64. The character of $(\mathbb{Q}, +, \cdot)$ is 0.
65. The characteristic of $(\mathbb{Z}_7, \oplus, \otimes)$ is 7.
66. The map $f: \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = x^2+3$ is not a ring homomorphism.
67. The map $f: \mathbb{Z} \to \mathbb{Z}$ defined $f(x) = 2x$ is group homomorphism.
68. Let $f: \mathbb{C} \to \mathbb{C}$ be defined by $f(z) = \bar{z}$. Then $\ker f$ is $\{0\}$.
69. Any ordered integral domain is of characteristic 0.
70. Let $R = \{a+ib \,/\, a, b \in \mathbb{Z}\}$. Then R is a ring under usual addition and multiplication. This ring is called the Ring of Gaussian integers.
71. $\{0\}$ with binary operations '+' and '.' defined as $0+0=0$ & $0 \cdot 0 = 0$ is a ring. This is called the null ring.
72. In a ring with identity the identity element is unique.
73. Let R be a ring with identity element. R is called a skew field or a division ring if every non-zero element in R is a unit.
74. A commutative skew field is called a field.
75. Let R be a ring. If $\exists$ a +ve integer $n \ni: na = 0$, $\forall a \in R$ then the least such +ve integer is called the characteristic of the ring R.
76. If no such +ve integer exists then the ring is said to be of characteristic zero.
77. $M_2(\mathbb{R})$ is a ring of characteristic zero.
78. Any Boolean ring is of characteristic 2.
79. The characteristic of an integral domain D is either 0 or a prime number.

$ab = 0$ (or) $ba = 0$

32. In this case a is called left zero divisor & b is called right zero divisor.

33. In the ring of integers no element is a zero divisor.

34. No skew fields has any zero divisors.

35. A commutative ring with identity having no zero divisor is called an Integral domain.

36. Z is an integral domain.

37. Z_7 is an integral domain.

38. $(Z_n, +, \cdot)$ is an integral domain.

39. Z_{12} is not an integral domain.

40. Z_n is an integral domain iff n is a prime.

41. Any unit in R cannot be a zero divisor.

42. A Ring R has no zero divisor iff cancellation law is valid in R.

43. The only idempotent elements of an integral domain are 0 & 1.

44. A ring R is called a boolean ring if $a^2 = a \; \forall \; a \in R$ and an element 'a' is called Idempotent element.

45. Any field F is an integral domain.

46. An integral domain need not be a field.

47. Z is an integral domain but not a field.

48. Any finite integral domain is a field.

49. An infinite integral domain is not a field.

50. Z_n is a field iff n is a prime.

A finite commutative ring R without zero divisor is a field.

The Algebraic structure which is not a ring is $(Z_n, \otimes, \oplus)$

The Algebraic structure which is not a ring is $(R, \cdot, +)$

The Algebraic structure which is a ring is $(P(S), \Delta, \cap)$

Which of the following pair of tables makes $(0, 1)$ is a ring?

a)

+	0	1
0	0	1
1	1	0

·	0	1
0	0	1
1	0	1

b)

+	0	1
0	0	1
1	1	0

·	0	1
0	0	0
1	0	1

c)

+	0	1
0	0	0
1	1	1

·	0	1
0	0	1
1	0	1

d)

+	0	1
0	0	1
1	1	1

·	0	1
0	0	0
1	0	1

56. A ring is called a Boolean ring if $a^2 = a \ \forall\, a \in R$.

57. In the ring $(Z_4, \oplus, \otimes)$, $(\{0, 2\}, \oplus, \otimes)$ is subring without identity.

58. In the ring $M_2(R)$ $\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$ is a unit.

59. In $(Z, +, \cdot)$ 1 & -1 are the only units.

60. An example of a finite commutative ring with identity but not an integral domain is $(Z_4, \oplus, \otimes)$

61. An example of an infinite commutative ring without identity is $(2Z, +, \cdot)$

62. Let R be a ring with identity. Then for all $a, b \in R$ we have $(a+b)^2 = a^2 + ab + ba + b^2$.

63. Let R be a commutative ring. Then for all $a, b \in R$ $(a+b)^2 = a^2 + 2ab + b^2$.

52. If H' is a subgroup of G', then $f^{-1}(H')$ is a subgroup of G.

53. If H' is normal in $f(G)$ then $f^{-1}(H')$ is normal in G.

Unit : 4

1. A non-empty set R together with two binary operation denoted '+' and '.' and called addition and multiplication which satisfy the following axioms is called a Ring.
2. $(R,+)$ is an abelian group.
3. '.' is an associative binary operation in R.
4. $a\cdot(b+c) = a\cdot b + a\cdot c$
5. $(a+b)\cdot c = a\cdot c + b\cdot c$
6. The unique identity of the additive group $(R,+)$ is denoted by 0 and is called the zero element of the ring.
7. The unique additive inverse of a is denoted by $-a$.
8. $(Z,+,\cdot), (Q,+,\cdot), (R,+,\cdot), (C,+,\cdot)$ are all rings.
9. $(2Z,+,\cdot)$ is a ring.
10. $(Z,+,\cdot)$ is a ring.
11. Let R be a ring and $a,b \in R$ then $0\cdot a = a\cdot 0 = 0$
12. $a(-b) = (-a)b = -(ab)$
13. $(-a)(-b) = ab$
14. $a(b-c) = ab-ac$
15. A ring R is called a Boolean ring if $a^2=a \ \forall \ a \in R$.
16. If $f: R\to R'$ is an isomorphism we say that, $R \cong R'$
17. Let R & R' be two rings & $f: R\to R'$ be an isomorphism then clearly, f is an isomorphism of the group $(R,+)$ to the group $(R',+)$. Hence $f(0)=0'$, $f(-a) = -f(a)$.

18. A ring R is said to be commutative if $ab = ba$ $\forall$ $a, b \in R$.

19. $(\mathbb{Z},+,\cdot)$, $(\mathbb{R},+,\cdot)$, $(\mathbb{C},+,\cdot)$, $(\mathbb{Q},+,\cdot)$ are commutative rings.

20. Let R be a ring we say that, R is a ring with identity if $\exists$ an element $1 \in R$ $\ni$: $a \cdot 1 = 1 \cdot a = a$ $\forall$ $a \in R$.

21. $(2\mathbb{Z},+,\cdot)$ is a ring without identity.

22. $M_2(R)$ is a ring without identity.

23. $(2\mathbb{Z},+,\cdot)$ is a commutative ring without identity.

24. $(M_2(R),+,\cdot)$ is a non-commutative ring with identity.

25. Let R be a ring with identity & an element, $u \in R$ is called a unit in R, if it has a multiplicative inverse in R.

26. In a skew field, the non-zero elements form a group under multiplication.

27. If $(F,+)$ is an abelian group, $(F-\{0\},\cdot)$ is an abelian group & $a\cdot(b+c) = a\cdot b + a \cdot c$ $\forall$ $a, b, c \in F$ then it is known as Field.

28. $(\mathbb{Z},+,\cdot)$ is a commutative ring with identity but not a field.

29. Let p be a prime, then $(\mathbb{Z}_p, \oplus, \cdot)$ is a field.

30. 1 & -1 are the only non-zero elements which have inverse.

31. Let R be a ring. A non-zero element $a \in R$ is said to be a zero divisor if $\exists$ a non-zero element $b \in R$ $\ni$: $ab = 0$ or $ba = 0$.

40. In $(Z,+)$ the order of every element other than 0 is infinite.

41. In the group $(\wp(S), \Delta)$ the order of every element other than ϕ is 2.

42. In V_4 the order of every element other than the identity is 2.

43. If $\alpha = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ then $\alpha\beta =$

$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$

44. If $\alpha = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ is a permutation the $\alpha^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

45. Any infinite cyclic group G is isomorphic to $(Z,+)$.

46. Any two infinite cyclic groups are isomorphic to each other.

47. Any two finite cyclic groups of the same order are isomorphic.

48. Let G be a group. The set of all automorphism of G is denoted by Aut G.

49. The set of all inner automorphisms of G is denoted by $I(G)$.

50. For any group G,

i) Aut G is a group under composition of functions.

ii) $I(G)$ is a normal subgroup of Aut G.

51. Every isomorphism is a homomorphism and a bijective homomorphism is an isomorphism.

One Words.
Unit: 3

1. A map $f: G \to G'$ is called a homomorphism if $f(ab) = f(a) \cdot f(b)$ $\forall a, b \in G$.

2. If $f: (G, \cdot) \to (G', +)$. Then $f(x, y) = f(x) + f(y)$.

3. If $f: (G, +) \to (G', \cdot)$ Then $f(x+y) = f(x) \cdot f(y)$

4. Let G be a group and N be a normal subgroup of G. Then prove $f: G \to G/N$ is defined by $f(a) = Na$ is homomorphism.

5. The homomorphism $f: G \to G/N$ is called Canonical homomorphism or Natural mapping.

6. Let $f: G \to G'$ be a homomorphism. If f is onto, then it is called an epimorphism.

7. Let $f: G \to G'$ be a homomorphism. If f is 1-1, then it is called a monomorphism.

8. If $f: G \to G'$ is an epimorphism then G' is called a homomorphic image of G.

9. A homomorphism of a group to itself is called an endomorphism.

Let $f: G \to G'$ be a homomorphism.

10. $f(e) = e'$

11. $f(a^{-1}) = (f(a))^{-1}$

12. If H is a subgroup of G, then $f(H)$ is a subgroup of G'

13. If H is normal in G, then $f(H)$ is normal in $f(G)$.

14. Let $f: G \to G'$ be a homomorphism.
Let $K = \{x / x \in G, f(x) = e'\}$ then K is called Kernal of f and is denoted by Kerf.

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29. The groups $(Z,+)$ and $(Q,+)$ are not isomorphic because $(Z,+)$ is cyclic but $(Q,+)$ is not cyclic.

30. The Kernal of a homomorphism $f: G \to G'$ is a normal subgroup of G.

31. The Kernal of the homomorphism $f:(Z,+) \to (R^*, \cdot)$ defined by $f(x) = 2^x$ is $\{0\}$.

32. The Kernal of the homomorphism $f:(Z,+) \to (Z,+)$ gn $f(x) = 2x$ is $\{0\}$.

33. The kernal of the homomorphism $f: (R^*, \cdot) \to (R^+, \cdot)$ defined by $f(x) = |x|$ is $\{1,-1\}$

34. For group $(Z_{12}, \oplus)$, the no. of generators is 4.

35. The set of all generators of group $(Z_{12}, \oplus)$ is 1, 5, 7, 11.

36. The number of automorphisms of a cyclic group of order n is $\phi(n)$.

37. In a group $b^5 = e$ and $aba^{-1} = a^2$ for some $a, b \in G$. The order of a is divisor of 10.

38. Let G be an abelian group and $O(a) = i$, $O(b) = j$ for $a, b \in G$. Let $gcd(i,j) = 1$. The $O(ab) = ij$

39. The incorrect answer from the following choices is

a) Any two groups of order two are isomorphic.

b) Any two groups of order three are isomorphic.

c) Any proper subgroup of $(Z,+)$ is isomorphic to $(Z,+)$.

d) $(Q,+) \cong (Q^*, \cdot)$

$(Q,+) \cong (Q^*, \cdot)$

15. Let G & G' be two groups. A map $f: G \to G'$ is called an isomorphism. If i) f is bijection. ii) $f(xy) = f(x) \cdot f(y)$ $\forall\ x, y \in G$.

16. If two groups G & G' are isomorphic then we write $G \cong G'$

17. Let $f: G \to G'$ be an isomorphism. If G is abelian then G' is also abelian.

18. Let $f: G \to G'$ be an isomorphism. If G is cyclic then G' is also cyclic.

19. Let G be any group. S.T. $f: G \to G$ is gn by $f(x) = x^{-1}$ is an isomorphism iff G is abelian.

20. Isomorphism is an equivalence relation among groups.

21. An isomorphism of a group G to itself is called an automorphism of G.

22. Any group G has atleast one automorphism namely, i_G.

23. The map $f: R \to R$ defined by $f(a) = a^{-1}$ is an automorphism.

24. The automorphism $\phi_a : G \to G$ defined by $\phi_a(x) = axa^{-1}$ is called an inner automorphism of the group G.

25. Let $f: G \to G'$ be a homomorphism then f is 1-1 iff Kernal $f = \{e\}$

26. Any finite cyclic group of order n is isomorphic to $(Z_n, \oplus)$

27. Any finite group is isomorphic to a group of permutation.

28. Every homomorphic image of a group G is isomorphic to some Quotient group G.