Hence, M_1 is not connected.

which is a $\Rightarrow\Leftarrow$

$\therefore$ A is connected.

$\therefore f(M_1)$ is connected.

Hence proved.

Theorem-8

INTERMEDIATE VALUE THEOREM

Statement:

Let f be a real valued continuous function defined on an interval I. Then f takes every value b/w any two values it assumes.

Proof:

Let $a, b \in I$

Let $f(a) \neq f(b)$

without lose of generality we assume that $f(a) < f(b)$.

Let C be $\ni$: $f(a) < C < f(b)$

Since I is an interval it is connected.

Then $f(I)$ is a connected subset of $\mathbb{R}$.

$\therefore$ $f(I)$ is an interval

Also $f(a), f(b) \in f(I)$.

Hence, $[f(a), f(b)] \subseteq f(I)$

$\therefore C \in f(I)$ $[\because f(a) < C < f(b)]$

$\therefore C = f(x)$

For some $x \in I$

$\therefore$ f takes every value b/w any 2 values it assumes.

Hence proved.

Problems:

1) P.1 If f is a non-constant real valued continuous function on $\mathbb{R}$ then the range of f is uncountable.

Sol:

W.k.T $\mathbb{R}$ is connected.

Since f is continuous on $\mathbb{R}$, $f(\mathbb{R})$ is connected.

$\therefore f(\mathbb{R})$ is an interval.

Since f is a non-constant function the interval $f(\mathbb{R})$