
Partition

Definition

A partition for a closed interval $[a, b]$ is a set of point $(x_0 \text{ to } x_n)$ such that:

$$a = x_0, \quad x_k < x_{k+1} \quad \text{and} \quad b = x_n$$

Partition subinterval

Definition

A sub interval of a partition $(x_0 \text{ to } x_n)$ is $x_{k+1} - x_k$

Norm of a partition

Definition

The norm of a partition P is the largest subinterval of a partition. It is donated by $\|P\|$

Reimans sum

Definition

The Reiman sum for a function f on a closed interval $[a, b]$ is

$$\sum_{k=1}^n f(c_k) \Delta_k$$

where: The set of Δ s is all subintervals of a partition P of $[a, b]$ and

$$c_k \in \left[\sum_{j=1}^{k-1} \Delta_j, \sum_{j=1}^k \Delta_j \right]$$

Definate intergral

Definition

The definite integral of a function f over a closed interval $[a, b]$ is:

$$\lim_{||P|| \rightarrow 0} \sum_{k=0}^n f(c_k) \Delta_k$$

where: $\sum_{k=0}^n f(c_k) \Delta_k$ is any Riemann sum of f on interval $[a, b]$

It is denoted by

$$\int_a^b f(x) dx$$

Continuous functions are integrable

Result

Continuous functions are integrable

Piecewise continuous function

Definition

A function is piecewise continuous if it has a finite number of jump discontinuities

Piecewise continuous functions are integrable

Result

Piecewise continuous functions are integrable

Norm of a partition

Result

If two Riemann sums of a function f over a closed interval $[a, b]$ are not equal, f is not integrable over $[a, b]$

Zero width interval

Result

$$\int_a^b f(x)dx \stackrel{\text{Intergal}}{=} - \int_b^a f(x)dx \quad \text{where } f \text{ is integratable of interval}[a, b]$$

Intergal Could not find a file

Zero width interval

Result

$$\int_a^a f(x)dx \stackrel{\text{Intergal}}{=} 0 \quad \text{where } f \text{ is integratable of interval}[a, b]$$

Intergal Could not find a file

Intergration constant

Result

$$\int_a^b kf(x)dx = k \int_a^b f(x)dx \quad \text{for any constant } k$$

Integration sum and difference

Result

$$\int_a^b f(x) \pm g(x)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

Additivity

Result

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

Max min inequality

Result

If f has maximum value f_{\max} and minimum value f_{\min}

$$f_{\min}(b-a) \leq \int_a^b f(x)dx \leq f_{\max}(b-a)$$

Domination

Result

$$f(x) \geq g(x) \forall x \in [a, b] \Rightarrow \int_a^b f(x) dx \geq \int_a^b g(x) dx$$

Area under a graph

Definition

The area A under a curve $y = f(x)$ over $[a, b]$ is equal to

$$\int_a^b f(x) dx$$

Mean value theorem for definite integrals

if f is continuous over $[a, b]$ then at some point c in $[a, b]$

$$f(c) = \frac{1}{b-a} \int_a^b f(x) dx$$

Antiderivatives integral

Definition

If $F(x) = \int_k^x f(t) dt$ ($F(x)$ is any antiderivative of f)

$$F'(x) = \frac{d}{dx} \int_a^x f(t) dt = f(x)$$

$$\int_a^b f(x) dx = F(b) - F(a)$$

Integral of an odd function

Definition

for an odd function defined

$$\int_{-a}^a f(x)dx = 0$$

Integral of an even function

Definition

for an even function

$$\int_{-a}^a f(x)dx = 2 \int_0^a f(x)dx$$

Area between curves

Result

The area between two curves $y = f(x)$ and $y = g(x)$ over a closed interval $[a, b]$ is equal to

$$\int_a^b |f(x) - g(x)| dx$$

Volumn

Definition

The volumn of a solid with integrable cross-sectional area $A(x)$ over the closed interval $[a, b]$ is

$$V = \int_a^b A(x)dx$$

Volumn slicing by disk

Result

The volumn obtained by rotating the area between $f(x)$ and $g(x)$ over the interval $[a, b]$ around the x axis is

$$V = \pi \int_a^b (f(x)^2 - g(x)^2) dx$$

Volume by shell

Result

The volume that the area under a $f(x)$ will sweep through when rotated θ radians around the axis $x = L$ is given by

$$V = \int_a^b 2\pi(x - L)(f(x))dx$$