

# The Linear Point Standard Ruler with DESI DR1 and DR2 Data

**N. Uberoi<sup>ID</sup>,<sup>a</sup> F. Nikakhtar<sup>ID</sup>,<sup>a</sup> N. Padmanabhan<sup>ID</sup>,<sup>a</sup> R. K. Sheth<sup>ID</sup>,<sup>b</sup> J. Aguilar,<sup>c</sup> S. Ahlen<sup>ID</sup>,<sup>d</sup> D. Bianchi<sup>ID</sup>,<sup>e,f</sup> D. Brooks,<sup>g</sup> F. J. Castander<sup>ID</sup>,<sup>h,i</sup> T. Claybaugh,<sup>c</sup> A. Cuceu<sup>ID</sup>,<sup>c</sup> A. de la Macorra<sup>ID</sup>,<sup>j</sup> Arjun Dey<sup>ID</sup>,<sup>k</sup> Biprateep Dey<sup>ID</sup>,<sup>l,m</sup> P. Doel,<sup>g</sup> J. E. Forero-Romero<sup>ID</sup>,<sup>n,o</sup> E. Gaztañaga<sup>ID</sup>,<sup>h,p,i</sup> S. Gontcho A Gontcho<sup>ID</sup>,<sup>c,q</sup> G. Gutierrez,<sup>r</sup> K. Honscheid<sup>ID</sup>,<sup>s,t,u</sup> C. Howlett<sup>ID</sup>,<sup>v</sup> M. Ishak<sup>ID</sup>,<sup>w</sup> R. Joyce<sup>ID</sup>,<sup>k</sup> D. Kirkby<sup>ID</sup>,<sup>x</sup> T. Kisner<sup>ID</sup>,<sup>c</sup> O. Lahav,<sup>g</sup> C. Lamman<sup>ID</sup>,<sup>u</sup> M. Landriau<sup>ID</sup>,<sup>c</sup> L. Le Guillou<sup>ID</sup>,<sup>y</sup> M. Manera<sup>ID</sup>,<sup>z,aa</sup> P. Martini<sup>ID</sup>,<sup>s,ab,u</sup> A. Meisner<sup>ID</sup>,<sup>k</sup> R. Miquel,<sup>ac,aa</sup> S. Nadathur<sup>ID</sup>,<sup>p</sup> W. J. Percival<sup>ID</sup>,<sup>ad,ae,af</sup> C. Poppett,<sup>c,ag,ah</sup> F. Prada<sup>ID</sup>,<sup>ai</sup> I. Pérez-Ràfols<sup>ID</sup>,<sup>aj</sup> G. Rossi,<sup>ak</sup> L. Samushia<sup>ID</sup>,<sup>al,am,an</sup> E. Sanchez<sup>ID</sup>,<sup>ao</sup> D. Schlegel,<sup>c</sup> M. Schubnell,<sup>ap,aq</sup> J. Silber<sup>ID</sup>,<sup>c</sup> D. Sprayberry,<sup>k</sup> G. Tarlé<sup>ID</sup>,<sup>aq</sup> B. A. Weaver,<sup>k</sup> H. Zou<sup>ID</sup>,<sup>ar</sup>**

<sup>a</sup>Physics Department, Yale University, P.O. Box 208120, New Haven, CT 06511, USA

<sup>b</sup>Center for Particle Cosmology, University of Pennsylvania, PA 19104, USA

<sup>c</sup>Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Berkeley, CA 94720, USA

<sup>d</sup>Department of Physics, Boston University, 590 Commonwealth Avenue, Boston, MA 02215 USA

<sup>e</sup>Dipartimento di Fisica “Aldo Pontremoli”, Università degli Studi di Milano, Via Celoria 16, I-20133 Milano, Italy

<sup>f</sup>INAF-Osservatorio Astronomico di Brera, Via Brera 28, 20122 Milano, Italy

<sup>g</sup>Department of Physics & Astronomy, University College London, Gower Street, London, WC1E 6BT, UK

<sup>h</sup>Institut d’Estudis Espacials de Catalunya (IEEC), c/ Esteve Terradas 1, Edifici RDIT, Campus PMT-UPC, 08860 Castelldefels, Spain

<sup>i</sup>Institute of Space Sciences, ICE-CSIC, Campus UAB, Carrer de Can Magrans s/n, 08913 Bellaterra, Barcelona, Spain

<sup>j</sup>Instituto de Física, Universidad Nacional Autónoma de México, Circuito de la Investigación Científica, Ciudad Universitaria, Cd. de México C. P. 04510, México

<sup>k</sup>NSF NOIRLab, 950 N. Cherry Ave., Tucson, AZ 85719, USA

<sup>l</sup>Department of Astronomy & Astrophysics, University of Toronto, Toronto, ON M5S 3H4, Canada

- <sup>m</sup>Department of Physics & Astronomy and Pittsburgh Particle Physics, Astrophysics, and Cosmology Center (PITT PACC), University of Pittsburgh, 3941 O'Hara Street, Pittsburgh, PA 15260, USA
- <sup>n</sup>Departamento de Física, Universidad de los Andes, Cra. 1 No. 18A-10, Edificio Ip, CP 111711, Bogotá, Colombia
- <sup>o</sup>Observatorio Astronómico, Universidad de los Andes, Cra. 1 No. 18A-10, Edificio H, CP 111711 Bogotá, Colombia
- <sup>p</sup>Institute of Cosmology and Gravitation, University of Portsmouth, Dennis Sciama Building, Portsmouth, PO1 3FX, UK
- <sup>q</sup>University of Virginia, Department of Astronomy, Charlottesville, VA 22904, USA
- <sup>r</sup>Fermi National Accelerator Laboratory, PO Box 500, Batavia, IL 60510, USA
- <sup>s</sup>Center for Cosmology and AstroParticle Physics, The Ohio State University, 191 West Woodruff Avenue, Columbus, OH 43210, USA
- <sup>t</sup>Department of Physics, The Ohio State University, 191 West Woodruff Avenue, Columbus, OH 43210, USA
- <sup>u</sup>The Ohio State University, Columbus, 43210 OH, USA
- <sup>v</sup>School of Mathematics and Physics, University of Queensland, Brisbane, QLD 4072, Australia
- <sup>w</sup>Department of Physics, The University of Texas at Dallas, 800 W. Campbell Rd., Richardson, TX 75080, USA
- <sup>x</sup>Department of Physics and Astronomy, University of California, Irvine, 92697, USA
- <sup>y</sup>Sorbonne Université, CNRS/IN2P3, Laboratoire de Physique Nucléaire et de Hautes Energies (LPNHE), FR-75005 Paris, France
- <sup>z</sup>Departament de Física, Serra Húnter, Universitat Autònoma de Barcelona, 08193 Bellaterra (Barcelona), Spain
- <sup>aa</sup>Institut de Física d'Altes Energies (IFAE), The Barcelona Institute of Science and Technology, Edifici Cn, Campus UAB, 08193, Bellaterra (Barcelona), Spain
- <sup>ab</sup>Department of Astronomy, The Ohio State University, 4055 McPherson Laboratory, 140 W 18th Avenue, Columbus, OH 43210, USA
- <sup>ac</sup>Institució Catalana de Recerca i Estudis Avançats, Passeig de Lluís Companys, 23, 08010 Barcelona, Spain
- <sup>ad</sup>Department of Physics and Astronomy, University of Waterloo, 200 University Ave W, Waterloo, ON N2L 3G1, Canada
- <sup>ae</sup>Perimeter Institute for Theoretical Physics, 31 Caroline St. North, Waterloo, ON N2L 2Y5, Canada
- <sup>af</sup>Waterloo Centre for Astrophysics, University of Waterloo, 200 University Ave W, Waterloo, ON N2L 3G1, Canada
- <sup>ag</sup>Space Sciences Laboratory, University of California, Berkeley, 7 Gauss Way, Berkeley, CA 94720, USA
- <sup>ah</sup>University of California, Berkeley, 110 Sproul Hall #5800 Berkeley, CA 94720, USA
- <sup>ai</sup>Instituto de Astrofísica de Andalucía (CSIC), Glorieta de la Astronomía, s/n, E-18008 Granada, Spain
- <sup>aj</sup>Departament de Física, EEBE, Universitat Politècnica de Catalunya, c/Eduard Maristany 10, 08930 Barcelona, Spain

<sup>ak</sup>Department of Physics and Astronomy, Sejong University, 209 Neungdong-ro, Gwangjin-gu, Seoul 05006, Republic of Korea

<sup>al</sup>Abastumani Astrophysical Observatory, Tbilisi, GE-0179, Georgia

<sup>am</sup>Department of Physics, Kansas State University, 116 Cardwell Hall, Manhattan, KS 66506, USA

<sup>an</sup>Faculty of Natural Sciences and Medicine, Ilia State University, 0194 Tbilisi, Georgia

<sup>ao</sup>CIEMAT, Avenida Complutense 40, E-28040 Madrid, Spain

<sup>ap</sup>Department of Physics, University of Michigan, 450 Church Street, Ann Arbor, MI 48109, USA

<sup>aq</sup>University of Michigan, 500 S. State Street, Ann Arbor, MI 48109, USA

<sup>ar</sup>National Astronomical Observatories, Chinese Academy of Sciences, A20 Datun Road, Chaoyang District, Beijing, 100101, P. R. China

E-mail: [navya.uberoi@yale.edu](mailto:navya.uberoi@yale.edu), [farnik.nikakhtar@yale.edu](mailto:farnik.nikakhtar@yale.edu)

**Abstract.** The linear point  $s_{LP}$ , a purely geometric feature in the monopole of the two-point correlation function, has been proposed as an alternative standard ruler. Compared to the peak in the correlation function, it is more robust to late-time nonlinear effects at the percent level. In light of improved simulations and high quality data, we revisit the claimed robustness of the linear point scale and use it as an alternative to template-based fitting approaches typically used in BAO analyses. We present the first measurements of the linear point on galaxy samples from the first and second data releases (DR1 and DR2) of the Dark Energy Spectroscopic Instrument (DESI) survey. We convert the linear point into a dimensionless parameter  $\alpha_{\text{iso},LP}$ , defined as the ratio of the linear point in the fiducial cosmology and the observed value, analogous to the isotropic BAO scaling parameter  $\alpha_{\text{iso},BAO}$  used in previous BAO measurements. Using the second generation of **AbacusSummit** large-scale structure mock catalogs (**Abacus-2**), we find that linear point measurements are more precise when calculated in the post-reconstruction regime with 15-60% smaller uncertainties than those pre-reconstruction. We also compare the mean  $\alpha_{\text{iso},LP}$  across the 25 **Abacus-2** mocks for each tracer to the mean  $\alpha_{\text{iso},BAO}$  found using the template-based fitting regime used in DESI BAO analyses. We find a systematic shift in the linear point measurements compared against the isotropic BAO measurements; we attribute this to the isotropic damping parameter responsible for smearing the linear point in the nonlinear regime. We propose a sample-dependent correction to  $\alpha_{\text{iso},LP}$  that mitigates the impact of these late-time nonlinear effects. While this introduces a cosmology dependence in an otherwise model-independent measurement, this correction is necessary given the sub-percent precision dictated by current cosmological surveys. Comparing  $\alpha_{\text{iso},LP}$  with isotropic BAO measurements made on the DESI DR1 and DR2 galaxy samples, we find excellent agreement after applying this correction, particularly post-reconstruction. We discuss future scope regarding cosmological inference with linear point measurements.

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## 1 Introduction

In the early universe, photons and baryons were coupled in a hot, dense plasma. In regions with slight overdensities of baryons, gravity and radiation pressure would counteract each other, resulting in pressure waves propagating through this plasma. These baryon acoustic oscillations (BAO) were halted at redshift  $z \approx 1100$  when the universe cooled enough for the photons to decouple. These oscillations imprint a characteristic peak in the matter (and galaxy) two-point correlation function<sup>1</sup> at a scale corresponding to the sound horizon at the drag epoch [1, 2]. The location of the peak in the correlation function serves as a standard ruler and is one of the most important probes used to study the expansion

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<sup>1</sup>In the power spectrum (the Fourier transform of the correlation function), these manifest as a series of oscillations

history of the universe [3, 4]. The BAO imprint was first observed in the SDSS [5] and 2dFGRS [6] surveys, with progressively higher significance measurements made in subsequent spectroscopic surveys like BOSS [7], eBOSS [8], 6dFGRS [9], WiggleZ [10], DES [11], and more recently the Dark Energy Spectroscopic Instrument (DESI, [12–14]). These surveys have used galaxy clustering to derive cosmological parameters from the BAO imprint in two-point statistics [15, 16]. Future large-scale structure surveys like Euclid [17], Roman [18], and Spec-S5 [19] will seek to further constrain the expansion history of the universe.

However, like most standard rulers, the BAO signal is an imperfect one. It is sensitive to nonlinear effects due to late-time structure formation, and as such the peak in the correlation function is smeared [20–22]. Simple peak finding algorithms are therefore insufficient to constrain the BAO scale, as the position of the peak shifts by more than 2%. Surveys have therefore used physically-motivated templates, initialized with a fiducial cosmology, that fit the correlation function to extract the BAO scale [23–30]. Density field reconstruction [23, 31, 32] has been extensively implemented in galaxy surveys [13, 33, 34] and has been shown to be incredibly effective in removing the shifts in the BAO peak caused by nonlinearities and galaxy biasing, reducing the systematic errors in BAO measurements [21, 35–37]. These methods require assuming a fiducial cosmology to convert galaxy redshifts to distances as well as a fiducial value for the growth rate of cosmic structure and the linear bias of the galaxy sample [23, 38]. While it is in principle a cosmology-dependent pipeline, significant work has been done to show that both template-based fits and standard reconstruction are robust and that the effects of assuming a fiducial cosmology are minimal [39–42]. However, it is desirable to examine the effects of alternative BAO fitting schemes, especially in light of the most recent findings by DESI hinting at evolving dark energy [43–46]. This includes considering alternative standard rulers like the linear point (the focus of this work) and the zero-crossing scale [47], and alternative reconstruction algorithms like Laguerre reconstruction [48, 49], optimal transport reconstruction [50–54], iterative algorithms [55–58], and hybrid methods involving machine learning [59–61].

The linear point  $s_{LP}$  [62, 63] is defined as the mean of the locations of the peak and the preceding dip in the monopole of the two-point correlation function. It is a purely geometric scale whose value can be inferred using a simple polynomial fit to a narrow region of the correlation function [64]. Its utility as a standard ruler is motivated by its weak sensitivity to smearing effects like late-time nonlinearities, redshift space distortions, and scale-dependent bias, which potentially mitigates the need for density field reconstruction altogether. An adjustment of 0.5% to the calculated linear point was predicted to correct for any weak sensitivity to the aforementioned effects. To further remove the effects of assuming a fiducial cosmology in calculating the correlation function, the linear point is often redefined as  $y_{LP} \equiv s_{LP}/D_V$ , where  $D_V$  is the isotropic volume distance for the fiducial cosmology. This is similar to how the standard BAO measurement is often denoted as the ratio of the BAO standard ruler scale  $r_d$  and the angular diameter distance  $D_A$ .

The linear point pipeline was validated in [64], and the first measurements of the linear point on data were made on the LOWZ and CMASS samples from the twelfth data release of the BOSS experiment [65] in [63]. Cosmological inference using the linear point was first described in [66], and preliminary measurements of the Hubble constant  $H_0$  and the matter density  $\Omega_m$  using BOSS DR12 linear point measurements were presented in [67]. With improved simulations and data, it is critical to revisit the linear point and verify its robustness as a standard ruler in the era of precision cosmology.

In what follows, we distinguish between two distinct ways of using the linear point. The

first is close to the original [62–64, 66, 68, 69]: to provide an estimate of the distance scale in as cosmology-independent a way as possible. This means that no cosmology-dependent template shapes are used when estimating the peak and dip scales in the two-point correlation function measured in the *pre*-reconstructed field, and the only correction for nonlinear effects is to multiply the raw  $s_{\text{LP}}$  estimate by a cosmology- and redshift-independent factor of 1.005 as done in [63, 64, 66]. The second is to use the linear point as an alternative to template-based estimates of the distance scale for providing constraints that are complementary to standard BAO analyses. In this case, we compromise on the strict cosmology-independence ideal – in this case, not only is the factor 1.005 allowed to be cosmology and redshift dependent, but estimating the linear point in the *post*-reconstructed field is potentially useful and interesting.

In this work, we present the linear point measurements made on the bright galaxy survey (BGS), and luminous red galaxy (LRG) and emission line galaxy (ELG) samples from the first and second data releases (DR1 and DR2) of the DESI survey. In Section 2, we describe the data and mocks used in this work, along with a brief discussion of the methods used in the primary DESI analysis pipeline. In Section 3, we introduce the linear point, discuss its robustness to nonlinear effects and smearing, and adjust free parameters in the pipeline. In Section 5, we use mock catalogs generated with the DESI fiducial cosmology to investigate the impact of reconstruction on the linear point and compare them to isotropic BAO measurements made on the same correlation functions using the fixed-template pipeline. Finally, we present the linear point measurements for DESI DR1 and DR2 samples, and summarize our findings and discuss future prospects in Section 6.

## 2 Data and Methods

The Dark Energy Spectroscopic Instrument (DESI) survey is a Stage IV dark energy experiment designed to measure the expansion history and growth of structure at redshifts  $z < 3$ . The instrument is a multi-fiber spectrograph installed on the Mayall 4m telescope located at the Kitt Peak National Observatory in Arizona [70]. DESI is conducting an eight-year survey [71] covering about 17,000 deg<sup>2</sup> of the sky, and can obtain simultaneous spectra of almost 5000 objects over a  $\sim$ 3° field [72–74]. The full survey is expected to measure spectra of 63 million galaxies and quasars [75], compared to the initial forecasts of 39 million. The first data release (DR1, [43]) includes observations from 5.7 million galaxies and quasars made in the first year of survey operations starting May 14, 2021, after a successful validation phase [76], continuing until June 14, 2022 and covering an effective volume of roughly 18 Gpc<sup>3</sup>. Now, with redshifts of more than 14 million galaxies and quasars, the second data release sample (DR2) is by far the largest spectroscopic galaxy sample to date, covering a cumulative effective volume of over 42 Gpc<sup>3</sup>. DESI adjusts its observing schedule based on conditions, running a “bright-time” program focused on the Bright Galaxy Survey [77] and a “dark-time” program that targets luminous red galaxies (LRGs [78]), emission-line galaxies (ELGs [79]), and quasars (QSOs [80]). A detailed overview of the spectroscopy, target selection, and creation of large-scale structure catalogs is provided in [75, 81–83].

In this section, we describe the DESI DR1 survey targets and mock catalogs used in this work, along with a brief discussion on the methods used to measure the two-point correlation function, perform reconstruction, compute covariance matrices, and perform BAO template fits.

## 2.1 Survey targets and mock catalogs

We use identical samples of the bright galaxy survey (BGS), luminous red galaxies (LRGs), and emission-line galaxies (ELGs) from the first and second data releases (DR1 and DR2) used in BAO analysis [13]. We list the redshift cuts, effective redshift, and linear bias for each target in Table 1. Our analysis also makes use mock catalogs simulating large-scale structure in DR1 with DESI survey geometry. Below is a brief description of these mocks; we refer the reader to [13] and references within for more details:

- The **Abacus mocks** [84] derive from the **AbacusSummit** N-body simulation suite [85] and produce highly accurate nonlinear structure in the DESI footprint. In this analysis, we use the 25 base simulation boxes for each tracer with a combined volume of  $200 h^{-3} \text{Gpc}^{-3}$ , generated using the Planck 2018  $\Lambda$ CDM cosmology [86]. The **Abacus** mocks used in this work were produced in two generations for DR1 analyses – the first generation used very early version of the DESI early data release (DESI-EDR [87]) to find the best fit halo occupation distribution model whereas the second generation (**Abacus-2** hereafter) used the final DESI-EDR after correcting for all the systematics and including a detailed model for DESI focal plane effects. These mocks are further produced in three variations of fiber assignment completeness: **complete** (no fiber assignment), **altmtl** (full fiber assignment pipeline [88]), and **ffa** (fast fiber assignment, a sampling-based process that is quicker to implement than the full pipeline [89]).
- The **Effective Zel'dovich mocks (EZmocks)** [90] use the Zel'dovich approximation [38] to produce 1000 computationally cheap simulation boxes. While they may not have accurate nonlinear physics, they are effective in calculating covariances between the 25 **Abacus-2** mocks. The latter is calculated using **EZmocks** with a box side of  $2 h^{-1} \text{Gpc}$  to match the size of **Abacus-2** boxes, whereas realizations with a box side of  $6 h^{-1} \text{Gpc}$  are used to validate covariance matrices for the full survey volume. Because of the large effective volume of these mocks, only the **ffa** pipeline has been applied to them. These mocks have been shown to have good agreement with large-scale clustering [91].

In this work, we use DR1 **Abacus-2** mocks with full fiber assignment (**altmtl**) for validation<sup>2</sup>, and use **EZmocks** for numerically computing covariance matrices when needed. We further discuss covariance matrices in the context of linear point analyses in Section 2.2.

## 2.2 Methods

In this section, we describe the methods used in the DESI DR1 and DR2 BAO analyses to measure two-point clustering statistics in configuration-space, perform reconstruction, and use the template-based pipeline to derive BAO measurements from the correlation function.

### 2.2.1 Two-point correlation function measurements

Two-point clustering measurements on DESI DR1 and DR2 samples are discussed in [45, 92], with the DR2 measurements validated using mock catalogs in [14]. In this work, we use the configuration-space measurements, namely the  $l = 0$  monopole component of two-point correlation function. The correlation function was calculated using the Landy-Szalay estimator

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<sup>2</sup>DR2 mocks were not available at the time this analysis was performed; however, a recent work [14] suggests strong consistency between BAO measurements made on DR1 and DR2 mocks, with smaller uncertainties in the latter. We therefore expect our validation tests to be consistent with DR2 mocks as well.

Tracer	Redshift range	Effective redshift $z_{\text{eff}}$	Linear bias $b$
BGS	0.1–0.4	0.30	1.5
LRG1	0.4–0.6	0.51	2.0
LRG2	0.6–0.8	0.71	2.0
LRG3	0.8–1.1	0.92	2.0
LRG3+ELG1	0.8–1.1	0.93	1.6
ELG1	0.8–1.1	0.95	1.2
ELG2	1.1–1.6	1.32	1.2

**Table 1:** Fiducial redshift range, effective redshift, linear bias, growth rate of structure, and smoothing scale assumed when reconstructing each DESI target sample used in this analysis. The choice of these parameters is motivated in [34, 92].

[93] and its modified version post-reconstruction [94]. Galaxies were weighted using ‘FKP’ weights, inspired by [95] and described in detail in [13, 43] in the context of DESI DR1 measurements and validated in [14] for DR2 BAO measurements. Clustering measurements were combined over the North and South Galactic Caps. The two-point correlation function multipoles were computed using `pycorr`<sup>3</sup>, a Python wrapper for the pair-counting code `corrfunc` [96, 97]. These measurements can be computed for a variety of bin widths; in this analysis, we sample the correlation function with integer bin widths ranging from  $1 \text{ } h^{-1}\text{Mpc}$  to  $5 \text{ } h^{-1}\text{Mpc}$ , and justify our choice of bin width in Appendix A. The linear theory correlation function for the DESI fiducial cosmology is predicted using `CLASS`<sup>4</sup> [98].

### 2.2.2 Reconstruction

DESI BAO analyses use a modification of the standard density-field reconstruction algorithm originally proposed in [23] and improved in [32]. An overview describing additional reconstruction algorithms in the context of DESI BAO results is presented in [99]. We use the iterative Fast Fourier Transform (iFFT) algorithm first presented in [100] and extensively described in [34] as the optimal reconstruction algorithm for DESI BAO analyses. In line with the BAO results presented in [13, 45], we use the `RecSym` convention [101], which shifts tracers and randoms in LSS catalogs by the same amount to preserve redshift-space distortions in post-reconstruction clustering. The smoothing scale used in the reconstruction pipeline is prescribed in [34] and listed in Table 1, along with the growth rate  $f$  and smoothing scale  $\Sigma_{\text{sm}}$  for each tracer. Reconstruction is numerically implemented using `pyrecon`<sup>5</sup>, a Python package developed by the DESI collaboration that offers a range of reconstruction algorithms and conventions.

### 2.2.3 BAO pipeline

The DESI BAO fitting pipeline is designed to extract the BAO feature in two-point clustering measurements by combining a physically motivated theory model from quasi-linear theory and a parameterized model to marginalize over nonlinearities [13]. The observed power spectrum in Fourier space is modeled as a function of the smooth (no-wiggles) component and the BAO (wiggles) component damped by nonlinear evolution. This template is

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<sup>3</sup><https://github.com/cosmodesi/pycorr>

<sup>4</sup>[https://github.com/lesgourg/class\\_public](https://github.com/lesgourg/class_public)

<sup>5</sup><https://github.com/cosmodesi/pyrecon>

Hankel-transformed to configuration space to yield the template for the correlation function multipoles. This template inherits model parameters from the power spectrum template, namely BAO dilation parameters, damping effects, linear galaxy bias, and growth of structure. The Python package `desilike`<sup>6</sup> provides a framework for writing DESI BAO theory templates and likelihoods, which are implemented using JAX [102]. Posterior estimates are made by analytically marginalizing over broadband parameters using MCMC sampling with the EMCEE package [103]. All model parameters in the pipeline are listed in [13] and are initialized with flat priors, with the exception of the transverse and line-of-sight damping parameters,  $\Sigma_{\perp}$  and  $\Sigma_{\parallel}$  respectively, which are sampled from Gaussian priors. The means and standard deviations of these Gaussian priors are derived using a combination of theoretical calculations and many realizations of measurements of the cross-correlation between pre- and post-reconstruction density fields in `Abacus-2` mocks; these values are listed [13] and restated in Table 2. By attempting to undo the effects of nonlinearities in structure formation, reconstruction aims to reduce the damping parameters; this is evident in Table 2. However, the effects of residual damping in the post-reconstruction field will affect comparisons between linear point and standard BAO measurements, which we discuss later in this work.

The DESI BAO measurements are made in terms of Alcock-Paczynski-like parameters [31, 104], namely isotropic and anisotropic BAO dilation parameter ( $\alpha_{\text{iso}}$  and  $\alpha_{\text{AP}}$  respectively). These are often redefined in terms of the apparent size of the BAO standard ruler perpendicular ( $\alpha_{\perp}$ ) and parallel ( $\alpha_{\parallel}$ ) to the line of sight:

$$\alpha_{\text{iso}} = (\alpha_{\perp}^2 \alpha_{\parallel})^{\frac{1}{3}}, \quad \alpha_{\text{AP}} = \alpha_{\parallel}/\alpha_{\perp} \quad (2.1)$$

These are further defined in terms of the angular diameter distance  $D_A(z)$  and the Hubble parameter  $H(z)$ :

$$\alpha_{\parallel} = \frac{H^{\text{fid}}(z)r_d^{\text{fid}}}{H(z)r_d}, \quad \alpha_{\perp} = \frac{D_A(z)r_d^{\text{fid}}}{D_A^{\text{fid}}(z)r_d} \quad (2.2)$$

Here, the superscript ‘fid’ refers to quantities measured in the fiducial cosmology, and  $r_d$  is the comoving scale of the BAO feature.

### 2.3 Covariance matrices

In this work, we use both analytic and numeric covariance matrices for the two-point correlation function.

Analytic covariance matrices for DESI are calculated using the `RascalC` code in configuration space [105–108] and using `TheCov` in Fourier space [109–111]. Since this work is performed exclusively in configuration space, we use the former. These covariance matrices are extensively validated in [112]. At the time of writing this paper, these covariance matrices are only available for correlation functions sampled with a bin width  $4 h^{-1}\text{Mpc}$ . A full discussion of analytical covariance matrices in the context of DR1 data is presented in [113]. Numerical covariances are calculated using the 1000 `EZmocks` and can be computed at any integer bin width  $\Delta s \geq 1 h^{-1}\text{Mpc}$ . The two flavors of covariance matrices are compared in [114] and are found to have a good level of agreement in configuration space. DESI Y1 BAO analyses are therefore performed using the analytic covariance matrices, which additionally allow tuning the covariances to match observed clustering in data and avoid discrepancies between clustering in mocks and data.

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<sup>6</sup><https://github.com/cosmodesi/desilike>

Parameter	Recon	BGS	LRGs	ELGs
$\Sigma_{\perp} [h^{-1}\text{Mpc}]$	Pre	$6.5 \pm 1.0$	$4.5 \pm 1.0$	$4.5 \pm 1.0$
$\Sigma_{\parallel} [h^{-1}\text{Mpc}]$	Pre	$10.0 \pm 2.0$	$9.0 \pm 2.0$	$8.5 \pm 2.0$
$\Sigma_{\text{iso}} [h^{-1}\text{Mpc}]$	Pre	$7.5 \pm 0.9$	$5.7 \pm 0.9$	$5.6 \pm 0.9$
$\Sigma_{\perp} [h^{-1}\text{Mpc}]$	Post	$3.0 \pm 1.0$	$3.0 \pm 1.0$	$3.0 \pm 1.0$
$\Sigma_{\parallel} [h^{-1}\text{Mpc}]$	Post	$8.0 \pm 2.0$	$6.0 \pm 2.0$	$6.0 \pm 2.0$
$\Sigma_{\text{iso}} [h^{-1}\text{Mpc}]$	Post	$4.2 \pm 0.9$	$3.8 \pm 0.9$	$3.8 \pm 0.9$

**Table 2:** Mean values and standard deviations of the Gaussian priors for the nonlinear BAO damping parameters across and along the line of sight ( $\Sigma_{\perp}$  and  $\Sigma_{\parallel}$ , respectively) used in the DESI BAO fitting pipeline, and the corresponding isotropic damping scale  $\Sigma_{\text{iso}} \approx (\Sigma_{\perp}^2 \Sigma_{\parallel})^{\frac{1}{3}}$ .

All previous linear point analyses have used the Gauss-Poisson approximation for covariance matrices [115–117], so using the numerical ones represents a change in methodology. We find no significant difference between linear points measured using numerical and analytical covariances with  $4 h^{-1}\text{Mpc}$  bin widths, which is expected given the excellent agreement found between the two flavors of covariances in [114]. We also refer the reader to [118] for a discussion of the dependence on bin size in the context of the linear point.

### 3 The Linear Point

The linear point is a geometric feature of the monopole of the two-point correlation function, defined as the mean of the distance scales corresponding to the peak in the correlation function and the preceding dip:

$$s_{\text{LP}} \equiv \frac{s_{\text{peak}} + s_{\text{dip}}}{2} \quad (3.1)$$

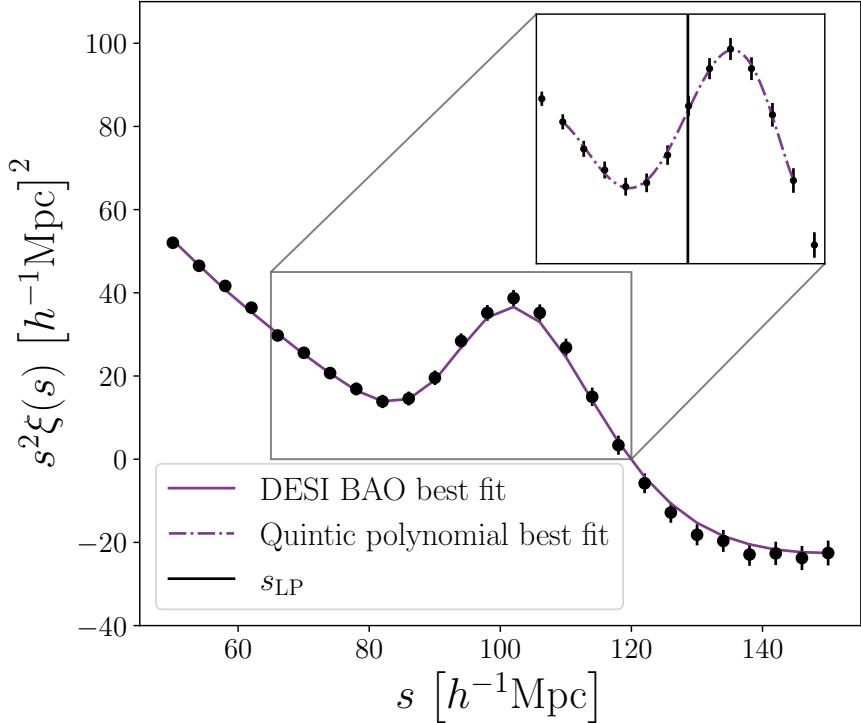
Since this is a purely geometric scale, its value can be inferred without assuming an underlying model for the correlation function. The utility of the linear point lies in its robustness to smearing in the correlation function monopole due to nonlinear structure formation and redshift space distortions, compared to the location of the BAO peak. The linear point was proposed as a purely geometric standard ruler to provide an alternative to cosmology-informed template fits which are typically used to extract the BAO location in the correlation function, thus making the calculation as cosmology- and model-agnostic as possible. This implies that we do not need to assume a template or model to fit the correlation function; rather, a model-independent fit over the region encompassing the peak and the dip (such that Eq. 3.1 can be computed) is sufficient. Figure 1 illustrates this polynomial fit and compares it to the much wider range used in the template-based fitting method.

Linear point measurements are often quoted in the form of an angle-like quantity [63, 68]:

$$y_{\text{LP}} \equiv \frac{s_{\text{LP}}}{D_V}, \quad (3.2)$$

where  $D_V$  is the isotropic volume distance defined as follows:

$$D_V(z) \equiv \left( (1+z)^2 D_A(z)^2 \frac{cz}{H(z)} \right)^{\frac{1}{3}} \quad (3.3)$$



**Figure 1:** Average two point correlation function of the 25 LRG1 Abacus-2 mocks in the typical BAO fitting range of  $50\text{--}150\,h^{-1}\text{Mpc}$ . The solid line represents the best-fit DESI BAO template to the points, whereas the inset plot shows a narrower range of  $70\text{--}115\,h^{-1}\text{Mpc}$  in which the correlation function can be approximated as an odd degree polynomial. A fifth degree polynomial fit is shown as a dotted-dashed line in the inset, and the vertical line indicates the location of the linear point.

This is analogous to the angular size of BAO standard ruler, defined as the ratio of the BAO radius  $r_d$  and the angular diameter distance  $D_A$ . The angular form of the linear point measurements  $y_{\text{LP}}$  further reduces the dependence on fiducial cosmology, since the fiducial and true cosmologies are related as follows: [63, 64, 66, 68, 69]

$$\frac{s_{\text{LP}}^{\text{fid}}}{D_V^{\text{fid}}} \approx \frac{s_{\text{LP}}^{\text{true}}}{D_V^{\text{true}}} \quad (3.4)$$

It is therefore natural to define a dimensionless parameter for the linear point, analogous to the BAO dilation parameters stated in Eq 2.1 and 2.2, as the ratio of the quantities stated above:

$$\alpha_{\text{iso}} = \frac{y_{\text{LP}}^{\text{fid}}}{y_{\text{LP}}^{\text{true}}} = \frac{s_{\text{LP}}^{\text{fid}}}{D_V^{\text{fid}}} \cdot \frac{D_V^{\text{true}}}{s_{\text{LP}}^{\text{true}}} \quad (3.5)$$

The subscript ‘iso’ follows the convention used in BAO analyses to indicate the isotropic size of the standard ruler; since there is no linear point-like feature in the quadrupole of the correlation function, we only concern ourselves with the relative size of the standard ruler compared to the fiducial cosmology. With this definition and the relation stated in Eq 3.4, we naturally expect  $\alpha_{\text{iso}} \approx 1$  if the fiducial cosmology used to calculate the correlation function is the same as (or indistinguishable from) the true underlying cosmology. However, the

observed angular size of the linear point is related to measured linear point  $s_{\text{LP}}^{\text{obs}}$  (simplified to  $s_{\text{LP}}$  hereon) as follows:

$$y_{\text{LP}}^{\text{obs}} = \frac{s_{\text{LP}}}{D_V^{\text{fid}}} \quad (3.6)$$

To calculate the isotropic size of the *observed* linear point standard ruler relative to the fiducial cosmology, we insert the above equation into Eq 3.5 to obtain:

$$\alpha_{\text{iso}} = \frac{s_{\text{LP}}^{\text{fid}}}{s_{\text{LP}}} \quad (3.7)$$

From here on, we refer to the quantity defined in Eq. 2.1 as  $\alpha_{\text{iso},\text{BAO}}$  and the quantity defined in Eq. 3.7 as  $\alpha_{\text{iso},\text{LP}}$ . The calculation of the fiducial linear point is detailed later in Section 3.2.1.

### 3.1 Robustness of the linear point

The motivation behind using the linear point as an alternative standard ruler is its robustness to nonlinear effects. To a good approximation, the nonlinear correlation function,  $\xi_{\text{NL}}$ , can be written as a Gaussian convolution of the linear theory correlation function  $\xi_{\text{lin.th.}}$  with an isotropic smoothing kernel  $\Sigma_{\text{iso}}$  [22]<sup>7</sup>:

$$\xi_{\text{NL}}(s) = \int_0^\infty \frac{dr}{\Sigma_{\text{iso}}^2} \frac{r^3 s}{(rs)^{\frac{3}{2}}} e^{-\frac{(r^2+s^2)}{2\Sigma_{\text{iso}}^2}} i_0 \left( \frac{rs}{\Sigma_{\text{iso}}^2} \right) \xi_{\text{lin.th.}}(r), \quad (3.8)$$

This smoothing kernel can be approximated as the isotropic damping parameter, which is roughly estimated using the transverse and line-of-sight damping parameters ( $\Sigma_\perp$  and  $\Sigma_\parallel$  respectively) used in standard template-based BAO fitting regimes:

$$\Sigma_{\text{iso}} \approx (\Sigma_\perp^2 \Sigma_\parallel)^{\frac{1}{3}}. \quad (3.9)$$

This smoothing makes the linear point in  $\xi_{\text{NL}}$  differ slightly from that in  $\xi_{\text{lin.th.}}$ . To illustrate, for the Planck 2018 cosmology, the linear point exhibits a 0.5% deviation from its fiducial value when  $\Sigma_{\text{iso}} \sim 3.5 h^{-1}\text{Mpc}$ , with larger  $\Sigma_{\text{iso}}$  producing larger differences. For this reason, [62] propose multiplying the value measured in the evolved field by a factor of 1.005 and argue that this obviates the need for reconstruction altogether, in the sense that any remaining difference from the linear theory value will always be sub-percent. This makes the linear point a useful alternative standard ruler [62–64]. Although sub-percent, we show in later sections that modeling this shift may be necessary to achieve agreement with isotropic BAO measurements and consequently derive cosmological parameters with similar precision as those derived from more conventional BAO measurements.

The DESI BAO fitting pipeline uses Gaussian priors on the transverse and line-of-sight damping parameters,  $\Sigma_\perp$  and  $\Sigma_\parallel$ , to initialize the correlation function template. Using the means and standard deviations of these priors, we can estimate the corresponding isotropic damping parameter  $\Sigma_{\text{iso}}$  from Eq. 3.9; we list these values for pre- and post-reconstruction templates in Table 2. The pre-reconstruction damping in DESI BGS evaluates to  $\Sigma_{\text{iso}} \sim 7.5 h^{-1}\text{Mpc}$  and that for LRGs and ELGs is  $\Sigma_{\text{iso}} \sim 5.7 h^{-1}\text{Mpc}$ , which will reduce the linear

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<sup>7</sup>The convolution produces a modified Bessel function of the first kind,  $I_0$ . In this equation,  $i_0$  is the modified *spherical* Bessel function, and is related to  $I_0$  in the same way that the spherical Bessel function  $j_0$  is related to  $J_0$ . See [119] for more details.

point from its fiducial value by up to 1% [48]. Multiplying by 1.005 will not entirely undo this shift, potentially compromising the constraining value of linear point measurements in the pre-reconstructed field. In fact, the (non-zero) post-reconstruction value of  $\Sigma_{\text{iso}}$  suggests that the linear point in the reconstructed correlation functions would still deviate at the 0.5% level from the fiducial unsmeared linear theory value. We explore this further in Section 5.

### 3.2 Estimating the linear point

We take advantage of the purely geometric nature of the linear point scale and use a linear combination of polynomials as our fitting function for the correlation function over a narrow range that includes the peak and the dip. Previous works [64] have shown that an odd-degree polynomial interpolation is a valid approximation for the correlation function in the region between  $60 h^{-1}\text{Mpc}$  and  $120 h^{-1}\text{Mpc}$ , and [117] provide a Bayesian framework for determining the appropriate order polynomial.

Using the DESI DR1 and DR2 LRG and ELG correlation functions and associated covariance matrices, we can use  $\chi^2$  likelihood minimization to fit a centered and scaled polynomial to the data points:

$$\xi_0(s) = \sum_{i=0}^n a_i \left( \frac{s - s_0}{\sigma} \right)^i, \quad (3.10)$$

where  $a_i$  are the fitting coefficients of an  $n$ th degree polynomial,  $s_0 = 93 h^{-1}\text{Mpc}$  is chosen to be roughly in the middle of the fitting range for centering, and  $\sigma = 15 h^{-1}\text{Mpc}$  is an arbitrary scaling parameter. Centering and scaling the polynomial fit ensures that the fitting coefficients  $a_i$  are unaffected by any floating point errors when such coefficients are too small.

We find the locations of the peak ( $s_{\text{peak}}$ ) and the dip ( $s_{\text{dip}}$ ) in the correlation function by analytically computing the roots of the polynomial interpolation and calculating the linear point,  $s_{\text{LP}}$ , using Eq. 3.1. We use a Monte-Carlo like approach to calculate the error on a single measurement of the linear point. Using the covariance matrix of the best-fit coefficients, we generate 1000 Gaussian random samples of the coefficients centered around the best-fit values. We compute the linear point for each realization and report their standard deviation as the error on the original measurement. This approach was shown to be more robust than standard error propagation in [67].

The linear point pipeline has a set of adjustable parameters including the order of the polynomial fitting function,  $n$ ; the range of the correlation function over which to fit the polynomial; and the spacing of the correlation function measurements,  $\Delta s$ . These parameters were optimized and the pipeline validated for a BOSS-like survey in [64] using QPM mocks [120]. We make adjustments as needed to optimize the pipeline for DR1 and DR2 DESI correlation functions and covariance matrices. We find that a quintic polynomial ( $n = 5$ ) fit to the correlation function calculated with bin width  $\Delta s = 4 h^{-1}\text{Mpc}$  in the range  $70-115 h^{-1}\text{Mpc}$  is the most appropriate choice. We justify these choices in Appendix A.

It is possible in some cases that the chosen polynomial interpolation does not yield an identifiable peak and/or dip in the range of the correlation function that we consider. It is also possible that multiple peaks and/or dips are identified, some of which could have no physical meaning and would not yield a reliable linear point estimate. To get a reliable linear point measurement, we identify the local minimum (maximum) closest to  $s_0$  as the dip (peak); spatially, the next (previous) root is therefore identified as the peak (dip). This ensures that we choose the peak that is most likely to capture the BAO signal. We reject any analytical roots that are found outside the fitting range. This can result in failure to identify the BAO

feature in the monopole of the correlation function which may have otherwise been measured with the template-based BAO fitting approach. We explore this further in Appendix B.

### 3.2.1 The linear point in linear theory

Knowing the fiducial linear point is essential for constraining cosmological parameters using the linear point estimated from observed correlation functions. The dimensionless parameter defined in Eq. 3.7 can be directly compared to the isotropic BAO measurements, and any deviation of this ratio from unity can be attributed to a true cosmology that differs from the assumed fiducial cosmology.

We define the fiducial linear point,  $s_{\text{LP}}^{\text{fid}}$ , to be the linear point calculated on the unsmeared linear theory correlation function at  $z = 0$  (i.e.  $\Sigma_{\text{iso}} = 0$  in Eq. 3.8). We generate the linear theory power spectrum at  $z = 0$  for the fiducial Planck 2018  $\Lambda$ CDM cosmology using CLASS, and compute the correlation function by performing a Fourier transformation.

We noted in Section 3.1 that the linear point deviates under nonlinear damping – for the DESI BGS sample, this results in a 1% (0.6%) shift pre- (post-)reconstruction, whereas for LRGs and ELGs this shift is 0.9% (0.5%) pre- (post-)reconstruction. This means that the observed linear point  $s_{\text{LP}}$  would result in inflated values of  $\alpha_{\text{iso},\text{LP}}$  if Eq. 3.7 is evaluated with  $s_{\text{LP}}^{\text{fid}} = s_{\text{LP}}^{\text{lin.th.}}$ . Given our two distinct views of how the linear point can be used, we have two distinct ways of accounting for smearing.

- Only work with the measured  $s_{\text{LP}}$  or the ratio  $\alpha_{\text{iso},\text{LP}} \equiv s_{\text{LP}}^{\text{lin.th.}}/s_{\text{LP}}$ , (perhaps with  $s_{\text{LP}}$  multiplied by 1.005), in the fiducial cosmology, as originally proposed. This preserves the cosmology independent nature of the pipeline, but, for comparison with template-based analyses, the effects of damping must then be propagated into the cosmological parameters inferred from these measurements.
- Modify the linear point  $s_{\text{LP}}$  measured on the observed pre- (post-)reconstruction correlation function by including a sample-dependent multiplicative factor; for example, for the DESI LRG and ELG samples, this factor would be 1.009 (1.005), and for the BGS sample, this would be 1.01 (1.006). This enables a more direct comparison with standard BAO analyses when the measurements are converted to  $\alpha_{\text{iso},\text{LP}}$ . Equivalently, we can modify the fiducial linear point  $s_{\text{LP}}^{\text{fid}}$  in Eq. 3.7 such that it reflects the effects of smearing, while keeping  $s_{\text{LP}}$  unchanged. When the pre- and post-reconstruction damping parameters  $\Sigma_{\text{iso}}$  from Table 2 are used, the two approaches are mathematically equivalent. The latter, however, allows us to preserve the measurements while changing the standard model against which we compare the measurements for the purpose of deriving cosmological parameters. Additionally, as we will see later in this section, this allows us to inflate the errors in the modified values of  $\alpha_{\text{iso},\text{LP}}$  that arises from the uncertainties in  $\Sigma_{\text{iso}}$ .

In this work, we attempt to correct the  $\alpha_{\text{iso},\text{LP}}$  measurements using the latter approach – re-defining the fiducial linear point by a redshift and bias dependent factor in the smeared regime. There is however a third possibility that involves an alternative reconstruction algorithm in which we use the shape of the observed nonlinear (smeared) correlation function to derive the linear correlation function. This is done by undoing the Gaussian smearing from Eq. 3.8, subject to a (de)convolution kernel which we derive from the pre-reconstruction prior on  $\Sigma_{\text{iso}}$ . When the linear point is measured on the resulting linear correlation function, no correction is required to either  $s_{\text{LP}}$  or  $s_{\text{LP}}^{\text{fid}}$  when calculating  $\alpha_{\text{iso},\text{LP}}$ . We discuss the third approach, known as Laguerre reconstruction, in Appendix D.

	$\Sigma_{\text{iso}} [h^{-1}\text{Mpc}]$	$s_{\text{LP}}^{\text{fid}} [h^{-1}\text{Mpc}]$
Linear theory	0	93.01
BGS, Pre-recon.	$7.5 \pm 0.9$	$92.04 \pm 0.03$
LRGs, Pre-recon.	$5.7 \pm 0.9$	$92.13 \pm 0.44$
ELGs, Pre-recon.	$5.6 \pm 0.9$	$92.11 \pm 0.12$
BGS, Post-recon.	$4.2 \pm 0.9$	$92.34 \pm 0.19$
LRGs, Post-recon.	$3.8 \pm 0.9$	$92.46 \pm 0.17$
ELGs, Post-recon.	$3.8 \pm 0.9$	$92.41 \pm 0.19$

**Table 3:** Values of the ‘corrected’ linear point scale,  $s_{\text{LP}}^{\text{fid}}$ , as a function of the isotropic smearing scale,  $\Sigma_{\text{iso}}$ , corresponding to the pre- and post-reconstruction regimes for DESI tracers. The linear point in unsmeared linear theory is provided for reference.

We can check the effect of damping on the fiducial linear point by performing a Gaussian convolution of the linear theory correlation function using the relevant isotropic kernel  $\Sigma_{\text{iso}}$ . We generate 1000 realizations of  $\Sigma_{\text{iso}}$  sampled from the Gaussian priors mentioned in Table 2; for each value, we generate a nonlinear correlation function  $\xi_{\text{NL}}(s)$  using Eq. 3.8. We calculate the linear point for each realization and compute the mean and standard deviation of the sample. We define the mean of the sample to be “corrected” fiducial linear point, and the uncertainty as the standard deviation. We list the pre- and post-reconstruction values of the “corrected”  $s_{\text{LP}}^{\text{fid}}$  in Table 3. We define  $\alpha_{\text{iso},\text{LP}}^{\text{corrected}}$  as the ratio of the smeared fiducial linear point to the observed linear point.

#### 4 Linear Point Measurements in Abacus-2 DR1 Mocks

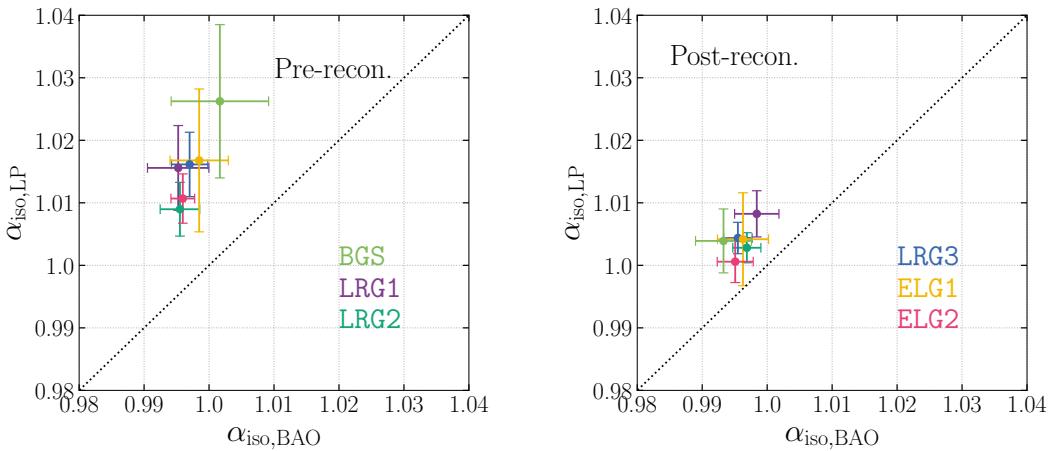
In this section, we run the linear point pipeline on Abacus-2 mock catalogs for DESI DR1 samples, and compare the linear point standard ruler to the BAO measurements derived using the standard template-based pipeline. We validate the robustness of the linear point to nonlinear effects and explore whether standard reconstruction is necessary to correct for some of the resulting smearing.

For each tracer, we measure the linear point on each of the 25 Abacus-2 mock correlation functions and convert these measurements to  $\alpha_{\text{iso},\text{LP}}$  using Eq. 3.7 with  $s_{\text{LP}}^{\text{fid}} = 93.01 h^{-1}\text{Mpc}$ . We present the mean linear point measurements and mean  $\alpha_{\text{iso},\text{LP}}$  values for each set of 25 Abacus-2 mocks in Table 4. The error bars in each column represent the error on the mean, which is the standard deviation of the sample divided by the square root of the number of sample. In Figure 2, we plot the mean values of  $\alpha_{\text{iso},\text{LP}}$  for each tracer against the mean isotropic BAO measurements,  $\alpha_{\text{iso},\text{BAO}}$ , made using the BAO template pipeline. The left panel shows these measurements pre-reconstruction (i.e.,  $\alpha_{\text{iso},\text{LP}}$  is the linear point ratio as originally intended, without multiplying by 1.005), and the right panel depicts post-reconstruction measurements (as a first step for estimating consistency with standard BAO analyses in reconstructed fields). The error bars in both panels show the errors on the mean.

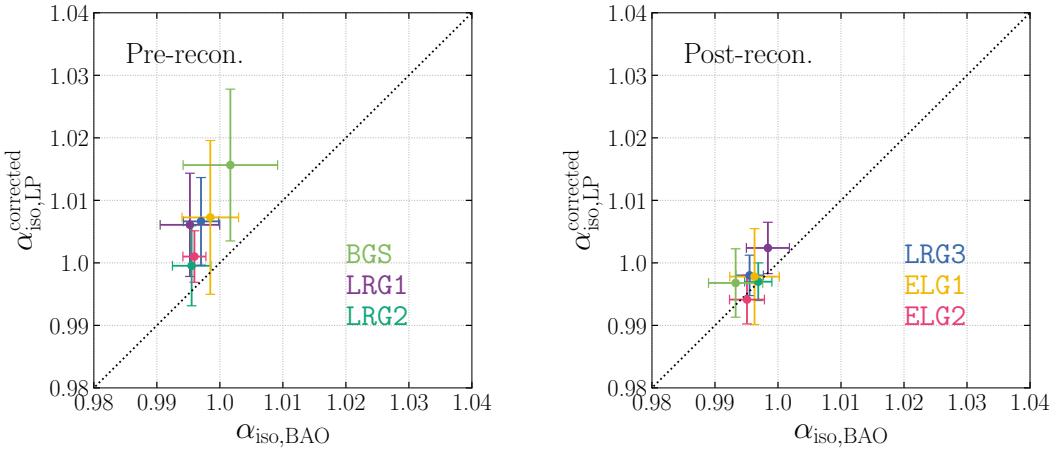
In both pre- and post-reconstruction regimes, the linear point measurements are systematically biased compared to BAO measurements made using the template-based pipeline, despite being otherwise well correlated. As pointed out in Section 3.1, this is likely a result of nonlinear damping effects that inflate the value of  $\alpha_{\text{iso},\text{LP}}$  by roughly 1% (0.5%) due to

Tracer	Redshift	Recon	$\langle s_{LP} \rangle [h^{-1}\text{Mpc}]$	$\langle \alpha_{\text{iso},\text{LP}} \rangle$	$\langle \alpha_{\text{iso},\text{LP}}^{\text{corrected}} \rangle$
BGS	0.1-0.4	Pre	$90.62 \pm 1.08$	$1.026 \pm 0.012$	$1.016 \pm 0.012$
LRG1	0.4-0.6	Pre	$91.52 \pm 0.61$	$1.016 \pm 0.007$	$1.006 \pm 0.008$
LRG2	0.6-0.8	Pre	$91.52 \pm 0.39$	$1.009 \pm 0.004$	$1.000 \pm 0.006$
LRG3	0.8-1.1	Pre	$91.52 \pm 0.46$	$1.016 \pm 0.005$	$1.007 \pm 0.007$
ELG1	0.8-1.1	Pre	$91.46 \pm 1.03$	$1.017 \pm 0.011$	$1.007 \pm 0.012$
ELG2	1.1-1.6	Pre	$91.46 \pm 0.36$	$1.011 \pm 0.004$	$1.001 \pm 0.004$
BGS	0.1-0.4	Post	$92.64 \pm 0.47$	$1.004 \pm 0.005$	$0.997 \pm 0.005$
LRG1	0.4-0.6	Post	$92.60 \pm 0.34$	$1.008 \pm 0.004$	$1.002 \pm 0.004$
LRG2	0.6-0.8	Post	$92.60 \pm 0.22$	$1.003 \pm 0.002$	$0.997 \pm 0.003$
LRG3	0.8-1.1	Post	$92.60 \pm 0.23$	$1.004 \pm 0.003$	$0.998 \pm 0.003$
ELG1	0.8-1.1	Post	$92.61 \pm 0.69$	$1.004 \pm 0.007$	$0.998 \pm 0.008$
ELG2	1.1-1.6	Post	$92.61 \pm 0.31$	$1.001 \pm 0.003$	$0.994 \pm 0.004$

**Table 4:** Mean linear point measurements  $s_{LP}$ , mean  $\alpha_{\text{iso},\text{LP}}$  values without any modifications to  $s_{LP}$  or  $s_{LP}^{\text{fid}}$ , and mean  $\alpha_{\text{iso},\text{LP}}$  values using modified (corrected)  $s_{LP}^{\text{fid}}$  from Table 3 for the 25 Abacus-2 mocks corresponding to each DR1 tracer. The errors in each column are the errors on the mean.



**Figure 2:** The mean isotropic BAO dilation parameter,  $\alpha_{\text{iso},\text{BAO}}$ , compared to the mean ratio of the fiducial linear point to the measured linear point, defined in Eq. 3.7 as  $\alpha_{\text{iso},\text{LP}}$ , computed on LRG and ELG Abacus-2 mock catalogs pre- (left) and post-reconstruction (right). Both panels have the same  $x$ -axis and  $y$ -axis range to illustrate the difference in size of error bars pre- and post-reconstruction.



**Figure 3:** Same as Figure 2, but using the “corrected”  $\alpha_{\text{iso},\text{LP}}$  values after taking into account the smearing in the fiducial linear point due to late-time nonlinear evolution. The correction only shifts the  $y$ -axis coordinates to lower values and slightly increases the size of error bars.

damping in the pre- (post-)reconstruction regime. Naively, we expect the pre-reconstruction  $\alpha_{\text{iso},\text{LP}}$  to always be larger than unity and to be larger at lower redshift. The left-side panel in Figure 2 shows that  $\alpha_{\text{iso},\text{LP}} \geq 1$ , but any trend with redshift is difficult to discern given the error bars. Since the pre-reconstruction  $\alpha_{\text{iso},\text{LP}}$  measurements are all greater than 1.005, the 0.5% correction to the linear point proposed in [62] will neither fully correct this offset nor bring the measurements into agreement with the post-reconstruction values.

The right-hand panel shows that  $\alpha_{\text{iso},\text{LP}}$  is substantially closer to unity in the post-reconstructed field. Here, some of the discrepancy from unity, and with respect to  $\alpha_{\text{iso},\text{BAO}}$ , arises from the fact that  $\Sigma_{\text{iso}} \neq 0$  in the post-reconstruction field. To address this, we correct each  $s_{\text{LP}}$  value by a sample-dependent amount which reflects the bias in the linear point that arises from smearing in the fiducial cosmology. This bias is larger in the pre-reconstructed fields, but recall that it is non-zero post-reconstruction as well.

Figure 3 shows the result of using these modified (corrected)  $s_{\text{LP}}$  values. We see that the offset from the  $\alpha_{\text{iso},\text{BAO}}$  values is reduced, with linear point measurements much closer to unity, indicating better agreement with the fiducial cosmology. However, we note an increase in the size of error bars of up to 29% (48%) in the pre- (post-)reconstruction regime when compared to the uncorrected  $\alpha_{\text{iso},\text{LP}}$  measurements; these arise from propagating the uncertainties on the appropriate value of  $\Sigma_{\text{iso}}$ .

The above correction, when applied to the pre-reconstruction linear point measurements, should in principle completely undo the effects of nonlinearities and result in measurements that agree with linear theory when the correct prior on the isotropic damping scale  $\Sigma_{\text{iso}}$  is chosen. It is evident from the difference in the left and right-side panels in Figure 3 that this is not the case. The scatter in pre-reconstruction  $\alpha_{\text{iso},\text{LP}}$  measurements on mocks is considerably larger than that post-reconstruction, with mean values that remain more than 0.5% biased from unity for most tracers. In contrast, the post-reconstruction  $\alpha_{\text{iso},\text{LP}}$  measurements are more strongly clustered around unity, indicating agreement between measured and fiducial linear point values, while also being more strongly correlated with BAO measurements. The improvement in the signal-to-noise ratio of the BAO signal post-reconstruction

Tracer	Redshift	Recon	$s_{LP} [h^{-1}\text{Mpc}]$	$\alpha_{\text{iso},LP}$	$y_{LP}$	$\chi^2/\text{dof}$
LRG1	0.4-0.6	Pre	$94.09 \pm 2.14$	$0.989 \pm 0.023$	$0.0740 \pm 0.0017$	3.33/6
LRG2	0.6-0.8	Pre	$95.60 \pm 1.70$	$0.973 \pm 0.017$	$0.0584 \pm 0.0010$	4.25/6
LRG3	0.8-1.1	Pre	$92.24 \pm 0.75$	$1.008 \pm 0.008$	$0.0471 \pm 0.0004$	2.89/6
LRG3+ELG1	0.8-1.1	Pre	$92.83 \pm 0.82$	$1.002 \pm 0.009$	$0.0465 \pm 0.0004$	8.53/6
ELG1	0.8-1.1	Pre	$96.00 \pm 3.61$	$0.969 \pm 0.036$	$0.0480 \pm 0.0018$	12.62/6
ELG2	1.1-1.6	Pre	$90.27 \pm 1.69$	$1.022 \pm 0.019$	$0.0372 \pm 0.0007$	6.66/6
BGS	0.1-0.4	Post	$96.09 \pm 3.95$	$0.968 \pm 0.040$	$0.1397 \pm 0.0057$	4.93/6
LRG1	0.4-0.6	Post	$93.87 \pm 0.87$	$0.991 \pm 0.009$	$0.0738 \pm 0.0007$	3.48/6
LRG2	0.6-0.8	Post	$95.86 \pm 1.24$	$0.970 \pm 0.013$	$0.0585 \pm 0.0008$	4.82/6
LRG3	0.8-1.1	Post	$92.21 \pm 0.54$	$1.009 \pm 0.006$	$0.0471 \pm 0.0003$	6.50/6
LRG3+ELG1	0.8-1.1	Post	$92.57 \pm 0.73$	$1.005 \pm 0.008$	$0.0463 \pm 0.0004$	0.96/6
ELG1	0.8-1.1	Post	$93.52 \pm 1.25$	$0.995 \pm 0.013$	$0.0468 \pm 0.0006$	9.58/6
ELG2	1.1-1.6	Post	$93.11 \pm 1.08$	$0.999 \pm 0.012$	$0.0384 \pm 0.0004$	7.48/6

**Table 5:** Linear point measurements on correlation functions of BGS, LRG, and ELG samples from the first DESI data release (DR1). Note that we were unable to measure the linear point on the pre-reconstruction BGS correlation function; that entry is therefore omitted and is discussed in Appendix B.

likely contributes to improved quality of polynomial fits to the correlation function, leading to smaller scatter across the 25 mocks. (See [118] for discussion of why this is expected.) Therefore, reconstruction seems to be a crucial step in ensuring higher precision in linear point measurements, lower scatter due to cosmic variance, and better agreement with isotropic BAO measurements. Combined with the need for a sample-dependent correction, the strictly model-independent nature of the linear point pipeline is compromised if we seek sub-percent precision and agreement with BAO results derived from template-based fits.

## 5 DESI Linear Point Measurements and Comparisons

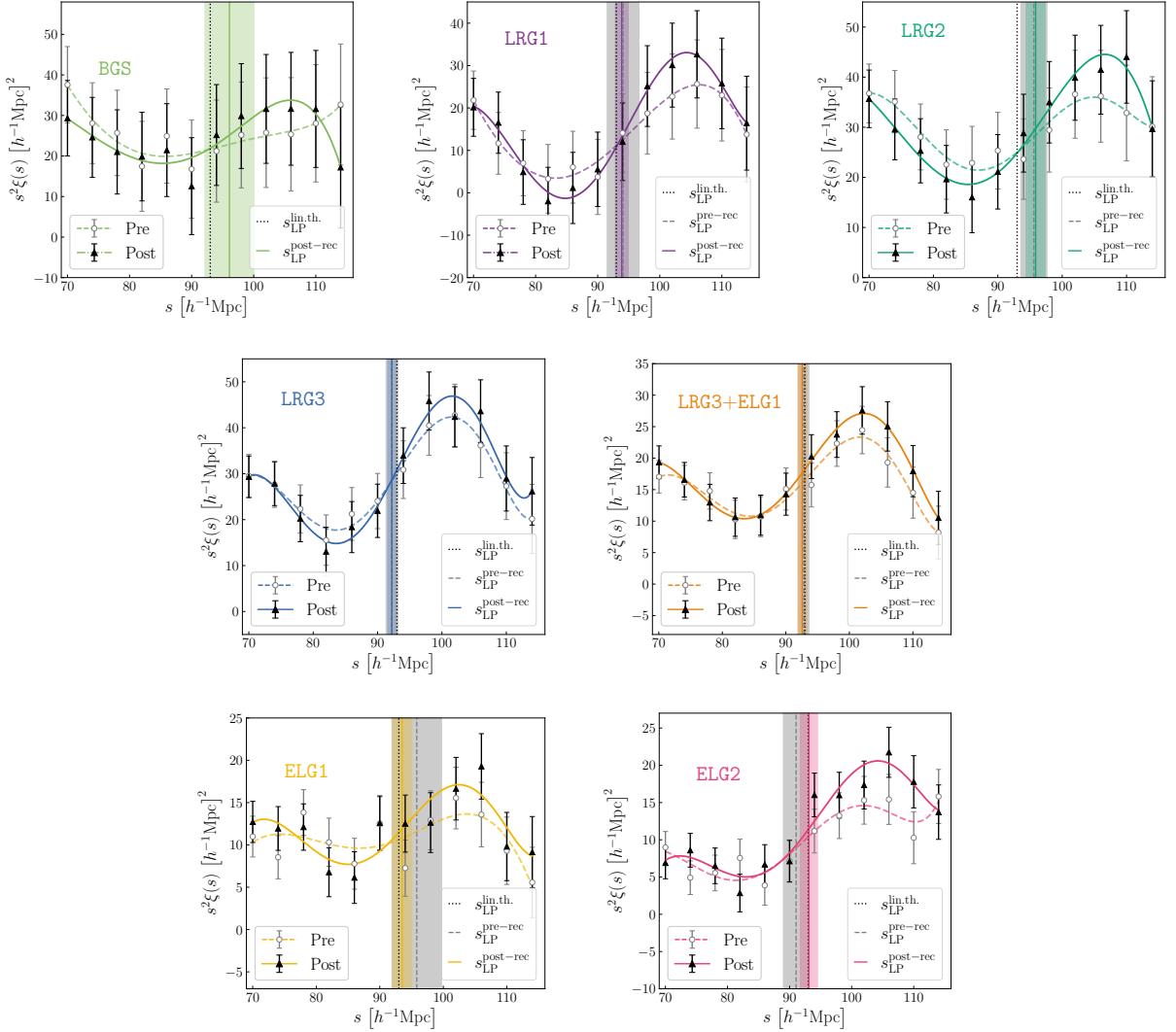
In this section, we present the linear point measurements for the BGS, LRG, and ELG targets that were a part of the first and second DESI data release (DR1 and DR2) using two-point correlation functions and covariance matrices presented in [13, 45]. We compare the linear point measurements to BAO measurements made using the DESI template-based BAO fitting pipeline [13, 45, 121].

### 5.1 DR1 results

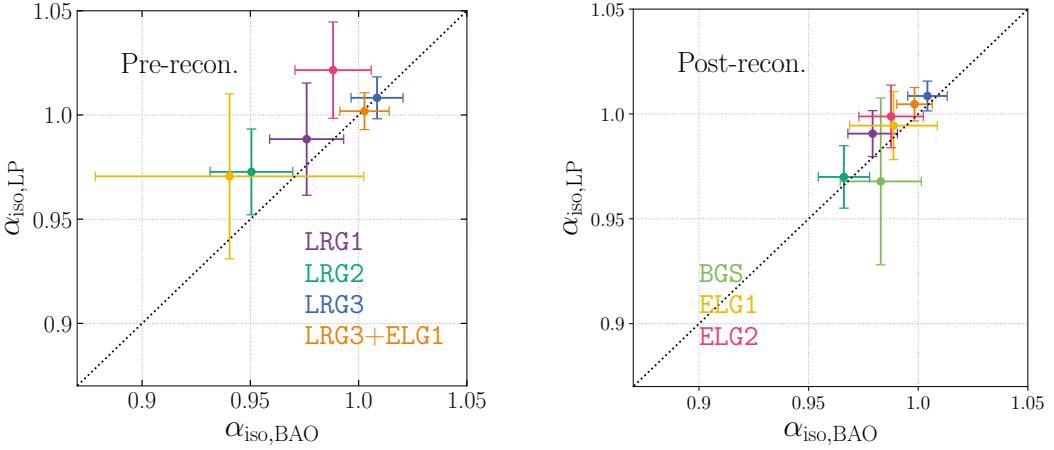
We present the  $s_{LP}$  estimates for correlation function measurements for DR1 samples in Table 5, along with the  $\alpha_{\text{iso},LP}$  and  $y_{LP}$  measurements and the  $\chi^2$  values corresponding to each fit<sup>8</sup>. We plot the pre- and post-reconstruction correlation functions, zoomed in on the region used in the linear point pipeline with quintic degree polynomial fits, in Figure 4, along with the measured linear points (gray band around dashed vertical line, and colored band around solid vertical line, respectively). The vertical dotted line shows  $s_{LP}^{\text{lin.th.}}$ .

We note that we were not able to measure a linear point on the pre-reconstruction BGS correlation function. We present its polynomial fit in Figure 4, where we see that no

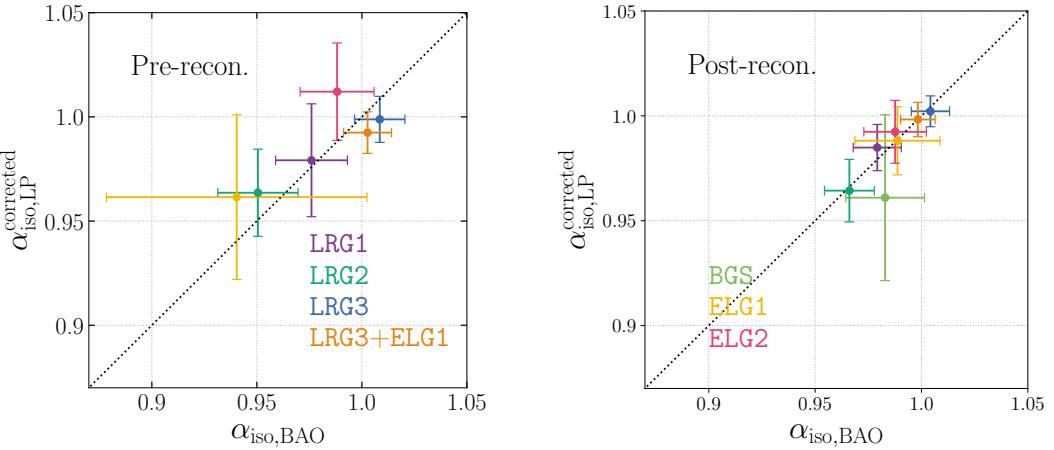
<sup>8</sup>The  $\chi^2$  value for the LRG3+ELG1 tracer post-reconstruction is observed to be smaller than that of other tracers. We estimate that this occurs in about 1% of samples; we therefore consider this a reasonable value.



**Figure 4:** Two-point correlation functions for different DESI DR1 tracers before (open circles) and after reconstruction (solid triangles). The overplotted curves are the best fit polynomial interpolations over the linear point fitting range ( $70 \text{ } h^{-1}\text{Mpc} < s < 115 \text{ } h^{-1}\text{Mpc}$ ) before (dashed lines) and after reconstruction (dotted-dashed lines). The pre-reconstruction (gray) post-reconstruction (colored) linear point measurements are shown as vertical lines with the shaded region representing the  $1\sigma$  error bar. We also plot, as a dotted vertical line, the linear point in the unsmeared linear theory correlation function obtained from **CLASS** for the DESI fiducial cosmology (same in all panels). Note that we were unable to measure the linear point on the pre-reconstruction BGS correlation function; the first panel is therefore missing the gray vertical line and shaded band.



**Figure 5:** The linear point measurements on DR1 tracers, converted to  $\alpha_{\text{iso},\text{LP}}$ , plotted against the BAO measurements made in [13], pre- (left) and post-reconstruction (right).



**Figure 6:** Same as Figure 5, but with the corrected  $\alpha_{\text{iso},\text{LP}}$  measurements accounting for the intrinsic smearing in the fiducial linear point value in linear theory.

peak was identified within the fitting range. This highlights a major drawback of the linear point pipeline – when the correlation function is poorly constrained due to low signal-to-noise ratio and/or large errors, a polynomial is not an optimal fitting function and the linear point feature simply cannot be identified. In these cases, reconstruction becomes essential for improving the signal-to-noise ratio and measuring the linear point. We explore this in more detail in Appendix B.

The naive expectation is that the linear point measured in pre-reconstructed fields (gray bands) should be slightly smaller at lower  $z$ . This was difficult to see in the mocks, and is not obviously the case in DR1; the higher redshift ELG2 sample especially, appears to be shifted to smaller values than the others. (Note that we have *not* multiplied any of our  $s_{\text{LP}}$  values by 1.005 as suggested by [62, 63].) The ELG1 sample has relatively noisier clustering measurements, and ELG2 has low completeness and irregular footprint for DESI DR1 [13, 92].

This increased noise is expected to distort the polynomial interpolation especially of the pre-reconstruction correlation functions (see  $\chi^2$  for pre-recon ELG1 in Table 5), ultimately leading to larger estimates of uncertainties on the linear point (this is especially evident for ELG1 in Fig 4).

The linear point  $\alpha_{\text{iso},\text{LP}}$  measurements are compared against the DR1 isotropic BAO dilation parameter measurements  $\alpha_{\text{iso},\text{BAO}}$  [13] in Figure 5. The error bars on  $\alpha_{\text{iso},\text{LP}}$  are generated using the sampling approach discussed in Section 3.2, while the errors on  $\alpha_{\text{iso},\text{BAO}}$  are an output of the standard fitting pipeline [13, 121]. Whereas Figure 5 is the analog of Figure 2, the analog of Figure 3, showing “corrected”  $\alpha_{\text{iso},\text{LP}}$  values are plotted against the same  $\alpha_{\text{iso},\text{BAO}}$  measurements in Figure 6. Similar to what we saw in the mocks, we see excellent agreement between the two sets of measurements, especially post-reconstruction, which also improves the precision of  $\alpha_{\text{iso},\text{LP}}$  measurements. The uncertainties on the  $s_{\text{LP}}$  measurements on data, however, are smaller than those measured on the isotropic BAO scale, in contrast to mocks, where the  $\alpha_{\text{iso},\text{LP}}$  errors (scatter in measurements) were larger than  $\alpha_{\text{iso},\text{BAO}}$  scatter. We study the discrepancy between mock errors and data errors in Appendix C. Nevertheless, it is striking that the uncorrected pre-reconstruction  $s_{\text{LP}}$  values are slightly smaller than that of the fiducial linear theory value; in the mocks they were slightly larger. Likewise, in the corrected, post-reconstruction values, the estimated distance scale seems, if anything, to be slightly smaller than the fiducial cosmology value, in agreement with the  $\alpha_{\text{iso},\text{BAO}}$  values. With this in mind, we now consider estimates in DESI DR2.

## 5.2 DR2 results

The linear point measurements on the DR2 BGS, LRG, and ELG samples are listed in Table 6, along with  $\alpha_{\text{iso},\text{LP}}$  and  $y_{\text{LP}}$  measurements and best-fit  $\chi^2$  values<sup>9</sup>. The correlation functions, plotted along with the best-fit quintic polynomial interpolation and the linear point location plotted as a vertical line, are shown in Figure 7.

We compare the linear point, converted to  $\alpha_{\text{iso},\text{LP}}$ , to the DR2 isotropic BAO measurements presented in [45],  $\alpha_{\text{iso},\text{BAO}}$ , in Figure 8. The left-side panel shows the  $\alpha_{\text{iso},\text{LP}}$  measurements using  $s_{\text{LP}}^{\text{fid}} = s_{\text{LP}}^{\text{lin.th}}$ , whereas the right-side panel uses the correction to modify the  $s_{\text{LP}}^{\text{fid}}$  using the appropriate post-reconstruction damping parameter from Table 2. As seen in the case of DR1, the two measurements are in good agreement, and the correction on the right-side panel fixes the bias observed in the left-side uncorrected  $\alpha_{\text{iso},\text{LP}}$  measurements. We note that improvements in the DR2 correlation function measurements result in smaller error bars on both linear point and BAO measurements compared to DR1.

The angular size of the linear point standard ruler,  $y_{\text{LP}}$ , is of particular interest, since it is analogous to the observable angular size of the BAO standard ruler. In the left panel of Figure 9, we plot the post-reconstruction linear point standard ruler  $y_{\text{LP}} \equiv s_{\text{LP}}/D_V$  as a function of redshift for the DR1 and DR2 tracers considered in this work. We calculate the isotropic volume distance  $D_V$  for the fiducial cosmology by first obtaining the angular diameter distance  $D_A$  using CLASS and then using Eq. 3.3. We plot  $y_{\text{LP}}^{\text{fid}} \equiv s_{\text{LP}}^{\text{fid}}/D_V$  as a dotted curve. In the right panel, we present a Hubble diagram analogous to the top left panel in Figure 13 in [45]. We calculate  $y_{\text{LP}}$  for the DESI DR2 best-fit  $\Lambda\text{CDM}$  cosmology and the

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<sup>9</sup>The  $\chi^2$  value for the LRG3 tracer pre-reconstruction is observed to be unusually smaller than that of other tracers. We estimate that this occurs in about 0.1% of samples. While this may occur due to overestimating the covariance, we do not have a way to test the statistical significance of this number. However, the  $\chi^2$  value for the same tracer post-reconstruction is reasonable. Similarly, the  $\chi^2$  value for the LRG3+ELG1 tracer post-reconstruction is large; this is estimated to occur about 4% of the time.

Tracer	Redshift	Recon	$s_{\text{LP}} [h^{-1}\text{Mpc}]$	$\alpha_{\text{iso},\text{LP}}$	$y_{\text{LP}}$	$\chi^2/\text{dof}$
BGS	0.1-0.4	Pre	$95.21 \pm 1.53$	$0.977 \pm 0.016$	$0.1193 \pm 0.0019$	13.80/6
LRG1	0.4-0.6	Pre	$91.74 \pm 1.69$	$1.014 \pm 0.019$	$0.0721 \pm 0.0013$	4.78/6
LRG2	0.6-0.8	Pre	$95.02 \pm 1.44$	$0.979 \pm 0.015$	$0.0580 \pm 0.0009$	4.27/6
LRG3	0.8-1.1	Pre	$92.62 \pm 0.80$	$1.004 \pm 0.009$	$0.0473 \pm 0.0004$	0.45/6
LRG3+ELG1	0.8-1.1	Pre	$92.81 \pm 0.89$	$1.002 \pm 0.010$	$0.0464 \pm 0.0004$	11.40/6
ELG1	0.8-1.1	Pre	$92.59 \pm 3.06$	$1.005 \pm 0.033$	$0.0463 \pm 0.0015$	9.80/6
ELG2	1.1-1.6	Pre	$94.03 \pm 1.09$	$0.989 \pm 0.012$	$0.0388 \pm 0.0005$	13.46/6
BGS	0.1-0.4	Post	$93.91 \pm 0.86$	$0.990 \pm 0.009$	$0.1176 \pm 0.0011$	8.44/6
LRG1	0.4-0.6	Post	$93.98 \pm 0.78$	$0.990 \pm 0.008$	$0.0739 \pm 0.0006$	6.89/6
LRG2	0.6-0.8	Post	$94.19 \pm 0.72$	$0.987 \pm 0.008$	$0.0575 \pm 0.0004$	3.68/6
LRG3	0.8-1.1	Post	$92.94 \pm 0.46$	$1.001 \pm 0.005$	$0.0475 \pm 0.0002$	7.90/6
LRG3+ELG1	0.8-1.1	Post	$93.31 \pm 0.42$	$0.997 \pm 0.004$	$0.0467 \pm 0.0002$	13.73/6
ELG1	0.8-1.1	Post	$92.31 \pm 0.99$	$1.008 \pm 0.011$	$0.0462 \pm 0.0005$	4.64/6
ELG2	1.1-1.6	Post	$92.52 \pm 0.55$	$1.005 \pm 0.006$	$0.0382 \pm 0.0002$	8.74/6

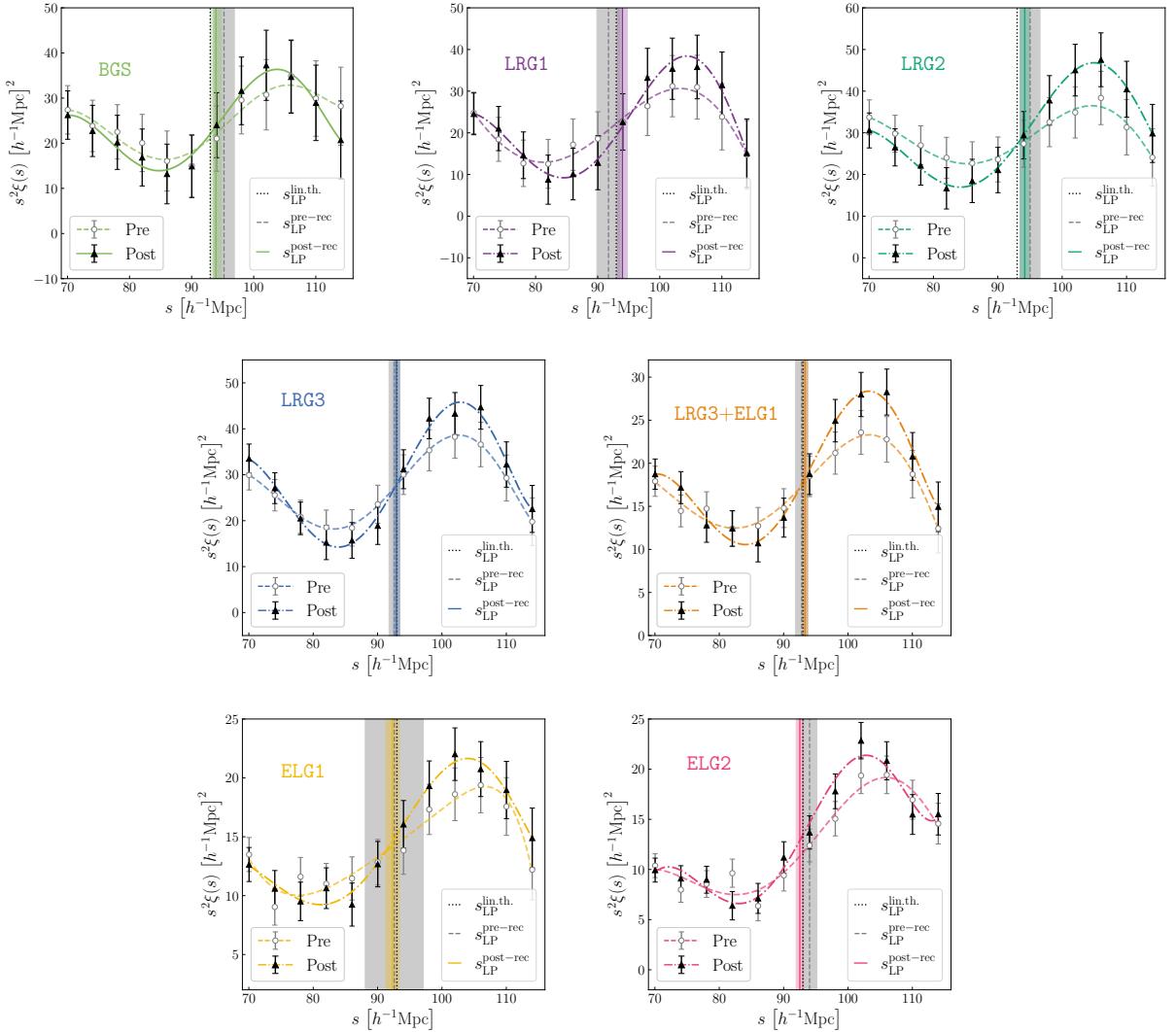
**Table 6:** Linear point measurements on reconstructed correlation functions of tracers from the second data release (DR2) [45] of the DESI survey.

DESI DR2 best-fit  $w_0w_a$ CDM cosmology [45] – we first compute the theoretical linear point using the methodology discussed in 3.2.1, and then  $D_V$  at different redshifts using CLASS as discussed above. We scale these values, and those plotted on the left panel, by  $y_{\text{LP}}^{\text{fid}}$ , and plot them here. As expected, we have improved precision in DR2. Evidently we cannot rule out  $\Lambda$ CDM or  $w_0w_a$ CDM from this plot alone; a full cosmological analysis using the linear point is beyond the scope of this work. We also note that none of the measurements in either panel have been multiplied by 1.005 or otherwise modified to account for smearing, which is expected to impact the consistency we expect to see with the DESI BAO results. Additionally, with the linear point we can only measure the isotropic dilation parameter of the standard ruler  $\alpha_{\text{iso}}$ , whereas the template-based BAO pipeline is able to extract more information by measuring the anisotropic parameter  $\alpha_{\text{AP}}$  as well (see remaining panels in Figure 13 in [45]). This further limits the scope of cosmological inference using the linear point. However, Figure 9 qualitatively corroborates the results presented in [45], additionally providing a model-independent approach to constraining cosmology in the future.

## 6 Conclusion and Discussion

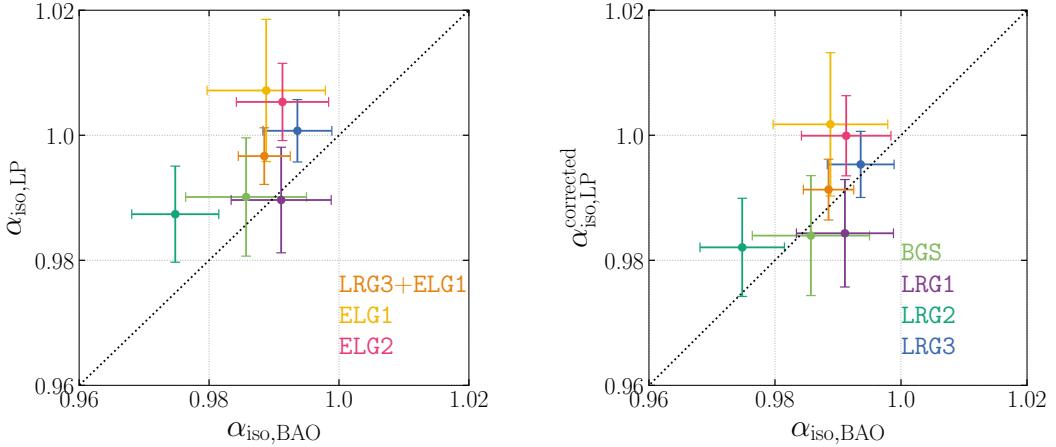
The linear point was proposed as an alternative to the BAO standard ruler due to its increased robustness to nonlinear effects in the two-point correlation function [62, 63]. Defined as the average of the scales corresponding to the BAO peak and the preceding dip in the correlation function, it is a purely geometric standard ruler, since its value depends only on the shape of the correlation function and avoids the need for cosmology-dependent template fitting. In this work, we measure the linear point using the correlation function measurements from the first and second data releases (DR1 and DR2) of DESI.

We validate the model-independent linear point pipeline, a simple polynomial fit to a small section of the correlation function, using Abacus-2 DR1 mock catalogs. We do this in both the evolved and reconstructed fields, when available.

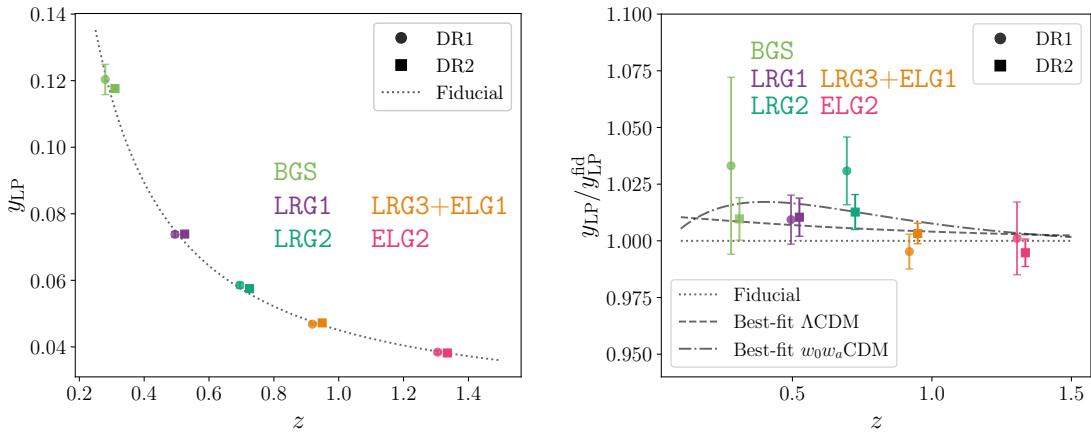


**Figure 7:** Two-point correlation functions for DESI DR2 tracers, with the best-fit quintic polynomial interpolation plotted in dashed (pre-reconstruction) and dotted-dashed lines (post-reconstruction). The black dotted vertical indicated the linear point in linear theory for the fiducial Planck 2018  $\Lambda$ CDM cosmology. The dashed gray and solid colored vertical lines indicate linear points measured pre- and post-reconstruction respectively, and the corresponding shaded regions represent the  $1\sigma$  error bars.

The linear point measurements in the pre-reconstructed field, converted to  $\alpha_{\text{iso},\text{LP}}$  without any modifications for damping, show shifts of up to 1% when compared to the analogous template-based BAO measurements. Even after applying the redshift- and cosmology-independent multiplicative correction of 1.005 prescribed in [62, 63], a residual offset remains that is still large enough to systematically bias constraints on the cosmological distance scale. Therefore, we considered techniques that compromise on the strictly model-independent nature of the linear point pipeline. These were of two flavors: (a) we applied physically motivated sample-dependent shifts to the pre-reconstructed field, and (b) we performed linear point



**Figure 8:** Post-reconstruction linear point measurements on DESI DR2 data, converted to  $\alpha_{\text{iso},\text{LP}}$  before (left) and after (right) applying the linear theory correction, compared to the BAO standard ruler measurements  $\alpha_{\text{iso},\text{BAO}}$  from [45]. Note that the range of values shown is 3× smaller than in Fig.2.



**Figure 9:** The angular form of the linear point standard ruler as a function of redshift. The left panel depicts post-reconstruction  $y_{\text{LP}} \equiv s_{\text{LP}}/D_V$  measurements on DR1 and DR2 tracers considered in this work. The circular markers represent DR1 values, and the square markers represent DR2 measurements. The values of  $y_{\text{LP}}^{\text{fid}}$ , predicted for the fiducial AbacusSummit base cosmology, are plotted as a dotted line. The right panel presents the same measurements as in the left panel, except these are scaled by  $y_{\text{LP}}^{\text{fid}}$ . We additionally compute  $y_{\text{LP}}$  for the best-fit  $\Lambda\text{CDM}$  cosmology and the best-fit  $w_0w_a\text{CDM}$  cosmology, both derived from [45], and plot these values scaled by  $y_{\text{LP}}^{\text{fid}}$ . This is analogous to the top left panel of Figure 13 in [45].

analyses in the post-reconstructed field. To address the former, we presented a preliminary method to correct for smearing due to nonlinear damping, namely using Gaussian convolution on the linear theory correlation function with a kernel sampled from a prior to obtain a fiducial nonlinear correlation function, computing the modified fiducial linear point, and using the ratio of this value and the measurement as the “corrected” parameter in cosmological inference. As for the latter, we noted that standard reconstruction corrects for some of these nonlinear effects, with increased precision and accuracy in the linear point measurements. However, there remains a non-zero damping parameter which biases the measurements in mocks by up to 0.5% compared to the expected value in the fiducial cosmology.

We presented the linear point measurements made on DESI DR1 tracers correlation functions in Table 5 and Figure 4, and those made on DR2 targets in Table 6 and Figure 7. When converted to  $\alpha_{\text{iso},\text{LP}}$  and corrected for the smearing using the aforementioned physically motivated shift, the linear point shows excellent agreement with isotropic template-based BAO measurements presented in [13]. We find that while the uncorrected  $\alpha_{\text{iso},\text{LP}}$  values are shifted by roughly 0.5% post-reconstruction for all tracers, the magnitudes of errors in  $\alpha_{\text{iso},\text{LP}}$  marginally increase by up to 4% due to the uncertainty in the damping scale propagating into these measurements.

The offset in the linear point compared to the standard template-based BAO measurements, while seemingly more pronounced in mocks than in the data, implies that this completely model-independent standard ruler will bias cosmological parameters if nonlinear smearing is unaccounted for. We also note that the linear point pipeline is more sensitive to the signal-to-noise ratio of the BAO peak than template-based fitting approaches; this is evident by the fact that no linear point was detected in the DR1 BGS correlation function pre-reconstruction. This further motivates the use of reconstruction in implementing linear point-inspired analyses.

This work demonstrates that the linear point is an attractive addition to cosmological distance scale analyses, particularly in tandem with standard reconstruction. It would be interesting to see if other reconstruction methods, e.g. the recent ones based on Optimal Transport [50–54], return reconstructions with smaller residual damping. As shown in [67], it is possible to extract  $H_0$  and  $\Omega_m$  using the linear point; while we leave these analyses for the future, it would be interesting to compare these parameters derived using the linear point and those derived using the DESI BAO measurements.

## 7 Data Availability

The data used in this analysis is part of the DESI Data Release 1, available at available at <https://data.desi.lbl.gov/doc/releases/dr1/>, and Data Release 2, which will be made public (details in <https://data.desi.lbl.gov/doc/releases/>).

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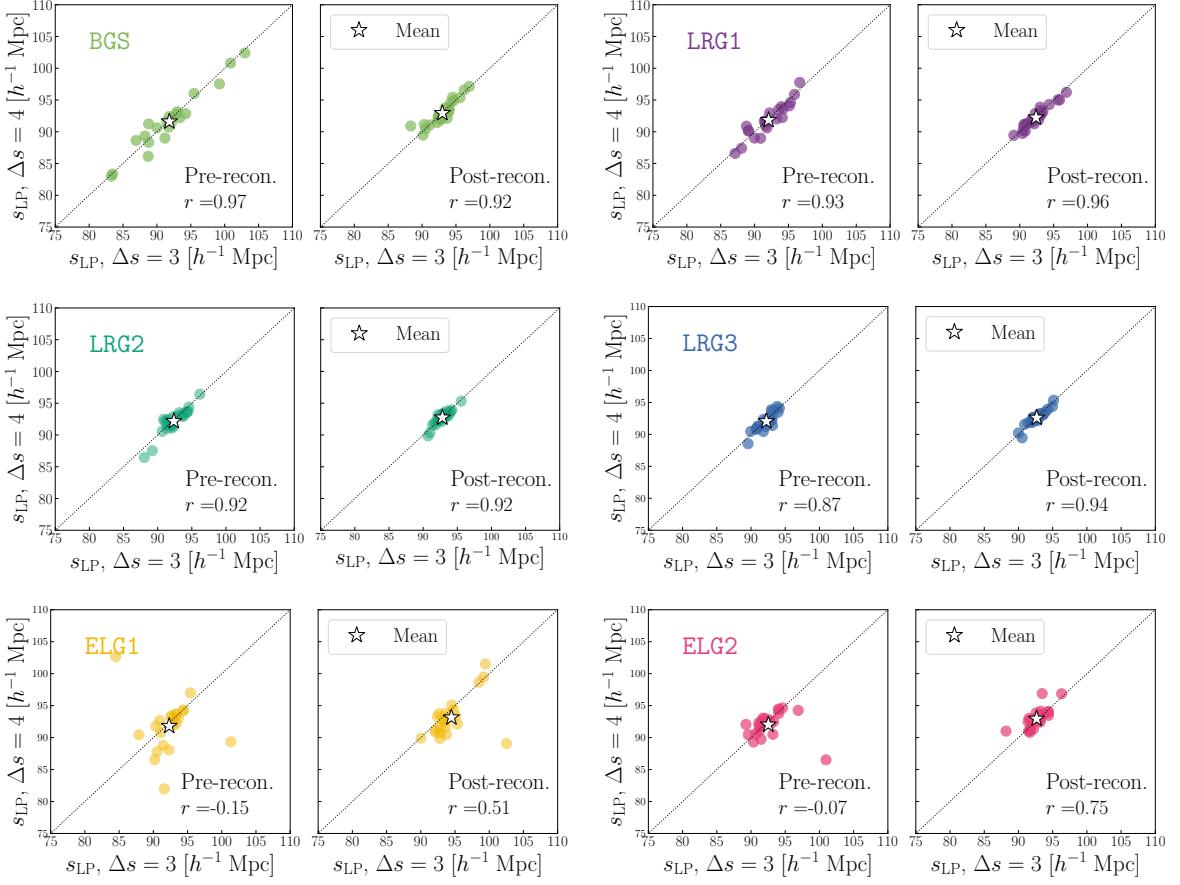
The authors are honored to be permitted to conduct scientific research on I’oligam Du’ag (Kitt Peak), a mountain with particular significance to the Tohono O’odham Nation.

## A Optimizing Free Parameters in the Linear Point Pipeline

The linear point pipeline consists of a model-independent fit to a narrow region of the two-point correlation function. In [64], a detailed validation process suggested that a quintic polynomial ( $n = 5$ ) fit to the correlation function calculated with a bin width  $\Delta s = 3 h^{-1}\text{Mpc}$  in the range  $60\text{--}120 h^{-1}\text{Mpc}$  was the most optimal choice of free parameters in the pipeline. Strictly speaking, their validation tests are applicable to a BOSS-like survey. However, the Bayesian analysis in [117] suggests that the same choices should remain appropriate at least for DESI **BGS**, **LRG** and potentially **ELG** samples. In this section, we make adjustments to the recommended optimal parameters in [64] for DESI correlation functions and covariance matrices to optimize the pipeline.

Since we scale and center the correlation function measurements using  $s_0$  and  $\sigma$  in Eq. 3.10, our pipeline is largely unaffected by the range of the correlation function, as long as a polynomial interpolation remains a valid approximation for the general shape. We choose to fit over the range  $70\text{--}115 h^{-1}\text{Mpc}$  in our pipeline, since it is more symmetric with respect to the linear point in linear theory for the fiducial  $\Lambda\text{CDM}$  cosmology and retains the polynomial nature of the correlation function.

All DESI BAO analyses have been performed with  $\Delta s = 4 h^{-1}\text{Mpc}$  given the availability of analytic covariance matrices in configuration space for this bin width. We compare the linear points measured on  $3 h^{-1}\text{Mpc}$  and  $4 h^{-1}\text{Mpc}$  binned correlation functions for the 25 **Abacus-2** mocks for each tracer using fifth degree polynomials in Figure 10. We compute the Pearson correlation coefficient,  $r$ , to quantify the scatter between these measurements. The **BGS** and **LRG** measurements are highly correlated both pre- and post-reconstruction, whereas the **ELG** measurements contain considerable scatter due to outliers. Upon visual inspection of the linear point fits of the outlier mocks, we find that random noise in the correlation function resulted in vastly different identification of roots. Higher signal-to-noise ratio in DR2 mocks and data is expected to mitigate this issue. Nevertheless, the mean linear point measurements (gray star) in mocks are in agreement regardless of our choice of bin width. We therefore sample the correlation function with  $4 h^{-1}\text{Mpc}$  bin widths in our linear point

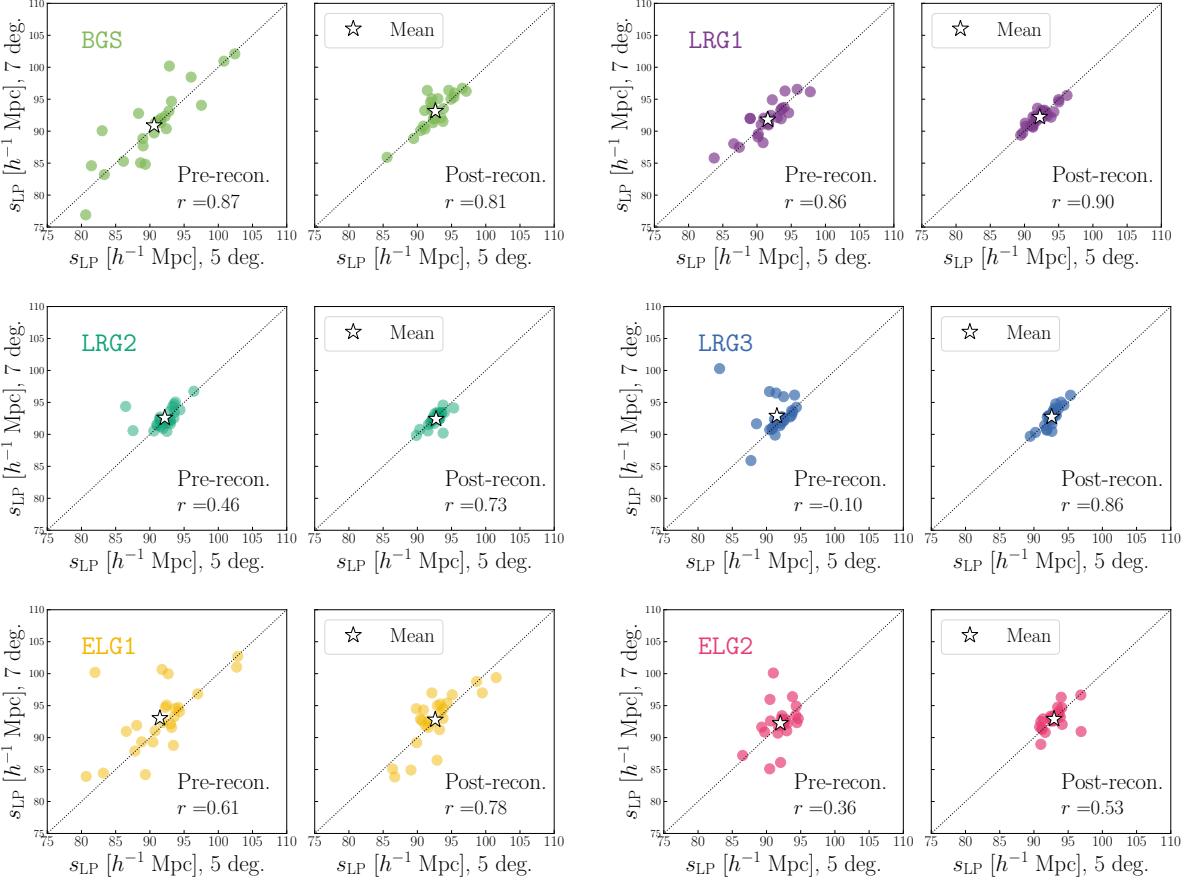


**Figure 10:** Linear point measurements on correlation functions measured on **Abacus-2** mocks with bins width  $\Delta s = 3 h^{-1}\text{Mpc}$  and  $4 h^{-1}\text{Mpc}$ . We use numerical covariance matrix calculated using the 1000 **EZmocks** for  $3 h^{-1}\text{Mpc}$  binned correlation functions and the analytical covariance matrix calculated using the **RascalC** code for  $4 h^{-1}\text{Mpc}$  binned correlation functions. In both cases, we use a fifth-degree polynomial interpolation to calculate the linear point. The left panel for each tracer depicts linear points calculated pre-reconstruction and the right panel, post-reconstruction. We additionally plot the average of the 25 mocks in each panel (white star marker).

pipeline. We confirm in Figure 11 that fifth degree polynomials are still an appropriate choice despite the change in bin width, given small scatter between the two measurements. Our findings are consistent with expectations about the order of polynomial [117] and dependence on binning [118].

## B The Linear Point with Poor Signal-to-Noise Ratio

A major drawback of the linear point pipeline, mentioned in Section 3.2, is that the polynomial fit may not be able to identify a peak and dip in the correlation function when the signal-to-noise ratio is poor, therefore unable to measure a linear point. This was explored in [67] with MultiDark-Patchy mocks [122], where the term ‘reliability’ was defined as the percentage of mocks that yielded a measurable linear point. We measure the reliability in



**Figure 11:** Linear point measurements on correlation functions with bin width  $4 h^{-1} \text{Mpc}$  measured on **Abacus-2** mocks with fifth degree ( $n = 5$ ) and seventh degree ( $n = 7$ ) polynomial interpolations. The left panel for each tracer depicts linear points calculated pre-reconstruction and the right panel, post-reconstruction. We additionally the average of the 25 mocks in each panel (white star marker).

**Abacus-2** mocks and find that post-reconstruction, all mocks for all tracers yield a measurement. However, pre-reconstruction we find 100% reliability in LRG2, LRG3, and both sets of ELG mocks, but find that one BGS mock and one LRG1 mock fail to yield a measurable linear point. A minimum of 96% reliability is encouraging; however, we fail to measure a linear point in the pre-reconstruction BGS DR1 correlation function. We explore this below.

In the context of data, we can interpret reliability through the signal-to-noise ratio of the data and the resulting polynomial fit. In Section 3.2, we mention that to calculate the error in a single linear point measurement, we generate multiple realizations of polynomial fit to the correlation function using the covariance matrix of the best-fit coefficients. We then calculate the linear point of each realization and report the standard deviation of the samples as the error in the original measurement. Not all realizations will result in a measurement, and so the percentage of failed measurements can serve as a qualitative measure of the signal-to-noise ratio of the data. We define the percentage of sampled polynomial fits that did *not* result in a linear point measurement as the failure rate, and present the DR1 failure rates in Table 7.

Tracer	Redshift	Failure Rate	
		Pre	Post
BGS	0.1-0.4	34.0%	7.9%
LRG1	0.4-0.6	0.6%	0.0%
LRG2	0.6-0.8	5.9%	0.2%
LRG3	0.8-1.1	0.0%	0.0%
LRG3+ELG1	0.8-1.1	0.0%	0.0%
ELG1	0.8-1.1	12.2%	0.5%
ELG2	1.1-1.6	0.9%	0.0%

**Table 7:** Failure rates, defined as the percentage of samples of the polynomial interpolations of the correlation functions, for DR1 tracers pre- and post-reconstruction.

We see that the **BGS** correlation function pre-reconstruction has a very high failure rate – over a third of the polynomial interpolations of the correlation function fail to identify a linear point, which may explain why we did not measure the linear point feature in the DR1 correlation function. In Figure 12, we present a histogram of the linear point values that were measured during this sampling process. For each tracer, we intuitively expect the distribution to be Gaussian centered around the mean of the sample, which should also agree with the linear point originally measured on the DR1 correlation function. While we see this trend in the post-reconstruction **BGS** samples (despite a relatively higher failure rate), the pre-reconstruction samples are in fact bimodal. This means that due to the poor fit to the correlation function, not only is the linear point not measurable, but the mean of the samples is also an unreliable estimate of the measurement.

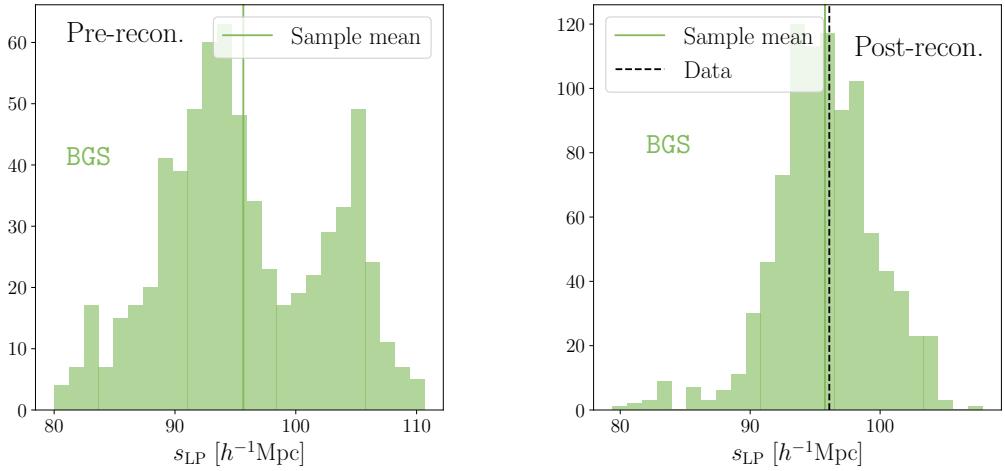
We also note a relatively higher failure rate for pre-reconstruction **ELG1** correlation function; while the linear point is still measured in this case, the relatively high  $\chi^2$  value of the fit and the large error bars on the linear point (presented in Table 5 and Figure 4) hint at poor signal-to-noise ratio.

The linear point pipeline, as it stands, is therefore not optimal for measuring a standard ruler when the correlation function is poorly measured or the errors on those measurements are large. In such cases, template-based fitting approaches may indeed be a necessity. An alternative could be modifying the fitting functions used in the linear point pipeline; in fact, [67] explore fitting a fifth polynomial to  $s^2\xi(s)$  instead of  $\xi(s)$ . They find that this improves reliability in Patchy mocks. We leave such implementations to future works.

## C Error Analysis in Y1 Mocks and Data

We have shown in Sections 4 and 5 that the linear point standard ruler, converted to  $\alpha_{\text{iso},LP}$ , shows excellent agreement with the BAO standard ruler measured using the standard fitting pipeline. However, the scatter in the linear point measurements on LRG and ELG mocks is 8–90% larger post-reconstruction, compared to the errors on  $\alpha_{\text{iso},LP}$  measured on DR1 tracers, which range from 7% larger to 50% smaller than those measured on the isotropic BAO scale post-reconstruction.

We study this discrepancy between mock errors and data errors by calculating the uncertainty  $\sigma_{s_{LP}}$  on the linear point measured on the 25 mocks and comparing their distribution to the error calculated on the data. We plot the distribution of post-reconstruction errors for the mocks in Figure 13 and indicate the error calculated on the data measurements as

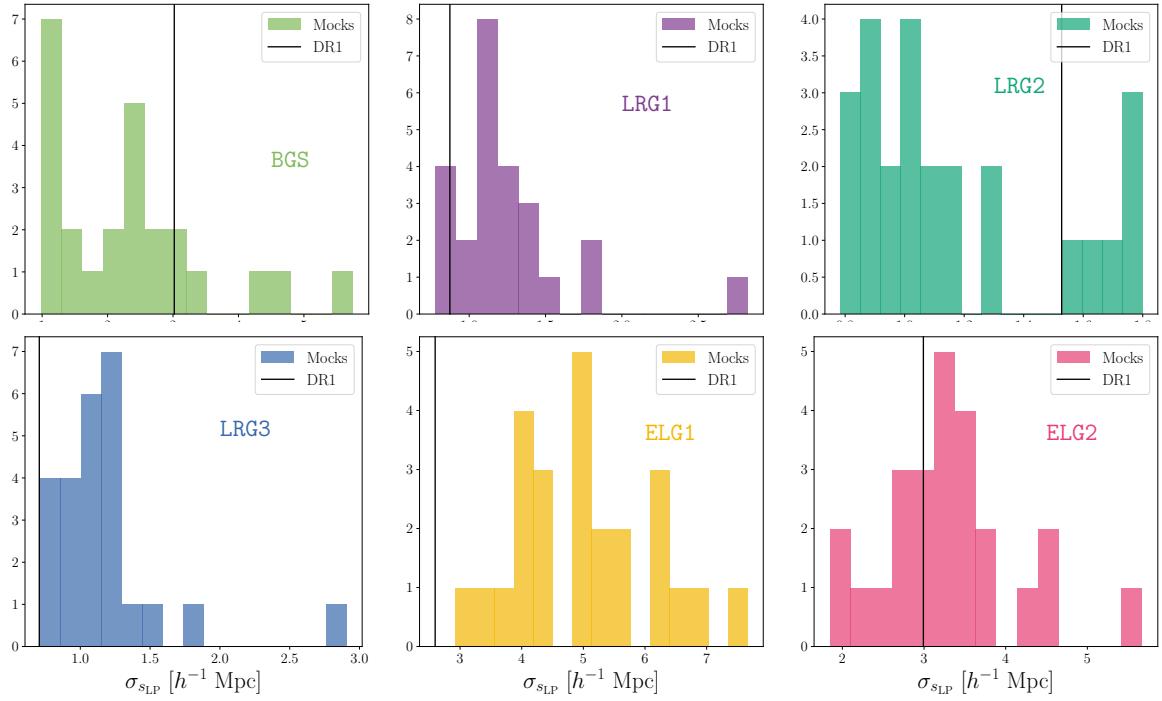


**Figure 12:** Histograms of the linear points measured on polynomial interpolations of the DR1 BGS correlation functions pre- (left) and post-reconstruction (right) sampled from the covariance matrix of the best-fit polynomial coefficients. The solid vertical line represents the mean of the sample, whereas the black dotted vertical line represents the linear point measured on the original correlation function. The latter is missing from the left panel because we were unable to measure a linear point on the pre-reconstruction BGS correlation function.

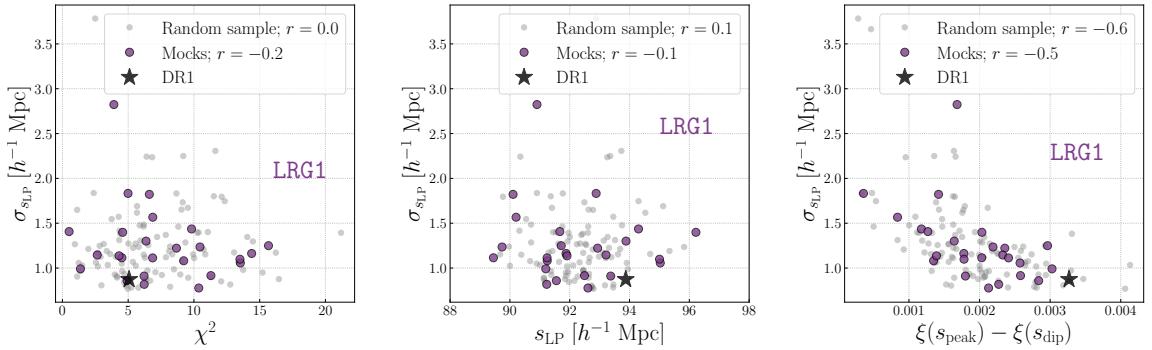
a vertical line. There is a large scatter in the error values, and only LRG1 and ELG2 exhibit somewhat Gaussian-like distributions. With the exception of LRG2 and ELG2, the errors on the data measurements are consistently on the lower end of the distribution of mock errors.

We further explore any correlations that may arise between  $\sigma_{s_{LP}}$  and factors that might affect the quality of our measurements, such as the  $\chi^2$  of the polynomial fit, the location of the linear point itself, and the amplitude of the BAO peak in the correlation function. We choose to perform these tests using the LRG1 mocks, since the distribution of mock errors is well-behaved and the data error is found on the lower end of the mock distribution. We plot the mock errors as a function of each of these quantities in Figure 14, and indicate the data error as a black star in each panel. We measure the Pearson correlation coefficient  $r$  in each panel to predict the correlation between the two quantities plotted in each panel of the figure.

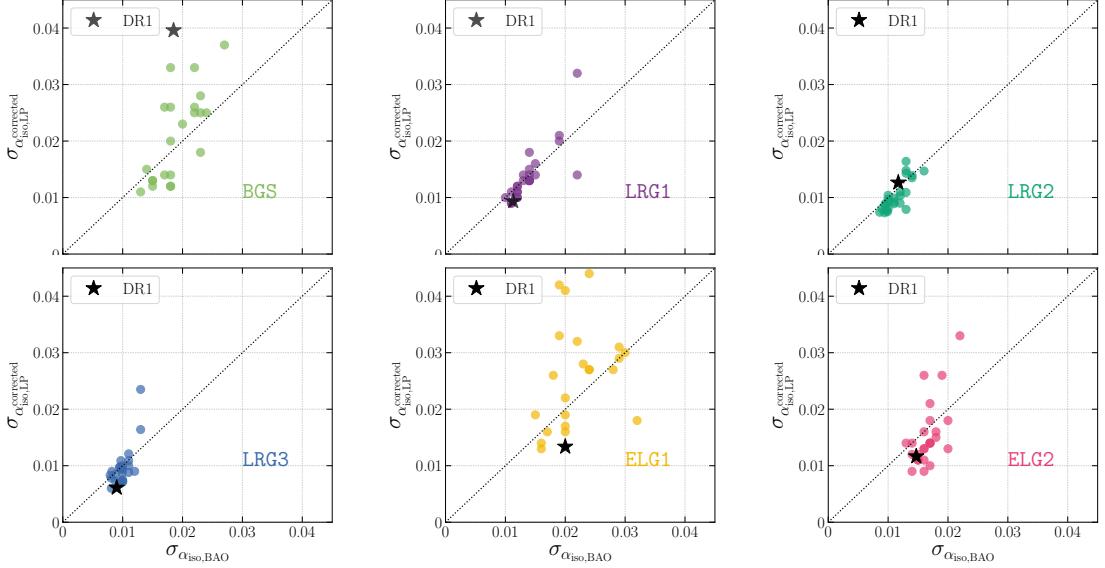
While we find little to no correlation between the  $\chi^2$  and the location of the linear point  $s_{LP}$  (left-most and central panel of Figure 14), we find that the error is negatively correlated with the absolute difference between the size of the BAO peak and the preceding dip,  $\xi(s_{\text{peak}}) - \xi(s_{\text{dip}})$ , as shown in the right-side panel of Figure 14. The scales corresponding to the peak and the dip,  $s_{\text{peak}}$  and  $s_{\text{dip}}$ , are computed using the roots of the best-fit polynomial; and the amplitudes,  $\xi(s_{\text{peak}})$  and  $\xi(s_{\text{dip}})$ , are computed by evaluating the polynomial interpolation of the correlation function at the peak and dip scales. As indicated in the figure, lower errors on the Y1 linear point measurements are a direct consequence of a more pronounced BAO peak in the data correlation function compared to those in the mocks. This is consistent with previous work [118] which argued that the error depends on the sharpness (i.e. curvature) of the peak and dip features: if the separation between both features



**Figure 13:** Histograms of the post-reconstruction errors on the linear point measurements on the 25 Abacus-2 mocks for the DESI DR1 BGS, LRG, and ELG samples. The error on the data linear point measurement is indicated with a solid black vertical line.



**Figure 14:** The post-reconstruction error on linear point measurements on each of the 25 LRG1 Abacus-2 mocks plotted as a function of the  $\chi^2$  of the polynomial fit (left), the linear point measurement  $s_{LP}$  (center), and the difference between the amplitudes of the BAO peak  $\xi(s_{peak})$  and the preceding dip  $\xi(s_{dip})$  (right). The same quantities calculated on the Y1 data from the first data release (DR1) are indicated using a black star marker in each panel.



**Figure 15:** The errors on the isotropic BAO parameter plotted against of the errors on the corresponding linear point parameter for each set of 25 Abacus-2 mocks. The black star marker indicates the same values calculated on DESI DR1 tracers.

is approximately fixed, then larger curvature implies larger  $\xi(s_{\text{peak}}) - \xi(s_{\text{dip}})$ . We perform the same tests using linear point errors calculated from a randomly generated sample of 100 correlation functions centered on the mean of the 25 LRG1 mock correlation functions using the covariance matrix. These errors are plotted in gray in Figure 14, and indicate that the trend seen in the mocks is an accurate representation of a random sample.

We also test whether smaller errors on the linear point translate to smaller errors in BAO measurements. We convert the post-reconstruction  $s_{\text{LP}}$  errors calculated on each mock to errors on corrected  $\alpha_{\text{iso},\text{LP}}$  values ( $\sigma_{\alpha_{\text{iso},\text{LP}}^{\text{corrected}}}$ ) and plot them against the errors on isotropic BAO measurements ( $\sigma_{\alpha_{\text{iso},\text{BAO}}}$ ) in Figure 15. We depict the errors in the Y1  $\alpha_{\text{iso},\text{LP}}$  and  $\alpha_{\text{iso},\text{BAO}}$  measurements with a black star marker in each panel. We see that the data BAO error is also on the lower end of the mock error distribution, which is consistent with the distribution of  $\alpha_{\text{iso},\text{LP}}$  errors.

## D Laguerre reconstruction

In large-scale cosmological analysis, the evolved correlation function on BAO scales can be modeled as a Gaussian convolution of the linear theory correlation function (c.f. Eq. 3.8). This suggests that an analytical deconvolution of the measured correlation function could serve as an alternative to standard reconstruction algorithms, which typically operate on the matter density field. That is, rather than reconstructing the field and moving particles, this approach directly works with the two-point statistics of the field to recover its linear form, potentially offering a much faster and simpler method. Within the specific range of 60–120  $h^{-1}$  Mpc, the linear theory correlation function can be accurately approximated by a simple odd-degree polynomial. Consequently, the corresponding nonlinear correlation function can be effectively fit using a linear combination of generalized Laguerre functions (i.e. Gaussian-

convolved polynomials), which are well-suited to capture its behavior in this range, providing a robust mathematical framework for this alternative reconstruction method.

In more detail, if the linear theory correlation function is approximated as a simple  $n$ th order polynomial

$$\xi_{\text{lin.th.}}(r) = \sum_{k=0}^n a_k \left(\frac{r}{\sigma}\right)^k, \quad (\text{D.1})$$

where  $\sigma$  is set to a fiducial value to keep the coefficients  $a_k$  dimensionless, and the nonlinear correlation function is just a convolution with a Gaussian of rms  $\Sigma$ , then

$$\xi_{\text{NL}}(s) = \sum_{k=0}^n c_k \mu_k(x), \quad \text{where } x \equiv s/\Sigma, \quad (\text{D.2})$$

$c_k \equiv a_k (\Sigma/\sigma)^k$ , and the modified Laguerre functions  $\mu_k$  are defined as follows:

$$\mu_{2n}(x) = 2n!! L_n^{(1/2)}(-x^2/2) \quad \mu_{2n-1}(x) = (2n-1)!! \sqrt{\frac{\pi}{2}} L_{n-1/2}^{(1/2)}(-x^2/2). \quad (\text{D.3})$$

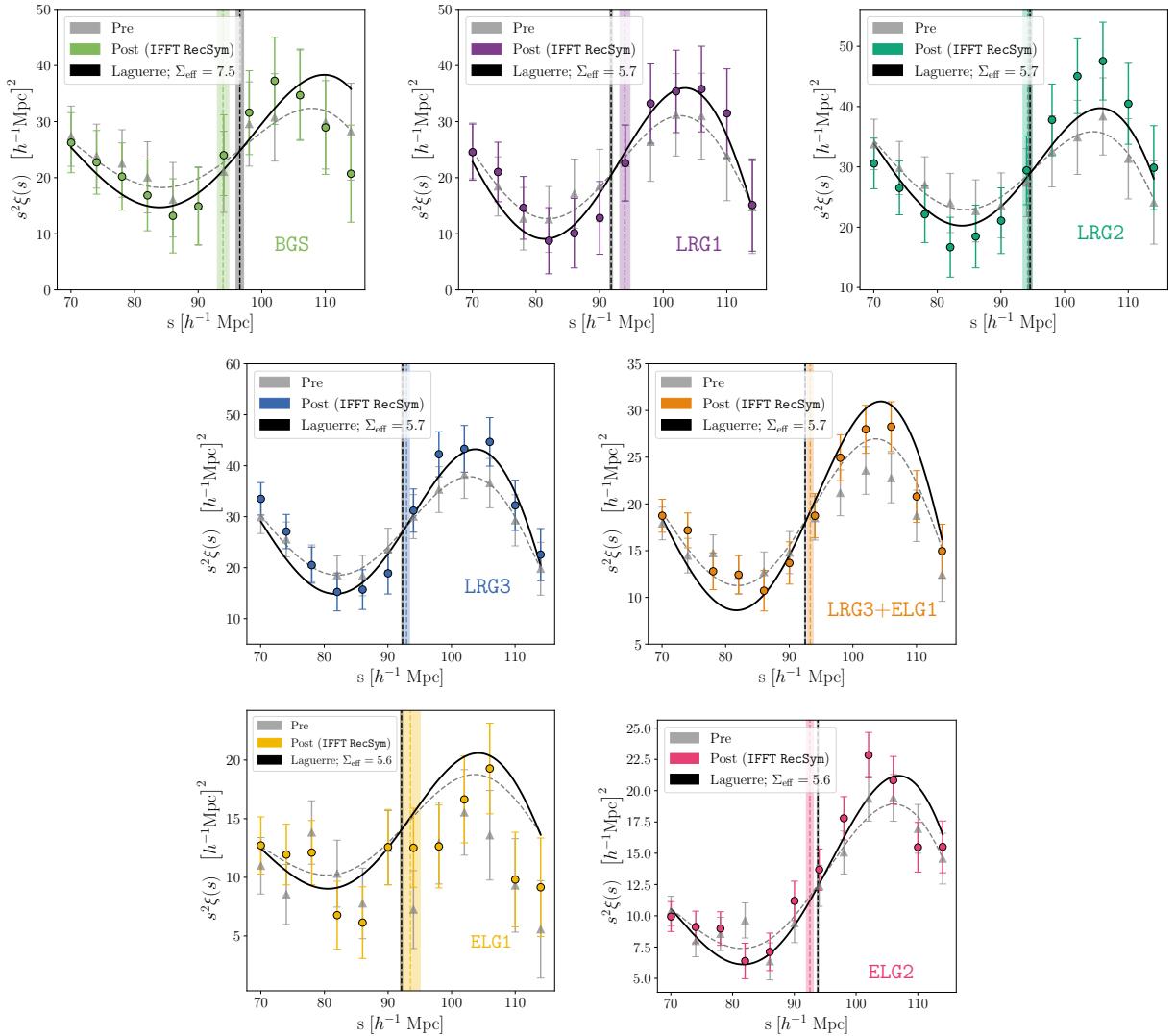
Here,  $L_\beta^{(\alpha)}$  are generalized Laguerre functions, listed in the Appendix of [48].

The Laguerre reconstruction pipeline leverages this as follows. When Eq. D.2 is fit to the nonlinear (observed) correlation function, then the best-fit Laguerre coefficients  $c_k$  are used to estimate  $a_k = c_k (\sigma/\Sigma)^k$ . Substitution of these  $a_k$  into Eq. D.1 yields the reconstructed linear correlation function [48, 49].

Unlike standard reconstruction algorithms that focus on reconstructing the matter density field, the Laguerre method targets the shape of the monopole of the correlation function [for initial attempts that extend this framework to higher order multipoles, see 123, 124]. Additionally, it complements the purely geometric nature of the linear point pipeline, enhancing the toolkit available for BAO analysis with a method that is both efficient and broadly applicable. While model-independent, the Laguerre pipeline contains a non-trivial dependence on the Gaussian smearing kernel  $\Sigma$ , which is completely degenerate with the fitting coefficients  $c_k$  in eq. D.2. Therefore, a rigorous implementation of Laguerre reconstruction requires the use of informed priors on  $\Sigma$ , which introduces some cosmology dependence in the pipeline.

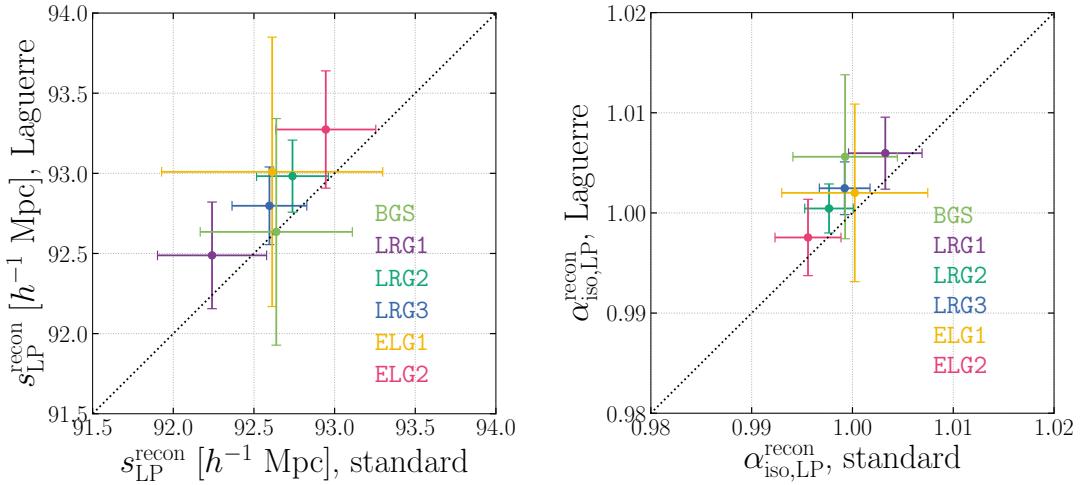
Here, we present preliminary reconstructions of the observed nonlinear correlation function measurements for DR2 targets using the Laguerre method. We use the pre-reconstruction template Gaussian prior on  $\Sigma_{\text{iso}}$  from Table 2 to inform our choice of  $\Sigma$  in the pipeline. The pre- and post-reconstruction correlation functions, along with the Laguerre reconstructed curve, are shown in Figure 16. We observe that Laguerre reconstruction, plotted as a black solid curve, seem to mostly agree with the correlation function calculated post standard reconstruction, shown as colored circles. However, evidently in the case of LRG2, the deconvolution falls short of the post-reconstruction correlation function, indicating a smaller  $\Sigma$  used than what was needed. On the other hand, in the case of LRG3+ELG1, we seem to have overestimated  $\Sigma$ . This highlights the drawback of the Laguerre reconstruction method – since the kernel is perfectly degenerate with the fitting coefficients, there is little we can do to mitigate this issue as of now.

We further compute the linear point on the Laguerre-reconstructed correlation functions in BGS, LRG, and ELG Abacus-2 mocks and compare these measurements to those made on standard post-reconstruction correlation functions in the left panel of Figure 17. The two measurements show remarkable agreement, albeit with a slight bias of  $\sim 0.2\%$ . This could be



**Figure 16:** Pre-reconstruction (gray triangles) and post-reconstruction correlation functions (colored circles) for DR2 DESI tracers , along with the Laguerre function fit to the pre-reconstruction correlation function (gray dashed curve) and Laguerre reconstructed (i.e. associated simple polynomial) correlation function (solid black curve). The linear points calculated post standard reconstruction (colored dashed vertical line) and post Laguerre reconstruction (black dashed vertical line) are plotted along with their  $1\sigma$  uncertainties.

attributed to the fact that standard reconstruction retains some nonlinear effects as evidenced by non-zero damping in Table 2. We convert the two sets of linear point measurements to  $\alpha_{\text{iso},\text{LP}}$  using Eq. 3.7 – we use  $s_{\text{LP}}^{\text{fid}} = 93.01 h^{-1}\text{Mpc}$  in the case of Laguerre-reconstructed measurements, and  $s_{\text{LP}}^{\text{fid}} = 92.51 h^{-1}\text{Mpc}$  for standard reconstruction, since they retain the effects of smearing. We plot these measurements in the right side panel of Figure 17, and note excellent correlation between the mean linear point measurements and the errors.



**Figure 17:** Comparing the linear point measurements made on LRG and ELG Abacus-2 mocks using standard reconstruction and Laguerre reconstruction. Left panel shows mean linear point measurements using standard post-reconstruction correlation functions (x-axis) and Laguerre-reconstructed correlation functions (y-axis) with  $\Sigma = 5.7 h^{-1} \text{Mpc}$ . Right panel shows the same measurements converted to  $\alpha_{\text{iso},\text{LP}}$  using eq. 3.7, with  $s_{\text{LP}}^{\text{fid}} = 92.51 h^{-1} \text{Mpc}$  for DESI values to reflect the non-zero smearing of the post-reconstruction regime, and retaining  $s_{\text{LP}}^{\text{fid}} = 93.01 h^{-1} \text{Mpc}$  for Laguerre values. The error bars on each data point are the standard deviations of the 25 measurements, scaled by the square root of the number of mocks.

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