A Basic Proportional Integral Derivative (PID) Algorithm

Definition of terms (memory elements [shift registers] defined in ***Italics***):

Counting indices:

j ≡ represents the index of the current cycle

$j-1$ ≡ represents the index of the previous cycle

Temperature measurement:

T_j ≡ current temperature of this cycle [^oC], it also can be written as just T when the index isn't relevant

$\mathit{T_{j-1}}$ ≡ temperature of the last cycle [^oC], this is stored temporarily in a shift register

δT ≡ $T_j - T_{j-1}$ temperature change since the last cycle

Time measurement:

t_j ≡ current time of this cycle (sec), also can be written as just t

$\mathit{t_{j-1}}$ ≡ time of the last cycle (sec)

δt ≡ $t_j - t_{j-1}$ elapsed time since the last cycle

Control Parameters:

T_s ≡ Setpoint (desired temperature) [^oC], note that this is often a constant, but it can be a function of time $T_s(t)$ as well, for instance in a situation where a particular temperature profile versus time is desired.

allowing us to define the error signal:

$\boldsymbol{\varepsilon_j}$	≡	$\mathbf{T_j - T_s}$	**Error signal** [^oC], it also can be written as just $\varepsilon = T - T_s$. This is the signal that the control system is trying to bring down to zero. Note that it can be positive or negative. It is the central variable in control theory.
$\mathit{\varepsilon_{j-1}}$	≡	$T_{j-1} - T_s$	Error signal for the previous cycle [^oC]
$\delta\varepsilon$	≡	$\varepsilon_j - \varepsilon_{j-1} = T_j - T_{j-1}$	Error change since the last cycle [^oC], also equal to the temperature change since the last cycle, <u>but only if the set point is constant</u>

Δ ≡ Control bandwidth [^oC], this is the range over which our proportional control swings from full power down to zero power, centered around the set point. It is the inverse of the control gain K more commonly used in control theory literature. We use Δ in this situation because of its intuitive connection to the range of linear control on the proportional band. Making it too large leads to stable behavior, but often at the expense of significant offset droop. The integral power has to make up more ground to pull it to the set point in such situations. Making it too small leads to oscillations. The optimal value of Δ, to make the system stable but still responsive, is roughly twice the value that causes oscillations.

S ≡ Settle bandwidth [ºC], this defines the range of temperatures close enough to the desired set point to consider the system settled. The user can make this tolerance as low as desired.

τ_i ≡ Integral time (sec). The integral power term is inversely proportional to the integral time, meaning that a long integral time produces a small result that is stable, but more sluggish than necessary to reach the set point, while an integral time too short leads to oscillatory instabilities. Using an integral time slightly longer than the response time of the system maintains stability while still being responsive to error fluctuations.

I_{on} ≡ Integral control on, Boolean, True = on, False = off, just a user switch

τ_d ≡ Derivative time (sec). The derivative power term is proportional to the derivative time. A small derivative time means that the system will be stable, but more sluggish than necessary to respond the rapid changes of the temperature, a derivative time too long leads to oscillatory instabilities; using a derivative time smaller than the response time of the system maintains stability while still being responsive to rapid temperature changes.

D_{on} ≡ Derivative control on, Boolean, True = on, False = off, just a user switch

t_s ≡ Settle time [sec], this defines the time that the system has to stay within the settle band to consider the system settled. The user can set this time as long if desired.

$T_H \equiv T_s + \Delta/2$ High limit of control band. The proportional power is zero here.

$T_L \equiv T_s - \Delta/2$ Low limit of control band. The proportional power is maximum here.

$T_{SH} \equiv T_s + S/2$ High limit of settle band.

$T_{SL} \equiv T_s - S/2$ Low limit of settle band

Power terms:

All of the power terms below are unitless by definition, and normalized so that $P_n = 1$ corresponds to the actual physical power delivered to the system being the maximum power P_{max} in units of Watts. It is just a simple linear scaling. The definition of normalized power in this regard is

$$P_n \equiv P/P_{max}.$$

All power terms defined below are normalized.

P_p ≡ Proportional power [unitless], the expression for it is below in the control algorithm

k_j ≡ current count of the number of cycles outside the control band

k_{j-1} ≡ count of the number of cycles outside the control band on the last cycle

P_{Ij} ≡ current integral power [unitless], also can be written as just P_I, the expression for it is below in the control algorithm

P_{Ij-1} ≡ integral power of last cycle

P_d ≡ Derivative power [unitless], the expression for it is below in the control algorithm

P_r ≡ Raw total power, before coercion

P_t ≡ Final total power, coerced [0-1]

Settle terms:

G_o ≡ Temperature is settled, Boolean, True = on, False = off, an indicator that allows the experiment to proceed to its next state.

t_{ej} ≡ current elapsed time in the settle band [sec], a measure of how long the system has stayed within the settle band without exiting

t_{ej-1} ≡ elapsed time in the settle band of the previous cycle [sec]

The Control Algorithm:

Proportional power: It is linear, going down from one to zero over the control band; it is exactly ½ at the set point when the error zero.

Pp	=	$1/2 - \varepsilon/\Delta$	if	$T_L \leq T_j \leq T_H$
	=	1	if	$T_j < T_L$
	=	0	if	$T_H < T_j$

Derivative power: It lowers the overall power if the temperature ramps up too quickly, and increases the power if the temperature drops too quickly.

P_d	=	$- [\tau_d/\Delta]*[\delta\varepsilon/\delta t]$	if	D_{on} = True		
	=	0	if	D_{on} = False	or	first call

Out counter: It counts how many measurements in a row are outside of the control band, it is used in the integral power algorithm below to clear the integral memory if it reaches 2 in a row

k_j	=	$k_{j-1} + 1$	if	$Tj < T_L$	or	$T_H < T_j$	out of band
	=	0	if	$T_L \leq T_j \leq T_H$			in band

Integral power: While active, it constantly tries to zero out the temperature error, adjusting the total power to compensate for the offset droop due to the proportional term, as well as any changes in the thermal environment, or fluctuations that pull the system away from the set point. It's the secret sauce of this approach.

This algorithm activates the integral in the control band, but zeroes it if two measurements in a row are outside of the control band. This avoids integral windup when changing set points, etc., while also not resetting and having to wait for the system to settle again due to one spurious measurement outside of the band, perhaps due to a noisy electrical environment.

$P_{Ij} \quad = \quad P_{Ij-1} - [(\varepsilon * \delta t)/[\Delta * \tau_i]$ if $k_i \leq 1$ and I_{on} = True

0 if $k_j > 1$ or I_{on} = False or first call

Raw power:

$P_r \quad = \quad P_p + P_I + P_d$

Final total power: just coercing the raw power to [0-1]

$P_t \quad = \quad P_r$ if $0 < P_r < 1$

$= \quad 1$ if $1 < P_r$

$= \quad 0$ if $P_r < 0$

The Settle Algorithm:

The system is considered settled if the temperature stays within the user-defined settle band for a time exceeding the user-defined settle time.

Settle time tracking:

$t_{ej} \quad = \quad t_{ej-1} + \delta t$ if $T_{SL} < T_j < T_{SH}$ in band

$= \quad 0$ if $T_j < T_{SL}$ or $T_{SH} < T_j$ out of band

Determination of settled condition:

$G_o \quad = \quad$ True if $t_s < t_{ej}$

Initialization:

It is important to make sure that none of the calculations above are trusted on the first call of the algorithm since all of the shift registers will have invalid values. In addition to sending out zero power on the first call (or a manual or automatic reset) for the integral and derivative terms, it also is important to make sure that all of the shift registers are fed their correct values on the first call as well, so that they will be valid and useful thereafter.

$T_{j-1} = T_j$

$t_{j-1} = t_j$

$k_{j-1} = 0$

$P_{Ij-1} = 0$

$t_{ej-1} = 0$

$P_d = 0$

$P_{Ij} = 0$

Stable starting control values for the "bug":

$\Delta = 4$ Control bandwidth [^oC]

$\tau_i = 30$ Integral time (sec)

$\tau_d = 0.5$ Derivative time (sec)

Reasonable settle values for the "bug":

$S = 1$ Settle bandwidth [^oC]

$t_s = 30$ Settle time [sec]