Assignment 10

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June 4, 2022

Outline

1. Question
2. Equations
3. Solution

Question

(a) Bernoulli trails : Using (7.52) We shall rederive the fundamental equation (3.13). We define the random variables $x_i = 1$ if heads shows at the ith trial and x_i =0 otherwise.
(b) Poisson theorem : We shall show that if p$\ll 1$, *then*
P(x=k) $\backsimeq \frac{e^{-np}(np)^k}{k!}$

Equations

equation 7.52

$\phi_z(\omega) = E(e^{jw(x_1+.....x_n)}) = \phi_1(\omega).....\phi_\pi(\omega)$

equation 3.13

Fundamental Theorem:

$p_n(k)$ =P(A occurs k times in any order) $= \binom{n}{k}$p$^k q^{n-k}$

Solution

(a):

$$P(x_i = 1) = P(h) = p \tag{1}$$

$$P(x_i = 0) = P(t) = q \tag{2}$$

$$\phi_z(\omega) = pe^{jw} + q \tag{3}$$

The random variables z=x_1+.....+x_n takes the values 0,1,......,n and (z=k) is the event (k heads in n tossing). Furthermore,

$$\phi_z(\omega) = E(e^{jwk}) = \sum_{k=0}^{n} P(z = k)e^{jkw} \tag{4}$$

The random variables x_i are independent because x_i depends only on the outcomes of the ith trial and the trials are independent . Hence

$$\phi_z(\omega) = (pe^{jw} + q)^n = \sum_{k=0}^{n} \binom{n}{k} p^k e^{jkw} q^{n-k} \tag{5}$$

concludes,

P(z=k) = P(k heads) = $\binom{n}{k}$p$^k q^{n-k}$

(b):
Suppose that the random variables x_i are independent and each takes the value 1 and 0 with respective probabilities p_i and $q_i =$ 1-p$_i . If p_i \ll 1, then,$

$$e^{p_i(e^{jw}-1)} \simeq 1 + p_i(e^{jw} - 1) = p_i e^{jw} + q_i = \phi_i(\omega) \tag{6}$$

With z=x_1+.....+x$_n, it follows that$

$$\phi_z(\omega) \simeq e^{p_i(e^{jw}-1} \dots e^{p_n(e^{jw}-1} = e^{a(e^{jw}-1)} \tag{7}$$

Where a $= p_1$+.....+p_n. This leads to the conclusion that the random variables z is approximately Poisson distributed with parameter a , It can be shows that the result is exact in the limit if
$p_i \rightarrow 0 and$
$p_1 + \dots\dots p_n \rightarrow a$
as n $\rightarrow \infty$