frac{9QR^2}{16} - \frac{QR^2}{16} = \frac{8QR^2}{16}$

$PQ^2 - PR^2 = \frac{1}{2} QR^2$

$2PQ^2 - 2PR^2 = QR^2$

$2PQ^2 = 2PR^2 + QR^2$

Hence proved.

37.

i) equation of median

$x = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-5+1}{2}, \frac{-1+9}{2}\right)$

$= \left(\frac{-4}{2}, \frac{8}{2}\right) = (-2, 4)$

$A(6, +2)$ $D(-2, 4)$.

equation of straight line

$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

$\frac{y - 2}{4 - 2} = \frac{x - 6}{-2 - 6}$.

$$\frac{y-2}{2} = \frac{x-6}{-8}$$

$$-8(y-2) = 2(x-6)$$

$$-8y+16 = 2x-12$$

$$-2x-8y+16+12=0$$

$$-2x-8y+28=0$$

$$2x+8y-28=0.$$

ii) altitude BC.

$$m_1 = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{9+1}{1+5} = \frac{10}{6} = \frac{5}{3}.$$

Perpendicular

$$m_1 \times m_2 = -1$$

$$\frac{5}{3} \times m_2 = -1$$

$$m_2 = \frac{-3}{5}$$

$A(6,2)$, $m = -3/5$

Slope of straight line

$$y-y_1 = m(x-x_1)$$

$$y-2 = -3/5(x-6)$$

$$5(y-2) = -3(x-6)$$

$$3x+5y-28=0.$$

38) $$\left[\frac{\cos^3 A - \sin^3 A}{\cos A - \sin A}\right] - \left[\frac{\cos^3 A + \sin^3 A}{\cos A + \sin A}\right]$$

$$= \left[\frac{(\cos A - \sin A)(\cos^2 A + \sin^2 A + \cos A \sin A)}{\cos A - \sin A}\right]$$

$$- \left[\frac{(\cos A + \sin A)(\cos^2 A + \sin^2 A - \cos A \sin A)}{\cos A + \sin A}\right]$$

$$= (1+\cos A \sin A) - (1-\cos A \sin A)$$

$$= 2\cos A \sin A$$

Hence Proved.

39) $h = 10cm$, $R = 28cm$, $r = 18cm$

CSA of Frustrum $= \pi l(R+r)$

$$l = \sqrt{h^2+(R-r)^2}$$

$$= \sqrt{10^2+(28-18)^2}$$

$$= \sqrt{100+100}$$

$$= \sqrt{2\times 100}$$

$$= 10\sqrt{2}$$

$$= 14.14$$

CSA $= \pi l(R+r)$

$$= \frac{22}{\cancel{7}} \times \cancel{14.14}^{2.02}(28-18)$$

$= 44.44 \times 10$

$= 444.40$.

40) $h = 15$ cm $\quad r = 6$ cm

Volume of cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 6 \times 6 \times 15$

$r_1 = 3$ cm $\quad h_1 = 9$ cm.

Volume of cone (ice cream) = Volume of cone + Volume of hemi sphere

$= \frac{1}{3}\pi r_1^2 h_1 + \frac{2}{3}\pi r_1^3$

$= \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 9 + \frac{2}{3} \times \frac{22}{7} \times 3^3$

Volume of ice cream $= \frac{22}{7} \times 45$

Required ice cream $= \dfrac{\text{Volume of cylinder}}{\text{volume of ice cream}}$

$= \dfrac{\frac{22}{7} \times 6 \times 6 \times 15}{\frac{22}{7} \times 45}$

$= 2 \times 2 \times 3$

No. of ice cream = 12.

41) $\bar{x} = 155$ cm

$\sigma^2 = 72.25^2$

SD $= 8.5$

co. of variation $= \frac{\sigma}{\bar{x}} \times 100$

$C.V = \frac{8.5}{155} \times 100$

$= 5.48\%$.

$= 46.50$ kg

$\sigma^2 = 28.09$ kg^2

SD $= 5.3$ kg

$C.V = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{5.3}{46.50} \times 100\%$.

$= 11.40\%$.

$CV_1 = 5.48\%$.

$CV_2 = 11.40\%$.

42. $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(S) = 8$.

exactly two heads.

$A = \{HHT, HTH, THH\}$

$n(A) = 3$

$P(A) = 3/8$

Atleast one tail

$B = \{HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(B) = 7$

$P(B) = 7/8$.

consecutively two heads.

$C = \{HHH, HHT, THH\}$

$n(C) = 3$

$P(C) = 3/8$.

$A \cap B = \{HHT, HTH, THH\}$

$n(A \cap B) = 3$

$P(A \cap B) = 3/8$.

$B \cap C = \{HHT, THH\}$

$n(B \cap C) = 2/8$

$C \cap A = \{HHT, THH\}$

$n(C \cap A) = 2$

$P(C \cap A) = 2/8$

$A \cap B \cap C = \{HHT, THH\}$

$n(A \cap B \cap C) = 2$

$P(A \cap B \cap C) = 2/8$.

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

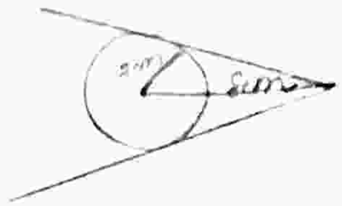
$= \frac{3}{8} + \frac{7}{8} + \frac{3}{8} - \frac{3}{8} - \frac{2}{8} - 2/8 + 2/8$

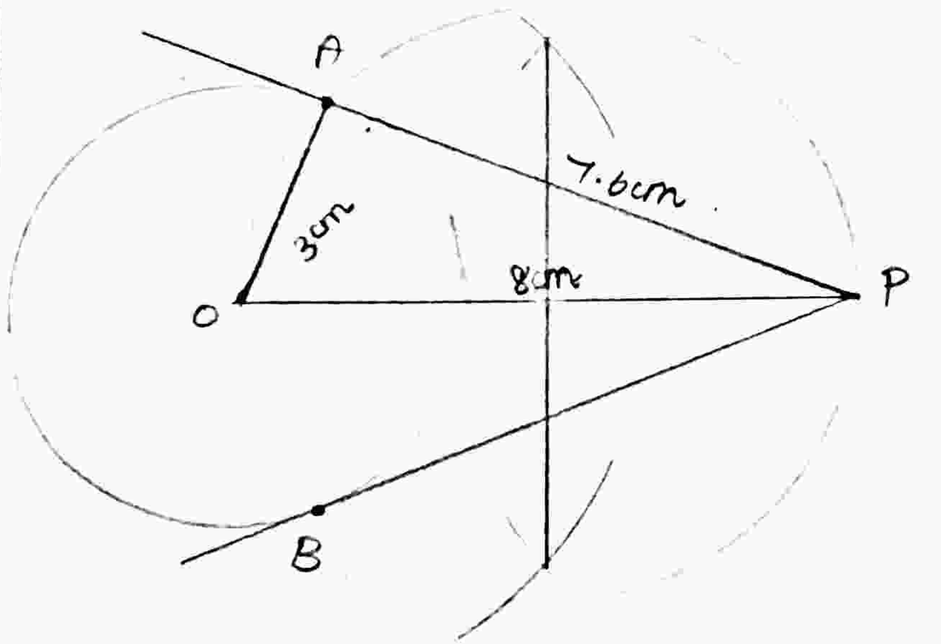
$= \frac{7}{8} + \frac{3}{8} - \frac{2}{8}$

$= \frac{8}{8} = 1$

44 b)

Rough Diagram.





By pyth.

$$OP^2 = OA^2 + AP^2.$$

$$OP^2 - OA^2 = AP^2$$

$$64 - 9 = AP^2$$

$$AP^2 = 55$$

$$AP = 7.6cm.$$

(F11)

3) 44) a) $\angle QRP$

$\angle QPR = 90°$ PS is the bisector of $\angle P$.

$ST \perp \angle PR$

Proof: In ΔPQR, PS is the bisector of $\angle P$

$$\frac{PQ}{PR} = \frac{QS}{SR}$$

Adding (1) on both side

$$1 + \frac{PQ}{PR} = 1 + \frac{QS}{SR}$$

$$\frac{PR + PQ}{PR} = \frac{SR + QS}{SR}$$

$$\frac{PR + PQ}{PR} = \frac{QR}{SR}$$

In ΔRST and ΔRQP

$\angle SRT = \angle QRP = \angle R$ (common)

$\therefore \angle QPR = \angle STR = 90°$

By AA similarity.

$\Delta RST \sim RQP$

$$\frac{SR}{QR} = \frac{ST}{PQ} \qquad \frac{QR}{SR} = \frac{PQ}{ST} \cdots (2)$$

From (1) and (2)

$$\frac{PQ + PR}{PR} = \frac{PQ}{ST}$$

$$ST(PQ + PR) = PQ \times PR.$$

44) b)

usual speed $= x$ km/hr

increase $= x + 25$ km/hr

distance $= 150$ km

Time $= \frac{\text{distance}}{\text{speed}}$

$$\frac{150}{x} - \frac{150}{x+25} = \frac{1}{2} \text{ hr} \left\{ 30 \text{ min} = \frac{1}{2} \text{h} \right.$$

$$\frac{150x + 150 \times 25 - 150x}{x(x+25)} = \frac{1}{2}$$

$$150 \times 25 \times 2 = x(x+25)$$

$$7500 = x^2 + 25x$$

$$x^2 + 25x - 750 = 0$$

$$(x - 75)(x + 100) = 0$$

$x = 75$ (or) $x = -100$

x is +ve

$\therefore x = 75$ km/hr

speed $= 75$ km/hr.

44) a) Graph.

$y = x^2 - 5x - 6$.

x	-2	-1	0	1	2	3	4	5	6	7
x^2	4	1	0	1	4	9	16	25	36	49
$-5x$	10	5	0	-5	-10	-15	-20	-25	-30	-35
-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
y	8	0	-6	-10	-12	-12	-10	-6	0	8

Pts are : (-3, 18) (-2, 8) (-1, 0) (0, -6) (1, -10) (2, -12)
(3, -12) (4, -10) (5, -6) (6, 0) (7, 8) (8, 18).

$y = x^2 - 5x - 6$

$0 = x^2 - 5x - 14$

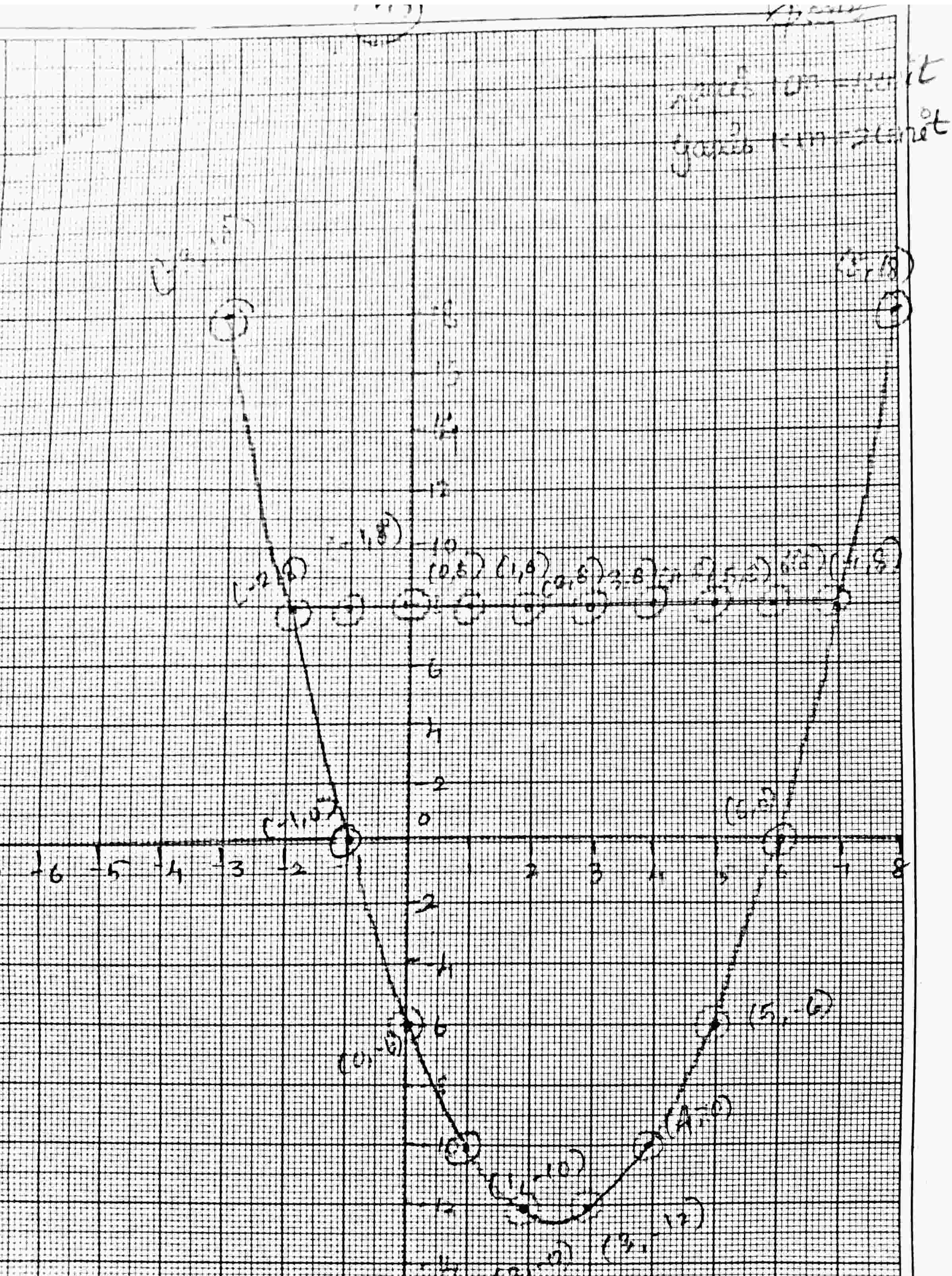
$(+)$

$y = \quad 8$

x	-2	-1	0	1	2	3	4	5	6	7
y	8	8	8	8	8	8	8	8	8	8

Pts are (-2, 8) (-1, 8) (0, 8) (1, 8) (2, 8) (3, 8) (4, 8)
(5, 8) (6, 8) (7, 8)

Solution : (-2, 7).



(5, -6)
(0, -6)

Maths PTA Question Key

Model - 2

M.A.Muthu Suttra Lakshmi
RDM 100%
Tuition Centre

PART - I

1. (a) 7
2. (b) 2
3. (b) $\frac{1}{27}$
4. (b) $16x^2$
5. (c) 4
6. (a) Straight line
7. (b) point of contract
8. (b) 25 Sq. units
9. (c) 3,5
10. (b) $\frac{1}{25}$
11. (b) 1
12. (b) 1:2
13. (c) 27
14. (c) $\frac{n+1}{2}$

PART - II

(2 marks)

15. $\{(x,y)/y=x^2+3, x\in\{0,1,2,3,4,5\}$

$x=0$
$y=0^2+3$
$=3$

$x=1$ $y=1^2+3$
$=4$

$x=2$ $y=2^2+3$
$=7$

$x=3$
$y=3^2+3$
$=12$

$x=4$
$y=4^2+3$
$=19$

$x=5$
$y=5^2+3$
$=28$

domain $=\{0,1,2,3,4,5\}$

Range $=\{3,4,7,12,19,28\}$

16. $f(x)=x^2-1$; $g(x)=x-2$

$g\circ f(a)=1$, find a

$g\circ f = g(f(a))=1$

$=g(a^2-1)=1$

$a^2-1-2-1$

$a=\pm\sqrt{4}$

$a=\pm 2$

$a=2$

17. $P(\bar{A})=0.5$, $P(A\cup B)=0.65$

$P(A\cap B)=0$ $P(B)=?$

$P(A)=1-P(\bar{A})$

$=1-0.45$

$P(A)=0.55$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$0.65=0.55+P(B)-0$

$0.65-0.55=P(B)$

$P(B)=0.10$

18. $P(x) = x^2 - 5x - 14$

$q(x) = ?$

$$\frac{P(x)}{q(x)} = \frac{(x-7)}{x+4}$$

$$\frac{x^2 - 5x - 14}{g(x)} = \frac{(x-7)}{x+2}$$

$$\frac{\cancel{(x-7)}(x+2)(x+2)}{\cancel{x-7}}$$

$= (x+2)^2$

$= x^2 + 4x + 4$

19. $A^T = \begin{bmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{bmatrix}$

$-A^T = \begin{bmatrix} -\sqrt{7} & \sqrt{5} & \sqrt{3} \\ 3 & -2 & 5 \end{bmatrix}$

20. $BC = 3cm$, $EF = 4cm$

$\Delta ABC = 54cm^2$

$$\frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \frac{BC^2}{EF^2}$$

$$\frac{54}{Area(\Delta DEF)} = \frac{3^2}{4^2}$$

$Area(\Delta DEF) = \frac{16 \times 54}{9}$

$Area(\Delta DEF) = 96cm^2$

21. $(\sin\theta - \cos\theta)$ & $(-\sin\theta, \cos\theta)$

Slope of a line $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{\cos\theta + \cos\theta}{-\sin\theta - \sin\theta}$

$= \frac{2\cos\theta}{-2\sin\theta}$

$= -\cot\theta$

22. $y - y_1 = m(x - x_1)$

$y - 3 = 1(x - 19)$

$y - 3 = x - 19$

$x - y - 16 = 0$

23. $x+6$, $x+12$, $x+15$ are in GP

$$\frac{t_2}{t_1} = \frac{t_3}{t_2}$$

$$\frac{x+12}{x+6} = \frac{x+15}{x+12}$$

$(x+12)(x+12) = (x+6)(x+15)$

$x^2 + 12x + 12x + 144 =$

$x^2 + 15x + 6x + 90$

$24x + 144 - 21x - 90 = 0$

$3x + 54 = 0$

$3x = -54$

$x = \frac{-54}{3}$

$x = -18$

24. $1 + 2 + 3 + \ldots + n$

$= \frac{n(n+1)}{2} = 666$

$n^2 + n - 1332 = 0$

$(n+37)(n-36) = 0$

$n = -37$ (or) $n = 36$

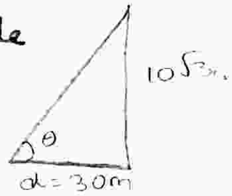
$n \neq -37$ $n = 36$

25. Distance = 30m

Height = $10\sqrt{3}$ m

$\theta = ?$

$\tan\theta = \dfrac{\text{opposite}}{\text{Adjacent side}}$



$= \dfrac{10\sqrt{3}}{30}$

$= \dfrac{\sqrt{3}}{3} = \dfrac{\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

$\tan\theta = \dfrac{1}{\sqrt{3}}$

$\theta = 30^\circ$

Angle of elevation $= 30^\circ$

26. Radius of the cone I = x

Radius of the cone II = $3x$

Height of the cone I = $3x$

Height of the cone II = $3x$

T.F : CSA of the cone

Slant height of the cone I

$l^2 = h^2 + r^2$

$= (3x)^2 + x^2$

$\Rightarrow 9x^2 + x^2$

$= 10x^2$

$l = x\sqrt{10}$

Slant height of the cone II

$l^2 = h^2 + r^2$

$l^2 = (3x)^2 + (3x)^2$

$= 9x^2 + 9x^2$

$l^2 = 18x^2$

$l = x\sqrt{18}$

CSA of the cone II

CSA $= \pi r l$

$= \pi \times 3x \times x\sqrt{18}$

$= \pi\ 3x^2\sqrt{18}$

Ratio of the CSA of cone I and II

$\pi x^2\sqrt{10} : \pi\ 3x^2\sqrt{18}$

$\sqrt{10} : 3\sqrt{18}$

$\sqrt{5\times 2} : 3\times 3\sqrt{2}$

$\sqrt{5} : 9$

27. $p = a^2b^3$, $q = a^3b$

HCF of $p, q = a^2b$

LCM of $p, q = a^3b^3$

LCM $\times$ HCF $= a^2b \times a^3b^3$

$= a^5b^4$

$pq = a^2b^3 \times a^3b$

$= a^5b^4$

$\therefore$ LCM $\times$ HCF of $p, q = pq$

28. diameter of spherical Lead shots $\}$ = 6cm

Radius = 3cm

Volume of cuboid $= l\times b\times h$

Volume of sphere $= \frac{4}{3}\pi r^3$

No. of lead shots $= \dfrac{\text{Volume of cuboid}}{\text{Volume of sphere}}$

No. of lead shots $= \dfrac{24\times 22\times 12}{\frac{4}{3}\times\frac{22}{7}\times 3\times 3\times 3}$

$= \dfrac{\cancel{24}\times\cancel{22}\times\cancel{12}\times\cancel{3}\times 7}{\cancel{4}\times\cancel{22}\times\cancel{3}\times\cancel{3}\times\cancel{3}}$

$= 56$

$\therefore$ No. of lead shots = 56

PART- III

(5 marks)

29. Area of the quadrilateral ABCD

$A = (-4,-8)$ $B(8,-4)$ $C = (6,10)$ $D = (-10,6)$

$= \frac{1}{2} \begin{Bmatrix} -4 & 8 & 6 & -10 & -4 \\ -8 & -4 & 10 & 6 & -8 \end{Bmatrix}$

$= \frac{1}{2} \{(16+80+36+80) - (-64-24-100-24)\}$

$= \frac{1}{2} \{212 - (-212)\}$

$= \frac{1}{2} \times 424$

$= 212$ Sq. units

Area of quadrilateral EFGH

$= \frac{1}{2} \begin{Bmatrix} -3 & 6 & 3 & -6 & -3 \\ -5 & -2 & 7 & 4 & -5 \end{Bmatrix}$

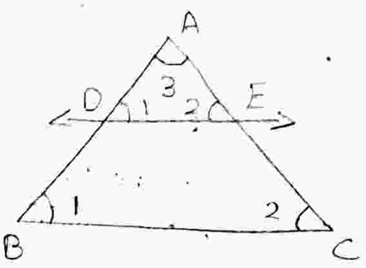
$= \frac{1}{2} \{(6+42+12+30) - (30-6-42-13)\}$

$= \frac{1}{2} \{(90) - (-90)\}$

$= \frac{1}{\cancel{2}_{1}} \times \cancel{180}^{90}$

$= 90$

30. Thales theorem



Statement :-

A Straight line drawn parallel to a Side of triangle intersecting the other two Sides, divides the Sides in the Same ratio.

Proof:

Given: In ΔABC, D is a point on AB and E is a Point on AC.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Draw a line $DE \parallel BC$

No	Statement	Reason
1.	$\angle ABC = \angle ADE = \angle 1$	corresponding angles are equal because DE ∥ BC
2.	$\angle ACB = \angle AED = \angle 2$	Corresponding angles are equal because DE ∥ BC
3.	$\angle DAE = \angle BAC = \angle 3$ $\Delta ABC \sim \Delta ADE$ $\frac{AB}{AD} = \frac{AC}{AE}$ $\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$	Both triangles have a common angle By A A Similarity corresponding Sides are proportional Split A B and AC by using the points D and E
4.	$1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$ $\frac{DB}{AD} = \frac{EC}{AE}$ $\frac{AD}{DB} = \frac{AE}{EC}$	on simplification cancelling 1 on both sides Taking reciprocals

∴ Hence proved.

31. $f(x) = x-4,\ g(x) = x^2,\ h(x) = 3x-5$

$(f o g) o h = f o (g o h)$

$(f o g)(x) = f(g(x))$
$= f(x^2)$
$= x^2 - 4$

$((f o g) o h) x = (f o g) h(x)$
$= (f o g)(3x-5)$
$= (3x-5)^2 - 4$
$= 9x^2 - 30x + 25 - 4$
$= 9x^2 - 30x + 21 \rightarrow ①$

$(g o h)(x) = g(h(x))$
$= g(3x-5)$
$= (3x-5)^2$
$= 9x^2 - 30x + 25$

$(f o (g o h))(x) = f(9x^2 - 30x + 25)$
$= 9x^2 - 30x + 25 - 4$
$= 9x^2 - 30x + 21 \rightarrow ②$

① = ②

$\therefore$ Hence proved.

33. Let the Senthil's house number be x

$1+2+3+\dots+(x-1) = (x+1)+(x+2)+\dots+49$

$1+2+3+\dots+(x-1) = [1+2+3+\dots+49] - [1+2+3+\dots+x]$

$\frac{x-1}{2}[1+(x-1)] = \frac{49}{2}[1+49] - \frac{x}{2}[1+x]$

$\frac{x(x-1)}{2} = \frac{49 \times 50}{2} - \frac{x(x+1)}{2}$

$x^2 - x = 2450 - x^2 - x$

$\Rightarrow 2x^2 = 2450$

$x^2 = 1225$

$x = 35$

32.

i) $67 + x \equiv 1 \pmod 4$

$67 + x - 1 = 4n$

$66 + x = 4n$

$66 + x$ is a multiple of 4

$\therefore$ The least value of x must be 2, Since 68 is the nearest multiple of 4 more than 66.

ii) $5x \equiv 4 \pmod 6$

$5x - 4$ is divisible by 6

$5 \times 2 - 4$ is divisible by 6

$x = 2, 8, 14, 20 \dots$

34. $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(S) = 8$

A $\rightarrow$ getting exactly two heads

$A = \{HHT, HTH, THH\}$

$n(A) = 3,\ P(A) = \frac{3}{8}$

B $\rightarrow$ atleast one tail

$B = \{HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(B) = 7$

$P(B) = \frac{7}{8}$

C $\rightarrow$ two consecutive Head

$C = \{HHH, HHT, THH\}$

$n(C) = 3,\ P(C) = \frac{3}{8}$

$A \cap B = \{HHT, HTH, THH\}$

$n(A \cap B) = 3,\ P(A \cap B) = \frac{3}{8}$

$B \cap C = \{HHT, THH\}$

$n(B \cap C) = 2, \; P(B \cap C) = \frac{2}{8}$

$A \cap C = \{HHT, THH\}$

$\therefore \; n(A \cap C) = 2, \; P(A \cap C) = \frac{2}{8}$

$A \cap B \cap C = \{HHT, THH\}$

$P(A \cap B \cap C) = \frac{2}{8}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{7}{8} + \cancel{\frac{3}{8}} - \cancel{\frac{3}{8}} - \frac{2}{8} - \cancel{\frac{2}{8}} + \cancel{\frac{2}{8}}$

$= \frac{8}{8} = 1$

35. City A

$A = 22$

x	$d = x - A$	d^2
18	$18 - 22 = -4$	16
20	$20 - 22 = -2$	4
22	$22 - 22 = 0$	0
24	$24 - 22 = 2$	4
26	$26 - 22 = 4$	16
$\Sigma x = 110$	$\Sigma d = 0$	$\Sigma d^2 = 40$

$\bar{x} = \frac{\Sigma x}{n} = \frac{110}{5} = 22$

$\sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2}$

$= \sqrt{\frac{40}{5} - 0}$

$\sigma = \sqrt{8} = 2\sqrt{2}$

$= 2(1.414)$

$\sigma = 2.828$

$C.V. = \frac{\sigma}{\bar{x}} \times 100\%$

$= \frac{2.828}{22} \times 100$

$= \frac{141.40}{11}$

$C.V. = 12.85\%$

City B

$A = 15$

x	$d = x - A$	d^2
11	$11 - 15 = -4$	16
14	$14 - 15 = -1$	1
15	$15 - 15 = 0$	0
17	$17 - 15 = 2$	4
18	$18 - 15 = 3$	9
$\Sigma x = 75$	$\Sigma d = 0$	$\Sigma d^2 = 30$

$\bar{x} = \frac{\Sigma x}{n} = \frac{\cancel{75}}{\cancel{5}} = 15$

$\sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2}$

$\sigma = \sqrt{\frac{30}{5} - 0}$

$\sigma = \sqrt{6}$

$= \sqrt{2} \times \sqrt{3}$

$\sigma = 2.4$

$C.V_2 = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{2.4}{15} \times \cancel{100}$

$= \frac{48}{\cancel{3}} = 16$

City A is more Consistent

36. $A = \{0,1\}$ $B = \{2,3,4\}$ $C = \{3,5\}$

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

LHS :

$A \times (B \cup C)$

$B \cup C = \{2,3,4\} \cup \{3,5\}$

$= \{2,3,4,5\}$

$A \times (B \cup C) = \{0,1\} \times \{2,3,4,5\}$

$= \{(0,2)(0,3)(0,4)(0,5)$
$(1,2)(1,3)(1,4)(1,5)\}$

RHS :

$(A \times B) \cup (A \times C)$

$A \times B = \{0,1\} \times \{2,3,4\}$

$= \{(0,2)(0,3)(0,4)$
$(1,2)(1,3)(1,4)\}$

$A \times C = \{0,1\} \times \{3,5\}$

$= \{(0,3)(0,5)(1,3)(1,5)\}$

$(A \times B) \cup (A \times C) = \{(0,2)(0,3)(0,4)$
$(0,5)(1,2)(1,3)$
$(1,4)(1,5)\}$

LHS = RHS

∴ Hence proved

37. Let vani's grand father's age $= x$

Let vani's Father's age $= y$

Let vani's age $= z$

$\frac{x+y+z}{3} = 53$

$x+y+z = 159 \rightarrow (1)$

$\frac{1}{2}x + \frac{1}{3}y + \frac{1}{4}z = 65 \rightarrow (2)$

$\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 65$

$\frac{6x+4y+3z}{12} = 65$

$6x+4y+3z = 65 \times 12$

$6x+4y+3z = 780 \rightarrow (2)$

$x-4 = 4(z-4)$

$x-4 = 4z-16$

$x-4-4z+16 = 0$

$x-4z+12 = 0$

$x-4z = -12 \rightarrow (3)$

$x+y+z = 159 \rightarrow (1)$

$6x+4y+3z = 780 \rightarrow (2)$

$x-4z = -12 \rightarrow (3)$

1×4

$4x + \cancel{4y} + 4z = 636$

$\underline{-}6x \, \underline{\cancel{+}}\cancel{4y} \, \underline{+}3z \, \underline{=}780$

$-2x \quad + z = -144 \rightarrow (4)$

$x - 4z = -12 \rightarrow (3)$

$-2x + z = -144 \rightarrow (4)$

$(3) \times 2$ $\quad \cancel{2x} - 8z = -24$

$-\cancel{2x} + z = -144$

$-7z = -168$

$z = \frac{\cancel{168}^{24}}{\cancel{7}}$

$z = 24$

$z = 24$ in ③ B - ⑨

$x - 4z = -12$

$x - 4 \times 24 = -12$

$x - 96 = -12$

$x = -12 + 96 = 84$

$x = 84$

$z = 24, x = 84$ in ①

$x + y + z = 159$

$84 + y + 24 = 159$

$108 + y = 159$

$y = 159 - 108$

$y = 51$

Vani's age $= 24$

Father's age $= 51$

grand Father's age $= 84$

$$= \begin{pmatrix} \sin^2\theta + 0 & 0+0 \\ 0+0 & 0+\sin^2\theta \end{pmatrix}$$

$$B^2 = \begin{pmatrix} \sin^2\theta & 0 \\ 0 & \sin^2\theta \end{pmatrix}$$

$$A^2 + B^2 = \begin{pmatrix} \cos^2\theta & 0 \\ 0 & \cos^2\theta \end{pmatrix} + \begin{pmatrix} \sin^2\theta & 0 \\ 0 & \sin^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\theta + \sin^2\theta & 0+0 \\ 0+0 & \cos^2\theta + \sin^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$= I$

$A^2 + B^2 = I$

38. $A = \begin{pmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{pmatrix}$

$B = \begin{pmatrix} \sin\theta & 0 \\ 0 & \sin\theta \end{pmatrix}$

ST:

$A^2 + B^2 = I$

$A^2 = A \times A$

$$= \begin{pmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{pmatrix}$$

$$A^2 = \begin{pmatrix} \cos^2\theta + 0 & 0+0 \\ 0+0 & 0+\cos^2\theta \end{pmatrix}$$

$B^2 = B \times B$

$$= \begin{pmatrix} \sin\theta & 0 \\ 0 & \sin\theta \end{pmatrix} \begin{pmatrix} \sin\theta & 0 \\ 0 & \sin\theta \end{pmatrix}$$

39. T.F: The volume of the cone $= \frac{1}{3}\pi r^2 h$

$\left.\begin{matrix}\text{Circumference of the} \\ \text{Sector}\end{matrix}\right\} = \text{Base of the cone}$

$$\frac{\theta}{360°} \times 2\pi r = 2\pi R$$

$$\frac{216}{360} \times 21 = R$$

$$\frac{9 \times 7}{5} = R$$

$R = 12.6$ cm

$l^2 = h^2 + r^2$

$h^2 = l^2 - r^2$

$h = \sqrt{l^2 - r^2}$

$= \sqrt{21^2 - 12.6^2}$

$(a^2 - b^2) = (a+b)(a-b)$

$= \sqrt{(21+12.6)(21-12.6)}$

$\sqrt{33.6 \times 8.4}$

$= \sqrt{\frac{33.6 \times 8.4 \times 100}{100}}$

$= \sqrt{\frac{336 \times 100 \times 84}{100}}$

$= \sqrt{\frac{3 \times 2 \times 2 \times 2 \times 2 \times 7 \times 2 \times 2 \times 7 \times 3}{10 \times 10}}$

$= \frac{3 \times 2 \times 2 \times 7 \times 2}{10}$

$= \frac{168}{10}$

$h = 16.8$ cm

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 12.6 \times 16.8$

$= 2794.18 \text{cm}^3$

40. Hemisphere:

$r = 1$ cm.

Frustum:

$R = 2.5$ cm, $r = 1$ cm, $h = 7 - 1 = 6$ cm

External Surface area

CSA of Frustum + CSA of hemisphere

$= \pi(R+r)\,l$

$l = \sqrt{(R-r)^2 + h^2}$

$= \sqrt{(2.5-1)^2 + 6^2}$

$= \sqrt{(1.5)^2 + 6^2}$

$= \sqrt{2.25 + 36}$

$= \sqrt{38.25}$

$l = 61.8$

41. Speed of Stream $= x$ km/hr

Speed of boat in still water $= 18$ km/hr

distance $= 24$ km

Speed of boat in upstream $= 18 - x$ km/hr

Speed of the boat in down stream $= 18 + x$ km/hr

$\text{Time} = \frac{\text{distance}}{\text{speed}}$

$\frac{24}{18-x} - \frac{24}{18+x} = 1$ hour

$\frac{24[18+x) - 24[18-x]}{(18-x)(18+x)} = 1$

$\cancel{24 \times 18} + 24x - \cancel{24 \times 18} + 24x = 18^2 - x^2$

$48x = 324 - x^2$

$x^2 + 48x - 324 = 0$

$(x+54)(x-6) = 0$

$x = -54, x = 6$

x is positive

Speed of Stream $= 6$ km/hr

42. $\tan 60° = \frac{87}{x}$

$\sqrt{3} = \frac{87}{x}$

$x\sqrt{3} = 87$

$x = \frac{87}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{87\sqrt{3}}{3}$

$= 29\sqrt{3}$

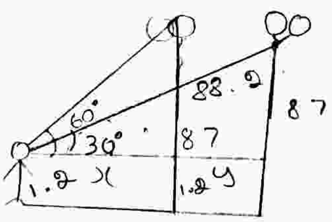
$\tan 30° = \frac{87}{x+y}$

$\frac{1}{\sqrt{3}} = \frac{87}{x+y}$

$x + y = 87\sqrt{3}$

$y = 87\sqrt{3} - 29\sqrt{3}$

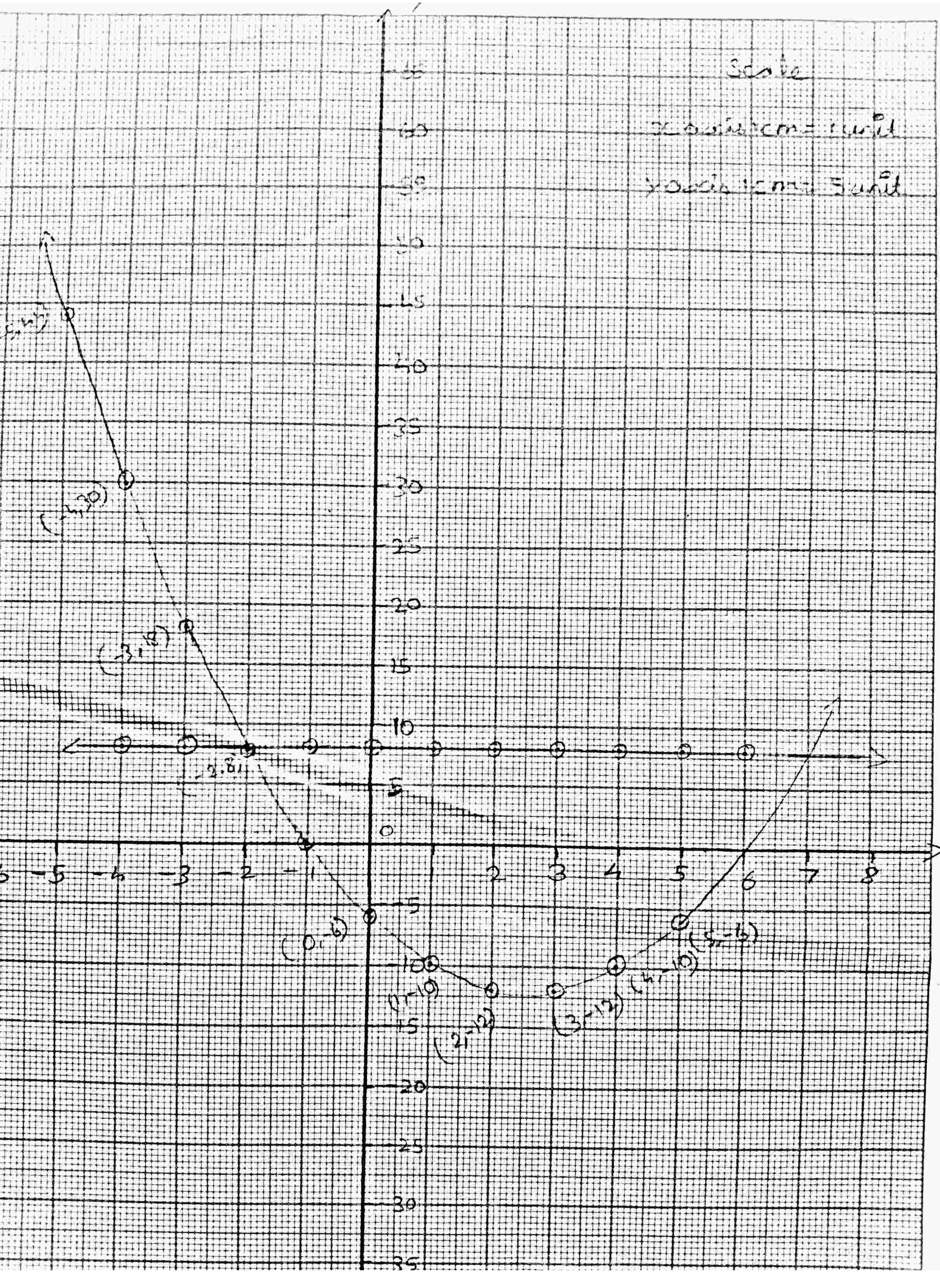
$y = 58\sqrt{3}$



$\therefore$ distance travelled by the balloon $= 58\sqrt{3}$

$= 58(1.732)$

$= 100.34$ m



Scale
x-axis 1cm = 1 unit
y-axis 1cm = 5 unit
(-5,44)
(-4,30)
(-3,18)
(-2,8)
(0,-6)
(1,-10)
(2,-12)
(3,-12)
(4,-10)
(5,-6)

13)

a) $y = x^2 - 5x - 6$

	-5	-4	-3	-2	-1	0	1	2	3	4	5
x^2	25	16	9	4	1	0	1	4	9	16	25
$-5x$	25	20	15	10	5	0	-5	-10	-15	-20	-25
-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
	44	30	18	8	0	-6	-10	-12	-12	-10	-6

The points are (-5,44) (-4,30) (-3,18) (-2,8) (-1,0) (0,-6) (1,-10) (2,-12) (3,-12) (4,-10) (5,-6)

$$y = \cancel{x^2} - \cancel{5x} - 6$$
$$0 = \underset{(-)}{\cancel{x^2}} \underset{(+)}{\cancel{- 5x}} \underset{(+)}{-14}$$
$$\underline{y = \qquad 8}$$

The points are (-4,8) (-3,8) (-2,8) (-1,8) (0,8) (1,8) (2,8) (3,8) (4,8) (5,8) (6,8)

The Solution is (-2,7)

b) $16x^4 - 24x^3 + (a-1)x^2 + (b+1)x + 49$

Find a and b

$4x^2 - 3x \pm 7$

$4x^2 \,|\, 16x^4 - 24x^3 + (a+1)x^2 + (b+1)x + 49$

$16x^4$

$8x^2 + 3x \,|\, -24x^3 + (a+1)x^2$

$-24x^3 + 9x^2$

$8x^2 - 6x \pm 7 \,|\, (a-8)x^2 + (b+1)x + 49$

$56x^2 - 42x + 49$

0

Case: I

$a - 8 - 56 = 0$ | $b + 1 + 42 = 0$

$\boxed{a = 64}$ | $\boxed{b = -43}$

Cas: II

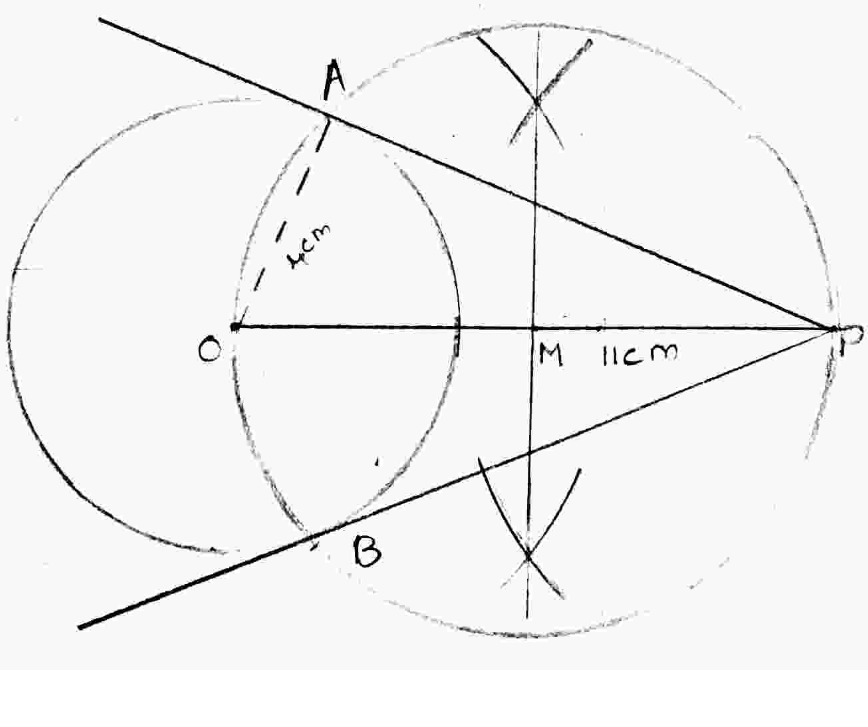
$a - 8 + 56 = 0$ | $b + 1 - 42 - 0$

$a + 48 = 0$ | $b - 41 = 0$

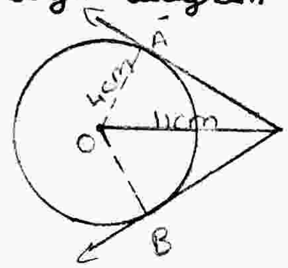
$\boxed{a = -48}$ | $\boxed{b = 41}$

$= |4x^2 - 3x \pm 7|$

44) a)



Rough Diagram



44) b) $\angle QPR = 90^\circ$; PS is the bisector of $\angle PST \perp \angle PR$

Proof : In $\triangle PQR$, PS is the bisector of $\angle A$

To prove: $ST \times (PQ + PR) = PQ \times PR$

$$\therefore \frac{PQ}{PR} = \frac{QS}{SR}$$

Adding (1) on both sides

$$1 + \frac{PQ}{PR} = 1 + \frac{QS}{SR}$$

$$\frac{PR + PQ}{PR} = \frac{SR + QS}{SR}$$

$$\frac{PQ + PR}{PR} = \frac{QR}{SR} \rightarrow ①$$

In $\triangle RST$ and $\triangle RQP$

$\angle SRT = \angle QRP = \angle R$ (common)

$\therefore \angle QPR = \angle STR = 90^\circ$

$\triangle RST \sim RQP$

$$\frac{SR}{QR} = \frac{ST}{PQ}$$

$$\frac{QR}{SR} = \frac{PQ}{ST} \rightarrow ②$$

From ① & ②

$$\frac{PQ + PR}{PR} = \frac{PQ}{ST}$$

$ST(PQ + PR) = PQ \times PR$

Maths PTA Question key
Model - 3

G. Karmega Rani
x
RDM 100%
Tuition centre

PART - I

1. c) 12
2. c) $\{1, -1\}$
3. c) 14280
4. b) 9
5. b) The slope is 5 and the y intercept is 1.6
6. d) $3x^2y$
7. a) 1.4 cm
8. d) 2
9. b) $x+y=3$; $3x+y=7$
10. d) $\cot\theta$
11. c) 3π
12. a) 3cm
13. a) 0
14. c) $\frac{23}{26}$.

PART - II

15. Given: $f(x) = 3x+2$

$x \in N$

To find: the preimage of 29, 53

i) Preimage of 29

$3x+2 = 29$

$3x = 29-2$

$3x = 27$

$x = 9$

ii) Preimage of 53

$3x+2 = 53$

$3x = 53-2$

$3x = 51$

$x = 17$.

16. Yes the given number is a composite number, because

$7\times5\times3\times2+3 = 3(7\times5\times2+1)$

$= 3\times71$

Since the given number can be factorized in terms of two prime, it is a composite number.

17. Given:

$a = 3+k, b = 18-k, c = 5k+1$

$2b = a+c$

$$2(18-k) = 3+k+5k+1$$
$$36-2k = 4+6k$$
$$-2k-6k = 4-36$$
$$-8k = -32$$
$$k = \frac{32}{8}$$
$$k = 4$$

18. $$1^3+2^3+3^3+\dots+k^3 = 169000$$
$$\left[\frac{k(k+1)}{2}\right]^2 = 16900$$
$$\frac{k(k+1)}{2} = \sqrt{16900}$$
$$= 2\times5\times13$$
$$= 130$$
$$1+2+3+\dots+k = 130$$

19. $$A = \begin{bmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{bmatrix}$$
$$2A = 2\begin{bmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{bmatrix}$$
$$= \begin{bmatrix} 14 & 16 & 12 \\ 2 & 6 & 18 \\ -8 & 6 & -2 \end{bmatrix}$$
$$2A+B = \begin{bmatrix} 14 & 16 & 12 \\ 2 & 6 & 18 \\ -8 & 6 & -2 \end{bmatrix} + \begin{bmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 18 & 27 & 9 \\ 1 & 8 & 22 \\ -1 & 11 & -2 \end{bmatrix}$$

20. $$3x^2+kx+81 = a \qquad a = 3,\ b = k,\ c = 81$$
$$\alpha, \alpha^2$$
$$\alpha+\alpha^2 = \frac{-b}{a}$$
$$\alpha(1+\alpha) = \frac{-k}{3} \quad \text{— (1)}$$
$$\alpha\times\alpha^2 = \frac{c}{a}$$
$$\alpha^3 = \frac{81}{3}$$
$$\alpha^3 = 27$$
$$\alpha = 3$$
From (1) $\Rightarrow 3(1+3) = \frac{-k}{3}$
$$3\times4 = \frac{-k}{3}$$
$$12\times3 = -k$$
$$k = -36$$

21. $$x = \frac{a^2+3a-4}{3a^2-3}$$
$$= \frac{(a-1)(a+4)}{3(a^2-1)} \Rightarrow \frac{(a-1)(a+4)}{3(a+1)(a-1)}$$
$$x = \frac{a+4}{3(a+1)}$$
$$y = \frac{a^2+2a-8}{2a^2-2a-4}$$
$$= \frac{(a+4)(a-2)}{2(a^2-2a-2)}$$
$$= \frac{(a+4)(a-2)}{2(a+1)(a-2)} \Rightarrow \frac{a+4}{2(a+1)}$$
$$x^2y^{-2} = \frac{x^2}{y^2}$$
$$= \frac{(a+4)^2}{9(a+1)^2} \div \frac{(a+4)^2}{4(a+1)^2}$$
$$= \frac{(a+4)^2}{9(a+1)^2} \times \frac{4(a+1)^2}{(a+4)^2}$$

$= \frac{4}{9}$

22. $AB = 10cm$, $AC = 14cm$, $BC = 6cm$

$BD = x$, $DE = 6 - x$

By Angle Bisector thm.

$\frac{AB}{AC} = \frac{BD}{DC}$

$\frac{10}{14} = \frac{x}{6-x}$

$\frac{5}{7} = \frac{x}{6-x}$

$5(6-x) = 7x$

$30 - 5x = 7x$

$30 = 12x$

$\frac{30}{12} = x$

$x = 2.5$

$BD(x) = 2.5cm$

$DC = 6 - x$

$= 6 - 2.5cm$

$= 3.5cm$

$m = 1$

$m = \tan\theta$

$1 = \tan\theta$

$\theta = 45°$

$BC = 20$

$AB = x$

$\theta = 60°$

$\tan\theta = \frac{\text{opp side}}{\text{Adj side}}$

$\tan 60° = \frac{BC}{AB}$

$\sqrt{3} = \frac{20}{x}$

$\sqrt{3}x = 20$

$x = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{20\sqrt{3}}{3} \Rightarrow \frac{20 \times 1.732}{3}$

$= 11.55m$

The distance between the foot of the tower and the ball is 11.55m

25. $h = 24cm$, $r = r$

Volume of cylinder = Volume of cone

$\pi r^2 h_2 = \frac{1}{3}\pi r^2 h_1$

$h_2 = \frac{1}{3} \times h_1$

$h_2 = \frac{1}{3} \times 24$

$h_2 = 8$

Height = 8cm

26. $P(A) : P(\bar{A}) = 17 : 15$

$\frac{P(A)}{P(\bar{A})} = \frac{17}{15}$

$\frac{1 - P(\bar{A})}{P(\bar{A})} = \frac{17}{15}$

$15 - 15(P(\bar{A})) = 17P(\bar{A})$

$15 = 17P(\bar{A}) + 15P(\bar{A})$

$15 = 32P(\bar{A})$

$$\frac{15}{32} = P(\bar{A})$$

27. $\bar{x} = 25.6$, $C.V = 18.75$

$$C.V = \frac{\sigma}{\bar{x}} \times 100\%$$

$$18.75 = \frac{\sigma}{\bar{x}} \times 100\%$$

$$18.75 = \frac{\sigma}{25.6} \times 100\%$$

$$\sigma = 4.8$$

28. $3x \overset{(-)}{\pm} 5y + 7 = 0$

$$\text{Slope } m_1 = \frac{-\text{coefficient of } x}{\text{coefficient of } y}$$

$$= \frac{-3x}{-5} \Rightarrow \frac{3}{5}.$$

$15x + 9y + 4 = 0$

$$\text{Slope } m_2 = \frac{-\text{coefficient of } x}{\text{coefficient of } y}$$

$$= \frac{-15}{9}.$$

$$\therefore m_1 \times m_2 = -1$$

$$\frac{3}{5} \times \frac{-15}{9} = -1$$

$$\frac{-3}{3} = -1$$

$$-1 = -1$$

Hence proved

PART - III

29. $A = \{1, 2, 3, 4\}$

$B = \{2, 5, 8, 11, 14\}$

Given. $f(x) = 3x - 1$

when $x = 1$,

$f(1) = 3(1) - 1 \Rightarrow 3 - 1$

$\Rightarrow 2$

when $x = 2$,

$f(2) = 3(2) - 1 \Rightarrow 6 - 1$

$\Rightarrow 5$

when $x = 3$,

$f(3) = 3(3) - 1 \Rightarrow 9 - 1$

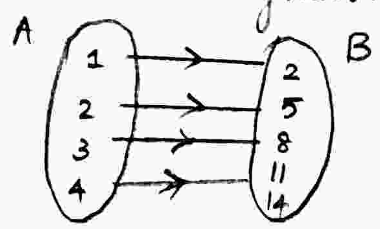
$= 8$

when $x = 4$.

$f(4) = 3(4) - 1 \Rightarrow 12 - 1$

$= 11$

i) Arrow diagram.



ii) table form

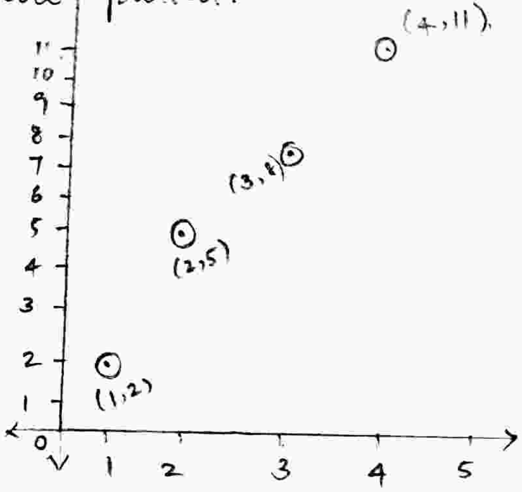
x	1	2	3	4
$f(x)$	2	5	8	11

iii) Set of ordered pairs:

$$f = \{(1,2),(2,5),(3,8),(4,11)\}.$$

iv) Graphical form:

$(1,2),(2,5),(3,8),(4,11)$ are plotted.



30. Given:

$$S(t) = \frac{1}{2}gt^2 + at + b$$

$$S(x) = S(y)$$

$$\frac{1}{2}gx^2 + ax + b = \frac{1}{2}gy^2 + ay + b$$

$$\frac{1}{2}gx^2 + ax + b - \frac{1}{2}gy^2 - ay - b = 0.$$

$$\frac{1}{2}g(x^2 - y^2) + a(x-y) = 0$$

$$\frac{1}{2}g(x+y)(x-y) + a(x-y) = 0$$

$$(x-y)\left[\frac{1}{2}g(x+y) + a\right] = 0$$

$$x - y = 0$$

$$x = y$$

Since, this is one-one function.

31. $A = \{1,2,3,4\}$, $B = \{0,1,2\}$
$C = \{0,1,2\}$.

To verify: $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

$A \times (B \cap C)$:

$$B \cap C = \{0,1,2\} \cap \{0,1,2\}.$$

$$B \cap C = \{0,1,2\}.$$

$$A \times (B \cap C) = \{1,2,3,4\} \times \{0,1,2\}.$$

$$= \{(1,0),(1,1),(1,2),(2,0),(2,1),(2,2),(3,0),(3,1),(3,2),(4,0),(4,1),(4,2)\}. \quad \text{——} \ (1)$$

$(A \times B) \cap (A \times C)$:

$$A \times B = \{1,2,3,4\} \times \{0,1,2\}.$$

$$= \{(1,0),(1,1),(1,2),(2,0),(2,1),(2,2),(3,0),(3,1),(3,2),(4,0),(4,1),(4,2)\}$$

$A \times C = \{1, 2, 3, 4\} \times \{0, 1, 2\}$.

$= \{(1,0), (1,1), (1,2), (2,0), (2,1), (2,2), (3,0), (3,1), (3,2), (4,0), (4,1), (4,2)\}$

$(A \times B) \cap (A \times C) = \{(1,0), (1,1), (1,2), (2,0), (2,1), (2,2), (3,0), (3,1), (3,2), (4,0), (4,1), (4,2)\}$ ——— 2

From 1 & 2, Hence verified.

32. $3 + 6 + 12 + \dots + 1536$

$a = 3, \quad r = \frac{6}{3} \Rightarrow 2.$

$t_n = 1536$

$t_n = ar^{n-1}$

$ar^{n-1} = 1536$

$3(2^{n-1}) = 1536$

$2^{n-1} = \frac{1536}{3}$

$2^{n-1} = 512$

$2^{n-1} = 2^9$

$n - 1 = 9$

$n = 10.$

No. of terms $= 10$.

33. $100 \dots 999$

3 digit no.s divisible by 9

$108, 117, 126 \dots 999$

$a = 108, \; d = 9, \; l = 999$

$n = \left(\frac{l-a}{d}\right) + 1$

$= \frac{999-108}{9} + 1$

$= \frac{891}{9} + 1 \Rightarrow 99 + 1$

$n = 100$

$S_n = \frac{n}{2}[a + l]$

$S_{100} = \frac{100}{2}[108 + 999]$

$= 50 \times 1107$

$S_{100} = 55350$

34. $\frac{2x}{y} + 5 - \frac{3y}{x}$

$\frac{2x}{y}$ | $\frac{4x^2}{y^2} + \frac{20x}{y} + 13 - \frac{30y}{x} + \frac{9y^2}{x^2}$

$\cancel{\frac{4x^2}{y^2}}$

$\frac{4x}{y} + 5$ | $\cancel{\frac{20x}{y}} + 13$

$\frac{20x}{y} + 25$

$-12 - \frac{30y}{x} + \frac{9y^2}{x^2}$

$-12 - \frac{30y}{x} + \frac{9y^2}{x^2}$

0

$$\frac{4x^2}{y^2} + \frac{20x}{y} + 13 - \frac{30y}{x} + \frac{9y^2}{x^2}$$

$$= \left|\frac{2x}{y} + 5 - \frac{3y}{x}\right|.$$

35. $\frac{5x+7}{x-1} = 3x+2$

$5x+7 = (3x+2)(x-1)$

$5x+7 = 3x^2-3x+2x-2$

$5x+7-3x^2+3x-2x+2=0.$

$-3x^2+6x+9=0$

$\div$ by (-3), $x^2-2x-3=0$

$x^2-2x=3$

$x^2-2x+1=3+1$

$x^2-2x+1=4$

Taking square root on both sides...

$\sqrt{(x-1)^2} = \sqrt{4}$

$x-1 = \pm 2$

$x-1 = 2$ $\quad$ $x-1=-2$

$x = 2+1$ $\quad$ $x=-2+1$

$x = 3$ $\quad$ $x=-1$

$x = 3$ and $x=-1$.

36. $A = \begin{bmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{bmatrix}$

To verify: $(AB)^T = B^TA^T$.

Given:

$A = \begin{bmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{bmatrix}$

$AB = \begin{bmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{bmatrix} \times \begin{bmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{bmatrix}$

$= \begin{bmatrix} 5+2+45 & 35+4-9 \\ 1+2+40 & 7+4-8 \end{bmatrix}$

$= \begin{bmatrix} 52 & 30 \\ 43 & 3 \end{bmatrix}$.

$(AB)^T = \begin{bmatrix} 52 & 43 \\ 30 & 3 \end{bmatrix}$ — (1)

$B^T = \begin{bmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{bmatrix}$

$A^T = \begin{bmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{bmatrix}$

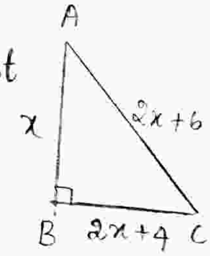
$B^TA^T = \begin{bmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{bmatrix} \times \begin{bmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{bmatrix}$

$= \begin{bmatrix} 5+2+45 & 1+2+40 \\ 35+4-9 & 7+4-8 \end{bmatrix}$

$$B^T A^T = \begin{bmatrix} 52 & 43 \\ 30 & 3 \end{bmatrix} \quad \text{--- (2)}$$

From (1) & (2), Hence verified.

37. Let the shortest side be x.



Hypotenuse $= 2x+6$

third side $= 2x+6-2$

$= 2x+4$.

In ΔABC,

$$AC^2 = AB^2 + BC^2$$

$$(2x+6)^2 = x^2 + (2x+4)^2$$

$$4x^2 + 36 + 24x = x^2 + 4x^2 + 16 + 16x$$

$$0 = x^2 - 24x + 16x - 36 + 16$$

$$\therefore x^2 - 8x - 20 = 0.$$

$$(x-10)(x+2) = 0.$$

$$x - 10 = 0.$$

$$x = 10$$

(or)

$$x + 2 = 0$$

$$x = -2$$

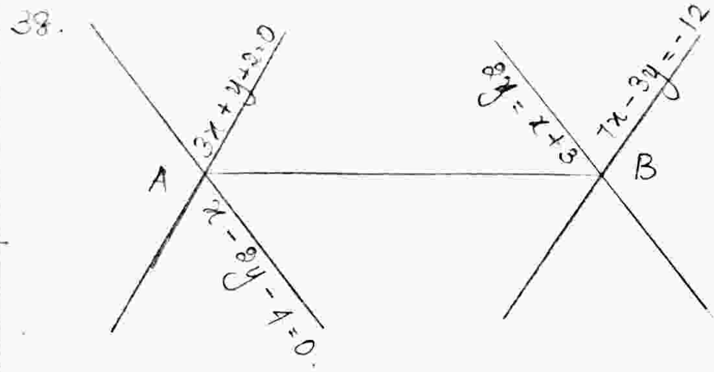
(Negative is omitted)

The side $AB = 10m$

The side $BC = 2(10)+4 = 24m$

Hypotenuse $AC = 2(10)+6 = 26m$

38.



The given lines are,

$$3x + y + 2 = 0$$

$$3x + y = -2 \quad \text{--- (1)}$$

$$x - 2y - 4 = 0$$

$$x - 2y = 4 \quad \text{--- (2)}$$

$$(1) \times 2 \Rightarrow 6x + 2y = -4$$

$$(2) \times 1 \Rightarrow x - 2y = 4$$

$$7x = 0.$$

$$x = \frac{0}{7}$$

$$\therefore x = 0$$

Substitute the value of $x = 0$ in (1)

$$3(0) + y = -2$$

$$y = -2 + 0$$

$$y = -2$$

The point of intersection $(0, -2)$.

The given equation is,

$$7x - 3y = -12 \quad \text{--- (5)}$$

$$2y = x + 3$$

$$-x + 2y = 3 \quad \text{--- (6)}$$

$$⑤ \times 1 \Rightarrow 7x - 3y = -12 \longrightarrow ⑦$$

$$⑥ \times 7 \Rightarrow -7x + 14y = 21 \longrightarrow ⑧$$

$$11y = 9.$$

$$y = \frac{9}{11}.$$

Substitute the value of $y = \frac{9}{11}$ in ⑥

$$-x + 2\left(\frac{9}{11}\right) = 3$$

$$-x + \frac{18}{11} = 3$$

$$-x = 3 - \frac{18}{11} \Rightarrow \frac{33-18}{11} = \frac{15}{4}$$

$$x = \frac{-15}{11}.$$

The point of intersection is $\left(\frac{-15}{11}, \frac{9}{11}\right)$.

Equation of the line joining the points $(0,-2)$ and $\left(\frac{-15}{11}, \frac{9}{11}\right)$ is,

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y+2}{\frac{9}{11}+2} = \frac{x-0}{\frac{-15}{11}-0}.$$

$$\frac{y+2}{\frac{9}{11}+2} = \frac{x}{\frac{-15}{11}}$$

$$\frac{y+2}{\frac{31}{11}} = \frac{x}{\frac{-15}{11}}$$

$$\frac{y+2}{\frac{31}{11}} = \frac{-11x}{15}$$

$(y+2)\frac{11}{31} = \frac{-11x}{15}$

$31\times(-11x) = 11\times 15(y+2)$

$31\times(-11x) = 165(y+2).$

$-341x = 165y+330$

$-341x-165y-330=0.$

$(\div \text{ by } -11),\ 31x+15y+30=0$

The required equation is $31x+15y+30=0$.

39. Given: $\sqrt{3}\sin\theta - \cos\theta = 0$

State that: $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

$\sqrt{3}\sin\theta - \cos\theta = 0.$

$\sqrt{3}\sin\theta = \cos\theta$

$\sqrt{3} = \frac{\cos\theta}{\sin\theta}$

Take reciprocal.

$\frac{1}{\sqrt{3}} = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\tan 30° = \frac{1}{\sqrt{3}}.$

$\theta = 30°$

To prove: $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

$\theta = 30°$

L.H.S:

$\tan 3(30°) = \tan 90°$

$\Rightarrow \infty$ undefined

R.H.S:

$= \frac{3\left(\frac{1}{\sqrt{3}}\right) - \left(\frac{1}{\sqrt{3}}\right)^3}{1-3\left(\frac{1}{\sqrt{3}}\right)^2}$

$= \frac{\sqrt{3}\times\sqrt{3}\left(\frac{1}{\sqrt{3}}\right) - \left(\frac{1}{\sqrt{3}}\right)^3}{1-3\frac{1}{3}}$

$= \frac{\sqrt{3}-3\sqrt{3}}{1-1} \Rightarrow \frac{\sqrt{3}-3\sqrt{3}}{0}$

$= \infty$ undefined

L.H.S = RHS

Hence proved.

40. Let r and h be the radius and height of the cone.

$r = 7m,\ h = 24m$

$l = \sqrt{r^2+h^2}$

$= \sqrt{49+576}$

$l = \sqrt{625}$

$l = 25m.$

CSA of the conical tent $= \pi r l$ sq.units

Area of the canvas

$= \frac{22}{7} \times 7 \times 25 \Rightarrow 550 m^2$

Length of the canvas

$= \frac{\text{Area of the canvas}}{\text{width}}$

$= \frac{550}{4}$

$= 137.5 m.$

Length of the canvas is 137.5 m

41. Total No. of cards = 52.

$n(S) = 52$

i) Let A be the event of getting a King card

$n(A) = 4$

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52}$

ii) Let B be the event of getting a heart card.

$n(B) = 13.$

$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52}$

iii) Let C be the event of getting a red card.

$n(C) = 26.$

$P(C) = \frac{n(C)}{n(S)} = \frac{26}{52}$

$P(A \cap B) = \frac{1}{52}$

$P(B \cap C) = \frac{13}{52}$

$P(A \cap C) = \frac{2}{52}$

$P(A \cap B \cap C) = \frac{1}{52}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C).$

$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{13}{52} - \frac{2}{52} + \frac{1}{52}$

$= \frac{28}{52}$

$= \frac{7}{13}.$

42.

x	x^2
18	324
20	400
15	225
12	144
25	625
$\Sigma x = 90$	$\Sigma x^2 = 1718$

$\bar{x} = \frac{90}{5}$

$\bar{x} = 18.$

$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

$= \sqrt{\frac{1718}{5} - \left(\frac{90}{5}\right)^2}$

$$= \sqrt{343.6 - 324}$$

$$= \sqrt{19.6}$$

$$\sigma \simeq 4.4$$

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{4.4}{18} \times 100 \Rightarrow \frac{440}{18}$$

$$C.V = 24.4\%$$

PART - IV

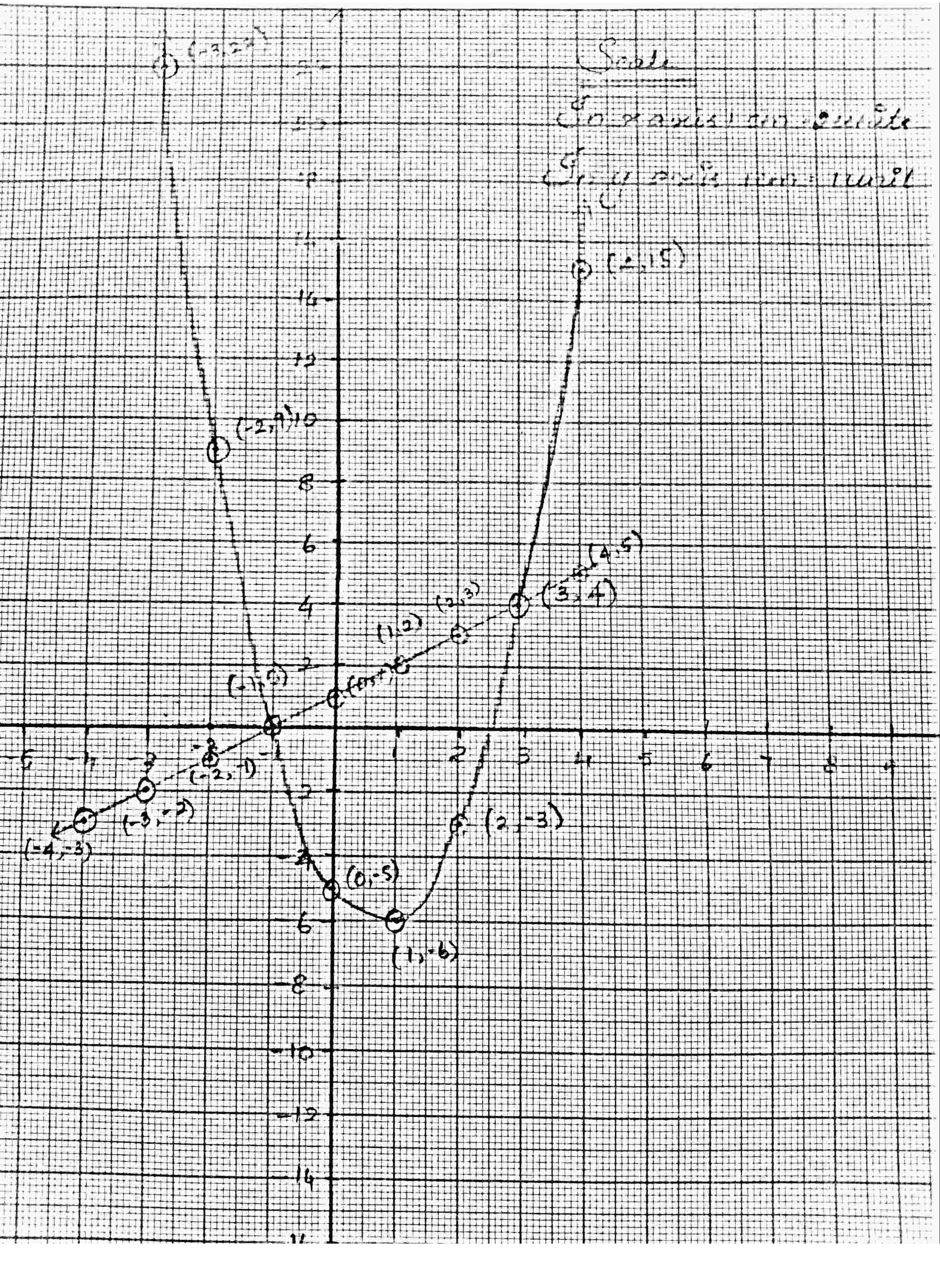
43. a) $y = 2x^2 - 3x - 5$

Solve : $2x^2 - 4x - 6 = 0$.

$$y = 2x^2 - 3x - 5$$

x	-3	-2	-1	0	1	2	3	4
x^2	9	4	1	0	1	4	9	16
$2x^2$	18	8	2	0	2	8	18	32
$-3x$	9	6	3	0	-3	-6	-9	-12
-5	-5	-5	-5	-5	-5	-5	-5	-5
y	22	9	0	-5	-6	-3	4	15

The points are (-3, 22), (-2, 9), (-1, 0), (0, -5), (1, -6), (2, -3), (3, 4), (4, 15).


Scale
(-3, 27)
(4, 15)
(-2, 9)
(4, 5)
(3, 4)
(2, 3)
(1, 2)
(0, 1)
(2, -3)
(0, -5)
(1, -6)
(-2, -1)
(-3, -2)
(-4, -3)

$$y = 2x^2 - 3x - 5$$
$$0 = 2x^2 - 4x - 6$$
$$(-) \quad (-) \quad (+) \quad (+)$$

$$y = x + 1$$

x	-4	-3	-2	-1	0	1	2	3	4
1	1	1	1	1	1	1	1	1	1
$x+1$	-3	-2	-1	0	1	2	3	4	5

The points are (-4, -3), (-3, -2), (-2, -1), (-1, 0), (0, 1), (1, 2), (2, 3), (3, 4), (4, 5).

b) $$\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$$

$$\frac{x+2+2x+2}{(x+1)(x+2)} = \frac{4}{x+4}$$

$$(3x+4)(x+4) = 4(x+1)(x+2)$$

$$3x^2+16x+16 = 4(x^2+3x+2)$$

$$3x^2+16x+16 = 4x^2+12x+8$$

$$3x^2+16x+16-4x^2-12x-8 = 0$$

$$-x^2+4x+8 = 0$$

$(\div$ by $-)$, $x^2-4x-8=0$

By formula method...

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1, \quad b = -4, \quad c = -8$

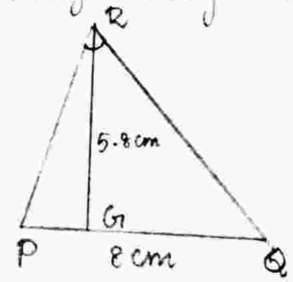
$$x = \frac{4 \pm \sqrt{16+32}}{2} \Rightarrow \frac{4 \pm \sqrt{48}}{2}$$

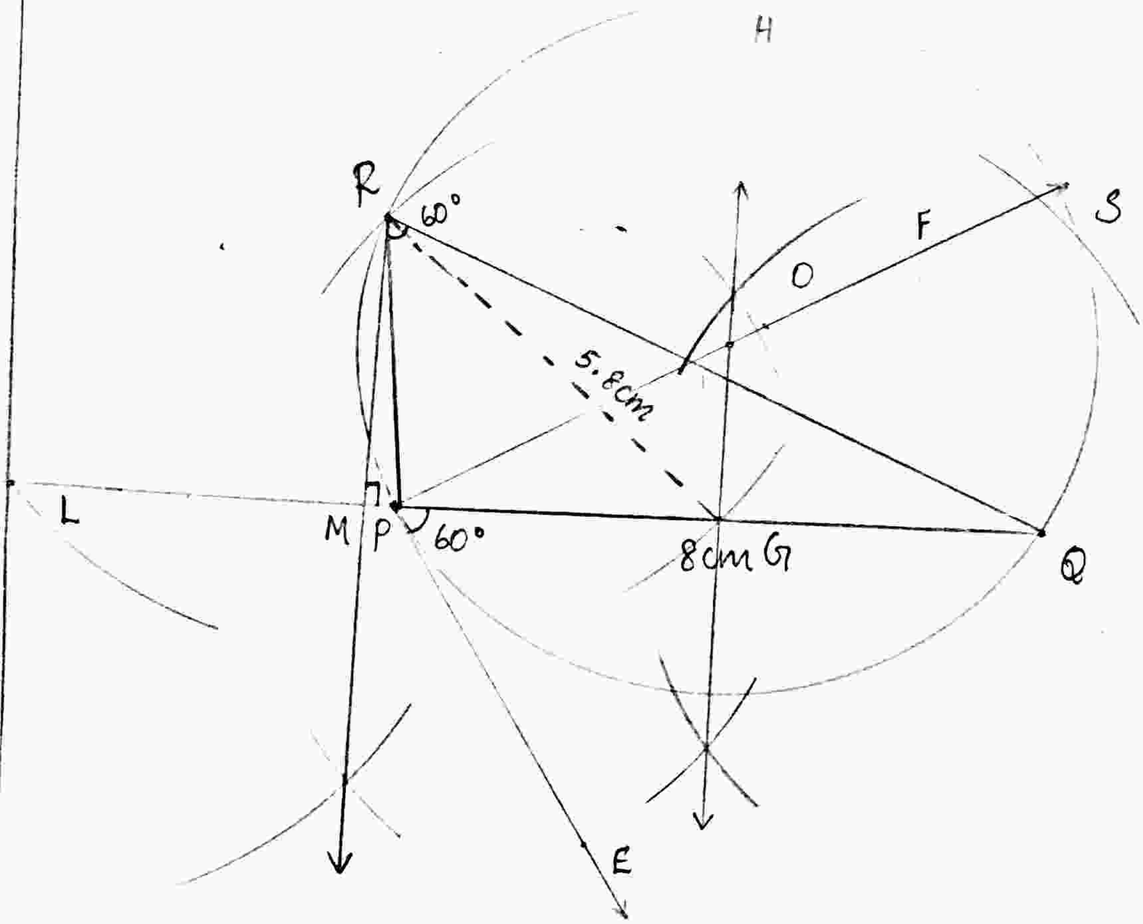
$$= \frac{4 \pm 4\sqrt{3}}{2}$$

$$x = 2 \pm 2\sqrt{3}$$

4. a)

Rough diagram

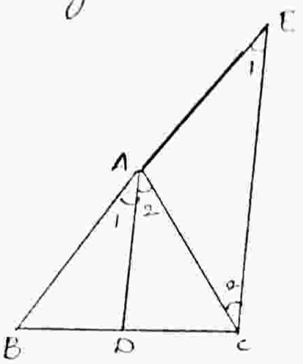




b) Converse of Angle Bisector theorem

Statement:

If a straight line through one vertex of a triangle divides the opposite side internally in the ratio of the other two sides then the line bisects the angle internally at the vertex.



Proof:

Given: ABC is a triangle. AD divides BC in the ratio of the sides containing the angles $\angle A$ to meet BC at D.

That is $\frac{AB}{AC} = \frac{BD}{DC}$ — (1)

To prove: AD bisects $\angle A$. i.e $\angle 1 = \angle 2$

Construction: Draw $CE \parallel DA$. Extend BA to meet A. E

No.	Statement	Reason
1.	Let $\angle BAD = \angle 1$ and $\angle DAC = \angle 2$	Assumption
2.	$\angle BAD = \angle AEC = \angle 1$	Since $DA \parallel CE$ and AC is transversal corresponding angles are equal.
3.	$\angle DAC = \angle ACE = \angle 2$	Since $DA \parallel CE$ and AC is transversal. Alternate angles are equal.
4.	$\frac{BA}{AE} = \frac{BD}{DC} \rightarrow (2)$	In ΔBCE by Thales Theorem

5.	$\frac{AB}{AC} = \frac{BD}{DC}$	From 1
6.	$\frac{AB}{AC} = \frac{BA}{AE}$	From 1 and 2
7.	$AC = AE \rightarrow 3$	Cancelling AB.
8.	$\angle 1 = \angle 2$	ΔACE is isosceles by 3
9.	AD bisects $\angle A$	Since $\angle 1 = \angle BAD = \angle 2 = \angle DAC$ Hence proved.

D → 1

Maths PTA Question key Model - 4	N. Swarna lakshmi RDM 100% Tuition centre

Part - I

1.) c.) {4, 9, 25, 49, 121}

2.) c.) one-one function

3.) a.) 0

4.) c.) 3

5.) a.) b

6.) b.) 5

7.) c.) even

8.) d.) $5\sqrt{2}$ cm

9.) b.) Two sides are parallel and other two side are non-parallel

10.) c.) $3x + 7y = 0$

11.) b.) $\frac{3}{2}$

12.) b.) 11200π cm^3

13.) a.) P (A) > 1

14.) b.) $\frac{1}{4}$

Part - II

15.) $f \circ f(k) = 5$ $\quad f(k) = 2k - 1$

$f \circ f(k) = 2(f(k)) - 1$

$= 2(2k-1) - 1$

$= 4k - 3$

$f \circ f(k) = 4k - 3$

$f \circ f(k) = 5$

$4k - 3 = 5$

$4k = 8$

$k = 2$

16.) let A = {1, 2, 3 ... 100}

R → is cube of A

$1^3 \in A$

$1^3 = 1$ (1, 1)

$2^3 \in A$

$2^3 = 8$ (2, 8)

$3^3 \in A$

$3^3 = 27$ (3, 27)

$4^3 \in A$

$4^3 = 64$ (4, 64)

$5^3 \notin A$

$5^3 = 125$. $\boxed{\text{Domain} = \{1, 2, 3, 4\}}$

R = {(1, 1) (2, 8) (3, 27) (4, 64)}

$\boxed{\text{Range} = \{1, 8, 27, 64\}}$

Number of seats in 1^{st} row $\Big\} = 20$

$t_1 = 20 \quad d = 2$

No of seats in 2^{nd} row $\Big\} = 20+2$

$= 22$

$a = 20 \; ; \; d = 2$

$n = 30$

$t_n = a + (n-1)d$

$t_{30} = 20 + 29(2)$

$= 20 + 58$

$t_{30} = 78$

18.) G.P $\frac{1}{4}, -\frac{1}{2}, 1, -2$

common ratio $= -2$

$a = \frac{1}{4}$

$t_n = ar^{n-1}$

$t_{10} = \frac{1}{4}(-2)^{10-1}$

$= \frac{1}{\cancel{4}} \times -\cancel{512}$ 256 128

$t_{10} = -128$

19.) $\frac{x^2+6x+8}{x^3+8} - p(x) = \frac{3}{x^2-2x+4}$

$p(x) = \frac{x^2+6x+8}{x^3+8} - \frac{3}{x^2-2x+4}$

(a^3+b^3)

8, 4, 2, 6

$= \frac{(x+4)(x+2)}{(x+2)(x^2-2x+4)} - \frac{3}{(x^2-2x+4)}$

$= \frac{x+4-3}{x^2-2x+4}$

$p(x) = \frac{x+1}{}$

20.) sum of roots $= -\frac{3}{2}$

product of roots $= -1$

$x^2 - \left(\begin{matrix}\text{sum of the}\\ \text{roots}\end{matrix}\right)x + \begin{matrix}\text{product}\\ \text{of roots}\end{matrix} = 0$

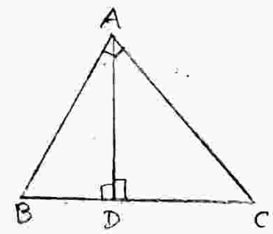
$x^2 - \left(\frac{-3}{2}\right)x + (-1) = 0$

$x^2 + \frac{3}{2}x - 1 = 0$

$\frac{2x^2 + 3x - 2}{2} = 0$

$2x^2 + 3x - 2 = 0$

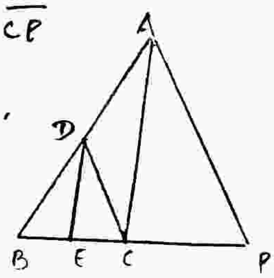
21.) Pythagoras Theorem:



In a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

22.) Given $DE \parallel AC$ and $DC \parallel AP$.

Prove that: $\frac{BE}{EC} = \frac{BC}{CP}$



In $\triangle BPA$, $DC \parallel AP$.

By BPT Theorem,

$\frac{BC}{CP} = \frac{BD}{DA} \rightarrow (1)$

In $\triangle BCA$, $DE \parallel AC$

By BPT Theorem,

$\frac{BE}{EC} = \frac{BD}{DA} \rightarrow (2)$

From (1) and (2)

$\frac{BE}{EC} = \frac{BC}{CP}$

Hence proved.

23.) $P(-1.5, 3)$ $Q(6, -2)$ $R(-3, 4)$

$$\left.\begin{array}{l}\text{Area of}\\ \triangle\ PQR\end{array}\right\} = \frac{1}{2}\left| \begin{array}{cccc} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{array} \right|$$

$$= \frac{1}{2}\left| \begin{array}{cccc} -1.5 & 6 & -3 & -1.5 \\ 3 & -2 & 4 & 3 \end{array} \right|$$

$$= \frac{1}{2}\{(3+24-9) - (18+6-6)\}$$

$$= \frac{1}{2}(18-18)$$

$$= 0$$

25.) radii of 2 cones are r_1, r_2

$r_1 = r_2 = r$

let height of cones are h_1, h_2

$$\frac{\text{Volume of cone}_1}{\text{Volume of cone}_2} = \frac{3600}{5040}$$

$$\frac{\frac{1}{3}\pi r^2 h_1}{\frac{1}{3}\pi r^2 h_2} = \frac{20}{30}$$

$$\frac{h_1}{h_2} = \frac{2}{3}$$

$$h_1 : h_2 = 2:3$$

24.)

$$\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$$

$$\frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} - \cos A} = \frac{\frac{\cos A - \sin A \cos A}{\sin A}}{\frac{\cos A + \cos A \sin A}{\sin A}}$$

$$= \frac{\cos A(1-\sin A)}{\cos A(1+\sin A)}$$

$$L.H.S = \frac{1-\sin A}{1+\sin A} \to 1$$

$$R.H.S = \frac{\frac{1}{\sin A} - 1}{\frac{1}{\sin A} + 1} \Rightarrow \frac{\frac{1-\sin A}{\sin A}}{\frac{1+\sin A}{\sin A}}$$

$$R.H.S = \frac{1-\sin A}{1+\sin A} \to 2$$

$L.H.S = R.H.S$

Hence proved

26.) Range $R = 13.67$

Largest value $L = 70.08$

Range $R = L - S$

$13.67 = 70.08 - S$

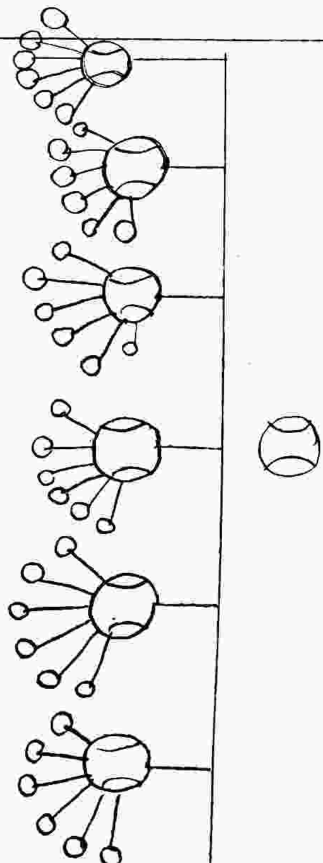
$S = 70.08 - 13.67$

$= 56.41$

Smallest Value $= 56.41$

27.)

Sample space = {(1,1)(1,2)(1,3)(1,4)(1,5)(1,6
(2,1)(2,2)(2,3)(2,4)(2,5)(2,6
(3,1) (3,2) (3,3) (3,4)(3,5)(3
(4,1) (4,2)(4,3)(4,4) (4,5)(4
(5,1) (5,2) (5,3)(5,4)(5,5)(5,
(6,1)(6,2)(6,3)(6,4)(6,5)(6,



28.) $8x^2 - 25 = 0$

$a = 8 \quad b = 0 \quad c = -25$

Sum of the proots $= \frac{-b}{a}$

$= \frac{0}{8}$

$= 0$

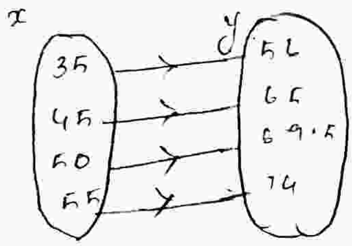
Products of the roots: $\frac{c}{a}$

$= \frac{-25}{8}$

Part - iii

I.) Answer any 10 questions:

29.) (i) $y = ax + b$



f is a function from x to Y + for different elements in x there are different images in Y. Hence f is a one-one function.

(ii) $y = ax + b$

$x = 35 \quad y = 56$

$56 = 35a + b \rightarrow ①$

$x = 45, \; y = 65$

$65 = 45a + b \rightarrow ②$

$(2) - (1) \Rightarrow 65 - 56 = 45a - 35a + b - b$

$9 = 10a$

$a = \frac{9}{10}$

$a = 0.9$

$① \Rightarrow 56 = 35\left(\frac{9}{10}\right) + b$

$56 = \frac{63}{2} + b$

$56 - \frac{63}{2} = b$

$b = \frac{112}{2} - \frac{63}{2}$

$b = \frac{49}{2} \quad b = 24.5$

(iii) $x = 40 \quad y = ?$

$y = (0.9)(40) + 24.5$

$y = 36.0 + 24.5$

$y = 60.5$

(iv) $y = 53.3$

$x = ?$

$53.3 = (0.9)x + 24.5$

$53.3 - 24.5 = (0.9)x$

$28.8 = (0.9)x$

$x = \frac{28.8}{0.9}$

$x = 32$

30.) $6x + 1 = (-5, -4, -3, -2 -1, 0, 1)$

$5x^2 - 1 = (2, 3, 4, 5)$

$3x - 4 = (6, 7, 8, 9)$

i) $f(7) - f(1)$

$= (3x - 4) - (6x + 1)$

$= (3(7) - 4) - (6(1) + 1)$

$= (21 - 4) - (6 + 1)$

$= (21 - 4) - 7$

$= 17 - 7$

$= 10$

(ii) $\dfrac{2f(-2) - f(6)}{f(4) - f(2)}$

$2f(-2) - f(6) = 2(6x+1) - (3x-4)$

$= 2\ (6(-2)+1) - (3(6)-4)$

$= 2\ (-12+1) - (18-4)$

$= 2(-11) - 14$

$= -22 - 14$

$= -36$

$f(4) + f(-2) = (5x^2-1)(6x+1)$

$= (5\ (6) - 1) + (6(-2)+1)$

$= (80-1) + (-12+1)$

$= 79 - 11$

$= 68$

$\dfrac{2f(-2) - f(6)}{f(4) - f(-2)} = \dfrac{-36}{68}$

$= \dfrac{-9}{17}$

31.) $5 + 55 + 555 \cdots + n$ terms.

$= 5(1 + 11 + 111 + \cdots n\text{ terms})$

$= \frac{5}{9}(9 + 99 + 999 + \cdots n\text{ terms})$

$= \frac{5}{9}((10-1) + (100-1) + (1000-1) \cdots + n\text{ terms})$

$= \frac{5}{9}((10 + 100 + 1000 \cdots + n\text{ terms}) - n)$

$= \frac{5}{9}\left(\dfrac{10(10^n-1)}{10-1} - n\right)$

$= \dfrac{50(10^n-1)}{81} - \dfrac{5n}{9}$

32.) let x be the girl's age and y be the sister's age

girl	x	$x+5$
sister	y	$y+5$

$x = 2y \rightarrow ①$

$(y+5)(x+5) = \cancel{0}\ 375$

$(y+5)(2y+5) = 375$

$2y^2 + 5y + 10y + 25 = 375$

$2y^2 + 15y + 25 - 375 = 0$

$2y^2 + 15y - 350 = 0$

$a = 2,\ b = 15,\ c = -350$

$y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-15 \pm \sqrt{(15)^2 - 4(2)(-350)}}{2(2)}$

$= \dfrac{-15 \pm \sqrt{225 + 2800}}{4}$

$= \dfrac{-15 \pm \sqrt{3025}}{4}$

$= \dfrac{-15 \pm 65}{4}$

(+)	(−)
$= \dfrac{-15+65}{4}$	$\dfrac{-15-65}{4}$
$= 10$	∴ Negative is not possible.
$y = 10$	

Sub y in ①

$2y = x$

$2(10) = x$

$20 = x$

33.) Find x

$$x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} x^2+8 & 24 \\ 10 & 6x \end{bmatrix}$$

$$\begin{bmatrix} 2x^2 & 2x \\ 3x & x^2 \end{bmatrix} + \begin{bmatrix} 16 & 10x \\ 8 & 8x \end{bmatrix} = \begin{bmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{bmatrix}$$

$$\begin{bmatrix} 2x^2+16 & 12x \\ 3x+8 & x^2+8 \end{bmatrix} = \begin{bmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{bmatrix}$$

$$12x = 48$$

$$x = \frac{48}{12} = 4$$

34.) $4x^4 - 12x^3 + 37x^2 + bx + a$

$$2x^2 - 3x + 7$$

$2x^2$ | $4x^4 - 12x^3 + 37x^2 + bx + a$

$(-)4x^4$

$4x^2 - 3x$ | $-12x^3 + 37x^2$

$-12x^3 + 9x^2$

$(+) \quad (-)$

$28x^2 + bx + a$

$28x^2 - 42x + 49$

$(-) \quad (+) \quad (-)$

0

$b + 42 = 0$

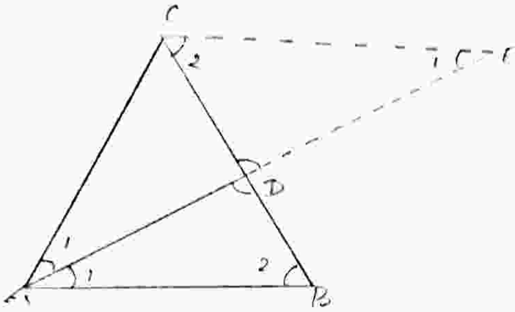
$b = -42$

$a - 49 = 0$

$a = 49$

35 STATEMENT:

The internal bisector of an angle of a triangle divides the opposite sides containing the angle



PROOF:

GIVEN: In ΔABC, AD is the internal bisector

To prove: $\frac{AB}{AC} = \frac{BD}{CD}$

CONSTRUCTION: Draw a line through C parallel to AE. Extend AD to meet line through C at E.

STATEMENT	REASON
$\angle AEC = \angle BAE = \angle 1$	Two parallel lines cut by a transversal make alternate angle equal.
ΔACE is isosceles $AC = CE \rightarrow (1)$	In ΔACE, $\angle CAE = \angle CEA$.
$\Delta ABD \sim \Delta ECD$ $\frac{AB}{CE} = \frac{BD}{CD}$	By AA similarity
$\frac{AB}{AC} = \frac{BD}{CD}$	From (1) $AC = CE$ Hence proved.

36 Gn: $QS = 2PR$

$S(1,1)$ $M(2,-1)$

T.F: The point $P(a,b)$

Slope of $SM = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-1-1}{2-1} = \frac{-2}{1}$

$= -2$

Slope of PM:

Given lines PM & SM are $\perp$r

$m_1 \times m_2 = -1$

$-2 \times m_2 = -1$

$m_2 = \frac{1}{2}$

Pt $(2,-1)$ Slope $(\frac{1}{2})$

$y_2 - y_1 = m(x_2 - x_1)$

$(b+1) = \frac{1}{2}(a-2)$

$2(b+1) = (a-2)$

$2b + 2 - a + 2 = 0$

$-a + 2b + 4 = 0$

$2b + 4 = a \rightarrow (1)$

Gn: $QS = 2PR$

$SM = 2PM$

Distance $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = 2\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$\sqrt{(2-1)^2 + (-1-1)^2} = 2\sqrt{(a-2)^2 + (b+1)^2}$

$\sqrt{1^2 + (-2)^2} = 2\sqrt{(2b+4-2)^2 + (b+1)^2}$

From (1) $a = 2b + 4$

$\sqrt{1+4} = 2\sqrt{(2b+2)^2 + (b+1)^2}$

$\sqrt{5} = 2\sqrt{(2b+2)^2 + (b+1)^2}$

Squaring on both side

$5 = 2^2 \{(2b+2)^2 + (b+1)^2\}$

$(a+b)^2$ $(a+b)^2$

$$5 = 4\{4b^2 + 2\times2\times b\times2 + 4 + b^2 + 2\times b\times 1 + 1^2\}$$

$$5 = 4\{5b^2 + 8b + 5 + 2b\}$$

$$5 = 4\{5b^2 + 10b + 5\}$$

$$5 = 4\times5\{b^2 + 2b + 1\}$$

$$5 = 20(b+1)^2$$

$$\frac{5}{20} = (b+1)^2$$

$$\frac{1}{4} = (b+1)^2$$

$$\sqrt{\frac{1}{4}} = \sqrt{(b+1)^2}$$

$$\pm\frac{1}{2} = b+1$$

$$b+1 = +\frac{1}{2}.$$

$$b = \frac{1}{2} - 1 = \frac{1-2}{2} = \frac{-1}{2}$$

$$a = 2b + 4$$

$$= 2\left(\frac{-1}{2}\right) + 4$$

$$= -1 + 4$$

$$a = 3.$$

$$b + 1 = \frac{-1}{2}$$

$$b = \frac{-1}{2} - 1 = \frac{-1-2}{2} = \frac{-3}{2}$$

$$b = \frac{-3}{2}.$$

when, $a = 2b + 4$

$$a = 2\left(\frac{3}{2}\right) + 4 = -3 + 4 = 1.$$

$(3, -\frac{1}{2})$ or $(1, -\frac{3}{2})$ $= -3 + 4 = 1.$

37)

Let the height of the statue be h m

Let AD be x

$EC = h - x$

In the righ ΔABD

$$\tan 34^\circ = \frac{AD}{AB}$$

$$0.6745 = \frac{x}{35}$$

$$x = 0.6745 \times 35$$

$$x = 23.61\,m$$

In right ΔDEC

$$\tan 24^\circ = \frac{EC}{DE}$$

$$0.4452 = \frac{h-x}{35}$$

$$h - x = 0.4452 \times 35$$

$$h - 23.61 = 15.58$$

$$h = 15.58 + 23.61$$

$$h = 39.19\,m$$

Height of the statue $= 39.19$ m.

38)

Volume of cylinder = Volume of Cone

$$\pi R^2 H = \frac{1}{3}\pi r^2 h$$

$$18 \times 18 \times 32 = \frac{1}{3} r^2 \times 24$$

$$r^2 = 18 \times 18 \times 4$$

$$r = 18 \times 2 = 36\ cm$$

$$l = \sqrt{r^2 + h^2}$$

$$= \sqrt{36^2 + 24^2}$$

$$= \sqrt{1296 + 576}$$

39.) Number of guavas, $n = 7$

Mean $\bar{x}_1 = \frac{30}{7} = 4.29$

x_i	x_i^2
3	9
5	25
6	36
4	16
3	9
5	25
4	16
$\Sigma x_i = 30$	$\Sigma x_i^2 = 136$

Standard deviation: $\sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$

$\sigma_1 = \sqrt{\frac{136}{7} - \left(\frac{30}{7}\right)^2}$

$= \sqrt{19.43 - 18.40}$

$\simeq 1.01$

$C.V_1 = \frac{\sigma_1}{\bar{x}_1} \times 100\% = \frac{1.01}{4.29} \times 100$

$C.V_1 = 23.54\%$

Number of oranges: $n = 7$

Mean $\bar{x}_2 = \frac{30}{7} = 4.29$

x_i	x_i^2
1	1
3	9
7	49
9	81
2	4
6	36
2	4
$\Sigma x_i = 30$	$\Sigma x_i^2 = 184$

Standard deviation $\sigma_2 = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$

$\sigma_2 = \sqrt{\frac{184}{7} - \left(\frac{30}{7}\right)^2}$

$= \sqrt{26.29 - 18.40}$

$= 2.81$

$C.V_2 = \frac{\sigma_2}{\bar{x}_1} \times 100\% = \frac{2.81}{4.29} \times 100\%$

$= 65.50\%$

$C.V_1 = 23.54\%$

$C.V_2 = 65.50\%$

$C.V_1 < C.V_2$

$\therefore$ Guavas is more consistent

40.) Total number of students $n(S) = 50$

Let A and B be the events of students opted for NCC and NSS respectively.

$n(A) = 28 \quad n(B) = 30 \quad n(A \cap B) = 18$

$P(A) = \frac{n(A)}{n(S)} = \frac{28}{50}$

$P(B) = \frac{n(B)}{n(S)} = \frac{30}{50}$

$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{18}{50}$

i) Probability of the students opted for NCC but not NSS.

$P(A \cap \bar{B}) = P(A) - P(A \cap B) = \frac{28}{50} - \frac{18}{50}$

$= \frac{10}{50} = \frac{1}{5}$

ii) Probability of the students opted for NSS but not NCC

$P(\bar{A} \cap B) = P(B) - P(A \cap B) = \frac{30}{50} - \frac{18}{50}$

$= \frac{6}{25}$

(iii) Probability of the students opted for exactly one of them.

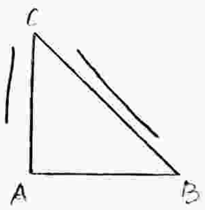
$= P[(A\cap\bar{B}) \cup P(\bar{A}\cap B)]$

$= P(A\cap\bar{B}) + P(\bar{A}\cap B)$

$= \frac{1}{5} + \frac{6}{25} = \frac{11}{25}$

41.) A $(1,-4)$ B $(2,-3)$ C $(4,-7)$

Slope of AB $= \frac{y_2-y_1}{x_2-x_1}$



$\Rightarrow \frac{-3-(-4)}{2-1}$

$= \frac{-3+4}{1} \Rightarrow \frac{1}{1}$

$m_1 = 1$

Slope of BC $= \frac{-7-(-3)}{4-2} \Rightarrow \frac{-7+3}{4-2}$

$= \frac{-4}{2} \Rightarrow -2$

$m_2 = -2$

Slope of AC $= \frac{-7-(-4)}{4-1} \Rightarrow \frac{-7+4}{4-1}$

$= \frac{-3}{3} \Rightarrow -1$

$m_3 = -1$

$m_1 \times m_3 = 1 \times (-1)$

$= -1$

AB is perpendicular to AC

$\angle A = 90°$. Therefore, ΔABC is a right angled Δ^{le}.

42.) Given: $S_n = 16500$

$n = 10$

$d = 100$

To find: $a = ?$

$S_n = \frac{n}{2}(2a+(n-1)d)$

$S_{10} = \frac{10}{2}(2a+(10-1)100)$

$16500 = 5(2a + 9\times100)$

$16500 = 2a + 900$

$3300 - 900 = 2a$

$\frac{2400}{2} = a$

$\therefore a = 1200$

Part - IV

43.) b.) $\frac{a^2-16}{a^3-8} \times \frac{2a^2-3a-2}{2a^2+9a+4} \div \frac{3a^2-11a-4}{a^2-2a+4}$

$\frac{(a+4)(a-4)}{(a-2)(a^2-2a-4)} \times \frac{(2a+1)(a-2)}{(2a+1)(a+4)} \times \frac{a^2-2a+4}{(3a+1)(a-4)}$

$= \frac{a^2-2a+4}{(3a+1)(a^2+2a+4)}$

a.) $y = 2x^2$

step : 1

x	-2	-1	0	1	2
y	8	2	0	2	8

To solve $2x^2-x-6=0$, Subtract $2x^2-x-6=0$ from $y=2x^2$

that is
$y = 2x^2$
$0 = 2x^2-x-6$

Step : 2 $y = x+6$

x	-2	-1	0	1	2
y	4	5	6	7	8

Step : 3 Mark the points of intersection of the curve $y=2x^2$ and the line $y=x+6$. That is $(-1.5, 4.5)$ and $(2, 8)$

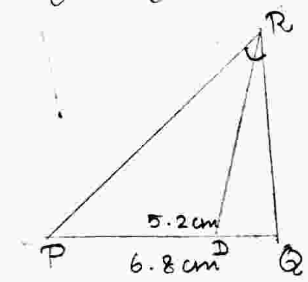
step : 4 The x coordinates of the respective points forms the solution set $(-1.5, 2)$ for $2x^2-x-6=0$

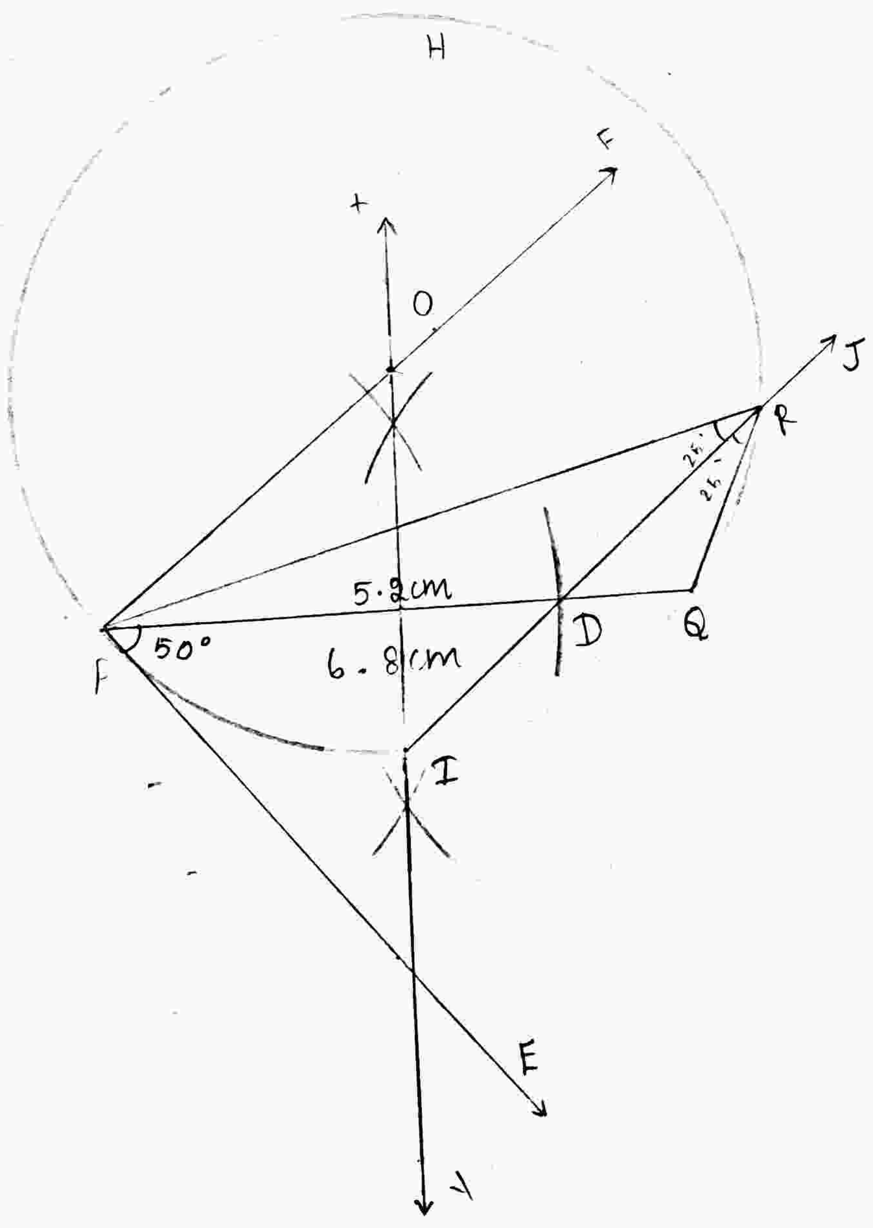
44.)

A.) PQ = 6.8 cm vertical angle = 50°

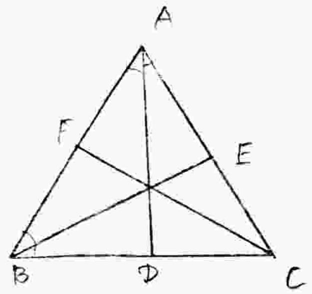
PD = 5.2 cm

Rough Diagram





44.) b.) To prove: angle bisectors are concurrent



Proof: In ΔABC

By angle bisector thm, BC is the angle bisector of $\angle A$.

$$\frac{AB}{AC} = \frac{BD}{DC} \rightarrow 1$$

In ΔABC, by angle bisector thm, AC is the Angle bisector of $\angle B$.

$$\frac{BC}{BA} = \frac{CE}{EA} \rightarrow 2$$

Multiply, 1 2 3

$$\frac{AB}{AC} \times \frac{BC}{BA} \times \frac{CA}{CB} = \frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB}$$

$$1 = \frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB}$$

From ceva's thm,

The angle bisectors are concurrent.

D