

## MINOR: COMPUTATIONAL MATHEMATICS

### Transcendental Equations

#### Q.1) Finding Real Root using Bisection Method in C Programming.

**AIM:** To write a C program to find the real root of an equation using the Bisection Method.

**Formula:**

$$x_r = \frac{a + b}{2}$$

**Programming Formula:**

$$xr = (a + b) / 2;$$

**Program:**

```
#include <stdio.h>
#include <math.h>
#include <conio.h>

float f(float x) {
    return (x * x * x - 4 * x - 9);
}

int main() {
    float a = 2, b = 3, xr, fx, fb, fxr;
    int intr = 0;
    float eps = 0.001;
    float prev1 = 0.0, prev2 = 0.0;
    int same_count = 0;
    clrscr();
    fx = f(a);
    fb = f(b);
    if (fx * fb > 0) {
        printf("Root not found in the given interval.\n");
        return 0;
    }
```

```

}

printf("\n\nIntr\t a\t b\t xr\t f(a)\t f(b)\t f(xr)\n");

printf("-----\n");

do {
    intr++;
    xr = (a + b) / 2;
    fxr = f(xr);
    fx = f(a);
    fb = f(b);
    printf("%d\t%.4f\t%.4f\t%.4f\t%.4f\t%.4f\n",
        intr, a, b, xr, fx, fb, fxr);
    if (fx * fxr < 0)
        b = xr;
    else
        a = xr;
    float xr3 = floor(xr * 1000 + 0.5) / 1000;
    if (fabs(xr3 - prev1) < 0.0001 && fabs(prev1 - prev2) < 0.0001)
        same_count++;
    else
        same_count = 0;
    prev2 = prev1;
    prev1 = xr3;
    if (same_count >= 1)
        break;
} while (fabs(fxr) > eps);

printf("\n\nApproximate root (correct up to 3 decimal places) = %.3f\n", prev1);

getch();

return 0;

}

```

**Output:**

Itr	a	b	xr	f(a)	f(b)	f(xr)
1	2.0000	3.0000	2.5000	-9.0000	6.0000	-3.3750
2	2.5000	3.0000	2.7500	-3.3750	6.0000	0.7969
3	2.5000	2.7500	2.6250	-3.3750	0.7969	-1.4121
4	2.6250	2.7500	2.6875	-1.4121	0.7969	-0.3391
5	2.6875	2.7500	2.7188	-0.3391	0.7969	0.2209
6	2.6875	2.7188	2.7031	-0.3391	0.2209	-0.0611
7	2.7031	2.7188	2.7109	-0.0611	0.2209	0.0794
8	2.7031	2.7109	2.7070	-0.0611	0.0794	0.0090
9	2.7031	2.7070	2.7051	-0.0611	0.0090	-0.0260
10	2.7051	2.7070	2.7061	-0.0260	0.0090	-0.0085
11	2.7061	2.7070	2.7065	-0.0085	0.0090	0.0003

Approximate root (correct up to 3 decimal places) = 2.707

—

**Conclusion:** The program was executed successfully.

**Q.2) Finding Real Root using Regular Falsi Method in C Programming.**

**Aim:** To write a C program to find the real root of an equation using the Regular Falsi Method.

**Formula:**

$$x_r = \frac{a \times f(b) - b \times f(a)}{f(b) - f(a)}$$

**Programming Formula:**

$$xr = (a * f(b) - b * f(a)) / (f(b) - f(a));$$

**Program:**

```
#include <stdio.h>
#include <conio.h>
#include <math.h>
float f(float x) {
    return (x * x * x - 4 * x - 9);
}
int main() {
    float a = 2, b = 3, xr, fx, fb, fxr;
    int i = 0;
    float eps = 0.001;
    float prev1 = 0.0, prev2 = 0.0;
    int same_count = 0;
    clrscr();
    fx = f(a);
    fb = f(b);
    if (fx * fb > 0) {
        printf("Root not found in the given interval.\n");
        getch();
        return 0;
    }
    printf("\nIter\t a\t b\t xr\t f(a)\t f(b)\t f(xr)\n");
    printf("-----\n");
```

```

do {
    i++;
    xr = (a * fb - b * fx) / (fb - fx);
    fxr = f(xr);
    fx = f(a);
    fb = f(b);
    printf("%d\t %.6f\t %.6f\t %.6f\t %.6f\t %.6f\t %.6f\n", i, a, b, xr, fx, fb, fxr);
    if (fx * fxr < 0)
        b = xr;
    else
        a = xr;
    float xr3 = floor(xr * 1000 + 0.5) / 1000;
    if (fabs(xr3 - prev1) < 0.0001 && fabs(prev1 - prev2) < 0.0001)
        same_count++;
    else
        same_count = 0;
    prev2 = prev1;
    prev1 = xr3;
    if (same_count >= 1)
        break;
} while (fabs(fxr) > eps);
printf("\nApproximate root (correct up to 3 decimal places) = %.3f\n", prev1);
getch();
return 0;
}

```

Output:

Itr	a	b	xr	f(a)	f(b)	f(xr)
1	2.0000	3.0000	2.5000	-9.0000	6.0000	-3.3750
2	2.5000	3.0000	2.7500	-3.3750	6.0000	0.7969
3	2.5000	2.7500	2.6250	-3.3750	0.7969	-1.4121
4	2.6250	2.7500	2.6875	-1.4121	0.7969	-0.3391
5	2.6875	2.7500	2.7188	-0.3391	0.7969	0.2209
6	2.6875	2.7188	2.7031	-0.3391	0.2209	-0.0611
7	2.7031	2.7188	2.7109	-0.0611	0.2209	0.0794
8	2.7031	2.7109	2.7070	-0.0611	0.0794	0.0090
9	2.7031	2.7070	2.7051	-0.0611	0.0090	-0.0260
10	2.7051	2.7070	2.7061	-0.0260	0.0090	-0.0085
11	2.7061	2.7070	2.7065	-0.0085	0.0090	0.0003

Approximate root (correct up to 3 decimal places) = 2.707

-

**Conclusion:** The program was executed successfully.

### Q.3) Finding Real Root using Newton–Raphson Method in C Programming

**Aim:** To write a C program to find the real root of an equation using the Newton–Raphson Method.

**Formula:**

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

**Programming Formula:**

$$x1 = x0 - (f(x0) / f1(x0));$$

**Program:**

```
#include <stdio.h>
#include <conio.h>
#include <math.h>
float f(float x) {
    return (x * x * x - 4 * x - 9);
}
float f1(float x) {
    return (3 * x * x - 4);
}
int main() {
    float x0 = 2, x1, f0, f1x;
    int i = 0;
    float eps = 0.001;
    float prev1 = 0.0, prev2 = 0.0;
    int same_count = 0;
    clrscr();
    printf("\nIter\t x0\t\t f(x0)\t\t f'(x0)\t\t x1\n");
    printf("-----\n");
    do {
        i++;
        f0 = f(x0);
        f1x = f1(x0);
```

```

x1 = x0 - (f0 / flx);
printf("%d\t %.6f\t %.6f\t %.6f\t %.6f\n", i, x0, f0, flx, x1);
float x13 = floor(x1 * 1000 + 0.5) / 1000;
if (fabs(x13 - prev1) < 0.0001 && fabs(prev1 - prev2) < 0.0001)
    same_count++;
else
    same_count = 0;
prev2 = prev1;
prev1 = x13;
if (same_count >= 1)
    break;
x0 = x1;
} while (fabs(f0) > eps);
printf("\nApproximate root (correct up to 3 decimal places) = %.3f\n", prev1);
getch();
return 0;
}

```

Output:

Iter	x0	f (x0)	f' (x0)	x1
1	2.000000	-9.000000	8.000000	3.125000
2	3.125000	9.017578	25.296875	2.768530
3	2.768530	1.145991	18.994274	2.708196
4	2.708196	0.030014	18.002983	2.706529
5	2.706529	0.000021	17.975901	2.706528

Approximate root (correct up to 3 decimal places) = 2.707

**Conclusion:** The program was executed successfully.



## Interpolation

### Q.1) Newton Forward Interpolation Table in C Programming

**Aim:** To create a Newton Forward Difference Table and interpolate a value using Newton's Forward Interpolation Formula.

**Formula:**

$$y(x) = y_0 + p\Delta y_0 + \frac{i(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \dots$$

$$p = \frac{x - x_0}{h}, h = x_0 - x_1$$

**Programming Formula:**

```
y_int = y_int + (p_term * diff[0][i]) / fact(i);
p_term = p_term * (p - i);
```

**Program:**

```
#include <stdio.h>
#include <conio.h>
float fact(int num)
{
    float f = 1;
    int i;
    for(i = 2; i <= num; i++)
        f = f * i;
    return f;
}
int main()
{
    int n = 5;
    float x[5] = {0, 1, 2, 3, 4};
    float y[5][5] = {
        {1}, {8}, {27}, {64}, {125}
```

```

};

float value = 2.5, h, p, y_int, p_term;

int i, j;

clrscr();

for(j = 1; j < n; j++)
    for(i = 0; i < n-j; i++)
        y[i][j] = y[i+1][j-1] - y[i][j-1];

printf("\nNewton Forward Difference Table:\n");
printf("x\tf(x)\ty\tDeltay\tDelta2y\tDelta3y\n");
for(i = 0; i < n; i++)
{
    printf("%.1f", x[i]);
    for(j = 0; j < n-i; j++)
        printf("\t%.2f", y[i][j]);
    printf("\n");
}

h = x[1] - x[0];
p = (value - x[0]) / h;
y_int = y[0][0];
p_term = 1;
for(i = 1; i < n; i++)
{
    p_term = p_term * (p - i + 1);
    y_int = y_int + (p_term * y[0][i]) / fact(i);
}

printf("\nValue at x = %.2f is %.4f\n", value, y_int);

getch();

return 0;
}

```

**Output:**

```
Newton Forward Difference Table:
x      f(x)    y      Deltay  Delta2y Delta3y
0.0    1.00    7.00    12.00   6.00    0.00
1.0    8.00    19.00   18.00   6.00
2.0    27.00   37.00   24.00
3.0    64.00   61.00
4.0    125.00

Value at x = 2.50 is 42.8750
-
```

**Conclusion:** The program was executed successfully.

## Q.2) Newton Backward Interpolation Table in C Programming

**Aim:** To write a C program to create a Newton Backward Difference Table and interpolate a value using Newton's Backward Interpolation Method.

**Formula:**

$$y(x) = y_n + p\Delta y_n + \frac{p(p+1)}{2!}\Delta^2 y_n + \frac{p(p+1)(p+2)}{3!}\Delta^3 y_n + \dots$$

$$p = \frac{x - x_0}{h}, h = x_0 - x_1$$

**Programming Formula:**

$$y\_int = y\_int + (p\_term * diff[n-i-1][i]) / fact(i);$$

$$p\_term = p\_term * (p + i);$$

**Program:**

```
#include <stdio.h>
#include <conio.h>
float fact(int num)
{
    float f = 1;
    int i;
    for(i = 2; i <= num; i++)
        f = f * i;
    return f;
}
int main()
{
    int n = 5;
    float x[5] = {0, 1, 2, 3, 4};
    float y[5][5] = {
        {1}, {8}, {27}, {64}, {125}
    };
    float value = 3.5, h, p, y_int, p_term;
    int i, j;
```

```

clrscr();
for(j = 1; j < n; j++)
    for(i = n-1; i >= j; i--)
        y[i][j] = y[i][j-1] - y[i-1][j-1];
printf("\nNewton Backward Difference Table:\n");
printf("x\tf(x)\tDelta y\tDelta^2 y\tDelta^3 y\tDelta^4 y\n");
for(i = 0; i < n; i++)
{
    printf("%.1f", x[i]);
    for(j = 0; j <= i; j++)
        printf("\t%.2f", y[i][j]);
    printf("\n");
}
h = x[1] - x[0];
p = (value - x[n-1]) / h;
y_int = y[n-1][0];
p_term = 1;
for(i = 1; i < n; i++)
{
    p_term = p_term * (p + i - 1);
    y_int = y_int + (p_term * y[n-1][i]) / fact(i);
}
printf("\nValue at x = %.2f is %.4f\n", value, y_int);
getch();
return 0;
}

```

**Output:**

```
Newton Backward Difference Table:
x      f(x)      Deltay      Delta2y      Delta3y      Delta4y
0.0      1.00
1.0      8.00      7.00
2.0     27.00     19.00     12.00
3.0     64.00     37.00     18.00     6.00
4.0    125.00     61.00     24.00     6.00     0.00

Value at x = 3.50 is 91.1250
```

**Conclusion:** The program was executed successfully.

## Numerical Integration

**Q.1) To find the approximate value of a definite integral using the Trapezoidal Rule.**

**Aim:** To find the approximate value of a definite integral using the Trapezoidal Rule.

**Formula:**

$$\int_a^b f(x) dx = \frac{h}{2} [y_0 + 2(y_1 + y_2 + y_3 + \dots + y_{n-1}) + y_n]$$

**Programming Formula :**

```
sum = y0 + yn;
for(i = 1; i < n; i++)
    sum += 2 * yi;
area = (h / 2) * sum;
```

**Program:**

```
#include <stdio.h>
#include <math.h>
#include <conio.h>
float f(float x) {
    return x*x;
}
int main() {
    int n = 4;
    float a = 0.0, b = 2.0;
    float h, area, sum = 0.0;
    int i;
    clrscr();
    h = (b - a) / n;
    sum = f(a) + f(b);
    for(i = 1; i < n; i++) {
        sum += 2 * f(a + i*h);
    }
```

```
    area = (h / 2) * sum;  
    printf("Approximate value of the integral: %.4f\n", area);  
    getch()  
    return 0;  
}
```

Output:



```
Approximate value of the integral: 2.7500  
-
```

**Conclusion:** The program was executed successfully.



**Q.2):To find the approximate value of a definite integral using Simpson's 1/3 Rule.**

**Aim:** To calculate the area under the curve using Simpson's 1/3 Rule.

**Formula:**

$$\int_a^b f(x)dx \approx \frac{h}{3} [y_0 + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2}) + y_n]$$

**Programming Formula:**

```
sum = y0 + yn;
for(i = 1; i < n; i++) {
    if(i % 2 == 0)
        sum += 2 * yi;
    else
        sum += 4 * yi;
}
integral = (h / 3) * sum;
```

**Program:**

```
#include <stdio.h>
#include <math.h>
#include <conio.h>
float f(float x) {
    return x*x;
}
int main() {
    int n = 4;
    float a = 0.0, b = 2.0;
    float h, area, sum = 0.0;
    int i;
    clrscr();
    h = (b - a) / n;
    sum = f(a) + f(b);
    for(i = 1; i < n; i++) {
```

```
    if(i % 2 == 0)
        sum += 2 * f(a + i*h);
    else
        sum += 4 * f(a + i*h);
}
area = (h / 3) * sum;
printf("Approximate value of the integral (Simpson 1/3): %.4f\n", area);
getch();
return 0;
}
```

**Output:**A screenshot of a terminal window with a black background. The text 'Approximate value of the integral (Simpson 1/3): 2.6667' is displayed in a light green or cyan monospaced font. Below this line, there is a single hyphen '-' character, also in the same color. The rest of the terminal area is empty.

Approximate value of the integral (Simpson 1/3): 2.6667  
-

**Conclusion:** The program was executed successfully.

**Q.3) To find the approximate value of a definite integral using Simpson's 3/8 Rule.**

**Aim:** To calculate the area under the curve using Simpson's 3/8 Rule.

**Formula:**

$$\int_a^b f(x)dx \approx \frac{3h}{8} [y_0 + 3(y_1 + y_2 + y_4 + y_5 + \dots) + 2(y_3 + y_6 + \dots) + y_n]$$

where

- $n$  is a multiple of 3
- $h = \frac{b-a}{n}$

**Programming Formula:**

```
sum = y0 + yn;
for(i = 1; i < n; i++) {
    if(i % 3 == 0)
        sum += 2 * yi;
    else
        sum += 3 * yi;
}
integral = (3*h/8) * sum;
```

**Program:**


```
#include <stdio.h>
#include <math.h>
#include <conio.h>

float f(float x) {
    return x*x;
}

int main() {
    int n = 6;
    float a = 0.0, b = 2.0;
    float h, area, sum = 0.0;
    int i;
    clrscr();
```

```
h = (b - a) / n;  
sum = f(a) + f(b);  
for(i = 1; i < n; i++) {  
    if(i % 3 == 0)  
        sum += 2 * f(a + i*h);  
    else  
        sum += 3 * f(a + i*h);  
}  
area = (3*h / 8) * sum;  
printf("Approximate value of the integral (Simpson 3/8): %.4f\n", area);  
getch();  
return 0;  
}
```

**Output:**



```
Approximate value of the integral (Simpson 3/8): 2.6667  
-
```

**Conclusion:** The program was executed successfully.

## Curve fitting

**Q.1) To find the best-fitting curve for a set of data points using the method of least squares.**

**Aim:** To determine the quadratic curve  $y = a + bx + cx^2$  that best fits the given data points.

**Formulas (Least Squares)**

$$\begin{aligned}\sum y_i &= na + b\sum x_i + c\sum x_i^2 \\ \{\sum x_i y_i &= a\sum x_i + b\sum x_i^2 + c\sum x_i^3 \\ \sum x_i^2 y_i &= a\sum x_i^2 + b\sum x_i^3 + c\sum x_i^4\end{aligned}$$

**Programming Formula :**

```
sum_x = sum_x2 = sum_x3 = sum_x4 = 0;
```

```
sum_y = sum_xy = sum_x2y = 0;
```

```
for(i=0; i<n; i++){
```

```
    sum_x += x[i];
```

```
    sum_x2 += x[i]*x[i];
```

```
    sum_x3 += x[i]*x[i]*x[i];
```

```
    sum_x4 += x[i]*x[i]*x[i]*x[i];
```

```
    sum_y += y[i];
```

```
    sum_xy += x[i]*y[i];
```

```
    sum_x2y += x[i]*x[i]*y[i];
```

```
}
```

```
a = Da / D;
```

```
b = Db / D;
```

```
c = Dc / D;
```

**Program:**

```
#include <stdio.h>
```

```
#include <conio.h>
```

```
int main() {
```

```
    clrscr();
```

```
    int n = 5;
```

```

float x[] = {1, 2, 3, 4, 5};
float y[] = {2.2, 2.8, 3.6, 4.5, 5.1};
float sum_x=0, sum_x2=0, sum_x3=0, sum_x4=0;
float sum_y=0, sum_xy=0, sum_x2y=0;
int i;
for(i=0; i<n; i++){
    sum_x += x[i];
    sum_x2 += x[i]*x[i];
    sum_x3 += x[i]*x[i]*x[i];
    sum_x4 += x[i]*x[i]*x[i]*x[i];
    sum_y += y[i];
    sum_xy += x[i]*y[i];
    sum_x2y += x[i]*x[i]*y[i];
}

float D = n*(sum_x2*sum_x4 - sum_x3*sum_x3) - sum_x*(sum_x*sum_x4 -
sum_x2*sum_x3) + sum_x2*(sum_x*sum_x3 - sum_x2*sum_x2);

float Da = sum_y*(sum_x2*sum_x4 - sum_x3*sum_x3) - sum_x*(sum_xy*sum_x4 -
sum_x2y*sum_x3) + sum_x2*(sum_xy*sum_x3 - sum_x2y*sum_x2);

float Db = n*(sum_xy*sum_x4 - sum_x2y*sum_x3) - sum_y*(sum_x*sum_x4 -
sum_x2*sum_x3) + sum_x2*(sum_x*sum_x2y - sum_y*sum_x2);

float Dc = n*(sum_x2*sum_x2y - sum_x3*sum_xy) - sum_x*(sum_x*sum_x2y -
sum_x3*sum_y) + sum_y*(sum_x*sum_x3 - sum_x2*sum_x2);

float a = Da / D;
float b = Db / D;
float c = Dc / D;

printf("Quadratic curve fitting equation: y = %.4f + %.4fx + %.4fx^2\n", a, b, c);
getch();
return 0;
}

```

**Output:**

```
Quadratic curve fitting equation:  $y = 1.4400 + 190.4178x + 14.5679x^2$ 
```

**Conclusion:** The program was executed successfully.

**Q.1) Runge–Kutta 4th Order Method in C Programming.**

**Aim:** To solve the differential equation  $y' = x + y$  using the Fourth Order Runge–Kutta Method and compute the values of  $y$  for the given step size.

**Formula:**

$$\begin{aligned}k_1 &= h f(x_0, y_0) \\k_2 &= h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) \\k_3 &= h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) \\k_4 &= h f(x_0 + h, y_0 + k_3)\end{aligned}$$

Weighted average slope:

$$k = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$$

Final update formula:

$$\begin{aligned}y_1 &= y_0 + k \\x_1 &= x_0 + h\end{aligned}$$

**Programming Formula:**

$k1 = H * f(x, y);$

$k2 = H * f(x + H/2, y + k1/2);$

$k3 = H * f(x + H/2, y + k2/2);$

$k4 = H * f(x + H, y + k3);$

$kavg = (k1 + 2*k2 + 2*k3 + k4) / 6;$

$y = y + kavg;$

$x = x + H;$

**Program:**

```
#include <stdio.h>
```

```
#include <math.h>
```

```
#include <conio.h>
```

```
double f(double x, double y) {
```



```

    return x + y;
}

void runge_kutta_4th_order_solve() {
    const double X0 = 0.0;
    const double Y0 = 1.0;
    const double H = 0.1;
    const int N = 5;
    double x = X0, y = Y0;
    double k1, k2, k3, k4;
    int i;
    clrscr();
    printf("---- RK4 Method ----\n");
    printf("y' = x + y , h=%.1f\n", H);
    printf("Step | x | y | k1 | k2 | k3 | k4 | kavg\n");
    printf("-----\n");
    printf("0 | %7.4f | %7.4f\n", x, y);
    for (i = 1; i <= N; i++) {
        k1 = H * f(x, y);
        k2 = H * f(x + H/2, y + k1/2);
        k3 = H * f(x + H/2, y + k2/2);
        k4 = H * f(x + H, y + k3);
        double kavg = (k1 + 2*k2 + 2*k3 + k4) / 6.0;
        y = y + kavg;
        x = x + H;
        printf("%-4d | %7.4f | %7.4f | %7.4f | %7.4f | %7.4f | %7.4f\n",
            i, x, y, k1, k2, k3, k4, kavg);
    }

    printf("-----\n");
    printf("Final y(%.1f) = %.6f", x, y);
}

```

```

}

int main() {
    runge_kutta_4th_order_solve();
    getch();
    return 0;
}

```

**Code:**

```

---- Runge-Kutta Method ----
y' = x + y , h=0.1
Step |  x   |  y   |  k1  |  k2  |  k3  |  k4  |  kavg
-----|-----|-----|-----|-----|-----|-----|-----
0    | 0.0000 | 1.0000 | 0.1000 | 0.1100 | 0.1105 | 0.1211 | 0.1103
1    | 0.1000 | 1.1103 | 0.1210 | 0.1321 | 0.1326 | 0.1443 | 0.1325
2    | 0.2000 | 1.2428 | 0.1443 | 0.1565 | 0.1571 | 0.1700 | 0.1569
3    | 0.3000 | 1.3997 | 0.1700 | 0.1835 | 0.1841 | 0.1984 | 0.1839
4    | 0.4000 | 1.5836 | 0.1984 | 0.2133 | 0.2140 | 0.2298 | 0.2138
5    | 0.5000 | 1.7974 | 0.1984 | 0.2133 | 0.2140 | 0.2298 | 0.2138
-----|-----|-----|-----|-----|-----|-----|-----
Final y(0.5) = 1.797441

```

**Conclusion:** The program was executed successfully.

## Solution Of Simultaneous Algebraic Equations

### Q.1) Gauss Elimination Method.

**Aim:** To write a C program to find the solution of **simultaneous algebraic equations** using the **Gauss Elimination Method**.

#### Formula:

For a system of 3 equations:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned}$$

Row transformation used:

$$R_j \rightarrow R_j - \left(\frac{a_{ji}}{a_{ii}}\right)R_i$$

#### Programming formula:

```
ratio = a[j][i] / a[i][i];
a[j][k] = a[j][k] - ratio * a[i][k];
```

#### Program:

```
#include <stdio.h>
#include <conio.h>
#include <math.h>

int main()
{
    float a[10][11] = {
        { 5, -3, 2, 12 },
        { -4, 6, 1, -7 },
        { 3, 2, -5, 10 }
    };

    float x[10], ratio, temp;
    int n = 3;
```

```

int i, j, k, max_row;
clrscr();
printf("\nSolving the augmented matrix:\n");
for(i = 0; i < n; i++)
{
    for(j = 0; j <= n; j++)
    {
        printf("%0.0f ", a[i][j]);
    }
    printf("\n");
}
for(i = 0; i < n; i++)
{
    max_row = i;
    for(k = i + 1; k < n; k++)
    {
        if(fabs(a[k][i]) > fabs(a[max_row][i]))
            max_row = k;
    }
    for(k = 0; k <= n; k++)
    {
        temp = a[i][k];
        a[i][k] = a[max_row][k];
        a[max_row][k] = temp;
    }
    if(a[i][i] == 0.0)
    {
        printf("\nMathematical Error: No unique solution\n");
        getch();
        return 0;
    }
}

```

```
}  
for(j = 0; j < n; j++)  
{  
    if(i != j)  
    {  
        ratio = a[j][i] / a[i][i];  
        for(k = 0; k <= n; k++)  
        {  
            a[j][k] = a[j][k] - ratio * a[i][k];  
        }  
    }  
}  
}  
for(i = 0; i < n; i++)  
{  
    x[i] = a[i][n] / a[i][i];  
}  
printf("\nThe solution is:\n");  
for(i = 0; i < n; i++)  
{  
    printf("x%d = %0.3f\n", i+1, x[i]);  
}  
  
getch();  
return 0;  
}
```

**Output:**

```
Solving the augmented matrix:  
5 -3 2 12  
-4 6 1 -7  
3 2 -5 10
```

```
The solution is:  
x1 = 2.839  
x2 = 0.727  
x3 = -0.006  
-
```

**Conclusion:** The program was executed successfully.

## Numerical solutions of first and second order differential equation

### Q.1) Euler's Method in C Programming.

**Aim:** To solve the differential equation  $y' = x + y$  using Standard Euler's Method and compute the approximate values of  $y$  for a given step size.

**Formula:**

$$y_{n+1} = y_n + h \cdot f(x_n, y_n)$$

**Programming Formula:**

slope = f(x, y);

y = y + STEP\_SIZE \* slope;

x = x + STEP\_SIZE;

**Program:**

```
#include <stdio.h>
```

```
#include <math.h>
```

```
#include <conio.h>
```

```
double f(double x, double y) {
    return x + y;
}
```

```
void eulers_method_solve() {
```

```
    const double X_INITIAL = 0.0;
```

```
    const double Y_INITIAL = 1.0;
```

```
    const double STEP_SIZE = 0.1;
```

```
    const int N_STEPS = 5;
```

```
    double x = X_INITIAL;
```

```
    double y = Y_INITIAL;
```

```
    double slope;
```

```
    int i;
```

```
    clrscr();
```

```
    printf("--- Standard Euler's Method (y' = x + y) ---\n");
```

```
    printf("Initial Condition: y(%.1f) = %.1f, Step Size h = %.1f\n", X_INITIAL, Y_INITIAL, STEP_SIZE);
```

```

printf("Step |   x   |   y   | f(x, y)\n");
printf("-----\n");
printf(" 0 | %10.6f | %10.6f\n", x, y);
for (i = 1; i <= N_STEPS; i++) {
    slope = f(x, y);
    y = y + STEP_SIZE * slope;
    x = x + STEP_SIZE;
    printf(" %d | %10.6f | %10.6f | %10.6f\n", i, x, y, slope);
}
printf("-----\n");
printf("Final Approximation at x=%.1f: y = %.6f\n", x, y);
}

int main() {
    eulers_method_solve();
    getch();
    return 0;
}

```

### Output:

```

--- Standard Euler's Method (y' = x + y) ---
Initial Condition: y(0.0) = 1.0, Step Size h = 0.1
Step |   x   |   y   | f(x, y)
-----
 0 | 0.000000 | 1.000000 |
 1 | 0.100000 | 1.100000 | 1.000000
 2 | 0.200000 | 1.220000 | 1.200000
 3 | 0.300000 | 1.362000 | 1.420000
 4 | 0.400000 | 1.528200 | 1.662000
 5 | 0.500000 | 1.721020 | 1.928200
-----
Final Approximation at x=0.5: y = 1.721020
-

```

**Conclusion:** The program was executed successfully.



## Q.2) Modified Euler's Method in C Programming

**Aim:** To solve the differential equation  $y' = x + y$  using the Modified Euler's Method and compute the approximate values of  $y$  for each step.

**Formula:**

$$y_{n+1} = y_n + \frac{h}{2} [f(x_n, y_n) + f(x_{n+1}, y_p)]$$

$$x_{n+1} = x_n + h$$

**Programming Formula:**

```
k1 = f(x, y);
```

```
y_predictor = y + H * k1;
```

```
k2 = f(x_next, y_predictor);
```

```
y = y + (H / 2.0) * (k1 + k2);
```

```
x = x_next;
```

**Program:**

```
#include <stdio.h>
```

```
#include <math.h>
```

```
#include <conio.h>
```

```
double f(double x, double y) {
```

```
    return x + y;
```

```
}
```

```
void modified_eulers_method_solve() {
```

```
    const double X_INITIAL = 0.0;
```

```
    const double Y_INITIAL = 1.0;
```

```
    const double H = 0.1;
```

```
    const int N_STEPS = 5;
```

```
    double x = X_INITIAL;
```

```
    double y = Y_INITIAL;
```

```

double x_next;

double y_predictor;

double k1, k2;

int i;

clrscr();

printf("--- Modified Euler's Method (y' = x + y) ---\n");

printf("Initial Condition: y(%.1f) = %.1f, Step Size h = %.1f\n", X_INITIAL, Y_INITIAL,
H);

printf("Step |   x   |   y   | Predictor y | Avg. Slope\n");
printf("-----\n");
printf(" 0 | %10.6f | %10.6f\n", x, y);
for (i = 1; i <= N_STEPS; i++) {
    k1 = f(x, y);
    x_next = x + H;
    y_predictor = y + H * k1;
    k2 = f(x_next, y_predictor);
    y = y + (H / 2.0) * (k1 + k2);
    x = x_next;
    printf(" %d | %10.6f | %10.6f | %10.6f | %10.6f\n",
        i, x, y, y_predictor, (k1 + k2) / 2.0);
}
printf("-----\n");
printf("Final Approximation at x=%.1f: y = %.6f\n", x, y);
}

int main() {
    modified_eulers_method_solve();

    getch();

    return 0;
}

```

**Output:**

```

--- Modified Euler's Method ( $y' = x + y$ ) ---
Initial Condition:  $y(0.0) = 1.0$ , Step Size  $h = 0.1$ 
Step |      x      |      y      | Predictor y | Avg. Slope
-----|-----|-----|-----|-----
0  | 0.000000 | 1.000000 | 1.100000 | 1.100000
1  | 0.100000 | 1.110000 | 1.231000 | 1.320500
2  | 0.200000 | 1.242050 | 1.386255 | 1.564153
3  | 0.300000 | 1.398465 | 1.568312 | 1.833389
4  | 0.400000 | 1.581804 | 1.779985 | 2.130894
5  | 0.500000 | 1.794894 | 1.779985 | 2.130894
-----|-----|-----|-----|-----
Final Approximation at  $x=0.5$ :  $y = 1.794894$ 
-

```

**Conclusion:** The program was executed successfully.

### Q. 3) Taylor Series Method

**Aim:** To solve the differential equation  $y' = x + y$  using the Taylor Series Method with given initial conditions.

**Formula:**

$$y_{i+1} = y_i + h y'_i + \frac{h^2}{2} y''_i$$

$$y'_i = f(x_i, y_i) = x_i + y_i$$

$$y''_i = f'(x_i, y_i) = 1 + x_i + y_i$$

**Programming Formula:**

```
y_prime = f(x, y);
y_double_prime = f_prime(x, y);
y = y + h * y_prime + (h * h / 2) * y_double_prime;
```

**Program:**

```
#include <stdio.h>
#include <math.h>
#include <conio.h>
double f(double x, double y) {
    return x + y;
}
double f_prime(double x, double y) {
    return 1.0 + x + y;
}
void taylor_series_solve() {
    const double X_INITIAL = 0.0;
    const double Y_INITIAL = 1.0;
    const double H = 0.1;
    const int N_STEPS = 5;
    double x = X_INITIAL;
    double y = Y_INITIAL;
    double y_prime;
```

```

double y_double_prime;

int i;

clrscr();

printf("--- Taylor Series Method (y' = x + y) ---\n");

printf("Initial Condition: y(%.1f) = %.1f, Step Size h = %.1f\n", X_INITIAL, Y_INITIAL,
H);

printf("Step |   x   |   y   |   y' (f)   |   y'' (f) \n");

printf("-----\n");

printf("0 | %10.6f | %10.6f | \n", x, y);

for (i = 1; i <= N_STEPS; i++) {
    y_prime = f(x, y);
    y_double_prime = f_prime(x, y);
    y = y + H * y_prime + (H * H / 2.0) * y_double_prime;
    x = x + H;
    printf(" %d | %10.6f | %10.6f | %10.6f | %10.6f\n", i, x, y, y_prime, y_double_prime);
}

printf("-----\n");

printf("Final Approximation at x=%.1f: y = %.6f\n", x, y);

}

int main() {
    taylor_series_solve();
    getch();
    return 0;
}

```

Output:

```

--- Taylor Series Method ( $y' = x + y$ ) ---
Initial Condition:  $y(0.0) = 1.0$ , Step Size  $h = 0.1$ 
Step |      x      |      y      |       $y' (f)$       |       $y'' (f')$ 
-----|-----|-----|-----|-----
0  |  0.000000  |  1.000000  |          |          |
1  |  0.100000  |  1.110000  |  1.000000 |  2.000000 |
2  |  0.200000  |  1.242050  |  1.210000 |  2.210000 |
3  |  0.300000  |  1.398465  |  1.442050 |  2.442050 |
4  |  0.400000  |  1.581804  |  1.698465 |  2.698465 |
5  |  0.500000  |  1.794894  |  1.981804 |  2.981804 |
-----|-----|-----|-----|-----
Final Approximation at  $x=0.5$ :  $y = 1.794894$ 

```

**Conclusion:** The program was executed successfully.

## Transportation problem

**Q.1) Transportation Problem** using the **Least Cost Method**.

**Aim:**

To write a C program to find the **initial feasible solution** of a **Transportation Problem** using the **Least Cost Method**, and to compute the **minimum transportation cost**.

**Formula:**

### 1. Least Cost Selection Rule

Choose the cell with:

$$\text{Minimum Cost} = \min (c_{ij})$$

### Allocation Rule

$$x_{ij} = \min (\text{Supply}_i, \text{Demand}_j)$$

### Update Supply & Demand

$$\begin{aligned} \text{Supply}_i &= \text{Supply}_i - x_{ij} \\ \text{Demand}_j &= \text{Demand}_j - x_{ij} \end{aligned}$$

### Total Transportation Cost

$$\text{Total Cost} = \sum (x_{ij} \times c_{ij})$$

### Programming Formula:

```
min_cost = temp_cost[i][j];
alloc = (supply_left[min_i] < demand_left[min_j]) ? supply_left[min_i] :
demand_left[min_j];
supply_left[min_i] -= alloc;
demand_left[min_j] -= alloc;
total_cost += alloc * cost[min_i][min_j];
```

### Program:

```
#include <stdio.h>
#include <conio.h>

int main() {
    int cost[3][4] = {
```

```

    { 4, 7, 6, 3 },
    { 8, 5, 9, 2 },
    { 1, 4, 3, 8 }
};

int supply[3] = { 40, 35, 45 };
int demand[4] = { 30, 25, 50, 15 };
int allocation[3][4];
int i, j;
int min_cost, min_i, min_j, alloc;
long total_cost = 0;
int items_to_alloc = 0;
int supply_left[3];
int demand_left[4];
int temp_cost[3][4];
clrscr();
printf("=====TRANSPORTATION PROBLEM=====\\n\\n");
printf("Cost Matrix:\\n");
for (i = 0; i < 3; i++) {
    for (j = 0; j < 4; j++) {
        printf("%4d", cost[i][j]);
    }
    printf(" | Supply: %d\\n", supply[i]);
}
printf("\\nDemand:");
for (j = 0; j < 4; j++) {
    printf("%4d", demand[j]);
}
printf("\\n\\n");
for (i = 0; i < 3; i++) {
    supply_left[i] = supply[i];

```



```

items_to_alloc += supply[i];
for (j = 0; j < 4; j++) {
    allocation[i][j] = 0;
    temp_cost[i][j] = cost[i][j];
}
}
for (j = 0; j < 4; j++) {
    demand_left[j] = demand[j];
}
while (items_to_alloc > 0) {
    min_cost = 9999;
    min_i = -1;
    min_j = -1;
    for (i = 0; i < 3; i++) {
        for (j = 0; j < 4; j++) {
            if (temp_cost[i][j] < min_cost) {
                min_cost = temp_cost[i][j];
                min_i = i;
                min_j = j;
            }
        }
    }
    if (supply_left[min_i] < demand_left[min_j]) {
        alloc = supply_left[min_i];
    } else {
        alloc = demand_left[min_j];
    }
    allocation[min_i][min_j] = alloc;
    total_cost += (long)alloc * cost[min_i][min_j];
    items_to_alloc -= alloc;
}

```

```

supply_left[min_i] -= alloc;
demand_left[min_j] -= alloc;
if (supply_left[min_i] == 0) {
    for (j = 0; j < 4; j++) {
        temp_cost[min_i][j] = 9999;
    }
}
if (demand_left[min_j] == 0) {
    for (i = 0; i < 3; i++) {
        temp_cost[i][min_j] = 9999;
    }
}
}
printf("====LEAST COST METHOD SOLUTION====\n\n");
printf("Allocation Matrix:\n");
for (i = 0; i < 3; i++) {
    for (j = 0; j < 4; j++) {
        if (allocation[i][j] > 0) {
            printf("%4d", allocation[i][j]);
        } else {
            printf("%4s", "-");
        }
    }
    printf("\n");
}
printf("\nTotal Transportation Cost: %ld", total_cost);
getch();
return 0;
}

```

**Output:**

```
=====TRANSPORTATION PROBLEM=====

Cost Matrix:
 4  7  6  3  | Supply: 40
 8  5  9  2  | Supply: 35
 1  4  3  8  | Supply: 45

Demand: 30 25 50 15

=====LEAST COST METHOD SOLUTION=====

Allocation Matrix:
  -  5 35  -
  - 20  - 15
30  - 15  -

Total Transportation Cost: 450
```

**Conclusion:** The program was executed successfully.