Linearity

A function f(x) is linear (in x) if f(ax +by) = af(x) + bf(y) $\forall x, y \in a$ and a, b scalars or real * The curl disrabutes over the sum proof by linearity of partial derivates **The sum of two linear functions id linear** * $\nabla \times (g\vec{F}) = (\nabla g) \times \vec{F} + g(\nabla \times \vec{F})$ * **g** \$ $= ()\nabla \times \vec{F}) \cdot \vec{G} - (\nabla \times \vec{G}) \cdot \vec{F}$

- Show that $\nabla \cdot (g\nabla f) = (\nabla g) \cdot (\nabla f) + g\nabla^2 f$ **Let $\nabla f = \vec{F}$
- Most preceding theorems are prooved of proceeding ones rather than from first principles.
- Revise definitions and properties of section 1.2 !!

Second order partial derivatives with respect to multiple veriable

- Let \$f:Rˆn $\to \mathbb{R} Than \frac{\partial^2 f}{\partial k_x \partial x_j} \overbrace{mostly} = \frac{\partial^2 f}{\partial x_k \partial x_j}$ **Problems arise for example when some of the limits don't exist**

Chain Rule

Partially derive in terms of x_1 to x_n before pluging in x = g(t) Onfriday

Use l'hoppital for dirrectional derivatives

Tangents and normals

cuts

We get cross sections 1 dimension less than the perant function $S\{\vec{x} \in \mathbb{R}^n : f(x) = c\}$ this can give curves or filled spaces and

- \$ $\in Sisaregularpointif F(\vec{x}) \neq)$
- \$ is a singular point if \$f()=0; If grad is a tangent zero grads make things wierd.
- Let S be a hyper surface in \$Rˆn. than n is a noraml to s at x_0 is $\forall \gamma : \mathbb{R} \to \mathbb{R}^n$ such that
  - $\gamma(t) \in S$
  - $\gamma(t_0) = x_0 for some t_0 in \mathbb{R}$
  - $\vec{n} \cdot \gamma'(t) = 0$

$d(\vec{x_0}) \cdot \vec{\gamma}'(t_0) = 0$ Every directional derivative is zero.

Proof $\forall \gamma R \rightarrow R^n | gammat_0 = x_0 forsomet_0 \in R$ and gamma(t) $\in S foralltinRwehavef(gammat = c), sothat(f(\gamma t)` = 0) \Rightarrow = f'(gammat)gamma'(t) = 0 = f(gammat) \cdot gamma't = 0$

in paticular at t0: $\gamma(t\_0) = x\_aand(f(gamma(t0)) = f_0, f(x_0) \cdot \gamma(t_0) = 0$

1.5.4 The set of tangent vectors to a hyper surface x at x zero is the set t at x_0 is all tangent vectors of cureve on the surface that are going through x0

Green's theorems

$ Let D be a region in ℝ