MAT9004 Assignment 3

Due 23:55pm Friday 20 October via online submission ONLY.

IMPORTANT INSTRUCTION: Present your solution so that the sequence of logical steps is clear, with succinct justfication for each step where it is not obvious. Note that marks will be deducted for solutions that do not show all the important logical steps and reasoning in reaching a conclusion.

1. To optimise a neural network can require selecting the best combination of a large set of parameters. For a simple case, suppose that a neural network has just two parameters x and y. The network is only feasible if $y \geq -1$, $x \leq 3$ and $y \leq x$. An analyst establishes that the performance function of the network is

$$f(x, y) = xy^2 + x^2 - 4xy.$$

   (a) Find $\nabla f(x, y)$.

   (b) Find the Hessian matrix $H(x, y)$ for f.

   (c) Locate and classify all stationary points of $f(x, y)$.

   (d) Draw a rough sketch showing the feasible region of parameters, along with the locations of the stationary points.

   (e) For what values of x and y is the maximum performance of the network achieved, and for what values is the minimum achieved?

   [Question 1 total: **15** marks]

2. The planner in charge of raw materials operations at a well known beer-producing company needs to obtain 5 truckloads of fresh water. The five trucks available can go to any of 11 sources of water (mainly dams and reservoirs). The basic question is how many possible ways this can be achieved, using each truck once. The 11 sources all have different kinds of minerals in the water, so it is always important how many truckloads come from which source, but there are different conditions as follows. Find the number of ways in each case. You may leave your answer expressed in terms of binomials, factorials, integers raised to integer powers, or products and quotients of these, unless otherwise requested.(*Important: make sure you explain what formulae you use, and why.*

   (a) It does not matter which truck brings which kind of water, and any sources can be used by any number of trucks. Express your answer as a single integer.

   (b) It does not matter which truck brings which kind of water, but no source can be used by more than one truck. In this case, give the answer as a single integer.

   (c) The company needs to keep track of how far each truck goes, so two arrangements are counted as different if they send different trucks to any given source. Still, no source can be used by more than one truck.

(d) The conditions in (c) hold, but additionally the company needs to record the order that the trucks return with the water. How many possible outcomes are there?

(e) The conditions in (a) hold, but additionally there is one of the sources that may only be used at most twice due to lack of supply. (The other sources are unrestricted). Express your answer as a single integer.

[Question 2 total: **11** marks]

3. A password is a sequence of letters (a–z) and digits (0–9). Find the number of passwords of length 10 under the constraints in (a), (b) or (c) (three separate problems). Express your answer using factorials and integers, products and ratios of them, and/or sums of such things.

   (a) There are 3 letters and 7 digits, and at most one '9'.

   (b) There are 6 letters and 4 digits, and no digit occurs twice.

   (c) No letters are used BUT the first four digits are not all the same AND the last four digits are not all the same, AND the 4th, 5th, 6th and 7th digits are not all the same.

   [Question 3 total: **10** marks]

4. A fair coin is flipped six times. The outcomes of the coin flips form a palindrome if the sequence of T's and H's reads the same forwards and backwards, e.g. THTTHT. Let A denote the event that the first, second and fourth flips are all 'T'. Let Z denote the event that the six flips form a palindrome.

   (a) Is A independent of Z?

   (b) Is $\overline{A}$ independent of Z?

   (c) A fair coin flipped six times and a certain property, Q, is being studied. Let Z be the event that the first three flips are all 'heads'. It is found that $\Pr(Q \mid Z) = 1/4$ and $\Pr(Q \mid \overline{Z}) = 2/7$. Show how to use *Bayes' Theorem* to find $\Pr(Z \mid Q)$.

   Reminder: as with all assignment questions, you must provide justification for your answers.

   [Question 4 total: **9** marks]

5. An integer N is chosen from 1 to 10 uniformly at random.

   Two random variables are defined:

   X is 1 plus the remainder on division of N by 3. So e.g. when $N = 5$, the remainder on division by 3 is 2, so $X = 3$.

   Y is $\lceil N/3 \rceil$. So e.g. when $N = 5$, $Y = 2$.

   (a) Find $\mathrm{E}[X]$, $\mathrm{E}[Y]$ and $\mathrm{Var}[Y]$.

   (b) Are X and Y independent?

   [Question 5 total: **7** marks]