Prerequisits

Common integrates

$$\frac{d}{dx}\tan(x) = \sec^2 x$$

$$\int uf'(x^2) = \frac{f(x^2)}{2}$$

$$\int x\sqrt{x^2+1} = \frac{1}{3}(t^2+1)^{\frac{3}{2}}$$

Integration of $sec^3\theta$ use inegration by part

$$u = \sec \quad v' = sec^2 \quad u' = \sec\tan \quad v = tan$$

$$\int sec^3 = \sec\tan - \int \sec\tan^2 = \sec\tan - \int \sec^3 - \sec^2$$

solveing for $\int \sec^3$

$$2\int sec^3 = \sec\tan + \tan$$

Integration of $\sqrt{ax^2+b}$

$$= \int \sqrt{b}\sqrt{\sqrt{\left(\frac{a}{b}x\right)^2}+1}$$

$$\frac{a}{b}x = \tan\theta \quad dx = \sqrt{\frac{b}{a}}\sec^2\theta$$

$$= \frac{b}{\sqrt{a}}\int \sec^3 x = \frac{b}{2\sqrt{a}}\,(sectan + sec)$$

Intergration by part

$$\int uv' = uv - \int v'u$$

Integrate of $\sqrt{ax^2+b}$

Scalar path integrals

Definition:
$$\int_\Gamma f ds = \int f(\gamma(t)||\gamma'(t)||dt$$

where Γ is a curve parameterized by $\gamma(t)$ where t is on the interval [a,b] and f is a real valued function defined on .

The scalar path integral does not depend on the orientation of the paramiterization

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Greens Theorem

$$\int\int_D \left(\frac{\partial Q}{\partial x} + \frac{\partial P}{\partial y}\right) = \int_{\partial D} Pdx + Qdy$$

Proof

Lemma 1

$$\int_\partial D \begin{pmatrix} f \\ 0 \end{pmatrix} \cdot dr = \int\int_D \frac{\partial f}{\partial y} dydx$$

Lemma 2

$$\int_\partial D \begin{pmatrix} 0 \\ f \end{pmatrix} \cdot dr = \int\int_D \frac{\partial f}{\partial x} dxdy$$