

**SECTION-I : ARITHMETICAL ABILITY****1. OPERATIONS ON NUMBERS**

**1. NUMBERS:** In Hindu-Arabic system, we have ten **digits**, namely 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called zero, one two, three, four, five, six, seven, eight and nine respectively.

A number is denoted by a group of digits, called **numeral**.

For denoting a numeral, we use the place-value chart, given below.

**Ex. 1.** Write each of the following numerals in words.

	Ten-Crores	Crores	Ten-Lacs	Lacs	Ten-Thousands	Thousands	Hundreds	Ten	Units
(i)				6	3	8	5	4	9
(ii)			2	3	8	0	9	1	7
(iii)		8	5	4	1	6	0	0	8
(iv)	5	6	1	3	0	7	0	9	0

**Sol.** The given numerals in words are:

- (i) Six lac thirty-eight thousand five hundred forty-nine.
- (ii) Twenty-three lac eighty thousand nine hundred seventeen.
- (iii) Eight crore fifty-four lac sixteen thousand eight.
- (iv) Fifty-six crore thirteen lac seven thousand ninety.

**Ex. 2.** Write each of the following numbers in figures:

- (i) Nine crore four lac six thousand two
- (ii) Twelve crore seven lac nine thousand two hundred seven.
- (iii) Four lac four thousand forty.
- (iv) Twenty-one crore sixty lac five thousand fourteen.

**Sol.** Using the place value chart, we may write

	Ten-Crores	Crores	Ten-Lacs	Lacs	Ten-Thousands	Thousands	Hundreds	Tens	Ones
(i)		9	0	4	0	6	0	0	2
(ii)	1	2	0	7	0	9	2	0	7
(iii)				4	0	4	0	4	0
(iv)	2	1	6	0	0	5	0	1	4

**2. Face value and Place value (or Local Value) of a Digit In a Numeral**

(i) The face value of a digit in a numeral is its own value, at whatever place it may be.

**Ex.** In the numeral 6872, the face value of 2 is 2, the face value of 7 is 7, the face value of 8 is 8 and the face value of 6 is 6.

(ii) In a given numeral:

$$\text{Place value of unit digit} = (\text{unit digit}) \times 1,$$

$$\text{Place value of tens digit} = (\text{tens digit}) \times 10,$$

$$\text{Place value of hundred's digit} = (\text{hundred's digit}) \times 100 \text{ and so on.}$$

**Ex.** In the numeral 70984, we have

$$\text{Place value of } 4 = (4 \times 1) = 4$$

**2**Place value of 8 =  $(8 \times 10) = 80$ ,Place value of 9 =  $(9 \times 100) = 900$ ,Place value of 7 =  $(7 \times 10000) = 70000$ .

**Note:** Place value of 0 in a given numeral is 0, at whatever place it may be.

**Ex. 3.** In the numeral 8734925, write down:

(i) Face value of 7

(ii) Face value of 9

(iii) Place value of 4

(iv) Place value of 3

(iv) Place value of 8

(v) Place value of 5

**Sol.** Writing the given numeral in place-value chart, we get

Ten-Lacs	Lacs	Ten-thousands	Thousands	Hundreds	Tens	Ones
8	7	3	4	9	2	5

(i) Face value of 7 is 7

(ii) Face value of 9 is 9

(iii) Place value of 4 =  $(4 \times 1000) = 4000$ (iv) Place value of 3 =  $(3 \times 10000) = 30000$ (v) Place value of 8 =  $(8 \times 1000000) = 8000000$ (vi) Place value of 5 =  $(5 \times 1) = 5$ 

### 3. Various Types of Numbers:

(i) **Natural Numbers:** Counting numbers are called natural numbers.

Thus 1, 2, 3, 4, 5, 6, .... are all natural numbers.

Clearly, every natural number is whole number and 0 is a whole number which is not a natural number.

(ii) **Whole Numbers:** All counting numbers and 0 form the set of whole numbers.

Thus 0, 1, 2, 3, 4, 5, .... etc. are whose numbers.

Clearly, every natural number is whole number and 0 is a whole number which is not a natural number.

(iii) **Integers:** All counting numbers, zero and negatives of counting numbers form the set of integers.

Thus, ..., -3, -2, -1, 0, 1, 2, 3, ... are all integers.

Set of positive integers = {1, 2, 3, 4, 5, 6, ...}

Set of negative integers = {-1, -2, -3, -4, ...}

Set of all non-negative integers = {0, 1, 2, 3, 4, 5, ...}.

### 4. Even And Odd Numbers:

(i) **Even Numbers:** A counting number divisible by 2 is called an even number.

Thus 0, 2, 4, 6, 8, 10, 12, .... etc. are all even numbers.

(ii) **Odd Numbers:** A counting number not divisible by 2 is called an odd number.

Thus 1, 3, 5, 7, 9, 11, 13, 15, .... etc. are all odd numbers.

**5. Prime Numbers:** A counting number is called a prime number if it has exactly two factors, namely itself and 1.

**Ex.** All prime numbers less than 100 are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

#### Test For a Number To be Prime:

Let  $p$  be a given number and let  $n$  be the smallest counting number such that  $n^2 \geq p$ .

Now, test whether  $p$  is divisible by any of the prime numbers less than or equal to  $n$ . If yes, the  $p$  is not prime otherwise,  $p$  is prime.

**Ex. 4.** Test, which of the following are prime numbers ?

- (i) 137 (ii) 173 (iii) 319 (iv) 437 (v) 811

**Sol.** (i) We know that  $(12)^2 > 137$ .

Prime numbers less than 2 are 2, 3, 5, 7, 11

Clearly, none of them divides 137.

$\therefore 137$  is a prime number.

(ii) We known that  $(14)^2 > 173$

Prime numbers less than 14 are 2, 3, 5, 7, 11, 13.

Clearly, none of them divides 173.

$\therefore 173$  is a prime number.

(iii) We know that  $(18)^2 > 319$ .

Prime numbers less than 18 are 2, 3, 5, 7, 11, 13, 17.

Out of these prime numbers, 11 divides 319 completely.

$\therefore 319$  is not a prime number.

(iv) We know that  $(21)^2 > 437$ .

Prime numbers less than 21 are 2, 3, 5, 7, 11, 13, 17, 19

Clearly, 437 is divisible by 19.

$\therefore 437$  is not a prime number.

(v) We know that  $(30)^2 > 811$ .

Prime numbers less than 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Clearly, none of these numbers divides 811.

$\therefore 811$  is a prime number.

**Composite Numbers:** The natural numbers which are not prime, are called composite numbers.

**6. Co Primes:** Two natural numbers  $a$  and  $b$  are said to be co-prime if their HCF is 1.

**Ex.** (2, 3), (4, 5), (7, 9), (8, 11) etc. are pairs of co-primes.

## TESTS OF DIVISIBILITY

### I. Divisibility By 2:

A number is divisible by 2 if its unit digit is any of 0, 2, 4, 6, 8.

**Ex.** 58694 is divisible by 2, while 86945 is not divisible by 2.

### II. Divisibility By 3:

A number is divisible by 3 only when the sum of its digits is divisible by 3.

**Ex.** (i) In the number 695421, the sum of digits = 27, which is divisible by 3.

$\therefore 695421$  is divisible by 3.

(ii) In the number 948653, the sum of digits = 35, which is not divisible by 3.

$\therefore 948653$  is not divisible by 3.

### III. Divisibility BY 9:

A number is divisible by 9 only when the sum of its digits is divisible by 9.

**Ex.** (i) In the number 246591, the sum of digits = 27, which is divisible by 9.

$\therefore 246591$  is divisible by 9.

(ii) In the number 734519, the sum of digits = 29, which is not divisible by 9.

$\therefore 734519$  is not divisible by 9.

### IV. Divisibility By 4:

A number is divisible by 4 is the sum of its last two digits is divisible by 4.

**Ex.** (i) 6879376 is divisible by 4, since 76 is divisible by 4.

(ii) 496138 is not divisible by 4, since 38 is not divisible by 4.

### V. Divisibility By 8:

A number is divisible by 8 if the number formed by hundred's ten's and unit's digit of the given number is divisible by 8.

**Ex.** (i) In the number 16789352, the number formed by last 3 digits, namely 352 is divisible by 8.

∴ 16789352 is divisible by 8.

(ii) In the number 576484, the number formed by last 3 digits, namely 484 is not divisible by 8.

∴ 576484 is not divisible by 8.

### VI. Divisibility By 10:

A number is divisible by 10 only when its unit digit is 0.

**Ex.** (i) 7849320 is divisible by 10, since its unit digit is 0.

(ii) 678405 is not divisible by 10, since its unit digit is not 0.

### VII. Divisibility By 5:

A number is divisible by 5 only when its unit digit is 0 or 5.

**Ex.** Each of the numbers 76895 and 68790 is divisible by 5.

### VIII. Divisibility By 11:

A number is divisible by 11 if the difference between the sum of its digits at odd places and the sum of its digits at even places is either 0 or a number divisible by 11.

**Ex.** (i) Consider the number 29435417

(Sum of its digits at odd places) – (Sum of its digits at even places)

$$= (7 + 4 + 3 + 9) - (1 + 5 + 4 + 2) = (23 - 12) = 11, \text{ which is divisible by 11.}$$

∴ 29435417 is divisible by 11.

(ii) Consider the number 57463822.

(Sum of its digits at odd places) – (Sum of its digits at even places)

$$= (2 + 8 + 6 + 7) - (2 + 3 + 4 + 5) = (23 - 14) = 9, \text{ which is not divisible by 11.}$$

∴ 57463822 is not divisible by 11.

### SOLVED EXAMPLES

**Ex. 1.**  $9587 - ? = 7429 - 4358$ .

**Sol.** Let  $9587 - x = 7429 - 4358$ . Then,

$$9587 - x = 3071 \Rightarrow x = 9587 - 3071 = 6516$$

$$\begin{array}{r} 7429 \\ - 4358 \\ \hline 3071 \end{array}$$

**Ex. 2.**  $5793405 \times 9999 = ?$

$$\begin{aligned} \text{Sol. } 5793405 \times 9999 &= 5793405 \times (10000 - 1) \\ &= 57934050000 - 5793405 \\ &= 57928256595 \end{aligned}$$

$$\begin{array}{r} 57934050000 \\ - 5793405 \\ \hline 57928256595 \end{array}$$

**Ex. 3.**  $839478 \times 625 = ?$

$$\text{Sol. } 839478 \times 625 = 839478 \times 5^4$$

$$= 839478 \times \left(\frac{10}{2}\right)^4 = \frac{839478 \times 10^4}{2^4}$$

$$= \frac{8394780000}{16} = 524673750$$

**Ex. 4.**  $976 \times 237 + 976 \times 763 = ?$

**Sol.** Using distributive law, we get:

$$\begin{aligned} 976 \times 237 + 976 \times 763 &= 976 \times (237 + 763) \\ &= 976 \times 1000 = 976000. \end{aligned}$$

**Ex. 5.**  $986 \times 307 - 986 \times 207 = ?$

**Sol.** By distributive law, we get

$$\begin{aligned} 986 \times 307 - 986 \times 207 &= 986 \times (307 - 207) \\ &= 986 \times 100 = 98600 \end{aligned}$$

**Ex. 6.**  $1607 \times 1607 = ?$

**Sol.**  $1607 \times 1607 = (1607)^2$

$$\begin{aligned} &= (1600 + 7)^2 = (1600)^2 + 7^2 + 2 \times 1600 \times 7 \\ &= 2560000 + 49 + 22400 = 2582449. \end{aligned}$$

2560000
+ 49
+ 22400
<hr/>
2582449

**Ex. 7.**  $1396 \times 1396 = ?$

**Sol.**  $1396 \times 1396 = (1396)^2$

$$\begin{aligned} &= (1400 - 4)^2 = (1400)^2 + 4^2 - 2 \times 1400 \times 4 \\ &= 1960000 + 16 - 11200 = 1948816 \end{aligned}$$

1960016
-11200
<hr/>
1948816

**Ex. 8.**  $(475 \times 475 + 125 \times 125) = ?$

**Sol.** We have  $(a^2 + b^2) = \frac{1}{2}[(a+b)^2 + (a-b)^2]$

$$\begin{aligned} \therefore (475)^2 + (125)^2 &= \frac{1}{2} \cdot [(475+125)^2 + (475-125)^2] \\ &= \frac{1}{2} \cdot [(600)^2 + (350)^2] = \frac{1}{2} \cdot [360000 + 122500] \\ &= \frac{1}{2} \times 482500 = 241250. \end{aligned}$$

**Ex. 9.**  $(796 \times 796 - 204 \times 204) = ?$

**Sol.**  $796 + 796 - 204 \times 204 = (796)^2 - (204)^2$

$= (796 + 204)(796 - 204)$

$= (1000 \times 592) = 592000.$

$[\because (a^2 - b^2) = (a + b)(a - b)]$
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**Ex. 10.**  $(387 \times 387 + 113 \times 113 + 2 \times 387 \times 113) = ?$

**Sol.** Given Exp. =  $(387)^2 + (113)^2 + 2 \times 387 + 113$

$= (a^2 + b^2 + 2ab), \text{ where } a = 387 \text{ and } b = 113$

$= (a + b)^2 = (387 + 113)^2 = (500)^2 = 250000$

**Ex. 11.**  $(87 \times 87 + 61 \times 61 - 2 \times 87 \times 61) = ?$

**Sol.** Given Exp. =  $(87)^2 + (61)^2 - 2 \times 87 \times 61$

$= (a^2 + b^2 - 2ab), \text{ where } a = 87 \text{ and } b = 61$

$= (a - b)^2 = (87 - 61)^2 = (26)^2 = (20 + 6)^2$

$= (20)^2 + 6^2 + 2 \times 20 \times 6 = (400 + 36 - 240)$

$= (436 - 240) = 196.$

**Ex. 12.** Find the least value of \* for which  $5967 * 13$  is divisible by 3.

**Sol.** Let the required value be  $x$ . Then,

$(5 + 9 + 6 + 7 + x + 1 + 3) = (31 + x) \text{ is divisible by 3.}$

$\therefore$  Least value of  $x$  is 2.

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**Ex. 13.** Find the least value of \* for which  $7^*5462$  is divisible by 3.

**Sol.** Let the required value be  $x$ . Then,

$$(7 + x + 5 + 4 + 6 + 2) = (24 + x) \text{ is divisible by 9.}$$

**Ex. 14.** Find the least value of \* for which  $4832^*18$  is divisible by 11.

**Sol.** (Sum of digits at odd places) - (Sum of digits at even places)

$$= (8 + x + 3 + 4) - (1 + 2 + 8) = (4 + x), \text{ which should be divisible by 11.}$$

$$\therefore x = 7.$$

**Ex. 15.** Show that  $52563744$  is divisible by 24.

**Sol.**  $24 = 3 \times 8$ , where 3 and 8 are co-prime.

Sum of digits = 36, which is divisible by 3.

So, the given number is divisible by 3.

The number formed by last 3 digits = 744, which is divisible by 8.

So, the given number is divisible by 8.

Hence, it is divisible by  $(3 \times 8)$ , i.e., 24.

**Ex. 16.** What least number must be subtracted from 1672 to obtain a number which is completely divisible by 17?

**Sol.** 17 ) 1672 ( 98

$$\begin{array}{r} 153 \\ 142 \\ \hline 136 \\ \hline 6 \end{array}$$

Number to be subtracted = 6.

**Ex. 17.** What least number must be added to 2010 to obtain a number which is completely divisible by 19?

**Sol.** 19 ) 2010 ( 105

$$\begin{array}{r} 19 \\ 110 \\ \hline 95 \\ \hline 15 \end{array}$$

Number to be added =  $(19 - 15) = 4$

**Ex. 18.** On dividing 12401 by a certain number, we get 76 as quotient and 13 as remainder. What is the divisor?

**Sol.** [(Divisor)  $\times$  (Quotient)] + Remainder = Dividend

$$\therefore \text{Divisor} = \frac{(\text{Dividend}) - (\text{Remainder})}{\text{Quotient}}$$

$$= \frac{(12401 - 13)}{76} = \frac{12388}{76} = \frac{3097}{19} = 163.$$

**Ex. 20.** On dividing a certain number by 342, we get 47 as remainder. If the same number is divided by 18, what will be the remainder?

**Sol.** Suppose that the on dividing the given number by 342, we get quotient =  $k$  and remainder = 47. Then,

$$\text{Number} = 342k + 47$$

$$= (18 \times 19k) + (18 \times 2) + 11$$

$$= 18 + (19k + 2) + 11.$$

So, the number when divided by 18 gives remainder = 11.

**Ex. 21. Simplify:**

$$\frac{789 \times 789 \times 789 + 211 \times 211 \times 211}{789 \times 789 - 789 \times 211 + 211 \times 211} = ?$$

$$\text{Sol. Given Exp. } = \frac{(789)^3 + (211)^3}{(789)^2 - (789 \times 211) + (211)^2}$$

$$= \frac{(a^3 + b^3)}{(a^2 - ab + b^2)}, \text{ where } a = 789 \text{ and } b = 211$$

$$= (a + b) = (789 + 211) = 1000.$$

**Ex. 22. Simplify:**

$$\frac{658 \times 658 \times 658 - 328 \times 328 \times 328}{658 \times 658 + 658 \times 328 + 328 \times 328} = ?$$

$$\text{Sol. Given Exp. } = \frac{(658)^3 - (328)^3}{(658)^2 + 658 \times 328 + (328)^2}$$

$$= \frac{(a^3 - b^3)}{(a^2 + ab + b^2)}, \text{ where } a = 658 \text{ and } b = 328$$

$$= (a - b) = (658 - 328) = 330.$$

**Ex. 23. Simplify:**

$$\frac{(893 + 786)^2 - (893 - 786)^2}{(893 \times 786)} = ?$$

$$\text{Sol. Given Exp. } = \frac{(a+b)^2 - (a-b)^2}{ab}, \text{ where } a = 893 \text{ and } b = 786$$

$$= \frac{4ab}{ab} = 4$$

**Ex. 24. What is the unit digit in the product  $(684 \times 759 \times 413 \times 676)$ ?**

**Sol.** Unit digit in the given product

$$= \text{Unit digit in the product } (4 \times 9 \times 3 \times 6) = 8.$$

**Ex. 25. What is the unit digit in the product  $(3547)^{153} \times (251)^{72}$ ?**

**Sol.** Required digit = unit digit in  $(7^{153} \times 1^{72})$

Now,  $7^4$  gives unit digit 1 and  $1^{72} = 1$ .

$$(7^{153} \times 1^{72}) = [(7^4)^{38} \times 7 \times 1]$$

$$\therefore \text{Required unit digit} = (1 \times 7 \times 1) = 7.$$

**Ex. 26. What is the unit digit in  $((264)^{102} + (264)^{103})$ ?**

$$\text{Sol. } (264)^{102} + (264)^{103} = (264)^{102} \{1 + 264\} = (264)^{102} + 265.$$

Required unit digit = unit digit in  $[(4)^{102} \times 5]$

$$= \text{unit digit in } [(4^4)^{25} + 4^2 \times 5]$$

$$= \text{unit digit in } (6 \times 6 \times 5) = 0.$$

**Ex. 27. Find the total number of prime factors in the product  $\{(4)^{11} \times 7^5 \times (11)^2\}$ .**

$$\text{Sol. } \{(4)^{11} \times 7^5 \times (11)^2\} = (2 \times 2)^{11} \times 7^5 \times (11)^2 = (2^2)^{11} \times 7^5 + (11)^2$$

$$= (2^{22} \times 7^5 \times 11^2)$$

$$\text{Required number of factors} = (22 + 5 + 2) = 29.$$

**Ex. 28.** Find the remainder when  $2^{31}$  is divided by 5.

$$\text{Sol. } 2^{31} = (2^{10} \times 2^{10} \times 2^{10}) \times 2 = (2^{10})^3 \times 2 = (1024)^3 \times 2$$

Unit digit in  $2^{31}$  = Unit digit in  $((1024)^3 \times 2) = (4 \times 2) = 8$ .

Now, 8 when divided by 5 gives 3 as remainder

$\therefore 2^{31}$  when divided by 5 gives remainder = 3.

**Ex. 29.** A number when successively divided by 3, 5 and 8 leaves remainders 1, 4 and 7 respectively. Find the respective remainders if the order of divisors be reversed.

**Sol.**

3	x
5	y - 1
8	z - 4
	1 - 7
8	238
5	29 - 6
3	5 - 4
	1 - 2

$$\therefore z = (8 \times 1 + 7) = 15, y = (5 \times z + 4) = (5 \times 15 + 4) = 79 \\ x = (3y + 1) = (3 \times 79 + 1) = (237 + 1) = 238.$$

Hence, the respective remainders are 6, 4, 2.

### Results On Some Series (Formulae)

$$(i) (1 + 2 + 3 + \dots + n) = \frac{1}{2}n(n + 1)$$

$$(ii) (1^2 + 2^2 + 3^2 + \dots + n^2) = \frac{1}{6}n(n + 1)(2n + 1)$$

$$(iii) (1^3 + 2^3 + 3^3 + \dots + n^3) = \frac{1}{4}n^2(n + 1)^2$$

#### (iv) Arithmetic Progression (A.P.)

a, a + d, a + 2d, a + 3d, ... are said to be in A.P. in which first term = a and common difference = d.

Let the nth term be  $t_n$  and let last term =  $t_n = l$ . Then

$$\text{I. } n\text{th term} = a + (n - 1)d$$

$$\text{II. Sum of } n \text{ terms} = \frac{n}{2}[2a + (n - 1)d]$$

$$\text{III. Sum of } n \text{ terms} = \frac{n}{2}(a + l), \text{ where } l \text{ is the last term.}$$

#### (v) Geometric Progression (G.P.)

a, ar, ar<sup>2</sup>, ar<sup>3</sup>, ... are said to be in G.P. in which first term = a and common ratio = r

$$\text{I. } n\text{th term} = ar^{n-1}$$

$$\text{II. Sum of } n \text{ terms} = \begin{cases} \frac{a(1 - r^n)}{(1 - r)}, & \text{when } r < 1 \\ \frac{a(r^n - 1)}{(r - 1)}, & \text{when } r > 1 \end{cases}$$

**Ex. 30.** How many natural numbers between 17 and 80 are divisible by 6?

**Sol.** These numbers are 18, 24, 30, 36, ..., 78.

This is an A.P. in which  $a = 18$ ,  $d = (24 - 18) = 6$  and  $l = 78$ .

Let the number of these terms be  $n$ . Then,

$$t_n = 78 \Rightarrow a + (n - 1)d = 78$$

$$\Rightarrow 18 + (n - 1) \times 6 = 78 \Rightarrow (n - 1) \times 6 = 60 \Rightarrow (n - 1) = 10 \Rightarrow n = 11$$

Required number of numbers = 11.

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**Ex. 31.** Find the sum of all even natural numbers less than 75.

**Sol.** Required sum =  $2 + 4 + 6 + \dots + 74$

This is an A.P. in which  $a = 2$ ,  $d = (4 - 2) = 2$ ,  $n = 74$

**Clearly,  $n = 37$ .**

$$\therefore \text{Required sum} = \frac{n}{2}(a+l) = \frac{37}{2} \times (2+74) = (37 \times 38)$$

$$= 37 \times (40 - 2) = (37 \times 40) - (37 \times 2)$$

$$= (1480 - 74) = 1406.$$

**Ex. 32.**  $(6 + 15 + 24 + 33 + \dots + 105) = ?$

**Sol.** Given series in an A.P. in which  $a = 6$ ,  $d = (15 - 6) = 9$  and  $l = 105$

Let the number of terms in it be  $n$ . Then,

$$\begin{aligned} a + (n - 1)d &= 105 \Rightarrow 6 + (n - 1) \times 9 = 105 \\ \Rightarrow (n - 1) \times 9 &= 99 \Rightarrow (n - 1) = 11 \Rightarrow n = 12 \end{aligned}$$

$$\text{Required sum} = \frac{n}{2}(a+l) = \frac{12}{2} \times (6+105) = (6 \times 111) = 666.$$

**Ex. 33.** Find the sum  $(2 + 2^2 + 2^3 + 2^4 + \dots + 2^{10})$

**Sol.** This is a G.P. in which  $a = 2$ ,  $r = \frac{2^2}{2} = \frac{4}{2} = 2$ .

$$\text{Required sum} = \frac{a(r^n - 1)}{(r - 1)} = \frac{2 \times (2^{10} - 1)}{(2 - 1)} = (2 \times 1023) = 2046$$

## EXERCISE

**Mark (✓) against the correct answer in each of the following:**

- 13.**  $(4300731) - ? = 2535618$
- (a) 1865113      (b) 1775123      (c) 1765113      (d) 1675123      (e) none of these  
**14.**  $(?) - 19657 - 33994 = 9999$
- (a) 63650      (b) 53760      (c) 59640      (d) 61560      (e) none of these  
**15.**  $3 + 33 + 333 + 3.33 = ?$
- (a) 362.3      (b) 372.33      (c) 702.33      (d) 702      (e) none of these  
**16.**  $9\frac{3}{4} + 7\frac{2}{17} - 9\frac{1}{15} = ?$
- (a)  $7\frac{719}{1020}$       (b)  $9\frac{817}{1020}$       (c)  $9\frac{719}{1020}$       (d)  $7\frac{817}{1020}$   
 (e) none of these  
**17.**  $-84 \times 29 + 365 = ?$
- (a) 2436      (b) 2801      (c) -2801      (d) -2071      (e) none of these  
**18.**  $(35423 + 7164 + 41720) - (317 \times 89) = ?$
- (a) 28213      (b) 84307      (c) 50694      (d) 56094      (e) none of these  
**19.**  $9548 + 7314 - 8362 = ?$
- (a) 8230      (b) 8410      (c) 8500      (d) 8600      (e) none of these  
**20.**  $(?) + 3699 + 1985 - 2047 = 31111$
- (a) 34748      (b) 27474      (c) 30154      (d) 27574      (e) none of these  
**21.**  $4500 \times ? = 3375$
- (a)  $\frac{2}{5}$       (b)  $\frac{3}{4}$       (c)  $\frac{1}{4}$       (d)  $\frac{3}{5}$   
 (e) none of these  
**22.** If  $1400 \times x = 1050$ . Then,  $x = ?$
- (a)  $\frac{1}{4}$       (b)  $\frac{3}{5}$       (c)  $\frac{2}{3}$       (d)  $\frac{3}{4}$   
 (e) none of these  
**23.**  $(1000)^9 + 10^{24} = ?$
- (a) 10000      (b) 1000      (c) 100      (d) 10      (e) none of these  
**24.**  $8988 \div 8 \div 4 = ?$
- (a) 4494      (b) 561.75      (c) 2247      (d) 280.875      (e) none of these  
**25.**  $666 \div 6 \div 3 = ?$
- (a) 37      (b) 333      (c) 111      (d) 84      (e) none of these  
**26.**  $(800 \div 64) \times (1296 \div 36) = ?$
- (a) 420      (b) 460      (c) 500      (d) 540      (e) none of these  
**27.**  $(12)^3 \times 6^4 \div 432 = ?$
- (a) 5184      (b) 5060      (c) 5148      (d) 5084      (e) none of these  
**28.**  $35 + 15 \times 1.5 = ?$
- (a) 75      (b) 51.5      (c) 57.5      (d) 5.25      (e) none of these

- 29.**  $5358 \times 51 = ?$   
 (a) 273258      (b) 273268      (c) 273348      (d) 273358      (R.B.I. 2003)
- 30.**  $587 \times 999 = ?$   
 (a) 586413      (b) 587523      (c) 614823      (d) 615173
- 31.**  $3897 \times 999 = ?$   
 (a) 3883203      (b) 3893103      (c) 3639403      (d) 3791203  
 (e) none of these
- 32.**  $72519 \times 9999 = ?$   
 (a) 725117481      (b) 674217481      (c) 685126481      (d) 696217481  
 (e) none of these
- 33.**  $2056 \times 987 = ?$   
 (a) 1936372      (b) 2029272      (c) 1896172      (d) 1923472  
 (e) none of these
- 34.**  $1904 \times 1904 = ?$   
 (a) 3654316      (b) 3632646      (c) 3625216      (d) 3623436  
 (e) none of these
- 35.**  $1397 \times 1397 = ?$   
 (a) 1951609      (b) 1981709      (c) 18362619      (d) 2031719  
 (e) none of these
- 36.**  $107 \times 107 + 93 \times 93 = ?$   
 (a) 19578      (b) 19418      (c) 20098      (d) 21908  
 (e) none of these
- 37.**  $217 \times 217 + 183 \times 183 = ?$   
 (a) 79698      (b) 80578      (c) 80698      (d) 81268      (Hotel Management 2002)
- 38.**  $106 \times 106 - 94 \times 94 = ?$   
 (a) 2400      (b) 2000      (c) 1904      (d) 1906  
 (e) none of these
- 39.**  $8796 \times 223 + 8796 \times 77 = ?$   
 (a) 2736900      (b) 2638800      (c) 2658560      (d) 2716740
- (e) none of these
- 40.**  $287 \times 287 + 269 \times 269 - 2 \times 287 \times 269 = ?$   
 (a) 534      (b) 446      (c) 354      (d) 324
- (e) none of these
- 41.**  $\{(476 + 424)^2 - 4 \times 476 \times 424\} = ?$   
 (a) 2906      (b) 3116      (c) 2704      (d) 2904  
 (e) none of these
- 42.**  $(112 \times 5^4) = ?$   
 (a) 67000      (b) 70000      (c) 76500      (d) 77200
- 43.**  $(935421 \times 625) = ?$   
 (a) 575648125      (b) 584638125      (c) 584649125      (d) 585628125
- 44.**  $(12345679 \times 72) = ?$   
 (a) 88888888      (b) 888888888      (c) 898989898      (d) 9999999998
- 45.**  $397 \times 397 + 104 \times 104 + 2 \times 397 \times 104 = ?$   
 (a) 250001      (b) 251001      (c) 260101      (d) 261001
- 46.** If  $(64)^2 - (36)^2 = 20 \times x$ , then  $x = ?$   
 (a) 70      (b) 120      (c) 180      (d) 140  
 (e) none of these
- 47.**  $\frac{(489 + 375)^2 - (489 - 375)^2}{(489 \times 375)} = ?$   
 (a) 144      (b) 864      (c) 2      (d) 4
- (e) none of these
- 48.**  $\frac{(963 + 476)^2 + (963 - 476)^2}{(963 \times 963 + 476 \times 476)} = ?$

- |  |  |                               |                                |
|--|--|-------------------------------|--------------------------------|
| <p>(a) 1449<br/>(e) none of these</p> <p><b>49.</b> <math>\frac{768 \times 768 \times 768 + 232 \times 232 \times 232}{768 \times 768 - 768 \times 232 + 232 \times 232} = ?</math></p> <p>(a) 1000<br/>(e) none of these</p> <p><b>50.</b> <math>\frac{854 \times 854 \times 854 - 276 \times 276 \times 276}{854 \times 854 + 854 \times 276 + 276 \times 276} = ?</math></p> <p>(a) 1130<br/>(e) none of these</p> <p><b>51.</b> <math>\frac{753 \times 753 + 247 \times 247 - 753 \times 247}{753 \times 753 \times 753 + 247 \times 247 \times 247} = ?</math></p> <p>(a) <math>\frac{1}{1000}</math><br/>(b) <math>\frac{1}{506}</math><br/>(c) <math>\frac{253}{500}</math><br/>(d) none of these</p> | <p>(b) 497</p> <p>(c) 2</p> <p>(d) 4</p> | <p>(c) 500</p> <p>(d) 268</p> | <p>(c) 565</p> <p>(d) 1156</p> |
|--|--|-------------------------------|--------------------------------|
- 52.** If the number  $517 * 324$  is completely divisible by 3, then the smallest whole number in place of \* will be
- (a) 0  
(b) 1  
(c) 2  
(d) none of these
- 53.** If the number  $481 * 673$  is completely divisible by 9, then the smallest whole number in place of \* will be:
- (a) 2  
(b) 5  
(c) 6  
(d) 7
- 54.** If the number  $97215 * 6$  is completely divisible by 11, then the smallest whole number in place of \* will be:
- (a) 3  
(b) 2  
(c) 1  
(d) 5
- 55.** If the number  $91876 * 2$  is completely divisible by 8, then the smallest whole number in place of \* will be:
- (a) 1  
(b) 2  
(c) 3  
(d) 4
- 56.** Which one of the following numbers is completely divisible by 45?
- (a) 181560  
(b) 331145  
(c) 202860  
(d) 2033550
- 57.** Which one of the following numbers is completely divisible by 99?
- (a) 3572404  
(b) 135792  
(c) 913464  
(d) 114345
- 58.** If the number  $42573 *$  is exactly divisible by 72, then the minimum value of \* is:
- (a) 4  
(b) 5  
(c) 6  
(d) 7  
(e) 8
- 59.** If  $x$  and  $y$  are the two digits of the number  $653 xy$  such that this number is divisible by 80, then  $x + y = ?$
- (a) 2  
(b) 3  
(c) 4  
(d) 6  
(e) none of these
- 60.** If the product  $4864 \times 9 P 2$  is divisible by 12, the value of  $P$  is:
- (a) 2  
(b) 5  
(c) 6  
(d) 6  
(e) none of these
- 61.** If the number  $5 * 2$  is divisible by 6, then  $* = ?$
- (a) 2  
(b) 3  
(c) 6  
(d) 8
- 62.** Which of the following numbers is divisible by 24?
- (a) 35718  
(b) 63810  
(c) 537804  
(d) 7
- 63.** How many of the following numbers are divisible by 132?
- 264, 396, 462, 792, 968, 2178, 5184, 6336
- (a) 4  
(b) 5  
(c) 6  
(d) 7  
(e) none of these
- 64.** 476 \*\* 0 is divisible by both 3 and 11. The non-zero digits in the hundred's and ten's places are respectively:
- (a) 7 and 4  
(b) 7 and 5  
(c) 8 and 5  
(d) none of these

(Hotel Management, 2002)

- 65.** Which one of the following numbers is exactly divisible by 11 ? (C.D.S. 2003)  
 (a) 235641 (b) 245642 (c) 315624 (d) 415624

**66.** How many 3 digit numbers are divisible by 6 in all ?  
 (a) 149 (b) 150 (c) 151 (d) 166

**67.** The sum of first 45 natural numbers is :  
 (a) 1035 (b) 1280 (c) 2070 (d) 2140

**68.** The sum of even numbers between 1 and 31 is :  
 (a) 6 (b) 128 (c) 240 (d) 512

**69.**  $(51 + 52 + 53 + \dots + 100) = ?$   
 (a) 2525 (b) 2975 (c) 3225 (d) 3775

**70.** The smallest prime number is:  
 (a) 0 (b) 1 (c) 2 (d) 3

**71.** The sum of first five prime numbers is :  
 (a) 11 (b) 18 (c) 26 (d) 28

**72.** How many prime numbers are less than 50 ?  
 (a) 16 (b) 15 (c) 14 (d) 18

**73.** Which of the following is a prime number ?  
 (a) 33 (b) 81 (c) 93 (d) 97

**74.** Which one of the following is not a prime number ?  
 (a) 31 (b) 61 (c) 71 (d) 91

**75.** Which one of the following is a prime number ?  
 (a) 161 (b) 221 (c) 373 (d) 437  
 (e) none of these

**76.** Which one of the following is a prime number ?  
 (a) 119 (b) 187 (c) 247 (d) 551  
 (e) none of these

**77.** The smallest 3-digit prime number is:  
 (a) 103 (b) 107 (c) 109 (d) 113

**78.** If  $a$  and  $b$  are odd numbers, then which of the following is even ?  
 (a)  $a + b$  (b)  $a + b + 1$  (c)  $ab$  (d)  $ab + 2$   
 (e) none of these

**79.** Which one of the following cannot be the square of a natural number ?  
 (a) 30976 (b) 75625 (c) 28561 (d) 143642  
 (e) none of these

**80.** Which one of the following cannot be the square of a natural number ?  
 (a) 32761 (b) 81225 (c) 42437 (d) 20164  
 (e) none of these

**81.** What smallest number should be added to 4456 so that the sum is completely divisible by 6 ?  
 (a) 4 (b) 3 (c) 2 (d) 1  
 (e) none of these

**82.** Which natural number is nearest to 9217, which is completely divisible by 88 ?  
 (a) 9152 (b) 9240 (c) 9064 (d) 9184  
 (e) none of these

**83.** Which natural number is nearest to 8485, which is completely divisible by 75 ?  
 (a) 8475 (b) 8500 (c) 8550 (d) 8525

**84.** The largest 4-digit number exactly divisible by 88 is  
 (a) 9944 (b) 9768 (c) 9988 (d) 8888  
 (e) none of these

**85.** The largest 5-digit number exactly divisible by 91 is  
 (a) 99921 (b) 99918 (c) 99981 (d) 99971  
 (e) none of these

**86.** What least number must be subtracted from 13601, so that the remainder is divisible by 87 ?  
 (a) 23 (b) 31 (c) 29 (d) 37  
 (e) 49

- 87.** What least number must be added to 1056, so that the sum is completely divisible by 23?  
 (a) 2      (b) 3      (c) 18      (d) 21
- 88.** The smallest 5-digit number exactly divisible by 41 is :  
 (a) 10041      (b) 10004      (c) 10045      (d) 10025
- 89.** The smallest 6-digit number exactly divisible by 111 is :  
 (a) 111111      (b) 110011      (c) 100011      (d) 110101
- 90.** In a division sum, the divisor is 10 times the quotient and 5 times the remainder. If the remainder is 46, what is the dividend ?  
 (a) 4236      (b) 4306      (c) 4336      (d) 5336  
 (e) none of these
- 91.** On dividing a number by 68, we get 269 as dividend and 0 as remainder. On dividing the same number by 67, what will be the remainder ?  
 (a) 0      (b) 1      (c) 2      (d) 3      (S.S.C. 2005)
- 92.** On dividing a number by 56, we get 29 as remainder. On dividing the same number by 8, what will be the remainder ?  
 (a) 4      (b) 5      (c) 6      (d) 7      (S.S.C. 2007)
- 93.** On dividing a number by 357, we get 39 as remainder. On dividing the same number by 17, what will be the remainder ?  
 (a) 0      (b) 3      (c) 5      (d) 11      (S.S.C. 2005)
- 94.** On dividing a number by 5, we get 3 as remainder. What will be the remainder when the square of this number is divided by 5 ?  
 (a) 0      (b) 1      (c) 2      (d) 4      (S.S.C. 2005)
- 95.** The difference of two numbers is 1365. On dividing the larger number by the smaller, we get 6 as quotient and 15 as remainder. What is the smaller number ?  
 (a) 240      (b) 270      (c) 295      (d) 360      (L.I.C. 2003)
- 96.** In a division sum, the remainder is 0. As student mistook the divisor by 12 instead of 21 and obtained 35 as quotient. What is the correct quotient ?  
 (a) 0      (b) 12      (c) 13      (d) 20      (S.S.C. 2003)
- 97.** The sum of the two numbers is 12 and their product is 35. What is the sum of the reciprocals of these numbers ?  
 (a)  $\frac{12}{35}$       (b)  $\frac{1}{35}$       (c)  $\frac{35}{8}$       (d)  $\frac{7}{32}$       (S.S.C. 2007)
- 98.** If  $60\% \text{ of } \frac{3}{5}$  of a number is 36, then the number is :  
 (a) 80      (b) 100      (c) 75      (d) 90
- 99.** The difference between a positive proper fraction and its reciprocal is  $\frac{9}{20}$ . The fraction is :  
 (a)  $\frac{3}{5}$       (b)  $\frac{3}{10}$       (c)  $\frac{4}{5}$       (d)  $\frac{5}{4}$       (S.S.C. 2006)
- 100.** On dividing 2272 as well as 875 by 3-digit number N, we get the same remainder. The sum of the digits of N is :  
 (a) 10      (b) 11      (c) 12      (d) 13      (S.S.C. 2007)
- 101.** On multiplying a number by 7, the product is a number each of whose digits is 3. The smallest such number is :  
 (a) 47619      (b) 47719      (c) 48619      (d) 47649      (S.S.C. 2006)
- 102.** The difference of the squares of two consecutive even integers is divisible by which of the following integers ?  
 (a) 3      (b) 4      (c) 6      (d) 7      (M.B.A. 2003)
- 103.** The difference of the squares of two consecutive odd integers is divisible by which of the following integers ?  
 (a) 3      (b) 6      (c) 7      (d) 8      (M.B.A. 2003)



- 124.** When a number is divided by 13, the remainder is 11. When the same number is divided by 17, the remainder is 9. What is the number ?  
 (a) 339      (b) 349      (c) 369  
 (d) data inadequate
- 125.** A number when divided by 296 leaves 75 as remainder. When the same number is divided by 37, the remainder will be : (C.B.I. 2003)  
 (a) 1      (b) 2      (c) 8      (d) 11
- 126.** A boy multiplied 987 by a certain number and obtained 559981 as his answer. If in the answer both 98 are wrong and the other digits are correct, then the correct answer would be :  
 (a) 553681      (b) 555181      (c) 555681      (d) 556581
- 127.** How many of the following numbers are completely divisible by 132 ?  
 264, 396, 462, 792, 968, 2178, 5184, 6336  
 (a) 4      (b) 5      (c) 6      (d) 7
- 128.** How many 3-digit numbers are completely divisible by 6 ?  
 (a) 149      (b) 150      (c) 151      (d) 166
- 129.** The sum of first 45 natural numbers is :  
 (a) 1035      (b) 1280      (c) 2070      (d) 2140
- 130.** The sum of all even natural numbers between 1 and 31 is  
 (a) 16      (b) 128      (c) 240      (d) 512
- 131.**  $(51 + 52 + 53 + \dots + 100) = ?$   
 (a) 2525      (b) 2975      (c) 3225      (d) 3775
- 132.**  $\left(1 - \frac{1}{n}\right) + \left(1 - \frac{2}{n}\right) + \left(1 - \frac{3}{n}\right) + \dots \text{ upto } n \text{ terms} = ?$   
 (a)  $\frac{1}{2}n$       (b)  $\frac{1}{2}(n-1)$       (c)  $\frac{1}{2}n(n-1)$       (d) none of these
- 133.** How many natural numbers are there between 23 and 100 which are exactly divisible by 6 ?  
 (a) 8      (b) 11      (c) 12      (d) 13  
 (e) none of these
- 134.** The sum of all two digit numbers divisible by 5 is  
 (a) 1035      (b) 1245      (c) 1230      (d) 945  
 (e) none of these
- 135.** The sum of how many terms of the series  $6 + 12 + 18 + 24 + \dots$  is 1800 ?  
 (a) 16      (b) 24      (c) 20      (d) 18  
 (e) 22
- 136.** How many terms are there in the G.P. 3, 6, 12, 24, ..., 384 ?  
 (a) 8      (b) 9      (c) 10      (d) 11  
 (e) 7
- 137.**  $2 + 2^2 + 2^3 + \dots + 2^9 = ?$   
 (a) 2044      (b) 1022      (c) 1056      (d) none of these
- 138.**  $(1^2 + 2^2 + 3^2 + \dots + 10^2) = ?$   
 (a) 330      (b) 345      (c) 365      (d) 385
- 139.**  $(2^2 + 4^2 + 6^2 + \dots + 20^2) = ?$   
 (a) 770      (b) 1155      (c) 1540      (d)  $385 \times 385$
- 140.**  $(11^2 + 12^2 + 13^2 + \dots + 20^2) = ?$   
 (a) 385      (b) 2485      (c) 2870      (d) 3255

## ANSWERS

1. (c)	2. (d)	3. (c)	4. (a)	5. (c)	6. (c)	7. (b)	8. (d)
9. (a)	10. (b)	11. (c)	12. (d)	13. (c)	14. (a)	15. (b)	16. (d)
17. (d)	18. (c)	19. (c)	20. (b)	21. (b)	22. (d)	23. (b)	24. (d)
25. (a)	26. (e)	27. (e)	28. (c)	29. (a)	30. (a)	31. (b)	32. (a)
33. (b)	34. (c)	35. (a)	36. (c)	37. (b)	38. (a)	39. (b)	40. (d)
41. (c)	42. (b)	43. (b)	44. (b)	45. (b)	46. (d)	47. (d)	48. (c)
49. (a)	50. (b)	51. (a)	52. (c)	53. (d)	54. (a)	55. (c)	56. (c)
57. (d)	58. (c)	59. (a)	60. (e)	61. (a)	62. (d)	63. (a)	64. (c)
65. (d)	66. (b)	67. (a)	68. (c)	69. (d)	70. (c)	71. (d)	72. (b)
73. (d)	74. (d)	75. (c)	76. (e)	77. (e)	78. (a)	79. (d)	80. (c)
81. (c)	82. (b)	83. (a)	84. (a)	85. (b)	86. (c)	87. (a)	88. (b)
89. (c)	90. (d)	91. (b)	92. (b)	93. (c)	94. (d)	95. (b)	96. (d)
97. (a)	98. (b)	99. (c)	100. (a)	101. (a)	102. (b)	103. (d)	104. (b)
105. (b)	106. (c)	107. (a)	108. (a)	109. (c)	110. (b)	111. (a)	112. (b)
113. (d)	114. (a)	115. (d)	116. (c)	117. (b)	118. (b)	119. (a)	120. (d)
121. (b)	122. (a)	123. (d)	124. (b)	125. (a)	126. (b)	127. (a)	128. (b)
129. (a)	130. (c)	131. (d)	132. (b)	133. (d)	134. (d)	135. (b)	136. (a)
137. (b)	138. (d)	139. (c)	140. (b)				

## SOLUTION

1. (Place value of 6) – (Face value of 6)  
 $= (6000 - 6) = 5994.$

2. (Local value of 7) – (Face value of 7)  
 $= (70000 - 7) = 69993.$

3. Required difference  $= (700000 - 70) = 699930.$

4. Unit digit in the given product = Unit digit in  $(4 \times 8 \times 7 \times 3) = 2$

5. Unit digit in  $7^{105}$  = Unit digit in  $[(7^4)^{26} \times 7]$

But, unit digit in  $(7^4)^{26} = 1$ .

$\therefore$  Unit digit in  $7^{105} = (1 \times 7) = 7.$

6. Unit digit in  $3^4 = 1 \Rightarrow$  Unit digit in  $(3^4)^{16} = 1$

$\therefore$  Unit digit in  $3^{65} =$  Unit digit in  $[(3^4)^{16} \times 3] = (1 \times 3) = 3$

Unit digit in  $6^{59} = 6$

Unit digit in  $7^4 = 1 \Rightarrow$  Unit digit in  $(7^4)^{17}$  is 1.

Unit digit in  $7^{71} =$  Unit digit in  $[(7^4)^{17} \times 7^3] = (1 \times 3) = 3$

$\therefore$  Required digit = Unit digit in  $(3 \times 6 \times 3) = 4.$

7. Unit digit in  $7^{95} =$  Unit digit in  $[(7^4)^{23} \times 7^3] = (1 \times 3) = 3$

Unit digit in  $3^{58} =$  Unit digit in  $[(3^4)^{14} \times 3^2] = (1 \times 9) = 9.$

Unit digit in  $(7^{95} - 3^{58}) = (13 - 9) = 4.$

8. Unit digit in  $(4137)^{754} =$  Unit digit in  $\left\{ \left[ (4137)^4 \right]^{188} \times (4137)^2 \right\} = (1 \times 9) = 9$

9. Unit digit in  $(6374)^{1793} =$  Unit digit in  $(4)^{1793}$

$=$  Unit digit in  $[(4^2)^{896} \times 4]$

$=$  Unit digit in  $(6 \times 4) = 4.$

Unit digit in  $(625)^{317} =$  Unit digit in  $(5)^{317} = 5.$

Unit digit in  $(341)^{491} =$  Unit digit in  $(1)^{491} = 1$

Required digit = Unit digit in  $(4 \times 5 \times 1) = 0.$

10. Let  $7589 - x = 3434$ .

$$\begin{array}{rcl} \text{Then, } x & = & 7589 \\ & = & 4155 \end{array} \quad \begin{array}{r} 7589 \\ - 3434 \\ \hline 4155 \end{array}$$

11. 7429 Let  $8597 - x = 3071$  8597

$$\begin{array}{r} -4358 \\ \hline 3071 \end{array} \quad \begin{array}{rcl} \text{Then, } x & = & 8597 - 3071 \\ & & \hline & & 5526 \end{array}$$

12. 3251 Let  $4207 - x = 3007$

$$\begin{array}{r} + 587 \\ + 369 \\ \hline 4207 \end{array} \quad \begin{array}{rcl} \text{Then, } x & = & 4207 - 3007 = 1200. \end{array}$$

13. Let  $4300731 - x = 2535618$

$$\text{Then, } x = 4300731 - 2535618$$

$$\begin{array}{r} 4300731 \\ - 2535618 \\ \hline 1765113 \end{array}$$

14. 19657 Let  $x - 53651 = 9999$

$$\begin{array}{r} 33994 \\ \hline 53651 \end{array} \quad \begin{array}{rcl} \text{Then, } x & = & 9999 + 53651 = 9999 + 1 + 53650 \\ & & = 10000 + 53650 = 63650. \end{array}$$

15. 3

$$\begin{array}{r} + 33 \\ + 333 \\ + 3.33 \\ \hline 372.33 \end{array}$$

16. Given sum =  $9 + \frac{3}{4} + 7 + \frac{2}{17} - \left( 9 + \frac{1}{15} \right)$

$$= (9 + 7 - 9) + \left( \frac{3}{4} + \frac{2}{17} - \frac{1}{15} \right)$$

$$= 7 + \frac{765 + 120 - 68}{1020} = 7 \frac{817}{1020}$$

17. Given Exp. =  $-84 \times (30 - 1) + 365$

$$= - (84 \times 30) + 84 + 365 \quad - 2520$$

$$= - 2520 + 449 = - 2071 \quad + 449$$

$$\underline{- 2071}$$

18. 35423  $317 \times 89 = 317 \times (90 - 1)$

$$+ 7164 = (317 \times 90 - 317)$$

$$+ 41720 = (28530 - 317) = 28213$$

$$\underline{84307}$$

$$\underline{- 28213}$$

$$\underline{54094}$$

19. 9548  $16862 = 8362 + x \Rightarrow x = 16862 - 8362 = 8500.$

$$\underline{+ 7314}$$

$$\underline{16862}$$

20.  $x + 3699 + 1985 - 2047 = 31111$

$$\begin{aligned} \Rightarrow & x + 3699 + 1985 = 31111 + 2047 & 33158 \\ \Rightarrow & x + 5684 = 33158 & -5684 \\ \Rightarrow & x = 33158 - 5684 = 27474. & \underline{\underline{27474}} \end{aligned}$$

21.  $4500 \times x = 3375 \Rightarrow x = \frac{3375}{4500} = \frac{3}{4}$

$$\begin{array}{r} 45 ) 3375( 75 \\ \underline{315} \\ 225 \\ \underline{225} \\ x \end{array}$$

22.  $1400 \times x = 1050 \Rightarrow x = \frac{1050}{1400} = \frac{3}{4}$

23. Given Exp.  $= \frac{(1000)^9}{10^{24}} = \frac{(10^3)^9}{10^{24}} = \frac{10^{27}}{10^{24}} = 10^{(27-24)} = 10^3 = 1000$ .

24. Given Exp.  $8988 \times \frac{1}{8} \times \frac{1}{4} = \frac{2247}{8} = 280.875$ .

25. Given Exp.  $= 666 \times \frac{1}{6} \times \frac{1}{3} = 37$ .

26. Given Exp.  $= \frac{300}{64} \times \frac{50}{4} \times \frac{1296}{36} \times \frac{216}{6} \times \frac{36}{4} = 450$ .

27. Given Exp.  $= \frac{(12)^3 \times 6^4}{432} = \frac{(12)^3 \times 6^4}{12 \times 6^2} = (12)^2 \times 6^2 = (72)^2 = 70$ .

28. Given Exp.  $= 35 + 15 \times \frac{3}{2} = 35 + \frac{45}{2} = 35 + 22.5 = 57.5$ .

29.  $5358 \times 51 = 5358 \times (50 + 1) = 5358 \times 50 + 5358 \times 1 = 267900 + 5358 = 273258$ .

$$\begin{array}{r} + 5358 \\ \hline 273258 \end{array}$$

30.  $587 \times 999 = 587 \times (1000 - 1) = 587 \times 1000 - 587 = 587000 - 587 = 586413$

31.  $3897 \times 999 = 3897 \times (1000 - 1) = 3897 \times 1000 - 3897 \times 1 = 3897000 - 3897 = 3893103$

$$\begin{array}{r} - 3897 \\ \hline 3893103 \end{array}$$

32.  $7259 \times 9999 = 72519 \times (10000 - 1) = 72519 \times 10000 - 72519 = 725190000 - 72519 = 725117481$

$$\begin{array}{r} -72519 \\ \hline 725117481 \end{array}$$

33.  $2056 \times 987 = 2056 \times (1000 - 13) = 2056 \times 1000 - 2056 \times 13 = 2056000 - 26728 = 2029272$

$$\begin{array}{r} -26728 \\ \hline 2029272 \end{array}$$

34.  $1904 \times 1904 = (1904)^2 = (1900 + 4)^2 = (1900)^2 + 4^2 + 2 \times 1900 \times 4 = 3610000 + 16 + 15200 = 3610016 + 15200 = 3625216$

- 35.**  $1397 \times 1397 = (1397)^2 = (1400 - 3)^2$  1960009  
 $= (1400)^2 + 3^2 - 2 \times 1400 \times 3 = 1960000 + 9 - 8400$  -8400  
 $= 1960009 - 8400 = 1951609.$  1951609
- 36.**  $107 \times 107 + 93 \times 93 = (107)^2 + (93)^2 = (100 + 7)^2 + (100 - 7)^2$  [ $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$ ]  
 $= 2 \times [(100)^2 + 7^2]$
- 37.**  $(217)^2 + (183)^2 = (200 + 17)^2 + (200 - 17)^2$  [( $a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$ ]  
 $= 2[(200)^2 + (17)^2]$   
 $= 2[40000 + 289] = (2 \times 40289) = 80578.$
- 38.**  $106 \times 106 - 94 \times 94 = (106)^2 - (94)^2 = (106 + 94)(106 - 94)$  [( $a^2 - b^2 = (a+b)(a-b)$ )]  
 $= (200 \times 12) = 2400.$
- 39.**  $8796 \times 223 + 8796 \times 77 = 8796 \times (223 + 77)$  [by distributive law]  
 $= (8796 \times 300) = 2638800.$
- 40.** Given Exp.  $= a^2 + b^2 - 2ab$ , where  $a = 287$  and  $b = 269$   
 $= (a - b)^2 = (287 - 269)^2 = (18)^2 = 324.$
- 41.** Given Exp.  $= [(a + b)^2 - 4ab]$ , where  $a = 476$  and  $b = 424$   
 $= [(476 + 424)^2 - 4 \times 476 \times 424]$   
 $= [(900)^2 - 807296] = 810000 - 807296 = 2704$
- 42.**  $(112 \times 5^4) = 112 \times \left(\frac{10}{2}\right)^4 = \frac{112 \times 10^4}{2^4} = \frac{1120000}{16} = 70000.$
- 43.**  $935421 \times 625 = 935421 \times 5^4 = 935421 \times \left(\frac{10}{2}\right)^4$   
 $\frac{935421 \times 10^4}{2^4} = \frac{9354210000}{16}$   
 $= 584638125$
- 44.**  $12345679 \times 72 = 12345679 \times (70 + 2)$   
 $= 12345679 \times 70 + 12345679 \times 2$   
 $= 864197530 + 24691358$  864197530  
 $= 88888888$  24691358  
 $\underline{88888888}$
- 45.** Given Exp.  $= (397)^2 + (104)^2 + 2 \times 397 \times 104$   
 $= (397 + 104)^2 = (501)^2 = (500 + 1)^2$   
 $= (500)^2 + 1^2 + 2 \times 500 \times 1 = 250000 + 1 + 1000$   
 $= 251001$
- 46.**  $20 \times x = (64 + 36)(64 - 36) = 100 \times 28$   
 $\Rightarrow x = \frac{100 \times 28}{20} = 140.$
- 47.** Given Exp.  $= \frac{(a+b)^2 - (a-b)^2}{ab} = \frac{4ab}{ab} = 4.$
- 48.** Given Exp.  $= \frac{(a+b)^2 + (a-b)^2}{(a^2 + b^2)} = \frac{2(a^2 + b^2)}{(a^2 + b^2)} = 2.$
- 49.** Given Exp.  $\frac{(a^3 + b^3)}{(a^2 - ab + b^2)} = (a+b) = (768 + 232) = 1000.$

50. Given Exp.  $\frac{(a^3 - b^3)}{(a^2 + ab + b^2)} = (a - b) = (854 - 276) = 578.$

51. Given Exp.  $\frac{(a^2 + b^2 - ab)}{(a^3 - b^3)} = \frac{1}{(a+b)} = \frac{1}{(753 + 247)} = \frac{1}{1000}.$

52. Sum of digits  $= (5 + 1 + 7 + x + 3 + 2 + 4) = (22 + x)$ , which must be divisible by 3.  
 $\therefore x = 2.$

53. Sum of digits  $= (4 + 8 + 1 + x + 6 + 7 + 3) = (29 + x)$ , which must be divisible by 9.  
 $\therefore x = 7.$

54. Given number  $= 97215x6$

$(6 + 5 + 2 + 9) - (x + 1 + 7) = (14 - x)$ , which must be divisible by 11.  
 $\therefore x = 3.$

55. The number  $6x2$  must be divisible by 8.

$\therefore x = 3$ , as 632 is divisible by 8.

56.  $45 = 5 \times 9$ , where 5 and 9 are co-primes.

Unit digit must be 0 or 5 and sum of digits must be divisible by 9.

Among given numbers, such number is 202860.

57.  $99 = 11 \times 9$ , where 11 and 9 are co-prime

By hit and trial, we find that 114345 is divisible by 11 as well as 9. So, it is divisible by 99.

58.  $72 = 9 \times 8$ , where 9 and 8 are co-prime.

The minimum value of  $x$  for which  $73x$  is divisible by 8 is,  $x = 6$ .

Sum of digits in 425736  $= (4 + 2 + 5 + 7 + 3 + 6) = 27$ , which is divisible by 9.

$\therefore$  Required value of \* is 6.

59.  $80 = 2 \times 5 \times 8$ .

Since 653 xy is divisible by 2 and 5 both, so  $y = 0$ .

Now, 653 x0 is divisible by 8, so  $3x0$  should be divisible by 8. This happens when  $x = 2$ .

$\therefore x + y = (2 + 0) = 2$ .

60. Clearly, 4864 is divisible by 4.

So, 9P2 must be divisible by 3. So,  $(9 + P + 2)$  must be divisible by 3.

$\therefore P = 1$ .

61.  $6 = 3 \times 2$ . Clearly,  $5 * 2$  is divisible by 2. Replace \* by x.

Then,  $(5 + x + 2)$  must be divisible by 3. So,  $x = 2$

62.  $24 = 3 \times 8$ , where 3 and 8 are co-prime

Clearly, 35718 is not divisible by 8, as 718 is not divisible by 8

Similarly, 63810 is not divisible by 8 and 537804 is not divisible by 8.

Consider part (d).

Sum of digits  $= (3 + 1 + 2 + 5 + 7 + 3 + 6) = 27$ , which is divisible by 3.

Also, 736 is divisible by 8

$\therefore 3125736$  is divisible by  $(3 \times 8)$ , i.e., 24.

63.  $132 = 11 \times 3 \times 4$ .

Clearly, 968 is not divisible by 3

None of 462 and 2178 is divisible by 4.

And, 5184 is not divisible by 11.

Each one of the remaining four numbers is divisible by each one of 4, 3 and 11. So, there are 4 such numbers.

**64.** Let the given number be  $476 \ xy \ 0$ .

Then  $(4 + 7 + 6 + x + y + 0) = (17 + x + y)$  must be divisible by 3.

And,  $(0 + x + 7) - (y + 6 + 4) = (x - y - 3)$  must be either 0 or 11.

$$x - y - 3 = 0 \Rightarrow y = x - 3$$

$$(17 + x + y) = (17 + x + x - 3) = (2x + 14) \Rightarrow x = 2 \text{ or } x = 8.$$

$$\therefore x = 8 \text{ and } y = 5.$$

**65.**  $(4 + 5 + 2) - (1 + 6 + 3) = 1$ , not divisible by 11.

$(2 + 6 + 4) - (4 + 5 + 2) = 1$ , not divisible by 11.

$(4 + 6 + 1) - (2 + 5 + 3) = 1$ , not divisible by 11.

$(4 + 6 + 1) - (2 + 5 + 4) = 0$ , So, 415624 is divisible by 11.

**66.** Required numbers are 102, 108, 114, ..., 996

This is an A.P. in which  $a = 102$ ,  $d = 6$  and  $l = 996$

Let the number of terms be  $n$ . Then,

$$a + (n - 1)d = 996 \Rightarrow 102 + (n - 1) \times 6 = 996$$

$$\Rightarrow 6 \times (n - 1) = 894 \Rightarrow (n - 1) = 149 \Rightarrow n = 150$$

Required number of terms = 150.

**67.** Let  $S_n = (1 + 2 + 3 + \dots + 45)$ . This is an A.P. in which  $a = 1$ ,  $d = 1$ ,  $n = 45$ .

$$S_n = \frac{n}{2} [2a + (n - 1)d] = \frac{45}{2} \times [2 \times 1 + (45 - 1) \times 1] = \left( \frac{45}{2} \times 46 \right) = (45 \times 23)$$

$$= 45 \times (20 + 3) = (45 \times 20 + 45 \times 3) = (900 + 135) = 1035.$$

**68.** Let  $S_n = (2 + 4 + 6 + \dots + 30)$ . This is an A.P. in which  $a = 2$ ,  $d = 2$  and  $l = 30$

Let the number of terms be  $n$ . Then,

$$a + (n - 1)d = 30 \Rightarrow 2 + (n - 1) \times 2 = 30 \Rightarrow n = 15.$$

$$\therefore S_n = \frac{n}{2} (a + l) = \frac{15}{2} \times (2 + 30) = (15 \times 16) = 240.$$

**69.**  $S_n = (1 + 2 + 3 + \dots + 50 + 51 + 52 + \dots + 100) - (1 + 2 + 3 + \dots + 50)$

$$= \frac{100}{2} \times (1 + 100) - \frac{50}{2} \times (1 + 50) = (50 \times 101) - (25 \times 51) = (5050 - 1275) = 3775.$$

**70.** The smallest prime number is 2.

**71.** Required sum =  $(2 + 3 + 5 + 7 + 11) = 28$ .

**72.** Prime numbers less than 50 are :

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

Their number is 15.

**73.** Clearly, 97 is a prime number.

**74.** 91 is divisible by 7. So, it is not a prime number.

**75.**  $\sqrt{437} > 22$

All prime numbers less than 22 are 2, 3, 5, 7, 11, 13, 17, 19.

161 is divisible by 7, and 221 is divisible by 13.

373 is not divisible by any of the above prime numbers.

$\therefore$  373 is prime.

**76.**  $\sqrt{551} > 24$

All prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19.

119 is divisible by 7; 187 is divisible by 11; 247 is divisible by 13 and 551 is divisible by 19.

So, none of the given numbers is prime.

77. The smallest 3-digit number is 100, which is divisible by 2.  
 $\therefore$  100 is not a prime number.

$\sqrt{101} < 11$  and 101 is not divisible by any of the prime numbers  
 2, 3, 5, 7, 11.

$\therefore$  101 is a prime number

Hence 101 is the smallest 3-digit prime number.

78. The sum of two odd number is even. So,  $a + b$  is even.

79. The square of a natural number never ends in 2.

$\therefore$  143642 is not the square of a natural number.

80. The square of a natural number never ends in 7.

$\therefore$  42437 is not the square of a natural number

81. 6 ) 4456 ( 742

$$\begin{array}{r} 42 \\ \hline 25 \end{array}$$

$$\begin{array}{r} 24 \quad \text{Required number} = (6 - 4) = 2 \\ \hline 16 \\ 12 \\ \hline 4 \end{array}$$

82. On dividing, we get

$$88 ) 9217 ( 104$$

$$\begin{array}{r} 88 \\ \hline 417 \\ 352 \\ \hline 65 \end{array}$$

$\therefore$  Required number =  $9217 + (88 - 65) = 9217 + 23 = 9240$

83. On dividing, we get: 75 ) 8485 ( 113

$$\begin{array}{r} 75 \\ \hline 98 \\ 75 \\ \hline 235 \\ 225 \\ \hline 10 \end{array}$$

$\therefore$  Required number =  $(8485 - 10) = 8475$ .

84. Largest 4-digit number = 9999

$$88 ) 9999 ( 113$$

$$\begin{array}{r} 88 \\ \hline 119 \\ 88 \\ \hline 319 \\ 264 \\ \hline 55 \end{array}$$

Required number =  $(9999 - 55) = 9944$

119 is divisible by 7; 187 is divisible by 11; 247 is divisible by 13 and 551 is divisible by 19.

So, none of the given numbers is prime.

77. The smallest 3-digit number is 100, which is divisible by 2.  
 $\therefore$  100 is not a prime number.

$\sqrt{101} < 11$  and 101 is not divisible by any of the prime numbers

2, 3, 5, 7, 11.

$\therefore$  101 is a prime number

Hence 101 is the smallest 3-digit prime number.

78. The sum of two odd number is even. So,  $a + b$  is even.

79. The square of a natural number never ends in 2.

$\therefore$  143642 is not the square of a natural number.

80. The square of a natural number never ends in 7.

$\therefore$  42437 is not the square of a natural number

81. 6 ) 4456 ( 742

$$\begin{array}{r} 42 \\ \hline 25 \\ 24 \quad \text{Required number} = (6 - 4) = 2 \\ \hline 16 \\ 12 \\ \hline 4 \end{array}$$

82. On dividing, we get

$$88 ) 9217 ( 104$$

$$\begin{array}{r} 88 \\ \hline 417 \\ 352 \\ \hline 65 \end{array}$$

$\therefore$  Required number =  $9217 + (88 - 65) = 9217 + 23 = 9240$

83. On dividing, we get: 75 ) 8485 ( 113

$$\begin{array}{r} 75 \\ \hline 98 \\ 75 \\ \hline 235 \\ 225 \\ \hline 10 \end{array}$$

$\therefore$  Required number =  $(8485 - 10) = 8475$ .

84. Largest 4-digit number = 9999

$$88 ) 9999 ( 113$$

$$\begin{array}{r} 88 \\ \hline 119 \\ 88 \\ \hline 319 \\ 264 \\ \hline 55 \end{array}$$

Required number =  $(9999 - 55) = 9944$

**85. Largest 5-digit number = 99999**

91 ) 99999 ( 1098

$$\begin{array}{r} 91 \\ \hline 899 \\ 819 \\ \hline 809 \\ 728 \\ \hline 81 \end{array}$$

Required number =  $(99999 - 81) = 99918$ .

**86. 87 ) 13601( 156**

$$\begin{array}{r} 87 \\ \hline 490 \\ 435 \\ \hline 551 \\ 522 \\ \hline 29 \end{array}$$

∴ Required number = 29

**87. 23 ) 1056 ( 45**

$$\begin{array}{r} 92 \\ \hline 136 \\ 115 \\ \hline 21 \end{array}$$

Required number =  $(23 - 21) = 2$

**88. The smallest 5-digit number = 10000.**

41 ) 10000 ( 243

$$\begin{array}{r} 82 \\ \hline 180 \\ 164 \\ \hline 160 \\ 123 \\ \hline 37 \end{array}$$

Required number =  $10000 + (41 - 37) = 10004$

**89. The smallest 6-digit number = 100000.**

111 ) 100000 ( 900

$$\begin{array}{r} 999 \\ \hline 100 \end{array}$$

Required number =  $100000 + (111 - 100) = 100011$ .

**90. Divisor =  $(5 \times 46) = 230$**

$$\therefore 10 \times \text{quotient} = 230 \Rightarrow \text{Quotient} = \frac{230}{10} = 23.$$

$$\begin{aligned} \text{Dividend} &= (\text{Divisor} \times \text{Quotient}) + \text{Remainder} \\ &= (230 \times 23) + 46 = (5290 + 46) = 5336. \end{aligned}$$

**91. Number =  $269 \times 68 + 0 = 18292$**

67 ) 18292 ( 273

$$\begin{array}{r} 134 \\ \hline 489 \\ 469 \\ \hline 202 \\ 201 \\ \hline 1 \end{array}$$

$\times 68$

$$\begin{array}{r} 2152 \\ 1614 \\ \hline 18292 \end{array}$$

∴ Required remainder = 1

93. Let  $x$  be the number and  $y$  be the quotient. Then,

$$\begin{aligned}x &= 357 \times y + 39 \\&= (17 \times 21 \times y) + (17 \times 2) + 5 \\&= 17 \times (21y + 2) + 5.\end{aligned}$$

$\therefore$  Required remainder = 5.

94. Let the number be  $x$  and on dividing  $x$  by 5, we get  $k$  as quotient and 3 as remainder.

$$\begin{aligned}\therefore x &= 5k + 3 \Rightarrow x^2 = (5k + 3)^2 = (25k^2 + 30k + 9) \\&= 5(5k^2 + 6k + 1) + 4\end{aligned}$$

$\therefore$  On dividing  $x^2$  by 5, we get 4 as remainder.

95. Let the smaller number be  $x$ . Then larger number =  $(x + 1365)$ .

$$\therefore x + 1365 = 6x + 15 \Rightarrow 5x = 1350 \Rightarrow x = 270$$

$\therefore$  Smaller number = 270.

96. Number =  $(12 \times 35) = 420$ .

$$\text{Correct Quotient} = 420 \div 21 = 20.$$

97. Let the numbers be  $a$  and  $b$ . Then,  $a + b = 12$  and  $ab = 35$ .

$$\therefore \frac{a+b}{ab} = \frac{12}{35} \quad \Rightarrow \quad \left(\frac{1}{b} + \frac{1}{a}\right) = \frac{12}{35}$$

$$\therefore \text{Sum of reciprocals of given numbers} = \frac{12}{35}$$

98. Let the number be  $x$ . Then

$$60\% \text{ of } \frac{3}{5} \text{ of } x = 36 \Rightarrow \frac{60}{100} \times \frac{3}{5} \times x = 36 \Rightarrow x = \left(36 \times \frac{25}{9}\right) = 100.$$

$\therefore$  Required number = 100.

99. Let the required fraction be  $x$ . Then,  $\frac{1}{x} - x = \frac{9}{20}$

$$\begin{aligned}\therefore \frac{1-x^2}{x} &= \frac{9}{20} \Rightarrow 20 - 20x^2 = 9x \Rightarrow 20x^2 + 9x - 20 = 0 \\&\Rightarrow 20x^2 + 25x - 16x - 20 = 0 \quad \Rightarrow 5x(4x+5) - 4(4x+5) = 0 \\&\Rightarrow (4x+5)(5x-4) = 0 \Rightarrow x = \frac{4}{5}.\end{aligned}$$

100. Clearly,  $(2272 - 875) = 1397$ , is exactly divisible by N.

$$\text{Now, } 1397 = 11 \times 127$$

$\therefore$  The required 3-digit number is 127, the sum of whose digits is 10

101. By hit and trial, we find that

$$47619 \times 7 = 333333.$$

$$7 ) \underline{\underline{333333}} \ ( 47619$$

28

$$\begin{array}{r} 53 \\ \hline 49 \end{array}$$

$$\begin{array}{r} 43 \\ \hline 42 \end{array}$$

$$\begin{array}{r} 13 \\ \hline 7 \end{array}$$

$$\begin{array}{r} 63 \\ \hline 63 \\ \hline x \end{array}$$

- 102.** Let the two consecutive even integers be  $2n$  and  $(2n + 2)$ . Then,  

$$(2n + 2)^2 - (2n)^2 = (2n + 2 + 2n)(2n + 2 - 2n) \\ = 2(4n + 2) = 4(2n + 1)$$
, which is divisible by 4.
- 103.** Let the two consecutive odd integers be  $(2n + 1)$  and  $(2n + 3)$ . Then,  

$$(2n + 3)^2 - (2n + 1)^2 = (2n + 3 + 2n + 1)(2n + 3 - 2n - 1) \\ = (4n + 4) \times 2 = 8(n + 1)$$
, which is divisible by 8.
- 104.**  $(6n^2 + 6n) = 6n(n + 1)$ , which is always divisible by 6 and 12 both, since  $n(n + 1)$  is always even.
- 105.** Let  $n = 4q + 3$ . Then,  $2n = 8q + 6 = 4(2q + 1) + 2$ .  
Thus, when  $2n$  is divided by 4, the remainder is 2.
- 106.**  $(x^n + 1)$  will be divisible by  $(x + 1)$  only when  $n$  is odd.  
 $\therefore (67^{67} + 1)$  will be divisible by  $(67 + 1)$   
 $\therefore (67^{67} + 1) + 66$ , when divided by 68 will give 66 as remainder.
- 107.**  $(x^n - 1)$  will be divisible by  $(x + 1)$  only when  $n$  is even.  
 $(49^{15} - 1) = [(7^2)^{15} - 1] = (7^{30} - 1)$ , which is divisible by  $(7 + 1)$ , i.e., 8.
- 108.** For every natural number  $n$ ,  $(x^n - a^n)$  is always divisible by  $(x - a)$ .
- 109.** When  $n$  is even,  $(x^n - a^n)$  is completely divisible by  $(x + a)$   
 $(17^{200} - 1^{200})$  is completely divisible by  $(17 + 1)$ , i.e., 18.  
 $\Rightarrow (17^{200} - 1)$  is completely divisible by 18.  
 $\Rightarrow$  On dividing  $17^{200}$  by 18, we get 1 as remainder.
- 110.** When  $n$  is odd,  $(x^n + a^n)$  is always divisible by  $(x + a)$ .  
 $\therefore$  Each one of  $(47^{43} + 43^{43})$  and  $(47^{47} + 43^{43})$  is divisible by  $(47 + 43)$ .
- 111.** Let  $2^{32} = x$ . Then,  $(2^{32} + 1) = (x + 1)$ .  
Let  $(x + 1)$  be completely divisible by the natural number N. Then,  
 $(2^{96} + 1) = [(2^{32})^3 + 1] = (x^3 + 1) = (x + 1)(x^2 - x + 1)$ , which is completely divisible by N, since  $(x + 1)$  is divisible by N.
- 112.**  $(4^{61} + 4^{62} + 4^{63} + 4^{64}) = 4^{61} \times (1 + 4 + 4^2 + 4^3) = 4^{61} \times 85 \\ = 4^{60} \times (4 \times 85) = (4^{60} \times 340)$ , which is divisible by 10.
- 113.**  $(3^{25} + 3^{26} + 3^{27} + 3^{28}) = 3^{25} \times (1 + 3 + 3^2 + 3^3) = 3^{25} \times 40 \\ = 3^{24} \times 3 \times 4 \times 10 = (3^{24} \times 4 \times 30)$ , which is divisible by 30.
- 114.** 
$$\left. \begin{array}{r} 4 \ a \ 3 \\ 9 \ 8 \ 4 \\ 13 \ b \ 7 \end{array} \right\} \Rightarrow a + 8 = b \Rightarrow b - a = 8$$
- Also,  $13b7$  is divisible by 11  $\Rightarrow (7 + 3) - (b + 1) = (9 - b) \Rightarrow (9 - b) = 0 \Rightarrow b = 9$   
 $\therefore (b = 9 \text{ and } a = 1) \Rightarrow (a + b) = 10$ .
- 115.** By hit and trial, we put  $x = 5$  and  $y = 1$  so that  $(3x + 7y) = (3 \times 5 + 7 \times 1) = 22$ , which is divisible by 11.  
 $\therefore (4x + 6y) = (4 \times 5 + 6 \times 1) = 26$ , which is not divisible by 11;  
 $(x + y + 4) = (5 + 1 + 4) = 10$ , which is not divisible by 11;  
 $(9x + 4y) = (9 \times 5 + 4 \times 1) = 49$ , which is not divisible by 11;  
 $(4x - 9y) = (4 \times 5 - 9 \times 1) = 11$ , which is divisible by 11.
- 116.**  $90 = 10 \times 9$   
Clearly, 653xy is divisible by 10, so  $y = 0$   
Now, 653x0 is divisible by 9. So,  $(6 + 5 + 3 + x + 0) = (14 + x)$  is divisible by 9. So,  $x = 4$ . Hence,  $(x + y) = (4 + 0) = 4$ .
- 117.** Marking (✓) those which are divisible by 3 but not by 9 and the others by (5), by taking

the sum of digits, we get:

$$\begin{array}{lllll} 2133 \rightarrow 9 (\times), & 2343 \rightarrow 12 (\checkmark), & 3474 \rightarrow 18 (\times) & 4131 \rightarrow 9 (\times), & 5286 \rightarrow 21 (\checkmark) \\ 5340 \rightarrow 12 (\checkmark), & 6336 \rightarrow 18 (\times) & 7347 \rightarrow 21 (\checkmark) & 8115 \rightarrow 15 (\checkmark) & 9276 \rightarrow 24 (\checkmark) \end{array}$$

Required number of numbers = 6

118. 639 is not divisible by 7.

2079 is divisible by each one of 3, 7, 9, 11.

119.  $132 = 4 \times 3 \times 11$

$$264 \rightarrow 11, 3, 4 (\checkmark); 396 \rightarrow 11, 3, 4 (\checkmark); 462 \rightarrow 11, 3 (\checkmark); 792 \rightarrow 11, 3, 4 (\checkmark)$$

$$968 \rightarrow 11, 4 (\times); 2178 \rightarrow 11, 3 (\times); 5184 \rightarrow 3, 4 (\times); 6336 \rightarrow 11, 3, 4 (\checkmark)$$

Required number of numbers = 4.

120. Let  $x = 6q + 3$ . Then,  $x^2 = (6q + 3)^2 = 36q^2 + 36q + 9 = 6(6q^2 + 6q + 1) + 3$

Thus, when  $x^2$  is divided by 6, then remainder = 3.

121.  $\begin{array}{r|rr} 4 & x \\ \hline & y = (5 \times 1 + 4) = 9 \end{array}$

$$\begin{array}{r|rr} 5 & y - 1 \\ \hline & x = (4 \times y + 1) = (4 \times 9 + 1) = 37 \\ \hline & 1 - 4 \end{array}$$

Now, 37 when divided successively by 5 and 4, we get

$$\begin{array}{r|rr} 5 & 37 \\ \hline & 7 - 2 \\ \hline & 1 - 3 \end{array}$$

Respective remainders are 2 and 3

122.  $\begin{array}{r|rr} 4 & x \\ \hline & z = 6 \times 1 + 4 = 10 \end{array}$

$$\begin{array}{r|rr} 5 & y - 2 \\ \hline & y = 5 \times z + 3 = 5 \times 10 + 3 = 53 \end{array}$$

$$\begin{array}{r|rr} 6 & z - 3 \\ \hline & x = 4 \times y + 2 = 4 \times 53 + 2 = 212 + 2 = 214. \\ \hline & 1 - 4 \end{array}$$

Hence, required number = 214.

123.  $\begin{array}{r|rr} 5 & x \\ \hline & z = 13 \times 1 + 12 = 25, \end{array}$

$$\begin{array}{r|rr} 9 & y - 4 \\ \hline & y = 9 \times z + 8 = 9 \times 25 + 8 = 233 \end{array}$$

$$\begin{array}{r|rr} 13 & z - 8 \\ \hline & x = 5 \times y + 4 = 5 \times 233 + 4 = 1169 \\ \hline & 1 - 12 \end{array}$$

$$585 ) 1169 ( 1$$

$$\begin{array}{r} 585 \\ \hline 584 \end{array}$$

$\therefore$  On dividing the number by 585, remainder = 584.

124.  $x = 13p + 11$  and  $x = 17q + 9$

$$\therefore 13p + 11 = 17q + 9 \Rightarrow 17q - 13p = 2 \Rightarrow q = \frac{2+13p}{17}$$

The least value of  $p$  for which  $q = \frac{2+13p}{17}$  is a whole number is  $p = 26$

$$\therefore x = (13 \times 26 + 11) = (338 + 11) = 349.$$

125. Let  $x = 296q + 75 = (37 + 8q + 37 \times 2) + 1$

$$= 37 + (8q + 2) + 1$$

Thus, when the number is divided by 37, the remainder is 1

126.  $987 = 3 \times 7 \times 47$ .

So, the required number must be divisible by each one of 3, 7, 47

$$553681 \rightarrow (\text{Sum of digits} = 28, \text{not divisible by 3})$$

**555181** → (Sum of digits = 25, not divisible by 3)

**555681** is divisible by each one of 3, 7, 47.

127.  $132 = 3 \times 4 \times 11$

$264 \rightarrow 3, 4, 11 (\checkmark)$ ;  $396 \rightarrow 3, 4, 11 (\checkmark)$ ;  $462 \rightarrow 3, 11 (\times)$ ;  $792 \rightarrow 3, 4, 11 (\checkmark)$

$968 \rightarrow 4, 11 (\times)$ ;  $2178 \rightarrow 3, 11 (\times)$ ;  $5184 \rightarrow 3, 4 (\times)$ ;  $6336 \rightarrow 3, 4, 11 (\checkmark)$

Required number of numbers = 4

128. 3-digit numbers divisible by 6 are:

102, 108, 114, ..., 996

This is an A.P. in which  $a = 102$ ,  $d = 6$  and  $l = 996$

Let the number of terms be  $n$ . Then  $t_n = 996$ .

$$\therefore a + (n - 1)d = 996 \Rightarrow 102 + (n - 1) \times 6 = 996$$

$$\Rightarrow 6 \times (n - 1) = 894 \Rightarrow (n - 1) = 149 \Rightarrow n = 150$$

∴ Number of terms = 150.

129. Let  $S_n = (1 + 2 + 3 + \dots + 45)$

This is an A.P. in which  $a = 1$ ,  $d = 1$ ,  $n = 45$  and  $l = 45$

$$\therefore S_n = \frac{2}{2}(a+l) = \frac{45}{2} \times (1+45) = (45 \times 23) = 45 \times (20+3)$$

$$= (45 \times 20) + (45 \times 3) = 900 + 135 = 1035.$$

Required sum = 1035.

130. Required sum =  $(2 + 4 + 6 + \dots + 30)$

This is an A.P. in which  $a = 2$ ,  $d = (4 - 2) = 2$  and  $l = 30$ .

Let the number of terms be  $n$ . Then

$$t_n = 30 \Rightarrow a + (n - 1)d = 30$$

$$\Rightarrow 2 + (n - 1) \times 2 = 30 \Rightarrow n - 1 = 14 \Rightarrow n = 15$$

$$\therefore S_n = \frac{n}{2}(a+l) = \frac{15}{2} \cdot (2+30) = 240.$$

131. This is an A.P. in which  $a = 51$ ,  $l = 100$  and  $n = 50$

$$\therefore \text{Sum} = \frac{n}{2}(a+l) = \frac{50}{2} \times (51+100) = (25 \times 151) = 3775.$$

132. Given sum =  $(1 + 1 + 1 + \dots \text{ to } n \text{ terms}) - \left( \frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \dots \text{ to } n \text{ terms} \right)$

$$= n - \frac{n}{2} \left( \frac{1}{n} + 1 \right) \quad [\because l = n^{\text{th}} \text{ terms} = \frac{n}{n} = 1]$$

$$= n - \frac{n+1}{2} = \frac{1}{2}(n-1).$$

133. Required numbers are 24, 30, 36, 42 ..., 96

This is an A.P. in which  $a = 24$ ,  $d = 6$  and  $l = 96$

Let the number of terms in it be  $n$ .

$$\text{Then, } t_n = 96 \Rightarrow a + (n - 1)d = 96$$

$$\Rightarrow 24 + (n - 1) \times 6 = 96 \Rightarrow (n - 1) \times 6 = 72 \Rightarrow (n - 1) = 12 \Rightarrow n = 13$$

Required number of numbers = 13.

134. Required numbers are 10, 15, 20, 25, ...., 95

This is an A.P. in which  $a = 10$ ,  $d = 5$  and  $l = 95$ .

Let the number of terms in it be  $n$ . Then

$$t_n = 95 \Rightarrow a + (n - 1)d = 95$$

$$\Rightarrow 10 + (n - 1) \times 5 = 95 \Rightarrow (n - 1) \times 5 = 85 \Rightarrow (n - 1) = 17 \Rightarrow n = 18$$

$$\text{Required sum} = \frac{n}{2}(a + l) = \frac{18}{2}(10 + 95) = (9 \times 105) = 945$$

135. This is an A.P. in which  $a = 6$ ,  $d = 6$  and  $S_n = 1800$

$$\text{Then, } \frac{n}{2}[2a + (n - 1)d] = 1800 \Rightarrow \frac{n}{2}[2 \times 6 + (n - 1) \times 6] = 1800$$

$$\Rightarrow 3n(n + 1) = 1800 \Rightarrow n(n + 1) = 600$$

$$\Rightarrow n^2 + n - 600 = 0 \Rightarrow n^2 + 25n - 24n - 600 = 0$$

$$\Rightarrow n(n + 25) - 24(n + 25) = 0 \Rightarrow (n + 25)(n - 24) = 0 \Rightarrow n = 24$$

Number of terms = 24

136. Here  $a = 3$  and  $r = \frac{6}{3} = 2$ . Let the number of terms be  $n$ .

$$\text{Then, } t_n = 384 \Rightarrow ar^{n-1} = 384$$

$$\Rightarrow 3 \times 2^{n-1} = 384 \Rightarrow 2^{n-1} = 128 = 2^7$$

$$\Rightarrow n - 1 = 7 \Rightarrow n = 8$$

$\therefore$  Number of terms = 8

137. This is a G.P. in which  $a = 2$ ,  $r = \frac{2^2}{2} = 2$  and  $n = 9$

$$S_n = \frac{a(r^n - 1)}{(r - 1)} = \frac{2 \times (2^9 - 1)}{(2 - 1)} = 2 \times (512 - 1) = 2 \times 511 = 1022.$$

138. We know that  $(1^2 + 2^2 + 3^2 + \dots + n^2) = \frac{1}{6}n(n+1)(2n+1)$

$$\text{Putting } n = 10, \text{ required sum} = \left( \frac{1}{6} \times 10 \times 11 \times 21 \right) = 385$$

$$\begin{aligned} 139. (2^2 + 4^2 + 6^2 + \dots + 20^2) &= (1 \times 2)^2 + (2 \times 2)^2 + (2 \times 3)^2 + \dots + (2 \times 10)^2 \\ &= (2^2 \times 1^2) + (2^2 \times 2^2) + (2^2 \times 3^2) + \dots + (2 \times 10)^2 \\ &= 2^2 \times [1^2 + 2^2 + 3^2 + \dots + 10^2] \end{aligned}$$

$$= \left( 4 \times \frac{1}{6} \times 10 \times 11 \times 21 \right) \quad \left[ \because (1^2 + 2^2 + 3^2 + \dots + n^2) = \frac{1}{6}n(n+1)(2n+1) \right]$$

$$= (4 \times 5 \times 77) = 1540.$$

$$140. (11^2 + 11^2 + 13^2 + \dots + 20^2) = (1^2 + 2^2 + \dots + 30^2) - (1^2 + 2^2 + \dots + 10^2)$$

$$= \left\{ \frac{20 \times 21 \times 41}{6} - \frac{10 \times 11 \times 21}{6} \right\} \quad \left[ \because (1^2 + 2^2 + \dots + n^2) = \frac{1}{6}n(n+1)(2n+1) \right]$$

$$= (2870 - 385) = 2485.$$