Introductory Analysis

Proof Techniques

Proof of uniquesness

Assume two seperate elements with the requiered property and than proove that they must be equal to eachother.

Proof by contradiction

Assume that a false stament is true and show that this results in a contradiction

Reals 1

Axioms of the reals

The real numbers are a field which also has ordered and diedekin complete

Field A set F of with two opperators $+$ and $\cdot$ such that $\forall f_1, f_2, f_3 \in F$

- Addition
  - **(A1)** $(f_1 + f_2) + f_3 = f_1 + (f_2 + f_3)$ Addition is associative
  - **(A2)** $f_1 + f_2 = f_1 + f_2$ Addition is comutitive
  - **(A3)** $\exists 0 | f_1 + 0 = f_1$ Indentity of addition
  - **(A3)** $\exists - f_1 | f_1 + (-f_1) = 0$
- Multiplication
  - **(M1)** $(f_1 f_2) f_3 = f_1 (f_2 f_3)$ Multiplication is associative
  - **(M2)** $f_1 f_2 = f_1 f_2$ Multiplication is comutitive
  - **(M3)** $\exists 1 \neq 0 | f_1 \dot{1} = f_1$ Indentity of addition
  - **(M4)** $(\forall f_1 \in F | f_1 \neq 0) \exists f_1^{-1} | f_1 (f_1^{-1}) = 1$
- Distrabution
  - **(D1)** $f_1(f_2 + f_3) = f_1 f_2 + f_1 f_2$ multiplication is distributive over addition

  Axioms of comparison

- Order
  - **(O1)** The trichotomy property of reals.
    - $(\forall r \in \mathbb{R})$ exactly one is true
    - $a > 0 \quad a = 0 \quad a < 0$

Let $r_1 r_2 \in \mathbb{R}$ then r_1 is called larger than r_2 written $r_1 > r_2$ if $r_1 - r_2 > 0$

- **(O2)** $a > 0 \wedge b > 0 \quad \rightarrow \quad a + b > 0$
- **(O3)** $a > 0 \wedge b > 0 \rightarrow ab > 0$

Diedekin completeness axiom

- Completeness
  - **(C)** All bounded sets have superium and infamium

Find the defintion of a monotonic transformation

Collararies

b) $a < b \wedge b < c \rightarrow a < c$ transitivty

sort of

- $r_1 \geq b \Leftrightarrow a > b \wedge a = b$
- $a \leq b \Leftrightarrow a = b \wedge a < b$

Proof

- $(S_s + \epsilon)^2 < a$

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