

3.

$$x^3 y' = x^4 y^2 - 2x^2 y - 1$$

$$y_1 = x^{-2}$$

$$y_1' = -2x^{-3}$$

$$-2 = 1 - 2 - 1$$

$$y = \underset{\substack{\downarrow \\ y_1}}{x^{-2}} + u^{-1}$$

$$y' = -2x^{-3} - u^{-2} u'$$

$$-2 - \underset{u'}{x^3 u^{-2}} = x^4 (x^{-2} + u^{-1})^2$$

$$-2x^2 (x^{-2} + u^{-1}) - 1$$

$$-1 - x^3 u^{-2} u' = x^4 (x^{-4} + 2x^{-2} u^{-1} + u^{-2})$$

$$-2 - 2x^2 u^{-1}$$

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$$1 - x^3 u^{-2} u' = 1 + 2x^2 u^{-1} + x^4 u^{-2}$$

$$-x^3 u^{-2} u' = x^4 u^{-2}$$

$$u' = -x$$

$$u = -\frac{x^2}{2} + C$$

$$y = x^{-2} + \left(-\frac{x^2}{2} + C\right)^{-1}$$

$$y = x^{-2} + \frac{2}{2C - x^2} \quad y(\sqrt{2}) = 0$$

$$0 = (\sqrt{2})^{-2} + \frac{2}{2C - (\sqrt{2})^2}$$

$$-2^{-1} = \frac{2}{2C - 2}$$

$$2C - 2 = -4$$

$$C = 3$$

4.

$$\frac{du}{dt} = u^q, \quad t \in [0, 10]$$

$$\int \frac{du}{u^q} = \int dt$$

$$q = 1$$

$$\ln u^q = t + C$$

$$\ln u = t + C$$

$$u = e^{t+C}$$

$$q \neq 1$$

$$\int u^{-q} du = \int dt$$

$$\frac{u^{-q+1}}{1-q} = t + C$$

$$q = 0$$

$$\downarrow$$

$$u = t + b$$

$$u^{-q+1} = (t+C)(1-q)$$

$$u = ((1-q)t + b)^{\frac{1}{1-q}}$$

2.

$$\vec{r}_{n+1} = 2\vec{r}_n - \vec{r}_{n-1} + \vec{a}(\vec{r}_n)h^2$$

~~\vec{r}_n~~

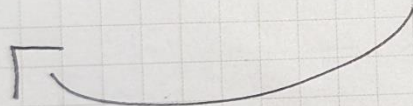
$$\vec{r}_{n+1} + \epsilon_{n+1} = 2\vec{r}_n + 2\epsilon_n - \vec{r}_{n-1} - \epsilon_{n-1}$$

$$+ \vec{a}(\vec{r}_n + \epsilon_n)h^2$$

$$\vec{a}(\vec{r}_n) + \epsilon_n \vec{a}'(\vec{r}_n)$$

$$\epsilon_{n+1} = 2\epsilon_n - \epsilon_{n-1} + \epsilon_n \hat{a}_n h^2$$

$$\epsilon_{n+1} = 2\epsilon_n - \epsilon_{n-1} + \epsilon_n \hat{a}_n h^2$$



$$\epsilon_{n+1} - 2\epsilon_n + \epsilon_{n-1} - \epsilon_n \hat{a}_n h^2 = 0$$

$$a = -\omega^2 x$$

$$a' = -\omega^2$$

$$\epsilon_{n+1} - \underbrace{(2 + (-\omega^2)h^2)}_{2R} \epsilon_n + \epsilon_{n-1} = 0$$

$$\epsilon_{n+1} - (2 - 2R) \epsilon_n + \epsilon_{n-1} = 0$$