Partition

Definition

A parition for a closed interval $[a, b]$ is a set of point (x_0 to x_n) such that:

$$a = x_0, \quad x_k < x_{k+1} \quad \text{and} \quad b = x_n$$

Partition subinterval

Definition

A sub interval of a partition(x_0 to x_n) is $x_{k+1} - x_k$

Norm of a partition

Definition

The norm of a partition P is the largest subinterval of a partition. It is donated by $||P||$

Reimans sum

Definition

The Reiman sum for a function f on a closed interval $[a, b]$ is

$$\sum_{k=1}^{n} f(c_k)\Delta_k$$

where: The set of Δs is all subintervals of a partition P of $[a, b]$ and

$$c_k \in \left[\sum_{j=1}^{k-1} \Delta_k, \sum_{j=1}^{k} \Delta_k\right]$$

Definate intergral

Definition

The definate intergral of a function f over a closed interval $[a, b]$ is:

$$\lim_{||P|| \to 0} \sum_{k=0}^{n} f(c_k)\Delta_k$$

where: $\sum_{k=0}^{n} f(c_k)\Delta_k$ is any reimmann sum of f on interval $[a, b]$
It is donaoted by

$$\int_a^b f(x)dx$$

Continues functions are intergratable

Result

Continues functions are intergratable

Piecewise continues function

Definition

A function is peicewise continues if it has a finite number of jump disconitiuities

Piecewise continues functions are itergratable

Result

Piecewise continues functions are intergratable

Norm of a partition

Result

If two rieman sums of a function f over and closed interval $[a, b]$ are not equal, f is not intergratable over $[a, b]$

Zero width interval

Result

$$\int_a^b f(x)dx \overbrace{=}^{\text{Intergal}} -\int_b^a f(x)dx \quad \text{where } f \text{ is integratable of interval}[a, b]$$

Intergal Could not find a file

Zero width interval

Result

$$\int_a^a f(x)dx \overbrace{=}^{\text{Intergal}} 0 \quad \text{where } f \text{ is integratable of interval}[a, b]$$

Intergal Could not find a file

Intergration constant

Result

$$\int_a^b kf(x)dx = k\int_a^b f(x)dx \quad \text{for any constant k}$$

Integration sum and difference

Result

$$\int_a^b f(x) \pm g(x)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

Additivity

Result

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

Max min inequality

Result

If f has maximum value $f_{\max}$ and minimium value $f_{\min}$

$$f_min(b-a) \leq \int_a^b f(x)dx \leq f_max(b-a)$$

Domination

Result

$$f(x) \geq g(x) \forall x \in [a, b] \Rightarrow \int_a^b f(x)dx \geq \int_a^b g(x)dx$$

Area under a graph

Definition

The area A under a curve $y = f(x)$ over $[a, b]$ is equal to

$$\int_a^b f(x)dx$$

Mean value theorem for definite intergrals

if f is continues over $[a, b]$ than at some point c in $[a, b$

$$f(c) = \frac{1}{b-a} \int_a^b f(x)dx$$

Antideivites intergral

Definition

If $F(x) = \displaystyle\int_k^x f(t)$ ($F(x)$ is any antideritive of f)

$$F'(x) = \frac{d}{dx} \int_a^x f(t)dt = f(x) \int_a^b f(x)dx$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

Integral of an odd function

Definition

for a odd function defined

$$\int_{-a}^{a} f(x)dx = 0dx$$

Integral of an even function

Definition

for an even function

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

Area between curves

Result

T he area between to curves $y = f(x)$ and $y = g(x)$ over a closed interval $[a, b]$is equal to

$$\int_{a}^{b} f(x) - g(x)dx$$

Volumn

Definition

The volumn of a solid with integrable cross-sectional area $A(x)$ over the closed interval $[a, b]$ is

$$V = \int_{a}^{b} A(x)dx$$

Volumn slicing by disk

Result

The volumn obtained by rotating the area between $f(x)$ and $g(x)$ over the interval $[a, b]$ around the x axis is

$$V = \pi\int_{a}^{b} f(x)^2 - g(x)^2$$

Volumn by shell

Result

The volumn that the area under a $f(x)$ will sweep through when rotated θ radians around the axis $x = L$ is given by

$$V = \int_a^b 2\pi(x - L)(f(x))dx$$