Tut 1

Q1

d

- **(i)** $\forall a \in \mathbb{R} - (-a) = a$
- $\iff$ **(ii)** $-(-a) - a = 0$
- Proof of **(ii)**
    - $-(-a) - a = -(-a) + (-a)$ **Definition**
    - $= 0$ **(A4)**

e

- **(i)** $(a + b) = -a - b$
- $\iff$ **(ii)** $$ -(a+b) +a + b = 0$
- Proof of **(ii)**
    - **(ii.i)** $-(a + b) + a + b = -(a + b) + (a + b)$ **(A1)**
    - **(ii.ii)** $= 0$ **(A4)**

f

- **(i)** $0 + 0 = 0$ **(A3)**
- **(ii)** $a + b = 0 \iff b = -a$ **(A4)** $\wedge$ **b**
- **(iii)** $-0 = 0$ **(i),(ii)**

Q2

a

- **(i)** $a1 = a \quad a1' = a$ **(M3)**
- **(ii)** $a1 = a1'$
- **(iii)** $1a = 1'a$ **(M2)**
- **(iv)** $1aa^{-1} = 1'aa^{-1}$
- **(v)** $1(aa^{-1}) = 1'(aa^{-1})$ **(M1)**
- **(vi)** $\quad 1 \cdot 1 = 1' \cdot 1$ **(M3)**
- **(vii)** $\quad 1 = 1'$ **So this is unique**

b

- **(i)** $aa^{-1} = 1 \quad a(a^{-1})'$ **(M4)**
- **(ii)** $aa^{-1} = a(a^{-1})'$
- **(iii)** $a^{-1}a = (a^{-1})'a$ **(M2)**
- **(iv)** $a^{-1}aa^{-1} = (a^{-1})'aa{-1}$
- **(v)** $a^{-1}(aa^{-1}) = (a^{-1})'(aa{-1})$ **(M1)**
- **(vi)** $a^{-1}1 = (a^{-1})'1$ **(M4)**
- **(vii)** $^a{-1} = (a^{-1})'$ **(M3) Unique**

c

- **(i)** $ax = b \quad ax' = b$ **2 solutions to the equaiton**
- **(ii)** $ax = ax'$
- **(iii)** $xa = x'a$ **(M2)**
- **(iv)** $xaa^{-1} = x'aa^{-1}$
- **(v)** $x(aa^{-1}) = x'(aa^{-1})$ **(M1)**
- **(vi)** $x1 = x'1$ **(M4)**
- **(vii)** $x = x'$ **(M3) Unique**

d

- **(i)** $1 = 1$
- **(ii)** $(a^{-1})(a^{-1})^{-1} = aa^{-1}$ **(M4)** $\neq 0$
- **(iii)** $aa^{-1}(a^{-1})^{-1} = aaa^{-1}$
- **(iv)** $(aa^{-1})(a^{-1})^{-1} = a(aa^{-1}$ **(M2)**
- **(v)** $1(a^{-1})^{-1} = a1$ **(M4)**
- **(vi)** $(a^1)^{-1}1 = a1$ **(M2)**
- **(vii)** $(a^{-1})^{-1} = a$ **(M3)**

e

- **(i)** $1 = 1$
- **(ii)** $(ab)(ab)^{-1} = aa^{-1} = aa{-1}bb^{-1}$ **(M4)**
- **(iii)** $(ab)^{-1}(ab) = a^{-1}b^{-1}ab = a^{-1}b^{-1}(ab)$ **(M1,M2)**
- **(iv)** $(ab)^{-1}(ab)(ab)^{-1} = a^{-1}b^{-1}(ab)(ab)^{-1}$
- **(v)** $(ab)^{-1}1 = a^{-1}b^{-1}1$ **(M4)**
- **(vi)** $(ab)^{-1} = a^{-1}b^{-1}$ **(M3)**

f

- **(i)** $(-a)^{-1} = -a^{-1}$

- $\iff$ **(ii)** $(-a)(-a)^{-1} = (-a) - a^{-1}$
- $\iff$ **(iii)** $1 = (-1)(-1)aa^{-1}$ **(Theorem 1.2 e, M2)**
- **(iv)** $1 = 1$ **(Theorem 1.2 f, M4)**

g

- **(i)** $1 \cdot 1 = 1$ **(M3)**
- **(ii)** $1^{-1} = 1$ **(M4, b)**

Q3

1. A rational number is any number that can be formed by deviding one integer by a none zero integer
   - All axioms exept completness
2. $x = \frac{x_1}{x_2} \quad x_1, x_2 \in \mathbb{Z} \quad y = \frac{y_1}{y_2} \quad y \in \mathbb{Z} \neq 0$

- $x + y = \frac{x_1 y_2 + x_2 y_1}{x_2 y_2}$ Both top and bottom are intgegers as integers are closed under addition and multiplication so the sum is a rational number
- $x + y = \frac{x_1 y_2}{x_2 y_2}$ **Same reasoning**

4

1

- $(a + b) - c$
- $= a + b - c$ **(A1)**
- $= a + b + (-c)$ **Def**
- $= a + (b + (-c))$ **(A1)**
- $= a + (b - c)$ **Def**

2

- $a - (b - c)$
- $= a + (-1)(b) + (-1)(-c)$ **Theorem 1.2 e, D**
- $= a + (-1)(-c) + (-1)(b)$ **(A1)**
- $= a + c - b$ **(Theorem 1.2 e and f)**
- $= (c + a) - b$ **(A1 A2)**

3

- $a((2b+1)+3)$
- $a(2b+1)+a3$ **D**
- $a2b+a+3a$ **D, M2**
- $2ab+1a+3a$ **M2 M3**
- $2ab+(1+3)a$ **D** $=$ 2ab +4a$

4

- $(-a)b$
- $= b(-a)$ **(M2)**