here $y_0=0$ $y_1=0.2588$ $y_2=0.5$ $y_3=0.7071$ $y_4=0.8660$

$y_5=0.9656$ $y_6=1$

WKT simposn's $1/3$rd rule is.

$$\int_a^b f(x)dx = \frac{h}{3}\{(y_0+y_n)+2(y_2+y_4+\cdots+y_{n-2})+4(y_1+y_3+\cdots+y_{n-1})\}$$

$$\int_0^{\pi/2} \sin x\,dx = \frac{h}{3}\{(y_0+y_1)+2(y_2+y_4)+4(y_1+y_3+y_5)\}$$

$$= \frac{\pi}{12\times 3}\{(0+1)+2(0.5+0.8660)+4(0.2588+0.7071+0.9656)$$

$$= \frac{180}{36}\{1+2.732+7.7272\}$$

$$= 5\{11.4592\}$$

$$= 57.296 \quad = \frac{22}{7\times 36}(1+2.732+7.7272)$$

$$= \frac{22}{252}(11.4592)$$

$$\int_0^{\pi/2} \sin x\,dx = 57.296 = \frac{252.1024}{252} = 1.0004 \text{ (Approx)}$$

4. Evaluate $\int_0^1 \frac{dx}{1+x}$ using i. trapezoidal rule ii. simpson's 3rd rule iii. simpson's $3/8$th rule iv. find the error in each method by comparing with actual integration upto 4 places of decimal pl take $n=1/6$ for all cases.

sol

$$h=\frac{b-a}{n}=\frac{1-0}{6}=1/6$$

Let $y(x)=1/(1+x)$ G.T $h=1/6$

The tabular column is as follows :-

x	0	$1/6$	$1/3$	$1/3$	$2/3$	$5/6$	1
$y=\frac{1}{1+x}$	1	$6/7$	$3/4$	$2/3$	$3/5$	$6/4$	0.5
		0.8571	0.75	0.6667	0.6	0.5455	

here $y_0=1$ $y_1=0.8571$ $y_2=0.75$ $y_3=0.6667$ $y_4=0.6$

$y_5=0.5455$ $y_6=0.5$

i. WKT trapezoidal rule is

$$\int_a^b f(x)dx = \frac{h}{2}\{(y_0+y_n)+2(y_1+y_2+\cdots+y_{n-1})\}$$

$$\therefore \int_0^1 \frac{dx}{1+x} = \frac{1/6}{2}\{(1+0.5)+2((1+0.8571)+0.75+0.6667+0.6+0.5455)\}$$

$$= \frac{1}{12}\{1.5+2(3.4193)\}$$

$$= \frac{1}{12}(8.3386)$$

$$= 0.69488$$

$$= 0.6949$$

ii. WKT the simpson's $1/3$rd rule is,

$$\int_a^b f(x)dx = \frac{h}{3}\{(y_0+y_n)+2(y_2+y_4+\cdots+y_{n-2})+4(y_1+y_3+\cdots+y_{n-1})\}$$

$$\int_0^1 \frac{dx}{1+x} = \frac{1/6}{3}\{(1.5)+2(0.75+0.6)+4((0.8571)+0.6667+0.5455)\}$$

$$= \frac{1}{18}\{1.5+2.7+8.2772\}$$

$$= \frac{1}{18}(12.4772)$$

$$= 0.69317$$

$$= 0.6932.$$

iii. WKT the simpson's $\frac{3}{8}$th rule is

$$\int_a^b f(x)dx = \frac{3h}{8}\{(y_0+y_6)+3(y_1+y_4)+3(y_2+y_5)+2y_3\}$$

$$\int_0^1 \frac{dx}{1+x} = \frac{3(1/6)}{8}\{(1.5)+3(0.8571+0.6)+3(0.75+0.5455)+2(0.6667)\}$$

$$= 1/16\{1.5+4.3731+3.8865+1.3334\}$$

$$= \frac{1}{16}\{11.0912\}$$

$$= 0.6932$$

iv. $$\int_0^1 \frac{dx}{1+x} = \int_0^1 \frac{d(1+x)}{1+x}$$

$$= [\log(1+x)]_0^1$$

$$= \log_e^2 - \log_e^1$$

$$= 0.6931.$$

$\log 2 = 0.3010$

$\log e = 0.4343$

$\log_e^2 = 0.6931.$

Trapezoidal rule, error = Exact value - Approx value

$= 0.6931 - 0.6949$

$= -0.0018$

simpson's $\frac{1}{3}$rd rule error $= 0.6931 - 0.6932$

$= -0.0001$

simpson's $\frac{3}{8}$th rule error $= 0.6931 - 0.6932$

$= -0.0001$.

WKT weddley's rule is,

$$\int_a^b f(x)dx = \frac{3h}{10}\{(y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5) + (2y_6 + 5y_7 + y_8 + 6y_9 + y_{10} + 5y_{11}) + \cdots + (2y_{n-6} + 5y_{n-5} + y_{n-4} + 6y_{n-3} + y_{n-2} + 5y_{n-1} + y_n)\}$$

Note :-

For weddle's rule put $n=6$ in newton's cote's quadrature formula.

problems

1\. i. Evaluate $\int_0^6 \frac{dx}{1+x^2} = \tan^{-1}x$ taking $h=1$ using

ii. Trapezoidal rule, iii. simpson's $\frac{1}{3}$rd rule

iv. simpson's $\frac{3}{8}$th rule iv. weddle's rule v. Also check up by integration which rule gives the value closets to the actual value.

sol) let $y(x) = \frac{1}{1+x^2} = \tan^{-1}x$ G.T $h=1$

The tabular column as follows

x	0	1	2	3	4	5	6
$y = \frac{1}{1+x^2}$	1	0.5	0.2	0.1	0.0588	0.0385	0.0270.

$y_0 = 1$ $y_1 = 0.5$ $y_2 = 0.2$ $y_3 = 0.1$ $y_4 = 0.0588$

$y_5 = 0.0385$ $y_6 = 0.0270$

i. WKT the trapezoidal rule is

$$\int_a^b f(x)dx = \frac{h}{2}\{(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})\}$$

$$\int_a^b \frac{dx}{1+x^2} = \frac{1}{2}\{(1+0.0385) + 2(0.5 + 0.2 + 0.1 + 0.0588 + 0.0385)\}$$

$= \frac{1}{2} (1.0270 + 2(0.8973))$

$= 0.5 (1.0270 + 1.7346)$

$= 0.5 (2.8216)$

$\int_0^b \frac{dx}{1+x^2} = 1.4108$

ii. WKT the simpson's $1/3$rd rule is

$\int_0^b \frac{dx}{1+x^2} = \frac{h}{3} \{(y_0 + y_6) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)\}$

$= \frac{1}{3} \{(1 + 0.0270) + 2(0.2 + 0.0588) + 4(0.5 + 0.1 + 0.0385)\}$

$= \frac{1}{3} \{1.0270 + 2(0.2588) + 4(0.03851)\}$

$= \frac{1}{3} (1.0270 + 0.5176 + 2.554)$

$= 4.0986/3$

$\int_0^b \frac{dx}{1+x^2} = 1.3662$

iii. WKT the simpson's $3/8$th rule is

$\int_0^6 \frac{dx}{1+x^2} = \frac{3h}{8} \{((y_0 + y_6) + 3(y_1 + y_4) + 3(y_2 + y_5) + 2y_3)\}$

$= \frac{3}{8} \{(1 + 0.0270) + 3(0.5 + 0.0588) + 3\ 0.2 + 0.0385 + 2(0.1)\}$

$= 3/8 \{1.0270 + 3(0.5588) + 3(0.2358) + 0.2\}$

$= 3/8 (3.6189)$

$= 1.3571.$

iv. WKT the weddles rule is,

$\int_0^6 \frac{dx}{1+x^2} = \frac{3h}{10} \{y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 4y_5 + y_6\}$

$= \frac{3}{10} \{1 + 5(0.5) + 0.2 + 6(0.1) + 0.0588 + 5(0.03846) + (0.027)\}$

$= 0.3 \{1 + 2.5 + 0.2 + 0.6 + 0.0588 + 0.1925 + 0.027\}$

$= 0.3 (4.5783)$

$\int_0^6 \frac{dx}{1+x^2} = 1.37\overset{3}{5}$

v. $\int_0^6 \frac{dx}{1+x^2} = [\tan^{-1}x]_0^6$

$= \tan^{-1}6 - \tan^{-1}0$

$= \tan^{-1}6 - \tan^{-1}(\tan 0)$

$= \tan^{-1}6.$

$\int_0^6 \frac{dx}{1+x^2} = 1.4056$ //.

1.