$$f(x) = \sum_{n=0}^{N} c_n P_n(x)$$

$$\int_{-1}^{1} f(x) P_m(x) dx = \sum_{n=0}^{N} c_n \int_{-1}^{1} P_n(x) P_m(x) dx$$

$$\int_{-1}^{1} f(x) P_n(x) dx = \frac{2}{2n+1} c_n$$

$$c_n = \frac{2n+1}{2} \int_{-1}^{1} f(x) P_n(x) dx \quad n = 0,1,2...N$$

http://www.sc.ehu.es/sbweb/fisica3/especial/legendre/legendre.html