Student's Name:
Enrolment No:
Date:

rishihood university

Mid Semester Examination – December 2025

Course Code:
Course Title:

Programme: BTech(CS & AI) **Semester: III**

Duration: 3 hrs **Max. Marks: 70**

- **Instructions:**
- Write your name & roll number on top immediately after receipt of this question paper
- Start each answer on a fresh page and write the number of your answers clearly.
- Answer all parts of the same question together and in sequence.
- Follow the word limit wherever applicable. Do not overwrite
- Please do not discuss anything while attempting the paper
- Scientific calculator is prohibited in the exam

Part A (7 Questions, 3 Marks Each)

1. Define a Vector Space. Verify whether the set of all 3×3 upper triangular matrices with determinant 0 forms a vector space under standard matrix addition and scalar multiplication. Justify your answer carefully. (3)

2. Determine if the set $S = \{(3, 6, 9), (6, 9, 12), (4, 7, 10)\}$ in $\mathbb{R}^3$ is linearly dependent or independent. Show your work. (3)

3. Find the Null Space of
$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ 1 & 1 & -1 \end{bmatrix}.$$
Express it in set-builder notation and state its dimension. (3)

4. Let P_3 be the vector space of all real polynomials of degree at most 3. Consider the set
$$S = \{\, 1 + x,\, x + x^2,\, x^2 + x^3 \,\}.$$
Does S form a basis of P_3? Justify your answer. (3)

5. Find the standard matrix of the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (4x + 2y, -2x + 5y)$. Further, determine whether T is invertible. (3)

6. Find the elementary matrix that represents the row operation $R_3 \leftarrow 2R_3 - R_1$ for a 3×3 matrix. (3)

7. What is an affine space? Give an example of a line in $\mathbb{R}^3$ that passes through $(1, 2, 3)$, show it is affine but not a subspace. (3)

Part B (7 Questions, 7 Marks Each)

8. Find the LU Decomposition of

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 3 & 3 \\ 6 & 5 & 9 \end{bmatrix}.$$

Using your decomposition, solve $Ax = b$ where $b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and comment on uniqueness of the solution. (7)

9. Let

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 3 \\ 1 & 3 & 2 \end{bmatrix}.$$

Find a sequence of elementary matrices $E_1, E_2, \ldots$ such that

$$E_k \cdots E_2 E_1 A = U,$$

where U is the row echelon form of A. Using it determine the rank of A, and find a basis for both the column space and the null space of A. Verify that the Rank–Nullity Theorem holds. (7)

10. Find the eigenvalues and eigenvectors of

$$B = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 5 \end{bmatrix}.$$

Is B diagonalizable? Justify. (7)

11. Let $v = (2, -1, 3)$ be a vector in $\mathbb{R}^3$ with respect to the standard basis

$$E = \{\, e_1 = (1, 0, 0),\ e_2 = (0, 1, 0),\ e_3 = (0, 0, 1) \,\}.$$

Consider another basis of $\mathbb{R}^3$ given by

$$B = \{\, b_1 = (1, 1, 0),\ b_2 = (0, 1, 1),\ b_3 = (1, 0, 1) \,\}.$$

Find the coordinate vector $[v]_B$ of v with respect to the basis B. Show all steps, including construction of the change-of-basis matrix. (7)

12. Let

$$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

be a vector in $\mathbb{R}^3$.

First rotate $\mathbf{v}$ about the x-axis by 60° counterclockwise. Then apply a shear in the xy-plane defined by

$$x \mapsto x + 2y, \qquad y \mapsto y, \qquad z \mapsto z.$$

Compute the resulting vector after these two operations. Show all steps (you may use $\cos 60^\circ = \frac{1}{2}$, $\sin 60^\circ = \frac{\sqrt{3}}{2}$) and briefly explain geometrically how the rotation and the shear affect the point. (7)

13. Show that

$$A = \begin{bmatrix} 6 & -2 \\ 2 & 1 \end{bmatrix}$$

is diagonalizable. Find P, D such that $A = PDP^{-1}$ and express A^{20} as a product (no need to expand). (7)

14. Let P_3 be the vector space of all real polynomials of degree at most 3.

Consider the subspace

$$W = \{\, p(x) \in P_3 : p(1) = 0,\ p'(1) = 0,\ \text{and } p''(1) = 0 \,\}.$$

Find a basis and the dimension of W. (7)

— End of Question Paper —