Queensland University of Technology

Black Scholes Model

# Extracted from *C++ for Financial Engineers*

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# 1 Rationale:

The purpose of this document to to gain an insight into how the Black Scholes Model functions, and how a C++ model can be developed to price call and put options using the Black Scholes Method.

To give some context, this report will be adapted from the works of Daniel J. Duffy, in his book *Introduction to C++ for Financial Engineers*. The goal of this book was to introduce the reader to the C++ programming language and its applications to the field of quantitative finance. There are three main parts to the book, which are detailed below:

1. C++ syntax
2. C++ design patterns, data structures, and libraries
3. C++ quantitative finance applications

# 2 Introduction

# 3 Black Scholes

We can derive the Black Scholes PDE using the delta hedging argument.

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0 \tag{1}$$

The key assumptions of this model are as follows:
The price of a stock follows a geometric brownian motion

$$dS_t = \mu S_t dt + \sigma S_t dw_t \tag{2}$$

Whereby, $\mu$ $\sigma$ are constants
The price of an option is a function of the following:
$V = V(T - t, S_t; r, \sigma, k)$ Whereby,

$$\begin{aligned} T - t &= Time to maturity of option \\ S_t &= Stock price \\ r &= Risk free rate \\ \sigma &= Volatility \\ K &= Strike price on option \end{aligned}$$

Given that $\sigma, r, K$ are constant, we can then rewrite;

$$V = V(T-t, S_t) \iff V_t \tag{3}$$

The value of a bank account amount has no stochastic property, and can therefore be written as:

$$dB = rBdt \tag{4}$$

Whereby,

$$r = riskfreerate.$$
$$B = Bankaccount.$$

Using the above assumptions, we can derive the Black Scholes PDE equation.

We know through Ito's Lemma, the follow is true;

$$dV = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial S}dS + \frac{1}{2}\frac{\partial^2 V}{\partial s^2}dS^2 \tag{5}$$

This asserts that the derivative of a stochastic function is equal to the above formula. We can use the above to substitute into Ito's Lemma Taking the 2nd order differential of $dS$ with respect to time;

$$\frac{dS^2}{dt} = \frac{dS}{dt}[\mu S dt] + \frac{dS}{dt}[\sigma S dW]$$

we can then use the product rule,

$$\frac{dS}{dt} = \sigma^2 S^2 dt$$

We ca then substitute into Ito's Lemma to get;