3.

$$x^3 y' = x^4 y^2 - 2x^2 y - 1$$

$y_1 = x^{-2}$

$y_1' = -2x^{-3}$

$-2 = 1 - 2 - 1$

$$y = \underbrace{x^{-2}}_{y_1} + u^{-1}$$

$$y' = -2x^{-3} - u^{-2} u'$$

$$-2 - x^3 u^{-2} u' = x^4 (x^{-2} + u^{-1})^2$$

$$- 2x^2 (x^{-2} + u^{-1}) - 1$$

$$-1 - x^3 u^{-2} u' = x^4 (x^{-4} + 2x^{-2} u^{-1} + u^{-2})$$
$$- 2 - 2x^2 u^{-1}$$

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$$1 - x^3 u^{-2} u' = 1 + 2x^2 u^{-1} + x^4 u^{-2}$$

$$1 - 2x^2 u^{-1}$$

$$-x^3 \cancel{u^{-2}} u' = x^4 \cancel{u^{-2}}$$

$$u' = -x$$

$$u = -\frac{x^2}{2} + c$$

$$y = x^{-2} + \left(-\frac{x^2}{2} + c\right)^{-1}$$

$$y = x^{-2} + \frac{2}{2c - x^2} \qquad y(\sqrt{2}) = 0$$

$$0 = (\sqrt{2})^{-2} + \frac{2}{2c - (\sqrt{2})^2}$$

$$-2^{-1} = \frac{2}{2c - 2} \qquad 2c - 2 = -4$$

$$c = 3$$

4.

$$\frac{du}{dt} = u^q \quad , \quad t \in [0, 10]$$

$$\int \frac{du}{u^q} = \int dt$$

$\boxed{q = 1}$

$$\ln u^q = t + C$$

$$\ln u = t + C$$

$$u = e^t C$$

$\boxed{q \neq 1}$

$$\int u^{-q} du = \int dt$$

$$\frac{u^{-q+1}}{1-q} = t + C$$

$$u^{-q+1} = (t + C)(1 - q)$$

$$u = ((1-q)t + b)^{\frac{1}{1-q}}$$

$q = 0$

$\downarrow$

$u = t + b$

2.

$$\vec{r}_{n+1} = 2\vec{r}_n - \vec{r}_{n-1} + \vec{a}(\vec{r}_n)h^2$$

$\cancel{\vec{r}_n +}$

$$\cancel{\vec{r}_{n+1}} + \epsilon_{n+1} = 2\vec{r}_n + 2\epsilon_n - \cancel{\vec{r}_{n-1}} - \epsilon_{n-1} + \underbrace{\vec{a}(\vec{r}_n + \epsilon_n)}_{\cancel{\vec{a}(\vec{r}_n)} + \epsilon_n \vec{a}'(\vec{r}_n)} h^2$$

$\cancel{\epsilon_{n+1} = 2}$

$$\epsilon_{n+1} = 2\epsilon_n - \epsilon_{n-1} + \epsilon_n \vec{a}'_n h^2$$

$$\epsilon_{n+1} - 2\epsilon_n + \epsilon_{n-1} - \epsilon_n a'_n h^2 = 0$$

$a = -\omega^2 x$

$a' = -\omega^2$

$$\epsilon_{n+1} - (2 + \underbrace{(-\omega^2)h^2}_{2R})\epsilon_n + \epsilon_{n-1} = 0$$

$$\epsilon_{n+1} - (2 - 2R)\epsilon_n + \epsilon_{n-1} = 0$$