Assignment 13

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Outline

Question

Show that if
$R_T(\tau) = \frac{1}{2T}\int_{-T+|\tau|/2}^{T-|\tau|/2} x(t+\frac{\tau}{2})x(t-\frac{\tau}{2})dt$
is the estimate of the autocorrelation $R(\tau)$ of a zero-mean normal process ,then
$\sigma_{R_T}^2 = \frac{1}{2T}\int_{-2T+|\tau|}^{2T-|\tau|} [R^2(\alpha) + R(\alpha+\tau)R(\alpha-\tau)](1-\frac{|\tau|+|\alpha|}{2T})d\alpha$

Solution

With c = T- $\frac{|\tau|}{2}$

$$z(t) = x(t + \frac{\mid \tau \mid}{2})x(t - \frac{\mid \tau \mid}{2}) \tag{1}$$

$$Fromequationf(x_1, ..., x_n) = f(x_n \mid x_{n-1}, ..., x_1).....f(x_2 \mid x_1)f(x_1) \tag{2}$$

From equation 2 we can write as

$$E(z(t_1)z(t_2)) - E(z(t_1))E(z(t_2))$$

$= \mathrm{R}^2(t_1 - t2) + R(t_1 - t_2 + \tau)R(t_1 - t_2 - \tau)(3)$

$$4T^2 VarR_T(\tau) = \int_{-c}^{c}\int_{-c}^{c}[R^2(t_1 - t_2) + R(t_1 - t_2 + \tau)R(t_1 - t_2 - \tau)]dt_1 dt_2 \tag{4}$$

$$= \int_{-2c}^{2c} [R^(\alpha) + R(\alpha+\tau)R(\alpha-\tau)](2T- \mid \tau \mid - \mid \alpha \mid)d\alpha$$