Tut 2

Section 1.1

1

a

- $\varphi' = \frac{\partial f}{\partial x_1} \quad \psi' = \frac{\partial g}{\partial x_1}$

b

- $\frac{\partial(af+bg)}{\partial x_1} = (a\varphi + b\psi)'$ **(Taking parital derivative with repect to x1 implease treat all other arguments as constant which means tha f and φ and equivelant and g and ψ are equivelant)**
- $= a\varphi' + b\psi'$ **(Result for single veriable derivatives)**
- $= a\frac{\partial f}{x_1} + b\frac{\partial(g)}{x_1}$ **(1.a)**

c

- $\frac{\partial fg}{\partial x_1} = (\varphi\psi)'$ **(Partial derivate of mulitveriate function and derivative of single veriable function are equivelant)**
- $= \varphi'\psi + \varphi\psi'$ **(Product rule for single veriable functions)**
- $= g\frac{\partial f}{\partial x_1} + f\frac{\partial g}{\partial x_1}$ **(1.a , communitivity of addition and multipaction of reals)**

2

a

- $\begin{pmatrix} t \\ t^2 \end{pmatrix}$ and $\begin{pmatrix} t^2 \\ t \end{pmatrix}$
- Intersections at $t = \{0, 1\}$
- Finding the derivates
  - $\begin{pmatrix} 1 \\ 2t \end{pmatrix}$ and $\begin{pmatrix} 2t \\ 1 \end{pmatrix}$
- Checking orthoganality at intersections
  - at t = 0
    * $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0$
    * orthognal
  - at t = 1

        * $\begin{pmatrix}1\\2\end{pmatrix} \cdot \begin{pmatrix}2\\1\end{pmatrix} = 4 \neq 0$
        * Not orthoganal

b

- $\begin{pmatrix}t\\t^2\end{pmatrix}$ and $\begin{pmatrix}s\\\frac{1}{2}-s^2\end{pmatrix}$
- intersections
    - same x coordinate $t = s$
    - same y cordinate $t^2 = \frac{1}{2} - t^2$ so $t = \pm\frac{1}{2}$
- Finding derivatives
    - $\begin{pmatrix}1\\2t\end{pmatrix}$ and $\begin{pmatrix}1\\-2t\end{pmatrix}$
- Checking orthoganality of vectors.
    - at $-\frac{1}{2}$
    - $\begin{pmatrix}1\\-1\end{pmatrix} \cdot \begin{pmatrix}1\\1\end{pmatrix} = 0$
    - Orthoganal
    - at $\frac{1}{2}$
        * $\begin{pmatrix}1\\1\end{pmatrix} \cdot \begin{pmatrix}1\\-1\end{pmatrix} = 0$
        * Orthoganal

6

a

7

b

- $(a\vec{F} + b\vec{G})' = \left(a\begin{pmatrix}f_1\\\vdots\\f_m\end{pmatrix} + b\begin{pmatrix}g_1\\\vdots\\g_m\end{pmatrix}\right)'$
- $= \left(\begin{pmatrix}af_1\\\vdots\\af_m\end{pmatrix} + \begin{pmatrix}bg_1\\\vdots\\bg_m\end{pmatrix}\right)'$
- $= \begin{pmatrix}af_1 + bg_1\\\vdots\\af_m + bg_m\end{pmatrix}'$

- $= \begin{pmatrix} \frac{\partial af_1+bg_1}{\partial x_1} & \cdots & \frac{\partial af_1+bg_1}{\partial x_n} \\ & \vdots & \\ \frac{\partial af_m+bg_m}{\partial x_1} & \cdots & \frac{\partial af_m+bg_m}{\partial x_m} \end{pmatrix}$
- $= \begin{pmatrix} \frac{\partial af_1}{\partial x_1} + \frac{\partial bg_1}{\partial x_1} & \cdots & \frac{\partial af_1}{\partial x_n} + \frac{\partial bg_1}{\partial x_n} \\ & \vdots & \\ \frac{\partial af_m}{\partial x_1} + \frac{\partial bf_m}{\partial x_1} & \cdots & \frac{\partial af_m}{\partial x_n} + \frac{\partial bg_m}{\partial x_n} \end{pmatrix}$
- $= \begin{pmatrix} \frac{\partial af_1}{\partial x_1} & \cdots & \frac{\partial af_1}{\partial x_n} \\ & \vdots & \\ \frac{\partial af_m}{\partial x_1} & \cdots & \frac{\partial af_n}{\partial x_n} \end{pmatrix} + \begin{pmatrix} \frac{\partial bg_1}{\partial x_1} & \cdots & \frac{\partial bg_1}{\partial x_n} \\ & \vdots & \\ \frac{\partial bg_m}{\partial x_1} & \cdots & \frac{\partial bg_m}{\partial x_n} \end{pmatrix}$
- $= \begin{pmatrix} a\frac{\partial f_1}{\partial x_1} & \cdots & a\frac{\partial f_1}{\partial x_n} \\ & \vdots & \\ a\frac{\partial f_m}{\partial x_1} & \cdots & a\frac{\partial f_n}{\partial x_n} \end{pmatrix} + \begin{pmatrix} b\frac{\partial g_1}{\partial x_1} & \cdots & b\frac{\partial g_1}{\partial x_n} \\ & \vdots & \\ b\frac{\partial g_m}{\partial x_1} & \cdots & b\frac{\partial g_m}{\partial x_n} \end{pmatrix}$
- $= a\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ & \vdots & \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix} + b\begin{pmatrix} \frac{\partial g_1}{\partial x_1} & \cdots & \frac{\partial g_1}{\partial x_n} \\ & \vdots & \\ \frac{\partial g_m}{\partial x_1} & \cdots & \frac{\partial g_m}{\partial x_n} \end{pmatrix}$
- $= a\vec{F}' + b\vec{G}'$