

UNIT-1

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- 1) Variable Separable an eqn of the form $f(x)dx + f(y)dy = 0$.
- 2) The sol is $\int f(x)dx + \int f(y)dy = C$.
- 3) Homogeneous eqn of the form $\frac{dy}{dx} = \frac{f_1(x,y)}{f_2(x,y)}$.
- 4) f_1 and f_2 are homogeneous functions of the same degree in x and y .
- 5) A differential equations is a eqn in which differentiable co-efficient occurs.
- 6) Differential eqns are of two types.
ordinary diff. eqn.
partial " " "
- 7) One (or) more of the derived function is called ordinary D.E.
- 8) Two (or) more independent variable a dependent variable and its partial derivatives is called a partial D.E.
- 9) The order and degree of the D.E. is a highest order and highest degree derivative occurs.
- 10) A D.E. of the form is,
 $f(x,y)dx + g(x,y)dy = 0$ is called a D.E.
of the 1st order and 1st degree.

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UNIT-1

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- 2) The sol is $\int f(x)dx + \int F(y)dy = C$.
- 3) Homogeneous eqn of the form $\frac{dy}{dx} = \frac{f_1(x,y)}{f_2(x,y)}$.
- 4) f_1 and f_2 are homogeneous functions of the same degree in x and y .
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- 9) The order and degree of the D.E. is a highest order and highest degree derivative occurs.
- 10) A D.E. of the form is,
 $f(x,y)dx + g(x,y)dy = 0$ is called a D.E.
of the 1st order and 1st degree.

11) A D.E. is said to be linear when the dependent variable and its derivatives occur only in the first degree. (2)

12) The linear eqn of the form $\frac{dy}{dx} + Py = Q$.

13) P and Q are functions of x on y.

14) The Sol is, I.F. = $e^{\int P dx}$.

$$y e^{\int P dx} = \int Q e^{\int P dx} + C$$

15) A non homogeneous eqn of the form, $\frac{dy}{dx} = \frac{ax+by+c}{a'x+b'y+c'}$

16) $d(xy) = xdy + ydx$.

19) $d(\tan^{-1}(y/x)) = \frac{xdy - ydx}{x^2 + y^2}$

17) $d(x/y) = \frac{ydx - xdy}{y^2}$

20) $d[\tan^{-1}(x/y)] = \frac{ydx - xdy}{x^2 + y^2}$

18) $d(y/x) = \frac{xdy - ydx}{x^2}$

21) Bernoulli's eqn of the form $\frac{dy}{dx} + Py = Q(y^n)$

22) The Sol is $\frac{dz}{dx} + Pz(1-n) = Q(1-n)$.

23) This eqn is linear in z.

24) An exact D.E. is obtained by equating an exact(or) perfect differential to zero.

25) $\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 0$ hypothesis.

26) $\frac{dy}{dx}$ here after by p.

27) The eqn of the 1st order and of the nth degree. be $P^n + P_1 P^{n-1} + P_2 P^{n-2} + \dots + P_n = 0$.

28) The Sol is, $\phi_1(x, y, c_1), \phi_2(x, y, c_2) \dots \phi_n(x, y, c_n) = 0$.

UNIT-2

1) If m_1 and m_2 are real & distinct,

$$y = Ae^{m_1 x} + Be^{m_2 x}$$

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12) The linear eqn of the form $\frac{dy}{dx} + Py = Q$.

13) P and Q are functions of x on y.

14) The sol is, I.F. = $e^{\int P dx}$

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27) The eqn of the 1st order and of the nth degree. be $P^n + P_1 P^{n-1} + P_2 P^{n-2} + \dots + P_n = 0$.

28) The sol is, $\phi_1(x, y, c_1), \phi_2(x, y, c_2), \dots, \phi_n(x, y, c_n) = 0$.

UNIT-2

1) If m_1 and m_2 are real & distinct,

$$y = Ae^{m_1 x} + Be^{m_2 x}$$

- 2) If m_1 and m_2 are real & equal $y = (A+Bx)e^{ax}$
- 3) If the roots are imaginary $y = e^{ax}(A \cos \beta x + B \sin \beta x)$
- 4) If $f(a) \neq 0$ $\frac{1}{f(D)} \cdot e^{ax}$ replace D by a .
- 5) If $f(a) = 0$, $f(D) = (D-a)^r \phi(D) \Rightarrow \frac{1}{f(D)} \cdot e^{ax} = \frac{1}{\phi(a)} \left(\frac{x^r e^{ax}}{r!} \right)$
- 6) $(D^2 + 6D + 9)y = 0 \Rightarrow y = (A+Bx)e^{-3x}$
- 7) $am^2 + bm + c = 0$ is called the aux. eqn. (3)
- 8) $(D^2 - 5D + 6)y = 2e^{4x} \Rightarrow y = Ae^{2x} + Be^{3x} + e^{4x}$
- 9) $(D^2 + 6D + 8)y = e^{-2x}$ C.F. = $Ae^{-4x} + Be^{-2x}$
- 10) $y = \frac{x}{f(D)}$ is called the particular Integral.

UNIT-3

1) An eqn involving more than two variables, only one independent vble, then the eqn is known as a set of ordinary simultaneous eqn.

2) A pair of simultaneous differential eqns of the 1st order and 1st degree may be written as,

$$P_1 \frac{dx}{dz} + Q_1 \frac{dy}{dz} + R_1 = 0, \quad P_2 \frac{dx}{dz} + Q_2 \frac{dy}{dz} + R_2 = 0.$$

where $P_1, P_2, Q_1, Q_2, R_1, R_2$ are functions of x, y & z .

3) The ratios of the differentials can be obtained as,

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} \text{ where } P, Q, R \text{ are functions of } x, y \text{ & } z.$$

4) $P \frac{\partial w}{\partial x} + Q \frac{\partial w}{\partial y} + R \frac{\partial w}{\partial z} = 0$ where $w = \phi(u, v)$, ϕ being an arbitrary function.

5) When two of the ratios in $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ involve only the two corresponding out of the three vbles x, y & z .

6) A part or the whole of the general Sol. of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ can also be found by the following method,

$$\frac{l dx + m dy + n dz}{lP + mQ + nR} \text{ where } l, m, n \text{ are multipliers}$$

We must also have $l dx + m dy + n dz = 0$.

7) If two different sets of such multipliers l, m, n can be obtained then $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ can be deduced by $u = \int (l dx + m dy + n dz)$.

8) If $u = C_1$ and $v = C_2$ are the general sol of the eqn $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ then it follows the system of curves

9) D stands for $\frac{d}{dt}$. (4)

10) The simplest case of two dependent vbles x & y the eqn can be written in the form,

$$f_1(D)x + \phi_1(D)y = T_1, f_2(D)x + \phi_2(D)y = T_2$$

11) If an integral included in the complementary functn of the given eqn be known the complete sol can be found in terms of, $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$ where P, Q, R are functns of x .

12) Some cases where in simple functns of x , like x and e^x are integrals of the eqn

$$P_2 \frac{d^2y}{dx^2} + P_1 \frac{dy}{dx} + P_0 y = 0 \text{ should be noted}$$

13) Reduction to the normal form is also known as removing the 1st derivative.

14) z such as the co-eff of $\frac{dy}{dz}$ i.e. $\frac{d^2z}{dz^2} + P\frac{dz}{dz}$ vanishes. $\therefore z = \int e^{-\int P dx} dx$.

15) When $\frac{d^2y}{dz^2} \left(\frac{dz}{dx}\right)^2 + \frac{dy}{dz} \left(\frac{d^2z}{dz^2} + P\frac{dz}{dz}\right) + Qy = R$ becomes immediately integrable is when $Q = \mu \left(\frac{dz}{dx}\right)^2$ where μ is a Const.

16) In 2nd case, when $\frac{d^2y}{dz^2} \left(\frac{dz}{dx}\right)^2 + \frac{dy}{dz} \left(\frac{d^2z}{dz^2} + P\frac{dz}{dz}\right) + Qy = R$ becomes integrable when $Qz^2 = \mu \left(\frac{dz}{dx}\right)^2$

Then $\frac{d^2y}{dz^2} \left(\frac{dz}{dx}\right)^2 + \frac{dy}{dz} \left(\frac{d^2z}{dz^2} + P\frac{dz}{dz}\right) + Qy = R$ becomes a homogeneous linear eqn.

UNIT-4

1) Partial D.E's are those which involves one or more partial derivatives.

2) The partial D.E. be $F(x, y, z, p, q) = 0$.

3) The sol of $F(x, y, z, p, q) = 0$ be $\phi(x, y, z, a, b) = 0$.

4) The sol of $\phi(x, y, z, a, b) = 0$ which contains independent vbles is called the Complete integral of $F(x, y, z, p, q) = 0$.

5) $\frac{\partial \phi}{\partial a} = 0$ and $\frac{\partial \phi}{\partial b} = 0$ is called the Singular integral.

6) Lagrange's method of solving the linear eqn of the subsidiary eqns $\frac{dx}{p} = \frac{dy}{q} = \frac{dz}{R}$.

7) The relation $\phi(u, v) = 0$ (or) $\phi(u_1, \dots, u_n) = 0$ contains all the integrals of the eqn which are not of the type called Singular.

8) Partial D.E. of all spheres whose centres lie on the plane $z=0$ and whose radius is const and equal to r is $z^2(1+p^2+q^2) = r^2$.

9) P.D.E. of all spheres whose centres lie on the z axis is $xq = yp$.

10) P.D.E. of all planes through the origin is $z = px + qy$.

11) P.D.E. of all " having equal x and y intercepts is $p = q$.

12) P.D.E. of all spheres of radius c having their centres on the xy -plane is $z^2(p^2 + q^2 + r^2) = c^2$.

13) P.D.E. of all planes which are at a constant distance a from the origin is $z = px + qy + a\sqrt{1+p^2+q^2}$.

UNIT-5

1) In std forms, Eqns in which the vbles do not occur explicitly can be written in the form $F(p, q) = 0$. (6)

2) The sol of $F(p, q)$ is $z = ax + by + c$.

3) The Complete integral is $z = ax + y f(a) + c$.

4) The singular integral is obtained by eliminating a and c between $z = ax + y f(a) + c$ is $0 = 1$.

5) The singular and general integrals must be indicated in every eqn besides the complete integral.

b) In std form, only one of the vbles x, y, z occurs explicitly such eqn can be written in one of the forms $F(x, p, q) = 0$, $F(y, p, q) = 0$, $F(z, p, q) = 0$.

7) The sol of $F(x, p, q) = 0$ is $z = \int \phi(x, a) dx + ay + b$.

8) " " " $F(y, p, q) = 0$ " $z = ax + \int \phi(y, a) dy + b$.

9) The sol of $F(z, p, q) = 0$ is $\int \frac{dz}{\phi(z, a)} = x + ay + b$.

10) The sol of $f_1(x, p) = f_2(y, q)$ is

$$z = \int \phi_1(a, x) dx + \int \phi_2(a, y) dy + b.$$

11) If $(x^m p)$ & $(y^n q)$ occur in the partial D.E. as in $F(x^m p, y^n q) = 0$ or in $F(z, x^m p, y^n q) = 0$.

12) $x^m p = (1-m) \frac{\partial z}{\partial x} = (1-m)p$ reduce to $F(p, q) = 0$

or to $F(z, p, q) = 0$.