Vector Analysis

`Check sakia for proof of this thoerem`

THEOREM 1.1.4

Proof[1]

$F, G : \mathbb{R}^n \to \mathbb{R}^m, g : \mathbb{R}^n \to \mathbb{R}$

1. $\vec{F}(\vec{x}) \cdot \vec{G}(\vec{x}))' = \vec{F}^T(\vec{x}) \cdot \vec{G}'(\vec{x}) + \vec{G}^T(\vec{x}) \cdot \vec{F}'(\vec{x})$ Product rule (check with 1 by 1 matrix)

2. $(g(x)F(x))' = g(x)F'(x) + F(x) \cdot g(x)$

DIRIVATIVES OF VECTOR FUNCTIONS

$\vec{F}(a+h) = \vec{F}(\vec{a}) + \vec{F}'(a) \cdot h$ for small enough h[2]

1. $F(x) = \begin{pmatrix} f_1(x) \\ f_2(x) \\ ... \\ f_n(x) \end{pmatrix}$

2. $f_1(a+h) = f1(a1+h1, a_2+h_2) + \frac{\partial F}{\partial x_1}(a_1, a_2+h_2, ..., a_n+h_n)h_1 + O(h_1^2)$[3]

   Now use Tayloar around $h_2 O(2)$ is ignored

3. $F(a_1, a_2 a_3 ... a_n) + \frac{\partial f_1}{\partial x_1}(a1, a2, a3+h3...1n)h1 + \frac{\partial f_2}{\partial x_2}(a1, a2, a3+h3, ...a_n + h_n) + \frac{\partial f_n}{\partial x_n}(a1...an)hn$

$F(a+h) = [F1(a+h)...]^T = [f1(a) + f1(a) \cdot h...]^t + O(h^2) = F(a) + F'(a) \cdot h$
$F(a) = [$
$frac\partial F1x_1(a)][h1, h2]^t$ Gradient : The gradient of a function $f : \mathbb{R}^n \to \mathbb{R}$ is written ∇

$$\nabla f \mp][$$

[1] product rule
[2] from taylor expansion with small $|h|$
[3] Taylor